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Instabilities of coherent beam-beam oscillations: theory and recent observations at LHC

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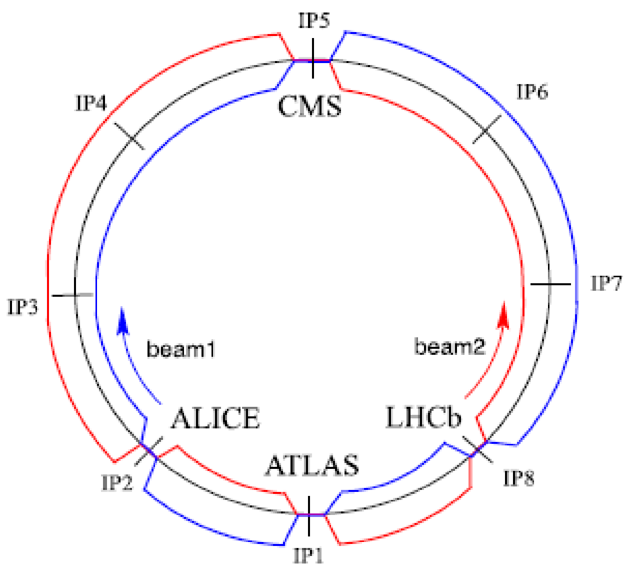
APC seminar

4 Feb 2016

LHC



IP	β* (m)	crossing angle (μrad)	separation (mm)
IP1	0.8	2×145 V	0
IP2	10	2×120 V	0.14 H
IP5	0.8	2×145 H	0
IP8	3	2×250 H	<0.1 V

Proton energy (TeV)	6.5
Np /bunch	1.15e11
Number of bunches	2808
Norm. emittance (μm)	3.75
RMS bunch length (cm)	7.55
Beam energy spread	1.1e-4
Head-on BB parameter	0.004
Chromaticity in collision	15
ADT gain @ top energy	0.002 (?)

Summary of 2012 Observations (T. Pieloni)

End of Squeeze instability present whole year 2012 and not yet understood!
Instabilities observed first around 3 m beta* (beginning of RUN)
Pushed at the end of the squeeze (End of Squeeze **EoS**)

No Instabilities during collapse of separation bumps

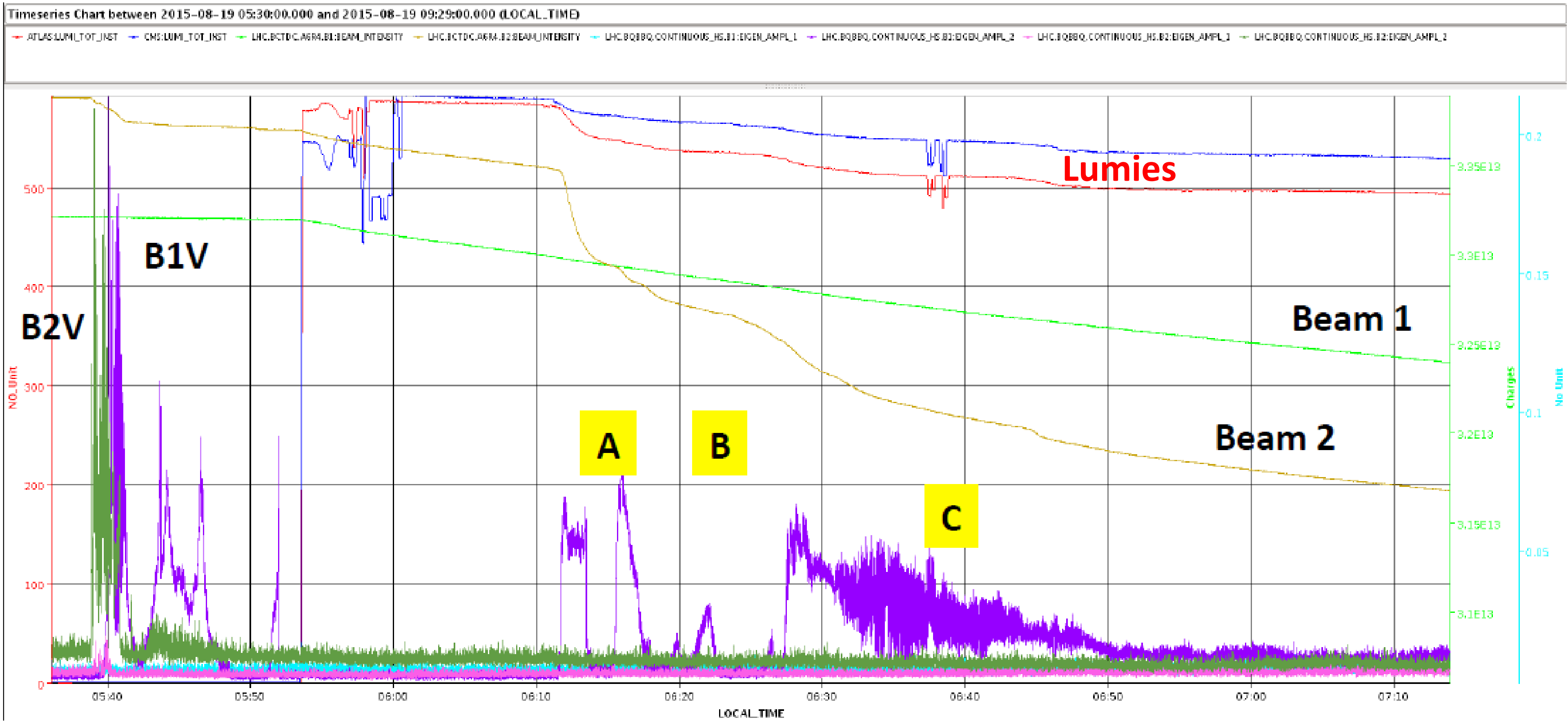
Instabilities in Stable beams

→Present during whole year: **only IP8 private bunches** NON-colliding (at separation of 1.5 σ)

→ **Never had instabilities on colliding bunches** (IP1&5 head-on collisions provided strong Landau damping)

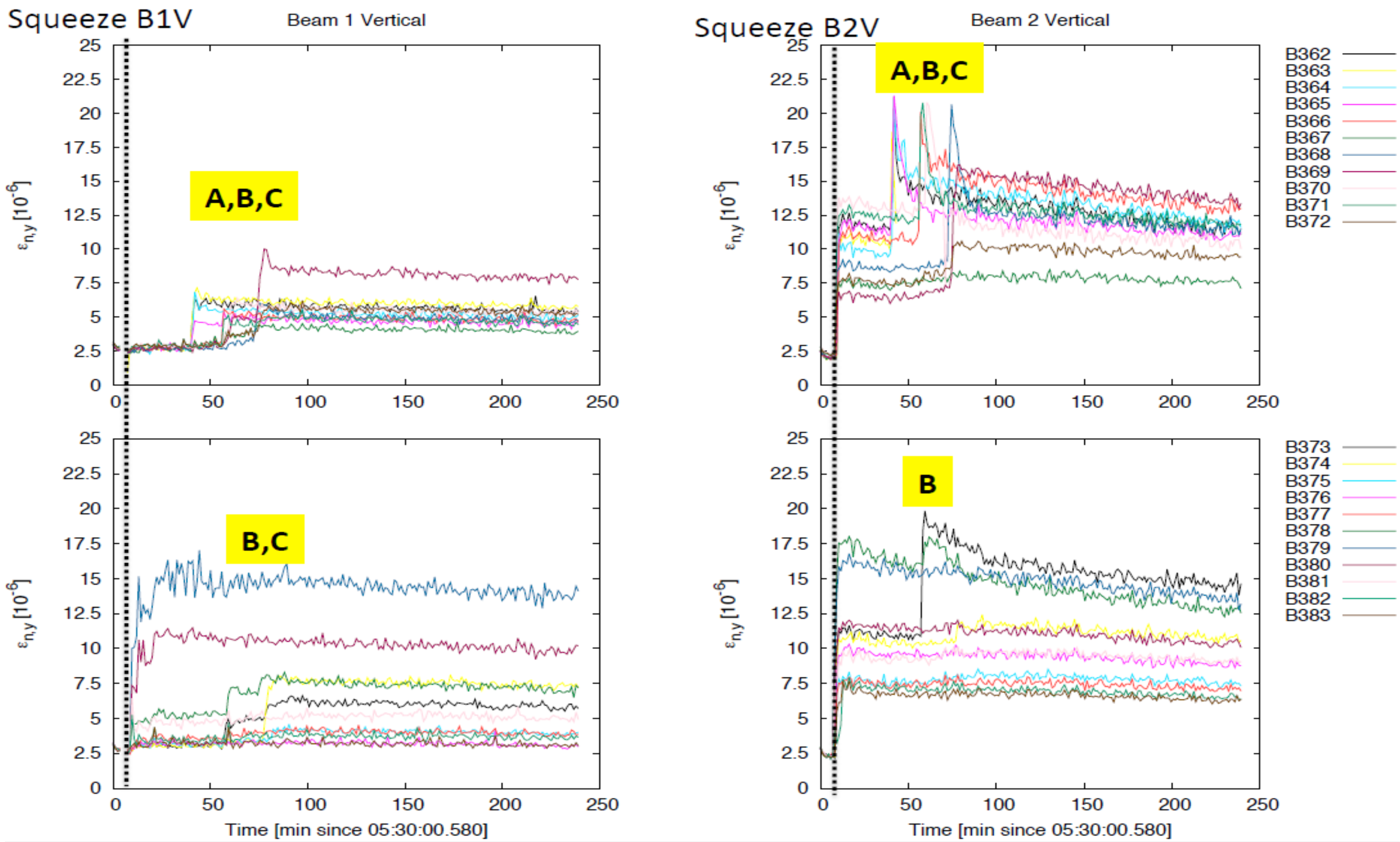
This changed in 2015!

