Decay of a bound muon

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CLFV2016
June 20-22, 2016
Charlottesville

Outline

- Muonic atoms
- Muon electron coherent conversion
- Spectrum of the bound muons
    - Central region
    - Endpoint region
    - Radiative correction to the spectrum

Muonic atoms &
Muon electron conversion

General characteristic

- One of the electrons is replaced by a muon
- Muon orbit is much smaller than the electron orbit $\frac{r_\mu}{r_e} \sim \frac{m_e}{m_\mu}$
  - Much larger momentum
  - Muons are more sensitive to the structure of the nucleus $\frac{1}{m_\mu} < r_N$
- Muon can be captured by the nucleus or it can decay

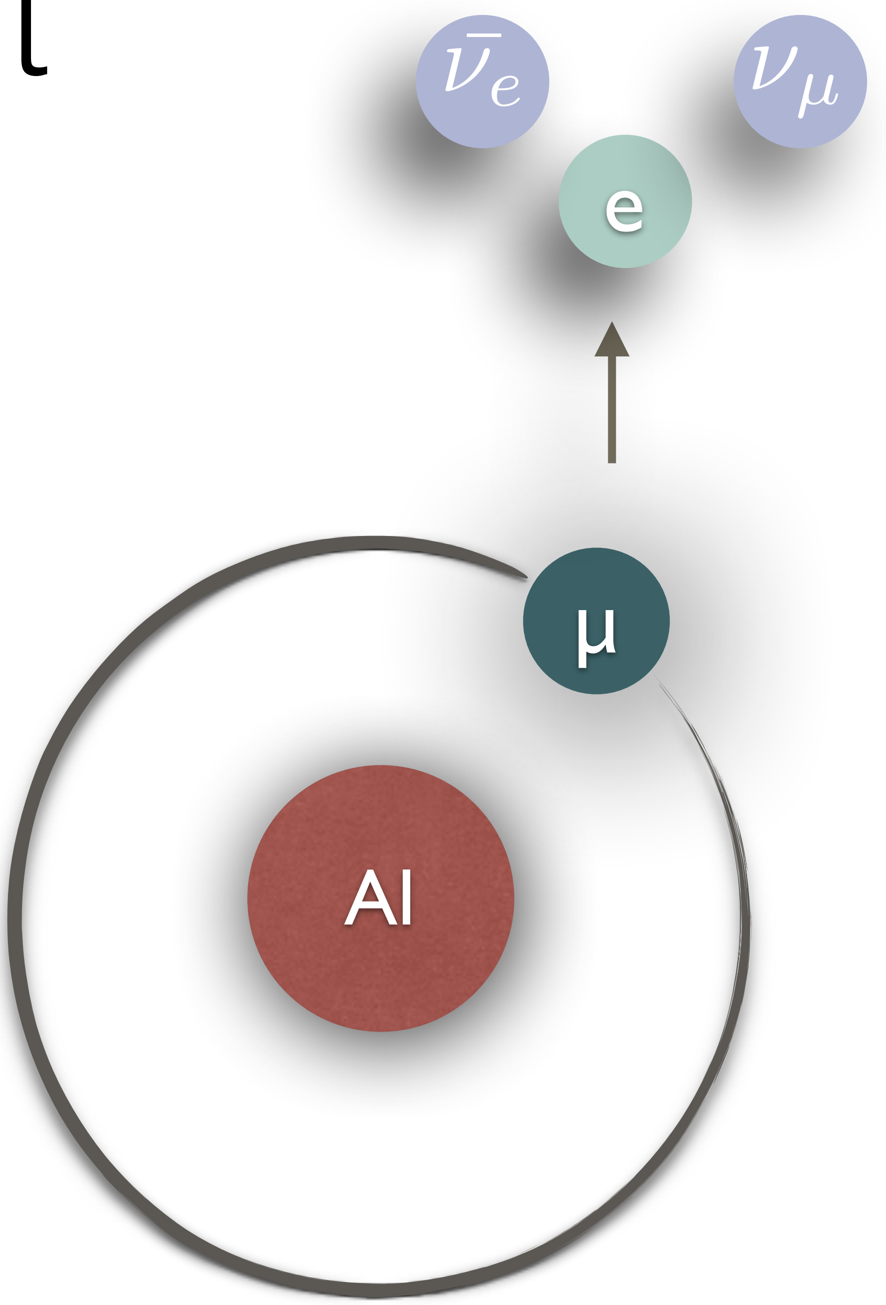
Muon DIO

~39%

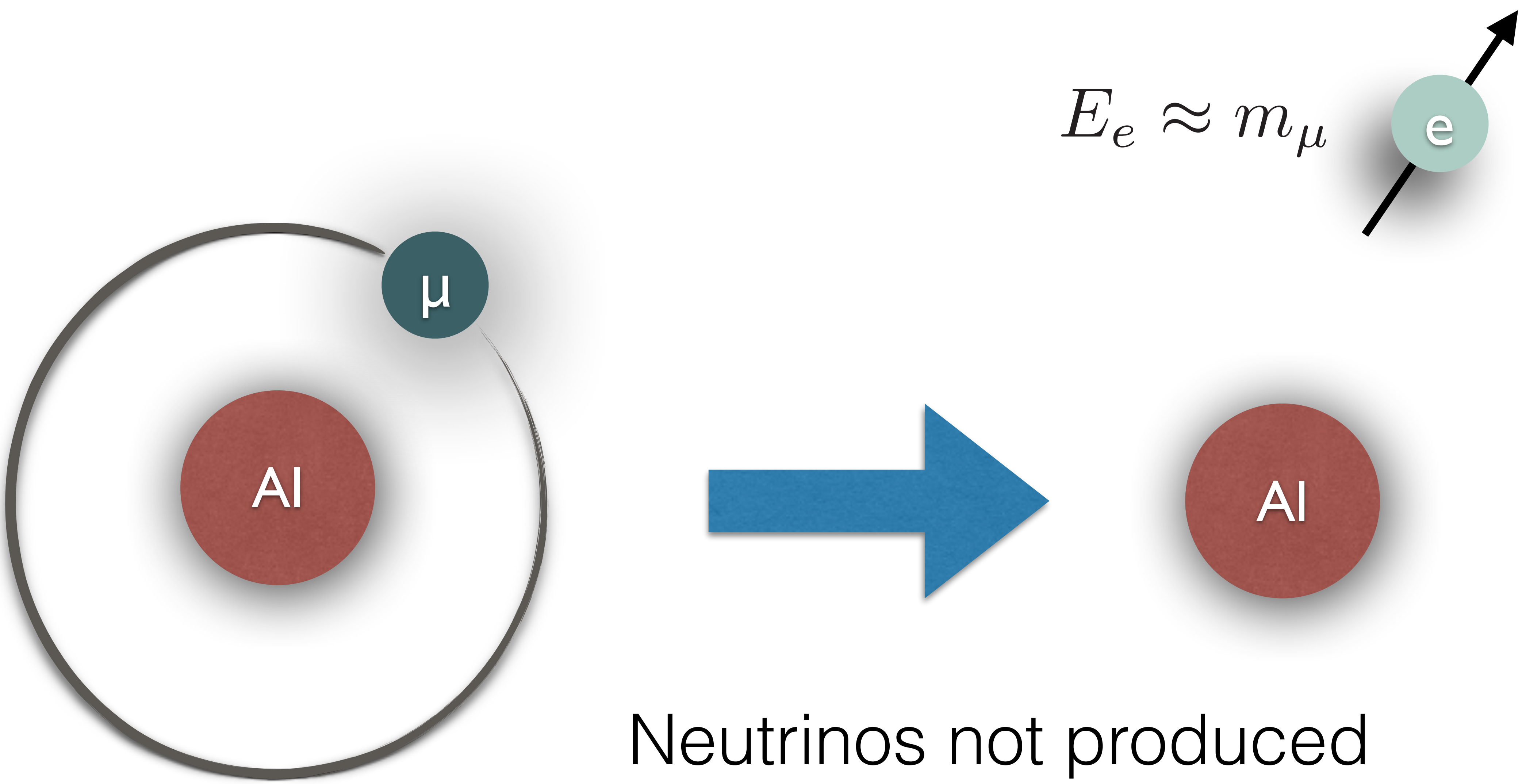
DIO — Decay In Orbit

- Muon DIO: standard muon decay into an electron and two neutrinos, with the muon and a nucleus forming a bound state

- For DIO momentum can be exchanged between the nucleus and both the muon and the electron



Muon electron coherent conversion



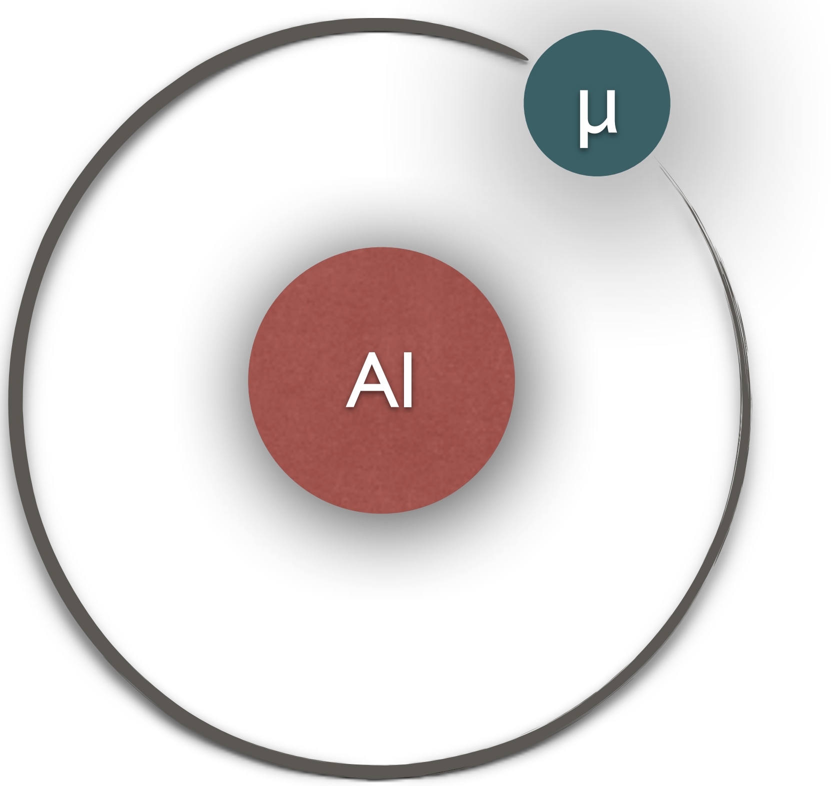
Neutrinos not produced
CLFV

Why conversion?

- Plethora of models gives large CLFV
- Muon g-2 discrepancy
  - 3.5σ Lattice calculation (Chakraborty, Davies, de Oliveira, Koponen, Lepage, 2016)
  - 3.3σ $e^+e^- + \tau$ (Jegerlehner, Szafron, 2011)
  - 3.3σ e^+e^- (Hagiwara, Liao, Martin, Nomura, Teubner, 2011)
- Proton radius puzzle
- Today's LHC statistical fluctuations

  * Signal is clean and the background is small
  * SM background can be well understood

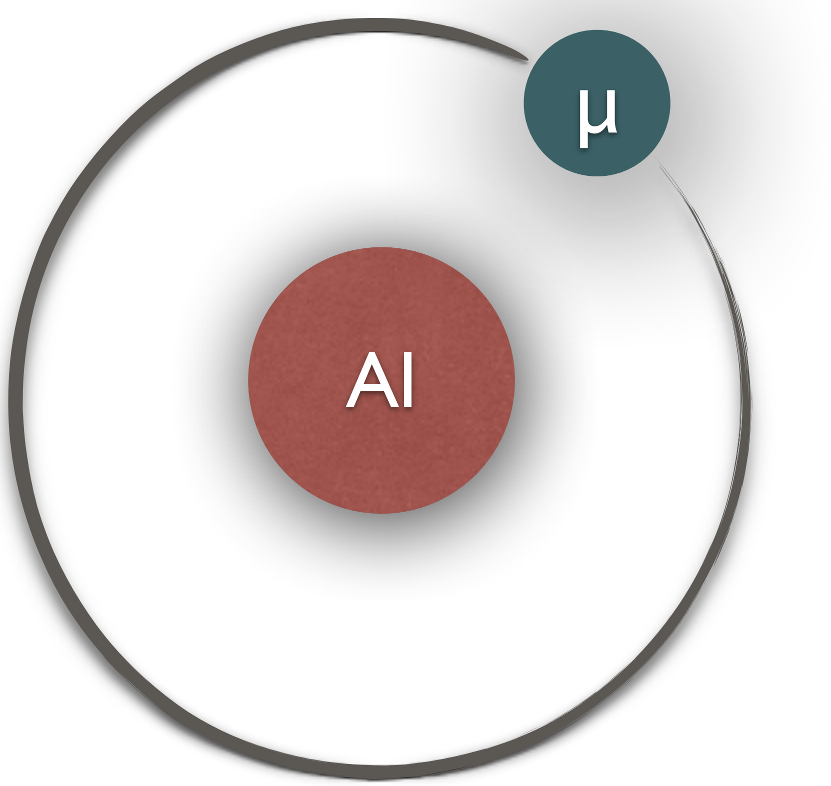
Characteristic scales of muonic atom



nucleus mass	M_{Al}
muon mass	m_μ
muon momentum	$Z\alpha m_\mu$
muon binding energy	$(Z\alpha)^2 m_\mu$
electron cloud	$\sim m_e$

$$M_{Al} \gg m_\mu \gg m_\mu Z\alpha \gg m_\mu (Z\alpha)^2$$

Characteristic scales of muonic atom



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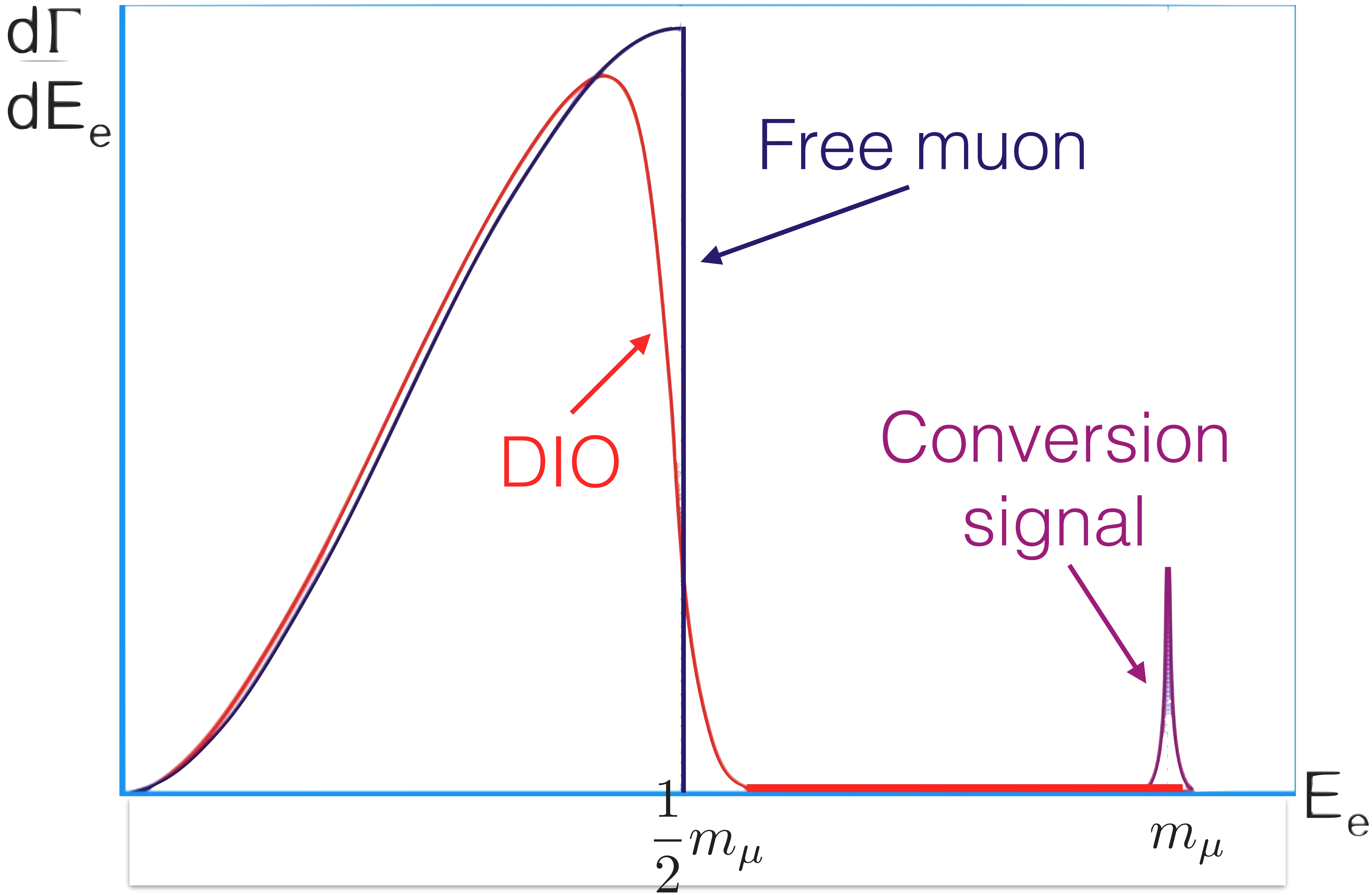
DIO Spectrum



