Determination of Magnetic Axis in a Sextupole magnet using Vibrating Wire Technique*

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Motivation

- Work was motivated by NSLS-II project requirement of precise alignment of a string of quadrupole and sextupole magnets on a ~6 m long girder within a tight tolerance of ±10-20 microns.

- Similar requirement is anticipated for ILC damping ring.

Sextupole magnet field properties

- Sextupole magnetic field:

$$B_y(x,y) = b_3(x^2 - y^2); \quad B_x(x,y) = -2b_3xy$$

- Horizontal scan:

$$B_y(x, y = y_{off}) = b_3x^2 - b_3y_{off}^2; \quad B_x(x,y) = -2b_3xy_{off}$$

- Vertical scan:

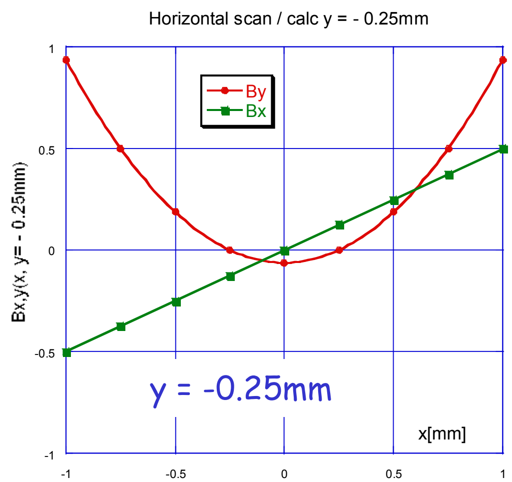
$$B_y(x_{off}, y) = -b_3y^2 + b_3x_{off}^2; \quad B_x(x,y) = -2b_3x_{off}y$$

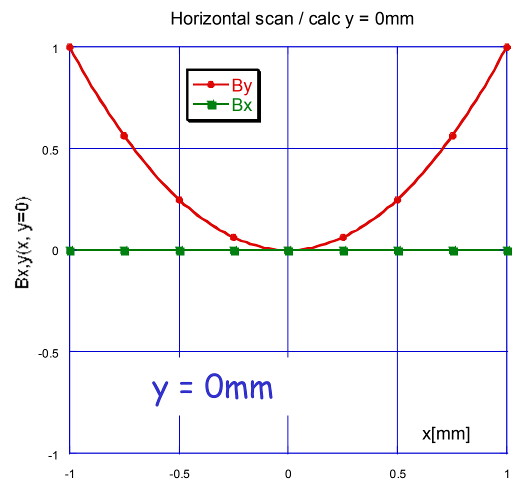
- The term which couples horizontal to vertical beam particles motion:

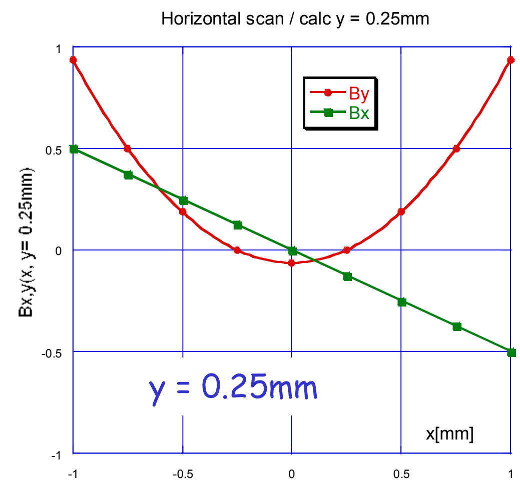
$$a_2(y_{off}) = \frac{\partial B_x(x, y_{off})}{\partial x} = -2b_3y_{off}$$

