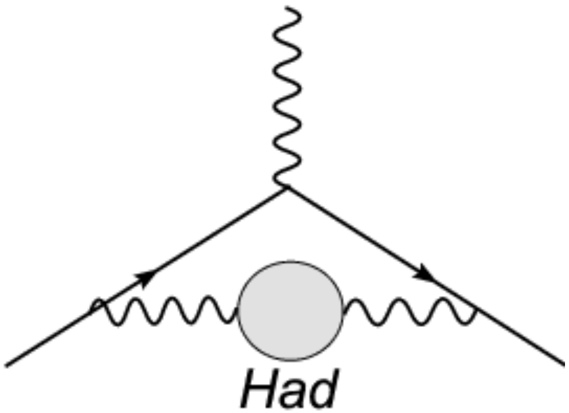
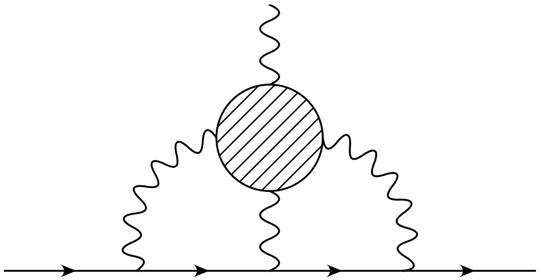
Status of g-2 (SM) theory



Taku Izubuchi
(RBC&UKQCD)



BROOKHAVEN NATIONAL LABORATORY

RIKEN BNL Research Center

2016-06-07 Flavor Physics and CP Violation (FPCP) 2016, Caltech, Pasadena, CA

Contents

To prepare for the new muon g-2 experiment at FNAL E989, and J-PARC E34, where x4 more accurate results are coming
[Themis Bowcock's talk]

- SM theory prediction for
  Hadronic Vacuum Polarization contributions (HVP)
  in addition to the determination using R-ratio from experiments
  [Liang Yan's talk]

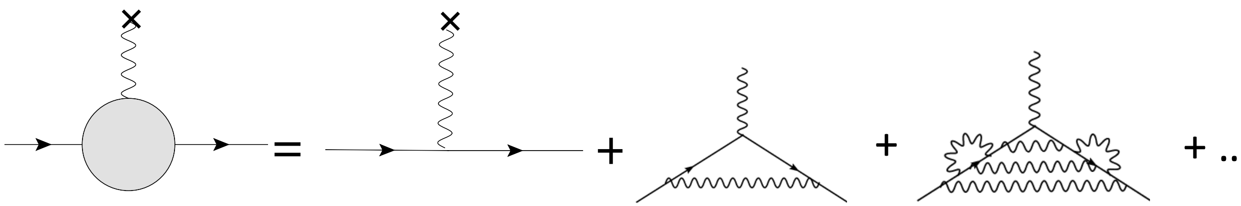
- SM theory predictions for Hadronic Light-by-Light contributions (HLbL)

(other related applications [e.g. K. Maltman's talk])

SM Theory

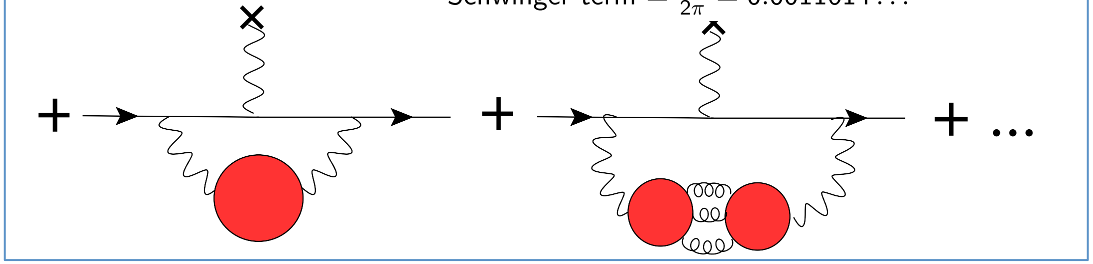
$$\gamma^{\mu} \rightarrow \Gamma^{\mu}(q) = \left(\gamma^{\mu} F_1(q^2) + \frac{i\,\sigma^{\mu\nu} q_{\nu}}{2m} F_2(q^2)\right)$$

- QED, hadronic, EW contributions

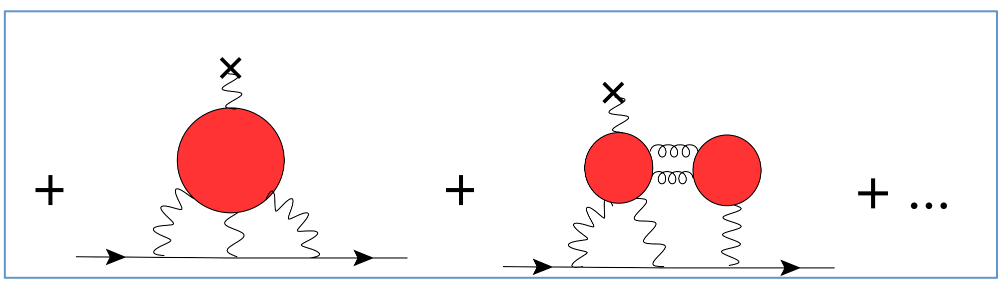


QED (5-loop !)
Aoyama et al.
PRL109,111808 (2012)

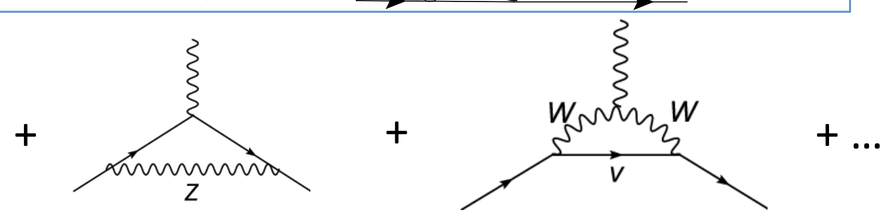
Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\ldots$



Hadronic vacuum polarization (HVP)



Hadronic light-by-light (Hlbl)



Electroweak (EW)
Knecht et al 02
Czarnecki et al. 02

$(g\text{-}2)_\mu$ SM Theory prediction

- QED, EW, Hadronic contributions

K. Hagiwara et al. , J. Phys. G: Nucl. Part. Phys. 38 (2011) 085003

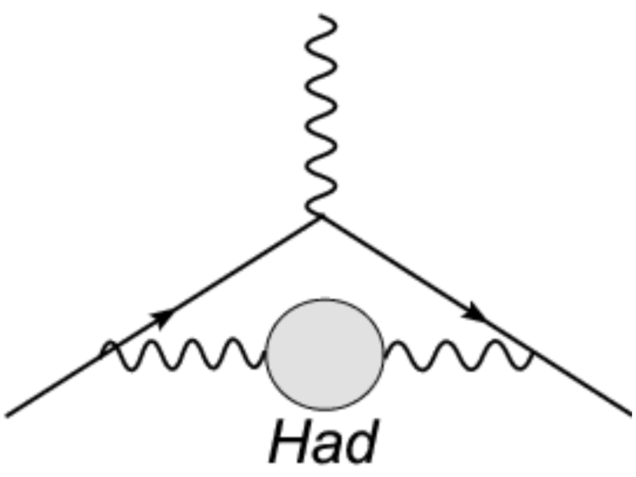
$$a_\mu^{\rm SM} = (11 \quad 659 \quad 182.8 \quad \pm 4.9 \quad) \times 10^{-10}$$

$$a_\mu^{\rm QED} = (11 \quad 658 \quad 471.808 \quad \pm 0.015 \quad) \times 10^{-10}$$
$$a_\mu^{\rm EW} = (\quad 15.4 \quad \pm 0.2 \quad) \times 10^{-10}$$
$$a_\mu^{\rm had,LOVP} = (\quad 694.91 \quad \pm 4.27 \quad) \times 10^{-10}$$
$$a_\mu^{\rm had,HOVP} = (\quad -9.84 \quad \pm 0.07 \quad) \times 10^{-10}$$
$$a_\mu^{\rm had,lbl} = (\quad 10.5 \quad \pm 2.6 \quad) \times 10^{-10}$$

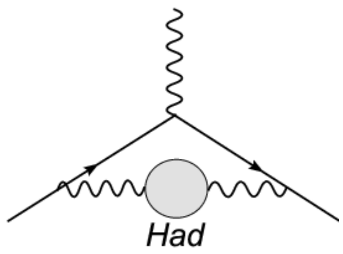
$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 28.8(6.3)_{\rm exp}(4.9)_{\rm SM} \times 10^{-10} \quad [3.6\sigma]$$

- Discrepancy between EXP and SM is larger than EW!
- Currently the dominant uncertainty comes from HVP, followed by HLbL
- x4 or more accurate experiment FNAL , J-PARC
- Goal : sub 1% accuracy for HVP (intermediate goal) → 10% accuracy for HLbL

Hadronic Vacuum Polarization (HVP) contributions



Leading order of hadronic contribution (HVP)



- Hadronic vacuum polarization (HVP)

V_μ  V_ν $= (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi_V(q^2)$

quark's EM current : $V_\mu = \sum_f Q_f \bar{f}\gamma_\mu f$

- Optical Theorem (Unitarity) $\mathrm{Im}\Pi_V(s) = \frac{s}{4\pi\alpha}\sigma_{\mathrm{tot}}(e^+e^- \to X)$

- Analyticity $\Pi(q^2) - \Pi(0) = \frac{1}{\pi}\int ds \frac{\mathrm{Im}\Pi(s)}{s(s-q^2)}$

- expedrimental determination [L. Yan's talk]

a_μ ~ 693(4) [0.6 % err , largest err in SM theory]

$$a_\mu^{\mathrm{HVP,LO}} = \frac{1}{4\pi^2}\int ds\, K(s)\sigma_{\mathrm{had}}(s)$$

[T. Blum PRL91 (2003) 052001]

HVP from Lattice

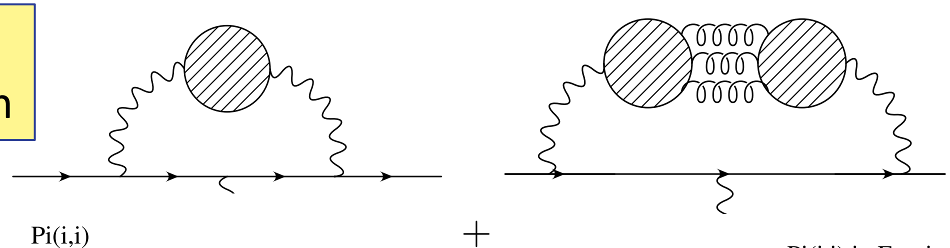
- Analytically continue to Euclidean/space-like momentum $K^2 = -q^2 > 0$
- Vector current 2pt function

$$a_\mu = \frac{g-2}{2} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dK^2 f(K^2)\hat{\Pi}(K^2)$$

$$\Pi^{\mu\nu}(q) = \int d^4x e^{iqx} \langle J^\mu(x) J^\nu(0) \rangle$$

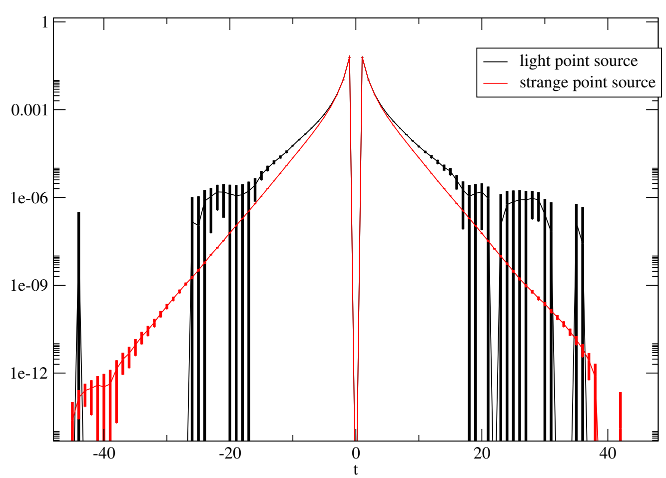
- Low Q2, or long distance, part of Π (Q2) is relevant for g-2

connected contribution

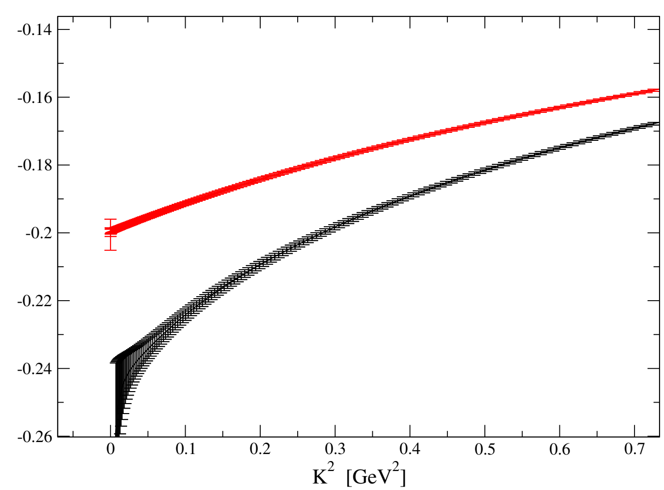


disconnected contribution

Pi(i,i)



Pi(i,i) in Fourier space vs K2



Current conservation, subtraction, and coordinate space representation

- Current conservation => transverse tensor

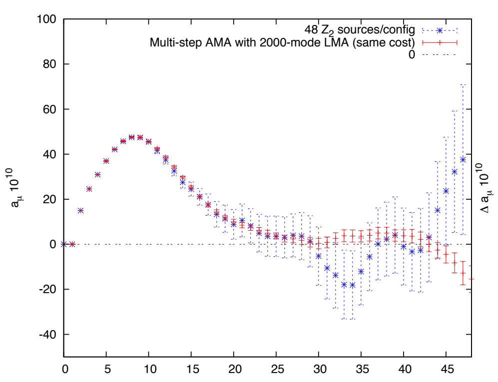
$$\sum_x e^{iQx}\langle J_\mu(x)J_\nu(0)\rangle = (\delta_{\mu\nu}Q^2 - Q_\mu Q_\nu)\Pi(Q^2)$$

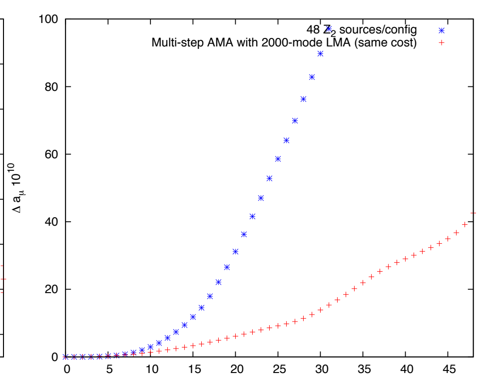
- Coordinate space vector 2 pt Green function C(t) is directly related to subtracted Π (Q2) [Bernecker-Meyer 2011, …]

$$\Pi(Q^2) - \Pi(0) = \sum_t \left(\frac{\cos(qt) - 1}{Q^2} + \frac{t^2}{2} \right) C(t)$$

- g-2 value is also related to C(t) with know kernel w(t) from QED.