Endpoint energy

$$E_{\mathrm{max}} = m_\mu + E_b + E_{\mathrm{rec}}$$

$$E_b \approx -m_\mu \frac{(Z\alpha)^2}{2}$$

Binding energy

(+ higher orders)

$$E_{rec} \approx -\frac{m_\mu^2}{2m_N}$$

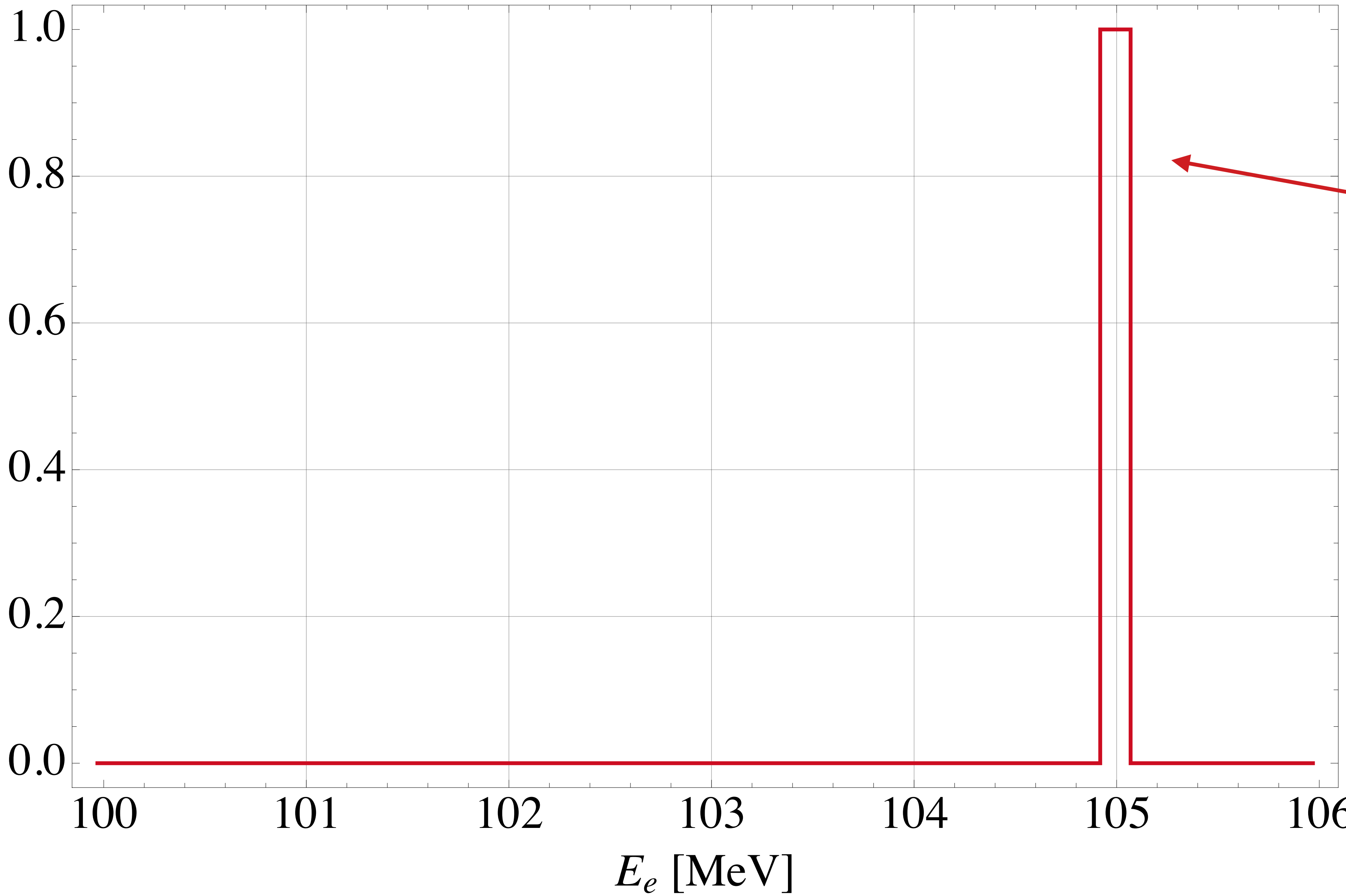
Recoil energy

(kinetic energy of the nucleus)

Both corrections decrease the endpoint energy

Conversion signal

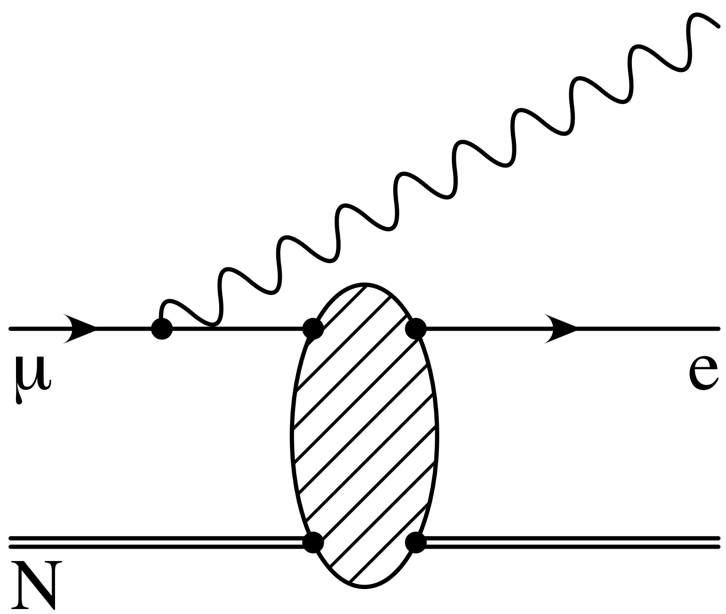
(theoretical perspective)

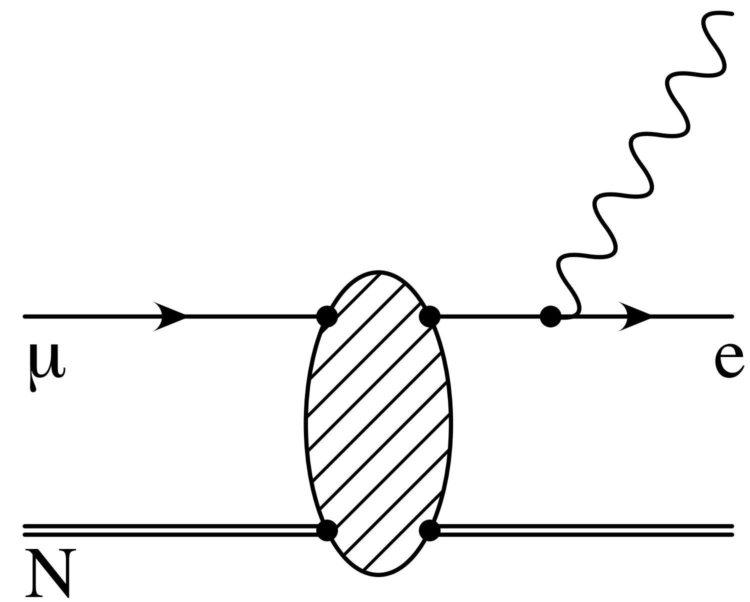


Conversion signal

…but energetic charged particles are accompanied by radiation…

Conversion spectrum





Emission of photons decreases the electron energy

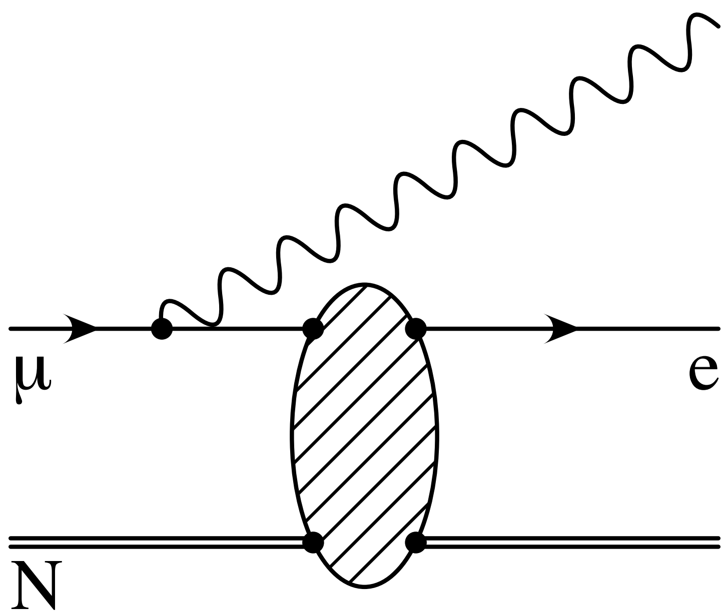
Types of photons:

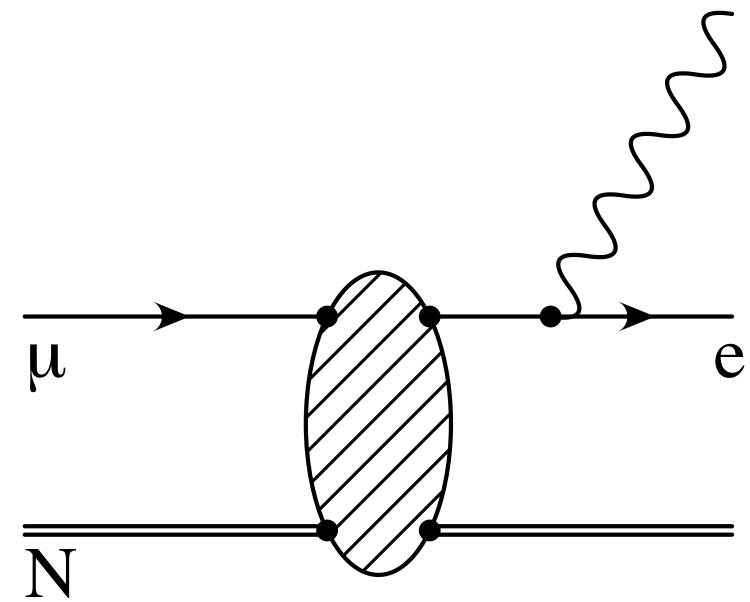
A. hard $E_\gamma \sim m_\mu$ ⟶ collinear $p_e p_\gamma \sim m_e^2$

B. soft $E_\gamma \sim m_\mu Z\alpha$

C. ultrasoft $E_\gamma \sim m_\mu (Z\alpha)^2$

Conversion spectrum





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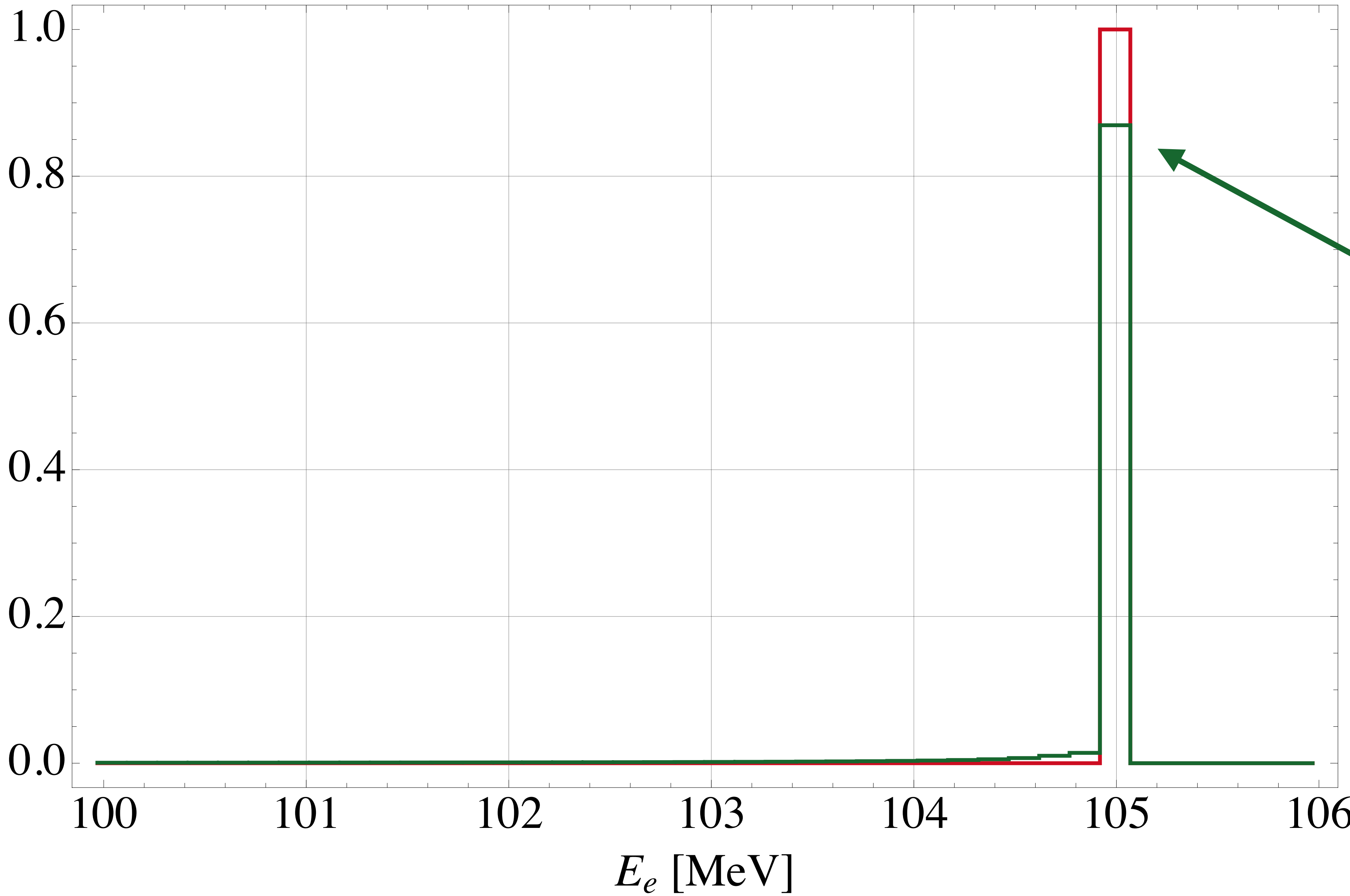
B. soft $E_\gamma \sim m_\mu Z\alpha$