Sextupole magnet field properties

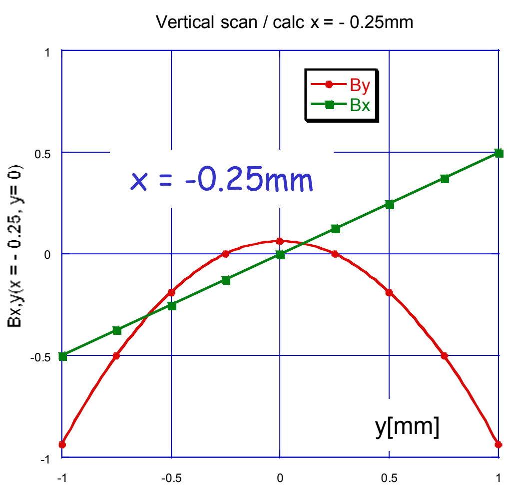
Horizontal scan

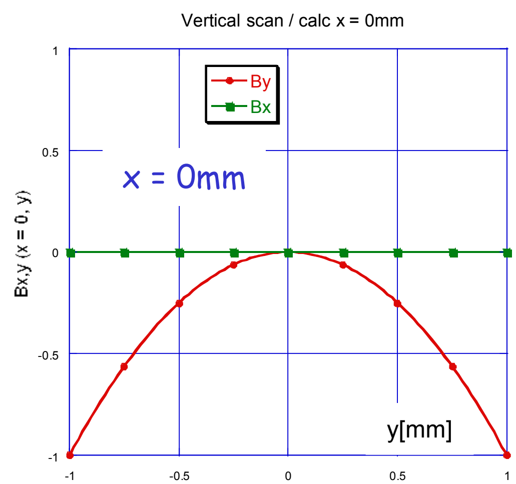


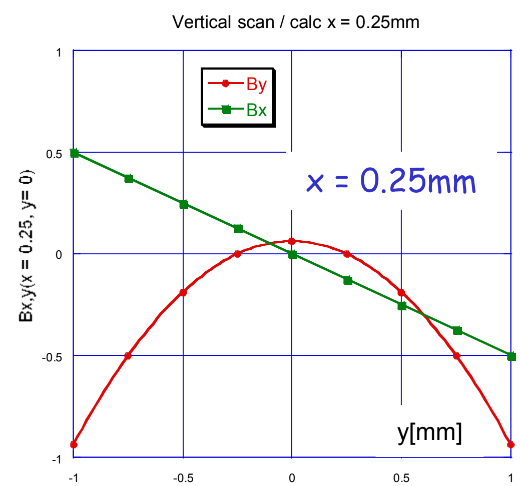




Vertical scan







- Algorithm for magnetic center finding from horizontal scan:

$B_{x,y}(x)$ - horizontal and vertical field components measured as a function of horizontal position.

$$B_y(x) = m_2 x^2 + m_1 x + m_0 = b_3 (x - x_c)^2 - b_3 y_c^2;$$

$$B_x(x) = n_1 x + n_0 = 2 b_3 x y_c;$$

$$x_c = -\frac{m_1}{2m_2}; \quad y_c = \frac{n_1}{2m_2}$$

Sextupole magnet field properties

- Algorithm for magnetic center finding from the vertical scan:

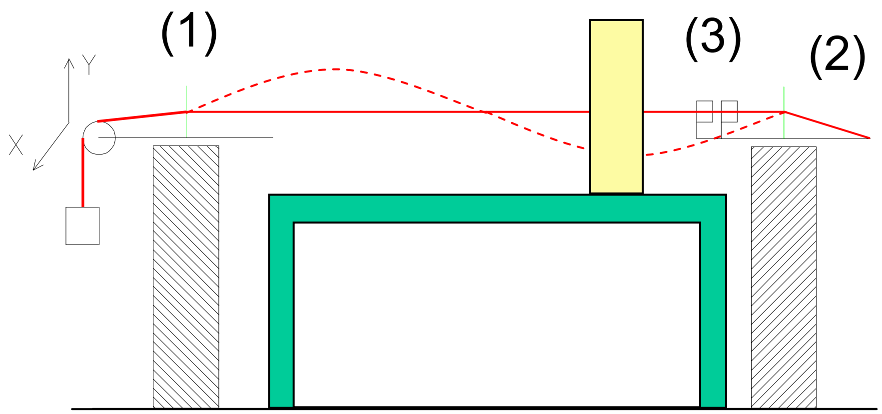
$B_{x,y}(x)$ - horizontal and vertical field components measured as a function of vertical position.

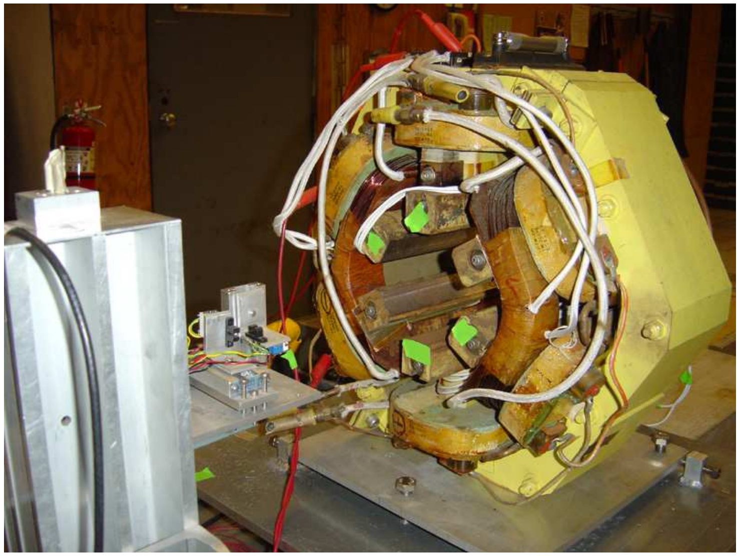
$$B_y(x) = m_2 y^2 + m_1 y + m_0 = b_3 x_c^2 - b_3 (y - y_c)^2;$$

$$B_x(x) = n_1 y + n_0 = 2 b_3 x_c y;$$

$$x_c = -\frac{n_1}{2m_2}; \quad y_c = -\frac{m_1}{2m_2}$$

Experimental Setup





Vibrating Wire:
Length ~ 2200mm
Driving current ~ 60mA (RMS)
Fundamental freq ~ 47Hz
For measurement used 2-nd vibrating mode

(1) Stage with tension mechanism
(2) Stage with wire position sensors
(3) Horizontal and vertical wire position sensors (LED-Phototransistor assemblies)

