

Charmless non-leptonic B decays - Theory

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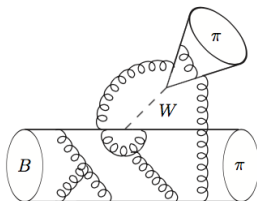
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:: Motivations

- ▶ Huge multiplicity of final states (2-body + multi-body), large data sets
- ▶ Important input in CKM studies (mostly angles)
- ▶ CP violation (SM and new physics)
- ▶ Non-trivial hadronic dynamics \Rightarrow Perturbative and non-perturbative QCD methods



:: Non-leptonic B -decay Amplitudes

- ▷ Effective Hamiltonian at the hadronic scale $\mu \sim m_B$

$$\mathcal{H}_{\text{eff}} = -\mathcal{L}_{QED+QCD} + \sum_i C_i(\mu) \mathcal{O}_i(\mu)$$

- ▷ C_i – Wilson coefficients (UV physics) \rightarrow perturbation theory

Known to NNLL: Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06.

- ▷ \mathcal{O}_i – Effective operators (IR physics) [e.g. $\mathcal{O} = (\bar{b}\gamma^\mu u)(\bar{u}\gamma_\mu d)$]

- ▷ Amplitudes:

$$\mathcal{A}(B \rightarrow M_1 M_2) = \sum_i C_i \langle M_1 M_2 | \mathcal{O}_i | B \rangle$$

The problem is to compute the **operator matrix elements**

\rightarrow non-perturbative, process dependent (non-universal)

:: OUTLINE

QCD FACTORIZATION

TWO-BODY DECAYS

- Perturbative calculation

- Tree and penguin decays

- Power corrections

THREE-BODY DECAYS

- Kinematics

- Factorization properties

- Hadronic input

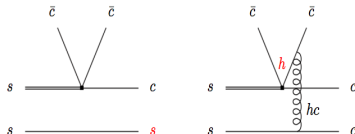
- Quasi-two-body decays

CHALLENGES

:: Multiscale problem

▷ 3 scales: m_b , $\sqrt{m_b \Lambda_{\text{QCD}}}$, Λ_{QCD} .

▷ 4 modes: **hard** ($p_h^2 \sim m_b^2$)
hard-collinear ($p_{hc}^2 \sim m_b \Lambda_{\text{QCD}}$)
collinear and soft ($p_{\bar{c},s}^2 \sim \Lambda_{\text{QCD}}^2$)



1. **QCD** → **SCET-1**: Integrate out **hard** modes

$$\text{▷ } \mathcal{O} = \int dt \tilde{T}^I(t) O^I(t) + \int dt ds \tilde{H}^{II}(t, s) O^{II}(t, s)$$

$$O^I(t) = [(\bar{\chi} W_{\bar{c}})(tn_-) \dots (W_{\bar{c}}^\dagger \chi)(0)] [(\bar{\xi} W_c)(0) \dots h_v(0)]$$

$$O^{II}(t, s) = [(\bar{\chi} W_{\bar{c}})(tn_-) \dots (W_{\bar{c}}^\dagger \chi)(0)] [(\bar{\xi} W_c)(0) \dots (W_c^\dagger i \not{D}_{\perp c} W_c)(sn_+) \dots h_v(0)]$$

▷ decoupling of anti-collinear modes. $\langle M_2 | [(\bar{\chi} W_{\bar{c}})(tn_-) \dots (W_{\bar{c}}^\dagger \chi)(0)] | 0 \rangle \sim \phi_{M_2}$

2. **SCET-1** → **SCET-2**: Integrate out **hard-collinear** modes

▷ $\langle M_1 | [(\bar{\xi} W_c)(0) \dots (W_c^\dagger i \not{D}_{\perp c} W_c)(sn_+) \dots h_v(0)] | B \rangle \sim J(s) \otimes \phi_B \otimes \phi_{M_1}$

▷ Hard-collinear factorization fails for $O^I(t)$.