$$a_\mu^{\mathrm{HVP}} = \sum_t w(t)C(t), \quad w(t) \propto t^4 \cdots$$



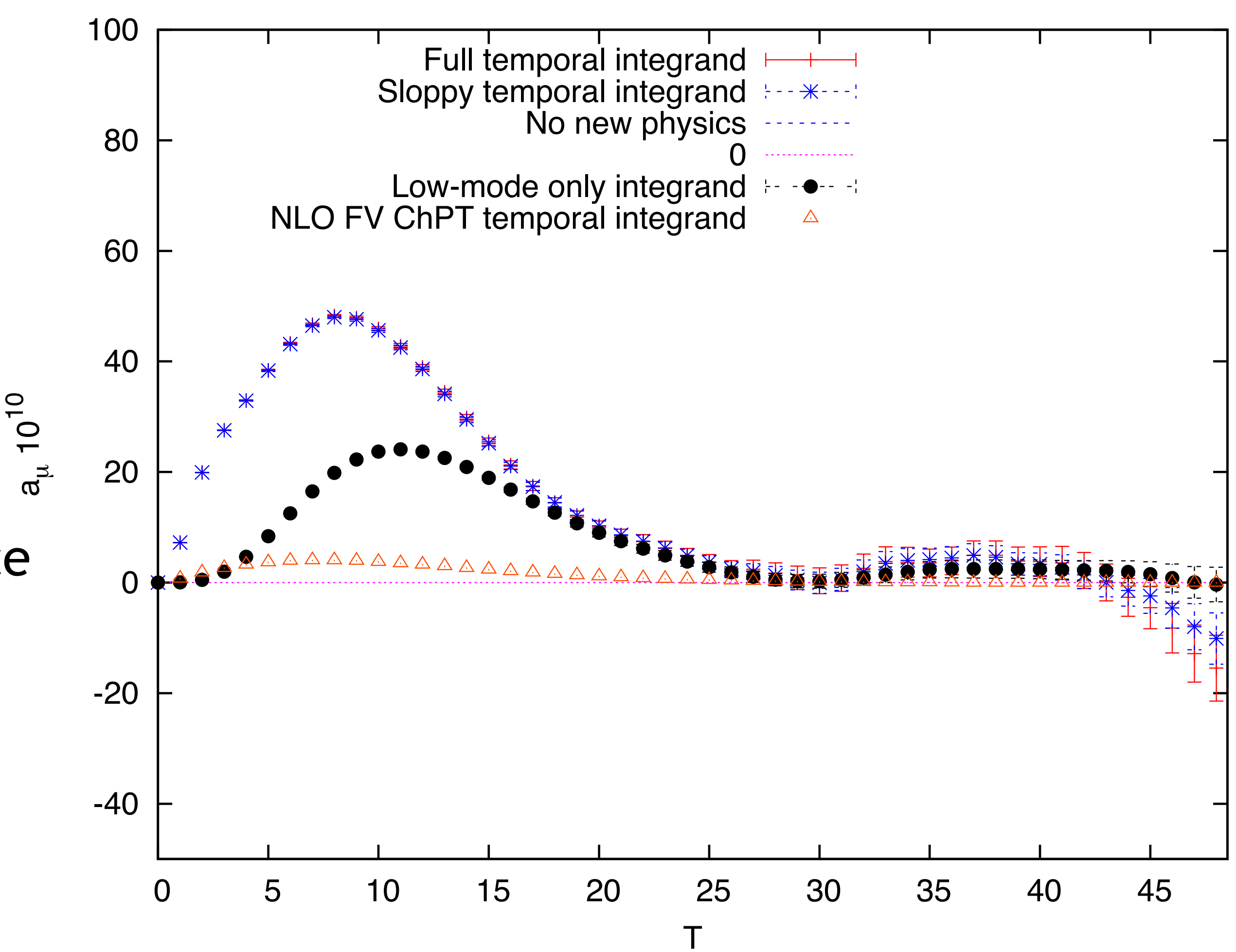


RBC/UKQCD
Chiral Lattice quark DWF
physical point
Quark Propagator Low Mode (A2A)
using All-Mode Averaging (AMA)

RBC/UKQCD Light contribution

- Use three stages of approximations with bias-correction
- Low mode approximation with sloppy calculation
- Low mode dominance for long distance
- compared with two pions model sQED.

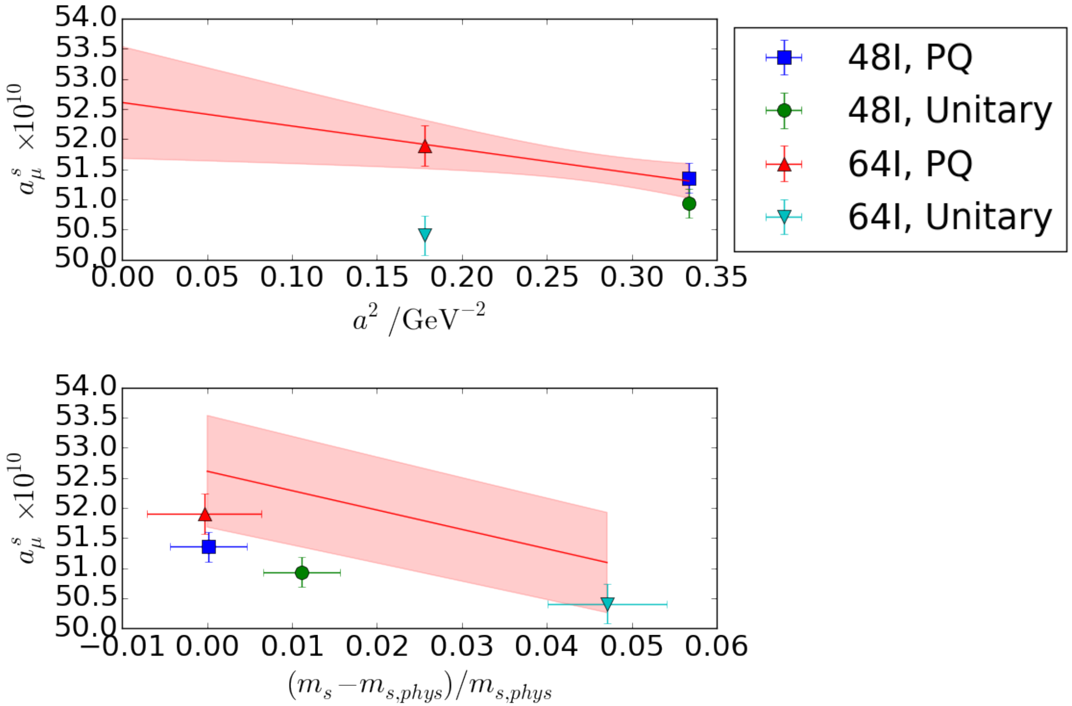
[C. Lehner preliminary]



Strange quark contribution

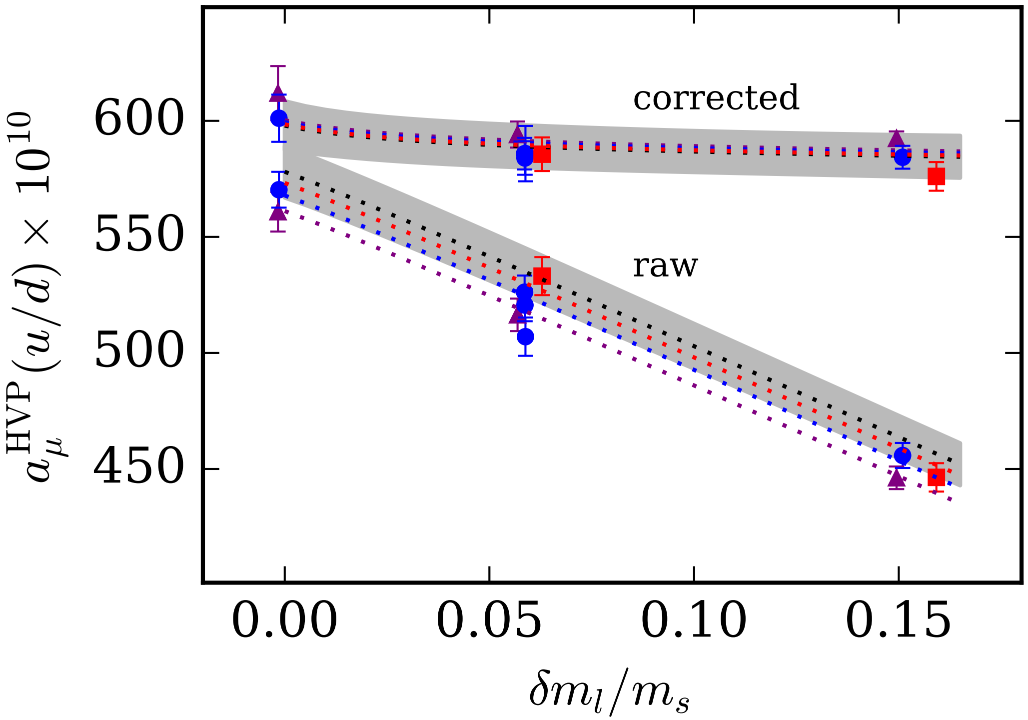
[RBC/UKQCD, JHEP 1604 (2016) 063]

- Mobius DWF, Nf=2+1,Physical mass, L=5.5fm, a=0.114, 0.09 fm
- Many fits, moment, and cuts are used to examine systematics
- parts of systematic errors are being estimated
- consistent with HPQCD's value (next page)



HPQCD light quark HVP

Chakraborty et al. arXiv:1601.03071, PRD93.074509, ...



- a=0.09, 0.12, 0.15 fm
- switch to multi-exp at t*=1.5fm
- sub 2% total error !
- Modeling ρ correction + ChPT pipi sub/add

→ a few percent correction at physical point

- Large finite volume effects, even for L~ 5.8fm, 5.1 fm at physical poit
- also from taste pion effects to pipi amplitude
- estimate disc. loop

$$\hat{\Pi}_j^{\mathrm{latt}} \to \left(\hat{\Pi}_j^{\mathrm{latt}} - \hat{\Pi}_j^{\mathrm{latt}}(\pi\pi)\right)\left[\frac{m_\rho^{2+2j}}{f_\rho^2}\right]_{\mathrm{latt}}\left[\frac{f_\rho^2}{m_\rho^{2+2j}}\right]_{\mathrm{expt}} + \hat{\Pi}_j^{\mathrm{cont}}(\pi\pi)$$

HPQCD g-2 HVP results

- Carried out up/down, strange, charm, bottom connected contributions

$$a_\mu^{\rm HVP,LO}\Big|_{\rm conn.} \times 10^{10} = \begin{cases} 598(11) & \text{from } u/d \text{ quarks} \\ 53.4(6) & \text{from } s \text{ quarks} \\ 14.4(4) & \text{from } c \text{ quarks} \\ 0.27(4) & \text{from } b \text{ quarks} \end{cases}$$

- together with disconnected

$$a_\mu^{\rm HVP,LO} = 666(6)(12) \times 10^{-10}$$ 2 % err, important to check

vs $a_\mu^{HVP,LO}(\text{R-ratio}) = 694.91(3.72)_{\rm exp}(2.10)_{\rm rad} \times 10^{-10}$ 0.6 % err [Hagiwara et al, 2011]

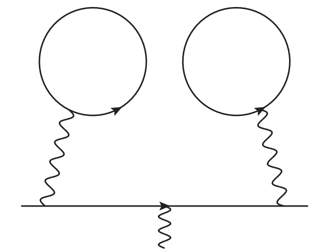
- QED/isospin breaking effects are folded into systematic error

	$a_\mu^{\rm HVP,LO}(u/d)$
QED corrections:	1.0 %
Isospin breaking corrections:	1.0 %
Staggered pions, finite volume:	0.7 %
Valence m_ℓ extrapolation:	0.4 %
Monte Carlo statistics:	0.4 %
Padé approximants:	0.4 %
$a^2 \to 0$ extrapolation:	0.3 %
Z_V uncertainty:	0.4 %
Correlator fits:	0.2 %
Tuning sea-quark masses:	0.2 %
Lattice spacing uncertainty:	< 0.05 %
Total:	1.8 %

disconnected quark loop contribution

- [C. Lehner et al. (RBC/UKQCD 2015, arXiv:1512.09054, PRL)]
- Very challenging calculation due to statistical noise
- Small contribution, vanishes in SU(3) limit,

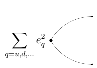
  Qu+Qd+Qs = 0

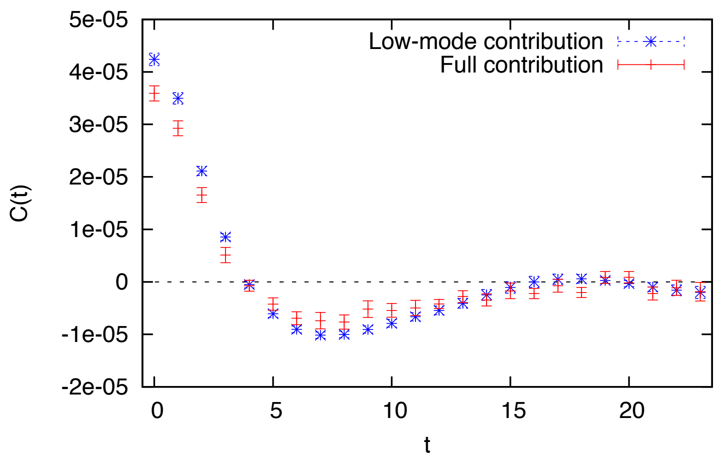


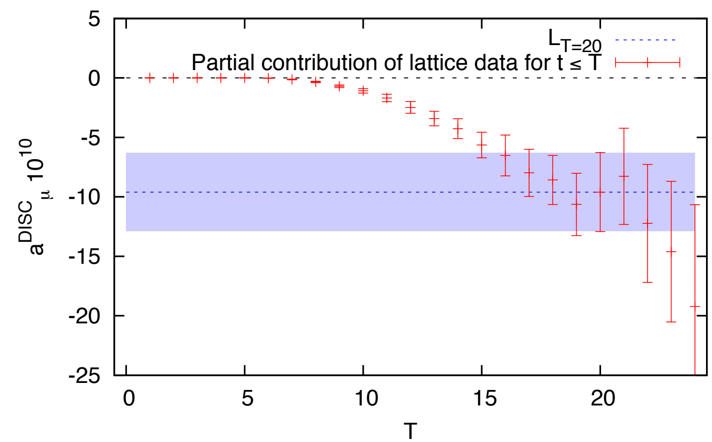
- Use low mode of quark propagator, treat it exactly

  (all-to-all propagator with sparse random source)
- First non-zero signal

$$a_\mu^{\mathrm{HVP\ (LO)\ DISC}} = -9.6(3.3)_{\mathrm{stat}}(2.3)_{\mathrm{sys}} \times 10^{-10}$$

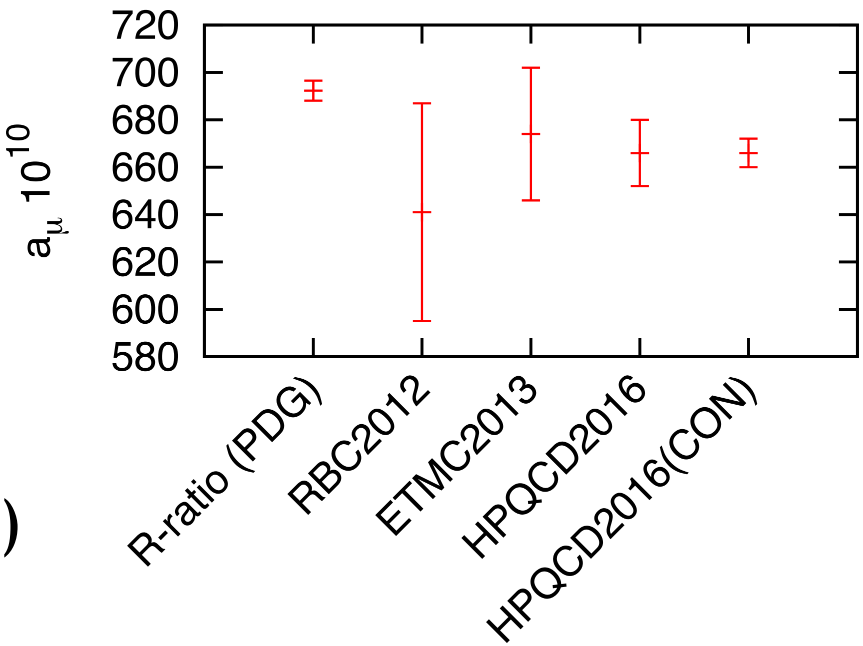






HVP on lattice summary

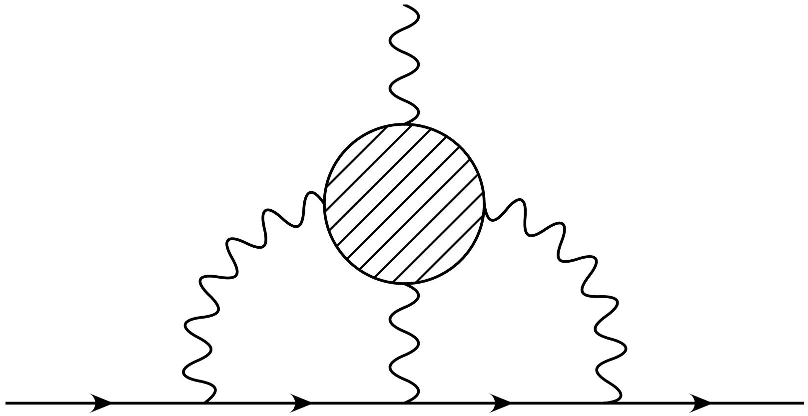
- First principle HVP from lattice making substantial progress by many groups