Sextupole magnet

b3l ~ 10.9Gm/cm^2
Length ~ 24cm
Bore radius ~5.5cm

Experimental Setup

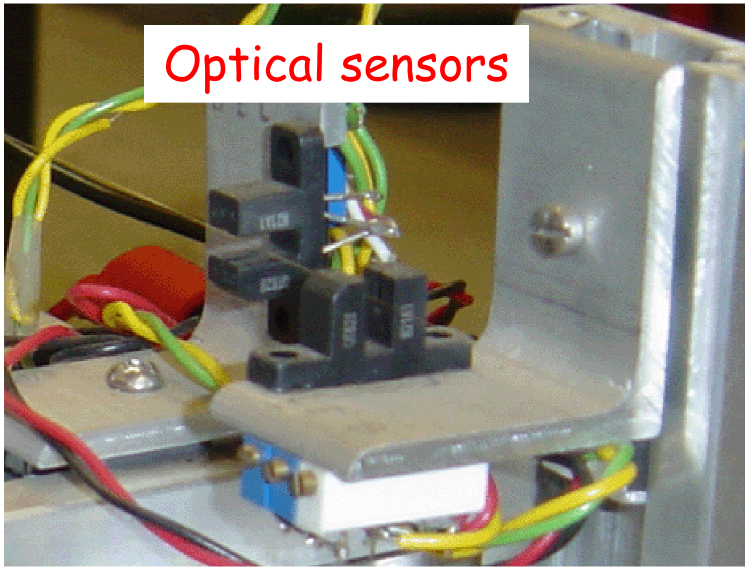
• In the VW technique the wire motion in vertical (horizontal) plane caused by the Lorentz forces between current flowing through the wire and horizontal (vertical) magnetic field.

• To separate horizontal and vertical field components measurement one should measure the wire motion in vertical and horizontal planes.

• The used assembly of the optical sensors show ~10% coupling between vertical and horizontal wire motion, we need ~1% or less.

• In a sextupole magnet, a X-Y coupling has the same effect as a roll of the magnet.

• Solution – assembly should be calibrated.



Optical wire position sensors calibration/decoupling procedure

• Stage (1) has been used for precise wire displacement (horizontal and vertical) at optical sensors and the sensors responds was measured.

$dS_{x,y}$ - horizontal/vertical sensor responces in Volts

dx_w, dy_w - horizontal and vertical wire displacment in mm

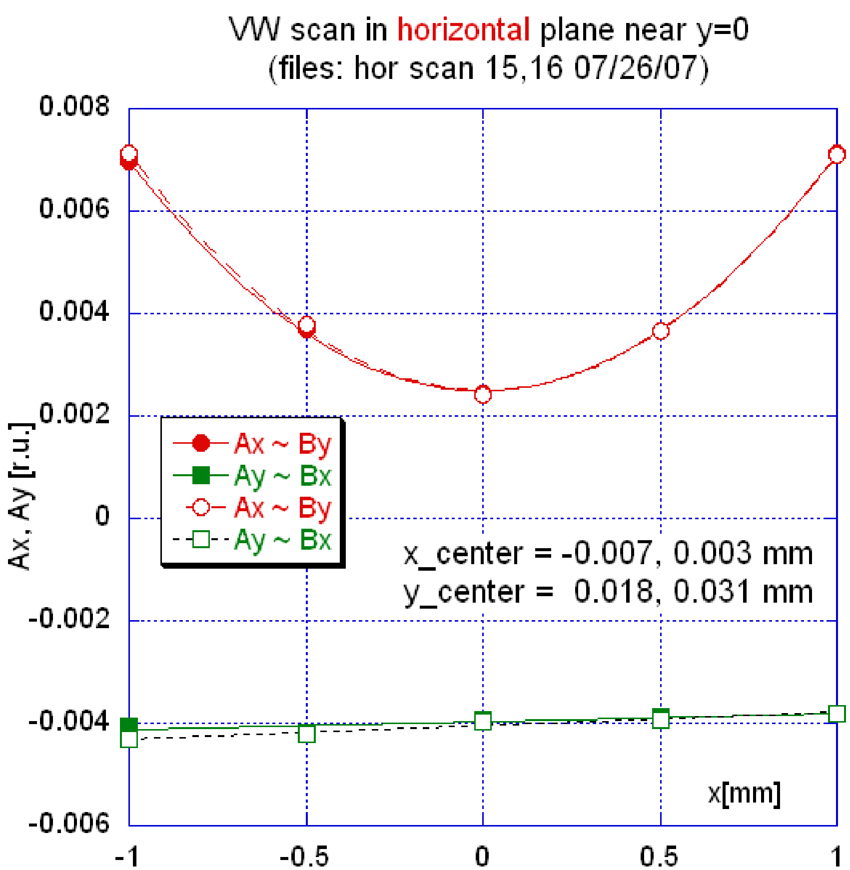
$$\begin{pmatrix} dS_x \\ dS_y \end{pmatrix} = M \begin{pmatrix} dx_w \\ dy_w \end{pmatrix}; \quad M = \begin{pmatrix} 1.43 & 0.116 \\ -0.045 & 1.32 \end{pmatrix}$$

