

Charged Lepton Flavor Violation (CLFV),
Anomalous Magnetic Moment ($g-2$)
and Electric Dipole Moment (EDM)

Kazuhiro Tobe (Nagoya University)

2nd International Conference on Charged Lepton Flavor Violation
@ Charlottesville, Virginia, USA.
June 20-22, 2016

Plan of my talk

- ▶ Introduction

Is CLFV interesting?

- ▶ Model independent study for CLFV,
g-2, and EDM

effective operator analysis

- ▶ CLFV, muon g-2 and EDM in a
general two Higgs doublet model
(as a concrete example)

Refs: JHEP 1505, 028 (2015), arXiv: 1511.08880

- ▶ Summary

Omura, Senaha, Tobe

Introduction

“Flavor” in the standard model (SM)

- ★ Fermion masses and mixings are free parameters
- ★ No principle nor theory for flavor

If there is any principle or theory for flavor,
it must be physics beyond the SM.

flavor is still a mystery in the SM

“Flavor” in the physics beyond the SM

★ there is additional origin of flavor violations

e.g. flavor violating squark/lepton masses
in SUSY models

extra Yukawa interactions in two Higgs doublet model

★ No principle nor theory for flavor

Typically it is difficult to make any definite predictions
in flavor violating processes

“Flavor” is difficult problem, but theoretical and
experimental studies for flavor will be important to make a
deeper understanding of flavor.

Why is Lepton Flavor Violation (LFV) interesting?

★ neutrino oscillation results suggest tiny neutrino masses and large flavor mixings

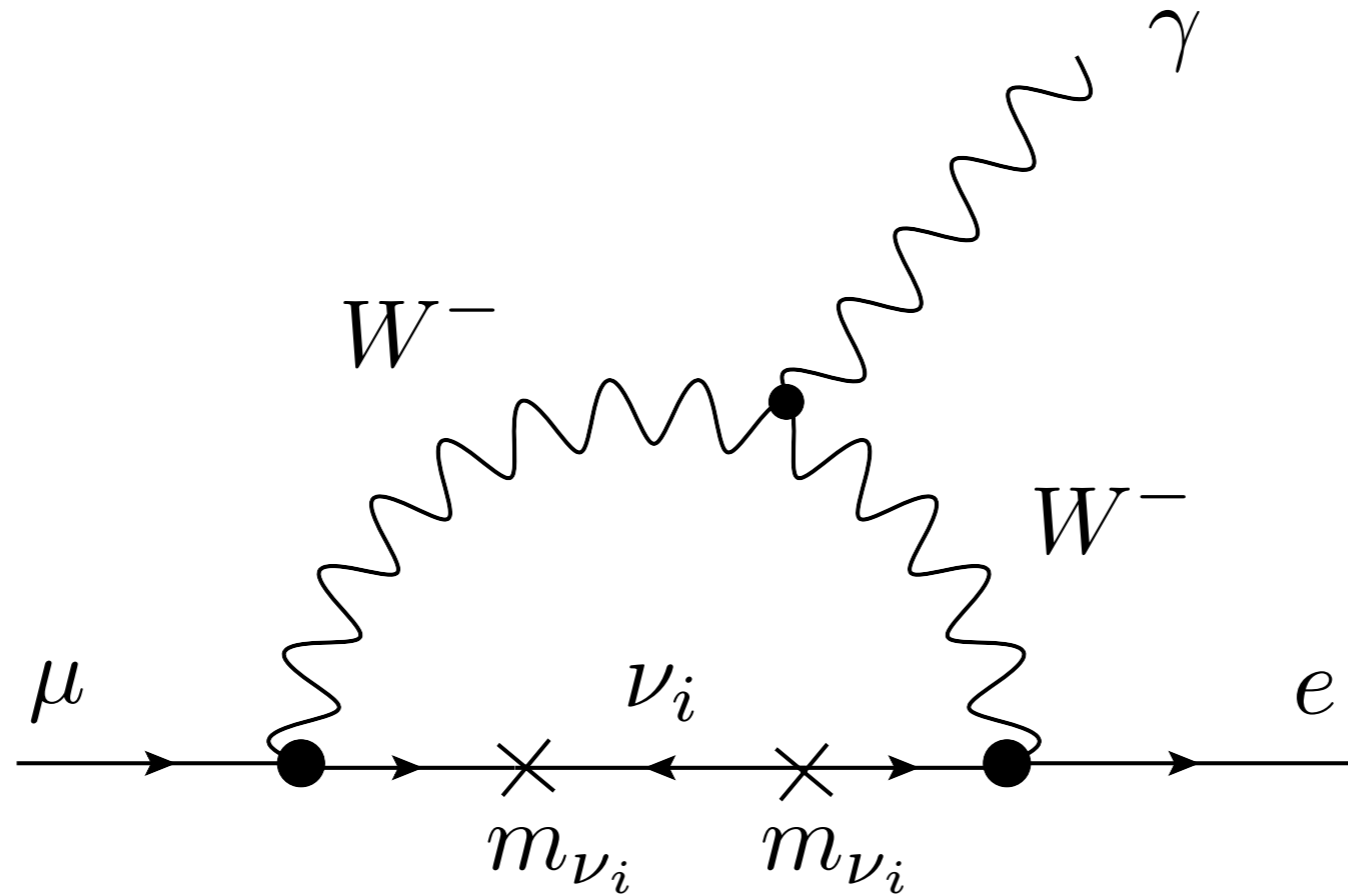
- each lepton flavor number is violated
- dimension 5 operator for neutrino masses

$$\mathcal{L}_{d=5} = \frac{1}{\Lambda} (LH)^c (LH)$$
$$\longrightarrow m_\nu = \frac{\langle H \rangle^2}{\Lambda}$$

$$\Lambda \sim 10^{15} \text{ GeV} \quad \text{for } m_\nu = \sqrt{\Delta m_{\text{atm}}^2}$$

very high scale!

In the standard model with tiny neutrino masses



$$\text{BR}(\mu \rightarrow e \gamma) \sim \frac{m_{\nu}^4}{m_W^4} < 10^{-50}$$

Charged lepton flavor violation, $\mu \rightarrow e \gamma$, is induced,
but very tiny

Is Charged LFV (CLFV) not interesting?

★ Various motivated new physics models

- Solutions to hierarchy problem

Supersymmetry, Little Higgs, extra dimension, etc

New physics scale \sim TeV

- WIMP dark matter

New physics scale \sim TeV

- Baryon asymmetry in the universe

New origin of CP violation

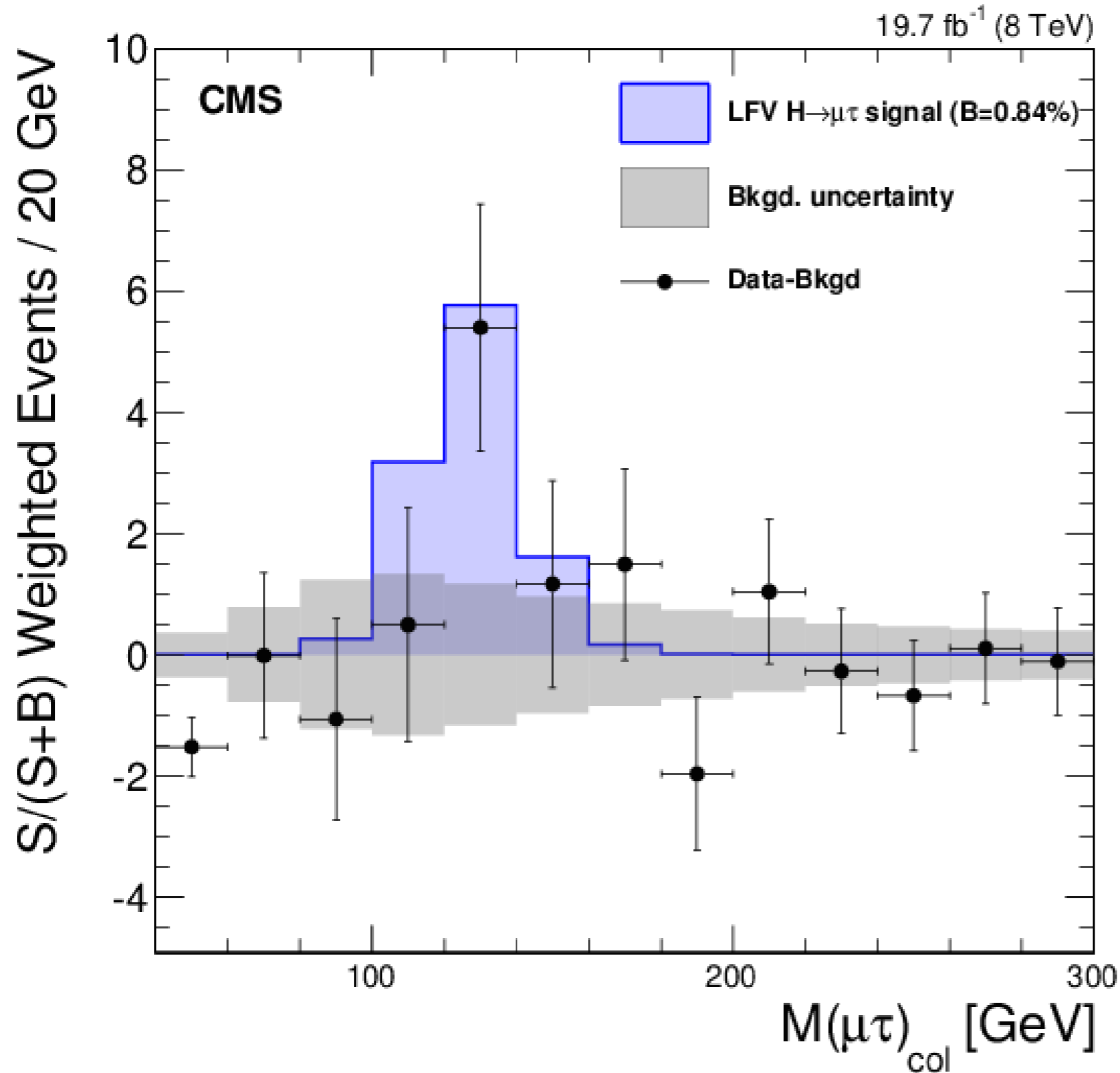
e.g. electroweak baryogenesis \sim TeV

★ Hints from experimental data related to lepton flavor??

- Lepton flavor violating Higgs boson decay $h \rightarrow \mu \tau$
reported by CMS (2.4σ excess)

CMS collaboration has reported an excess in $h \rightarrow \mu\tau$

CMS: arXiv: 1502.07400

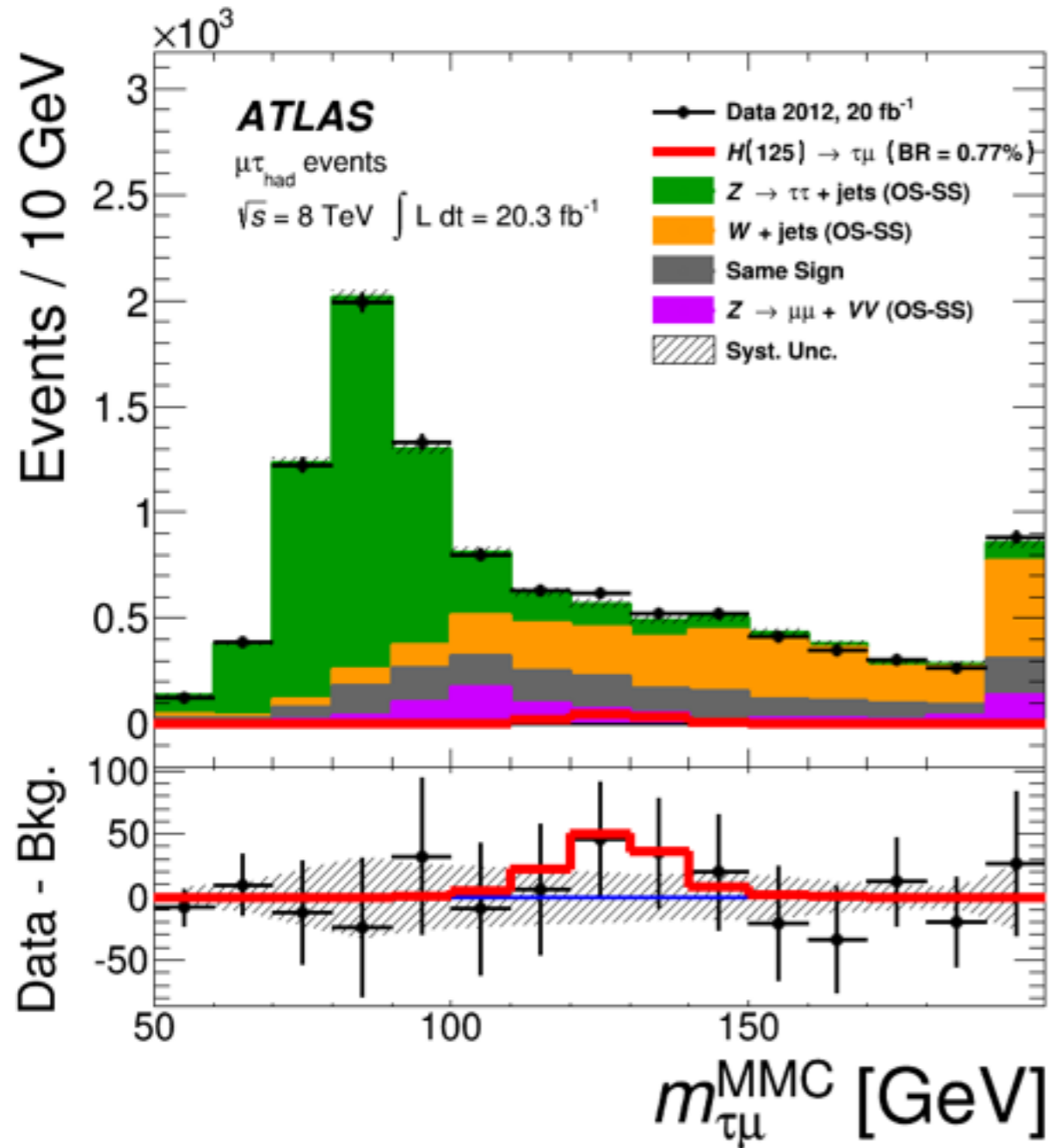


CMS best fit:

$$\text{BR}(h \rightarrow \mu\tau) = (0.84^{+0.39}_{-0.37})\%$$

2.4 σ excess

Talk by P. Onyisi
@FPCP 2015



ATLAS

$$\text{BR}(h \rightarrow \mu\tau) = (0.77 \pm 0.62)\%$$

ATLAS: arXiv: 1508.03372

In Moriond EW 2016

ATLAS:

$$\text{BR} = 0.53 \pm 0.51\% < 1.43\% \text{ (95\% CL)}$$

consistent with CMS

CMS best fit:

$$\text{BR}(h \rightarrow \mu\tau) = (0.84_{-0.37}^{+0.39})\%$$

2.4 σ excess

ATLAS: arXiv: 1508.03372

Hint for new physics!