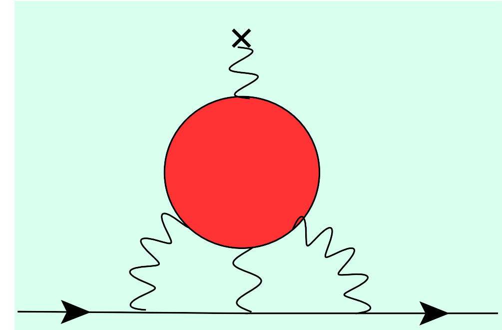
[Plot from C. Lehner]

- Challenges
  - Statistics (→ low mode)
  - Disconnected
    (→ SU(3), low mode + space src.)
  - Finite volume (→ ππ models ?)
  - QED and isospin breaking
- Other applications : CKM Vus from τ inclusive decay [K. Maltmann's talk] , $\alpha_{QED}(s)$, sin $\theta_W(s)$ running

Hadronic Light-by-Light (HLbL) contributions



Hadronic Light-by-Light



- 4pt function of EM currents
- No experimental data directly help
- Dispersive approach

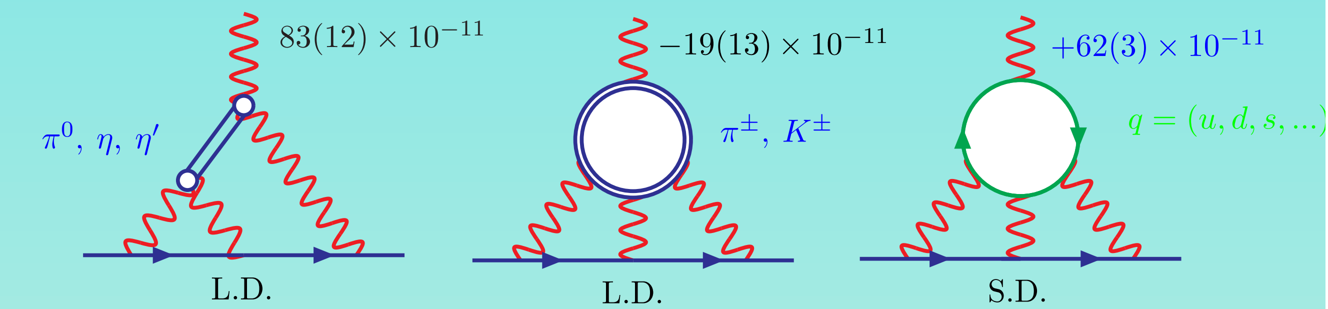
$$\Gamma_\mu^{(\mathrm{Hlbl})}(p_2,p_1) = ie^6 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2)}{k_1^2\,k_2^2\,k_3^2} \times\gamma_\nu S^{(\mu)}(\not p_2+\not k_2)\gamma_\rho S^{(\mu)}(\not p_1+\not k_1)\gamma_\sigma$$

$$\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_3,k_2) = \int d^4x_1\,d^4x_2\,d^4x_3\,\exp[-i(k_1\cdot x_1+k_2\cdot x_2+k_3\cdot x_3)] \times\langle 0|T[j_\mu(0)j_\nu(x_1)j_\rho(x_2)j_\sigma(x_3)]|0\rangle$$

$$\text{Form factor}:\ \Gamma_\mu(q)=\gamma_\mu\,F_1(q^2)+\frac{i\sigma^{\mu\nu}q_\nu}{2\,m_l}\,F_2(q^2)$$

HLbL from Models

- Model estimate with non-perturbative constraints at the chiral / low energy limits using anomaly : (9—12) x 10^{-10} with 25-40% uncertainty



Jegerlehner & Nyffeler 09

Contribution	BPP	HKS	KN	MV	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	0±10	−19±19	−19±13
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	15±10	22± 5
scalars	−6.8±2.0	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	105±26	116±39

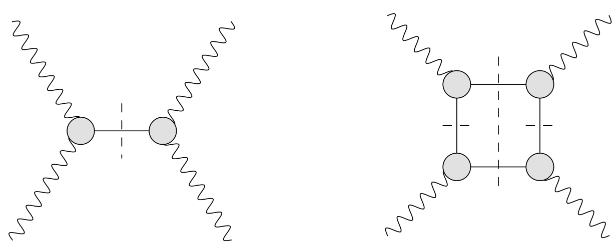
Dispersive analysis for HLbL

[Colangelo et al. 2014, 2015, Pauk&Vanderhaeghen 2014]

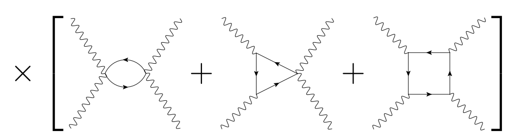
- Using crossing-symmetry, gauge invariance, 138 form factors are reduced to 12 scalars relevant for g-2 LbL

$$a_\mu^{\mathrm{HLbL}} = e^6 \int \frac{d^4 q_1}{(2\pi)^4} \frac{d^4 q_2}{(2\pi)^4} \frac{\sum_{i=1}^{12} \hat{T}_i(q_1, q_2; p) \hat{\Pi}_i(q_1, q_2, -q_1 - q_2)}{q_1^2 q_2^2 (q_1 + q_2)^2 [(p+q_1)^2 - m_\mu^2][(p-q_2)^2 - m_\mu^2]}$$

- Formalism for Pion exchange, and Pion box diagram. Latter is related sQED with pion's vector form factor



$$\equiv F_\pi^V(q_1^2) F_\pi^V(q_2^2) F_\pi^V(q_3^2)$$



- Other contributions neglected

Direct 4pt calculation for selected kinematical range

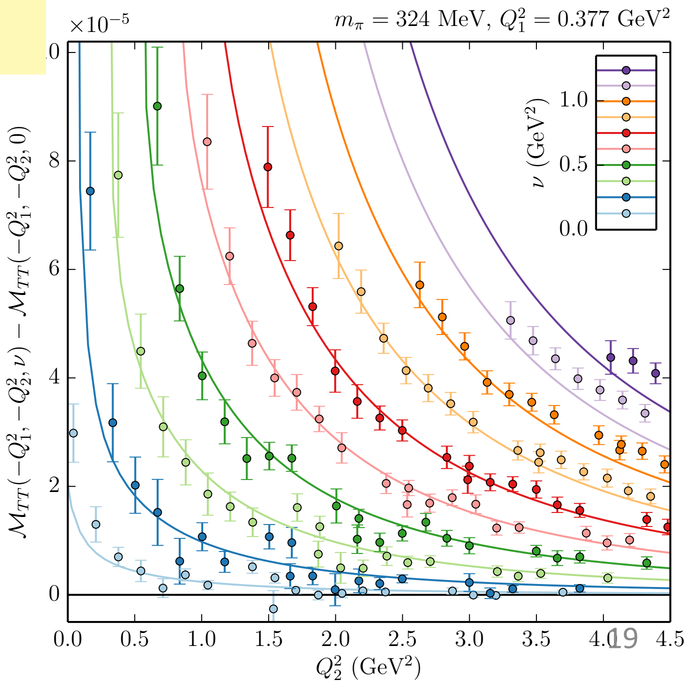
[J. Green et al. Mainz group, Phys. Rev. Lett 115, 222003(2015)]

- Compute connected contribution of 4 pt function in momentum space
- forward amplitudes related to $\gamma^*(Q1)\gamma^*(Q2)$ -> hadron cross sections via dispersion relation, allowed comparison among lattice and experiments/ phenomenological models

$$\mathcal{M}_{\text{had}}\left(\gamma^*(Q_1)\gamma^*(Q_2) \to \gamma^*(Q_1)\gamma^*(Q_2)\right)$$
$$\leftrightarrow \quad \sigma_{0,2}\left(\gamma^*(Q_1)\gamma^*(Q_2) \to \text{had.}\right)$$

$$\nu = -Q_1 \cdot Q_2$$

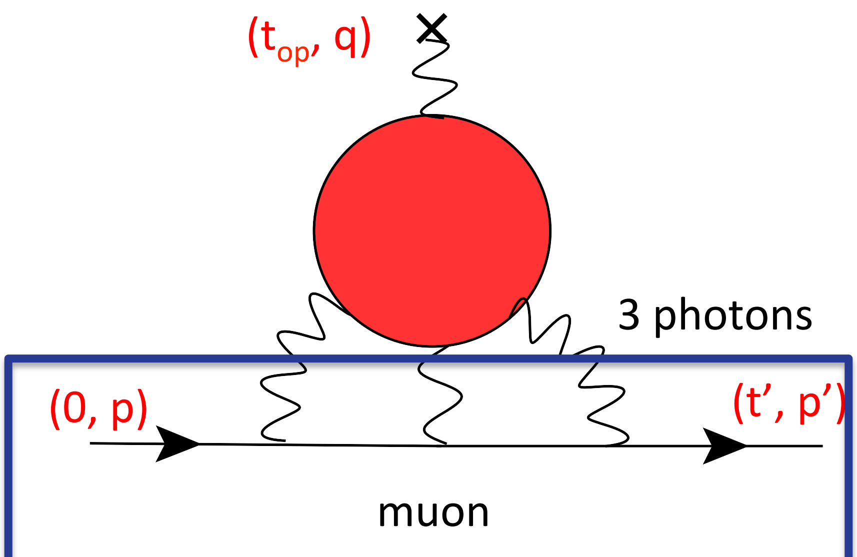
- Solid curve : model prediction
- π^0 exchange is seen to be not dominant, possibly due to heavy quark mass in the simulation (Mπ = 324 MeV)
- disconnected quark loop in progress (2016)



Our Basic strategy : Lattice QCD+QED system

- 4pt function has too much information to parameterize (?)
- Do Monte Carlo integration for QED two-loop with 4 pt function $\pi^{(4)}$ which is sampled in lattice QCD
- Photon & lepton part of diagram is derived either in lattice QED+QCD [Blum et al 2014] (stat noise from QED), or exactly derive for given loop momenta [L. Jin et al 2015] (no noise from QED+lepton).

$$\Gamma_\mu^{(\mathrm{Hlbl})}(p_2,p_1) = ie^6 \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\Pi^{(4)}_{\mu\nu\rho\sigma}(q,k_1,k_2,k_3)$$
$$\times\left[S(p_2)\gamma_\nu S(p_2+k_2)\gamma_\rho S(p_1+k_1)\gamma_\sigma S(p_1) + (\mathrm{perm.})\right]$$

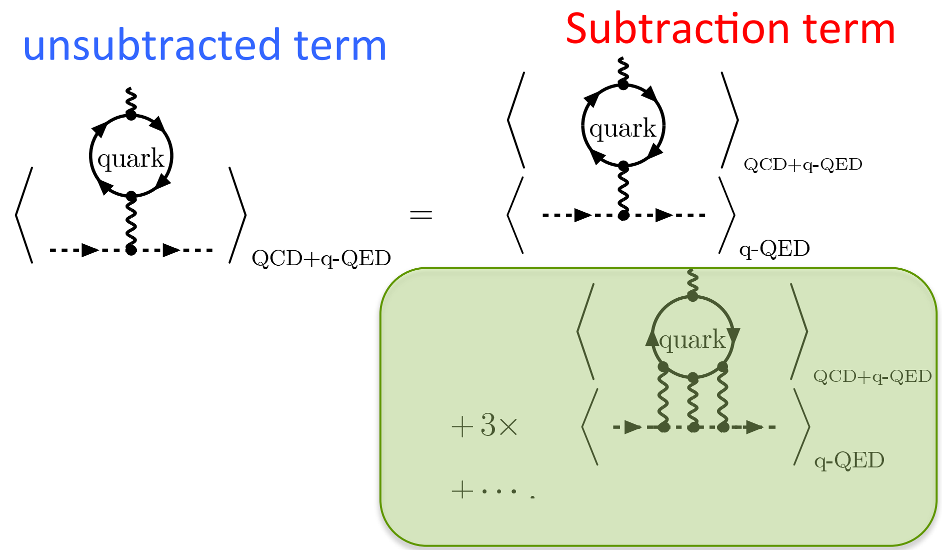


- set spacial momentum for
  - external EM vertex q
  - in- and out- muon p, p’
  q = p-p’
- set time slice of muon source(t=0), sink(t’) and operator (t_{op})
- take large time separation for ground state matrix element

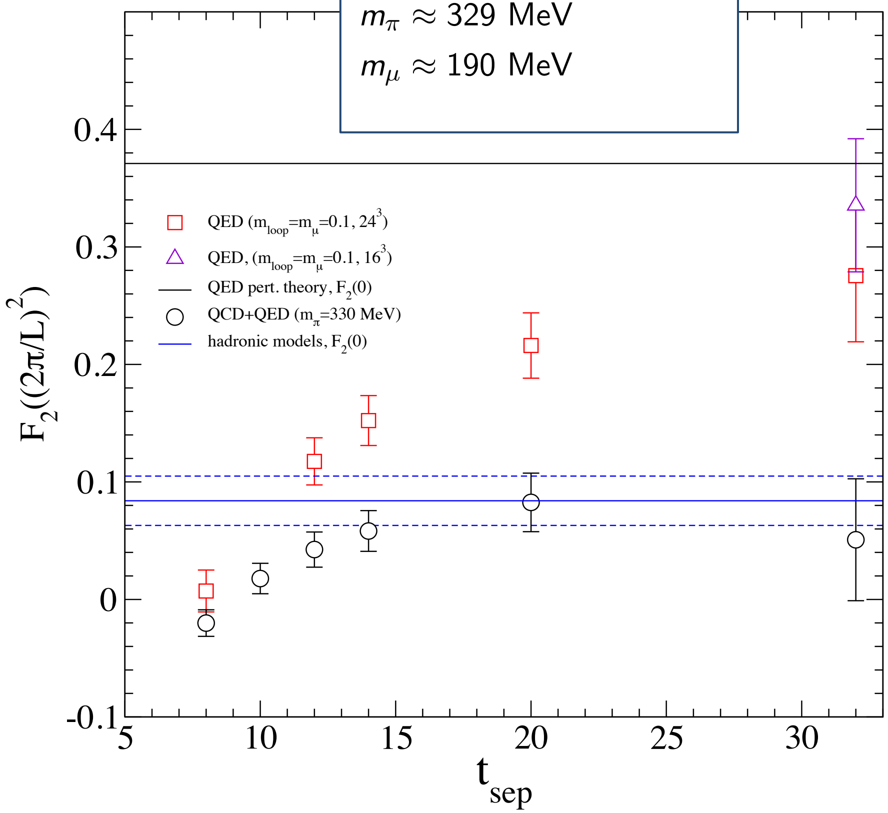
QCD+QED method

[Blum et al PRL 114, 012001 (2015)]

- One photon is treated analytically
- other two sampled stochastically
- needs subtraction
- use AMA for error reduction
- use Furry's theoretm to reduce α^2 noise



24^3 lattice size
$Q^2 = 0.11$ and 0.18 GeV^2
$m_\pi \approx 329$ MeV
$m_\mu \approx 190$ MeV



- Connected part only

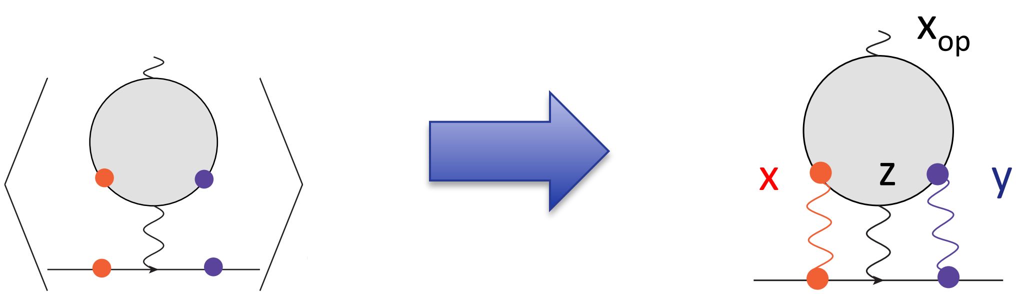
- QED only calculation consistent with QED loop calculation for larger volume

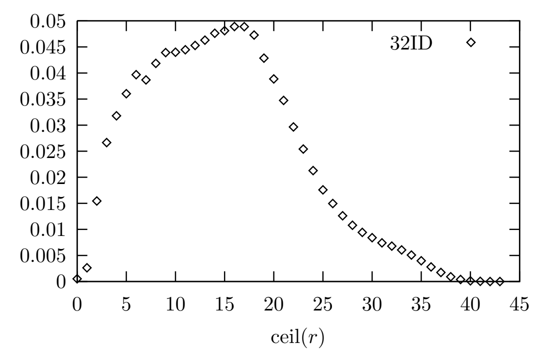
- QED+QCD
  - ball park of model values
  - significant exited state effects ?