2015 Observations: Fill # 4231 (somewhat unique)



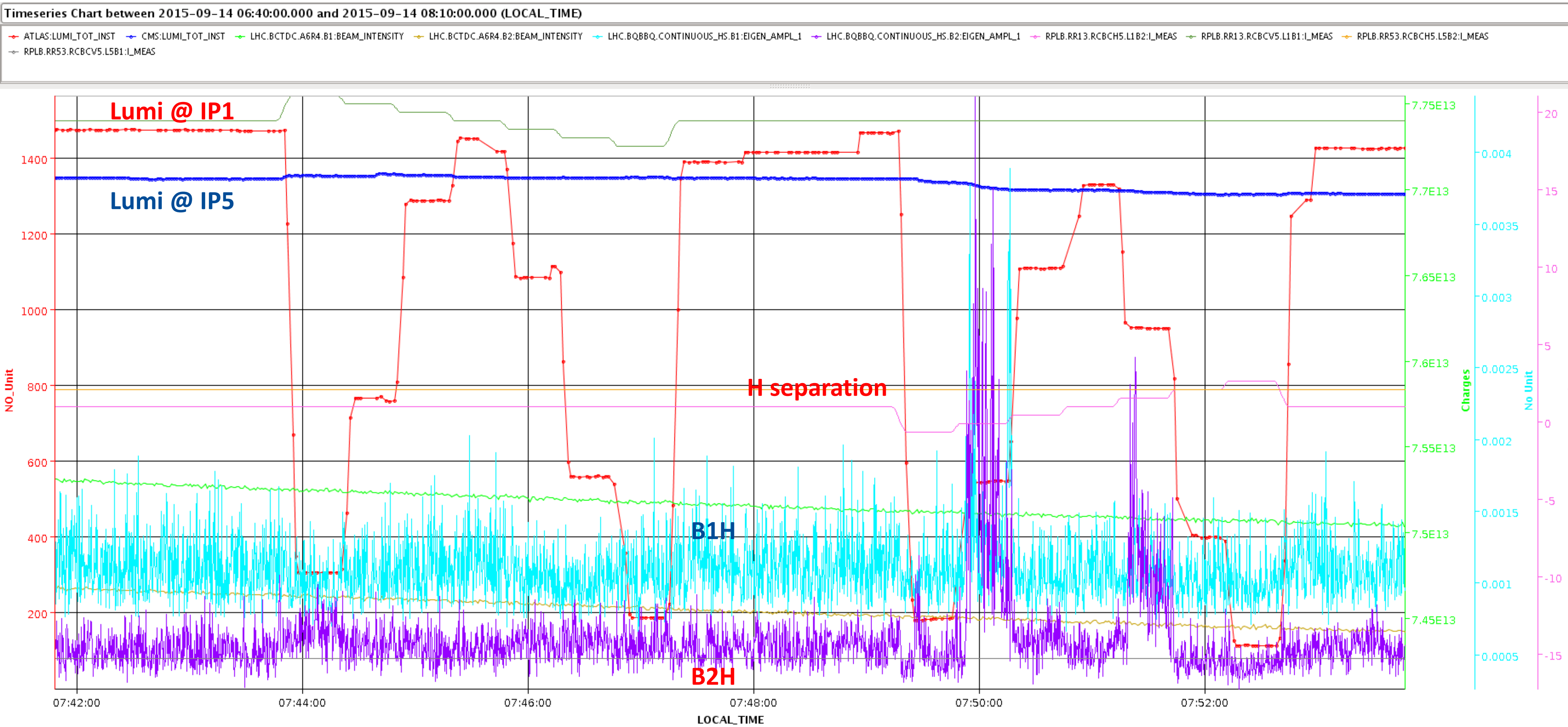
- During the squeeze first B2 then B1 go unstable – does beam-beam play a role?
- At moment “A” B1 again goes unstable, but it is B2 that suffers losses

2015 Observations: Fill # 4231 - emittances



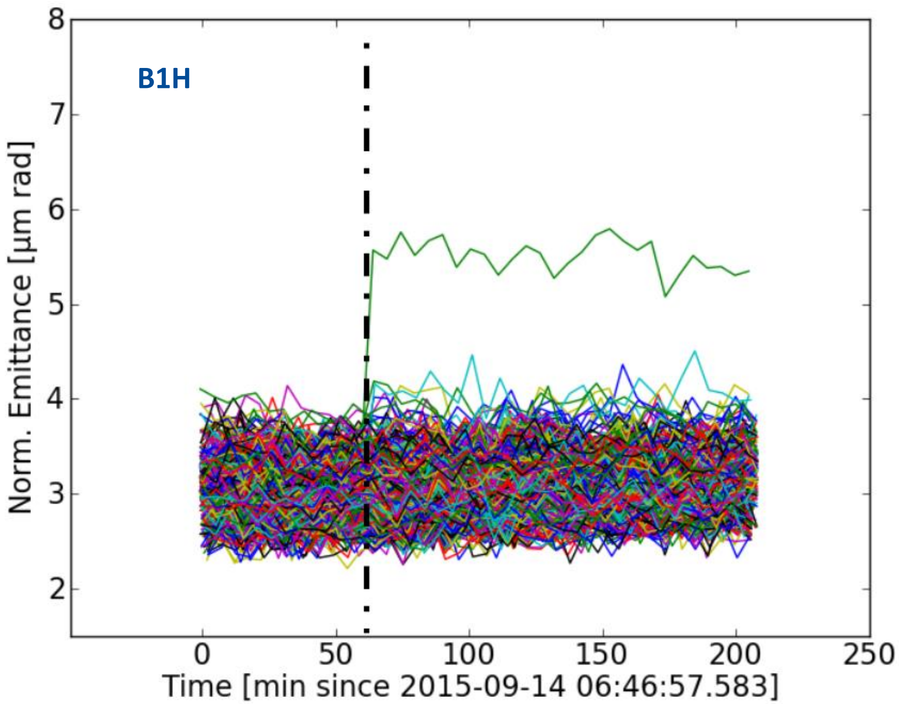
- B2 was blown up during the squeeze instability and in collision does not provide for B1 Landau damping by nonlinear tunespread
- B2 suffers losses because of strongly nonlinear B1 field

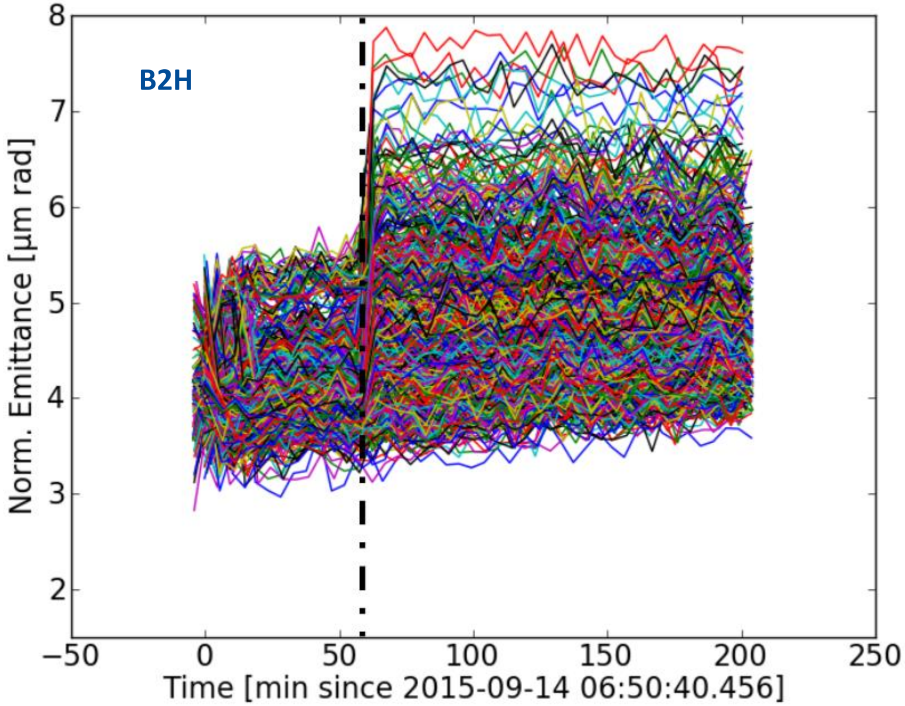
Observations during “OP scans” (T. Pieloni)



- No instability during vertical scan
- Instability happened during horizontal scan at intermediate separation ~1.5sigmas
- Asymmetry between B1 and B2 - the instability is more pronounced in B2

Observations during "OP scans" - emittances





- Just one bunch in B1 was blown up
- B2 – which experienced higher oscillation amplitude – suffered more
- Asymmetry between B1 and B2 was traced to a faulty damper (half the gain)
- After fixing the B2H ADT no instability during scans
- Will the ADT gain be sufficient for the upgrade parameters?