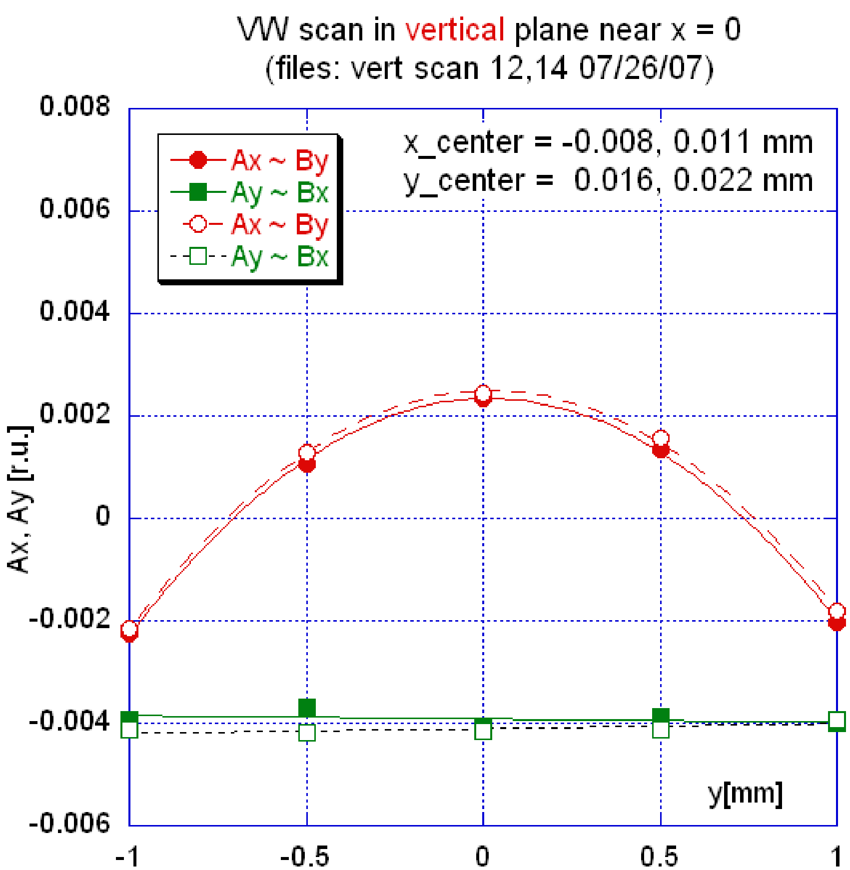
• In the course of measurement the sensor signals were converted to the wire displacement:

$$\begin{pmatrix} dx_w \\ dy_w \end{pmatrix} = M^{-1} \begin{pmatrix} dS_x \\ dS_y \end{pmatrix} = \begin{pmatrix} 0.697 & -0.061 \\ 0.024 & 0.755 \end{pmatrix} \begin{pmatrix} dS_x \\ dS_y \end{pmatrix}$$