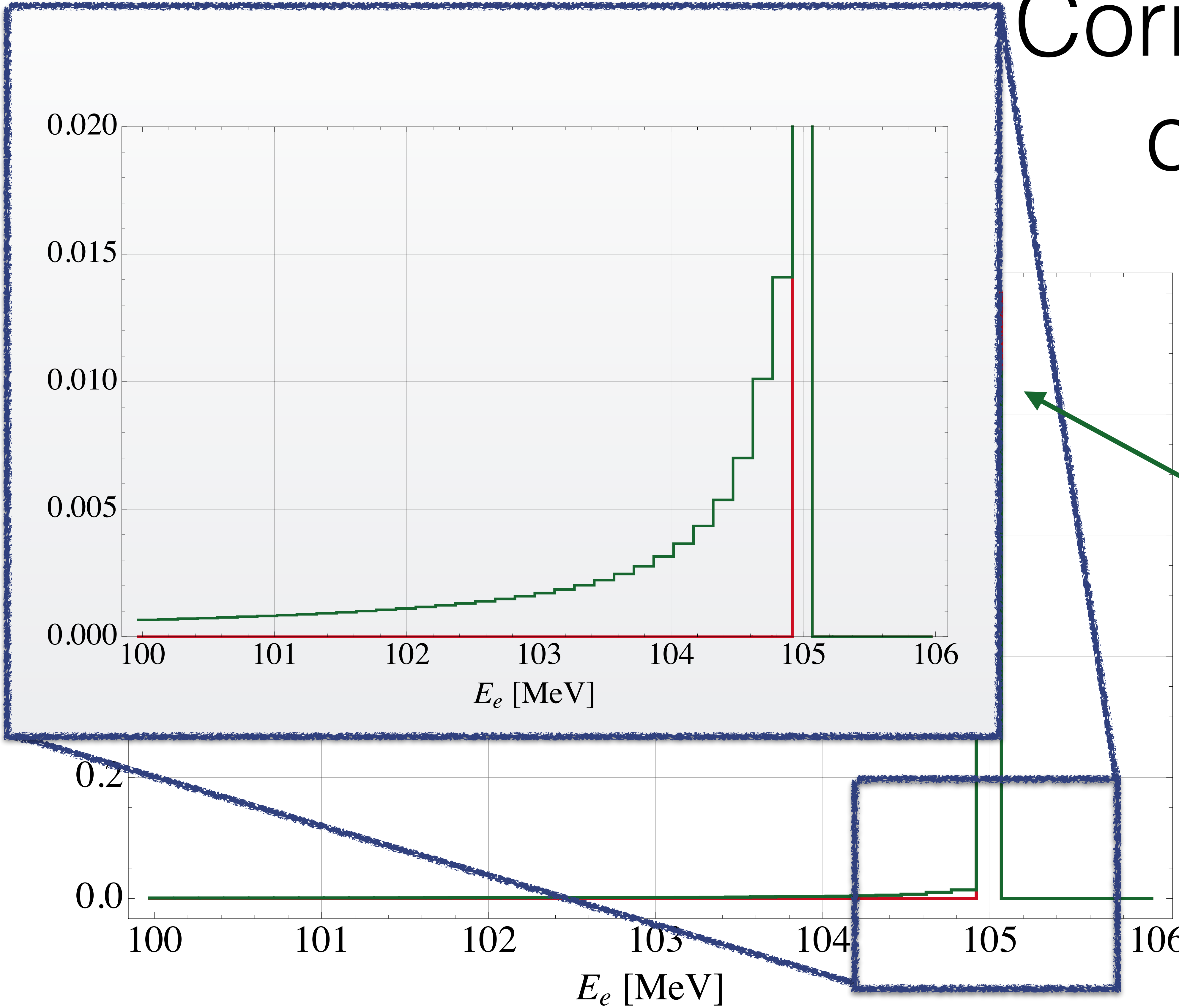
C. ultrasoft $E_\gamma \sim m_\mu (Z\alpha)^2$ ⟷ Requires NRQED (see orthopositronium case)

Corrections to conversion signal



Signal is reduced by the radiative corrections

Corrections to conversion signal



Signal is reduced by the radiative corrections

Corrections to the conversion signal

Signal window [MeV]	0.1	1.5	2.0
Universal part	0.861	0.923	0.930
Model I	0.861	0.923	0.930
Model II	0.861	0.924	0.930
Model III	0.858	0.921	0.927

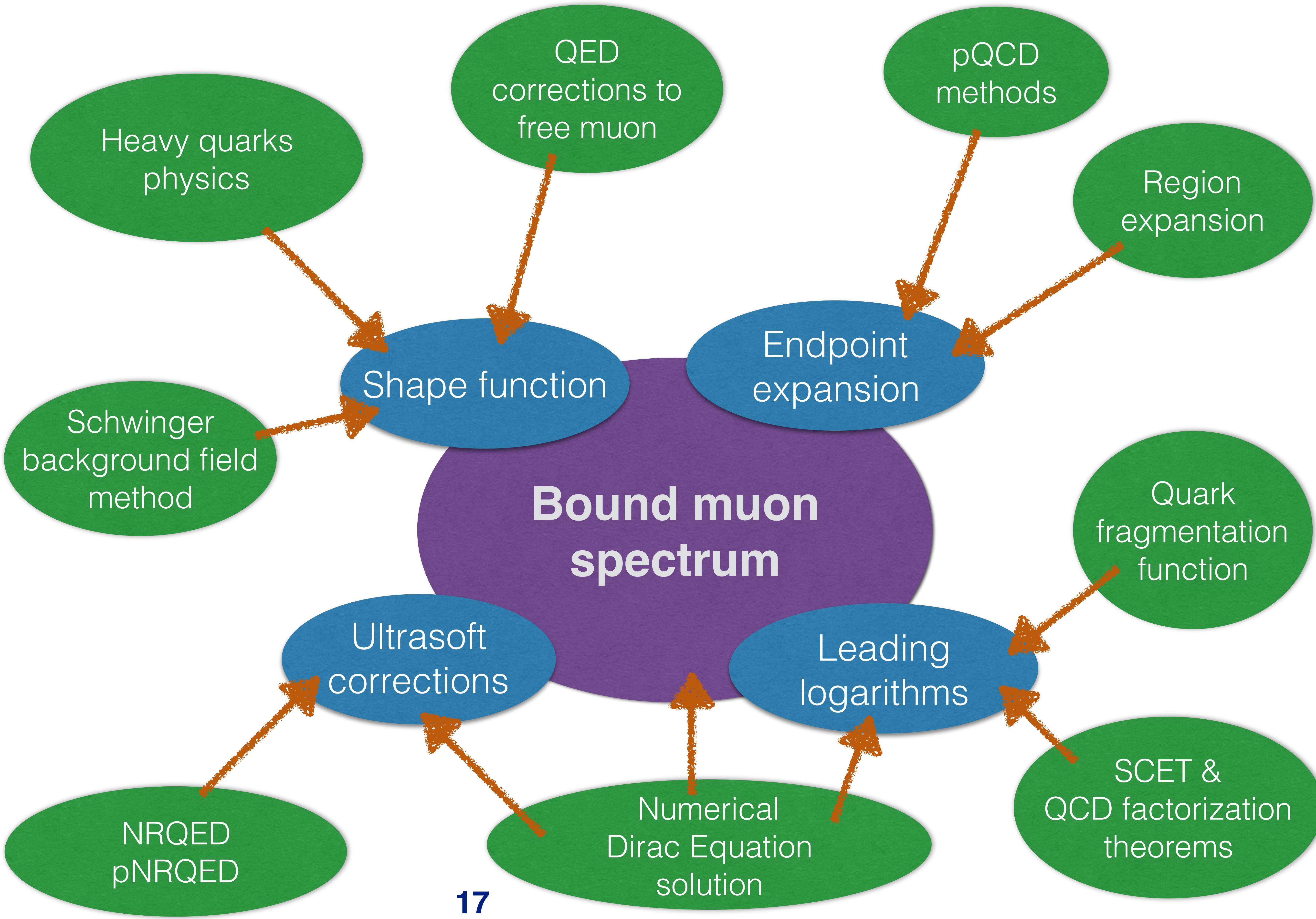
Number of electrons that can reach the detector per one conversion

Details of the model are not important!

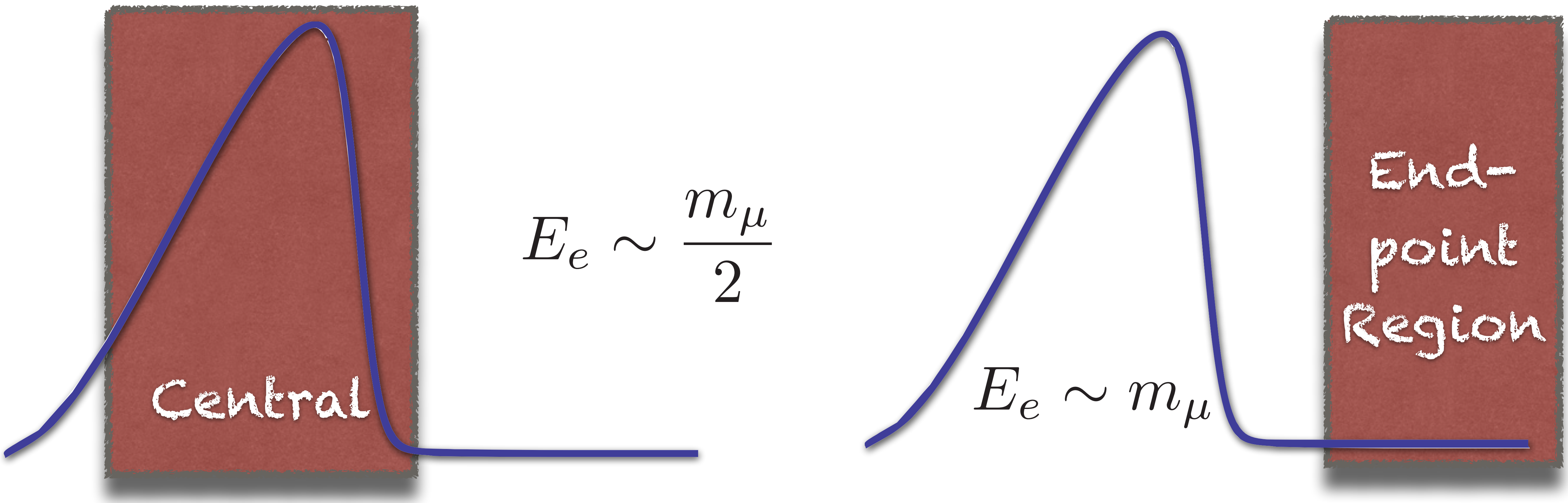
Bound muon spectrum

Why study bound muon spectrum?

- Background for a conversion process
- If not CLFV is found then at least we will have precise measurement of the DIO spectrum
- Underlying physics!
- Many similarities with the heavy quark decay where the perturbation theory breaks down at a scale $\sim \Lambda_{QCD}$
- For muons, pure theoretical calculation is possible without input from experiments


QED corrections to free muon
pQCD methods
Heavy quarks physics
Region expansion
Shape function
Endpoint expansion
Schwinger background field method
Bound muon spectrum
Quark fragmentation function
Ultrasoft corrections
Leading logarithms
NRQED pNRQED
Numerical Dirac Equation solution
SCET & QCD factorization theorems

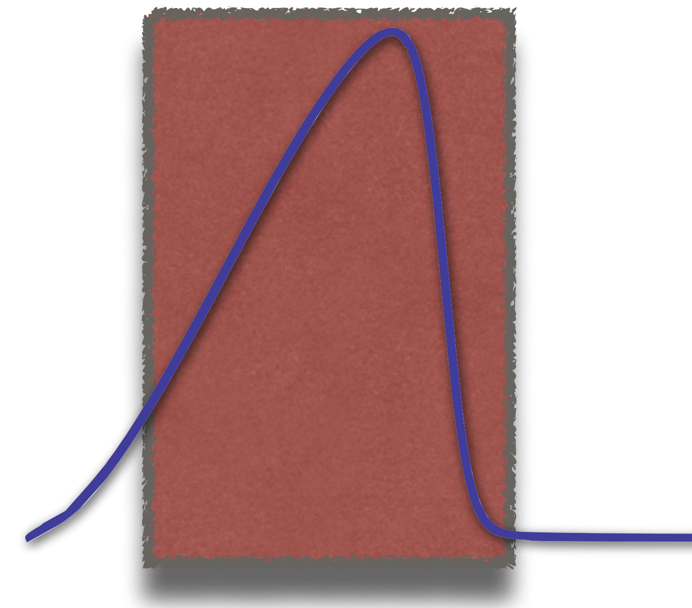
DIO spectrum regions



- Measured by the TWIST experiment in 2009
- Muon motion dominates

- Background for the conversion experiments
- Will be measured in conversion experiments

Central region



- Free muon decay is the Leading Order effect
- Binding effects are only a correction
- Typical momentum transfer between nucleus and muon is of the order of $m_\mu Z\alpha$
- Binding effects need to be re-summed; wave-function cannot be expanded

$$\psi(q) \sim \frac{1}{\left[q^2 + m_\mu^2 (Z\alpha)^2\right]^2}$$

Factorization

(shape function)

QCD case:
Neubert 1993; Mannel, Neubert 1994; Bigi, Shifman, Uraltsev, Vainshtein, 1994