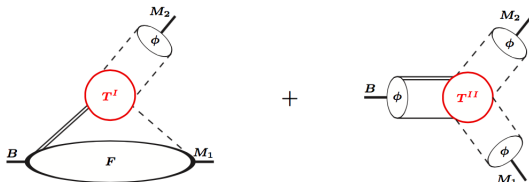
▷ End-point divergences can be absorbed into form factor F^{BM_1} .

:: Factorization formula for $B \rightarrow M_1 M_2$

To leading power in the heavy-quark expansion

Beneke, Buchalla, Neubert, Sachrajda '99

$$\langle M_1 M_2 | \mathcal{O} | B \rangle = F^{BM_1} \int du T^I(u) \phi_{M_2}(u) + \int d\omega du dv T^{II}(\omega, u, v) \phi_B(\omega) \phi_{M_1}(u) \phi_{M_2}(v)$$





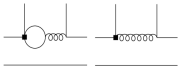





- ▷ Vertex corrections: $T^I(u) = 1 + \mathcal{O}(\alpha_s)$
- ▷ Spectator scattering: $T^{II}(\omega, u, v) = \mathcal{O}(\alpha_s)$ – (power suppressed if M_1 is heavy)
- ▷ Strong phases are perturbative [$\mathcal{O}(\alpha_s)$] or power suppressed [$\mathcal{O}(\Lambda/m_b)$].

:: Perturbative calculation

Two hard-scattering kernels for each operator insertion: T^I (vertex), T^{II} (spectator)

$$\langle M_1 M_2 | \mathcal{O}_i | B \rangle \simeq F^{BM_1} T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

and two classes of topological amplitudes: "Tree", "Penguin".

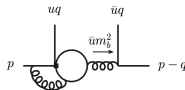
	T^I , tree	T^I , penguin	T^{II} , tree	T^{II} , penguin
LO: $\mathcal{O}(1)$				
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'04				
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07,'09 Beneke, Huber, Li '09	 Kim, Yoon '11, Bell Beneke, Huber, Li '15	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

:: Perturbative calculation

Motivation for NNLO: first correction to CP asymmetries

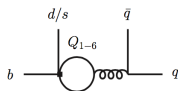
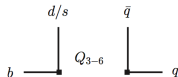
NNLO: non-trivial calculation

- ▷ $\mathcal{O}(70)$ diagrams
- ▷ 2 loops, 3 scales (m_b , um_b , m_c), 4 legs
- ▷ charm contribution has non-trivial threshold at $\bar{u}m_b^2 \gtrsim 4m_c^2$



Missing NNLO pieces:

- ▷ 2-loop tree insertions of penguin operators \mathcal{O}_{3-6}
Similar to $\mathcal{O}_{1,2}^u$ calculation, easier than $\mathcal{O}_{1,2}^c$
- ▷ 2-loop penguin insertions of penguin operators \mathcal{O}_{3-6}
Additional topology with “closed” quark loop.



$$\begin{aligned}
 T \equiv a_1(\pi\pi) &= 1.009 + [0.023 + 0.010i]_{\text{NLO}} + [0.026 + 0.028i]_{\text{NNLO}} \\
 &\quad - \left[\frac{r_{\text{sp}}}{0.485} \right] \left\{ [0.015]_{\text{LOsp}} + [0.037 + 0.029i]_{\text{NLOsp}} + [0.009]_{\text{tw3}} \right\} \\
 &= 1.00 + 0.01i \quad \rightarrow \quad 0.93 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}})
 \end{aligned}$$

$$r_{\text{sp}} = \frac{9f_{M_1}\hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B}$$

$$\begin{aligned}
 C \equiv a_2(\pi\pi) &= 0.220 - [0.179 + 0.077i]_{\text{NLO}} - [0.031 + 0.050i]_{\text{NNLO}} \\
 &\quad + \left[\frac{r_{\text{sp}}}{0.485} \right] \left\{ [0.123]_{\text{LOsp}} + [0.053 + 0.054i]_{\text{NLOsp}} + [0.072]_{\text{tw3}} \right\} \\
 &= 0.26 - 0.07i \quad \rightarrow \quad 0.51 - 0.02i \quad (\text{if } 2 \times r_{\text{sp}})
 \end{aligned}$$