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Vlasov Perturbation Theory

Motivation:

Exact solution of Vlasov equation provides correct results for:

- coherent mode tunes (as shown by K. Yokoya)
- beam response to external excitation (kick, harmonic excitation, noise) $\Rightarrow$
- energy sharing between discrete and Van Kampen modes
- Landau damping rate
- longitudinal dependence of transverse amplitude $\Rightarrow$
- coupling to external impedances

Also it can be used for benchmarking multi-particle tracking codes

Limitations:

- Small amplitude of coherent oscillations
- Stability of unperturbed particle motion – so that the normal forms exist
- All work was done for Gaussian equilibrium distribution (exponential in action variables):

$$F_0 = \frac{1}{(2\pi)^3 V} \exp\left(-\underline{\varepsilon}^{-1} \cdot \underline{I}\right) \quad \underline{\varepsilon} = \langle \underline{I} \rangle, \quad V = \varepsilon_x \varepsilon_y \varepsilon_s, \quad \underline{\varepsilon}^{-1} = (\varepsilon_x^{-1}, \varepsilon_y^{-1}, \varepsilon_s^{-1})$$

Vlasov Perturbation Theory

small perturbation
to be treated last

$$\frac{\partial}{\partial\theta}F_1^{(k)}+\underline{Q}^{(k)}(\underline{I})\frac{\partial}{\partial\underline{\psi}}F_1^{(k)}=-\frac{r_p N_{3-k}}{\gamma}\delta_{\mathrm{p}}(\theta)F_0\underline{\varepsilon}^{-1}\cdot\frac{\partial}{\partial\underline{\psi}}\int G^{(k)}F_1^{(3-k)}d^3I'd^3\psi'+\underline{\varepsilon}^{-1}\cdot\underline{\dot{I}}^{(ext)}F_0$$

$$G=-\ln\left\{[x-x'+(\alpha+\frac{p_x+p_x'}{2})(z-z')]^2+[y-y'+\frac{p_y+p_y'}{2}(z-z')]^2\right\}$$

k =1,2 is beam number,
z = long. displacement from the bunch center
α = half crossing angle (horizontal here)

finite bunch length effect

The incoherent tunes $Q = Q(I)$ may include contribution from lattice multipoles.

There is no attempt to replace the integral operator with something else, but apart from coherent resonances $\delta_{\mathrm{p}}(\theta)\rightarrow 1/2\pi$

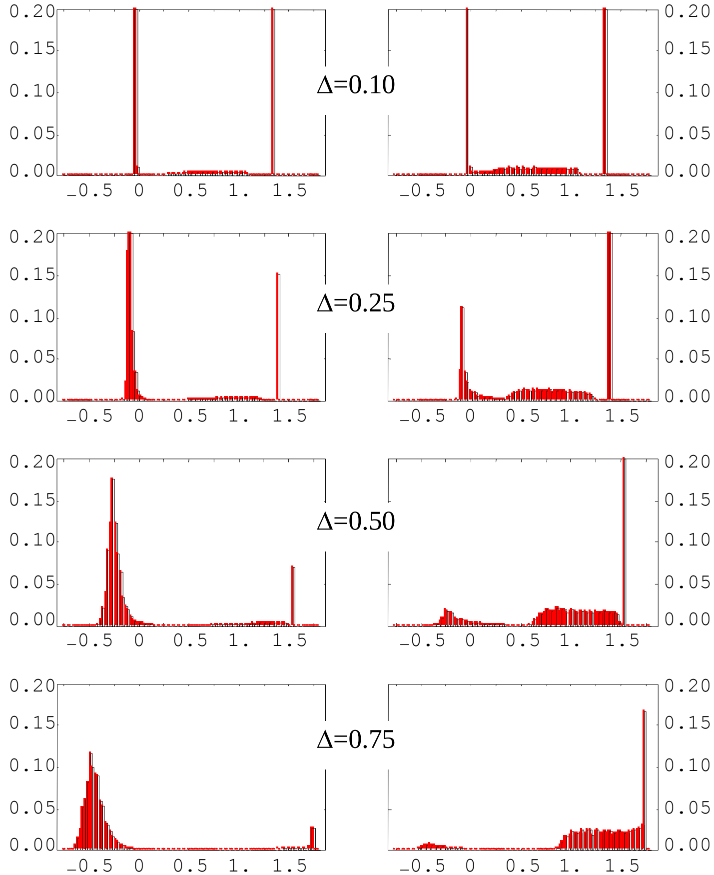
Expansion in angle variables ψ (azimuthal mode expansion)

$$F_1^{(k)}=\exp\left(-\underline{\varepsilon}^{-1}\cdot\underline{I}/2\right)\sum_{\underline{m}}\exp(i\underline{m}\cdot\underline{\psi})f_{\underline{m}}^{(k)}(\underline{I},\theta)$$

The integral operator becomes a matrix operator which couples azimuthal modes (all of them in the case of offsets!):

$$G_{\underline{m}\underline{m}'}=\frac{1}{(2\pi)^6}\int_0^{2\pi}Ge^{-i\underline{m}\cdot\underline{\psi}+i\underline{m}'\cdot\underline{\psi}'}d^3\psi d^3\psi'$$

Example: flat beams with split tunes



Spectra of oscillations excited by a dipole kick at the 1st beam (left) and the 2nd beam (right),

$\Delta = (Q_{x0}^{(2)} - Q_{x0}^{(1)}) / 2\xi$

The essence of the method: finding the system of eigenfunctions of the Vlasov operator

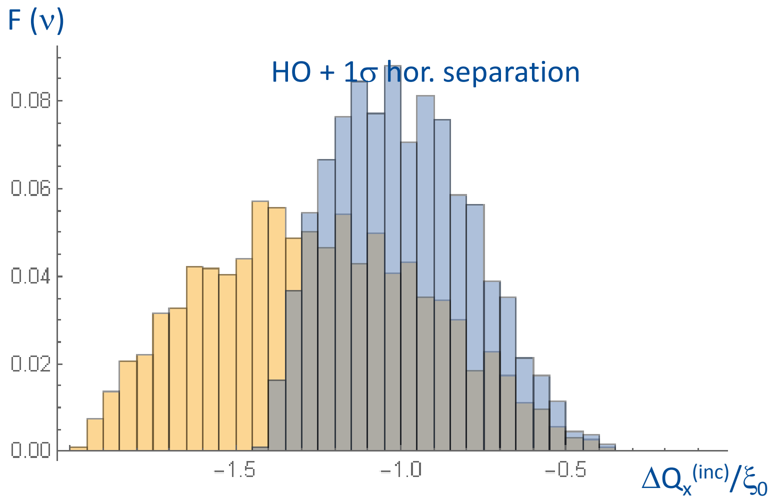
$$f_{\underline{m}} = \sum_n e^{-i\xi\lambda_n\theta} a_n \Phi_{\underline{m},n}(\underline{I})$$

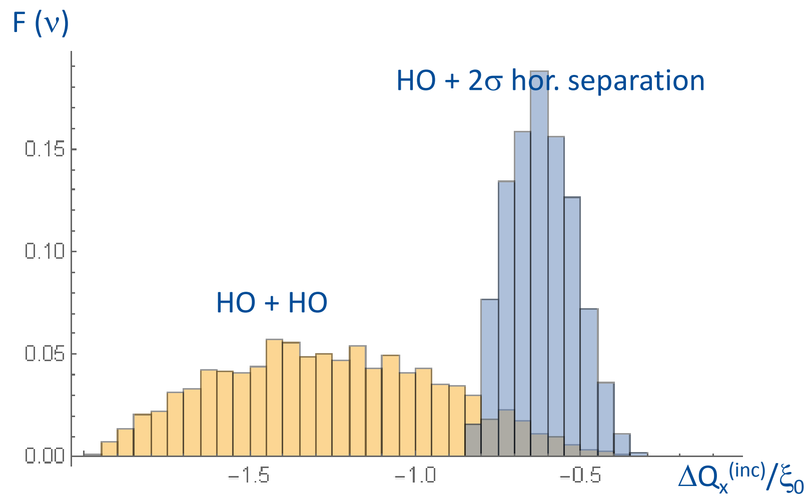
The eigenvalues λ_n comprise discrete values (really coherent modes) and continuum (Van Kampen modes).