Experimental results

Horizontal and vertical scans near sextupole magnetic center

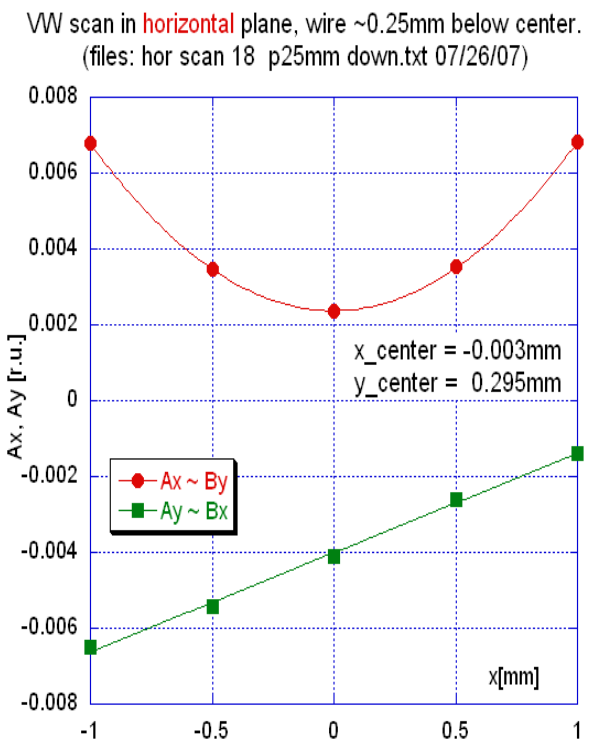


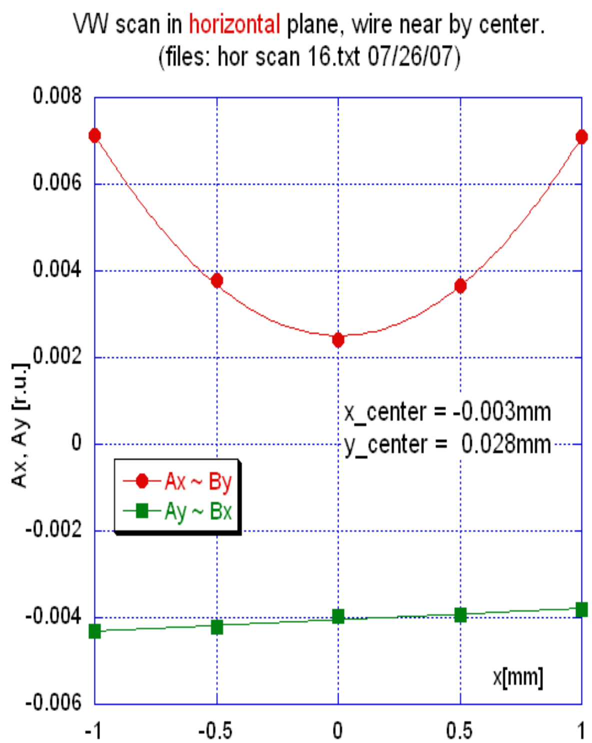


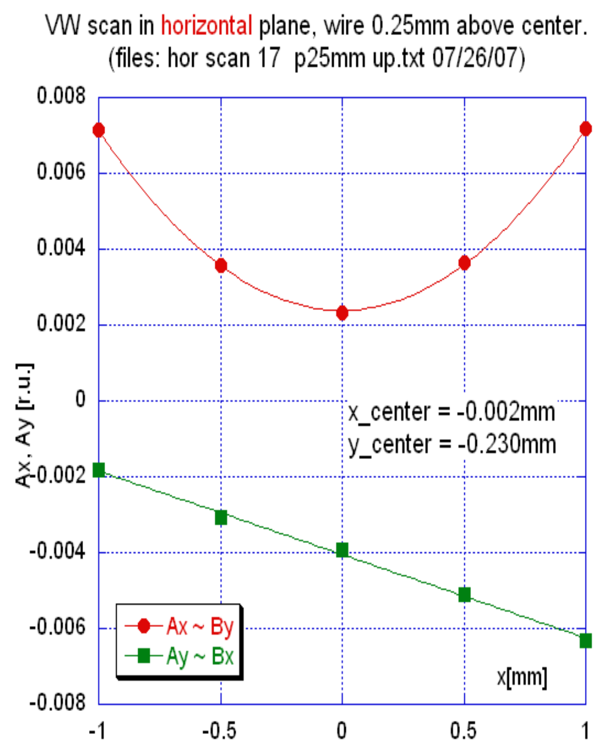
Two independent runs are shown

Experimental results

Horizontal scans at different vertical position



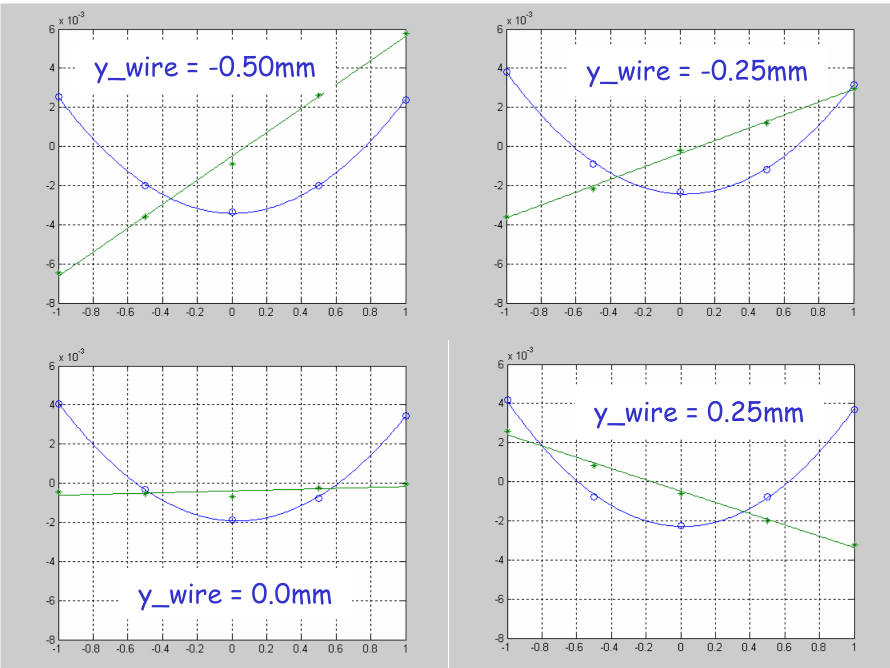


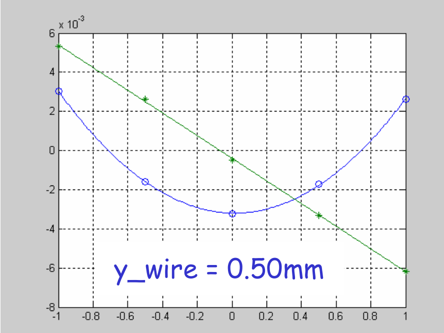


It is well consisted with theoretical prediction

Experimental results

Precision test: Horizontal scans at different vertical position