- ▷ Individual NNLO corrections large, but cancellations between FF and sp. terms.
- ▷ Perturbative expansion well behaved (remember color suppression).
- ▷ Color suppressed $a_2(\pi\pi)$ dominated by spectator scattering [larger uncertainty]
Can be large if λ_B is small.
- ▷ Relative phase $\arg(C/T)$ remains small.

	Theory I		Theory II		Experiment
$B^- \rightarrow \pi^- \pi^0$	$5.43^{+0.06+1.45}_{-0.06-0.84}$ (*)		$5.82^{+0.07+1.42}_{-0.06-1.35}$ (*)		$5.59^{+0.41}_{-0.40}$
$\bar{B}_d^0 \rightarrow \pi^+ \pi^-$	$7.37^{+0.86+1.22}_{-0.69-0.97}$ (*)		$5.70^{+0.70+1.16}_{-0.55-0.97}$ (*)		5.16 ± 0.22
$\bar{B}_d^0 \rightarrow \pi^0 \pi^0$	$0.33^{+0.11+0.42}_{-0.08-0.17}$		$0.63^{+0.12+0.64}_{-0.10-0.42}$		1.55 ± 0.19
			BELLE CKM 14:		0.90 ± 0.16
$B^- \rightarrow \pi^- \rho^0$	$8.68^{+0.42+2.71}_{-0.41-1.56}$ (**)		$9.84^{+0.41+2.54}_{-0.40-2.52}$ (**)		$8.3^{+1.2}_{-1.3}$
$B^- \rightarrow \pi^0 \rho^-$	$12.38^{+0.90+2.18}_{-0.77-1.41}$ (*)		$12.13^{+0.85+2.23}_{-0.73-2.17}$ (*)		$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \rightarrow \pi^+ \rho^-$	$17.80^{+0.62+1.76}_{-0.56-2.10}$ (*)		$13.76^{+0.49+1.77}_{-0.44-2.18}$ (*)		15.7 ± 1.8
$\bar{B}^0 \rightarrow \pi^- \rho^+$	$10.28^{+0.39+1.37}_{-0.39-1.42}$ (**)		$8.14^{+0.34+1.35}_{-0.33-1.49}$ (**)		7.3 ± 1.2
$\bar{B}^0 \rightarrow \pi^\pm \rho^\mp$	$28.08^{+0.27+3.82}_{-0.19-3.50}$ (†)		$21.90^{+0.20+3.06}_{-0.12-3.55}$ (†)		23.0 ± 2.3
$\bar{B}^0 \rightarrow \pi^0 \rho^0$	$0.52^{+0.04+1.11}_{-0.03-0.43}$		$1.49^{+0.07+1.77}_{-0.07-1.29}$		2.0 ± 0.5
$B^- \rightarrow \rho_L^- \rho_L^0$	$18.42^{+0.23+3.92}_{-0.21-2.55}$ (**)		$19.06^{+0.24+4.59}_{-0.22-4.22}$ (**)		$22.8^{+1.8}_{-1.9}$
$\bar{B}_d^0 \rightarrow \rho_L^+ \rho_L^-$	$25.98^{+0.85+2.93}_{-0.77-3.43}$ (**)		$20.66^{+0.68+2.99}_{-0.62-3.75}$ (**)		$23.7^{+3.1}_{-3.2}$
$\bar{B}_d^0 \rightarrow \rho_L^0 \rho_L^0$	$0.39^{+0.03+0.83}_{-0.03-0.36}$		$1.05^{+0.05+1.62}_{-0.04-1.04}$		$0.55^{+0.22}_{-0.24}$