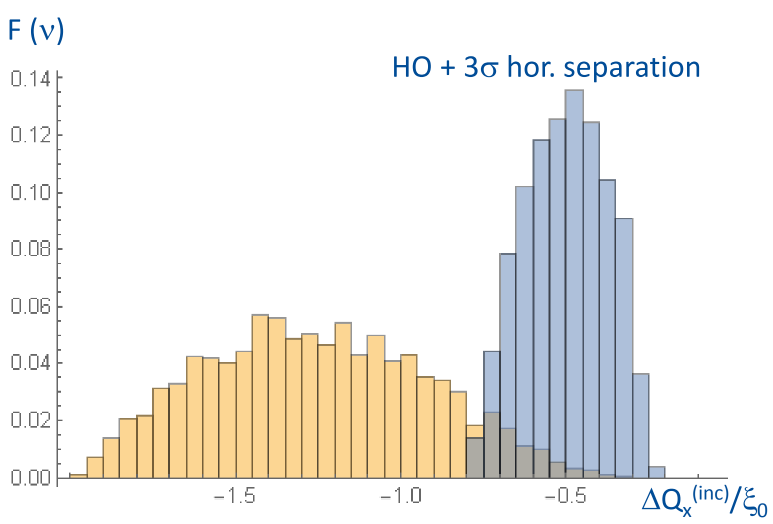
The simple example on the left shows:

- π-mode can be shifted far from incoherent tunes (Y=1.33 for Δ=0)
- Landau damping appears as a natural property of the eigensystem, no extra dispersion relation is required
- The damping rate can be inferred from the width of the peak
- even when coherent modes are not damped, a significant amount of energy of the kick can be carried away by Van Kampen modes (absent in a rigid bunch approximation)

Incoherent tunes with 2 IPs (head-on + offset)







The incoherent tunespread is at minimum for ~ 2 σ separation – as was first observed by Claudia Tambasco by tracking.

What about coherent tunes?

ξ_0 = beam-beam parameter / IP (≈ 0.004)

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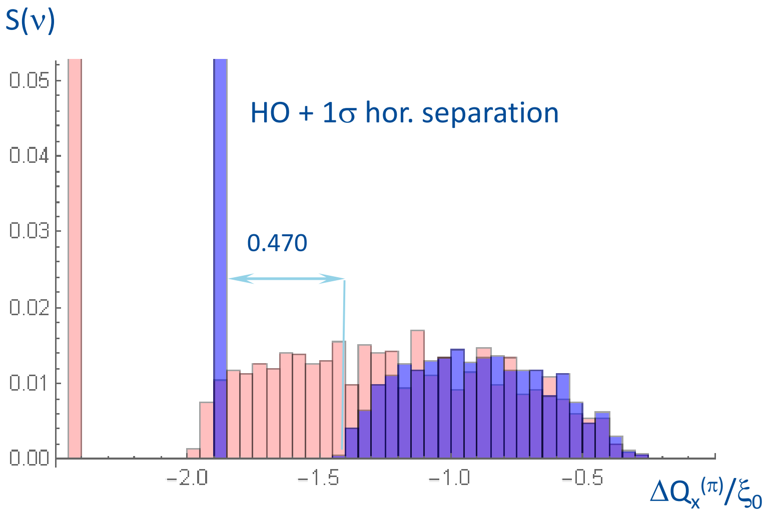
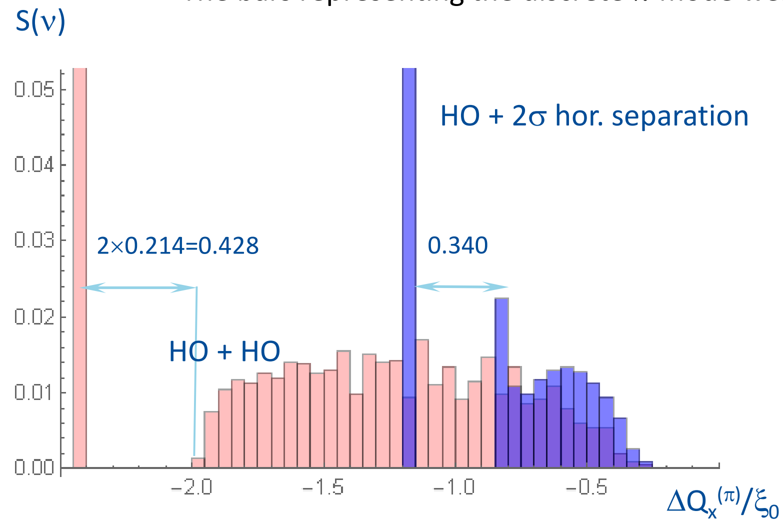
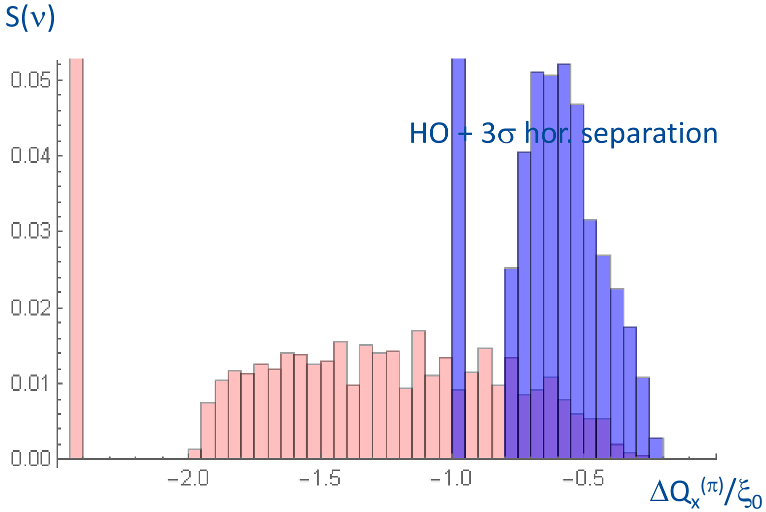
Spectra of π-mode for equal phase advances between IPs

The bars representing the discrete π-mode were cropped.







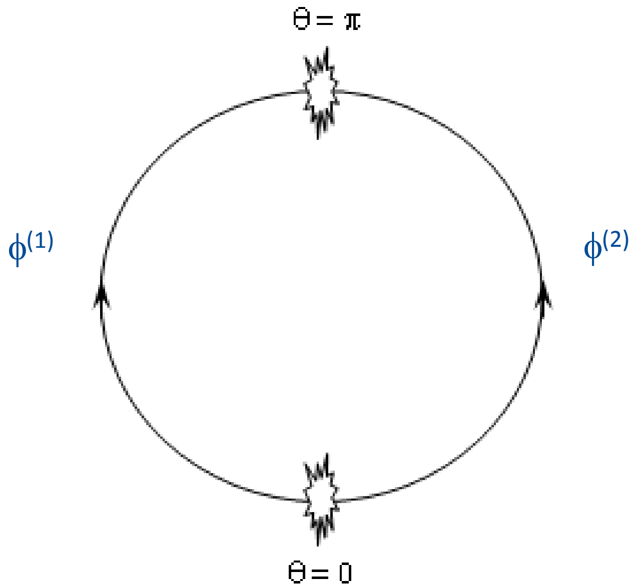
The gap between the discrete π-mode and the incoherent tunes is largest at HO + 1σ offset IP: worst case for Landau damping.

The spectral weight of the discrete π-mode is: 0.645 in the case of two HO IPs, 0.785 for HO + 1σ, 0.875 for HO + 2σ and 0.621 for HO + 3σ offset – in qualitative agreement with Xavier's results (next slide).

The nascence of a second discrete π-mode is clearly visible in the latter case – the beam-beam interaction became more coherent.

The discrete Σ-mode is of course unshifted and becomes more easily damped with the offset since the incoherent distribution is shifted towards it.

Effect of unequal phase advances for the two beams



If the phase advances between identical IPs are not equal, the coherent modes are weakened and – when $\phi^{(1)} - \phi^{(2)} = \pm\pi$ – are completely suppressed (A.Temnykh, J.Welch, 1995),
$\phi_x = \varphi_x(\theta) - Q_{x0}\cdot\theta$ is the periodic phase advance function.

In the LHC:

$\phi_x^{(1)} - \phi_x^{(2)} = 0.54\,\pi$, $\phi_y^{(1)} - \phi_y^{(2)} = -0.18\,\pi$,

A bit of theory:

The Vlasov equation is reduced to an eigenvalue problem for operator

$$\hat{A} = \xi \begin{pmatrix} \sum_{IP} \Delta Q_{IP}^{(inc)} & -\sum_{IP} e^{i(\phi_{IP}^{(1)}-\phi_{IP}^{(2)})}\hat{G}_{IP} \\ -\sum_{IP} e^{-i(\phi_{IP}^{(1)}-\phi_{IP}^{(2)})}\hat{G}_{IP} & \sum_{IP} \Delta Q_{IP}^{(inc)} \end{pmatrix} = \xi \begin{pmatrix} \Delta Q_{total}^{(inc)} & -(\hat{G}_{IP1} + e^{-i\delta\phi}\hat{G}_{IP5}) \\ -(\hat{G}_{IP1} + e^{+i\delta\phi}\hat{G}_{IP5}) & \Delta Q_{total}^{(inc)} \end{pmatrix}$$