New 13 TeV result from CMS

CMS PAS HIG-16-005

No excess is observed

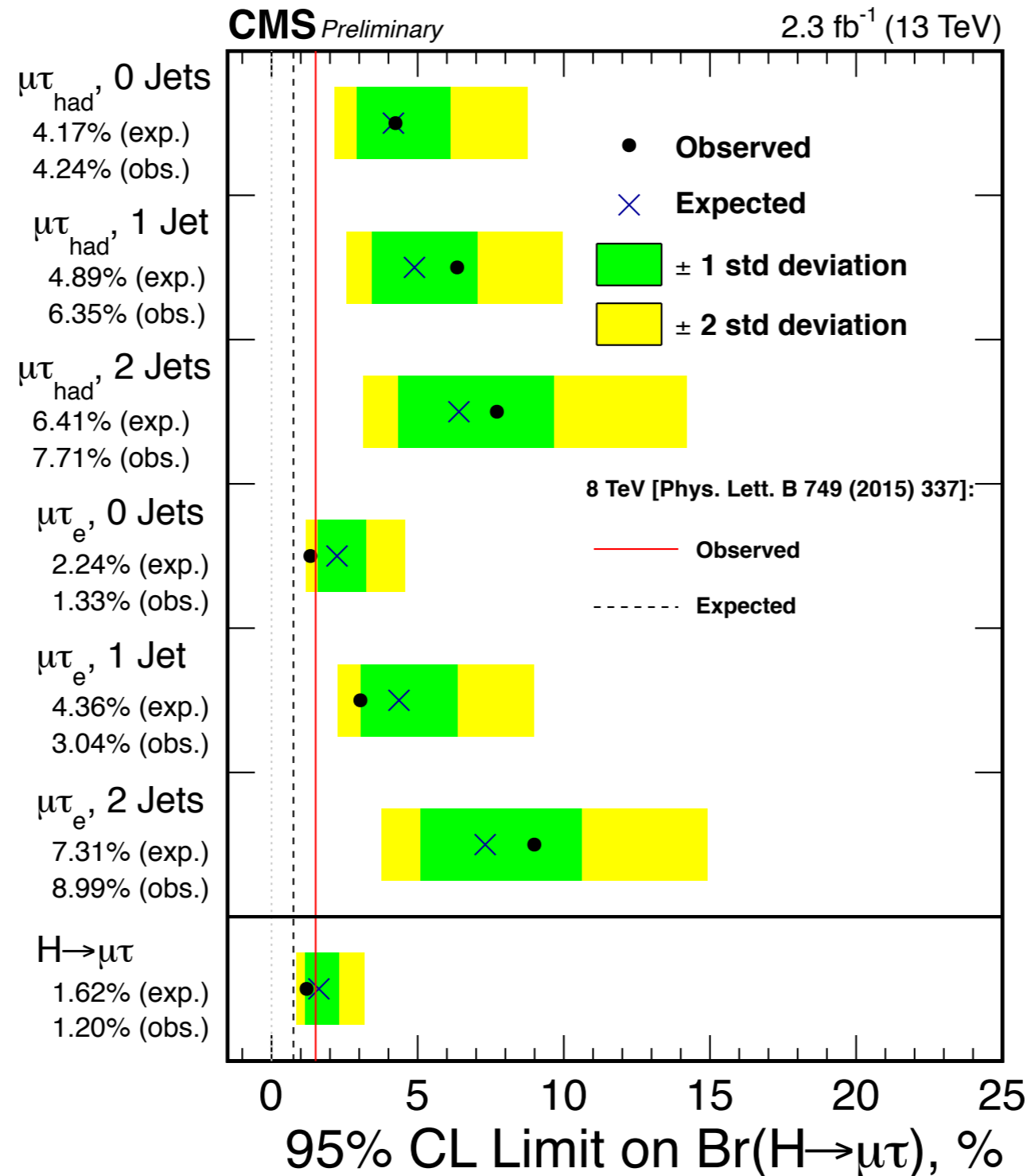


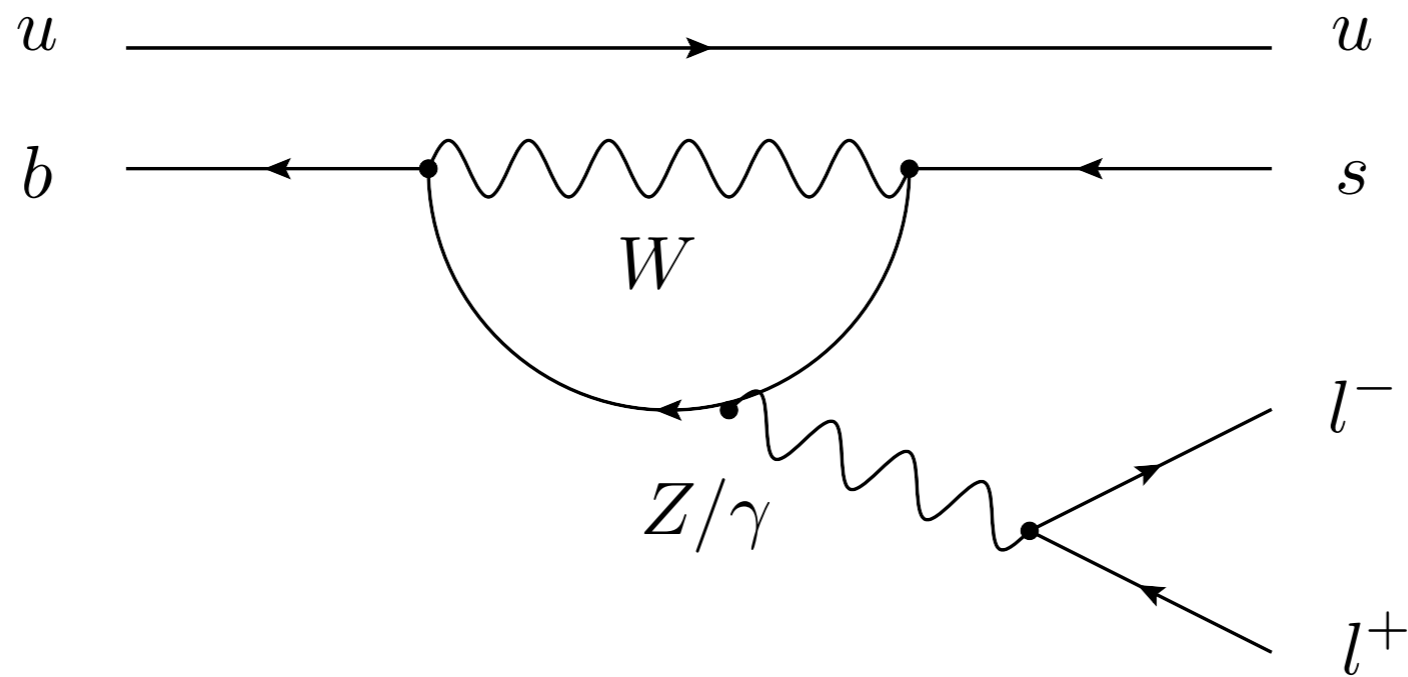
Figure 4: Observed and expected 95% CL upper limits on the $\mathcal{B}(H \rightarrow \mu\tau)$ for each individual category and combined. The solid red and dashed black vertical lines correspond, respectively, to the observed and expected 95% CL upper limits obtained at $\sqrt{s} = 8 \text{ TeV}$ [23].

Need more data

★ Hints from experimental data related to lepton flavor??

- Lepton flavor violating Higgs boson decay $h \rightarrow \mu \tau$
reported by CMS (2.4σ excess)
- Lepton universality in $B^+ \rightarrow K^+ l^+ l^-$ ($l = e, \mu$)

Lepton universality ~ gauge interactions ~



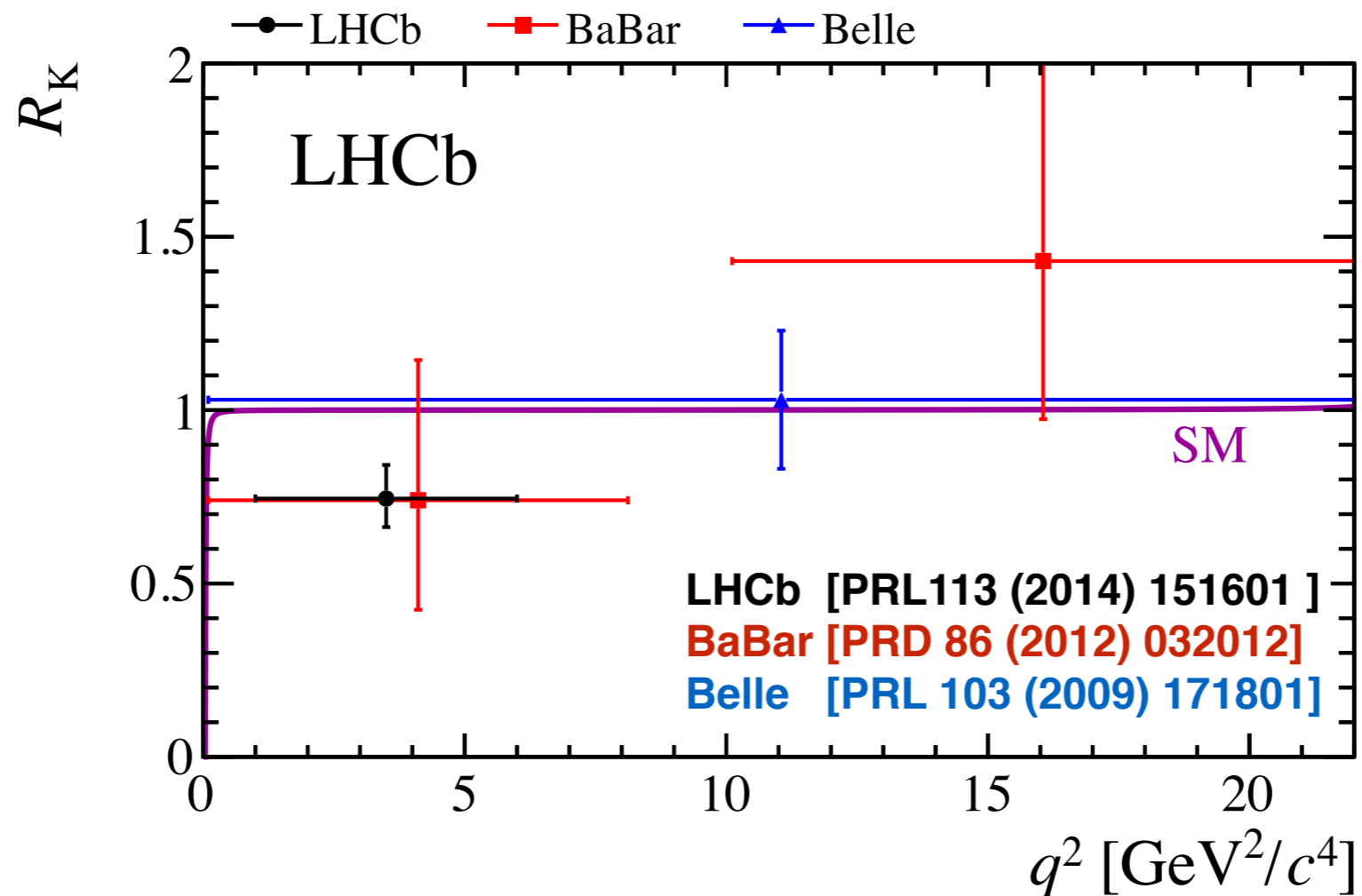
$$R_K = \frac{\text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\text{BR}(B^+ \rightarrow K^+ e^+ e^-)}$$

hadronic uncertainties cancel in the ratio

LHCb measures with 3fb^{-1}

$$R_K = \frac{BR(B^+ \rightarrow K^+ \mu^+ \mu^-)}{BR(B^+ \rightarrow K^+ e^+ e^-)} = 0.745 \begin{array}{l} +0.090 \\ -0.074 \end{array} \begin{array}{l} (stat) \\ (syst) \end{array} \pm 0.036(syst)$$

(SM: $R_K=1.00$, consistent at 2.6σ)

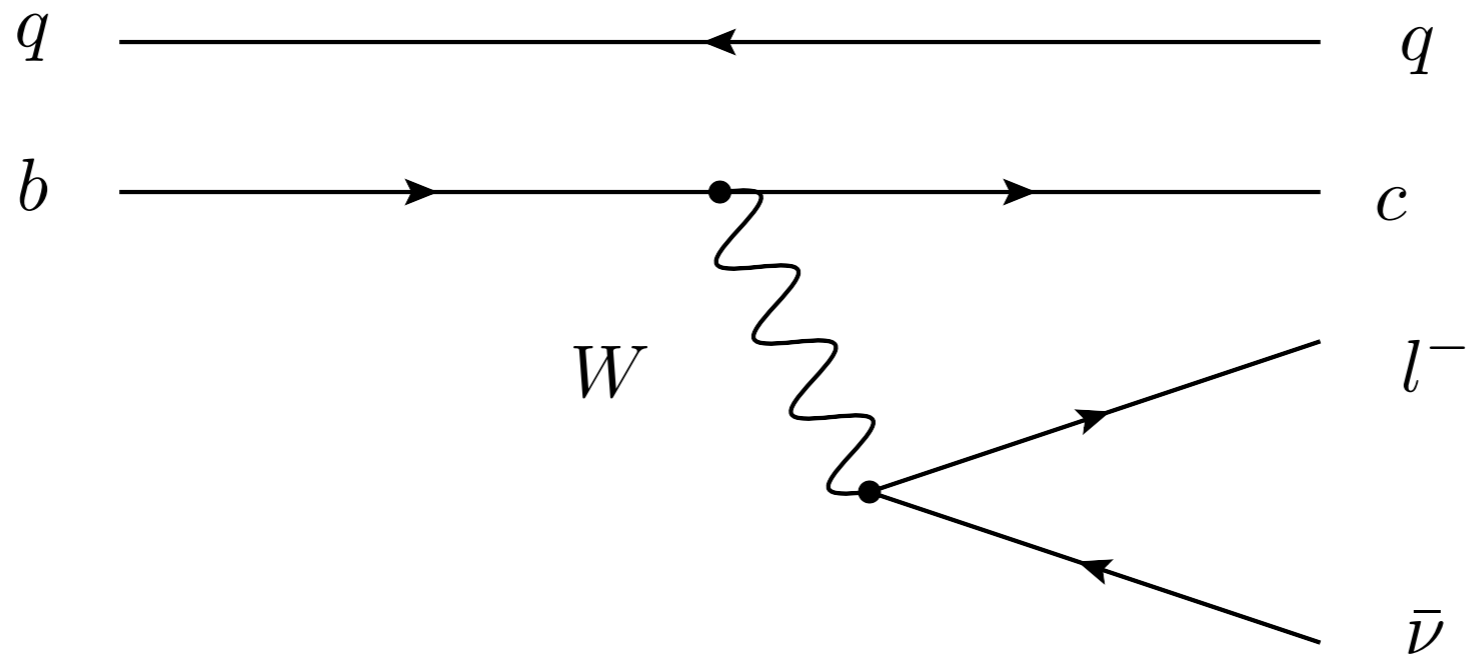


Talk by Johannes Albrecht @ Moriond 2016

★ Hints from experimental data related to lepton flavor??

- Lepton flavor violating Higgs boson decay $h \rightarrow \mu \tau$
reported by CMS (2.4σ excess)
- Lepton universality in $B^+ \rightarrow K^+ l^+ l^-$ ($l = e, \mu$)
- Lepton universality in $B^0 \rightarrow D^{(*)+} l^- \bar{\nu}$

Lepton universality



$$R_{D^{(*)}} = \frac{\text{BR}(B^0 \rightarrow D^{(*)+} \tau^- \bar{\nu})}{\text{BR}(B^0 \rightarrow D^{(*)+} l^- \bar{\nu})}$$

hadronic uncertainties cancel in the ratio

$$R_{D^*} = \frac{BR(B^0 \rightarrow D^{*+} \tau^- \bar{\nu})}{BR(B^0 \rightarrow D^{*+} \mu^- \bar{\nu})}$$

Belle

$$R(D) = 0.375 \pm 0.064 \pm 0.026$$

$$R(D^*) = 0.293 \pm 0.038 \pm 0.015$$

LHCb

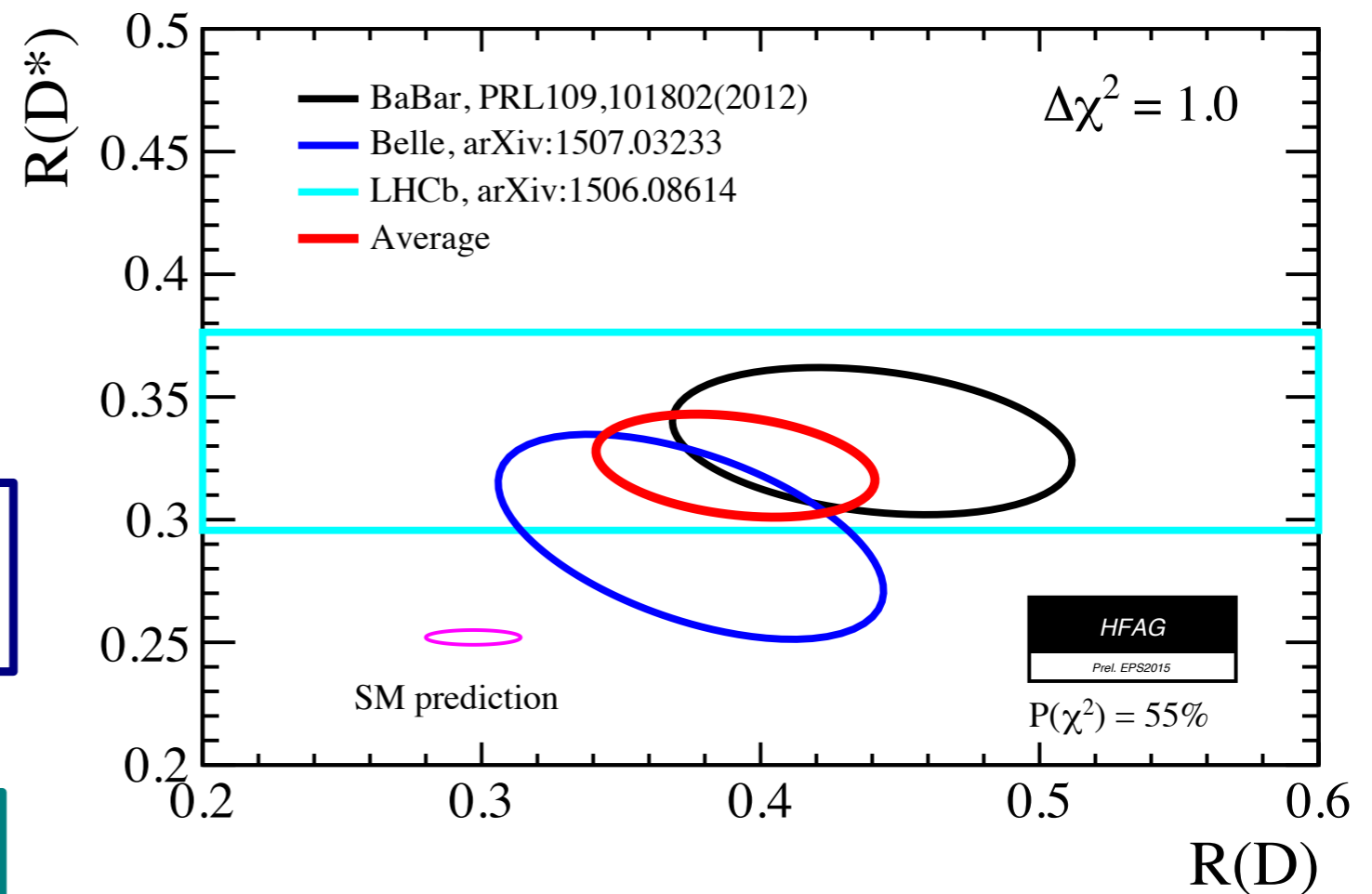
$$R(D^*) = 0.336 \pm 0.027 \pm 0.030$$

PRL 115(2015)111803

- Combination is 3.9σ from the SM expectation:

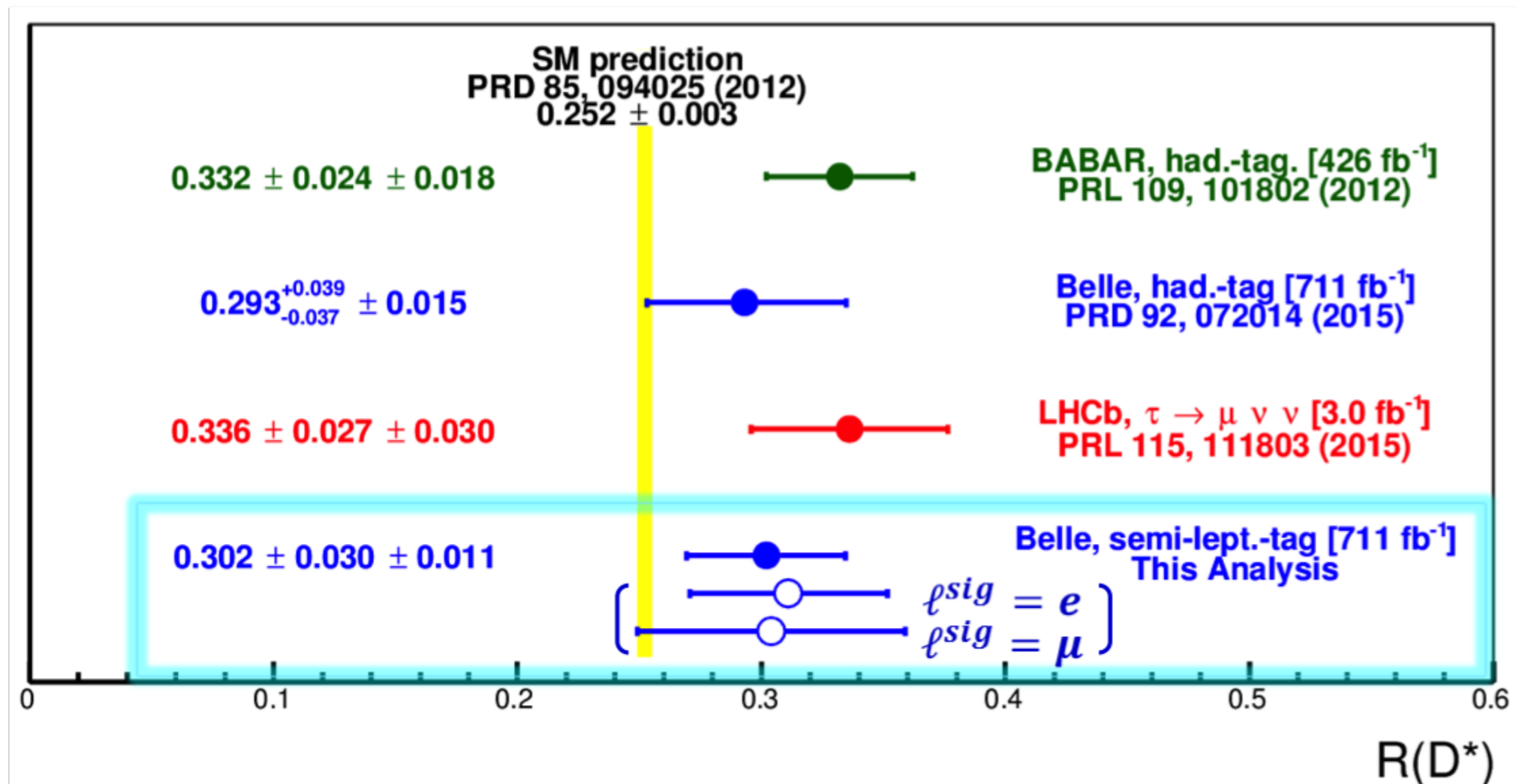
$$R(D^*) = 0.252 \pm 0.003$$

[Kamenik et al. Phys. Rev. D78 014003 (2008), S. Jajfer et al. Phys. Rev. D85 094025 (2012)]



Talk by Johannes Albrecht @ Moriond 2016

Preliminary



Central value close to Belle hadronic tag result.