Theory I: $f_+^{B\pi}(0) = 0.25 \pm 0.05$, $A_0^{B\rho}(0) = 0.30 \pm 0.05$, $\lambda_B(1 \text{ GeV}) = 0.35 \pm 0.15 \text{ GeV}$

Theory II: $f_+^{B\pi}(0) = 0.23 \pm 0.03$, $A_0^{B\rho}(0) = 0.28 \pm 0.03$, $\lambda_B(1 \text{ GeV}) = 0.20^{+0.05}_{-0.00} \text{ GeV}$

First error γ , $|V_{cb}| \cdot |V_{ub}|$ uncertainty *not* included. Second error from hadronic inputs.

Brackets: form factor uncertainty not included.

:: Impact of λ_B

G. Bell

B-meson LCDA inverse moment: $\lambda_B^{-1}(\mu) = \int_0^{\infty} \frac{d\omega}{\omega} \phi_B(\omega, \mu)$

Dominant parametric uncertainty in QCDF

▶ QCD sum rule estimate $\lambda_B(1\text{GeV}) \simeq (460 \pm 110) \text{ MeV}$

[Braun, Ivanov, Korchemsky 03]

▶ $\pi\pi/\pi\rho/\rho\rho$ data seems to prefer $\sim 200 \text{ MeV}$?

λ_B can be measured in $B \rightarrow \gamma \ell \nu$ decays

▶ state-of-the-art analysis (NLL, tree-level $1/m_b$)

[Beneke, Rohrwild 11; Braun, Khodjamirian 12]

▶ Babar 09 data ($E_\gamma > 1\text{GeV}$) $\Rightarrow \lambda_B(1\text{GeV}) > 115 \text{ MeV}$

▶ Belle 15 data ($E_\gamma > 1\text{GeV}$) $\Rightarrow \lambda_B(1\text{GeV}) > 238 \text{ MeV}$

▶ good prospects to measure λ_B at Belle-II

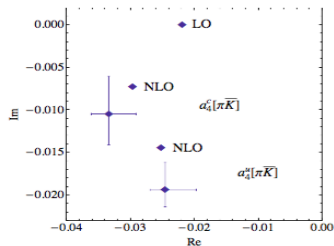
:: Penguin decays

Bell, Beneke, Huber, Li '15

$$\begin{aligned} a_4^u(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{V_1} + [0.49 - 1.32i]_{P_1} - [0.32 + 0.71i]_{P_2} \\ &\quad + \left[\frac{r_{sp}}{0.434} \right] \left\{ [0.13]_{LO} + [0.14 + 0.12i]_{HV} - [0.01 - 0.05i]_{HP} + [0.07]_{tw3} \right\} \\ &= (-2.46^{+0.49}_{-0.24}) + (-1.94^{+0.32}_{-0.20})i \end{aligned} \quad r_{sp} = \frac{9f_{M_1}\hat{f}_B}{m_b f_+^B(0)\lambda_B}$$

$$\begin{aligned} a_4^c(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{V_1} + [0.05 - 0.62i]_{P_1} - [0.77 + 0.50i]_{P_2} \\ &\quad + \left[\frac{r_{sp}}{0.434} \right] \left\{ [0.13]_{LO} + [0.14 + 0.12i]_{HV} + [0.01 + 0.03i]_{HP} + [0.07]_{tw3} \right\} \\ &= (-3.34^{+0.43}_{-0.27}) + (-1.05^{+0.45}_{-0.36})i \end{aligned}$$

- Two-loop is 40% (15%) of the imaginary (real) part of $a_4^u(\pi\bar{K})$, and 50% (25%) in the case of $a_4^c(\pi\bar{K})$.
- Spectator-scattering not relevant.