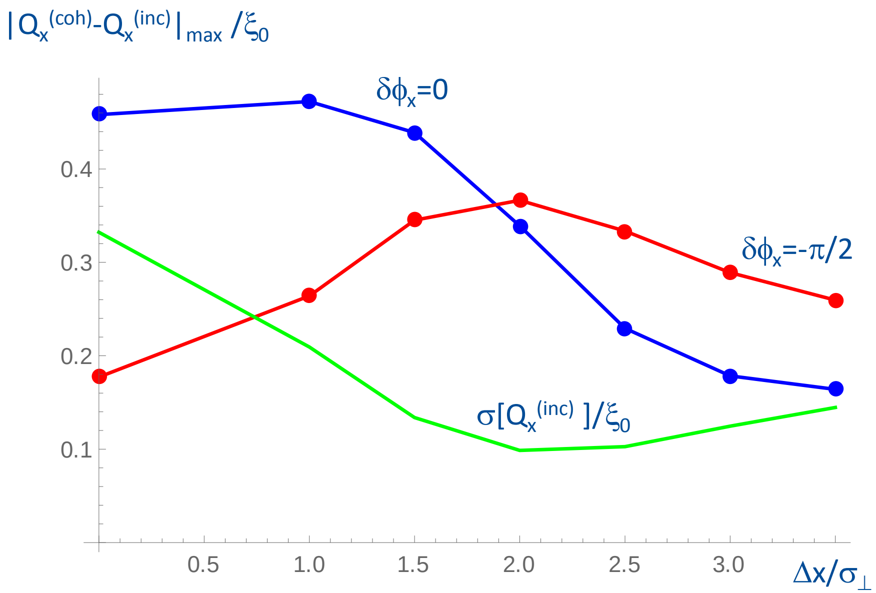
$$\text{if}\quad \hat{G}_{IP5} \equiv \hat{G}_{IP1} \quad\text{then}\quad \hat{G}_{IP1} + e^{\pm i\delta\phi}\hat{G}_{IP5} = 2e^{\pm i\delta\phi/2}\cos(\delta\phi/2)\hat{G}_{IP1}$$

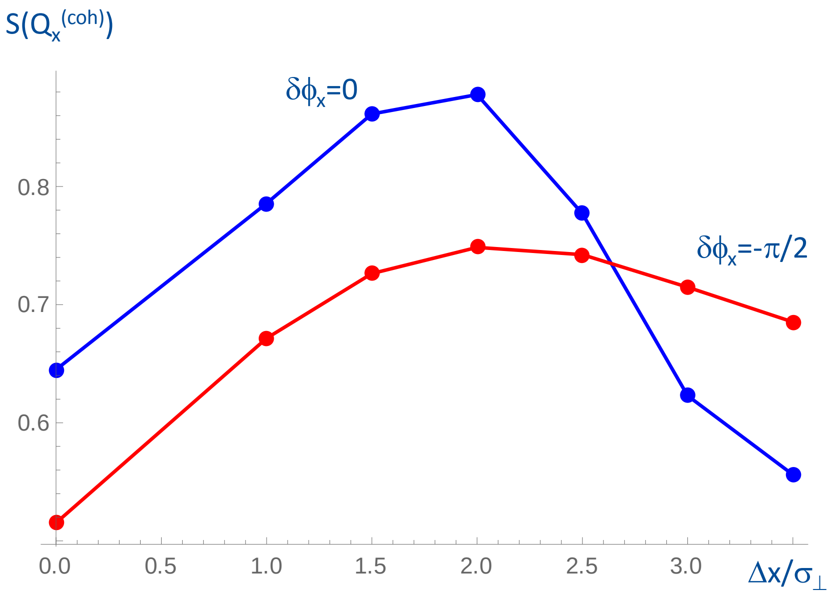
In this case any $\delta\phi \neq 0$ weakens coherent modes.

But if $\hat{G}_{IP5}$ and $\hat{G}_{IP1}$ have opposite signs (one is head-on, the other with offset) then coherent tuneshift will be increased.

Joint effect of unequal phase advances and offset

Still just 2 IPs, no crossing angle, chromaticity, octupoles.





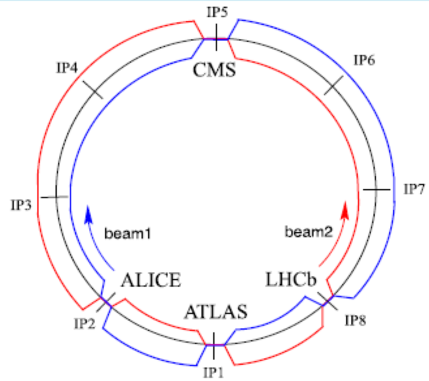
With the phase advance difference between beam 1 and 2, the gap between the horizontal discrete π-mode and the incoherent tunes reaches maximum at ~ 2 σ separation, just where the incoherent tunespread is at minimum.

Why the vertical π-mode is stable with much smaller phase advance difference?

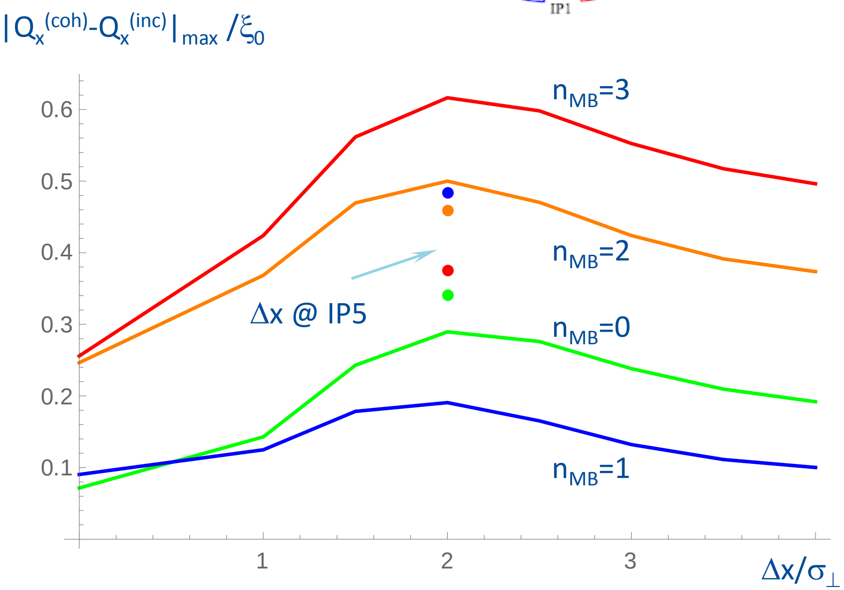
- Let us look at the effect of the other 2 IPs on 4 + 4 multibunch oscillations

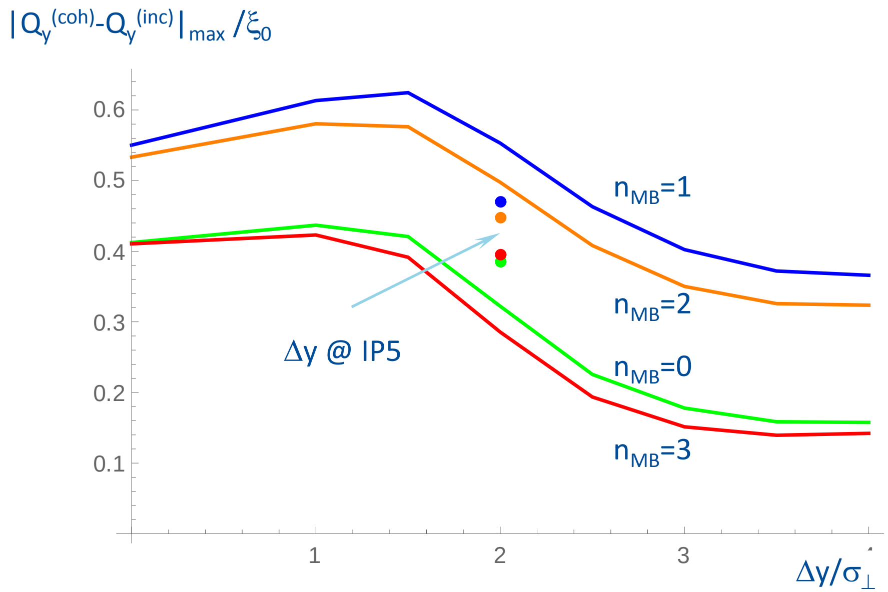
4 + 4 Multi-Bunch Modes

still no crossing angle, chromaticity, octupoles



IP	$\phi_x^{(2)} - \phi_x^{(1)}$	$\phi_y^{(2)} - \phi_y^{(1)}$	separation, σ
IP1	0	0	0
IP2	$1.118\ \pi$	$1.742\ \pi$	4 H
IP5	$-0.542\ \pi$	$0.182\ \pi$	0
IP8	$-1.185\ \pi$	$-1.888\ \pi$	3 V