Precision improvement over Belle hadronic tag and LHCb results.

★ Hints from experimental data related to lepton flavor??

- Lepton flavor violating Higgs boson decay $h \rightarrow \mu \tau$
reported by CMS (2.4σ excess)
- Lepton universality in $B^+ \rightarrow K^+ l^+ l^-$ ($l = e, \mu$)
- Lepton universality in $B^0 \rightarrow D^{(*)+} l^- \bar{\nu}$
- muon anomalous magnetic moment (muon $g-2$)

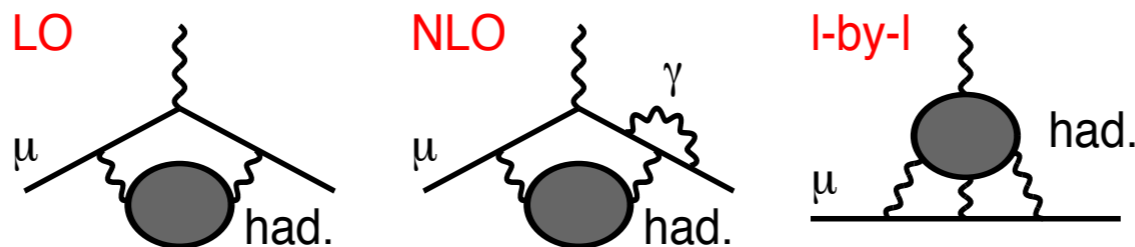
Status of muon $g-2$

.895 (0.008): 5-loop calculation (Aoyama et al '12)

QED contribution	11 658 471.808 (0.015)	Kinoshita & Nio, Aoyama et al
EW contribution	<u>15.4 (0.2)</u>	Czarnecki et al
Hadronic contributions	→ 15.4 (0.1): Higgs mass fixed (Grendiger et al '13)	
LO hadronic	694.9 (4.3)	HLMNT11
NLO hadronic	-9.8 (0.1)	HLMNT11
light-by-light	10.5 (2.6)	Prades, de Rafael & Vainshtein
Theory TOTAL	11 659 182.8 (4.9)	
Experiment	11 659 208.9 (6.3)	world avg
Exp – Theory	26.1 (8.0)	3.3 σ discrepancy

(in units of 10^{-10} . Numbers taken from HLMNT11, arXiv:1105.3149)

n.b.: hadronic contributions:



muon g-2 anomaly

Difference between the experimental value
and the SM prediction

$a_{\mu}^{\text{Exp}} [10^{-10}]$	$\delta a_{\mu} = a_{\mu}^{\text{Exp}} - a_{\mu}^{\text{SM}} [10^{-10}]$
11659208.9 ± 6.3 (~0.54 ppm)	$26.1 \pm 8.0 (3.3\sigma)$
	$31.6 \pm 7.9 (4.0\sigma)$
	$33.5 \pm 8.2 (4.1\sigma)$
	$28.3 \pm 8.7 (3.3\sigma)$
	$29.0 \pm 9.0 (3.2\sigma)$
	$28.7 \pm 8.0 (3.6\sigma)$

HLMNT11
THLMN10
BDDJ12
JS11
JN09
DHMZ12

3-4 σ deviation

possibly an evidence of new physics

If this anomaly is due to new physics,

The size of anomaly

$$\delta a_\mu = (26.1 \pm 8.0) \times 10^{-10}$$

is comparable to the electroweak contribution

$$a_\mu^{\text{EW}} = (15.4 \pm 0.1) \times 10^{-10}$$

we expect new particles with EW scale mass

- strong constraints from EW precision data
- good target at near future experiments

We may be able to discover the new physics

before new experiment or/and new (improved) calculation for muon g-2.

So, we should study it NOW!

★ Hints from experimental data related to lepton flavor??

- Lepton flavor violating Higgs boson decay $h \rightarrow \mu \tau$
reported by CMS (2.4σ excess)
- Lepton universality in $B^+ \rightarrow K^+ l^+ l^-$ ($l = e, \mu$)
- Lepton universality in $B^0 \rightarrow D^{(*)+} l^- \bar{\nu}$
- muon anomalous magnetic moment (muon $g-2$)
weak~TeV scale new physics
- (750 GeV resonance in diphoton mode?? and more)

these are not conclusive yet,

some of them may be hints for new physics and CLFV

Learn from the history.....

e.g. Kobayashi-Maskawa

Motivated from the observation of
CP violation in Kaon system

Three generations in the standard model

Good experimental data lead us to
the right answer!

★ Hints from experimental data related to lepton flavor??

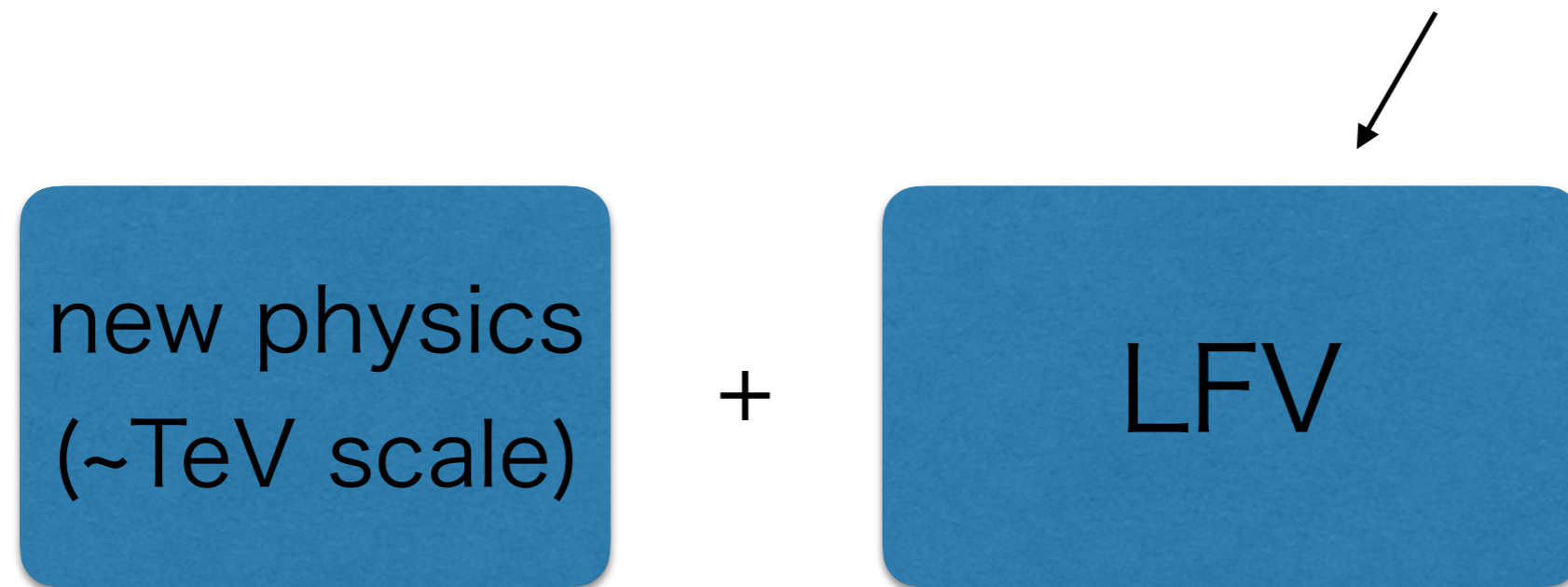
- Lepton flavor violating Higgs boson decay $h \rightarrow \mu \tau$
reported by CMS (2.4σ excess)
- Lepton universality in $B^+ \rightarrow K^+ l^+ l^-$ ($l = e, \mu$)
- Lepton universality in $B^0 \rightarrow D^{(*)+} l^- \bar{\nu}$
- muon anomalous magnetic moment (muon $g-2$)
weak~TeV scale new physics
- (750 GeV resonance in diphoton mode?? and more)

these are not conclusive yet,
some of them may be hints for new physics

Interplay between LHC and flavor physics will be important

If we have a new physics ...

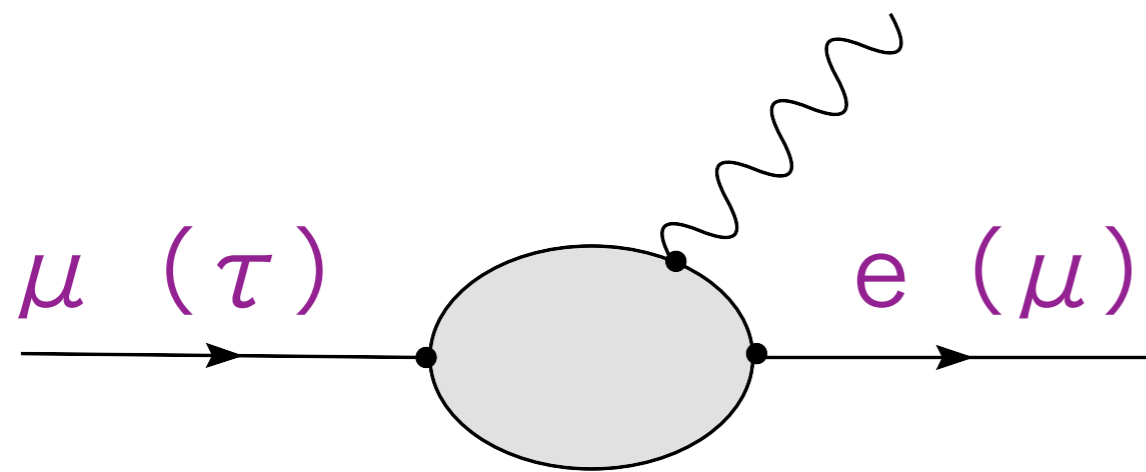
neutrino oscillation
(+extra source of LFV)



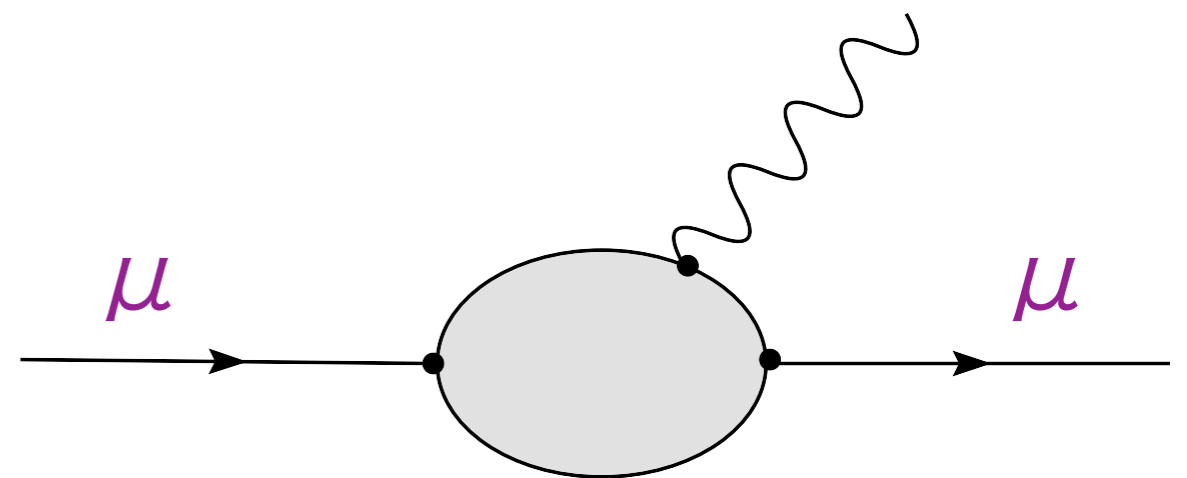
Is CLFV interesting?

Model independent study for CLFV, g-2 and EDM

~ effective operator analysis ~



$\mu \rightarrow e \gamma$ ($\tau \rightarrow \mu \gamma$)



muon g-2, muon EDM

sensitivity of new physics scale, flavor (CP) violation?

Model independent study for CLFV

Effective Lagrangian for $\mu \rightarrow e\gamma$

$$\mathcal{L}_{\text{LFV}} = y \frac{em_\mu}{\Lambda^2} \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + \text{h.c.} + \dots \quad \Lambda : \text{new physics scale}$$

$$\text{BR}(\mu \rightarrow e\gamma) = y^2 \frac{3(4\pi)^3 \alpha}{G_F^2 \Lambda^4} \quad \text{BR}(\mu \rightarrow e\gamma)_{\text{exp}} < 4.2 \times 10^{-13} \text{ (90\% C.L.)}$$

MEG: arXiv: 1605.05081

- **if** $y \simeq 1$, (The LFV operator is induced at tree level.)

$$\text{BR}(\mu \rightarrow e\gamma) = 3 \times 10^{-13} \times \left(\frac{1000 \text{ TeV}}{\Lambda} \right)^4 \left(\frac{y}{1.0} \right)^2$$

- **if** $y = \frac{g^2}{16\pi^2} \theta_{\mu e}$, (The LFV operator is generated at loop level)

$$\text{BR}(\mu \rightarrow e\gamma) = 3 \times 10^{-13} \times \left(\frac{5 \text{ TeV}}{\Lambda} \right)^4 \left(\frac{\theta_{\mu e}}{10^{-2}} \right)^2 \quad \text{e.g. SUSY}$$

Current search for $\mu \rightarrow e\gamma$ can be sensitive
to TeV scale new physics

Effective Lagrangian for $\tau \rightarrow \mu \gamma$

$$\mathcal{L}_{\text{LFV}} = y \frac{em_\tau}{\Lambda^2} \bar{\tau}_R \sigma^{\mu\nu} \mu_L F_{\mu\nu}$$

$$\text{BR}(\tau \rightarrow \mu \gamma) = 0.174 \times y^2 \frac{3(4\pi)^3 \alpha}{G_F^2 \Lambda^4}$$

$$\text{BR}(\tau \rightarrow \mu \gamma)_{\text{exp}} < 4.4 \times 10^{-8} \text{ (90\% C.L.)}$$

BaBar: 2010

- If $y \simeq 1$, (The LFV operator is induced at tree level.)

$$\text{BR}(\tau \rightarrow \mu \gamma) = 4 \times 10^{-8} \times \left(\frac{35 \text{ TeV}}{\Lambda} \right)^4 \left(\frac{y}{1.0} \right)^2$$

- If $y = \frac{g^2}{16\pi^2} \theta_{\tau\mu}$, (The LFV operator is generated at loop level)

$$\text{BR}(\tau \rightarrow \mu \gamma) = 3 \times 10^{-8} \times \left(\frac{1 \text{ TeV}}{\Lambda} \right)^4 \left(\frac{\theta_{\tau\mu}}{0.3} \right)^2$$

Current search for $\tau \rightarrow \mu \gamma$ can also be sensitive to TeV scale new physics

Effective Lagrangian for muon g-2

$$\mathcal{L}_{g-2} = \frac{em_\mu}{2} \frac{y_{g-2}}{\Lambda^2} \bar{\mu}_L \sigma_{\mu\nu} \mu_R F^{\mu\nu} + \text{h.c.}$$

$$\delta a_\mu = \frac{2y_{g-2}m_\mu^2}{\Lambda^2}$$

$$\delta a_\mu = (2.6 \pm 0.8) \times 10^{-9}$$

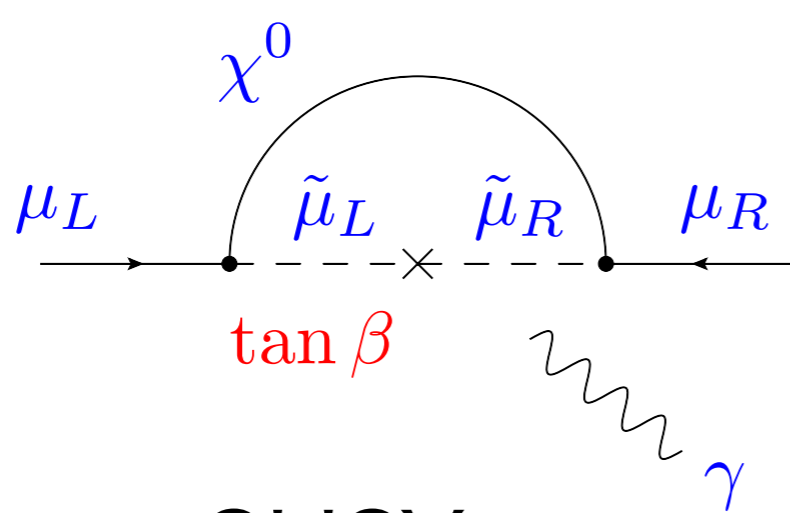
Hagiwara et al, 2011

• If $y_{g-2} = \frac{g^2}{16\pi^2}$, $\delta a_\mu = 3 \times 10^{-9} \times \left(\frac{150 \text{ GeV}}{\Lambda} \right)^2$

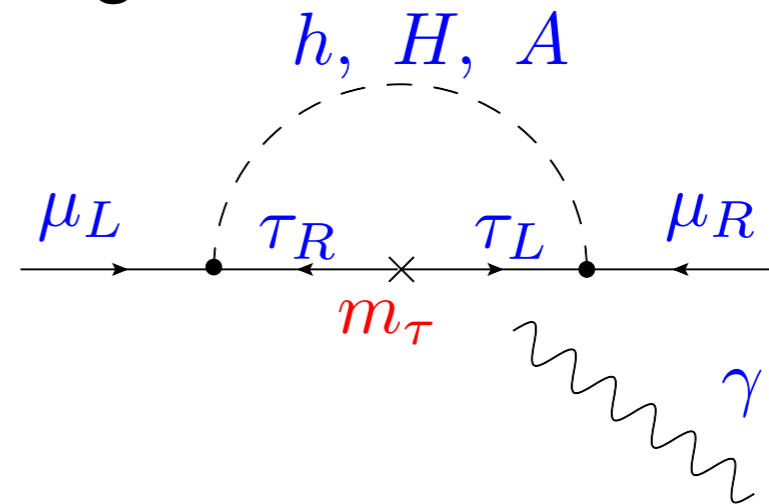
new physics scale needs to be close to weak scale

Note: muon chirality has to be flipped in muon g-2

If there is a mechanism to enhance the chirality flipping, the muon g-2 can get a large correction