Coordinate space Point photon method

[Luchang Jin et al. , PRD93, 014503 (2016)]

- Treat all 3 photon propagators exactly (3 analytical photons) , which makes the quark loop and the lepton line connected :
  disconnected problem in Lattice QED+QCD -> connected problem with analytic photon
- QED 2-loop in coordinate space. Stochastically sample, two of quark-photon vertex location x,y, z and x_{op} is summed over space-time exactly

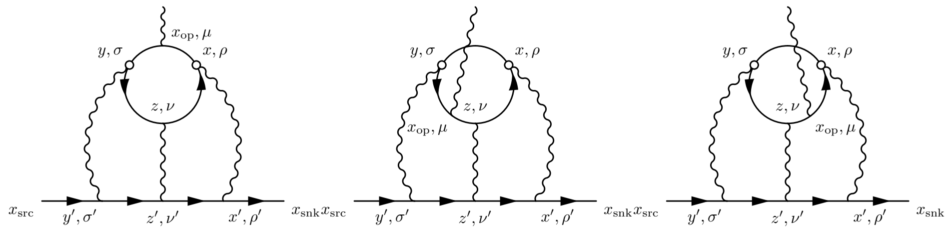




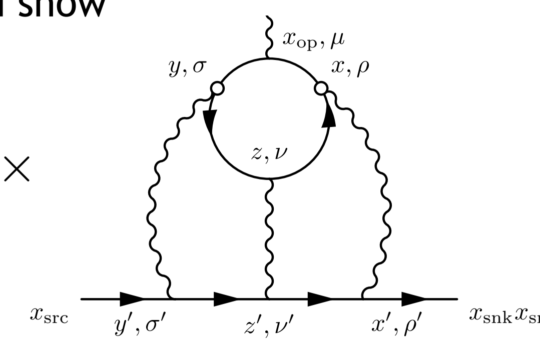
- Short separations, Min[|x-z|,|y-z|,|x-y|] < R ~ O(0.5) fm, which has a large contribution due to confinement, are summed for all pairs
- longer separations, Min[|x-z|,|y-z|,|x-y|] >= R, are done stochastically with a probability shown above (Adaptive Monte Carlo sampling)

Conserved current & moment method

- [conserved current method at finite q2] To tame UV divergence, one of quark-photon vertex (external current) is set to be conserved current (other three are local currents). All possible insertion are made to realize conservation of external currents config-by-config.



- [moment method , q2→0] By exploiting the translational covariance for fixed external momentum of lepton and external EM field, q->0 limit value is directly computed via the first moment of the relative coordinate, xop – (x+y)/2, one could show



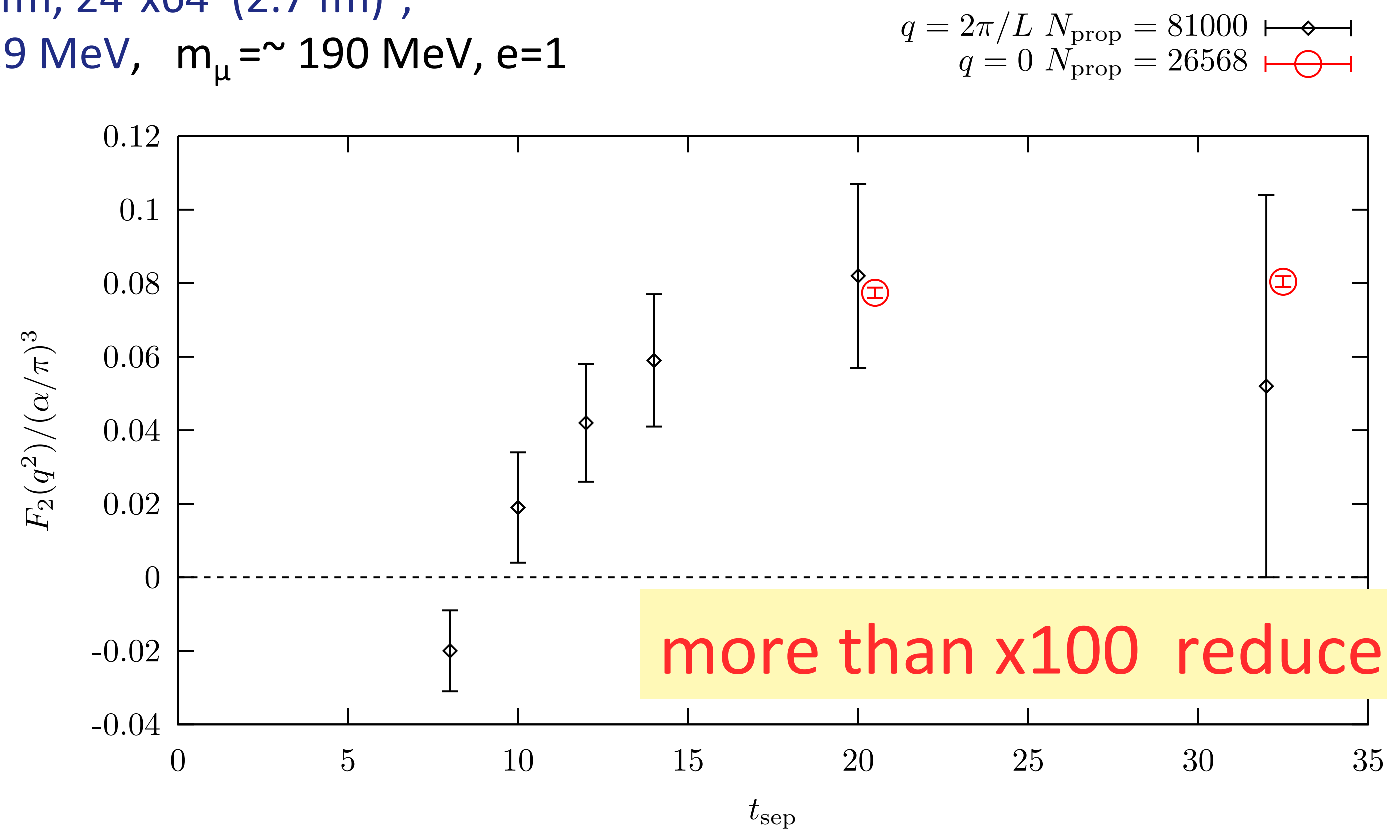
$$\frac{\partial}{\partial q_i}\mathcal{M}_\nu(\vec{q})|_{\vec{q}=0} = i \sum_{x,y,z,x_{op}} (x_{op} - (x+y)/2)_i \times$$

to directly get $F_2(0)$ without extrapolation.

$$\text{Form factor : } \Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2\, m_l} F_2(q^2)$$

Dramatic Improvement ! Luchang Jin

a=0.11 fm, 24^3x64 (2.7 fm)^3,
m_π = 329 MeV, m_μ =~ 190 MeV, e=1



more than x100 reduced cost !

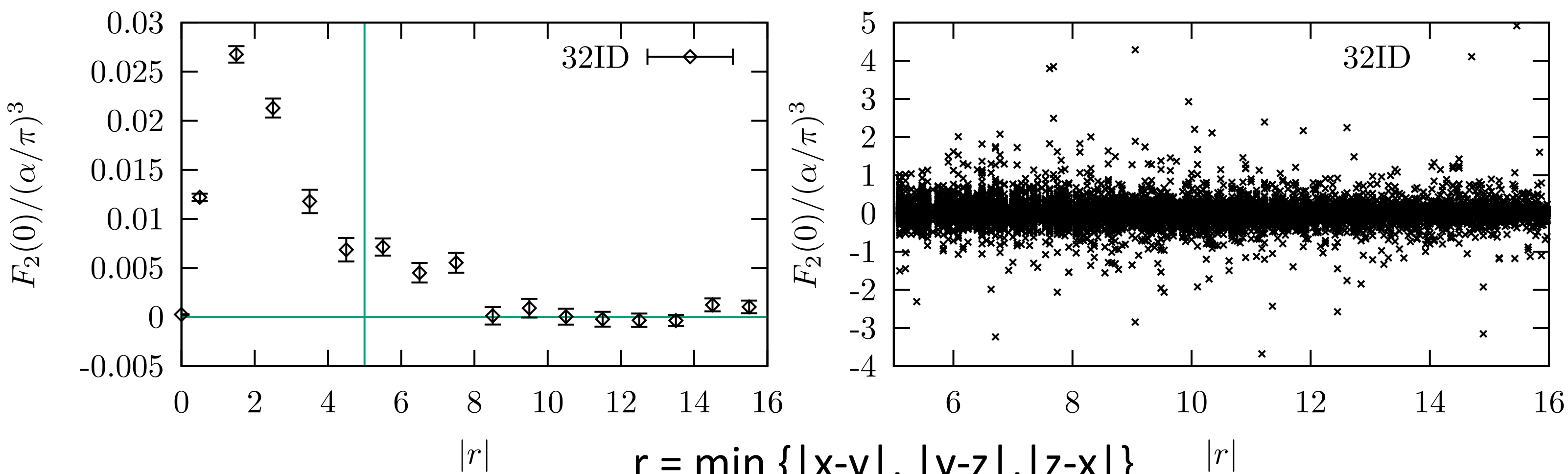
Method	$F_2/(\alpha/\pi)^3$	N_{conf}	N_{prop}	$\sqrt{\text{Var}}$
Conserved	0.0825(32)	12	$(118+128)\times 2\times 7$	0.65
Mom.	0.0804(15)	18	$(118+128)\times 2\times 3$	0.24

M_π=170 MeV cHLbL result

[Luchang Jin et al. , PRD93, 014503 (2016)]

- V=(4.6 fm)^3, a = 0.14 fm, m_μ=130 MeV, 23 conf
- pair-point sampling with AMA (1000 eigV, 100CG) , > 6000 meas/conf
  - |x-y| <= 5, all pairs, x2-5 samples for shorter distances, 217 pairs (10 AMA-exact)
  - |x-y| > 5, 512 pairs (48 AMA-exact)
- 13.2 BG/Q Rack-days

Integrand of F2



r = min {|x-y|, |y-z|,|z-x|}

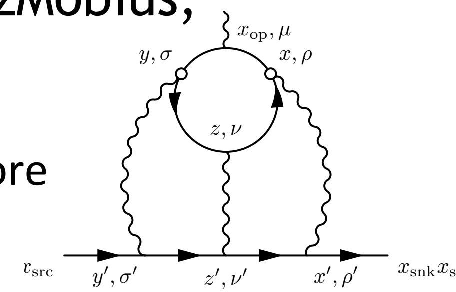
$$\frac{(g_\mu - 2)_{\rm cHLbL}}{2} = (0.1054 \pm 0.0054)(\alpha/\pi)^3 = (132.1 \pm 6.8) \times 10^{-11}.$$

Strange contribution : (0.0011± 0.005) $(\alpha/\pi)^3$

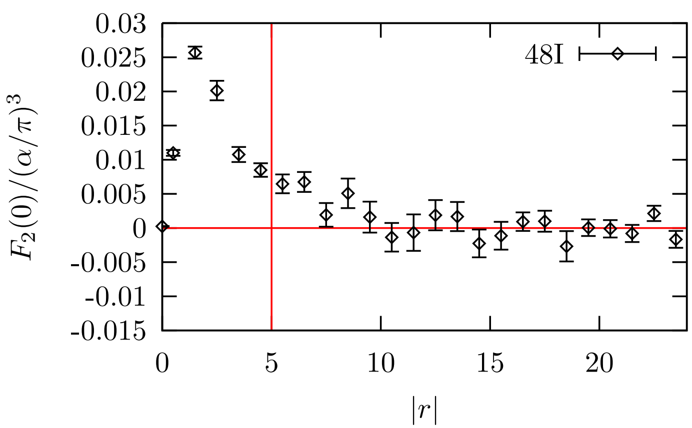
physical M_π=140 MeV cHLbL result

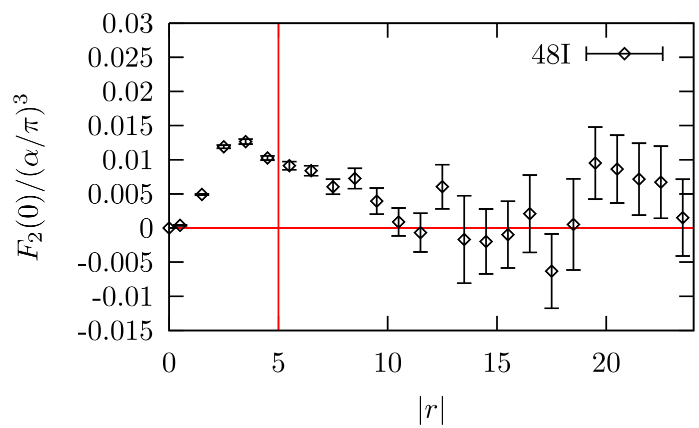
[Luchang Jin et al. , preliminary]

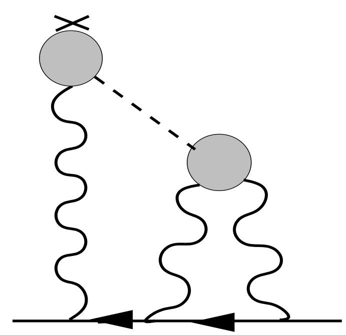
- V=(5.5 fm)^3, a = 0.11 fm, m_μ=106 MeV, 69 conf [RBC/UKQCD]
- Two stage AMA (2000 eigV, 200CG and 400 CG) using zMobius, ~4500 meas/conf
- 160 BG/Q Rack-days



integrand safely suppressed before reaching r ~ L/2







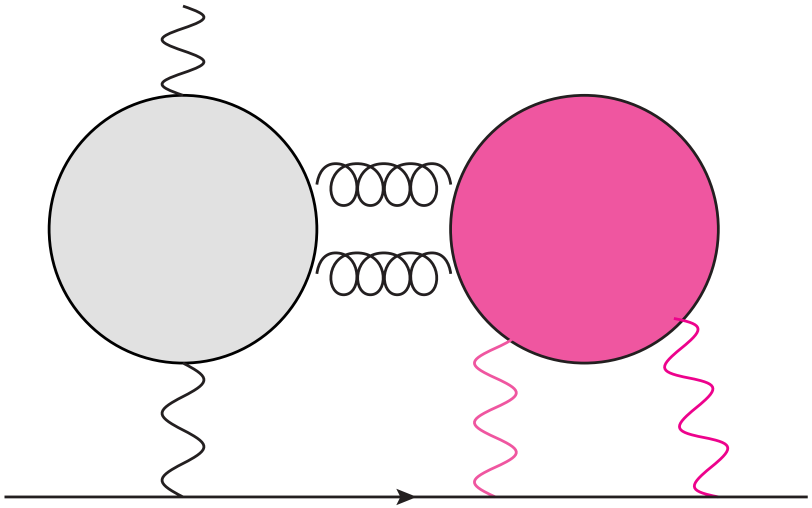
r = min {|x-y|, |y-z|,|z-x|}   r = max{|x-y|, |y-z|,|z-x|}

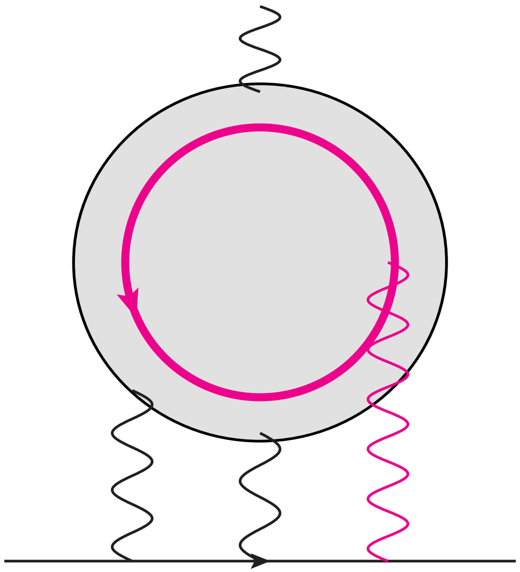
$$\frac{(g_\mu - 2)_{\mathrm{cHLbL}}}{2} = (0.933 \pm 0.0073)(\alpha/\pi)^3 = (116.9 \pm 9.1) \times 10^{-11}$$