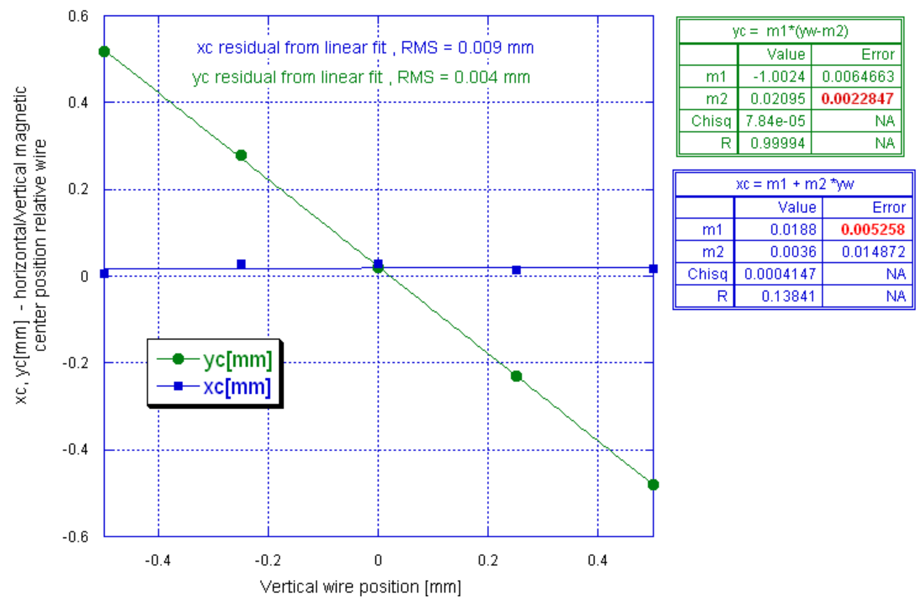




Data fitted using MatLab

Precision better than 10 microns !!!

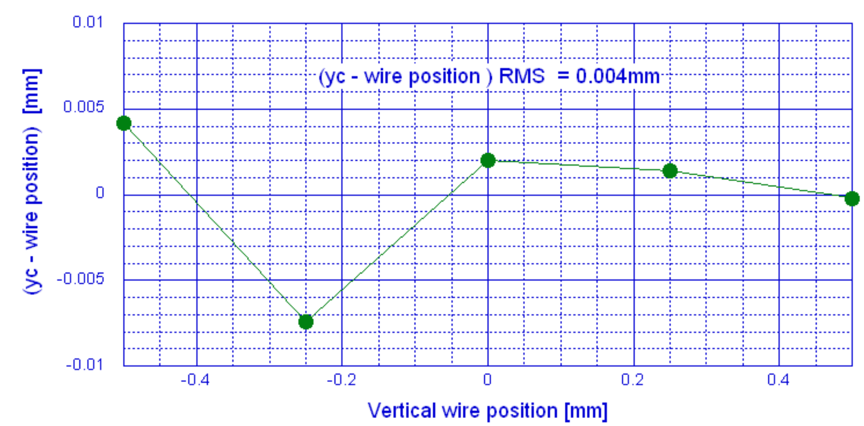
Sextupole magnetic center position relative wire as a function of the vertical wire position, files: hor scan (15, ... 19) 5/7/07



yc = m1*(yw-m2)		
	Value	Error
m1	-1.0024	0.0064663
m2	0.02095	0.0022847
Chisq	7.84e-05	NA
R	0.99994	NA

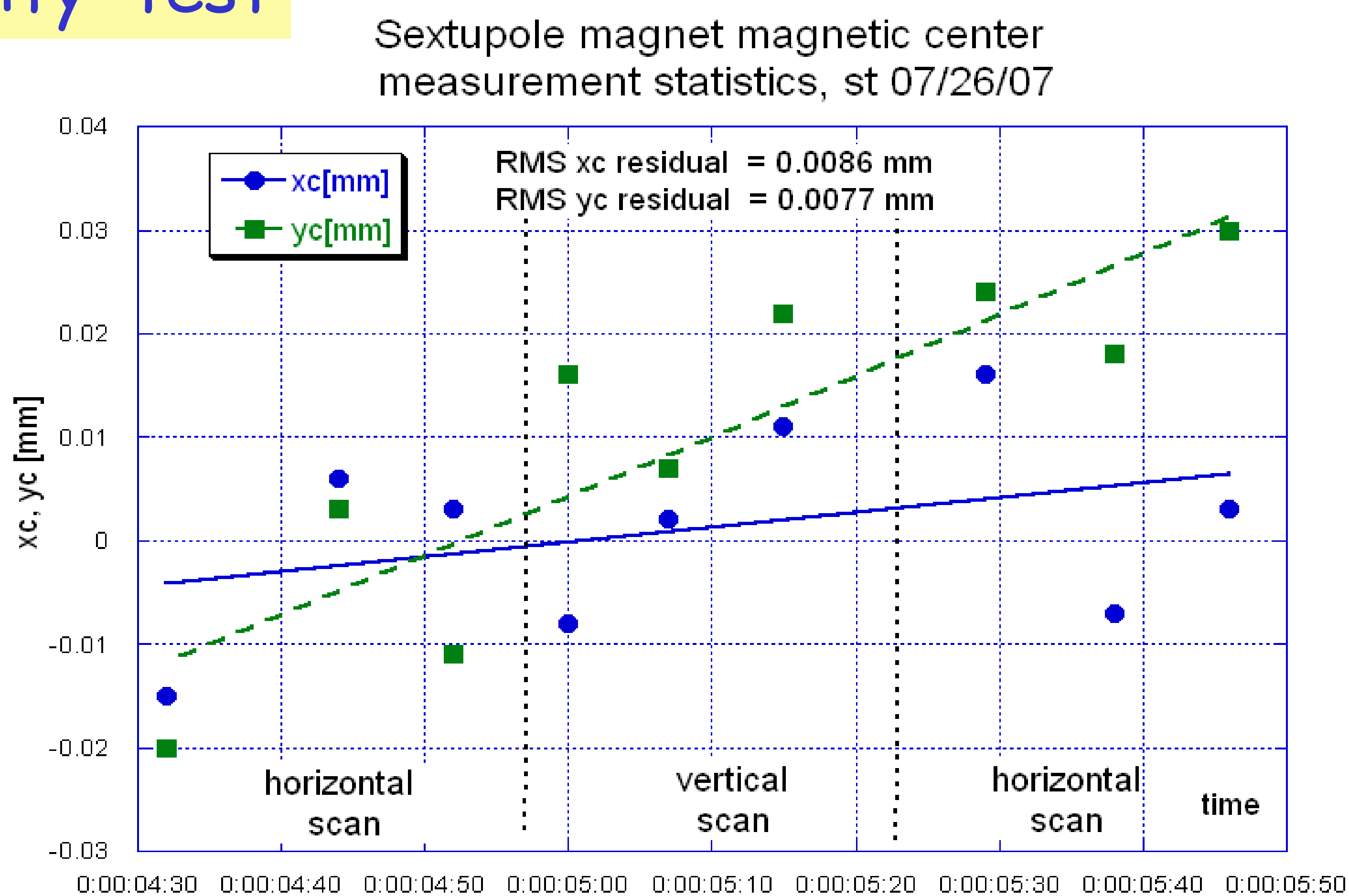
xc = m1 + m2 *yw		
	Value	Error
m1	0.0188	0.005258
m2	0.0036	0.014872
Chisq	0.0004147	NA
R	0.13841	NA