SUSY



2HDM with μ - τ mixing

Effective Lagrangian for muon g-2

$$\mathcal{L}_{g-2} = \frac{em_\mu}{2} \frac{y_{g-2}}{\Lambda^2} \bar{\mu}_L \sigma_{\mu\nu} \mu_R F^{\mu\nu} + \text{h.c.}$$

$$\delta a_\mu = \frac{2y_{g-2}m_\mu^2}{\Lambda^2}$$

$$\delta a_\mu = (2.6 \pm 0.8) \times 10^{-9}$$

Hagiwara et al, 2011

$$\bullet \text{If } y_{g-2} = \frac{g^2}{16\pi^2}, \quad \delta a_\mu = 3 \times 10^{-9} \times \left(\frac{150 \text{ GeV}}{\Lambda} \right)^2$$

new physics scale needs to be close to weak scale

muon g-2 vs. $\mu \rightarrow e\gamma$

$$\bullet \text{If } y_{\mu \rightarrow e\gamma} = y_{g-2} \theta_{\mu e},$$

$$\begin{aligned} \text{BR}(\mu \rightarrow e\gamma) &= \frac{3(4\pi)^3 \alpha}{4G_F^2 m_\mu^4} (\delta a_\mu)^2 \theta_{\mu e}^2 \\ &\simeq 3 \times 10^{-13} \times \left(\frac{\delta a_\mu}{10^{-9}} \right)^2 \left(\frac{\theta_{\mu e}}{3 \times 10^{-5}} \right)^2 \end{aligned}$$

sensitive to very small flavor violation

Effective Lagrangian for muon g-2

$$\mathcal{L}_{g-2} = \frac{em_\mu}{2} \frac{y_{g-2}}{\Lambda^2} \bar{\mu}_L \sigma_{\mu\nu} \mu_R F^{\mu\nu} + \text{h.c.}$$

$$\delta a_\mu = \frac{2y_{g-2}m_\mu^2}{\Lambda^2}$$

$$\delta a_\mu = (2.6 \pm 0.8) \times 10^{-9}$$

Hagiwara et al, 2011

$$\bullet \text{ If } y_{g-2} = \frac{g^2}{16\pi^2}, \quad \delta a_\mu = 3 \times 10^{-9} \times \left(\frac{150 \text{ GeV}}{\Lambda} \right)^2$$

new physics scale needs to be close to weak scale

muon g-2 vs. $\tau \rightarrow \mu\gamma$

$$\bullet \text{ If } y_{\tau \rightarrow \mu\gamma} = y_{g-2} \theta_{\mu\tau},$$

$$\text{BR}(\tau \rightarrow \mu\gamma) = 4 \times 10^{-8} \times \left(\frac{\delta a_\mu}{10^{-9}} \right)^2 \left(\frac{\theta_{\mu\tau}}{2 \times 10^{-2}} \right)^2$$

If muon g-2 anomaly is due to new physics, CLFV search can be sensitive to very small lepton flavor violation

muon g-2 vs. muon EDM

$$\mathcal{L} = \frac{em_\mu}{2} \frac{\hat{y}}{\Lambda^2} \bar{\mu}_R \sigma_{\mu\nu} \mu_L F^{\mu\nu} + \text{h.c.}$$

$$\hat{y} = y_R + iy_I$$

$$\mathcal{L} = \frac{em_\mu}{2} \frac{y_R}{\Lambda^2} \bar{\mu} \sigma_{\mu\nu} \mu F^{\mu\nu} - \frac{iem_\mu}{2} \frac{y_I}{\Lambda^2} \bar{\mu} \sigma_{\mu\nu} \gamma_5 \mu F^{\mu\nu}$$

$$\delta a_\mu = y_R \frac{2m_\mu^2}{\Lambda^2} \quad \delta d_\mu = y_I \frac{em_\mu}{\Lambda^2}$$

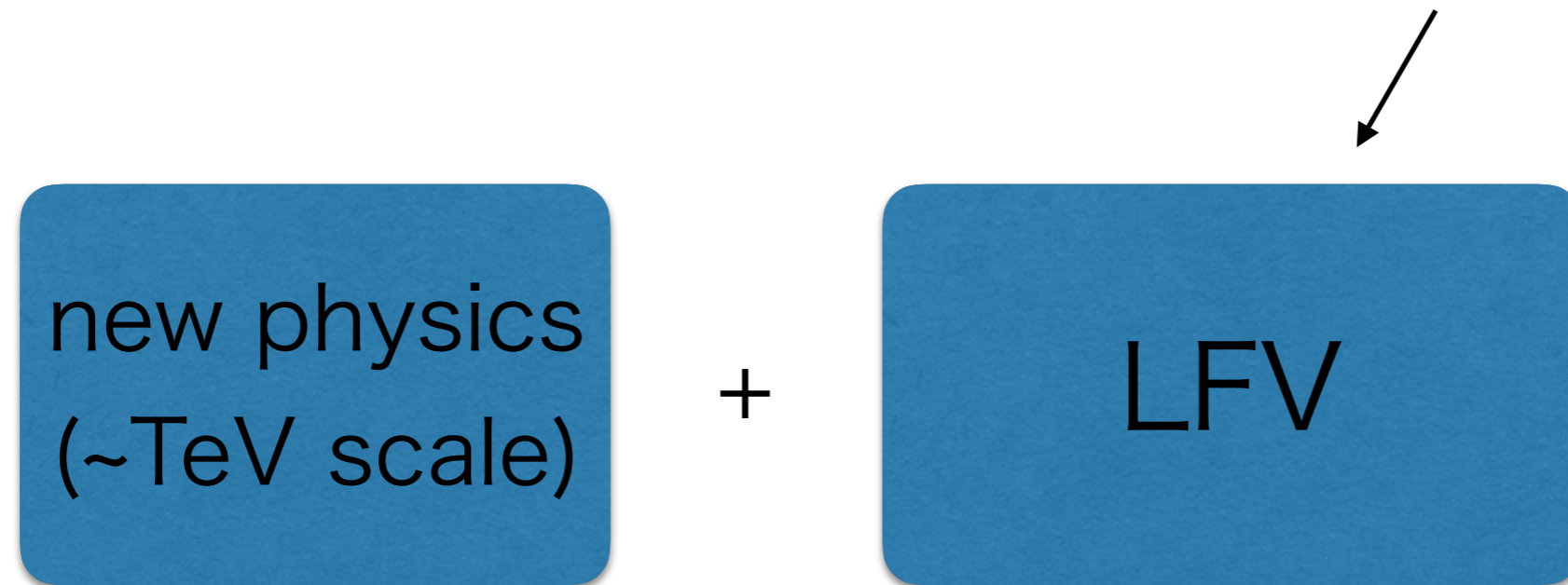
• If $y_I = y_R \theta_{\text{CP}}$,

$$d_\mu = \frac{e\theta_{\text{CP}}}{2m_\mu} \delta a_\mu = 3 \times 10^{-22} \left(\frac{\delta a_\mu}{3 \times 10^{-9}} \right) \left(\frac{\theta_{\text{CP}}}{1.0} \right)$$

The future measurement at the level of 10^{-22}
would be interesting

If we have a new physics ...

neutrino oscillation
(+extra source of LFV)



Is CLFV interesting?

Yes, the CLFV search will be sensitive to new physics around TeV.
If muon $g-2$ anomaly is due to new physics, the search for CLFV will put strong constraints on the new physics model.

incomplete list of Refs

★SUSY

CLFV (EDM)

GUT:

Gabbiani, Maniero, PLB209, 289 (1988); Hagelin, Kelley, Tanaka, NPB415, 293 (1994); Barbieri, Hall, PLB338, 212 (1994); Barbieri, Hall, Strumia, NPB445, 219 (1995) ...

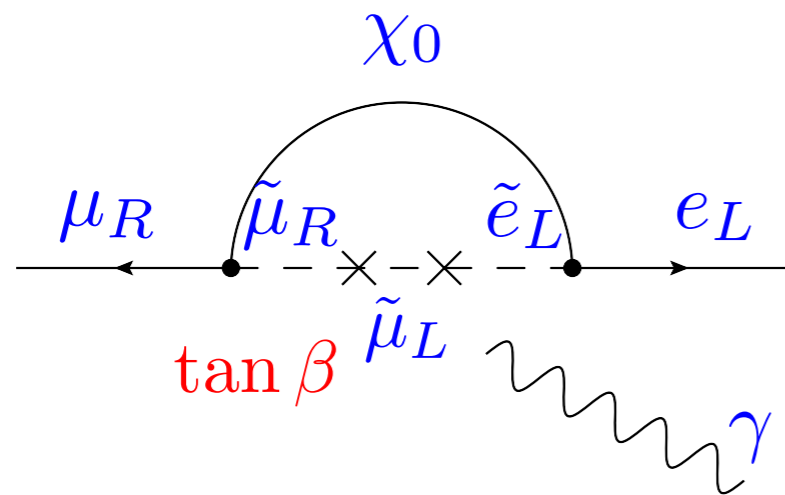
ν :

Borzumati, Masiero, PRL57, 961 (1986); Hisano, Moroi, Tobe, Yamaguchi, Yanagida, PLB357, 579 (1995) ...

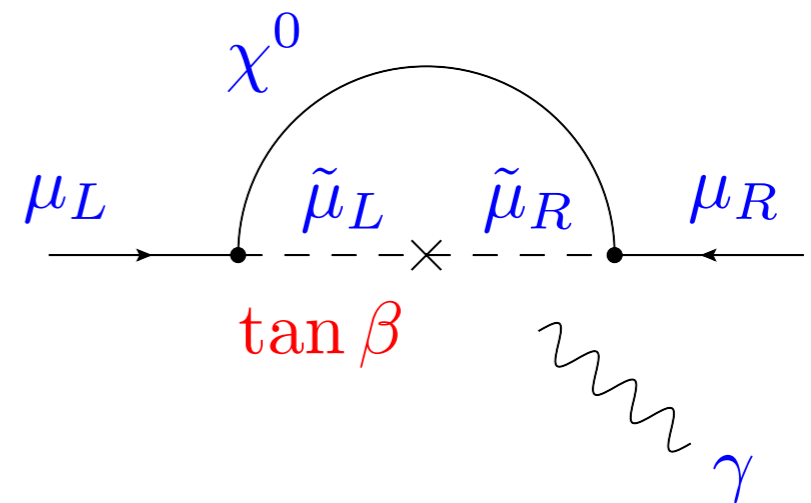
muon g-2

Faye 1980, Grifols, Mendez 1982, Ellis, Hagelin, Nanopoulos 1982, Barbieri, Maiani 1982, Kosower, Krauss, Sakai 1983, Yuan, Arnowitt, Chamseddine, Nath 1984, Romao, Barroso, Bento, Branco 1985, Moroi 1996, ...

CLFV



muon g-2



GUT, right-handed ν interactions can induce large LFV in slepton masses

$\tan \beta$ enhancement

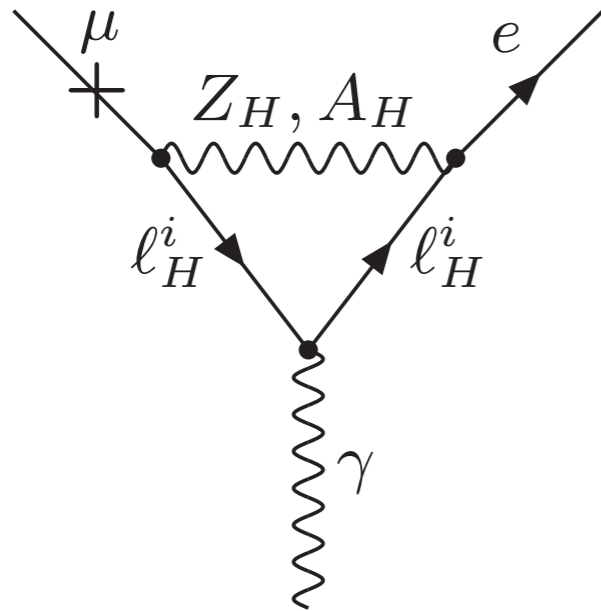
incomplete list of Refs

★ Little Higgs

CLFV, muon $g-2$

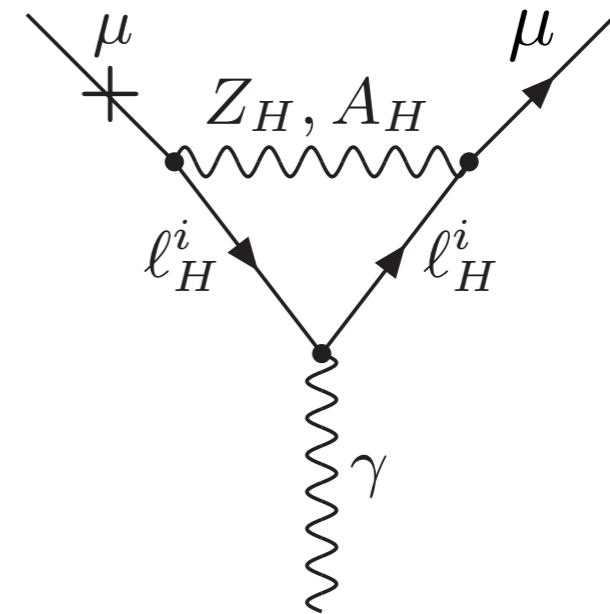
Chowdhury, Cornell, Deandrea, Gaur, Goyal PRD75, 055011 (2007);
Blanke, Buras, Duling, Poschenrieder, Trantino, JHEP0705, 013 (2007);
Agila, Illana, Jenkins JHEP0901, 080 (2009); Goto, Okada, Yamamoto
PRD83, 053011 (2011), ...

CLFV



LFV interactions of Little Higgs partners induce CLFV

$g-2$



no enhancement
typically small

ratio	LHT	MSSM (dipole)	MSSM (Higgs)
$\frac{Br(\mu^- \rightarrow e^- e^+ e^-)}{Br(\mu \rightarrow e \gamma)}$	0.4...2.5	$\sim 6 \cdot 10^{-3}$	$\sim 6 \cdot 10^{-3}$
$\frac{Br(\tau^- \rightarrow e^- e^+ e^-)}{Br(\tau \rightarrow e \gamma)}$	0.4...2.3	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$
$\frac{Br(\tau^- \rightarrow \mu^- \mu^+ \mu^-)}{Br(\tau \rightarrow \mu \gamma)}$	0.4...2.3	$\sim 2 \cdot 10^{-3}$	0.06...0.1
$\frac{Br(\tau^- \rightarrow e^- \mu^+ \mu^-)}{Br(\tau \rightarrow e \gamma)}$	0.3...1.6	$\sim 2 \cdot 10^{-3}$	0.02...0.04
$\frac{Br(\tau^- \rightarrow \mu^- e^+ e^-)}{Br(\tau \rightarrow \mu \gamma)}$	0.3...1.6	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$
$\frac{Br(\tau^- \rightarrow e^- e^+ e^-)}{Br(\tau^- \rightarrow e^- \mu^+ \mu^-)}$	1.3...1.7	~ 5	0.3...0.5
$\frac{Br(\tau^- \rightarrow \mu^- \mu^+ \mu^-)}{Br(\tau^- \rightarrow \mu^- e^+ e^-)}$	1.2...1.6	~ 0.2	5...10
$\frac{R(\mu Ti \rightarrow e Ti)}{Br(\mu \rightarrow e \gamma)}$	$10^{-2} \dots 10^2$	$\sim 5 \cdot 10^{-3}$	0.08...0.15

Table 3: Comparison of various ratios of branching ratios in the LHT model and in the MSSM without and with significant Higgs contributions.

CLFV searches are not only sensitive to these models,
but also able to distinguish them.

Some models motivated by the exp. data

(sorry for only incomplete list of Refs)

★ Lepton-specific (type X) two Higgs doublet model

A. Crivellin, J. Heeck, P. Stoffer, PRL 116, 081801 (2016)

“muon g-2” + $R(D^{(*)})$ (+“h → μ τ”)

→ light H, t → Hc, (τ → μ γ) ⋯

★ $L_\mu - L_\tau$ model

W. Altmannshoher, M. Carena, A. Crivellin, 1604.0822

“muon g-2” + “h → μ τ” + “ R_K ”

→ τ → 3 μ, h → μ μ, ⋯

★ Leptoquark model

S. Baek, K. Nishiwaki, PRD93, 015002 (2016)

“muon g-2” + “h → μ τ”

→ τ → μ γ, ⋯

★ General (type-III) two Higgs doublet model

Y. Omura, E. Senaha, K. Tobe, 1511.08880, JHEP 1505, 028 (2015)

“muon g-2” + “h → μ τ”

→ τ → μ γ, tau decay, ⋯

CLFV, muon $g-2$ and EDM in a general two Higgs doublet model

(both Higgs doublets couple to all fermions)

Refs: JHEP 1505, 028 (2015), arXiv: 1511.08880

Omura, Senaha, Tobe

+ work in progress

A basis where one Higgs doublet has vev (“Higgs basis”)

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v + \phi_1 + iG}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{\phi_2 + iA}{\sqrt{2}} \end{pmatrix},$$

G^+ , G : Nambu-Goldstone bosons

H^+ , A : charged and CP-odd Higgs bosons

In fermion mass eigenbasis (lepton sector)

$$\mathcal{L} = -\bar{L}_L^i H_1 y_e^i e_R^i - \bar{L}_L^i H_2 \rho_e^{ij} e_R^j$$

$$L = \begin{pmatrix} V_{\text{MNS}} \nu_L \\ e_L \end{pmatrix}$$

ρ_f ($f = d, u, e$) : flavor violating Yukawa couplings

scalar mixing

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{\beta\alpha} & \sin \theta_{\beta\alpha} \\ -\sin \theta_{\beta\alpha} & \cos \theta_{\beta\alpha} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}.$$

SM limit

$$c_{\beta\alpha} \rightarrow 0$$

mass eigenstates

$$s_{\beta\alpha} = \sin \theta_{\beta\alpha}, \quad c_{\beta\alpha} = \cos \theta_{\beta\alpha}$$

General 2HDM predicts

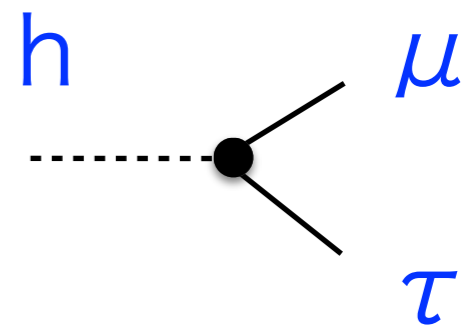
Flavor-changing phenomena mediated
by neutral Higgs bosons

Bjorken and Weinberg, PRL 38, 622 (1977)

This may be a problem if we do not observe any
flavor-changing phenomena beyond the SM.

But, now....

CMS result suggests



The diagram shows a central black vertex. A dashed line labeled 'h' enters from the left. Two solid lines exit to the right: the upper one is labeled 'μ' and the lower one is labeled 'τ'.

$$y_{hij} = \frac{m_f^i}{v} s_{\beta\alpha} \delta_{ij} + \frac{\rho_f^{ij}}{\sqrt{2}} c_{\beta\alpha},$$

$$\text{BR}(h \rightarrow \mu\tau) = (0.84^{+0.39}_{-0.37})\%$$

Sierra and Vicente, 1409.7690, Crivellin et al., 1501.00993,
Lima et al., 1501.06923, ...