Following QCD approach a factorization theorem can be derived

$$\frac{d\Gamma_{\rm DIO}}{dE_e} = \frac{d\Gamma_{\rm free}}{dE_e} \otimes S$$

Free muon spectrum
It is associated with the hard scale m_μ

QED Shape function
It is associated with the soft scale $m_\mu Z\alpha$

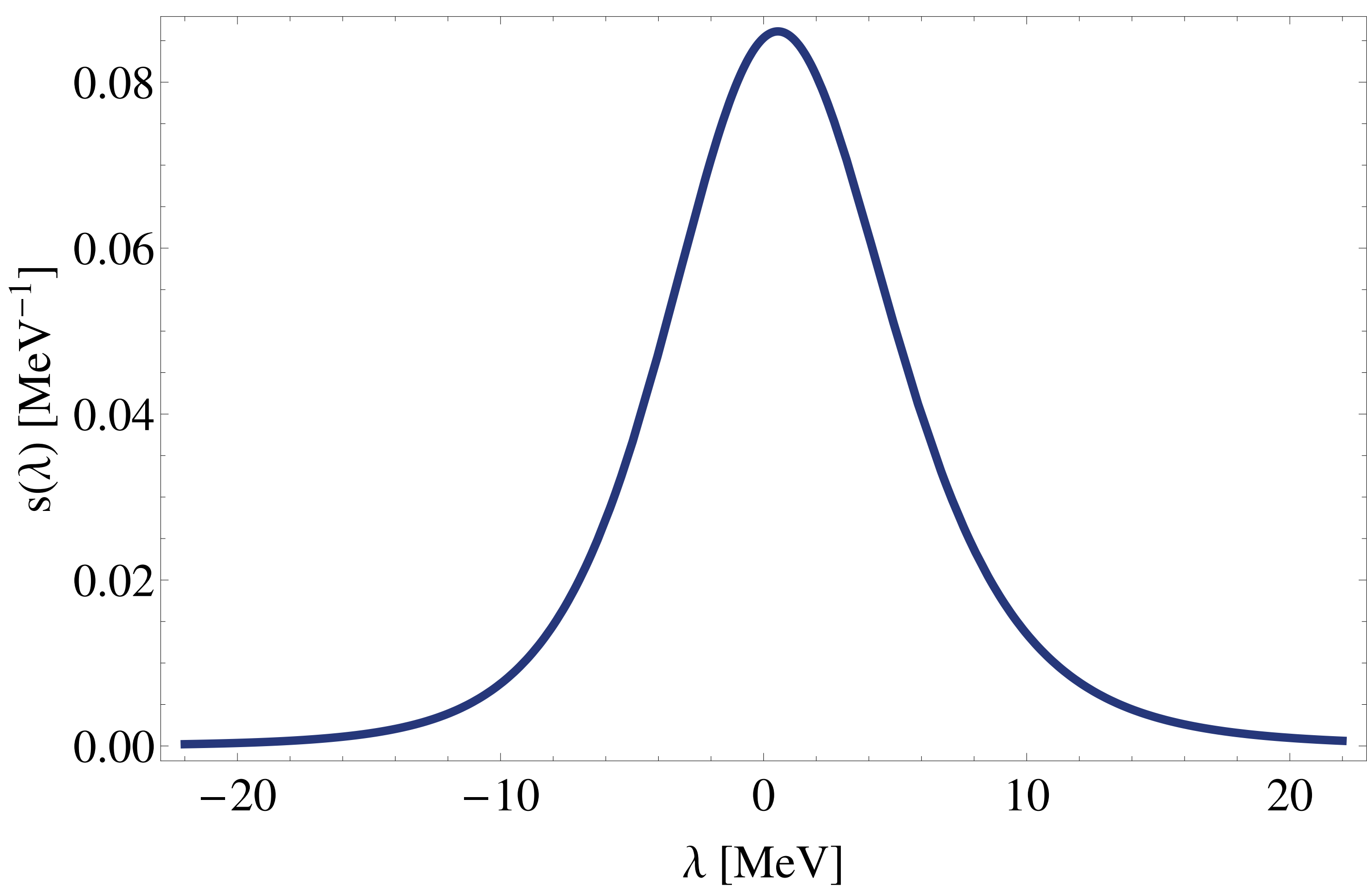
Separation of scales $m_\mu Z\alpha \ll m_\mu$

Shape function

For a point-like nucleus, the LO shape function can be calculated analytically

Szafron, Czarnecki, 2015

$$S(\lambda) = \frac{8m_\mu^5 Z^5 \alpha^5}{3\pi\left[\lambda^2 + m_\mu^2 Z^2 \alpha^2\right]^3}.$$



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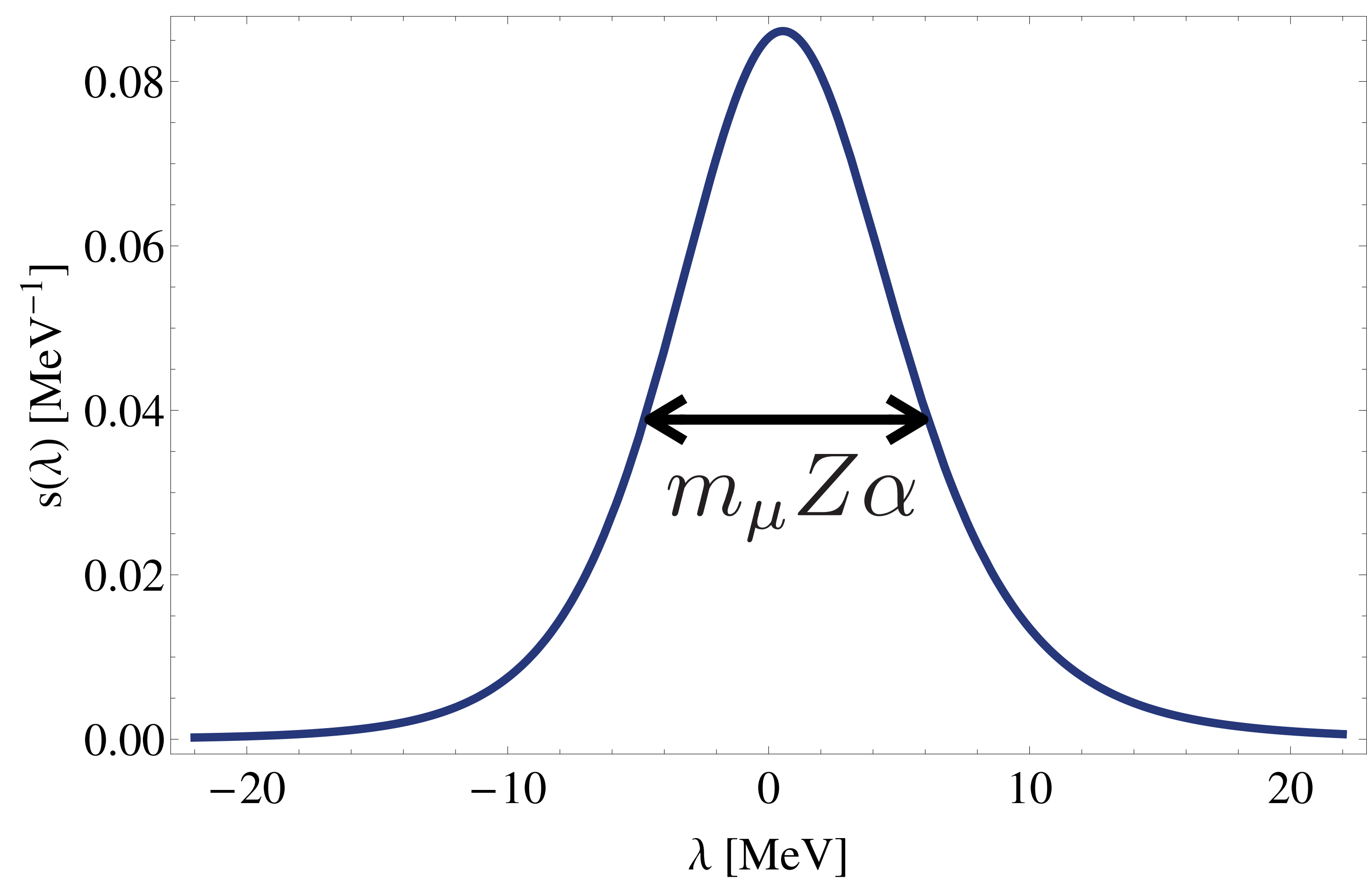

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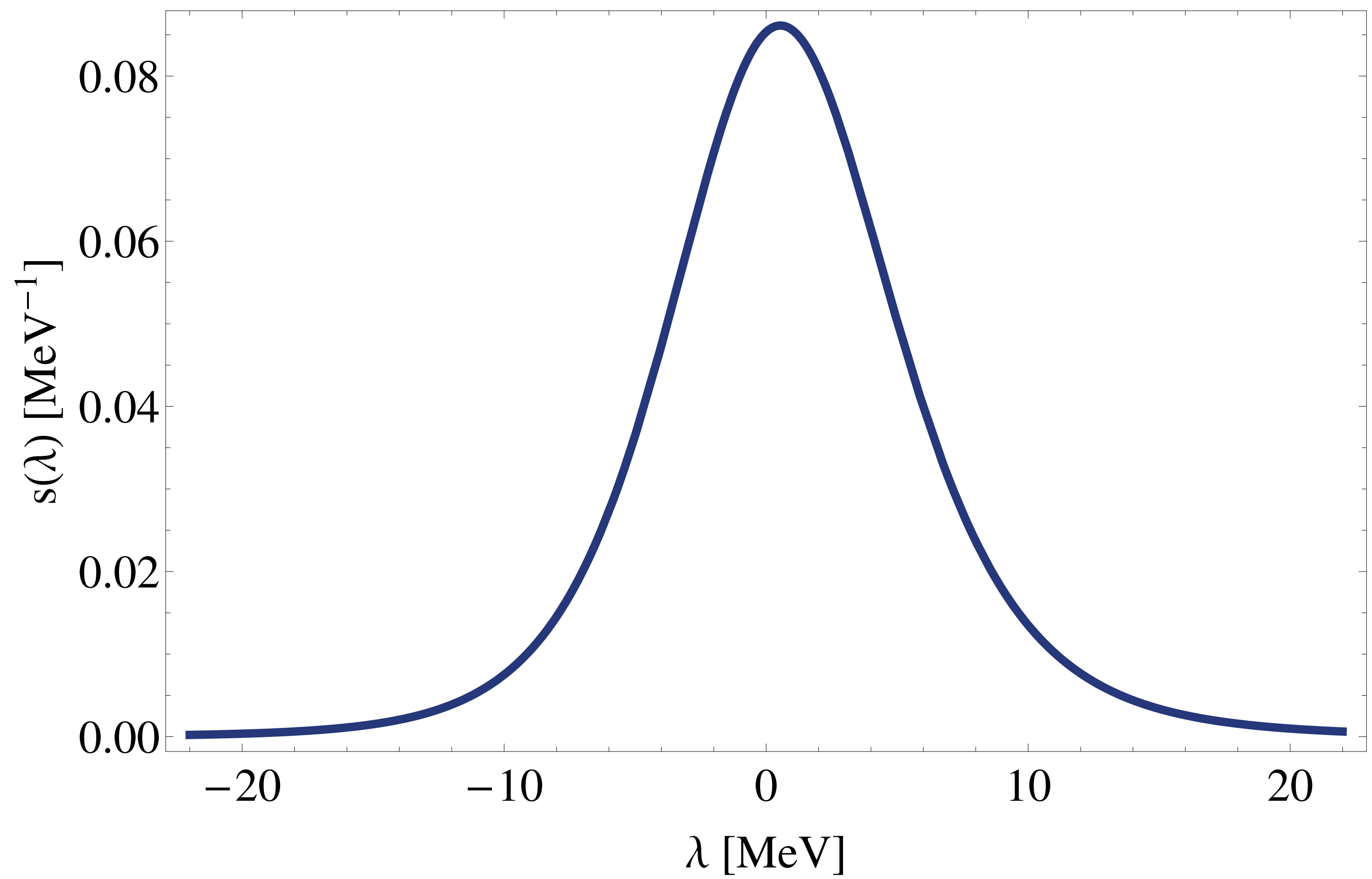

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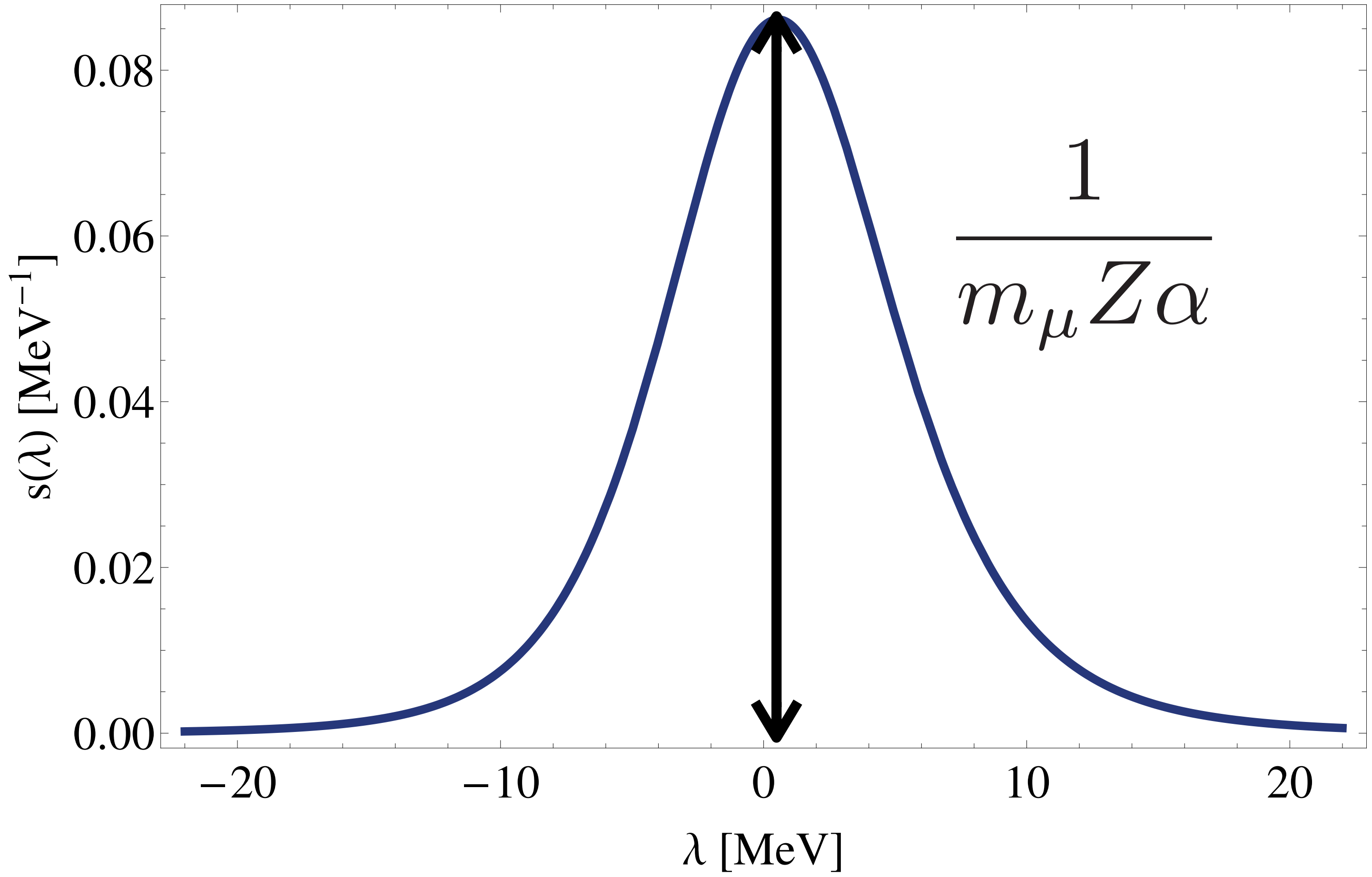