Sextupole center vertical position in the frame coupled with magnet



Experimental results

Stability test



Position drift: vertical plane ~ 35 micron per hour
horizontal plane ~ 5 micron per hour

Sextupole magnet magnetic axes finding in the presence of the field errors

$\tilde{b}_i, \tilde{a}_i$ - field errors, R - reference radius

$$B_y + i B_x = (\tilde{b}_1 + i\tilde{a}_1) + (\tilde{b}_2 + i\tilde{a}_2)\left(\frac{x+iy}{R}\right) + (b_3 + i\tilde{a}_3)\left(\frac{x+iy}{R}\right)^2$$

$$B_y = \tilde{b}_1 + \frac{\tilde{b}_2 x - \tilde{a}_2 y}{R} + b_3\frac{x^2-y^2}{R^2} - \tilde{a}_3\frac{2xy}{R^2};$$

$$B_x = \tilde{a}_1 + \frac{\tilde{a}_2 x + \tilde{b}_2 y}{R} + b_3\frac{2xy}{R^2} + \tilde{a}_3\frac{x^2-y^2}{R^2};$$

Magnetic center position from B_y measurement

(extrim in quadratic fit):

a) x_c from horizontal scan, $\tilde{y}$ – wire position in vertical plane.

$$\frac{\partial B_y}{\partial x} = \frac{\tilde{b}_2}{R} + b_3\frac{2x_c}{R^2} - \tilde{a}_3\frac{2\tilde{y}}{R^2} = 0 \Rightarrow$$

$$x_c = -\frac{\tilde{b}_2}{2b_3}R + \frac{\tilde{a}_3}{b_3}\tilde{y} \approx -\frac{\tilde{b}_2}{2b_3}R$$

b) y_c from vertical scan, $\tilde{x}$ – wire position in vertical plane.

$$\frac{\partial B_y}{\partial y} = -\frac{\tilde{a}_2}{R} - b_3\frac{2y_c}{R^2} - \tilde{a}_3\frac{2\tilde{x}}{R^2} = 0 \Rightarrow$$

$$y_c = -\frac{\tilde{a}_2}{2b_3}R - \frac{\tilde{a}_3}{b_3}\tilde{x} \approx -\frac{\tilde{a}_2}{2b_3}R$$

Magnetic center position from B_x measurement

("zero" sloop):

a) x_c from vertical scan, $\tilde{y}$ – averaged vertical wire position (small).

$$\frac{\partial B_x}{\partial y} = \frac{\tilde{b}_2}{R} + b_3\frac{2x_c}{R^2} - \tilde{a}_3\frac{2\tilde{y}}{R^2} = 0 \Rightarrow$$

$$x_c = -\frac{\tilde{b}_2}{2b_3}R + \frac{\tilde{a}_3}{b_3}\tilde{y} \approx -\frac{\tilde{b}_2}{2b_3}R$$

b) y_c from horizontal scan, $\tilde{x}$ – averaged horizontal wire position (small)

$$\frac{\partial B_x}{\partial x} = \frac{\tilde{a}_2}{R} + b_3\frac{2y_c}{R^2} + \tilde{a}_3\frac{2\tilde{x}}{R^2} = 0 \Rightarrow$$

$$y_c = -\frac{\tilde{a}_2}{2b_3}R - \frac{\tilde{a}_3}{b_3}\tilde{x} \approx -\frac{\tilde{a}_2}{2b_3}R$$

Conclusion:
Both methods should give the same results. It can be used for cross checking.

Summary / Conclusion

- Vibrating wire technique has been applied to sextupole magnet magnetic axes finding.
- Demonstrated precision, ~10microns, is adequate to the alignment requirement of NSLS - II project and, very likely, of ILC damping ring.