$h \rightarrow \mu\tau$

Before the CMS excess, see Pilaftsis, PLB 285, 68 (1992);
Assamagan et al, PRD 67, 035001 (2003); Brignole and
Rossi, PLB 566, 217 (2003); Kanemura et al, PLB 599, 83
(2004); Arganda et al, PRD 71, 035011 (2005);,
Blankenburg, Ellis, Isidori, PLB712, 386 (2012),

CMS result

$$\text{BR}(h \rightarrow \mu\tau) = (0.84_{-0.37}^{+0.39})\%$$

2HDM prediction

$$\text{BR}(h \rightarrow \mu\tau) = \frac{c_{\beta\alpha}^2 (|\rho_e^{\mu\tau}|^2 + |\rho_e^{\tau\mu}|^2) m_h}{16\pi\Gamma_h},$$

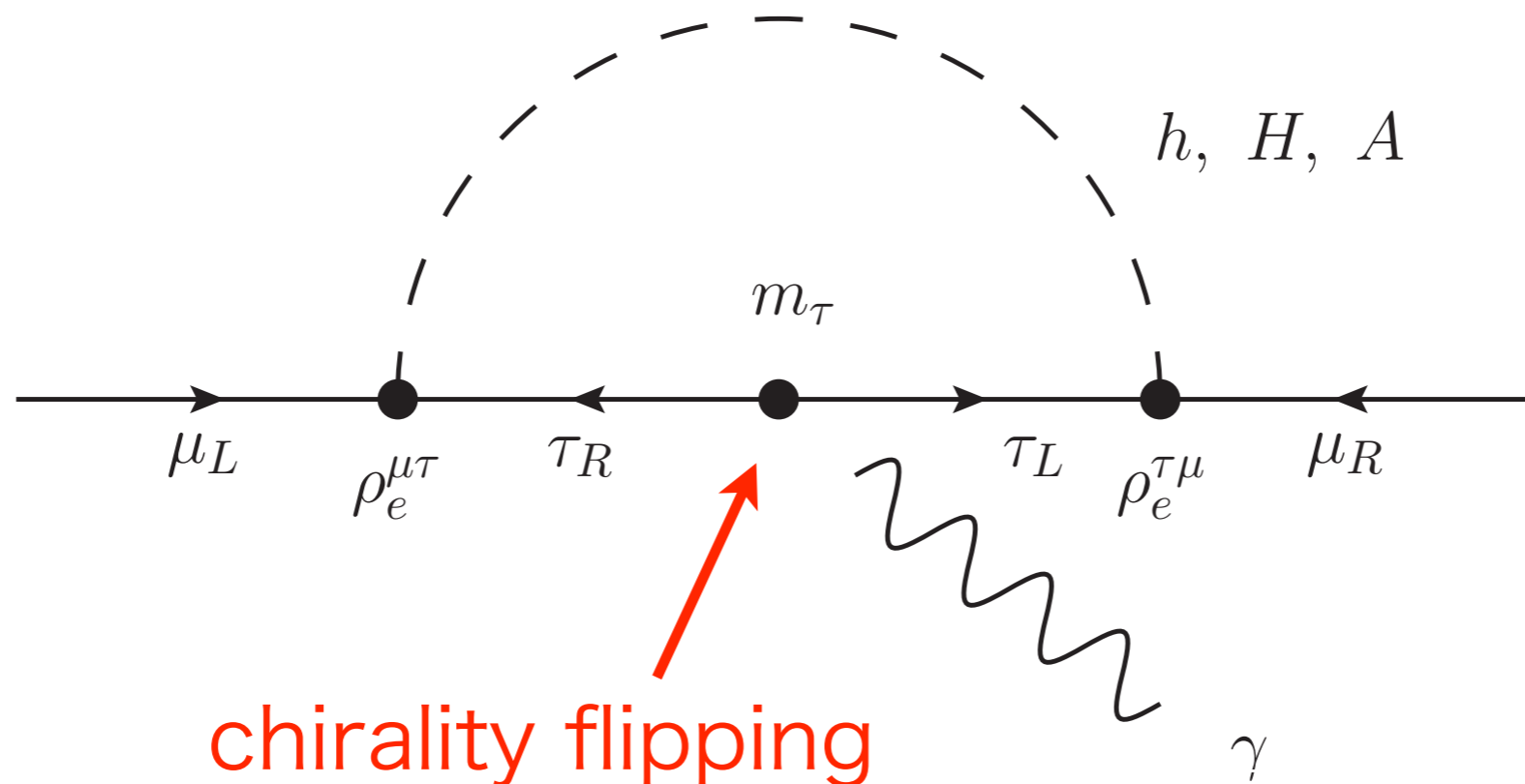
result

$$\begin{aligned} \bar{\rho}^{\mu\tau} &\equiv \sqrt{\frac{|\rho_e^{\mu\tau}|^2 + |\rho_e^{\tau\mu}|^2}{2}} \\ &\simeq 0.26 \left(\frac{|0.01|}{c_{\beta\alpha}} \right) \sqrt{\frac{\text{BR}(h \rightarrow \mu\tau)}{0.84 \times 10^{-2}}}. \end{aligned}$$

General 2HDM can explain it easily

muon g-2

induced by the μ - τ flavor violating coupling



$O\left(\frac{m_\tau}{m_\mu}\right)$ enhancement

The $\mu - \tau$ flavor-violating coupling
can enhance the muon g-2

neutral Higgs mass spectrum from tree level potential

For $c_{\beta\alpha} \ll 1$

$$m_{H^+}^2 = m_A^2 + \frac{\lambda_5 - \lambda_4}{2} v^2$$

$$m_H^2 \simeq m_A^2 + \lambda_5 v^2$$

$\lambda_{4,5}$; Higgs quartic couplings

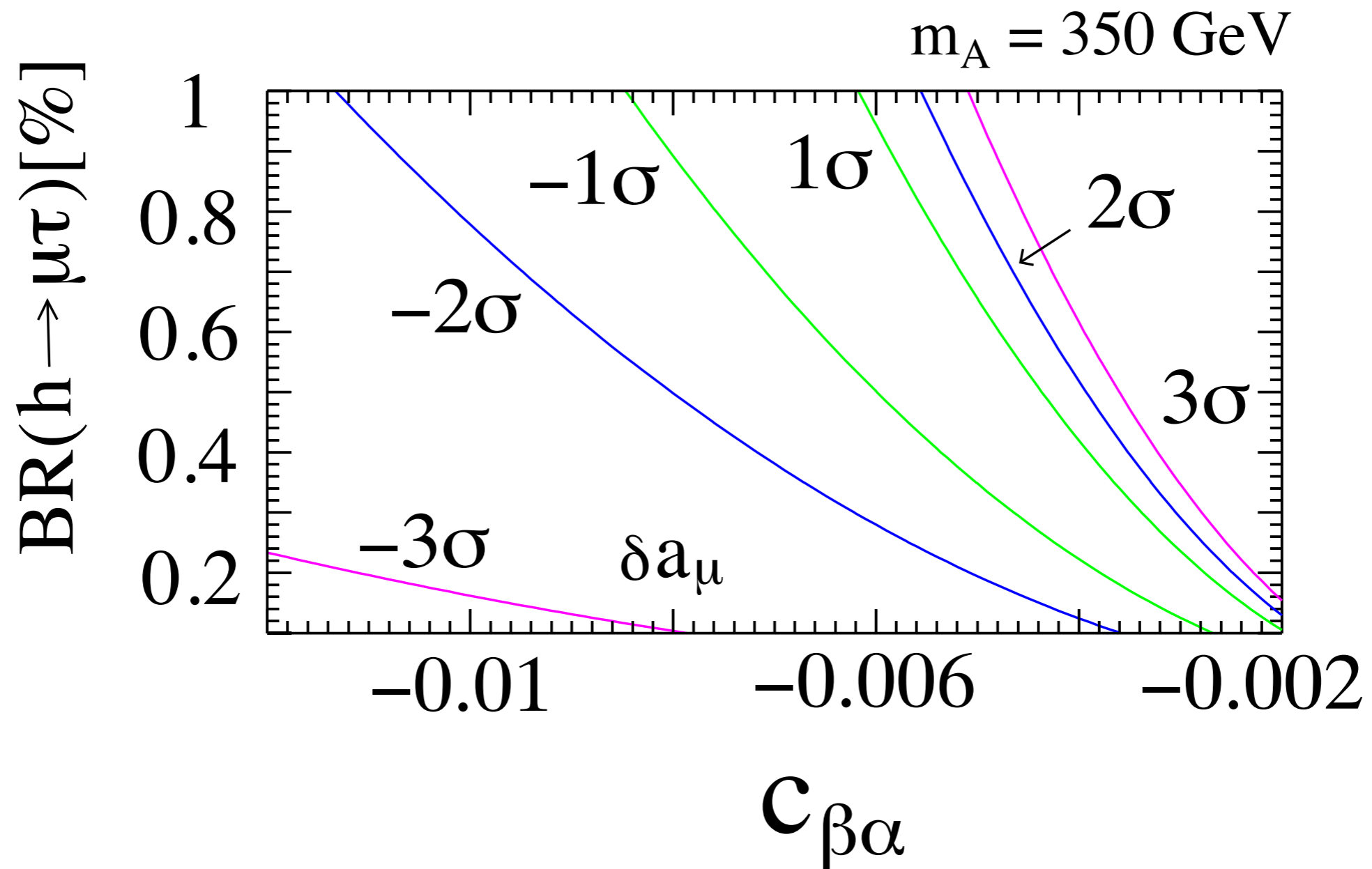
Note: correction to Peskin-Takeuchi T parameter

When $m_A \simeq m_{H^+}$,

the small $c_{\beta\alpha}$ suppresses the correction

Here, we mainly consider a case with $\lambda_4 = \lambda_5 = 0.5$

muon $g-2$

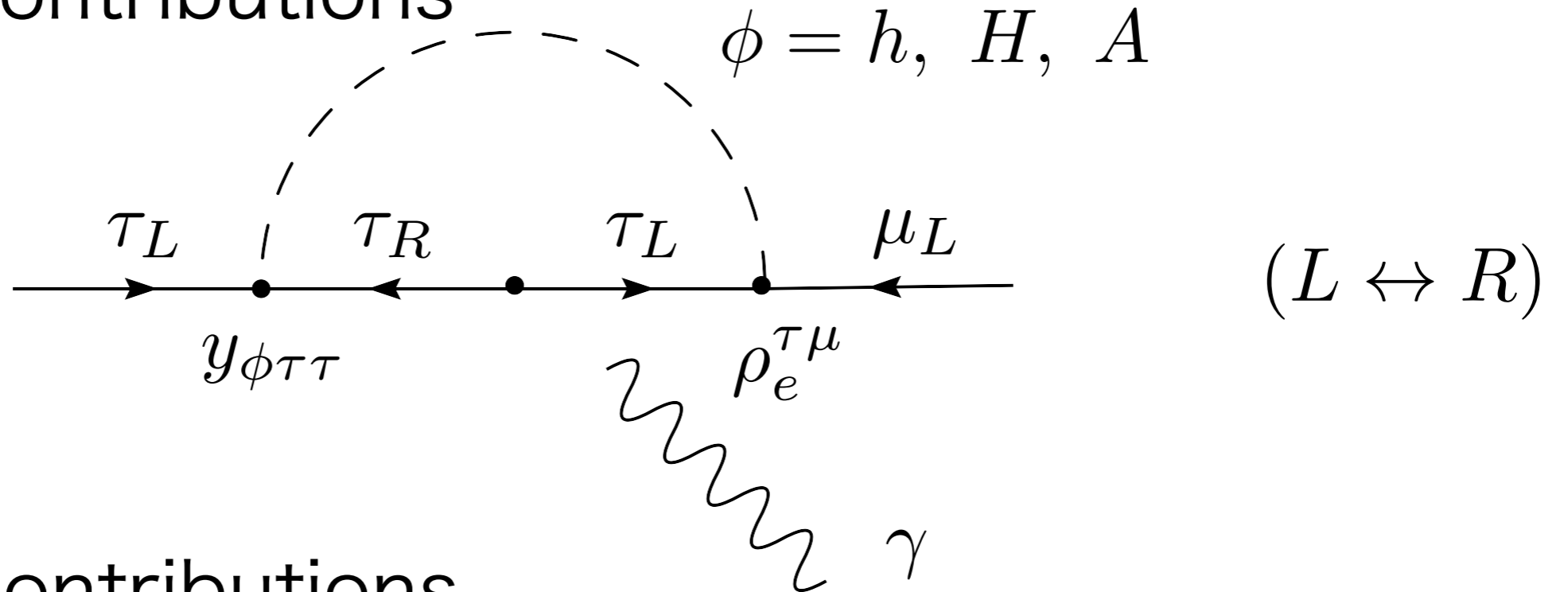


Both anomalies in the muon $g-2$ and $h \rightarrow \mu\tau$ can be accommodated in the general 2HDM

Predictions (constraints)?