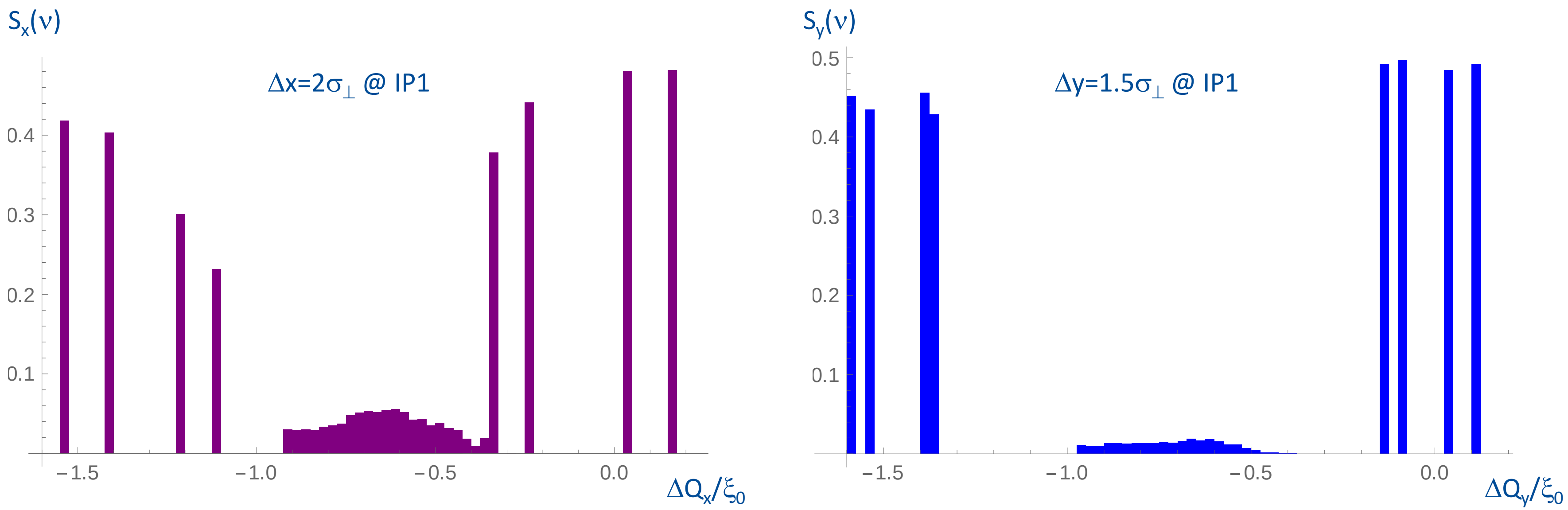




Maximum gap between the π-like multibunch modes and incoherent tunes as function of separation at IP1.
Left: horizontal modes vs horizontal separation, right: vertical modes vs vertical separation.
Dots present results for 2 σ separation at IP5.
Separation at IP1 is potentially more detrimental!

ξ_0 = beam-beam parameter / IP ($\approx$ 0.004)

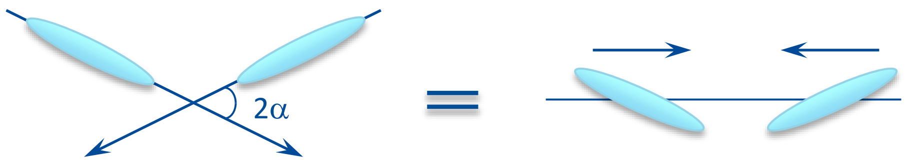
Spectral density of Multi-Bunch oscillations

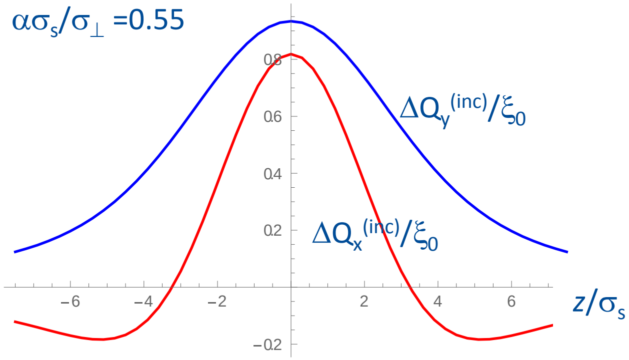


From the beam-beam viewpoint the vertical modes should be ~ as unstable as the horizontal ones, still only the horizontal instability was seen during the scans.
- Differences in the impedance? Problems with the dampers?

The major player is – of course – the large crossing angle not taken into account yet, but it is unlikely to affect the two planes (very) differently.

Effect of the crossing angle

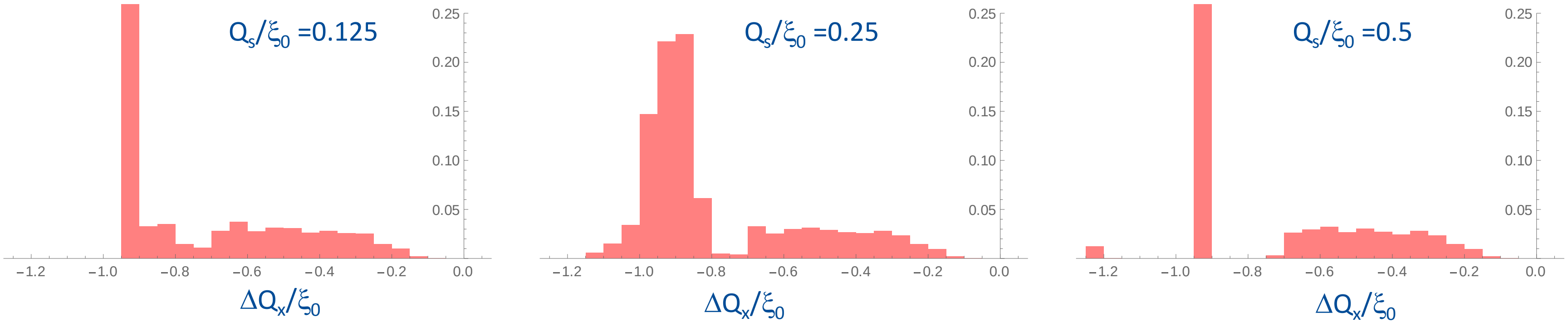




- Both incoherent tuneshift (plotted) and coherent kick due to the opposing bunch perturbation depend strongly on position *z* along the bunch

  ⇒ Reduction in the π-mode tuneshift

  ⇒ Coupling between even-order synchrotron sidebands of the betatron tunes which can completely suppresses the π-mode



Spectra of oscillations excited by a dipole kick in the case of 1 IP with Piwinsky (half) angle = 0.55 in the plane of oscillations (x). If the π-mode is overlapped by 2nd sideband it is Landau damped.

Landau damping by sidebands

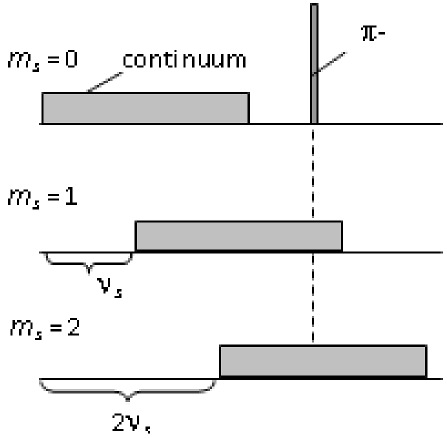


Figure 1. Principle of damping by synchrotron sidebands of the continuum modes.

Damping rate due to coupling to sidebands of incoherent tunes can be obtained analytically if the eigenfunctions of uncoupled modes are known

ms=0 discrete mode   ms≠0 continuum

$$\hat{A} = \begin{pmatrix} \hat{A}_0 & \hat{B} \\ \hat{B}^* & \hat{A}_1 \end{pmatrix} \qquad \hat{A}_0 \Psi_0 = \lambda_0 \Psi_0 \qquad \hat{A}_1 \Phi_\lambda = \lambda \Phi_\lambda, \quad \lambda \in \boldsymbol{C}$$

due to coupling to continuum the discrete mode gets finite width (i.e. damping rate)

$$a_0^2(\lambda) = \frac{|B_\lambda|^2}{\pi^2 |B_\lambda|^4 + (\lambda - \lambda_0 + \mathrm{p.v.}\int_C \frac{|B_\mu|^2}{\mu - \lambda} d\mu)^2} \qquad B_\mu \equiv (\Psi_0, \hat{B}\Phi_\mu).$$

This (perturbation) approach allows us to obtain the Landau damping rate without finding for the eigenvalues of the fully coupled system which may be prohibitive from the computational point of view

Effect of chromaticity

Chromaticity (of betatron tunes) is transformed away when the normal forms are introduced

$$a_x = \hat{T}^{-1} A_x \approx A_x \exp(-\frac{iQ'_x}{\alpha_c R} z) \ - \text{ Courant} - \text{Snyder variable, } \ x = \sqrt{2\beta_x}\,\mathrm{Re}[a_x \exp(i\phi_x)]$$

$$A_x = \sqrt{J_x}\exp(i\psi_x) \ - \text{ normal form variable}$$

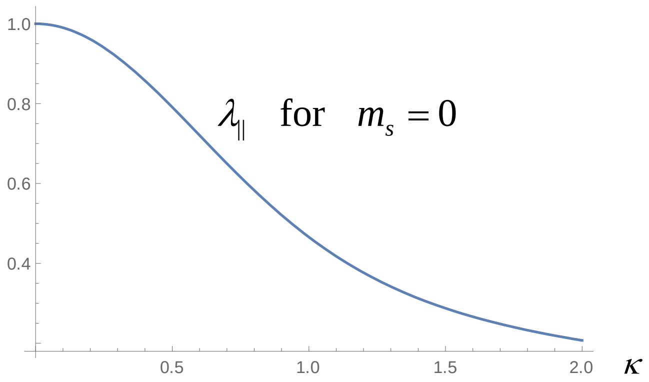
but it modifies Green's function as $G \to G e^{i\kappa(z-z')/\sigma_s}$ weakening the contribution from the integral.