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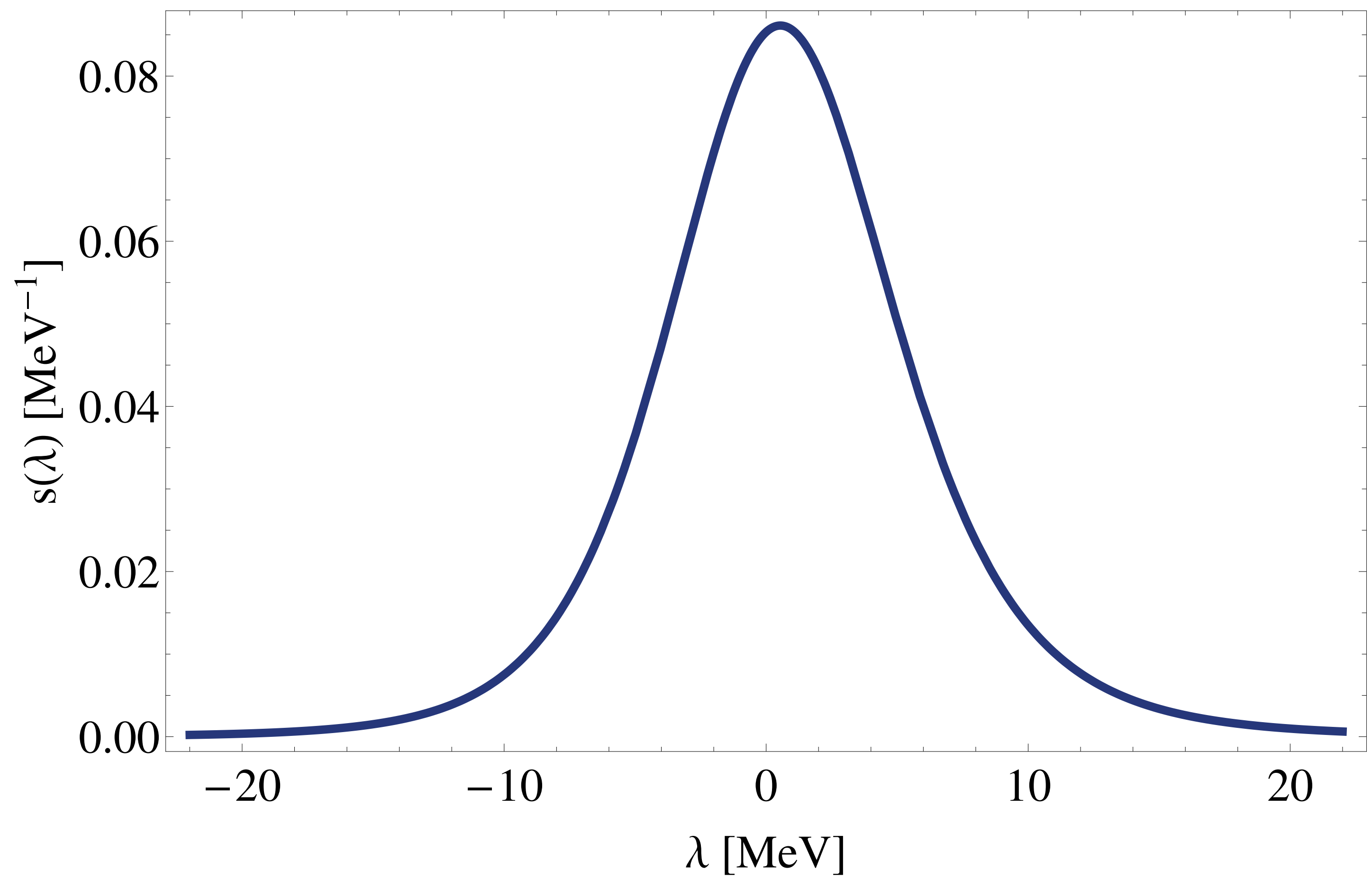

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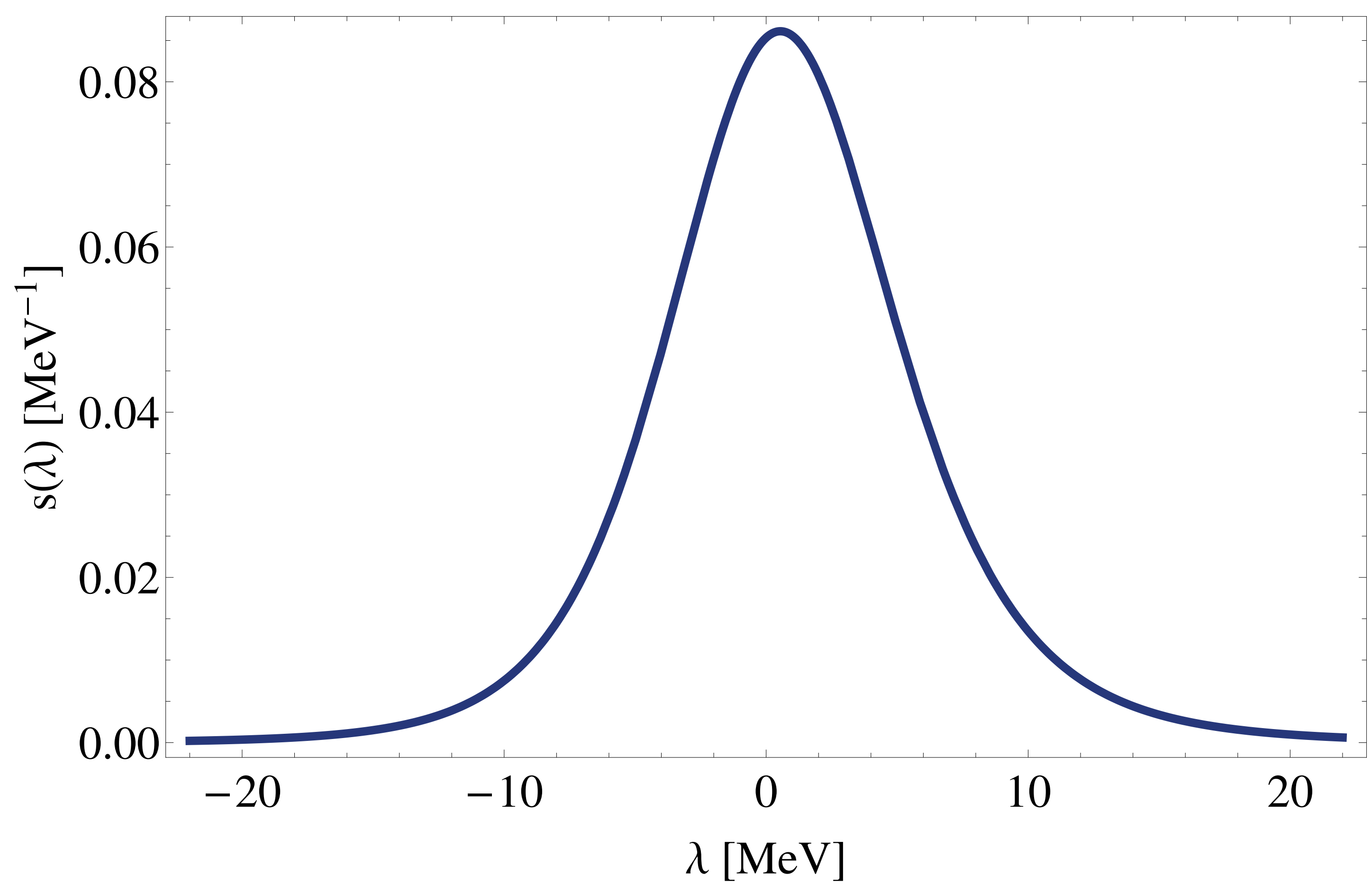
Scaling $\lambda \sim m_\mu Z\alpha$

First moment is zero

$$\int d\lambda \lambda S(\lambda) = 0$$

$$\downarrow$$

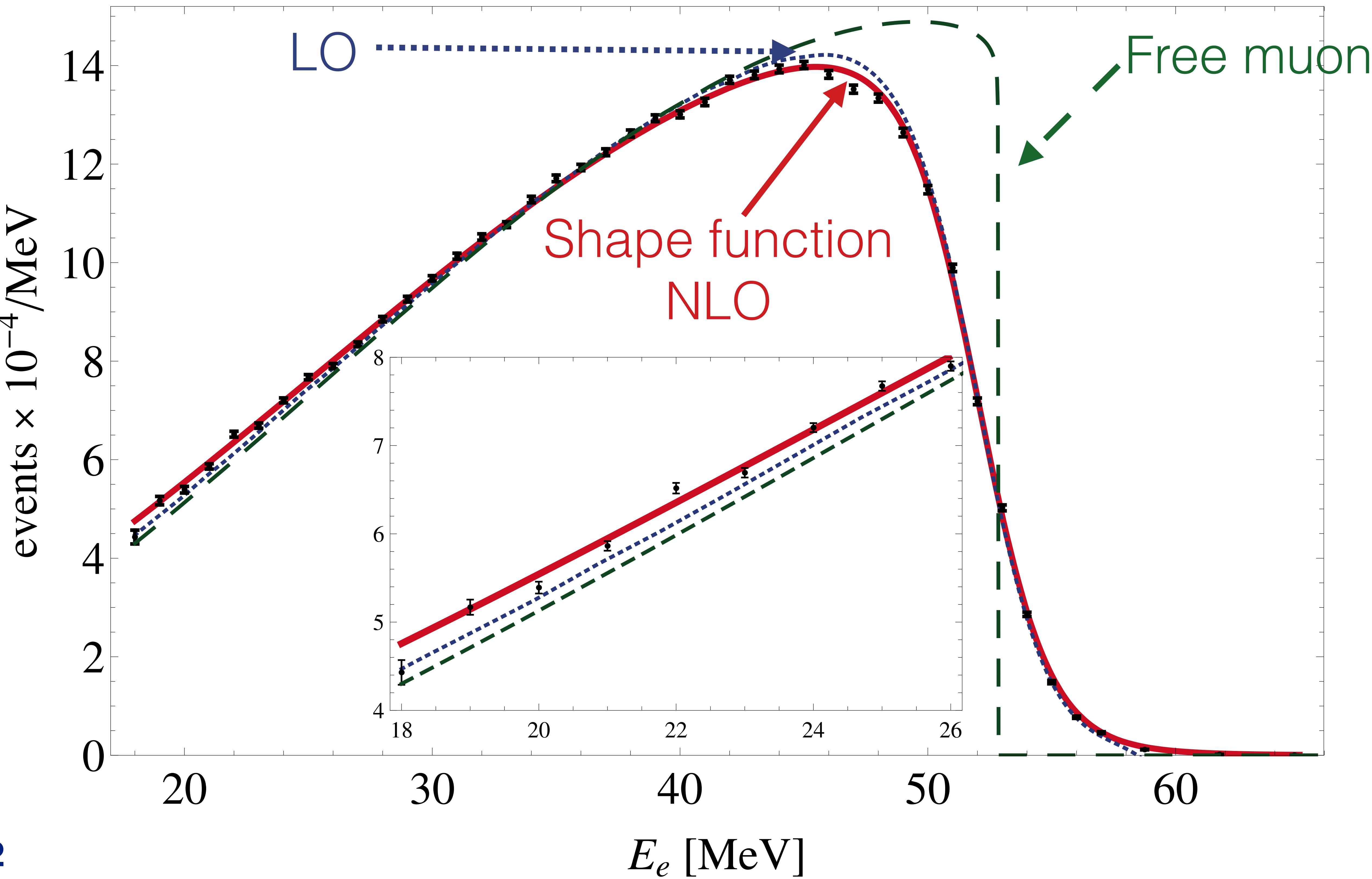
$$\Gamma_{\mathrm{DIO}} = \Gamma_0 + \mathcal{O}(Z^2\alpha^2)$$



Results for real atom

Czarnecki, Dowling,
Garcia i Tormo,
Marciano, Szafron; 2014

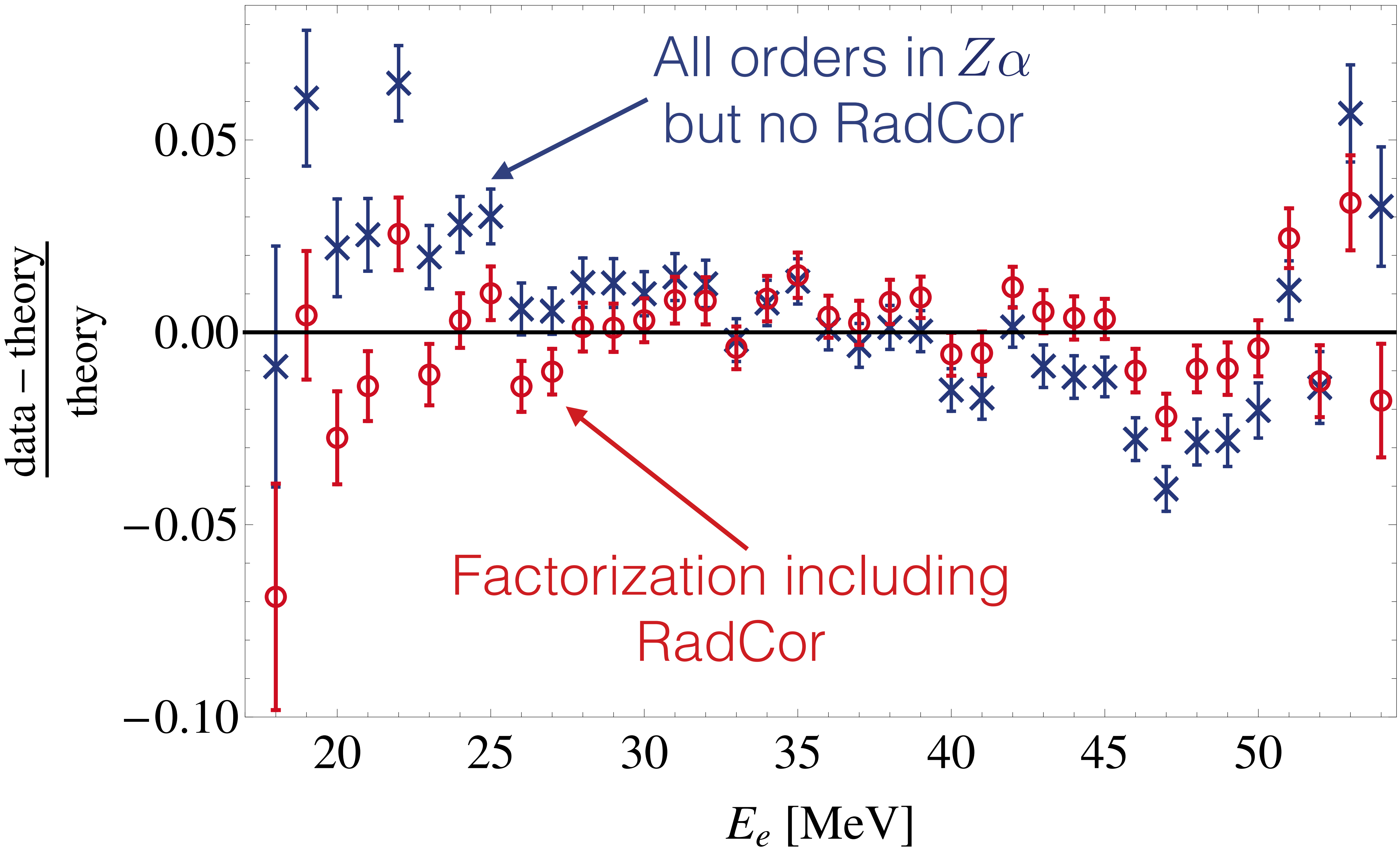
and their relation to the TWIST data



Leading Corrections

and their relation to the TWIST data

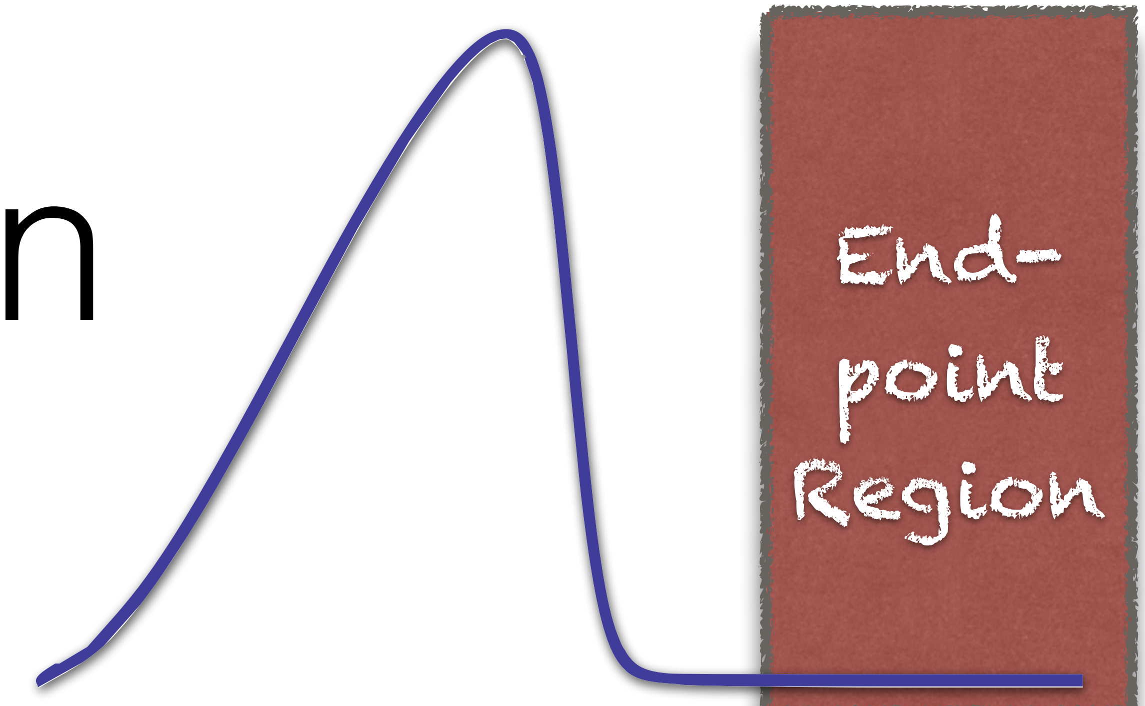
Czarnecki, Dowling, Garcia i Tormo, Marciano, Szafron 2014