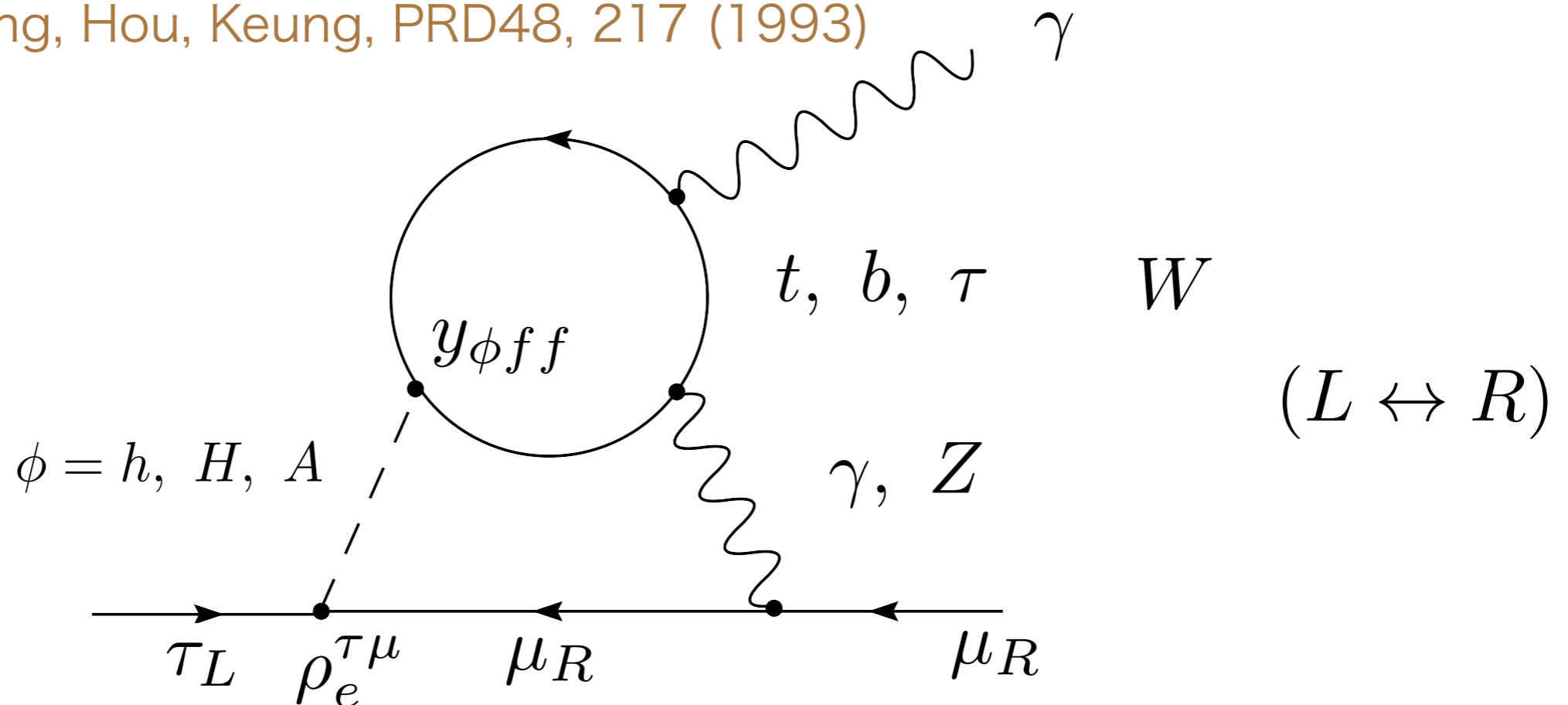
$$\tau \rightarrow \mu \gamma$$

1-loop contributions

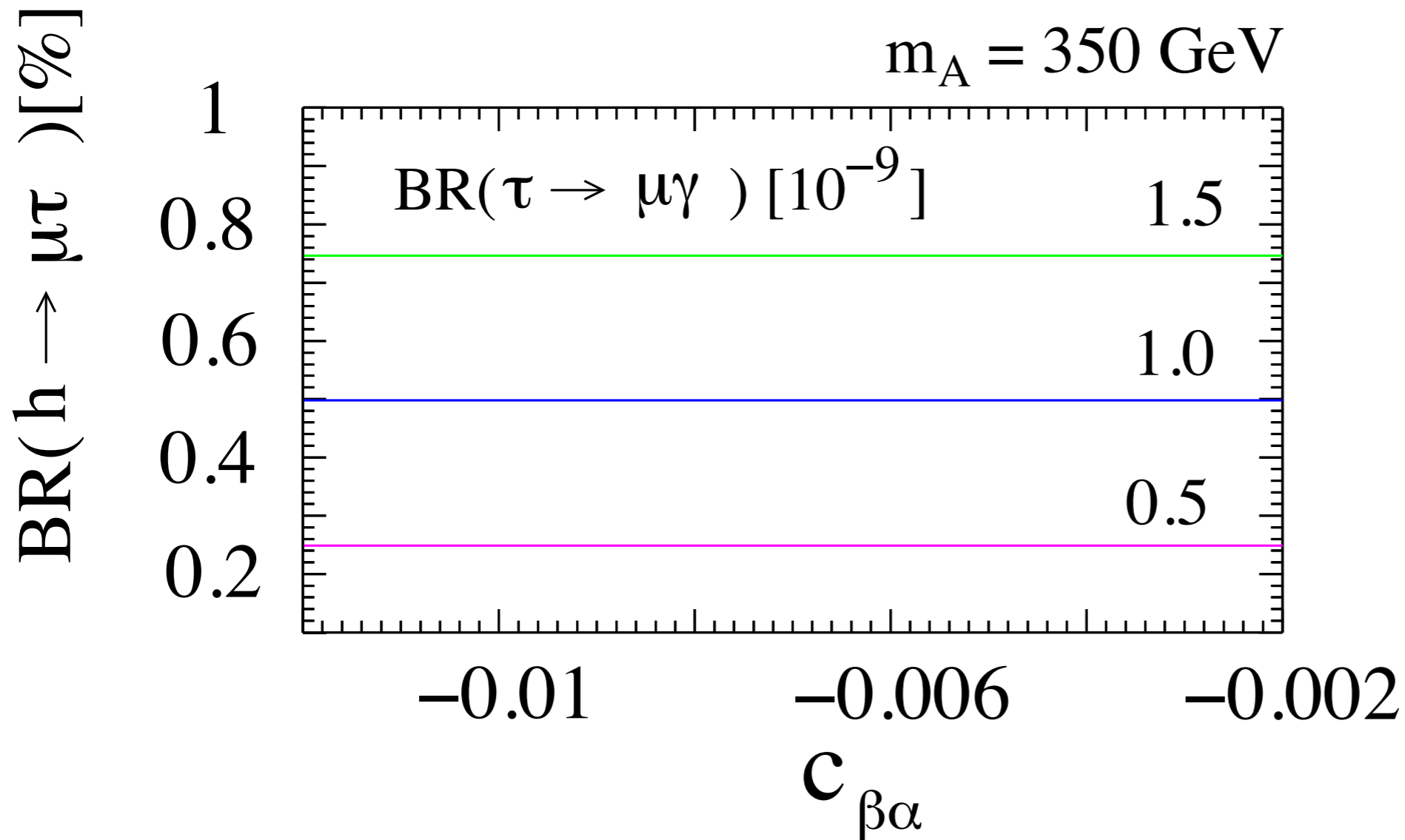
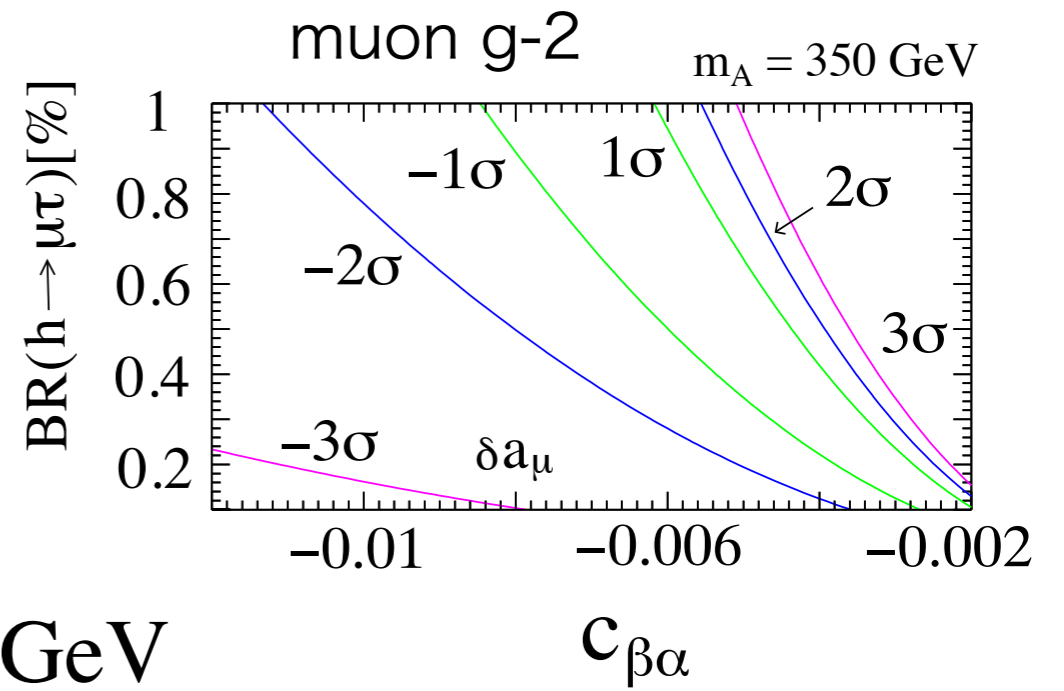


2-loop contributions

Chang, Hou, Keung, PRD48, 217 (1993)



For a case with $\rho_e^{\tau\tau} = \rho_u^{tt} = 0$



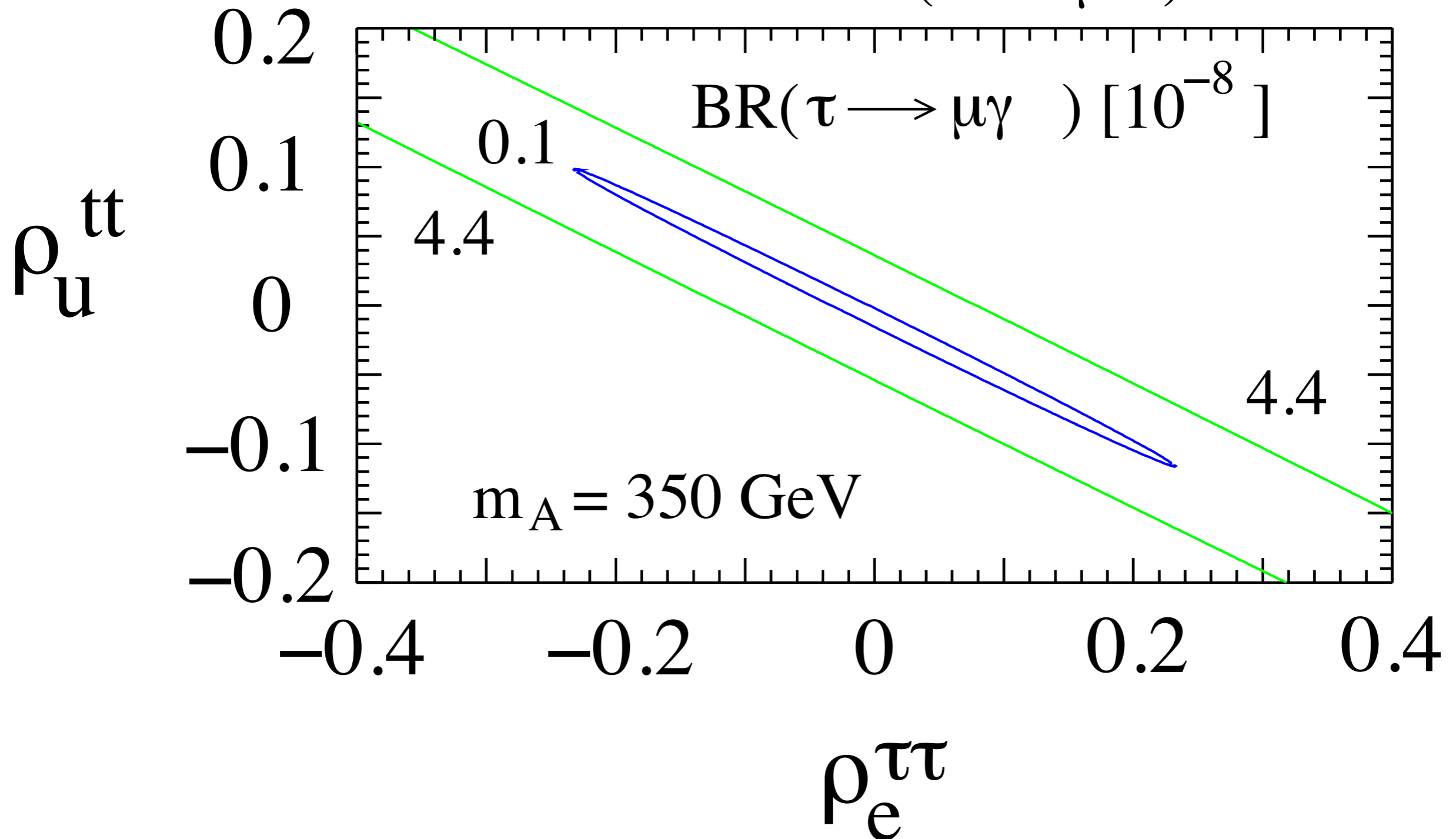
$$\text{BR}(\tau \rightarrow \mu\gamma)_{\text{exp.}} < 4.4 \times 10^{-8}$$

For a case with $\rho_e^{\tau\tau} \neq 0, \rho_u^{tt} \neq 0$

$$c_{\beta\alpha} = -0.007$$

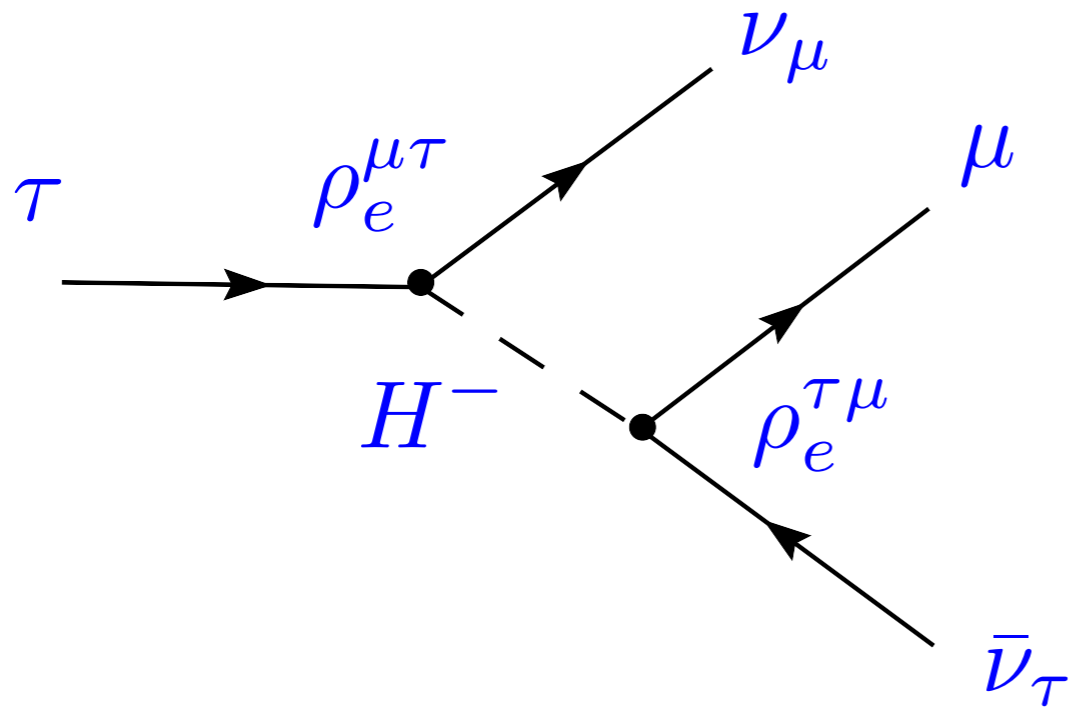
$$\delta a_\mu = 2.2 \times 10^{-9}$$

$$\text{BR}(h \rightarrow \mu\tau) = 0.84\%$$



The size of the rate can be within the reach of the future B-factory

Correction to $\tau \rightarrow \mu\nu\bar{\nu}$ decay



$$\Gamma(\tau \rightarrow \mu\nu\bar{\nu}) = \frac{m_\tau^5 G_F^2}{192\pi^3} (1 + \delta),$$

$$\delta = \frac{|\rho_e^{\mu\tau}|^2 |\rho_e^{\tau\mu}|^2}{32G_F^2 m_{H^+}^4}.$$

Michel parameters in τ decay

$$\frac{d\Gamma(\tau^- \rightarrow \mu^- \nu\bar{\nu})}{dx d\cos\theta_\mu} = \frac{m_\tau w^4}{2\pi^3} \sqrt{x^2 - x_0^2} G_{F_\mu}^2 [F_1(x) - F_2(x) \mathcal{P}_\tau \cos\theta_\mu]$$

$$F_1(x) = x(1-x) + \frac{2\rho}{9} (4x^2 - 3x - x_0^2) + \eta x_0 (1-x)$$

$$F_2(x) = -\frac{\xi \sqrt{x^2 - x_0^2}}{3} \left[1 - x + \frac{2\delta(4x - 4 + \sqrt{1 - x_0^2})}{3} \right]$$

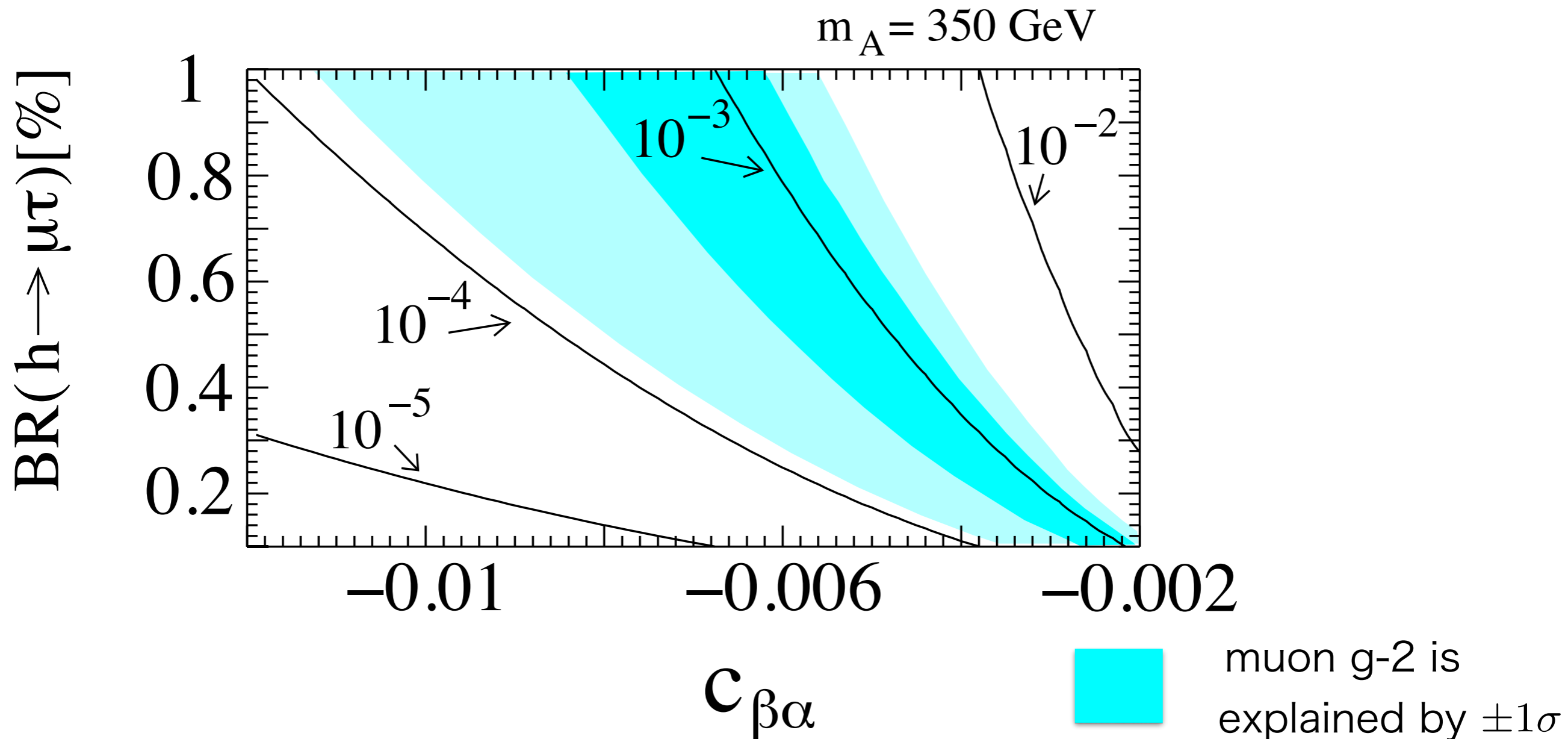
$$w = \frac{m_\tau^2 + m_\mu^2}{2m_\tau}$$

$$x = E_\mu/w, \quad x_0 = m_\mu/w$$

\mathcal{P}_τ : tau polarization

$$\underline{\xi \simeq -2\delta}$$

Michel parameter $|\xi|$



There are interesting correlation between muon $g-2$ and ξ
If future B-factory can measure the Michel parameter,
this will be very important for this scenario

Note: BABAR collaboration

lepton universality measurement

PRL 105, 051602 (2010)

$$\left(\frac{g_\mu}{g_e}\right)_\tau^2 = \frac{\mathcal{B}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) f(m_e^2/m_\tau^2)}{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) f(m_\mu^2/m_\tau^2)},$$

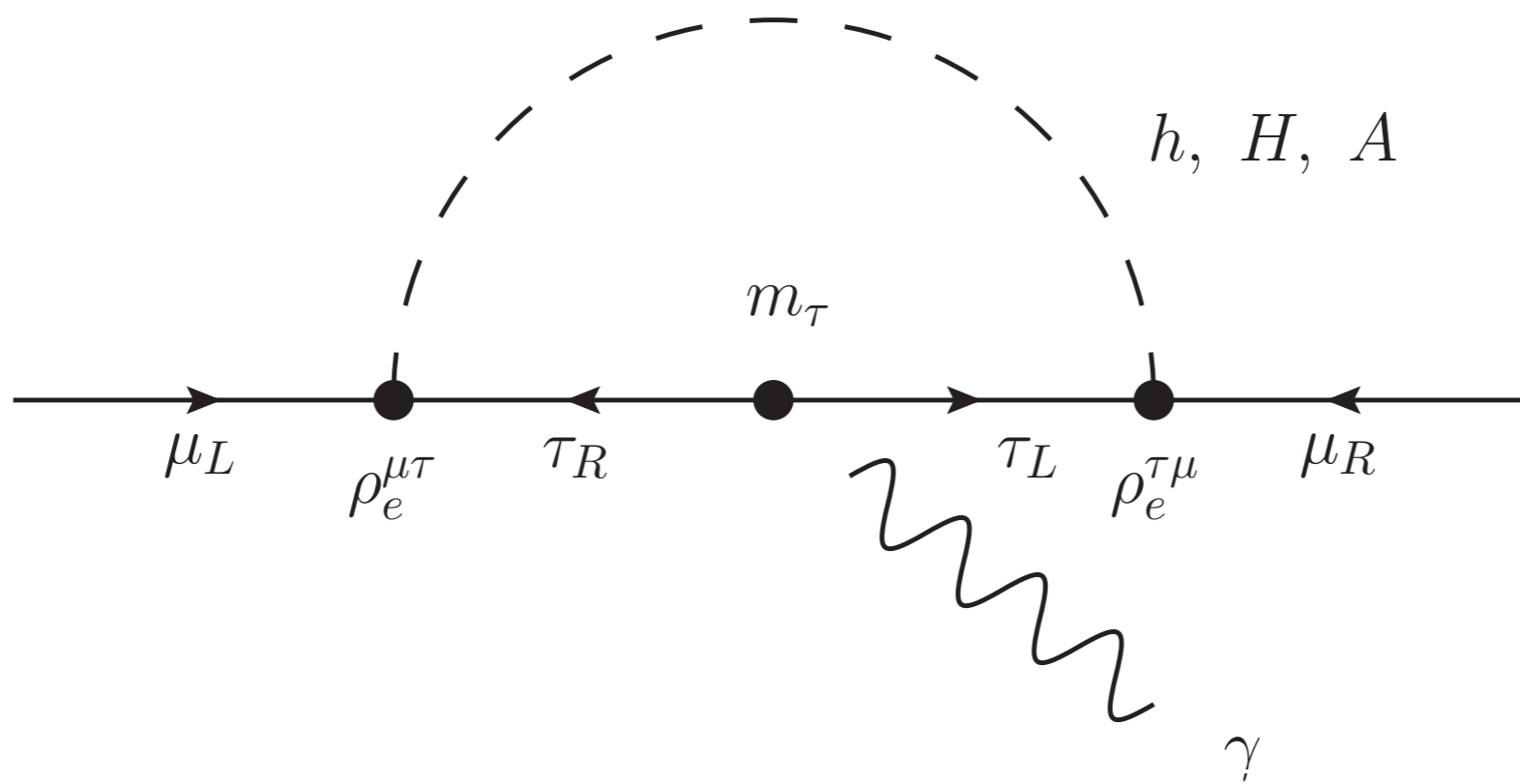
$$\left(\frac{g_\mu}{g_e}\right)_\tau = 1.0036 \pm 0.0020 \text{ (BaBar)}$$

$$= 1 + \frac{\delta}{2} = 1 - \frac{\xi}{4} \text{ (2HDM)}$$

The precise measurement will be important.