- This is a principal difference with Space Charge which is not affected by chromaticity.

The above-mentioned "weakening" of m_s (head-tail) modes is described by factor in the integral part of the Vlasov operator

$$\lambda_{||} = e^{-\kappa^2}\, \mathrm{I}_{m_s}(\kappa^2)$$

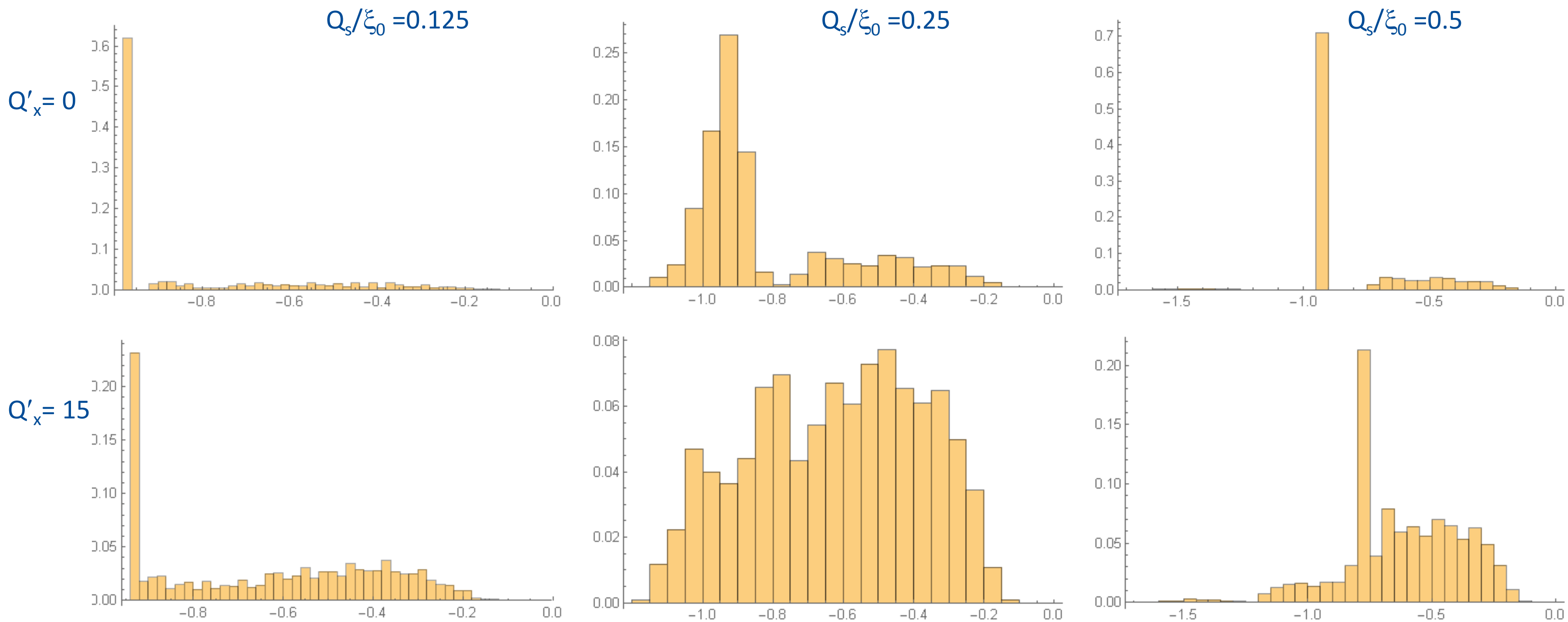
$$\kappa = (\frac{1}{\beta_x^*} - \frac{Q'_x}{\alpha_c R})\sigma_s \approx -Q'_x \sigma_E \approx -0.065 Q'_x$$



"Finite bunch length" correction to chromaticity is only -1.5 for β^*=0.8m, it will become more significant in the future

For head-tail modes ($m_s \neq 0$) $\lambda_{||}$ is small – no coherent HT modes in collision?

Chromaticity & crossing angle @ single IP



- Coherent tuneshifts are reduced → facilitated Landau damping
- The peak height is reduced by a factor > 3 → equivalent to reduced impedance
- Chromaticity ~15 should be enough unless there is large beam-beam contribution to chromaticity. There are 3 sources for such contribution:
  dispersion @ offset IPs, chromatic β-beat, finite bunch length @ low-β IPs (see previous slide)

- This is a major subject for studies!

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Long-range interactions

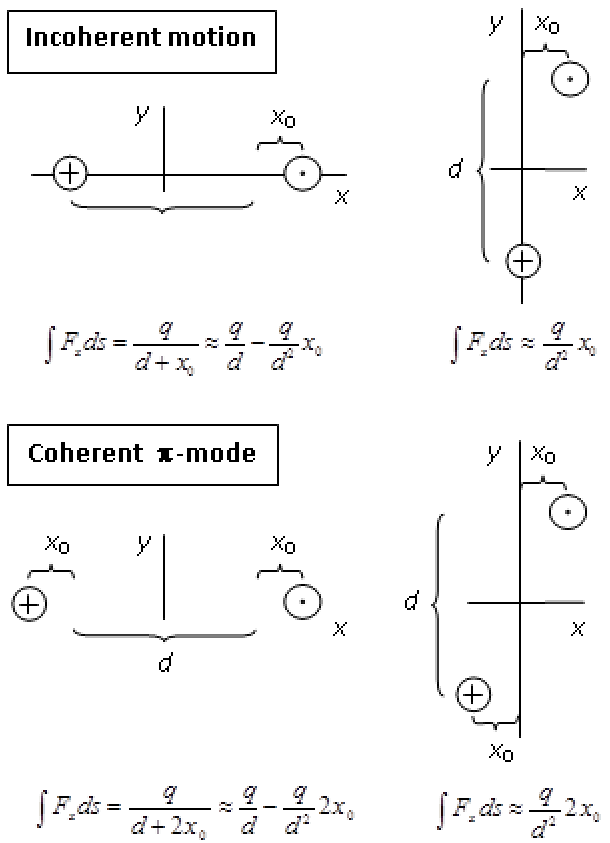


Figure 1. Schematic picture of incoherent and coherent effects of long-range interactions of pencil beams; $q = 2e^2N/\gamma$.

- The Yokoya factor for long-range interactions with separation in just one plane would be *Y* = 2 with both horizontal and vertical separation.
- With alternating separation the incoherent tuneshift cancels out, but the coherent kicks do not due to phase advance difference for beams 1 and 2 – potentially a trouble.

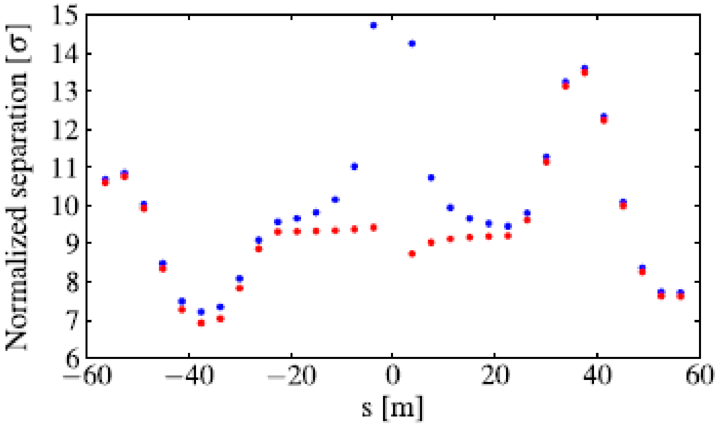
- According to Tatiana Pieloni the end-of-squeeze instability is still observed (from time to time) despite
  - octupoles at full current (550A)
  - high chromaticity (~10)
  - dampers on

Can the beam-beam effect set the stage for instability?

Long-range interactions @ LHC

from X. Buffat et al. PRSTAB 17, 111002 (2014)

Long-range interactions at IP1 & IP5



$$d_{average} = 1/\sqrt{\sum_i 1/d_i^2} \approx 9.3\sigma$$ (two nearest PIPs excluded)

Scaling with *E*, β* and ε gives for β*=0.8m and ε=3.5e-6 $\quad d_{average} \approx 11.2\sigma$

I had it 12σ and to have the same effect increased the number of lumped NLR 28 → 32