Endpoint Region

(conversion background)

$E_e \sim m_\mu$



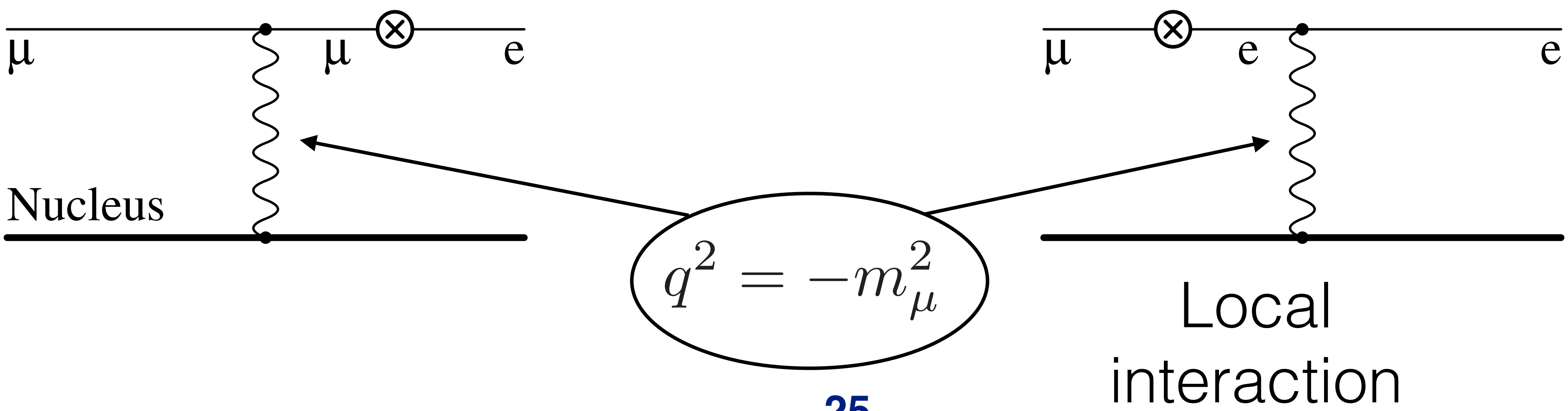
- Free muon spectrum is nonexistent in this region
- Binding effects constitute the LO terms
- Typical momentum transfer between the nucleus and the muon is large ($q^2 \sim m_\mu^2$)
- Both wave functions and propagators can be expanded in powers of $Z\alpha$

Endpoint expansion

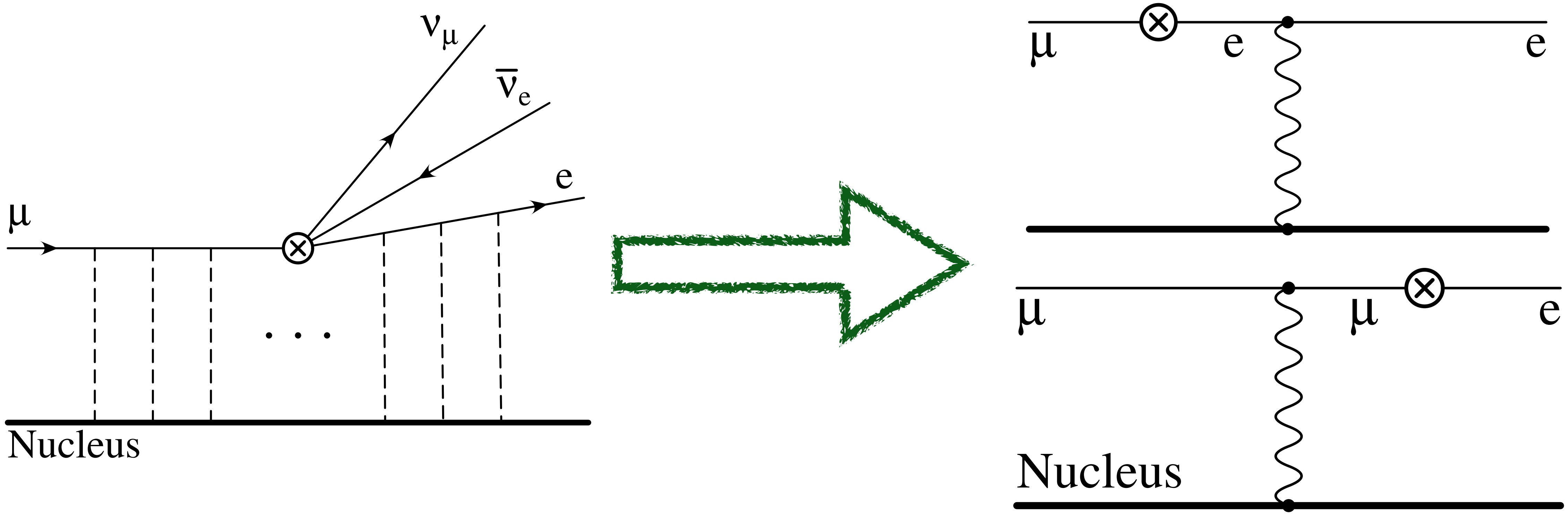
Near the endpoint, the dominant contribution comes from the exchange of hard virtual photons. Szafron, Czarnecki; 2015

$$\frac{m_\mu}{\Gamma_{Free}}\frac{d\Gamma}{dE_e} \approx \frac{1024}{5\pi}(Z\alpha)^5\left(\frac{\Delta}{m_\mu}\right)^5$$

$$\Delta = E_{max} - E_e$$



Phase space suppression

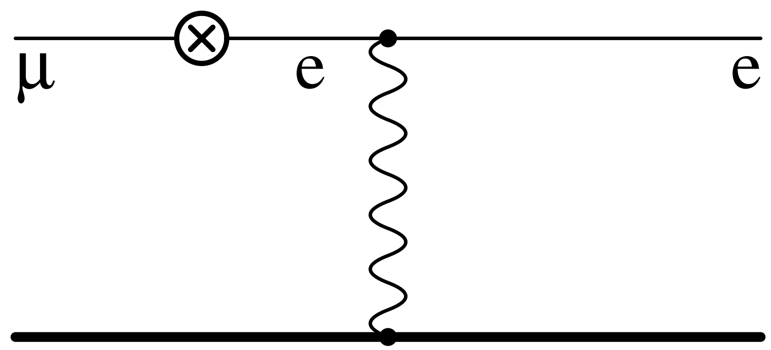


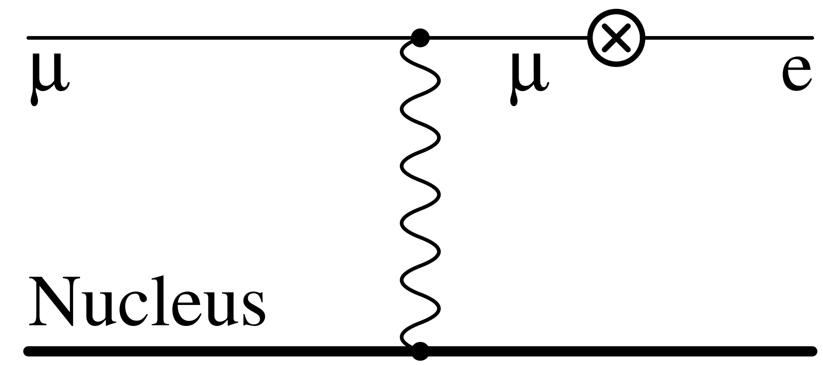
$$\int \frac{\mathrm{d}^3\nu}{\nu_0}\frac{\mathrm{d}^3\overline{\nu}_0}{\overline{\nu}_0}\delta\left(\Delta - \nu_0 - \overline{\nu}_0\right)\ldots\psi\ldots\overline{\psi} \sim \Delta^5$$

Each neutrino gives 3 powers of Δ

Can be used to constrain effective BSM operators!

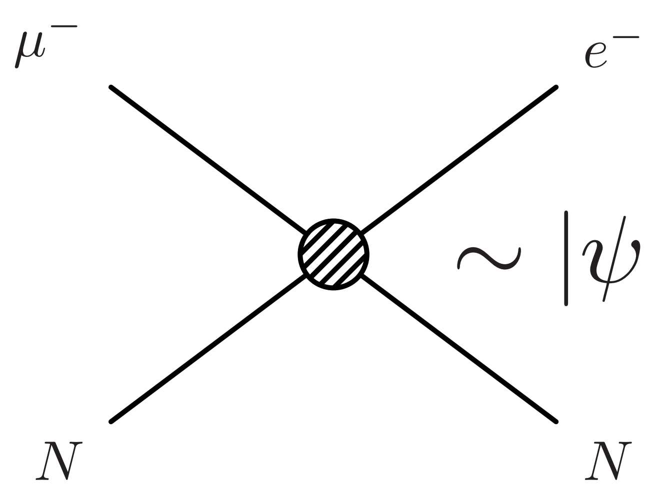
Binding suppression





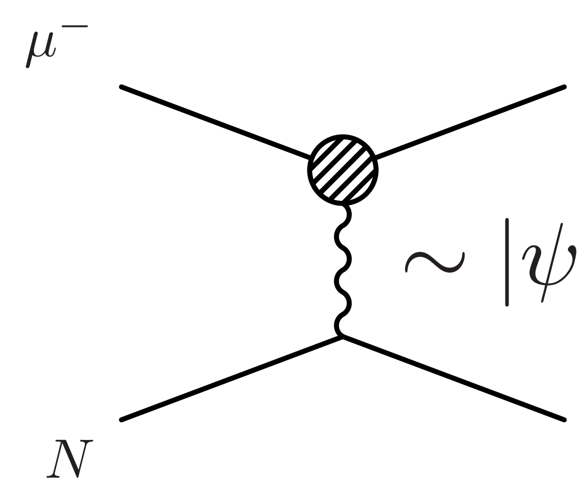
$$|\mathcal{M}|^2 \sim |\psi(0)|^2 \times \left|V(m_\mu^2)\right|^2 \sim (Z\alpha)^3 \times (Z\alpha)^2$$

$$|\psi(0)|^2 \sim (Z\alpha)^3$$



$$\sim |\psi(0)|^2$$

$$V(k^2) \sim -\frac{Z\alpha}{k^2}$$



$$\sim |\psi(0)|^2 \times \left|V(m_\mu^2)\right|^2$$

Endpoint Radiative Correction

Background suppression ~15%

$$\Delta = E_{max} - E_e$$

$$\frac{1}{\Gamma_{Free}}\frac{d\Gamma}{dE_e} = \Delta^5 \frac{1024}{5\pi m_\mu^6}(Z\alpha)^5 \left(\frac{\Delta}{m_\mu}\right)^{\frac{\alpha}{\pi}\delta_S} (1 + \frac{\alpha}{\pi}\delta_{VP} + \frac{\alpha}{\pi}\delta_H)$$

Endpoint Radiative Correction

- Soft vacuum polarization correction to the muon wave-function at the origin (running to $m_\mu Z\alpha$)

Background suppression ~15%

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Endpoint Radiative Correction

- Soft vacuum polarization correction to the muon wave-function at the origin (running to $m_\mu Z\alpha$)
- Hard vacuum polarization

Background suppression ~15%

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Endpoint Radiative Correction

- Soft vacuum polarization correction to the muon wave-function at the origin (running to $m_\mu Z\alpha$)
- Hard vacuum polarization
- Soft photon emission

Background suppression ~15%

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Endpoint Radiative Correction

- Soft vacuum polarization correction to the muon wave-function at the origin (running to $m_\mu Z\alpha$)
- Hard vacuum polarization
- Soft photon emission
- Hard correction

Background suppression ~15%

$$\Delta = E_{max} - E_e$$

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Interpolating between regions

- We also need to know the spectrum for intermediate electron energies
- We have identified the leading corrections and it is possible to calculate them!