Belle and future B-factory result will be very interesting.

Muon electric dipole moment (muon EDM)



$$\rho_e^{\mu\tau} \rho_e^{\tau\mu} = |\rho_e^{\mu\tau} \rho_e^{\tau\mu}| e^{i\phi}$$

imaginary parts of the Yukawas induce the muon EDM

$$\mathcal{L} = \bar{\mu} \sigma^{\mu\nu} \left(\frac{e}{4m_\mu} \delta a_\mu - \frac{i}{2} \delta d_\mu \gamma_5 \right) \mu F_{\mu\nu}.$$

A relation between δa_μ and δd_μ

$$\frac{\delta d_\mu}{\delta a_\mu} = -\frac{e \tan \phi}{2m_\mu}.$$

“Prediction”

$$\delta d_\mu = -3 \times 10^{-22} e \cdot \text{cm} \times \left(\frac{\tan \phi}{1.0} \right) \left(\frac{\delta a_\mu}{3 \times 10^{-9}} \right)$$

Future (J-PARC)

$$d_\mu \sim 10^{-24} e \cdot \text{cm}$$

future J-PARC experiment may have a sensitivity

Process	typical value	observability
muon g-2	$\delta a_\mu = (2.6 \pm 0.8) \times 10^{-9}$	(input)
$\tau \rightarrow \mu\gamma$	$\text{BR} \leq 10^{-9}$	○
$\tau \rightarrow e\gamma$	small	×
$\tau \rightarrow \mu l^+ l^-$ ($l = e, \mu$)	depends on $\rho_e^{\mu\mu}$ and ρ_e^{ee}	(○)
$\tau^- \rightarrow e^- l^+ l^-, e^- \mu^+ e^-, \mu^- e^+ \mu^-$	small	×
$\tau \rightarrow \mu\eta$	depends on ρ_d^{ss}	(○)
$\tau \rightarrow \mu\nu\bar{\nu}$	$\delta \leq 10^{-3}$, lepton non-universality	△
$\tau \rightarrow e\nu\bar{\nu}$	small, lepton non-universality	△
$\mu \rightarrow e\gamma$	depends on $\rho_e^{\tau e(e\tau)}$ and $\rho_e^{\mu e(e\mu)}$	(○)
$\mu - e$ conversion	depends on $\rho_e^{\mu e(e\mu)}$ and $\rho_{d,u}^{ij}$	(○)
$\mu \rightarrow 3e$	$\text{BR} \leq 10^{-13}$	(○)
muon EDM	$ \delta d_\mu \leq 10^{-22} e \cdot \text{cm}$	(○)
electron g-2	small	×

Summary

- ★ Experimental and theoretical studies for flavor are important to understand a mystery of flavor in the SM.
- ★ Searches for CLFV (EDM) will be sensitive to (well-motivated) new physics models around TeV scale.
- ★ In the LHC era, the interplay between LHC physics and flavor physics will be important since it provides interesting ideas for new physics sometimes.
- ★ General 2HDM with μ - τ flavor violation can explain both CMS excess in $h \rightarrow \mu \tau$ and muon $g-2$ anomaly. The rate of $\tau \rightarrow \mu \gamma$ can be within the reach of the future B factory. The precision measurement of τ decay will also provide a crucial test of this scenario. Furthermore, unknown flavor structure in this model will provide a rich flavor phenomenology.

Backup

Table 5: The observed and expected upper limits and the best-fit branching fractions for different n -jet categories for the $H \rightarrow \mu\tau$ process.

Expected limits				
	0-jet (%)	1-jet (%)	2-jets (%)	Combined (%)
$\mu\tau_h$	<4.17	<4.89	<6.41	<2.98
$\mu\tau_e$	<2.24	<4.36	<7.31	<1.96
$\mu\tau$	<1.62 %			
Observed limits				
	0-jet (%)	1-jet (%)	2-jets (%)	Combined (%)
$\mu\tau_h$	<4.24	<6.35	<7.71	<3.81
$\mu\tau_e$	<1.33	<3.04	<8.99	<1.15
$\mu\tau$	<1.20 %			
Best-fit branching fractions				
	0-jet (%)	1-jet (%)	2-jets (%)	Combined (%)
$\mu\tau_h$	$0.12^{+2.02}_{-1.91}$	$1.70^{+2.41}_{-2.52}$	$1.54^{+3.12}_{-2.71}$	$1.12^{+1.45}_{-1.40}$
$\mu\tau_e$	$-2.11^{+1.30}_{-1.89}$	$-2.18^{+1.99}_{-2.05}$	$2.04^{+2.96}_{-3.31}$	$-1.81^{+1.07}_{-1.32}$
$\mu\tau$	$-0.76^{+0.81}_{-0.84}\%$			

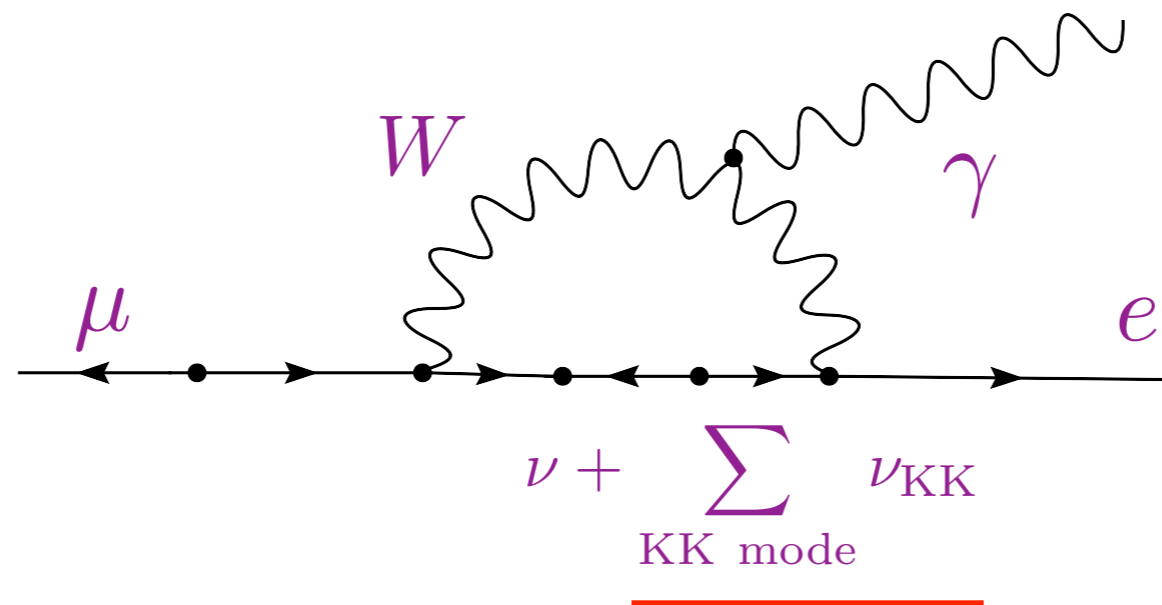
★ Extra dimension

incomplete list of Refs

CLFV, muon $g-2$

Faraggi, Pospelov, PLB458, 237 (1999); Kitano, PLB481, 39 (2000); Ioannian, Pilaftsis, PRD62, 066001 (2001), Cheng, Li, PLB502, 152 (2001) De Gouvea, Giudice, Strumia, Tobe, NPB623, 395 (2002),...

e.g. right-handed ν in extra dimension



The event rate can be sizable because of many contributions from the KK mode

incomplete list of Refs

★SUSY

CLFV (EDM)

GUT:

Gabbiani, Maniero, PLB209, 289 (1988); Hagelin, Kelley, Tanaka, NPB415, 293 (1994); Barbieri, Hall, PLB338, 212 (1994); Barbieri, Hall, Strumia, NPB445, 219 (1995) ...

ν :

Borzumati, Masiero, PRL57, 961 (1986); Hisano, Moroi, Tobe, Yamaguchi, Yanagida, PLB357, 579 (1995) ...

muon $g-2$

Faye 1980, Grifols, Mendez 1982, Ellis, Hagelin, Nanopoulos 1982, Barbieri, Maiani 1982, Kosower, Krauss, Sakai 1983, Yuan, Arnowitt, Chamseddine, Nath 1984, Romao, Barroso, Bento, Branco 1985, Moroi 1996, ...

★Little Higgs

CLFV, muon $g-2$

Chowdhury, Cornell, Deandrea, Gaur, Goyal PRD75, 055011 (2007); Blanke, Buras, Duling, Poschenrieder, Trantino, JHEP0705, 013 (2007); Agila, Illana, Jenkins JHEP0901, 080 (2009); Goto, Okada, Yamamoto PRD83, 053011 (2011), ...

(muon $g-2$ is typically small)

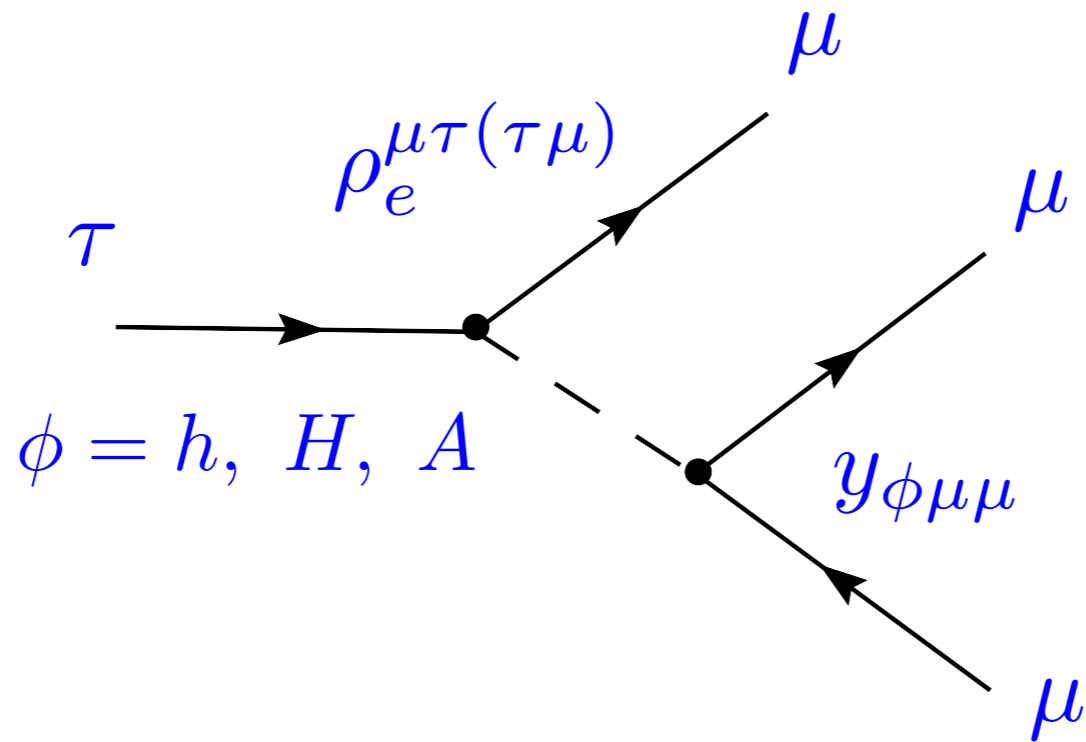
★Extra dimension

CLFV, muon $g-2$

Faraggi, Pospelov, PLB458, 237 (1999); Kitano, PLB481, 39 (2000); Ioannian, Pilaftsis, PRD62, 066001 (2001), Cheng, Li, PLB502, 152 (2001) De Gouvea, Giudice, Strumia, Tobe, NPB623, 395 (2002), ...

Other constraints (predictions) in 2HDM
with μ - τ flavor violation

$$\tau \rightarrow 3\mu, (\mu e^+ e^-)$$

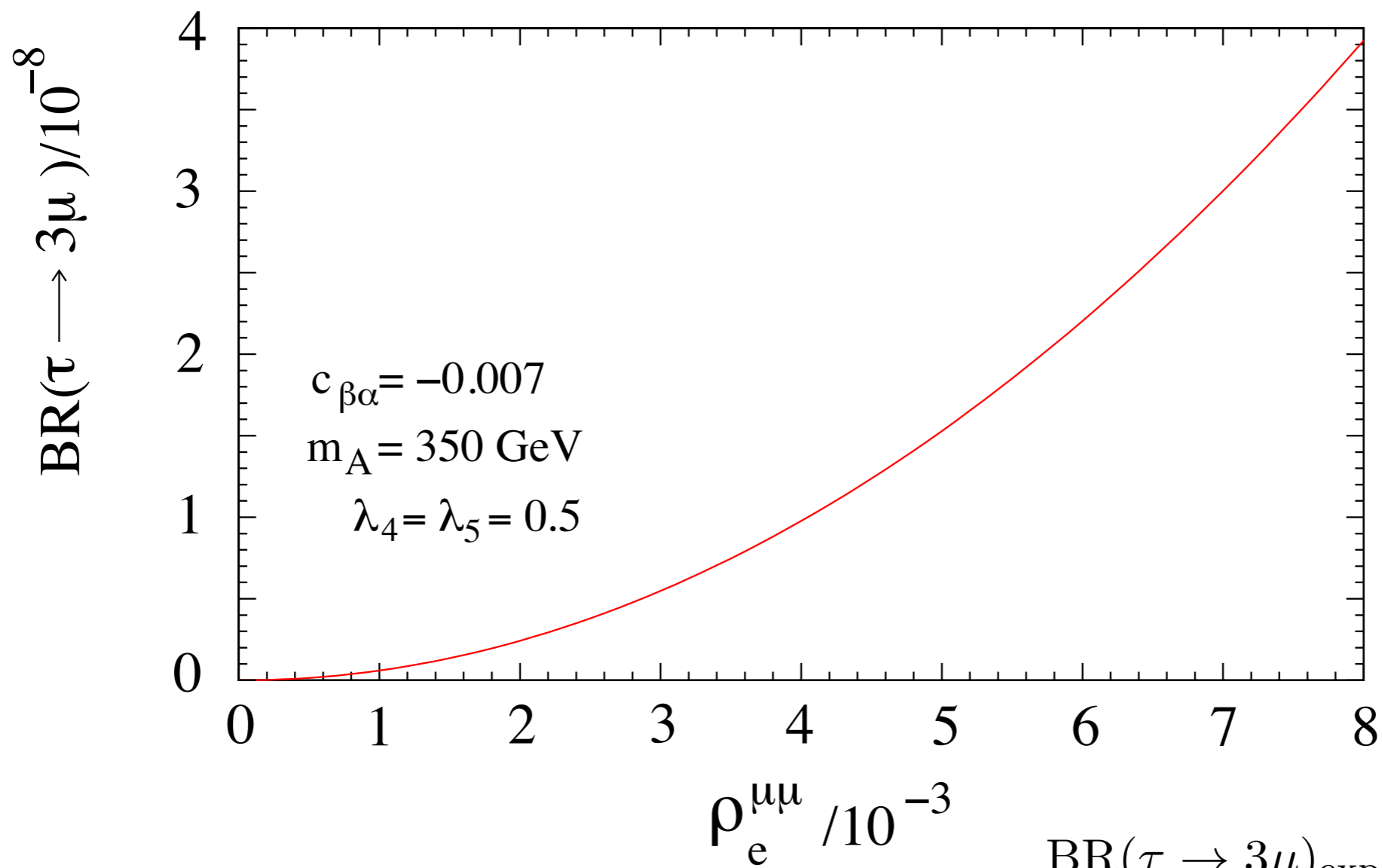
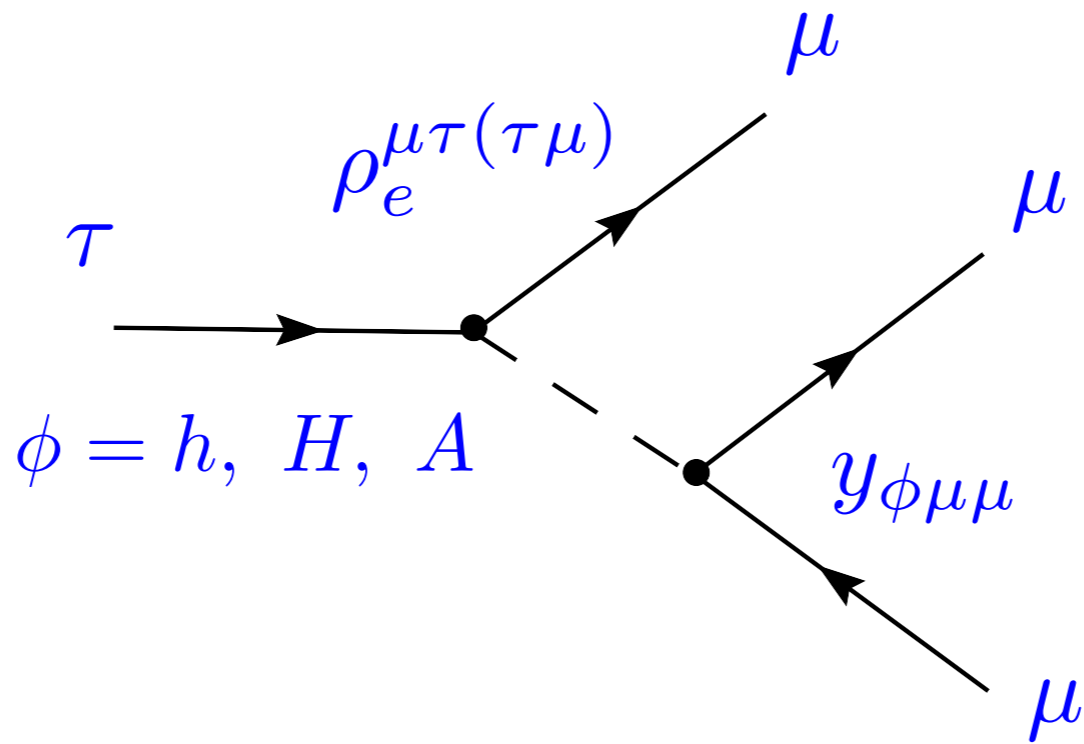


Even if other ρ_f (other than $\rho_e^{\mu\tau(\tau\mu)}$) are negligible,
 non-zero rate of $\tau \rightarrow 3\mu$ is predicted

$$\text{BR}(\tau \rightarrow 3\mu)_{\text{exp.}} < 2.1 \times 10^{-8}$$

but it is very small $O(10^{-13} - 10^{-12})$
 (since muon Yukawa is very small)