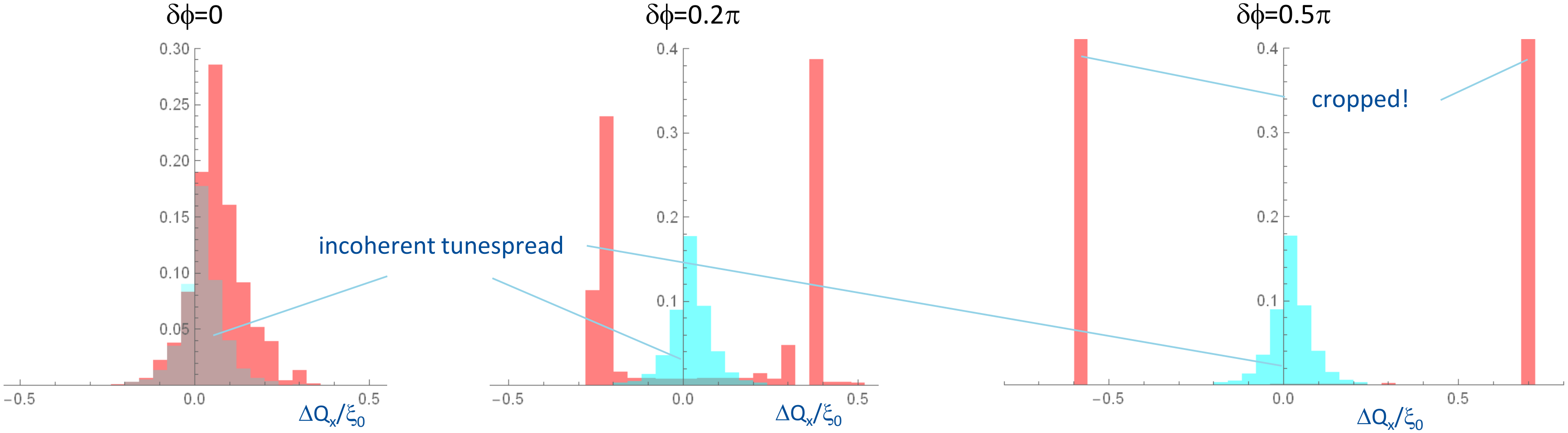
Octupoles

$$\Delta Q_x = a \cdot J_x + b \cdot J_y$$
$$\Delta Q_y = b \cdot J_x + a \cdot J_y$$

$$a = 3.28 \cdot \frac{I_{\rm oct}[\rm A] \cdot \epsilon[\rm m]}{E^2_{\rm beam}[\rm TeV^2]}$$ = 1.5 e-4 for I_{oct} = 550 A

$$b = -2.32 \cdot \frac{I_{\rm oct}[\rm A] \cdot \epsilon[\rm m]}{E^2_{\rm beam}[\rm TeV^2]}$$ = - 0.71 *a*

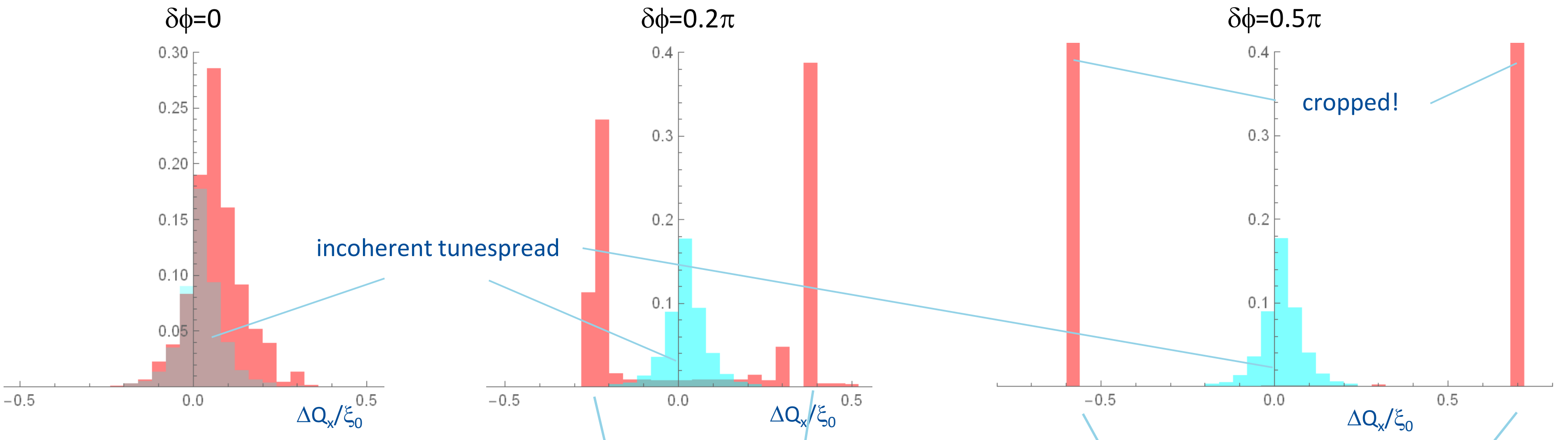
Alternating Crossing + Unequal Phase Advance



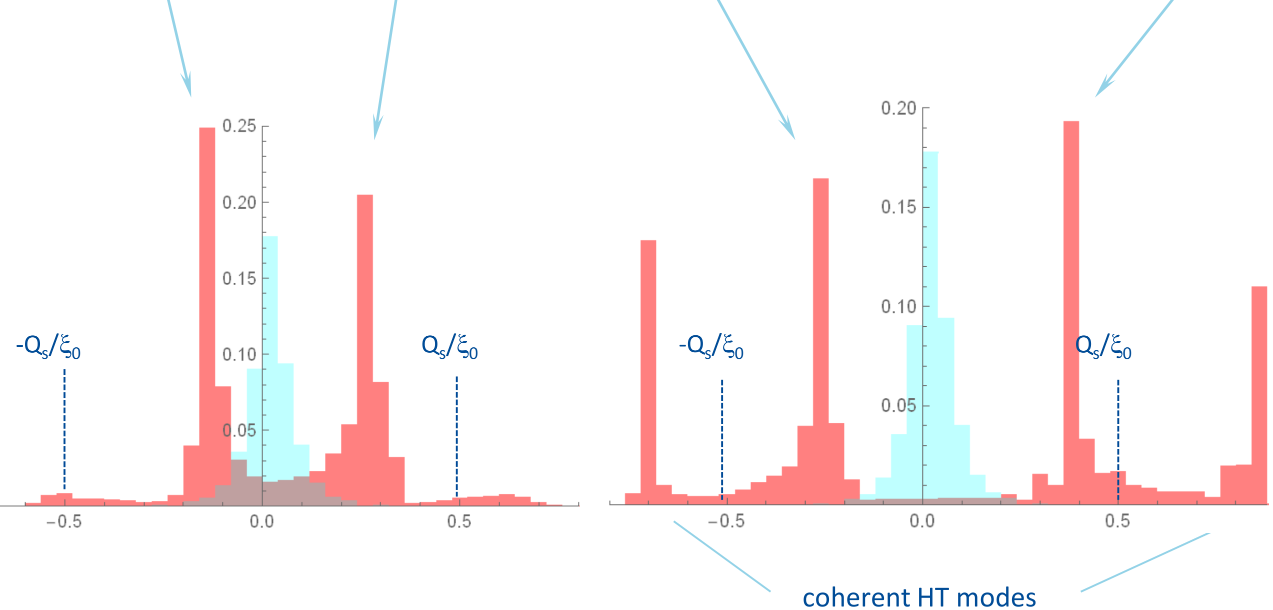
- Very large coherent tuneshifts → Landau damping switched off
- Large peak height → strong coherency
- Octupoles do not help much (included in calculations)
- There are coherent head-tail modes ($m_s \neq 0$) – see next slide

ξ_0 = beam-beam parameter / head-on IP (≈ 0.004)

… + Chromaticity = 10



- Coherent tuneshifts are significantly reduced → facilitated Landau damping
- The peak height is also reduced by a factor >2 → equivalent to reduced impedance
- Chromaticity ~10 should be enough unless there is:
  - large tuneshift by impedance
  - large beam-beam contribution to chromaticity



Summary & Outlook

- Difference in phase advance between IP1 and IP5 for the two beams is the key factor in LHC explaining the coherent beam-beam modes appearance.
- The presented analysis raises question of the beam-beam contribution to chromaticity (in the Tevatron it was up to 10 units).
- Generally, the approach based on the Vlasov eigenfunctions provides a systematic way to solve numerous problems of colliding beams stability (effects of noise, mode coupling, Landau damping etc.).
- The method is sufficiently advanced and shown to produce sensible results even with use of programs in *Mathematica* on a small laptop.
- Transition to Fortran (or C) and large computers will allow to greatly improve the quality of the results and address more demanding cases of a larger number of IPs and head-tail (synchrotron) modes (would be a good PhD thesis).

External impedance

In the presence of external impedance (wake W_1) the last term in the Vlasov operator from slide 9 becomes

$$\dot{I}_x^{(\text{ext})} = (\alpha_x x + \beta_x p_x)\,\dot{p}_x^{(\text{ext})} = -(\alpha_x x + \beta_x p_x)\frac{e^2 N_k}{2\pi}\int d^3 I' d^3\psi'\, W_1(z-z')\, x' F_1^{(k)}(I',\psi',\theta)$$

$$\xi\hat{A}^{(\text{ext})} = \frac{\beta_x e^2 N}{8\pi^2}\Psi_0(\underline{J})\int d^3 J' d\psi_s' \Psi_0(\underline{J'})\, W_1(z-z') e^{-i\chi(z-z')}$$

this is an integral operator

$$\Psi_0 = \sqrt{J_x}\, e^{-(J_x+J_y+J_s)/2}$$

(Complex) tuneshift due to the impedance is found according to the general perturbation theory formula

$$\xi\lambda^{(1)} = (\Phi_\lambda, \xi\hat{A}^{(\text{ext})}\Phi_\lambda)$$

The longitudinal profile of the eigenfunctions can be important