1. Real radiation can be approximated by taking into account collinear photon emission
2. Vacuum polarization can be included when we solve the Dirac equation numerically

Vacuum polarization

$$V(r) = -\frac{Z\alpha}{r} + Z\alpha\frac{\alpha}{\pi}V_U(r, m_e)$$

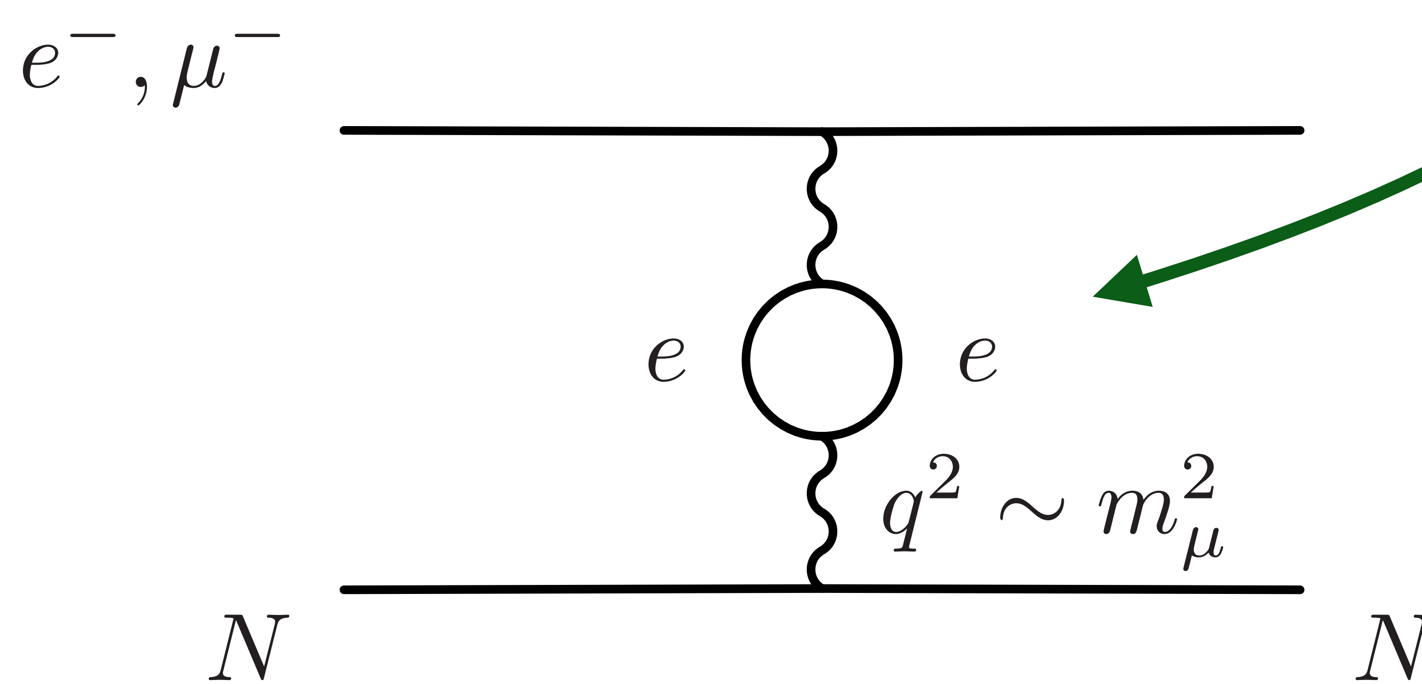
Electron loop generates long distance potential and this leads to large logarithmic corrections

$$r \sim \frac{1}{m_e} \gg \frac{1}{m_\mu Z\alpha}$$

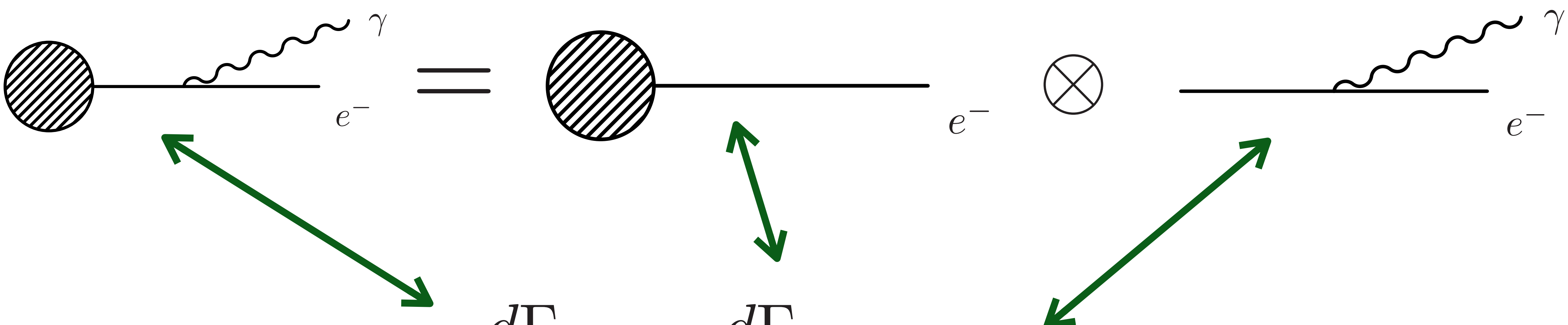
Correction range — Atom size

$$\ln\frac{Z\alpha m_\mu}{m_e}$$

$$\ln\frac{m_\mu}{m_e}$$



Soft-Collinear Factorization

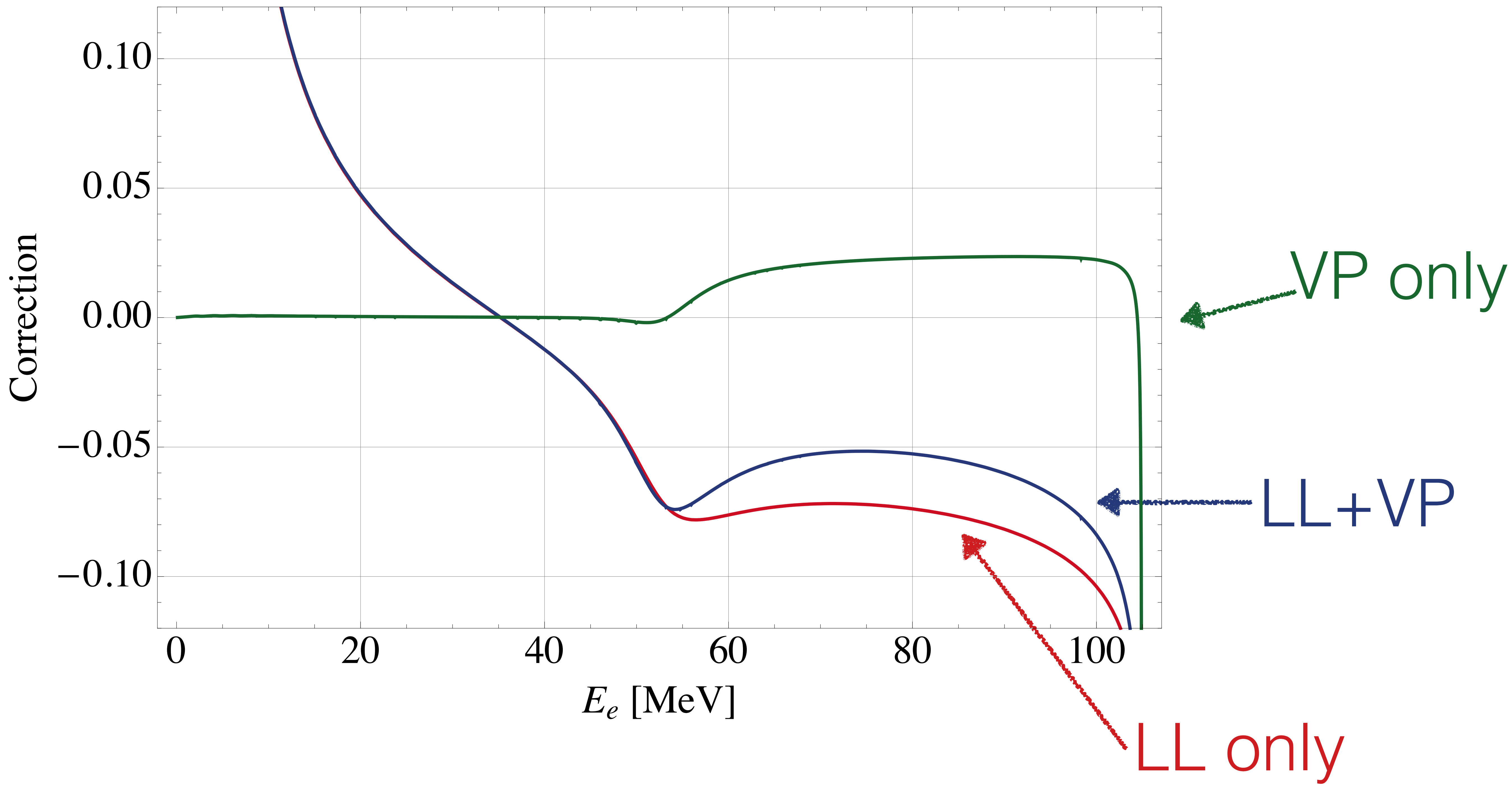


$$\frac{d\Gamma_{\mathrm{LL}}}{dE_e} = \frac{d\Gamma_{\mathrm{LO}}}{dE_e} \otimes D_e$$

with the perturbative fragmentation function

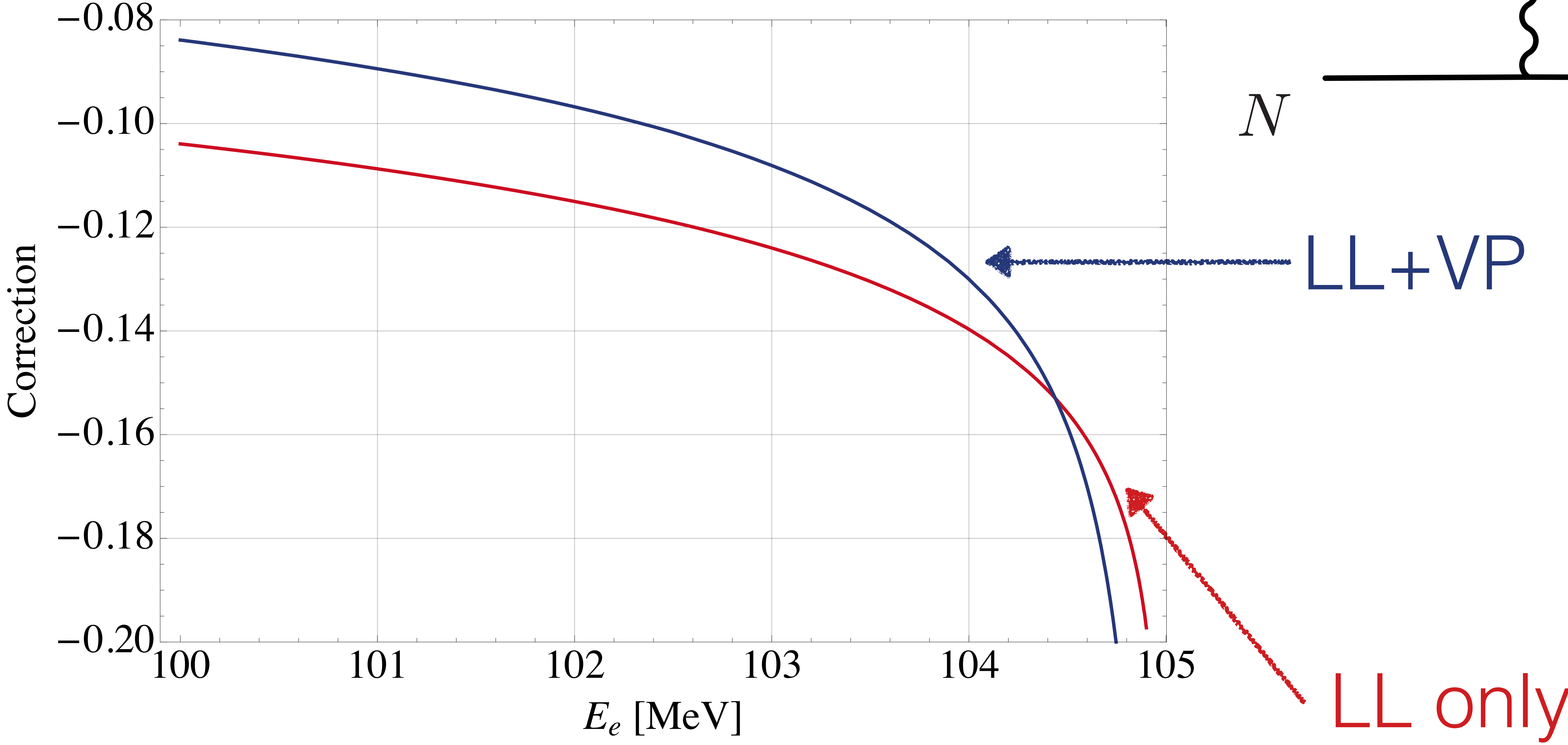
$$D_e(x) = \delta(1-x) + \frac{\alpha}{2\pi} \ln\left(\frac{m_\mu^2}{m_e^2}\right) P_{ee}^{(0)}(x) + \dots$$

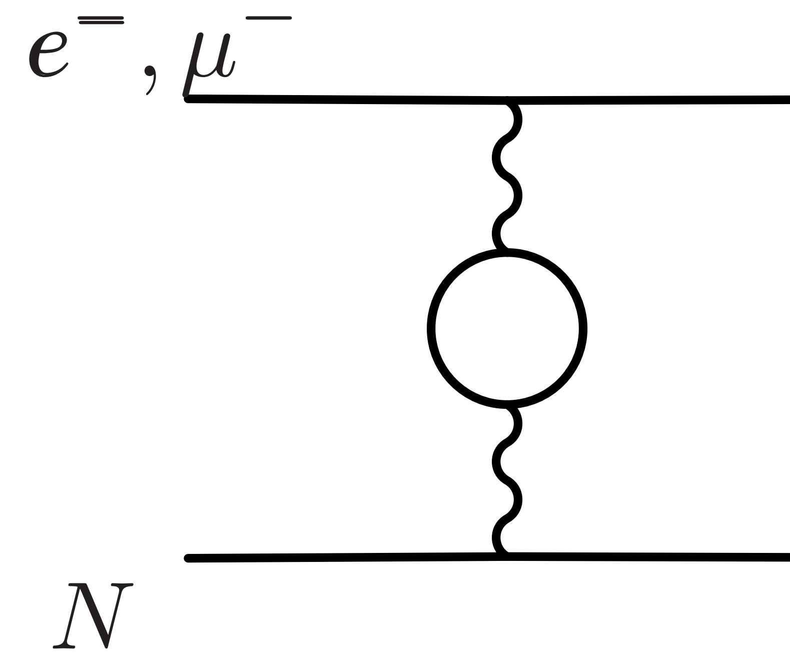
Correction to the DIO spectrum



Endpoint region

Vacuum Polarization correction is very important!