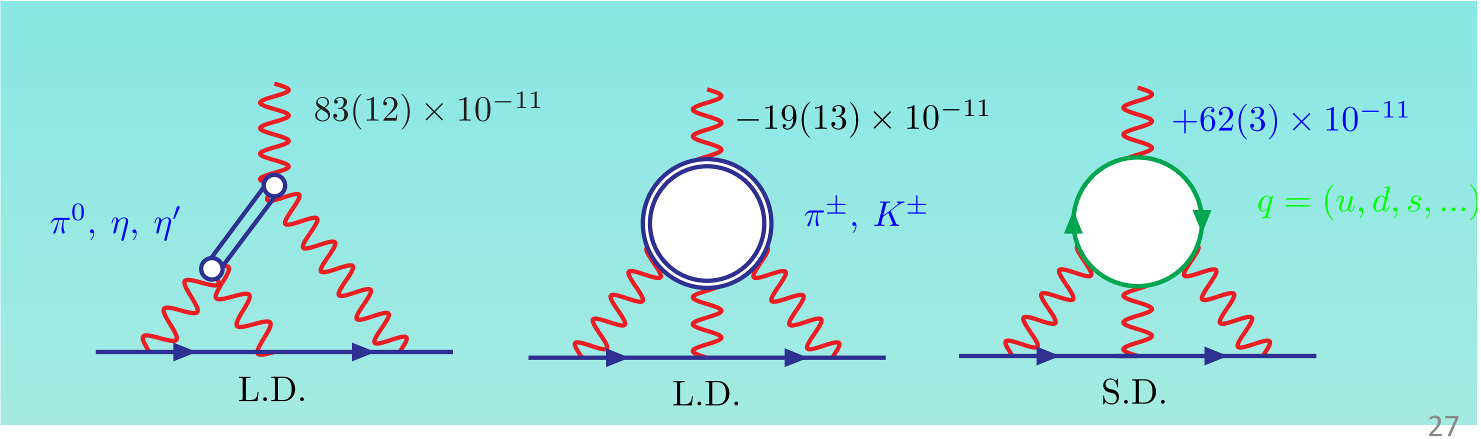
(preliminary, stat err only)

Disconnected diagrams in HLbL

- Disconnected diagrams

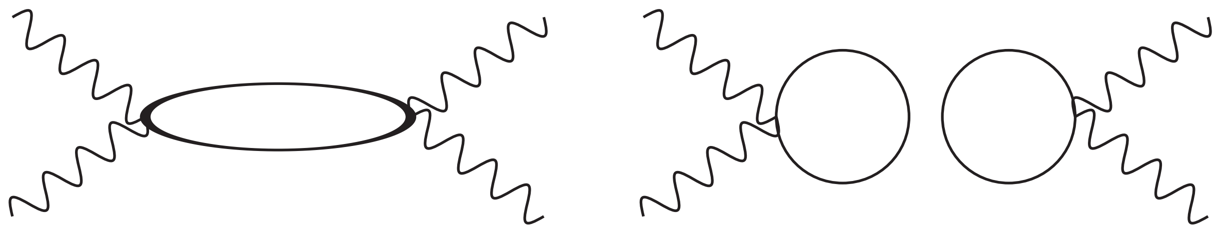






Disconnected HLbL would be non-negligible

- The major contribution, single π^0 (and η, η') exchange diagrams through $\gamma^*\gamma^*\to\pi^0$, would have both connected and disconnected contributions.

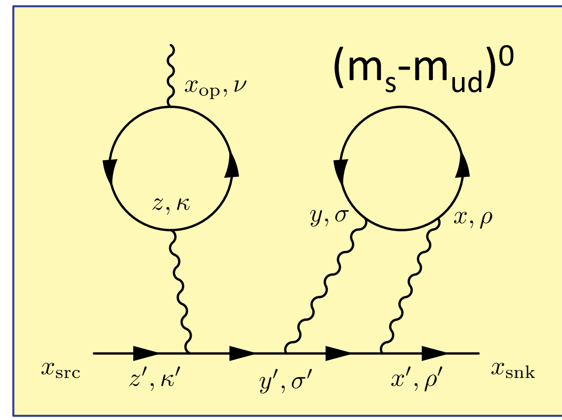


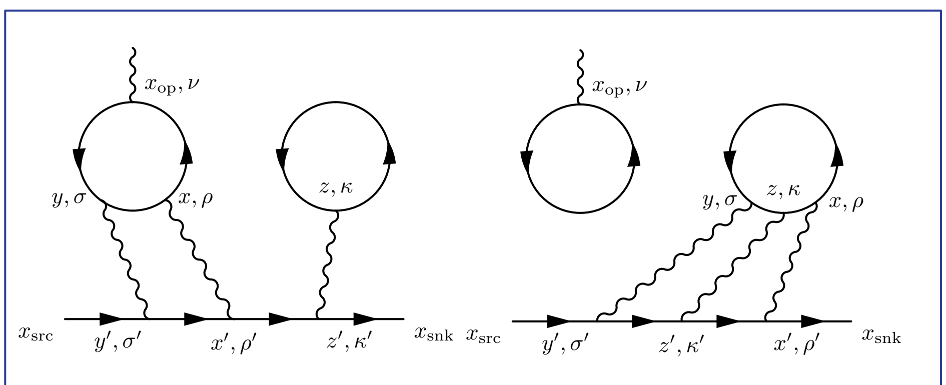
- Simple quark model consideration for LbL pi0 exchange turns out to be Con : DisCon roughly same size with opposite sign (34:-25)

- Good news : it's not η' (only), so S/N would not grow exponentially with the propagation length.
- Bad news : it's disconnected quark loops, and many of them.

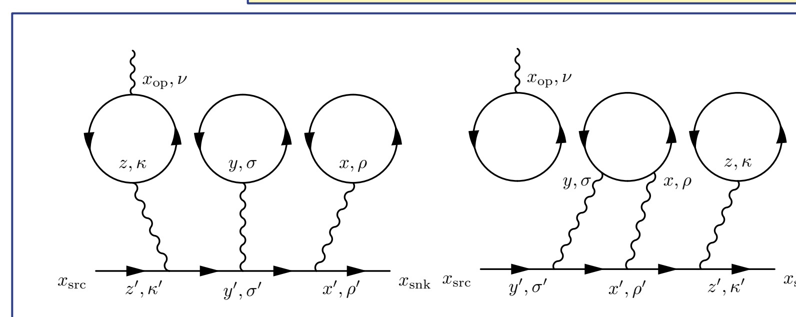
SU(3) hierarchies for d-HLbL

- At $m_s = m_{ud}$ limit, following type of dHLbL survives due to
  Qu + Qd + Qs = 0
- Physical point run is in progress using similar techniques to c-HLbL.
  preliminary result
  a negative value with ~30% stat err
- $O(m_s - m_{ud})/3$ and $O((m_s - m_{ud})^2)$





$(m_s - m_{ud})^1$



$(m_s - m_{ud})^2$

HLbL Systematic errors

- Missing disconnected diagrams
  → compute them

- Finite volume

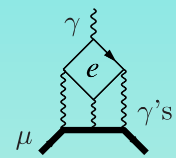
- Discretization error

  → a scaling study for 1/a = 2.7 GeV, 64 cube lattice at physical quark mass will be done on ALCC at Argonne

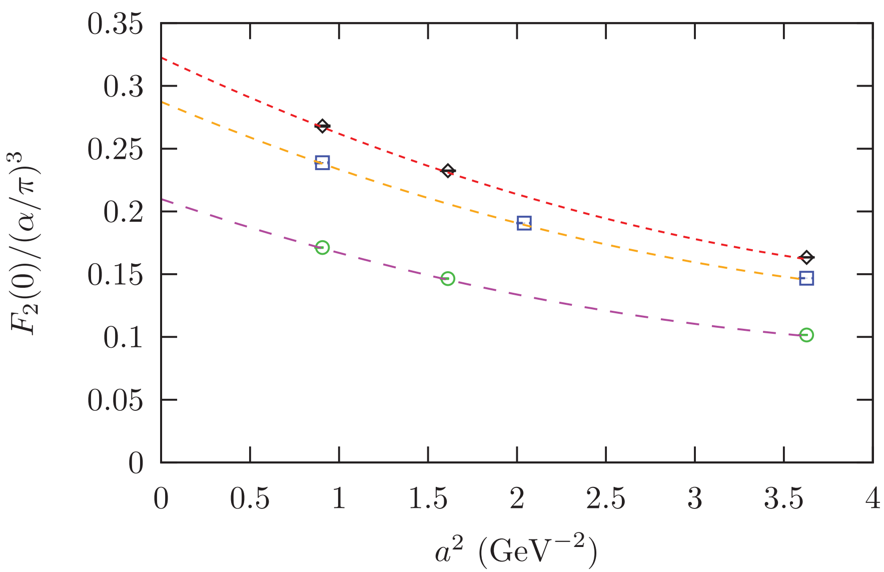
- ...

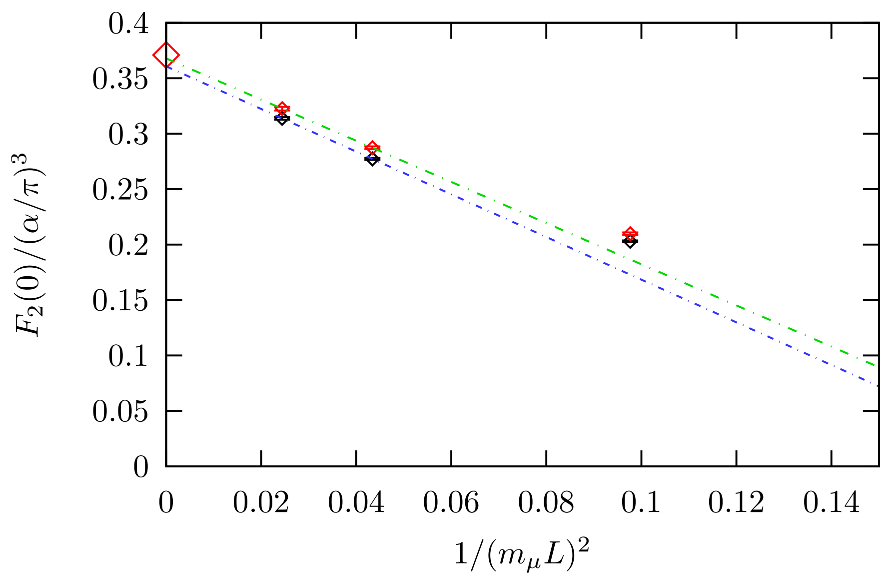
Systematic effects in QED only study

- muon loop, muon line
- a = a m_μ / (106 MeV)
- L= 11.9, 8.9, 5.9 fm
- known result : F2 = 0.371 (diamond) correctly reproduced (good check)



$$a_\mu^{(6)}(\mathrm{lbl}, e) = \left[\frac{2}{3}\pi^2 \ln\frac{m_\mu}{m_e} + \frac{59}{270}\pi^4 - 3\,\zeta(3) - \frac{10}{3}\pi^2 + \frac{2}{3} + O\left(\frac{m_e}{m_\mu}\ln\frac{m_\mu}{m_e}\right)\right]\left(\frac{\alpha}{\pi}\right)^3$$

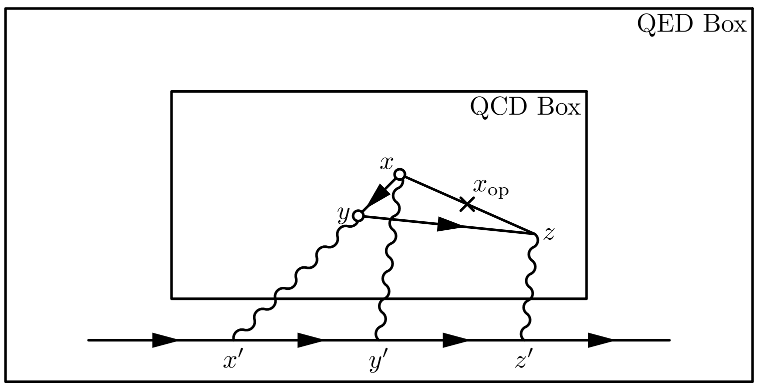


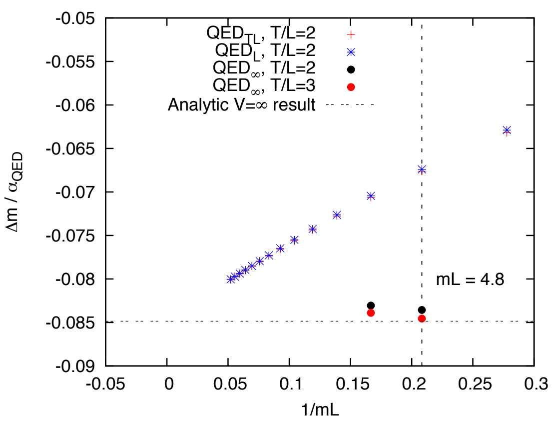


FV and discretization error could be as large as 20-30 % ? ,
similar discretization error seen from QCD+QED study

QCD box in QED box

- FV from quark is exponentially suppressed ~ $\exp(-M_\pi L_{QCD})$
- Dominant FV effects would be from photon
- Let photon and muon propagate in larger (or infinite) box than that of quark
-





- We could examine different lepton/photon in the off-line manner e.g. QED_L (Hayakwa-Uno 2008) with larger box, Twisting Averaging [Lehner TI LATTICE14] or Infinite Vol. Photon propagators [C. Lehner, L.Jin, TI LATTICE15], or continuum formula [Mainz LAT15]

HLbL on Lattice Summary

- Connected HLbL calculation is improved very rapidly
- Many orders of magnitudes improvements
  - coordinate-space integral using analytic photon propagator with adaptive probability (point photon method)
  - config-by-config conserved external current
  - take moment of relative coordinate to directly take q→0
  - AMA

→ 8 % stat. error at physical point (preliminary, connected, stat err only)

$$\frac{(g_\mu - 2)_{\mathrm{cHLbL}}}{2} = (0.933 \pm 0.0073)(\alpha/\pi)^3 = (116.9 \pm 9.1) \times 10^{-11}$$

- SU(3) unsuppressed disconnected diagram has signal also at physical point

- Still large systematic errors (missing disconnected, FV, discretization error, ...)

- Direct calculation of HLbL is in progress [Mainz group]
- Goal : 10% total error

g-2 (SM) theory status summary

- Uncertainty from Hadronic contributions dominate error
- Hadronic Vacuum Polarization (HVP)
  - Determination from R-ratio experiment ~ 0.6 % error
  - Lattice determinations, rapidly reducing errors ~ 2% error
  - One full (continuum, infinite volume) calculation by HPQCD, important to check assumptions
  - Disconnected diagram has definite error
  - Finite Volume, QED/isospin breaking effects
- Hadronic Light-by-light (HLbL)
  - Dispersive approaches are proposed
  - Rapidly making progress for connected diagram on Lattice
  - Lattice spacing error, Finite Volume error will be removed
  - Direct calculation of HLbL on lattice
- **Very exciting moment for g-2 Physics**

Collaborators

- HVP & DWF simulations
  RBC/UKQCD (next page), M. Spraggs, A. Porttelli, K. Maltman

- HLbL

  Tom Blum, Norman Christ, Masashi Hayakawa, **<u>Luchang Jin</u>**, Chulwoo Jung, Christoph Lehner, ...