M. Beneke, talk at *Future challenges in non-leptonic B decays* (2016)


:: Penguin decays (CPAs)

Bell, Beneke, Huber, Li '15

f	NLO	NNLO	NNLO + LD	Exp
$\pi^- \bar{K}^0$	$0.71^{+0.13+0.21}_{-0.14-0.19}$	$0.77^{+0.14+0.23}_{-0.15-0.22}$	$0.10^{+0.02+1.24}_{-0.02-0.27}$	-1.7 ± 1.6
$\pi^0 K^-$	$9.42^{+1.77+1.87}_{-1.76-1.88}$	$10.18^{+1.91+2.03}_{-1.90-2.62}$	$-1.17^{+0.22+20.00}_{-0.22-6.62}$	4.0 ± 2.1
$\pi^+ K^-$	$7.25^{+1.36+2.13}_{-1.36-2.58}$	$8.08^{+1.52+2.52}_{-1.51-2.65}$	$-3.23^{+0.61+19.17}_{-0.61-3.36}$	-8.2 ± 0.6
$\pi^0 \bar{K}^0$	$-4.27^{+0.83+1.48}_{-0.77-2.23}$	$-4.33^{+0.84+3.29}_{-0.78-2.32}$	$-1.41^{+0.27+5.54}_{-0.25-6.10}$	1 ± 10
$\delta(\pi \bar{K})$	$2.17^{+0.40+1.39}_{-0.40-0.74}$	$2.10^{+0.39+1.40}_{-0.39-2.86}$	$2.07^{+0.39+2.76}_{-0.39-4.55}$	12.2 ± 2.2
$\Delta(\pi \bar{K})$	$-1.15^{+0.21+0.55}_{-0.22-0.84}$	$-0.88^{+0.16+1.31}_{-0.17-0.91}$	$-0.48^{+0.09+1.09}_{-0.09-1.15}$	-14 ± 11
$\pi^- \bar{K}^{*0}$	$1.36^{+0.25+0.60}_{-0.26-0.47}$	$1.49^{+0.27+0.69}_{-0.29-0.56}$	$0.27^{+0.05+3.18}_{-0.05-0.67}$	-3.8 ± 4.2
$\pi^0 K^{*-}$	$13.85^{+2.40+5.84}_{-2.70-5.86}$	$18.16^{+3.11+7.79}_{-3.52-10.57}$	$-15.81^{+3.01+69.35}_{-2.83-15.39}$	-6 ± 24
$\pi^+ K^{*-}$	$11.18^{+2.00+9.75}_{-2.15-10.62}$	$19.70^{+3.37+10.54}_{-3.80-11.42}$	$-23.07^{+4.35+86.20}_{-4.05-20.64}$	-23 ± 6
$\pi^0 \bar{K}^{*0}$	$-17.23^{+3.33+7.59}_{-3.00-12.57}$	$-15.11^{+2.93+12.34}_{-2.65-10.64}$	$2.16^{+0.39+17.53}_{-0.42-36.80}$	-15 ± 13
$\delta(\pi \bar{K}^*)$	$2.68^{+0.72+5.44}_{-0.67-4.30}$	$-1.54^{+0.45+4.60}_{-0.58-9.19}$	$7.26^{+1.21+12.78}_{-1.34-20.65}$	17 ± 25
$\Delta(\pi \bar{K}^*)$	$-7.18^{+1.38+3.38}_{-1.28-5.35}$	$-3.45^{+0.67+9.48}_{-0.59-4.95}$	$-1.02^{+0.19+4.32}_{-0.18-7.86}$	-5 ± 45

- ▷ Overall, large experimental and/or theory uncertainties
- ▷ $\delta(\pi K)$ remains a puzzle.