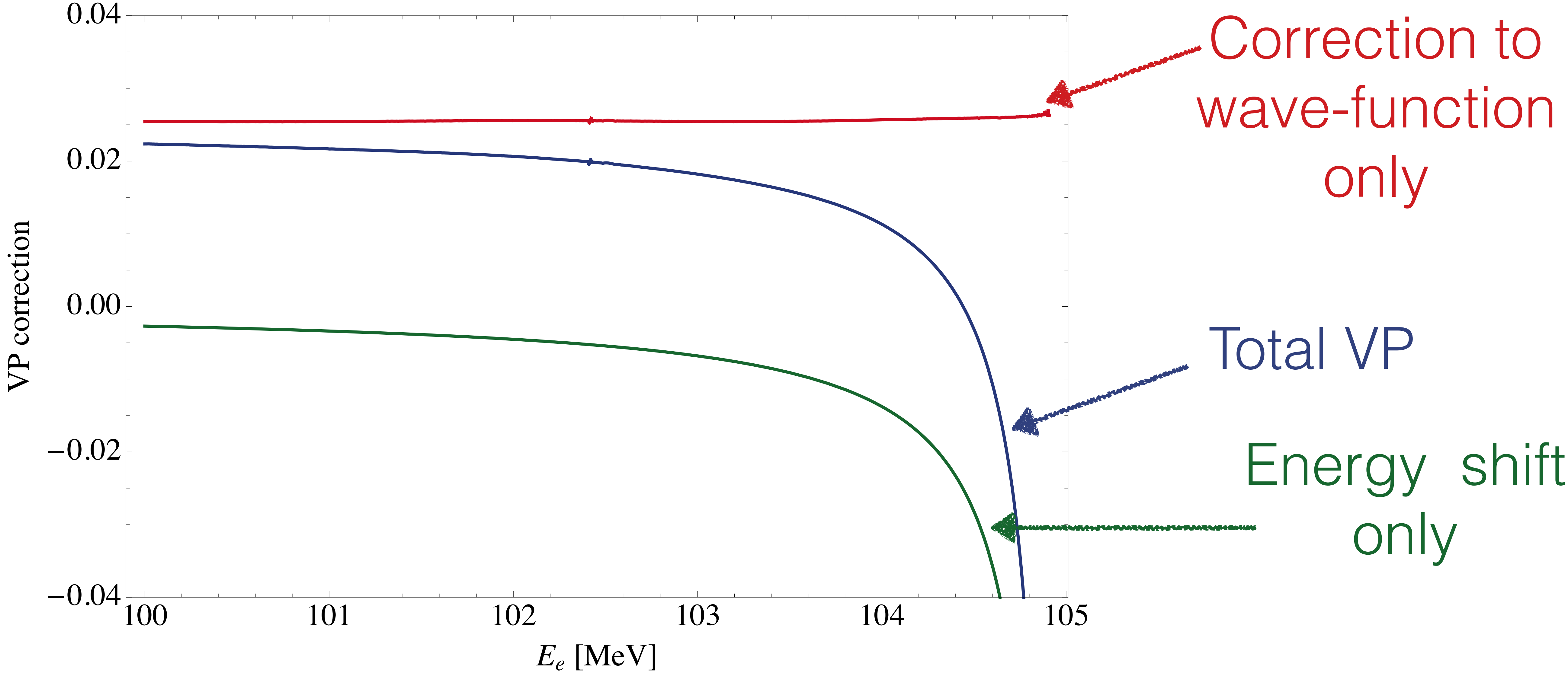
Vacuum polarization correction

$$E_b \to E_b + \frac{\alpha}{\pi}\delta E_b$$

Correction to the endpoint energy

$$\psi(p) \to \psi(p) + \frac{\alpha}{\pi}\delta\psi(p)$$

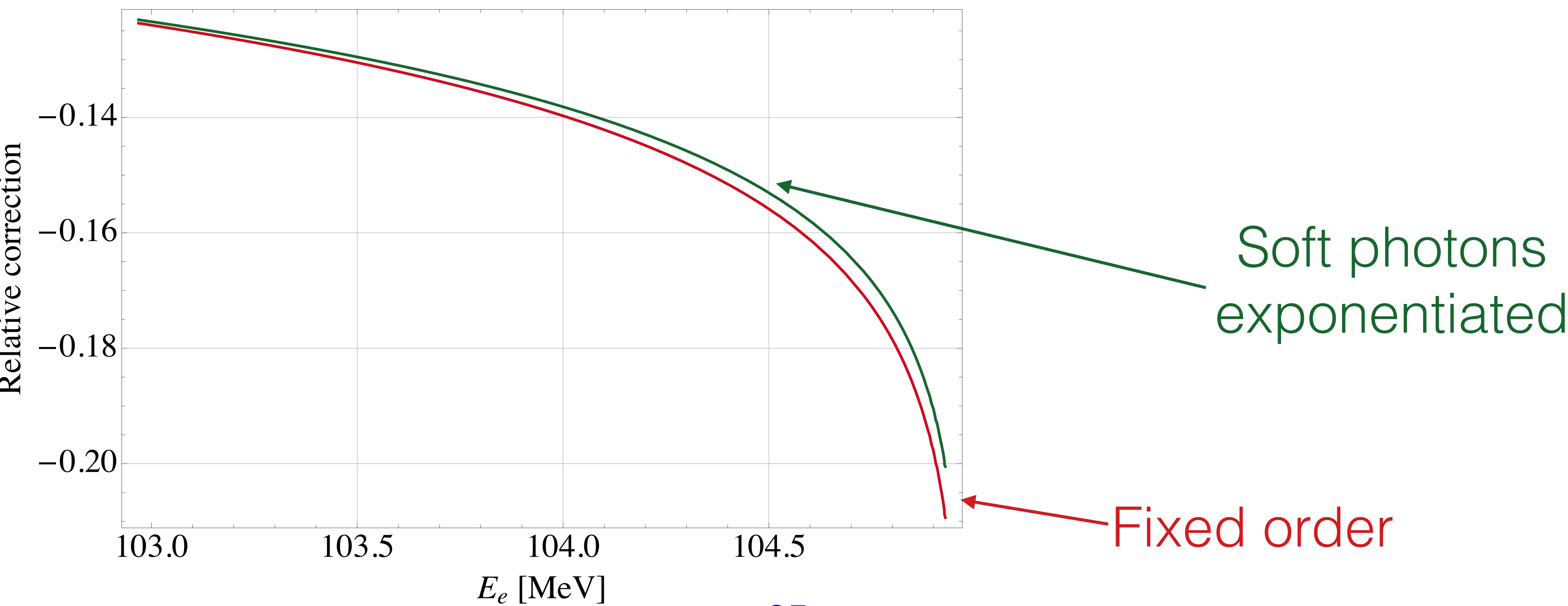
Corrections to the wave-functions



Exponentiation

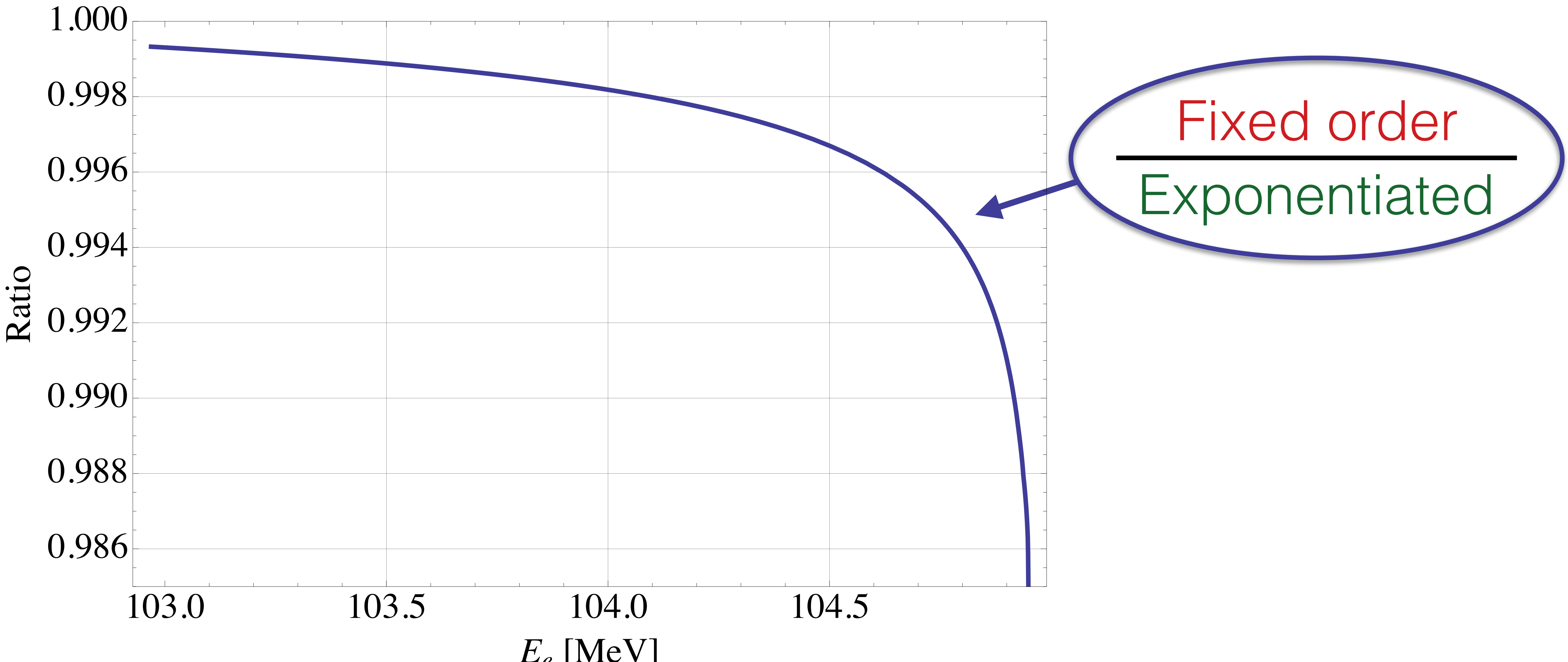
Near the endpoint emission of soft photons is logarithmically enhanced

$$\frac{\alpha}{\pi}\delta_S \ln\left(\frac{E_{\max}-E_e}{E_{\max}}\right) \to \left(\frac{E_{\max}-E_e}{E_{\max}}\right)^{\frac{\alpha}{\pi}\delta_S}$$



Exponentiation

Bin Size	1 MeV	0.5 MeV	0.1 MeV
Fixed order/ Exponentiated	0.2%	0.4%	0.8%



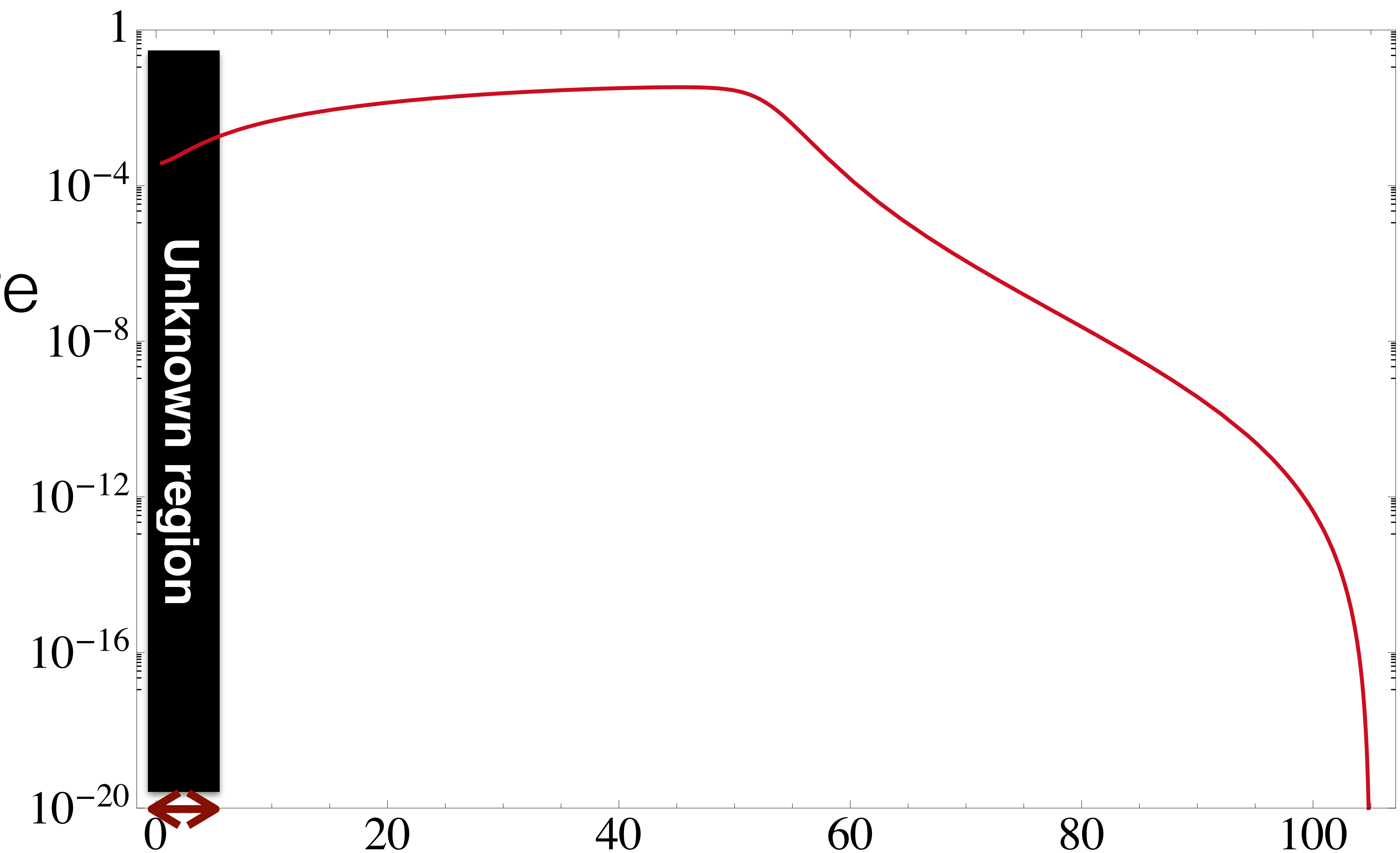
Summary

- Conversion spectrum is not sensitive to the BSM model
- We can correctly reproduce TWIST measurement
- Vacuum polarization gives large, nonfactorizable correction to the DIO spectrum
- Endpoint spectrum is very sensitive to the binding energy (Lamb shift)
- Large finite nucleus size effects

Summary

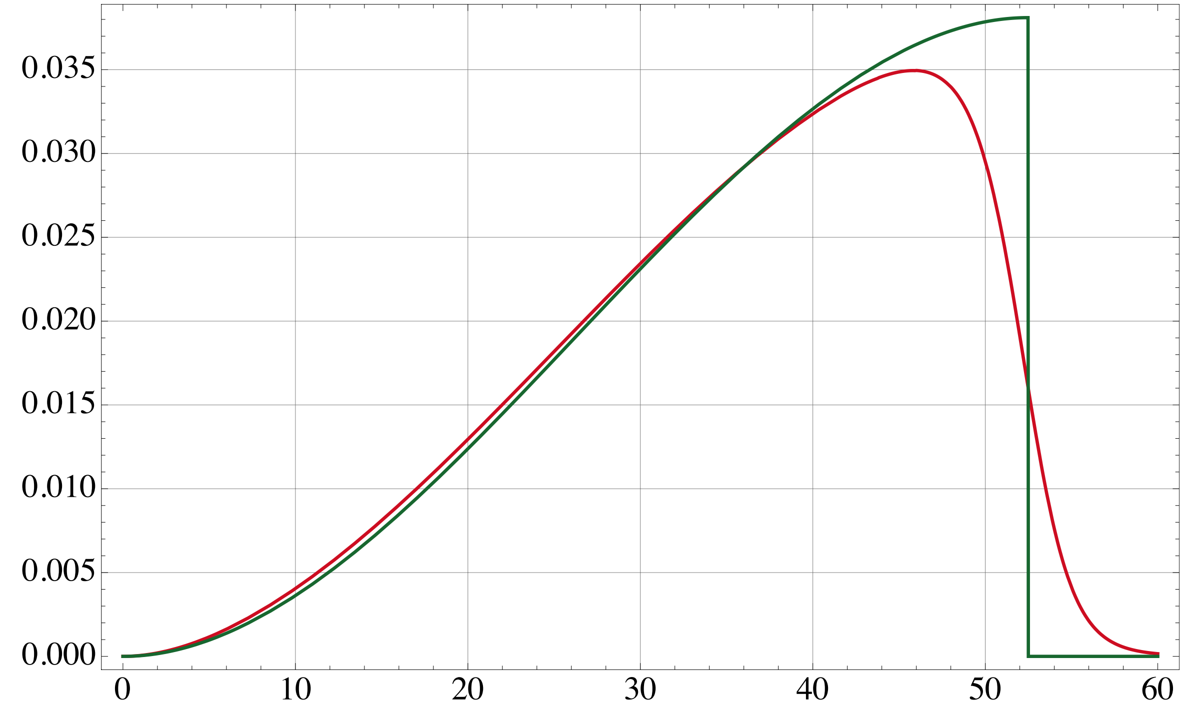
If you want to discover New Physics, first you have to understand the Standard Model

DIO spectrum — a quantity that is changing by more than 16 orders is calculated including the leading corrections



Backup

Free / Bound



Ratio
Free/Bound