The field error effect

Sextupole magnet magnetic axes in the presence of the field errors

5/23/07 (6)

General case, including Quad terms:

$$\left(B_y+iB_x\right)=\left(B_1+iA_1\right)+\left(B_2+iA_2\right)\left(\frac{x+iy}{R_{ref}}\right)+\left(B_3+iA_3\right)\left(\frac{x+iy}{R_{ref}}\right)^2$$

$$B_y = B_1+\left(B_2x-A_2y\right)/R_{ref}+B_3\left(\frac{x^2-y^2}{R_{ref}^2}\right)-2A_3\left(\frac{xy}{R_{ref}^2}\right)$$

$$B_x = A_1+\left(A_2x+B_2y\right)/R_{ref}+B_3\left(\frac{2xy}{R_{ref}^2}\right)+A_3\left(\frac{x^2-y^2}{R_{ref}^2}\right)$$

The "center" is defined as a point where B_2 & A_2 terms are zero:

$$B_2'+iA_2' = \left(B_2+iA_2\right)+2\left(\frac{x_c+iy_c}{R_{ref}}\right)\left(B_3+iA_3\right)=0$$

$$\boxed{x_c+iy_c = -\frac{\left(B_2+iA_2\right)R_{ref}}{2\left(B_3+iA_3\right)}}\qquad x_c=-\frac{B_2B_3+A_2A_3}{2\left(B_3^2+A_3^2\right)}R_{ref}\qquad y_c=-\frac{A_2B_3-B_2A_3}{2\left(B_3^2+A_3^2\right)}R_{ref}$$

if $A_3=0$, $x_c=-\frac{1}{2}\left(B_2/B_3\right)R_{ref}$; $y_c=-\frac{1}{2}\left(A_2/B_3\right)R_{ref}$

$$\left.dB_y/dx\right|_{y=y_0}=\frac{B_2}{R_{ref}}+2B_3\frac{x}{R_{ref}^2}-2A_3y_0/R_{ref}^2$$

$dB_y/dx=0$ at $x_c'=-\left[\frac{B_2-2A_3\left(y_0/R_{ref}\right)}{2B_3}\right]R_{ref}$

neglecting A_3^2 in comparison to B_3^2, $x_c\simeq\left[-\frac{1}{2}\left(B_2/B_3\right)-\frac{1}{2}\left(\frac{A_2A_3}{B_3^2}\right)\right]R_{ref}$

$\therefore$ error in $x_c=\Delta x_c\simeq\left[+\left(\frac{A_3}{B_3}\right)\left(\frac{y_0}{R_{ref}}\right)+\frac{1}{2}\left(\frac{A_2}{B_3}\right)\left(\frac{A_3}{B_3}\right)\right]R_{ref}$

$\because\frac{1}{2}A_2/B_3\simeq-y_c/R_{ref}$ $\quad\boxed{\Delta x_c\simeq\left(A_3/B_3\right)\left(y_0-y_c\right)}$

The error is minimized if
i) either $A_3\simeq0$
or ii) $y_0\simeq y_c$; i.e. dB_y/dx is evaluated at $y\simeq y_c$.

5/23/07 (7)

$$\left.\frac{dB_y}{dy}\right|_{x=x_0}=-A_2/R_{ref}-2B_3\,y/R_{ref}^2-2A_3x_0/R_{ref}^2$$

This is zero at $y=y_c'=-\left(\frac{A_2+2A_3\left(x_0/R_{ref}\right)}{2B_3}\right)R_{ref}$

neglecting A_3^2 in comparison to B_3^2, $y_c\simeq\left[-\frac{1}{2}\left(\frac{A_2}{B_3}\right)+\frac{1}{2}\left(\frac{B_2}{B_3}\right)\left(\frac{A_3}{B_3}\right)\right]R_{ref}$

$$\simeq\left[-\frac{1}{2}\left(\frac{A_2}{B_3}\right)-\left(\frac{x_c}{R_{ref}}\right)\left(A_3/B_3\right)\right]R_{ref}$$

$\therefore$ Error in y_c is

$$\boxed{\Delta y_c=y_c'-y_c=\left(\frac{A_3}{B_3}\right)\left(x_c-x_0\right)}$$

once again, the error is negligible if either
i) $A_3\simeq0$
or ii) $x_0\simeq x_c$; i.e. $\frac{\partial B_y}{\partial y}$ is evaluated at $x_0\simeq x_c$.

$$\left.\frac{dB_x}{dx}\right|_{y_0}=\frac{A_2}{R_{ref}}+2B_3y_0/R_{ref}^2+2A_3x/R_{ref}^2$$

it can be seen that $\left.\frac{dB_y}{dy}\right|_{x_0,y_0}=-\left.\frac{dB_x}{dx}\right|_{x_0,y_0}$, as expected.

~~dB_x/dx is zero at~~ ~~$x=\frac{-A}{\ldots}$~~

For a given y_0, $\left(\frac{dB_x}{dx}\right)$ is not strictly constant (i.e. B_x is not linear with x) due to the A_3 term.

~~for $2A_3x/R_{ref}^2<<\frac{A_2}{R_{ref}}+2B_3y_0/R_{ref}^2$~~

$x<<\left(\frac{A_2}{2A_3}\right)R_{ref}$

$$\left.\frac{dB_x}{dx}\right|_{y_0}=\frac{A_2}{R_{ref}^2}+2B_3y_0/R_{ref}^2\left[1+\left(\frac{A_3}{B_3}\right)\left(\frac{x}{y_0}\right)\right]$$

This can be large, specially near $y_0\simeq0$

$\therefore y_0\simeq0$ should be avoided for studying $\frac{dB_x}{dx}$.