- DWF simulations including HVP
  RBC/UKQCD Collaboration

Part of related calculation are done by resources from
USQCD (DOE), XSEDE, ANL BG/Q Mira (DOE, ALCC), Edinburgh BG/Q,
BNL BG/Q, RIKEN BG/Q and Cluster (RICC, HOKUSAI)

Support from US DOE, RIKEN, BNL, and JSPS

Backup slides / for discussion

interplays between dispersive approach and Lattice

- g-2 HVP
- Vus from strangeness τ inclusive decay

Use of Time-Moments

[HPQCD, PRD89(2014)114501]

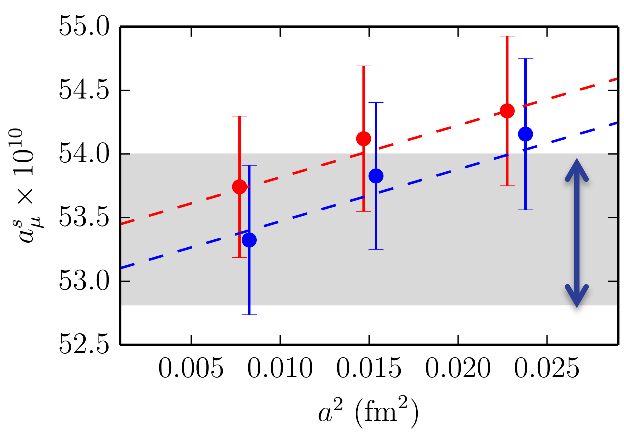
- Compute Time-moments of 2pt

$$G_{2n} \equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^i(\vec{x},t) j^i(0) \rangle = (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}(q^2) \bigg|_{q^2=0} .$$

$$\hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j} \Pi_j$$

$$\Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!} .$$

- subtractions by taking derivatives, use local currents
- Pade approximation, <u>determined</u> from Π j , for high q2 integration



1.1 %

Strange

$$a_\mu^s = 53.41(59) \times 10^{-10}.$$

[1.1% ~ lattice spacing error]

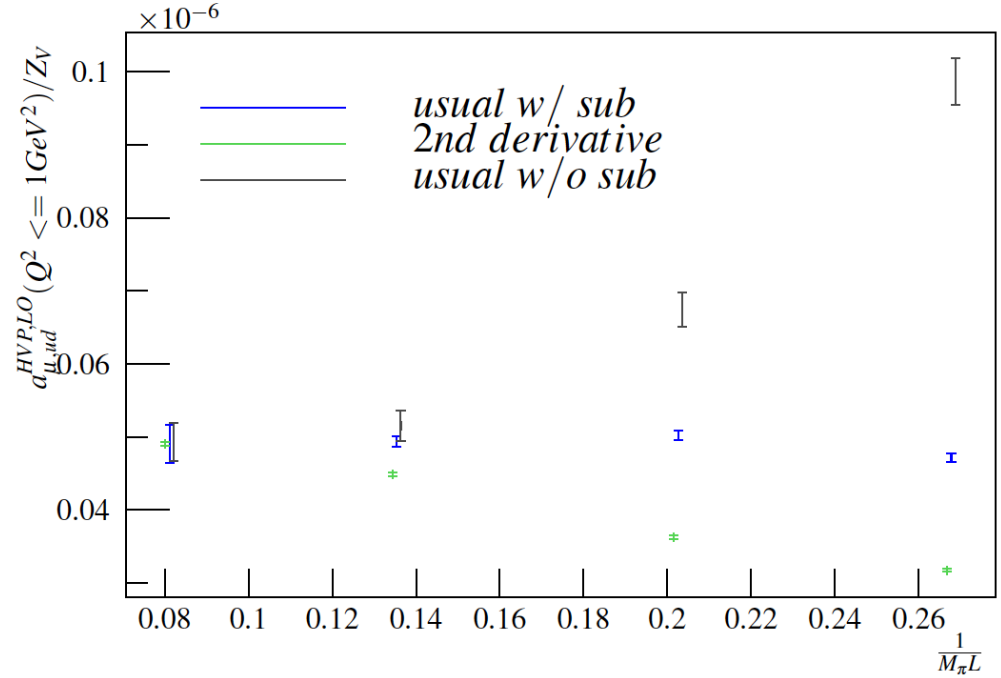
charm

$$a_\mu^c = 14.42(39) \times 10^{-10}.$$

[2.7% ~ Z_V error]

Finite Volume effects

- Malak et al. (15, BMWc)
- w/o $\Pi_{\mu\nu}(0)$ subtraction,
  +40% FVE at Mpi L=5
- FVE for $\Pi_{\mu\nu}(0)$ subtracted
  ones get small
- t^2 moment undershoots
  -30% or so at Mpi L =5



- Maarten Golterman [Tue, 17:30]

Compares different H4 Irrepps, find 10+% difference.
Also ChPT analysis for different FV treatment (Irreps, subtractions)

A_1:

[0,1] Padé: $a_\mu^{\rm HVP}(1\ {\rm GeV}^2) = 8.4(4) \times 10^{-8}$

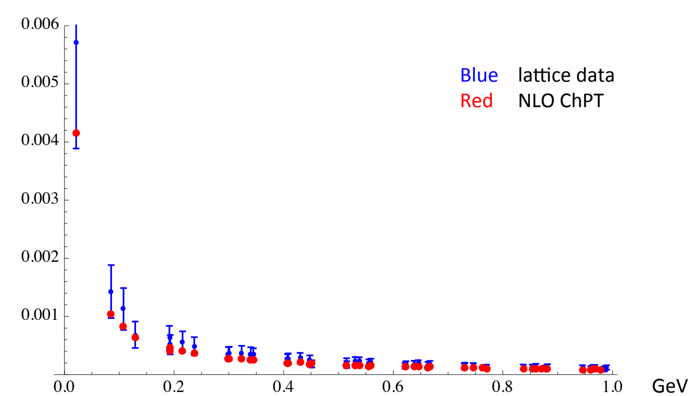
quadr. conf. pol.: $a_\mu^{\rm HVP}(1\ {\rm GeV}^2) = 8.4(5) \times 10^{-8}$

A_1^{44}:

[0,1] Padé: $a_\mu^{\rm HVP}(1\ {\rm GeV}^2) = 9.2(3) \times 10^{-8}$

quadr. conf. pol.: $a_\mu^{\rm HVP}(1\ {\rm GeV}^2) = 9.6(4) \times 10^{-8}$

Difference of 9 – 13% as a consequence of finite volume effects



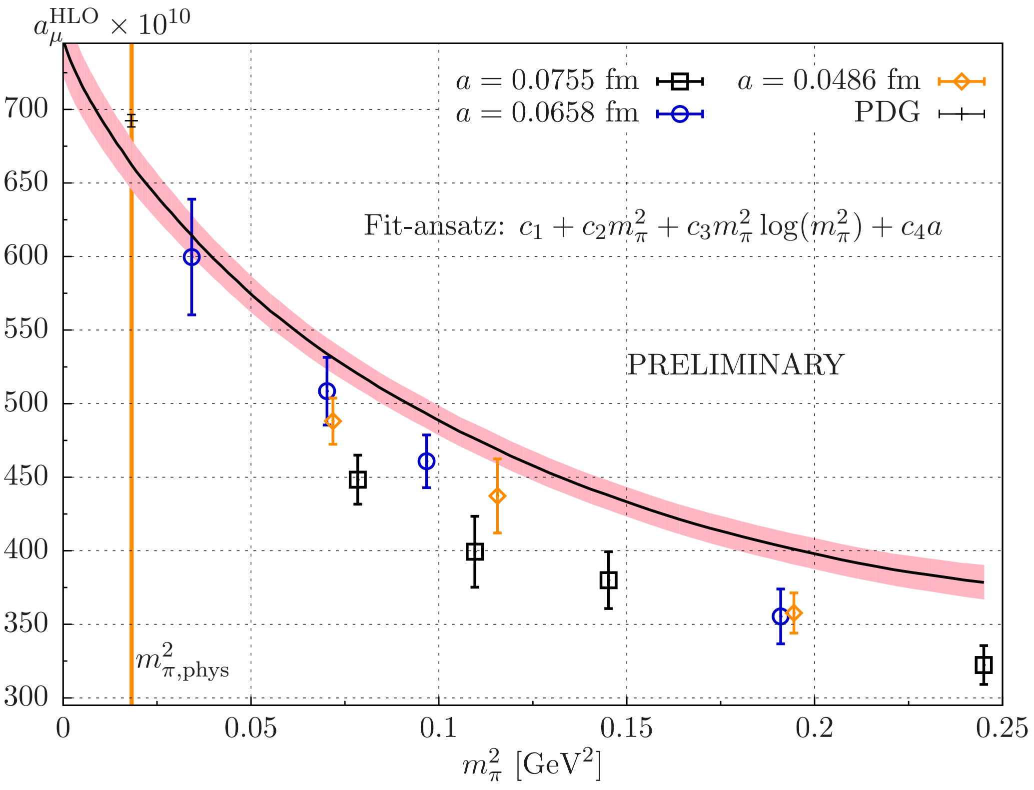
Difference of $\Pi_{A_1}(Q^2)$ (unsubtracted) and $\hat{\Pi}_{A_1}(Q^2)$ (subtracted)

Mainz HVP

Hanno Horch et al.

- Nf=2 O(a)-imp Wilson, CLS,
  Mpi = 185-495 MeV, a=0.05,0.07, 0.08 fm
- TBC
- ETMC rho rescaling
- extended frequentist's method

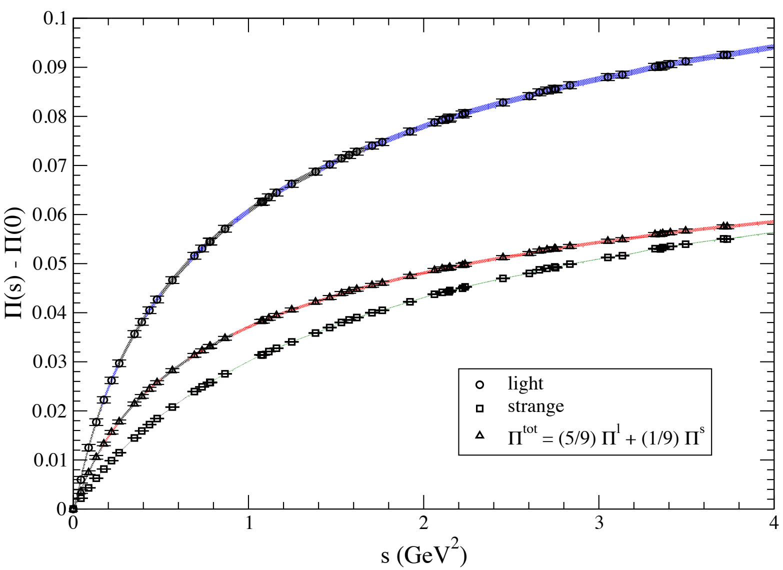
- Large chiral extrapolation error

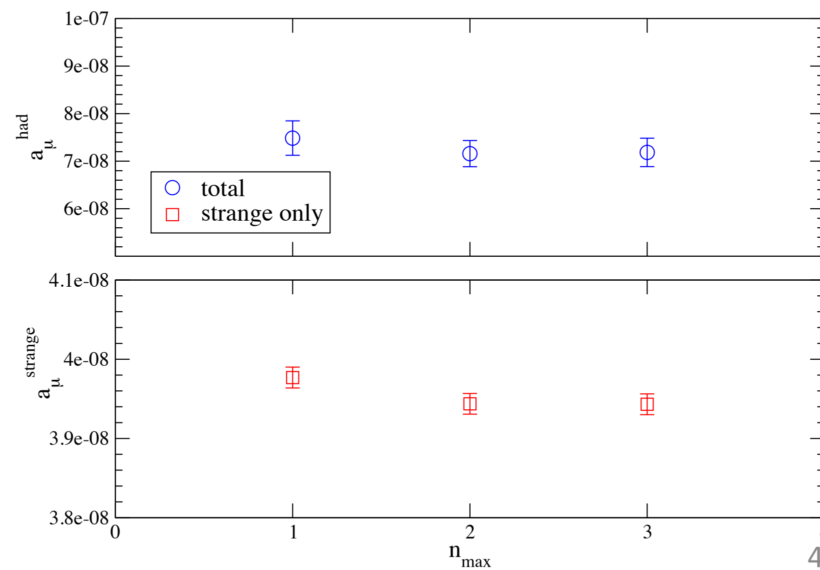


HVP on BMWc ensemble

Eric Gregory

- Extract smooth function π(s) from Taylor expansion, with derivatives measured from vector correlator.
- 1065 config @ physical Mpi, 1/a=2.131 GeV, ~6fm, 2HEX-smeared Wilson-type
- strange contribution ~15% smaller than HPQCD, RBC/UKQCD



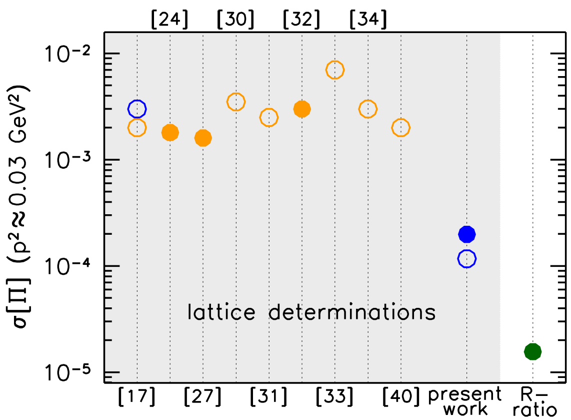


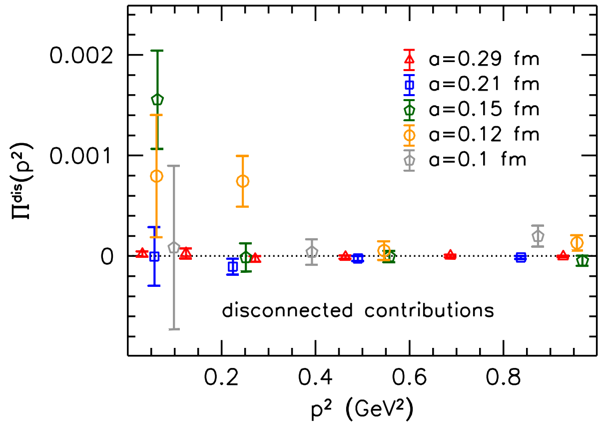
HVP & magnetic susceptibilities

- Gunnar Bali, Gergely Endrodi,arXiv:1506.08638
  Relates magnetic susceptibilities with oscillatory magnetic background and constant one, extract HVP. Also include disconnected loop

$$2\chi_p = \Pi(p^2)\,, \qquad \chi_0 = \Pi(0)\,. \qquad \chi_p = -\frac{\partial^2 f[\mathbf{B}^p]}{\partial(eB)^2}\,.$$

$$\mathbf{B}^p(x) = B\sin(px)\,\mathbf{e}_3\,, \qquad \mathbf{B}^0 = B\,\mathbf{e}_3\,,$$



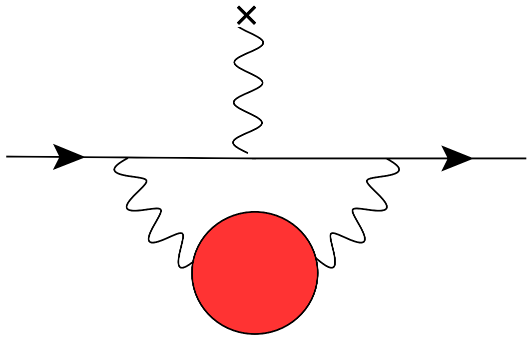


Compared to older results

QED effects

- From experimental e+ e- total cross section σ_{total}(e+e-) and dispersion relation

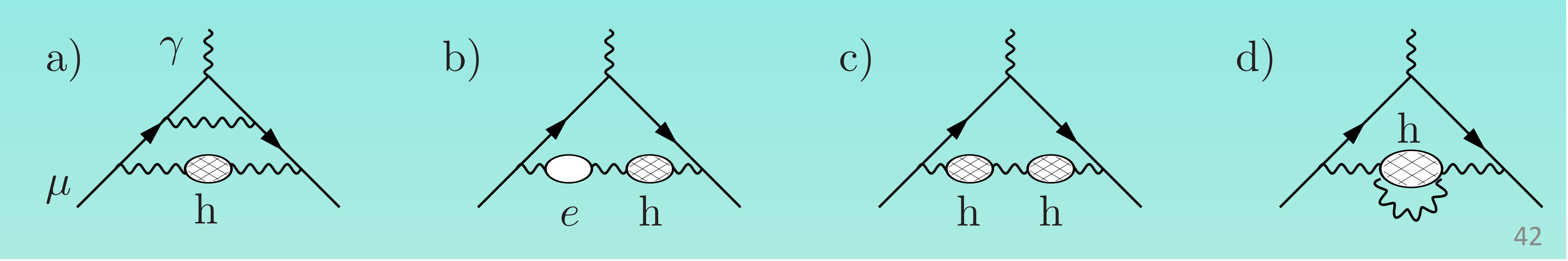
$$a_\mu^{\text{HVP}} = \frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} ds K(s)\sigma_{\text{total}}(s)$$



time like $q^2 = s >= 4\, m_\pi^2$

$$a_\mu^{\text{HVP,LO}} = (694.91 \pm 4.27) \times 10^{-10}$$

$$a_\mu^{\text{HVP,HO}} = (-9.84 \pm 0.07) \times 10^{-10}$$

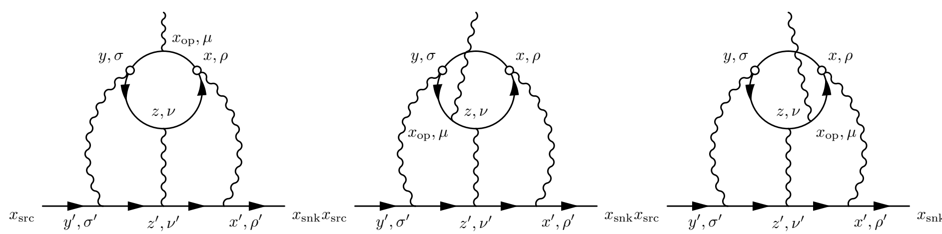


M_π=170 MeV cHLbL result (contd.)