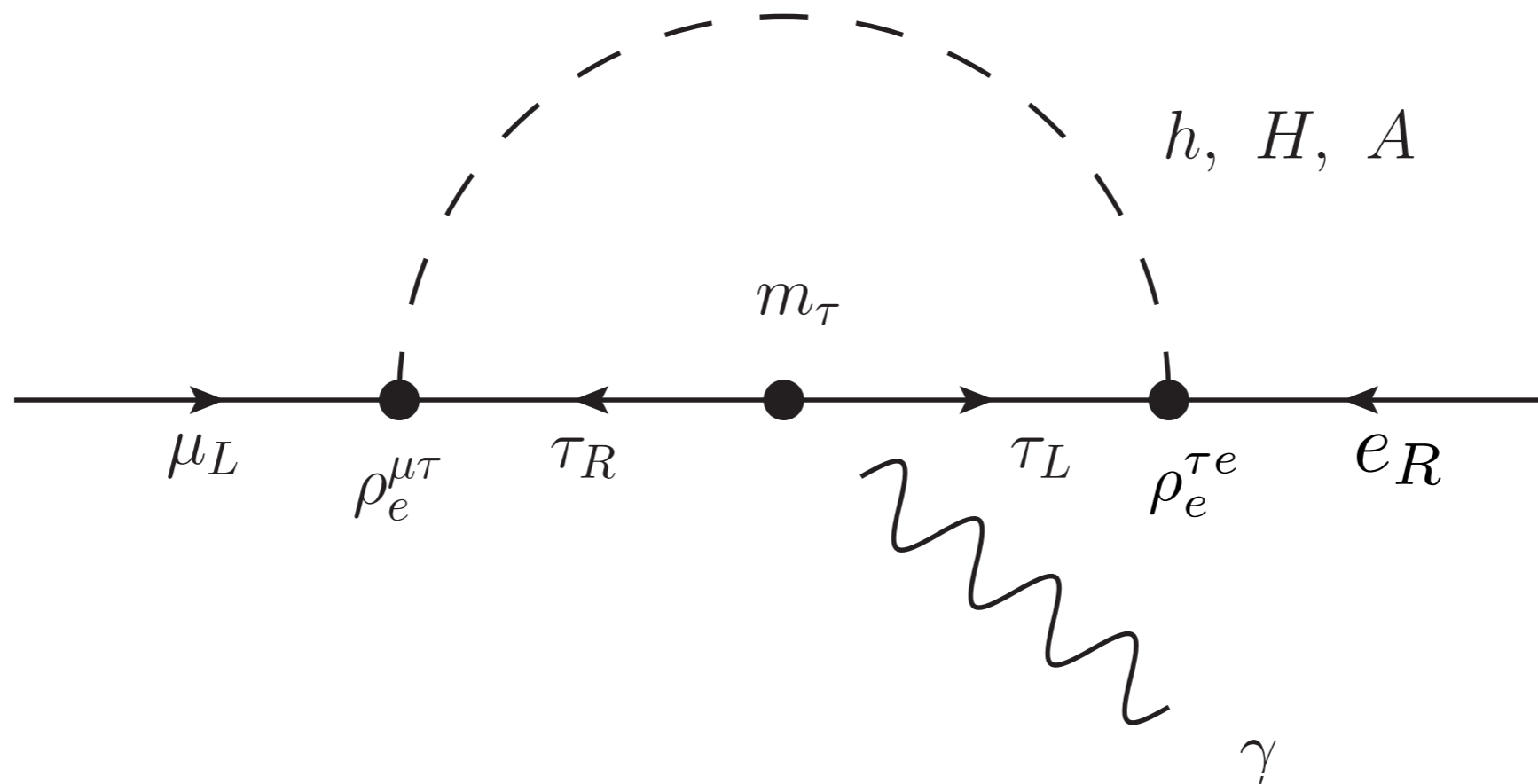
$$y_\mu = \frac{\sqrt{2}m_\mu}{v} \sim 6 \times 10^{-4}$$



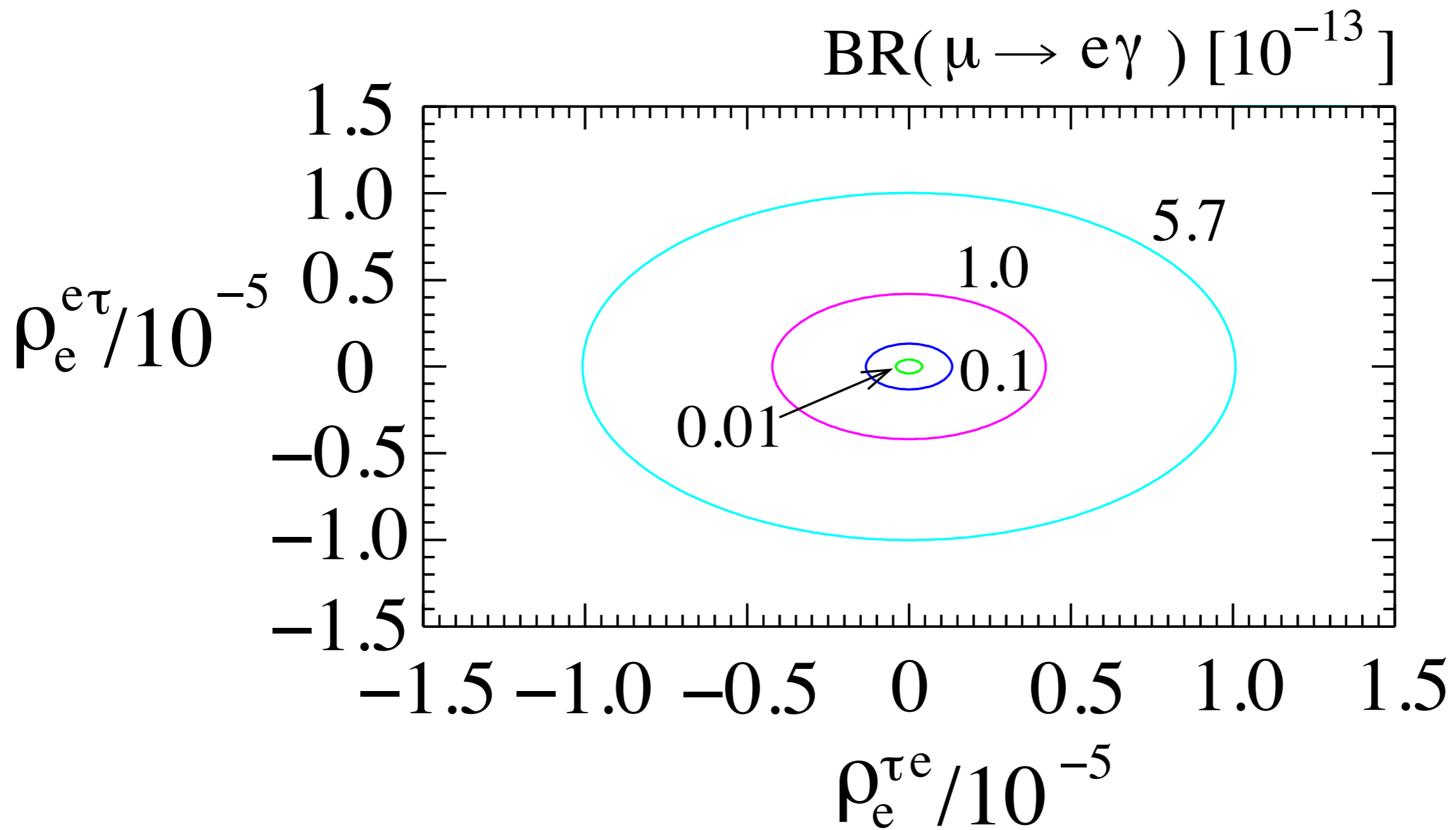
$$\mu \rightarrow e\gamma$$

Other lepton flavor violating Yukawa couplings
(e - τ , e - μ couplings) are strongly constrained
from $\mu \rightarrow e \gamma$ process

e - τ flavor violation



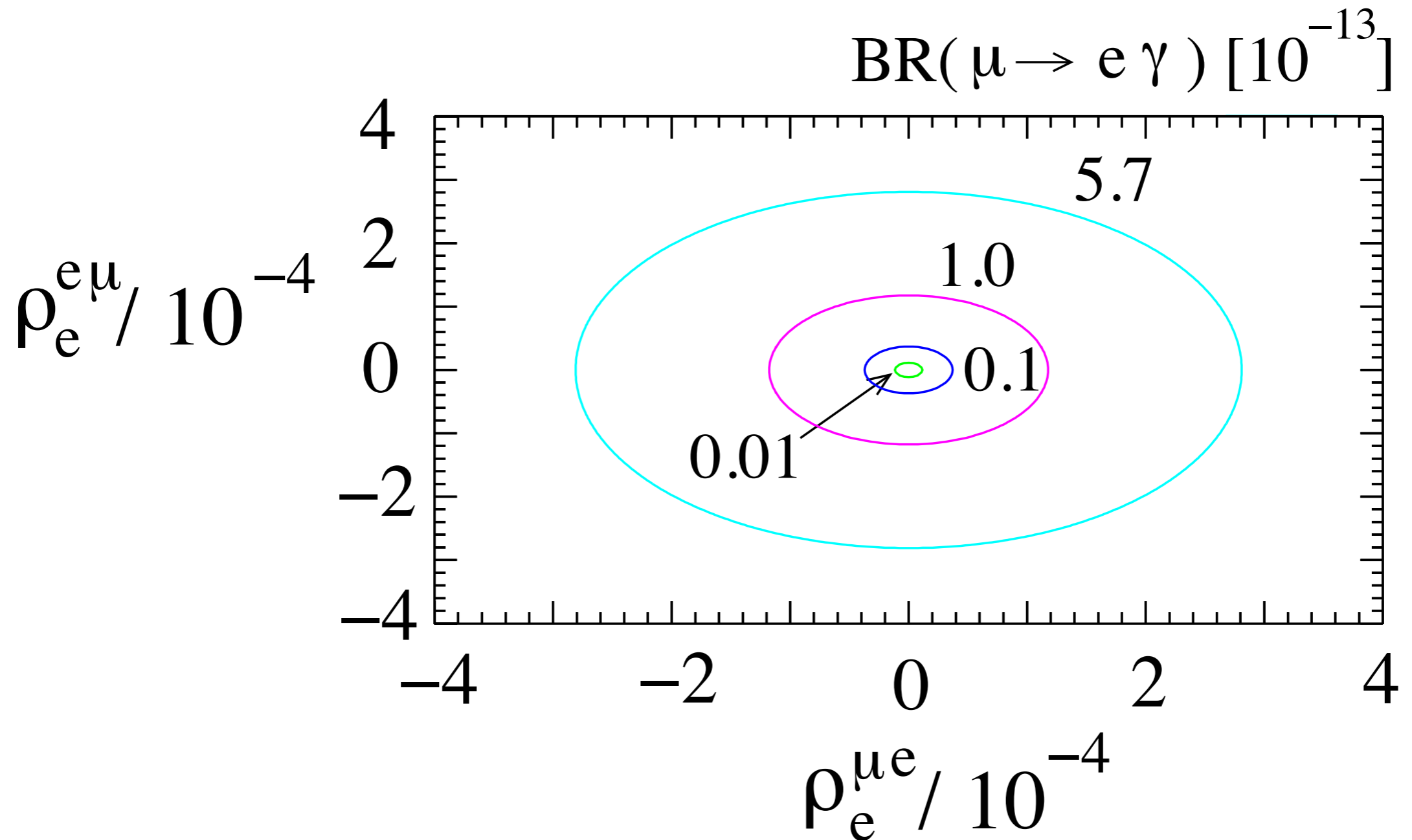
large enhancement



strongly constrained

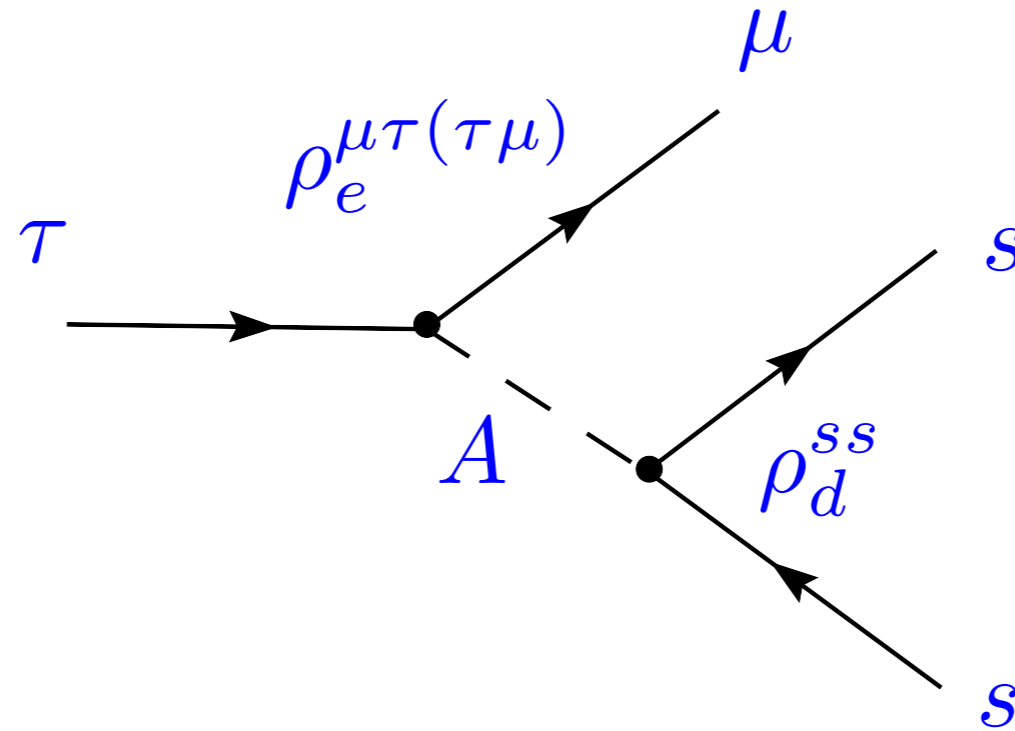
e- μ flavor violation

similar to $\tau \rightarrow \mu \gamma$, 2-loop contributions are important



The e - τ and e - μ flavor violating Yukawa couplings are already strongly constrained.

$$\tau \rightarrow \mu \eta$$



$$\text{BR}(\tau \rightarrow \mu \eta)_{\text{exp.}} < 6.5 \times 10^{-8}$$

$$|\rho_d^{ss}| < 0.007 \left(\frac{0.3}{\bar{\rho}^{\mu\tau}} \right) \left(\frac{m_A}{350 \text{ GeV}} \right)^2$$

Note: $y_s = \frac{\sqrt{s} m_s}{v} \simeq 5 \times 10^{-4}$

$\mu \rightarrow eee$

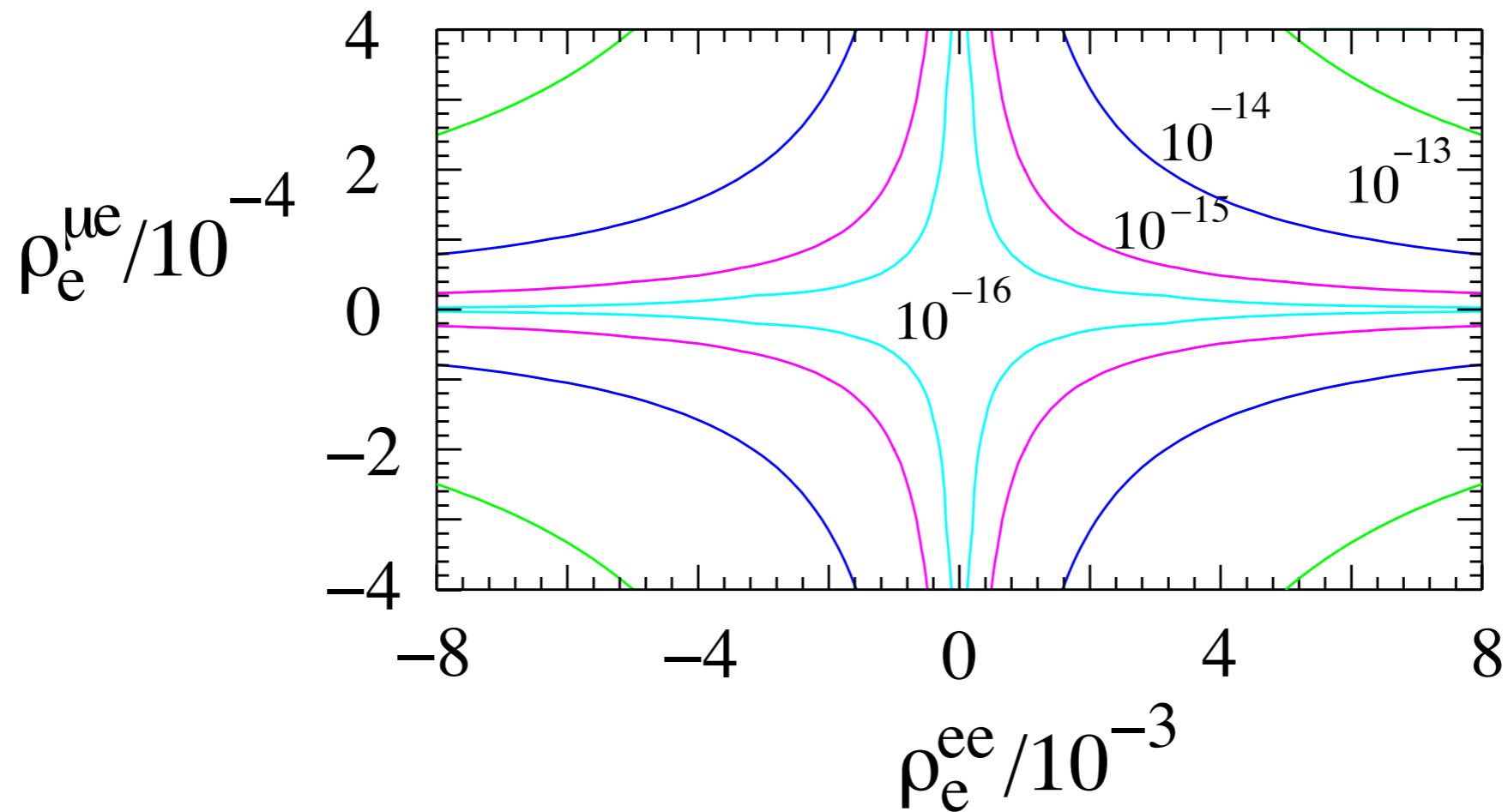


FIG. 12: $\text{BR}(\mu \rightarrow 3e)$ as a function of ρ_e^{ee} and $\rho_e^{\mu e}$. Here we have assumed that $\rho_e^{\mu e} = \rho_e^{e\mu}$, $c_{\beta\alpha} = -0.007$ and $m_A = 350$ GeV with $\lambda_4 = \lambda_5 = 0.5$.

Future Mu3e experiment ($\text{BR} \sim 10^{-16}$) may have
a sensitivity

Implication to Higgs physics

Is the CMS excess in $h \rightarrow \mu \tau$ real?

If it is real, can other lepton flavor violating decay modes be observed?

Because of the strong constraint of $\mu \rightarrow e \gamma$,
 $h \rightarrow e \mu$ and $h \rightarrow e \tau$ are strongly suppressed

LFV Higgs decay mode	BR	observability
$h \rightarrow \mu\tau$	$\text{BR} = (0.84^{+0.39}_{-0.37})\%$	(input)
$h \rightarrow e\tau$	small	×
$h \rightarrow e\mu$	small	×

LHC run2 data for $h \rightarrow \mu\tau$ will be crucial