"Exact" … q = 2pi / L,

"Conserved (current)" … q=2pi/L, 3 diagrams

"Mom" … moment method q->0, with AMA

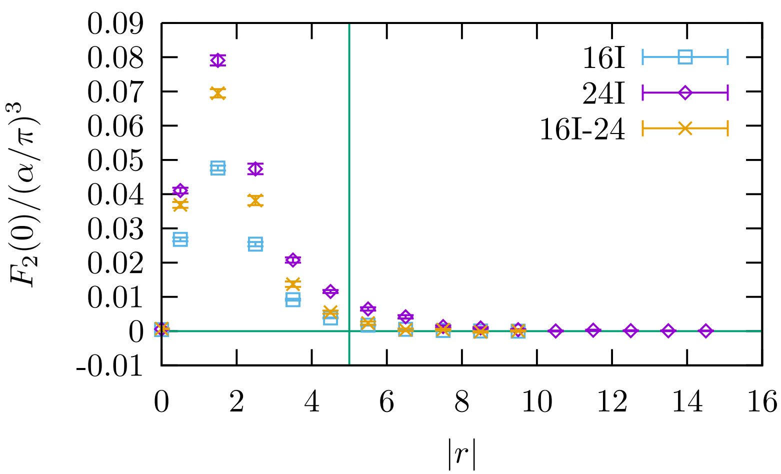


Method	$F_2/(\alpha/\pi)^3$	N_{conf}	N_{prop}	$\sqrt{\text{Var}}$	r_{max}	SD	LD	ind-pair
Exact	0.0693(218)	47	$58 + 8 \times 16$	2.04	3	−0.0152(17)	0.0845(218)	0.0186
Conserved	0.1022(137)	13	$(58 + 8 \times 16) \times 7$	1.78	3	0.0637(34)	0.0385(114)	0.0093
Mom. (approx)	0.0994(29)	23	$(217 + 512) \times 2 \times 4$	1.08	5	0.0791(18)	0.0203(26)	0.0028
Mom. (corr)	0.0060(43)	23	$(10 + 48) \times 2 \times 4$	0.44	2	0.0024(6)	0.0036(44)	0.0045
Mom. (tot)	0.1054(54)	23						

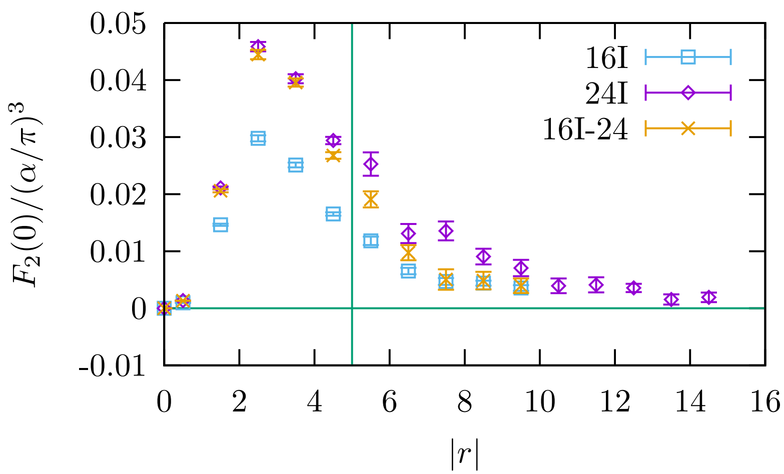
QED box in QCD box (contd.)

- Mπ=420 MeV, mμ=330 MeV, 1/a=1.7 GeV
- $(16)^3 = (1.8\text{ fm})^3$ QCD box in $(24)^3 = (2.7\text{ fm})^3$ QED box

Ensemble	$m_\pi L$	QCD Size	QED Size	$\frac{F_2(q^2=0)}{(\alpha/\pi)^3}$
16I	3.87	$16^3 \times 32$	$16^3 \times 32$	0.1158(8)
24I	5.81	$24^3 \times 64$	$24^3 \times 64$	0.2144(27)
16I-24		$16^3 \times 32$	$24^3 \times 64$	0.1674(22)



r = min {|x-y|, |y-z|,|z-x|}

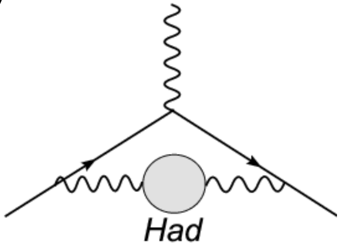


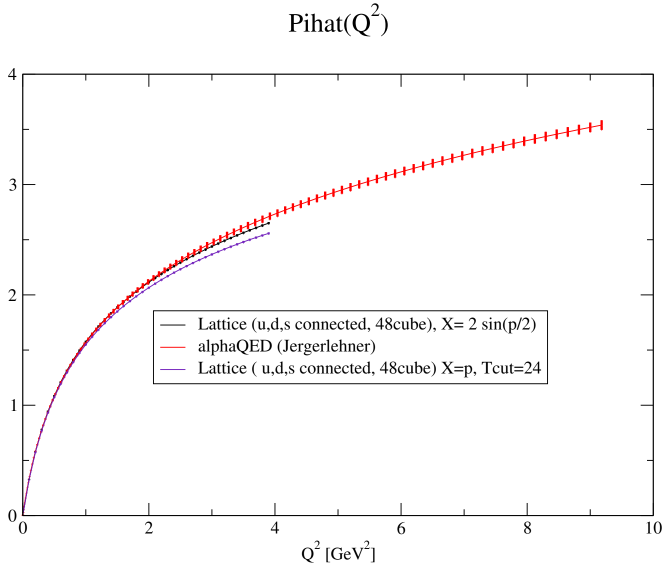
r = max{|x-y|, |y-z|,|z-x|}

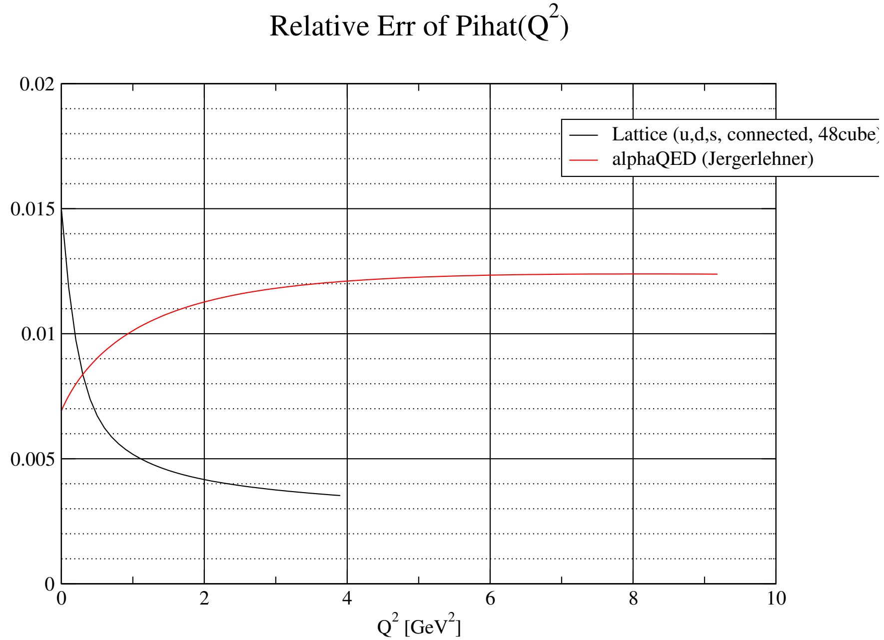
(plan B) Interplays between lattice and dispersive approach g-2

- Dispersive approach from R-ratio R(s)

$$\hat{\Pi}(Q^2) = \frac{Q^2}{3}\int_{s_0} ds \frac{R(s)}{s(s+Q^2)}$$



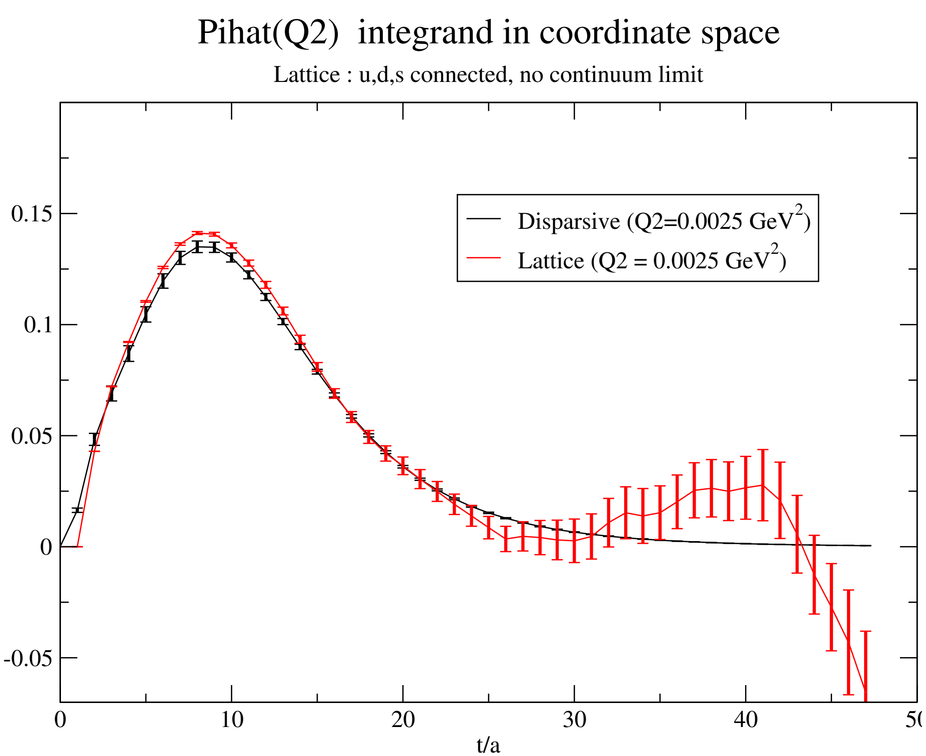


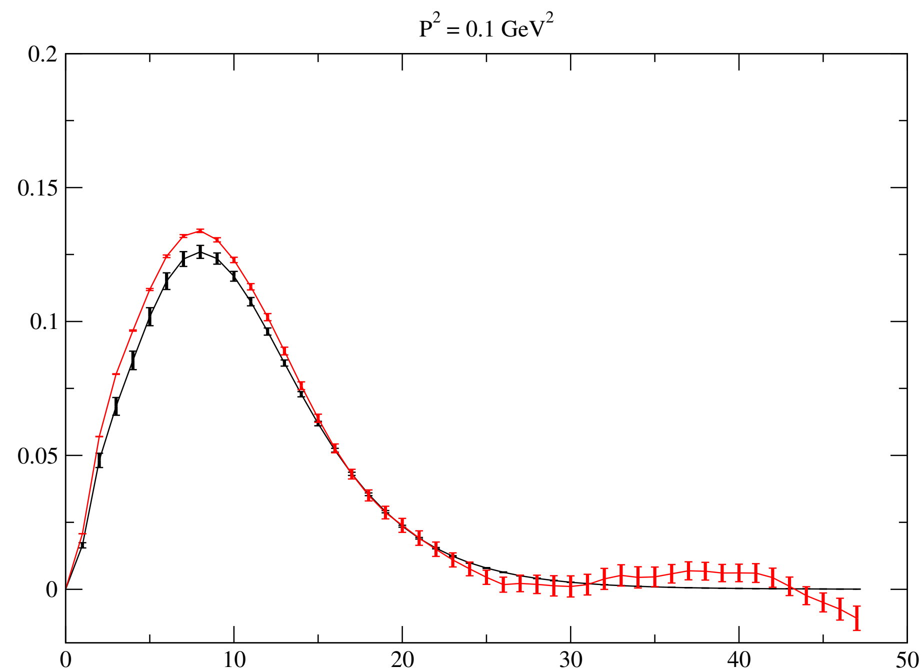


also [ETMC, Mainz, ...]

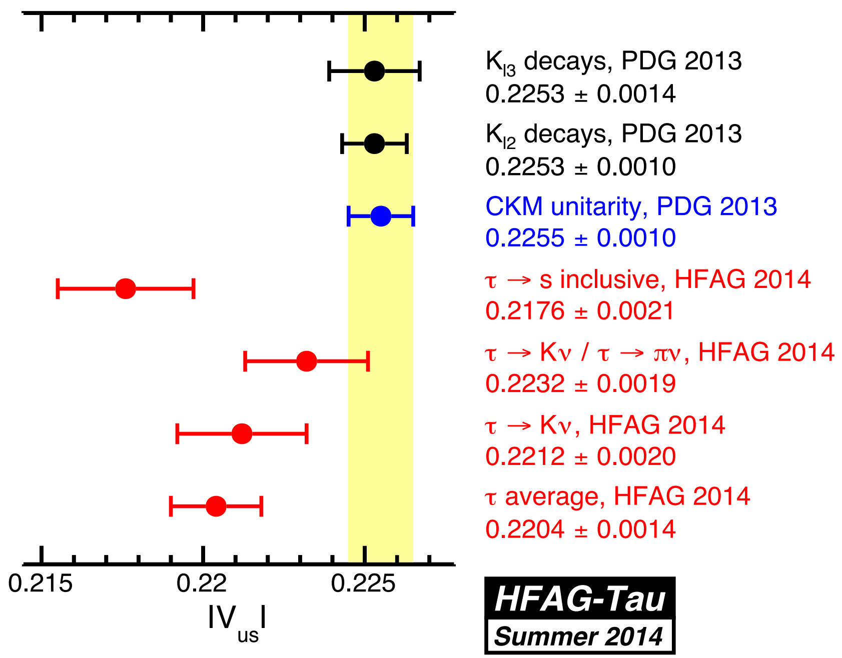
- Can we combine dispersive & lattice and get more precise (g-2)HVP than both ? [2011 Bernecker Meyer]
- Inverse Fourier trans to Euclidean vector correlator
- Relevant for g-2 $Q^2 = (m_\mu/2)^2 = 0.0025\ \text{GeV}^2$
- It may be interesting to think

$$\frac{\hat{\Pi}(Q^2)}{Q^2} = \left[\frac{\hat{\Pi}(Q^2)}{Q^2} - \frac{\hat{\Pi}(P^2)}{P^2}\right]^{\text{Exp}} + \left[\frac{\hat{\Pi}(P^2)}{P^2}\right]^{\text{Lat}}$$





V_{us} extraction strangeness tau inclusive decay



Tau decay

- $\tau \to \nu + had$ through V-A vertex
- Apply the optical theorem to related to VV and AA hadronic vacuum polarization (HVP)
- For hadrons with strangeness -1, CKM matrix elements V_{us} is multiplied
- ν takes energy away, makes differential cross section is related to the HVPs (c.f. in e^+e^- case, the total cross section is directly related to HVP)

$$R_{ij} = \frac{\Gamma(\tau^- \to \text{hadrons}_{ij}\ \nu_\tau)}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau)}$$

$$= \frac{12\pi|V_{ij}^2|S_{EW}}{m_\tau^2}\int_0^{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)\underbrace{\left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\Pi^{(1)}(s)+\text{Im}\Pi^{(0)}(s)\right]}_{\equiv\ \text{Im}\,\Pi(s)}$$

- The Spin=0 and 1, vacuum polarization, Vector(V) or Axial (A) current-current two point

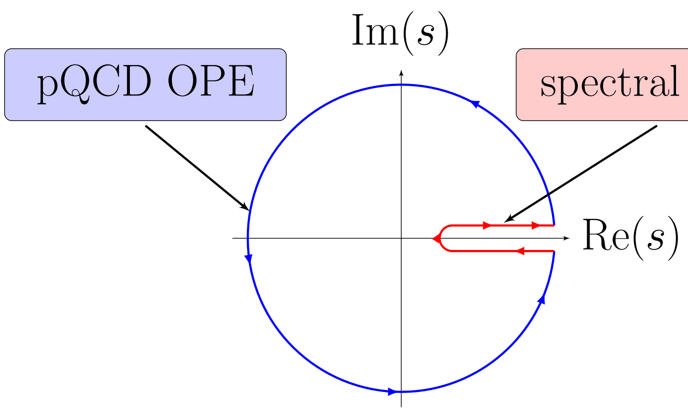
$$\Pi^{\mu\nu}_{ij;V/A}(q^2) = i\int d^4x e^{iqx}\left\langle 0|TJ^{\mu}_{ij;V/A}(x)J^{\dagger\mu}_{ij;V/A}(0)|0\right\rangle$$

$$= (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi^{(1)}_{ij;V/A}(q^2) + q^\mu q^\nu \Pi^{(0)}_{ij;V/A}$$

Finite Energy Sum Rule (FESR)

- Do the finite radius contour integral
- Real axis integral from experimental R_τ
- Use pQCD and OPE for the large circle integral
- Any analytic weight function $w(s)$

$$\int_{s_{th}}^{s_0} \mathrm{Im}\Pi(s)w(s) = \frac{i}{2}\oint_{|s|=s_0} ds\Pi(s)w(s)$$



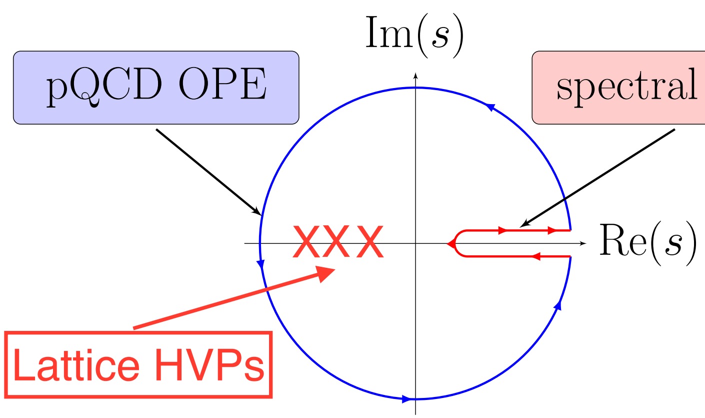
Combining FESR and Lattice

- If we have a reliable estimate for $\Pi(s)$ in Euclidean (space-like) points, $s = -Q_k^2 < 0$, we could extend the FESR with weight function $w(s)$ to have poles there,

$$\int_{s_{th}}^{\infty} w(s)\text{Im}\Pi(s) = \pi \sum_k^{N_p} \text{Res}_k[w(s)\Pi(s)]_{s=-Q_k^2}$$

$$\Pi(s) = \left(1 + 2\frac{s}{m_\tau^2}\right) \text{Im}\Pi^{(1)}(s) + \text{Im}\Pi^{(0)}(s) \propto s \quad (|s| \to \infty)$$

- For $N_p \geq 3$, the $|s| \to \infty$ circle integral vanishes.



weight function $w(s)$

- Example of weight function

$$w(s) = \prod_k^{N_p} \frac{1}{(s+Q_k^2)} = \sum_k a_k \frac{1}{s+Q_k^2}, \quad a_k = \sum_{j\neq k} \frac{1}{Q_k^2 - Q_j^2}$$
$$\Longrightarrow \sum_k (Q_k)^M a_k = 0 \ \ (M = 0, 1, \cdots, N_p - 2)$$

- The residue constraints automatically subtracts $\Pi^{(0,1)}(0)$ and $s\Pi^{(1)}(0)$ terms.
- For experimental data, $w(s) \sim 1/s^n, n \geq 3$ suppresses
  - ▷ *larger error from higher multiplicity final states at larger* $s < m_\tau^2$
  - ▷ *uncertanties due to pQCD+OPE at* $m_\tau^2 < s$
- For lattice, Q_k^2 should be not too small to avoid large stat. error, $Q^2 \to 0$ extrapolation, Finite Volume error(?). Also not too larger than m_τ^2 to make the suppression in time-like $0 < s < m_\tau^2$ working.
- Other $w(s)$ could be useful to enhance some region $s > 0$ which may be usable for $(g-2)_\mu$ HVP (?)
- c.f. HPQCD's HVP moments works

Preliminary results

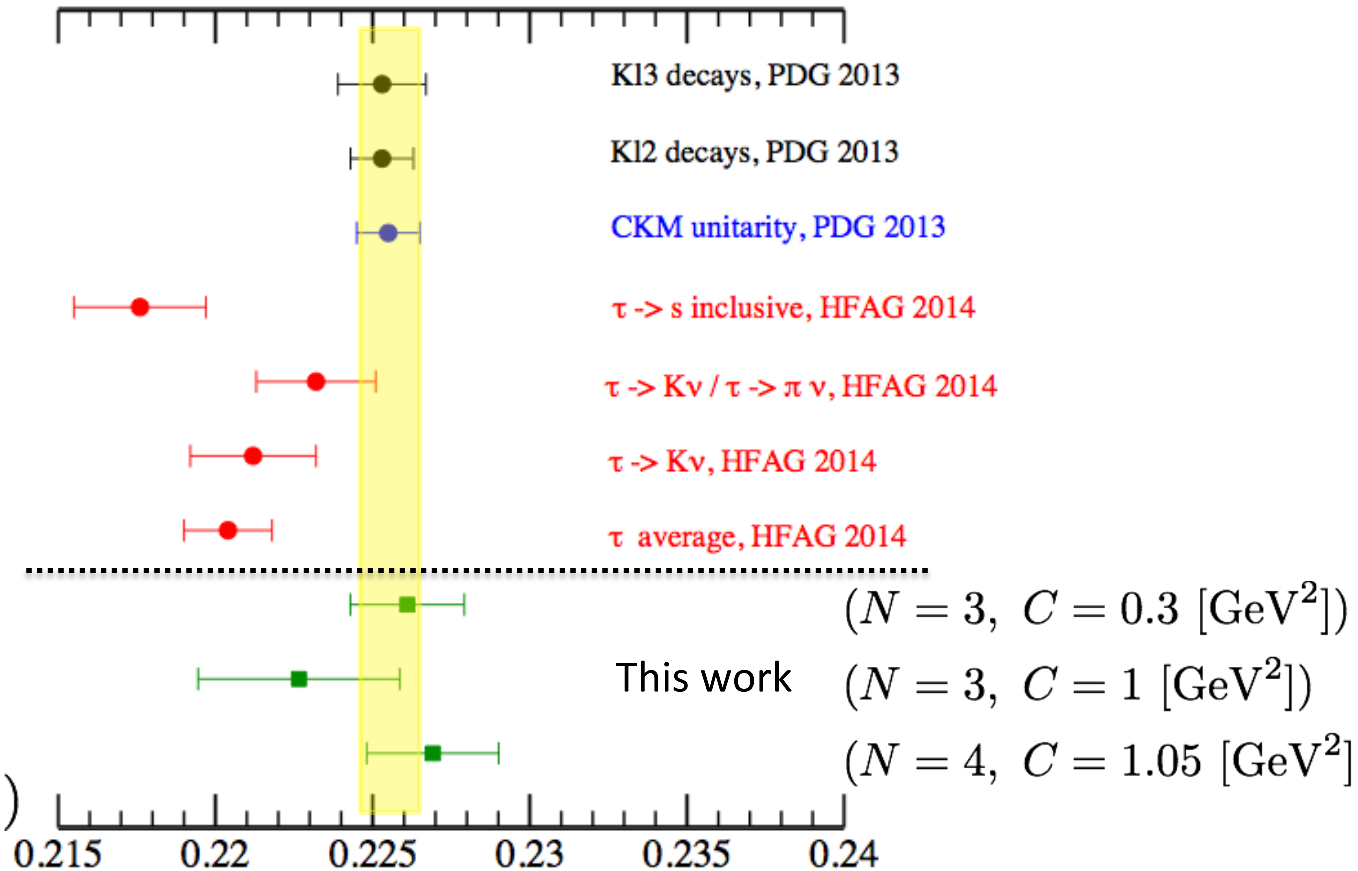
[H. Ohki, A. Juttner, C. Lehner, K. Maltman et al.]

very preliminary

Our result

for all channels

$(V_1 + V_0 + A_1 + A_0)$



All our results (C<1, N=3,4) are consistent with each other.

Note : Other systematic errors of sea quark mass chiral extrapolation, lattice O(a^4) discretization,

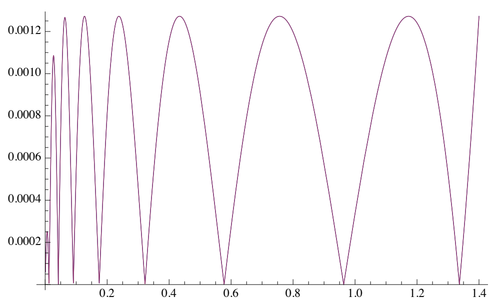
and higher order OPE have not been included. These must be assessed in a future study.

AMA+MADWF(fastPV)+zMobius accelerations

- We utilize complexified 5d hopping term of Mobius action [Brower, Neff, Orginos], zMobius, for a better approximation of the sign function.

$$\epsilon_L(h_M) = \frac{\prod_s^L(1+\omega_s^{-1}h_M) - \prod_s^L(1-\omega_s^{-1}h_M)}{\prod_s^L(1+\omega_s^{-1}h_M) + \prod_s^L(1-\omega_s^{-1}h_M)}, \qquad \omega_s^{-1} = b + c \in \mathbb{C}$$

- 1/a~2 GeV, Ls=48 Shamir ~ Ls=24 Mobius (b=1.5, c=0.5) ~ Ls=10 zMobius (b_s, c_s complex varying) ~5 times saving for cost AND memory



Ls	|eps(48cube) – eps(zMobius)|
6	0.0124
8	0.00127
10	0.000110
12	8.05e-6

- The even/odd preconditioning is optimized (sym2 precondition) to suppress the growth of condition number due to order of magnitudes hierarchy of b_s, c_s [also Neff found this]

$$\texttt{sym2}: \quad 1 - \kappa_b M_4 M_5^{-1} \kappa_b M_4 M_5^{-1}$$

- Fast Pauli Villars (mf=1) solve, needed for the exact solve of AMA via MADWF (Yin, Mawhinney) is speed up by a factor of 4 or more by Fourier acceleration in 5D [Edward, Heller]
- All in all, sloppy solve compared to the traditional CG is 160 times faster on the physical point 48 cube case. And ~100 and 200 times for the 32 cube, Mpi=170 MeV, 140, in this proposal (1,200 eigenV for 32cube) .

$$\underbrace{\frac{20,000}{600}}_{\text{MADWF+zMobius+deflation}} \times \underbrace{\frac{600*32/10}{300}}_{\text{AMA+zMobius}} = 33.3 \times 6.4 = \underline{210 \text{ times faster}}$$

[Blum, TI, Shintani PRD 88 (2013) 094503]

Covariant Approximation Averaging (CAA) a new class of Error reduction techniques

Original

$$\mathcal{O} = \mathcal{O}^{(\mathrm{appx})} + \mathcal{O}^{(\mathrm{rest})}$$

unbiased imporved

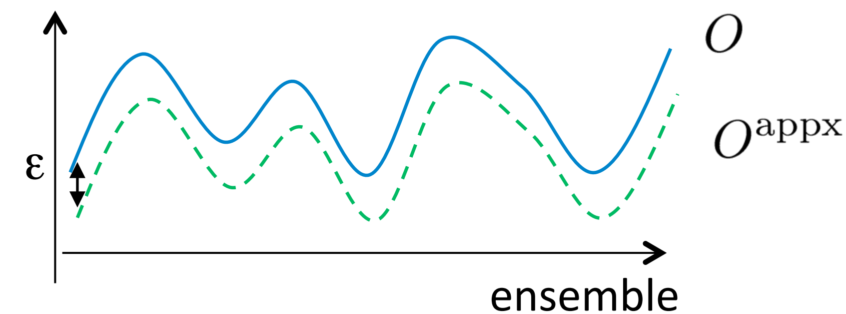
$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G} \sum_{g \in G} \mathcal{O}^{(\mathrm{appx}),g}$$

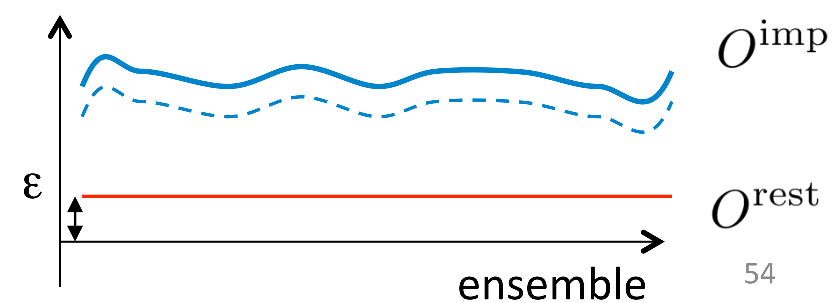
Lattice Symmetry

Expensive : infrequently measured

Cheap : frequently measured

- $O^{(\mathrm{imp})}$ has smaller error
  $O^{(\mathrm{appx})}$ need to be cheap & not to be too accurate
  N_G suppresses the bulk part of noise cheaply





New bias-free estimator even without covariant approximation by a stochastic choice of source location for the exact/rest computation is now available : Appendix D of arXiv:1402.0244

Examples of Covariant Approximations (contd.)

- All Mode Averaging **AMA**

  Sloppy CG or Polynomial approximations

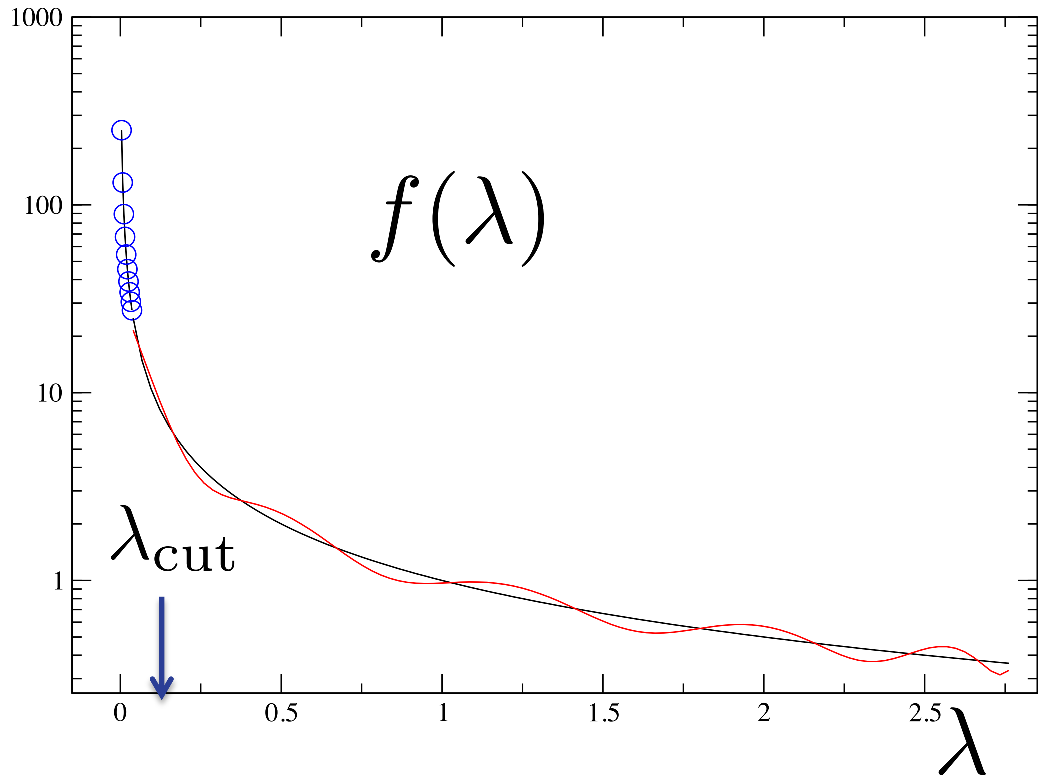
$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_{\lambda} v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \begin{cases} \frac{1}{\lambda}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

$$P_n(\lambda) \approx \frac{1}{\lambda}$$

If quark mass is heavy, e.g. ~ strange, low mode isolation may be unneccesary



accuracy control :
- low mode part : # of eig-mode
- mid-high mode : degree of poly.