Main limitation of QCDF approach, e.g. weak annihilation



$$\sim \int d\omega du dv T(\omega, u, v) \phi_B(\omega) \phi_{M_1}(v) \phi_{M_2}(u) \quad ?$$

- ▶ convolutions diverge at endpoints \Rightarrow **non-factorisation in SCET-2**
- ▶ currently modelled with arbitrary soft rescattering phase

Pure annihilation decays

$$10^6 \text{ Br}(B_d \rightarrow K^+ K^-) = 0.13 \pm 0.05 \quad (\Delta D = 1, \text{ exchange topology})$$

$$10^6 \text{ Br}(B_s \rightarrow \pi^+ \pi^-) = 0.76 \pm 0.13 \quad (\Delta S = 1, \text{ penguin annihilation})$$

\Rightarrow extract weak annihilation amplitudes from data

[Wang, Zhu 13; Bobeth, Gorbahn, Vickers 14;
Chang, Sun, Yang, Li 14]

▷ Or use “clean” combinations, e.g. $\Delta = T - P$ in penguin mediated decays

[Descotes-Genon, Matias, JV '06,'07,'11]

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QCD FACTORIZATION

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- Perturbative calculation

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THREE-BODY DECAYS

- Kinematics

- Factorization properties

- Hadronic input

- Quasi-two-body decays

CHALLENGES

:: Three-body B decays

- ▶ Model-independent treatment of vector resonances:
 - ▶ $B \rightarrow \rho l \nu \longrightarrow B \rightarrow [\pi\pi] l \nu$
 - ▶ $B \rightarrow K^* l l \longrightarrow B \rightarrow [K\pi] l l$
 - ▶ Finite-width effects, interference (S-wave pollution, etc.)
- ▶ More complicated kinematics \longrightarrow more observables
- ▶ Larger phase space: different kinematic regimes, different theory descriptions
- ▶ Kinematic distributions \longrightarrow tests of EFT expansions & Factorization
- ▶ E -dependent rescattering effects \longrightarrow large strong phases
 \longrightarrow Large localized CP asymmetries
- ▶ Huge data sets
- ▶ Many applications: CKM parameters, spectroscopy, etc.

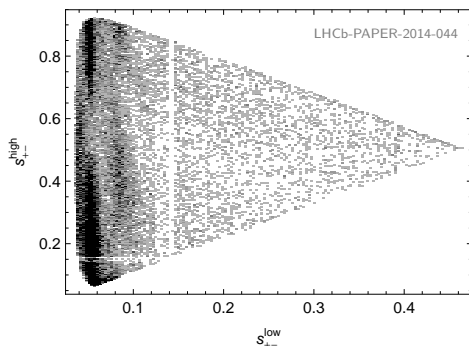
:: Kinematics

$$B^-(p) \rightarrow \pi^-(k_1)\pi^+(k_2)\pi^-(k_3)$$

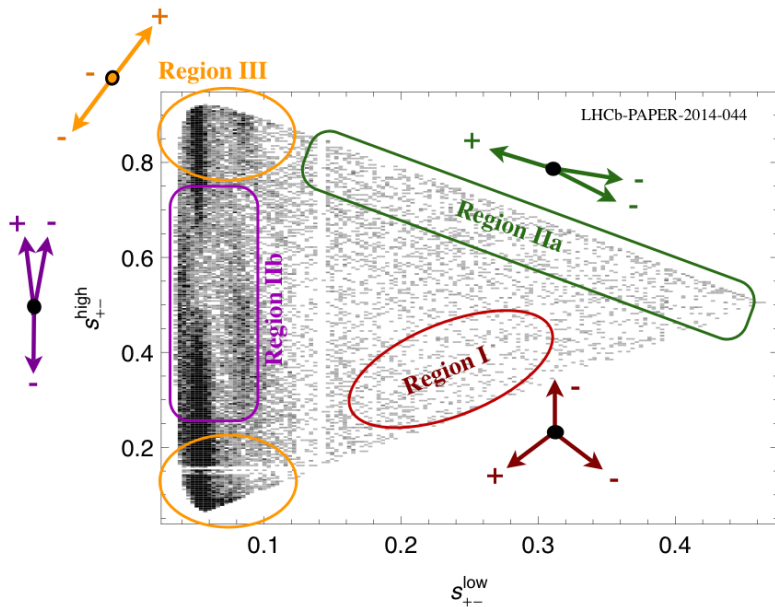
Kinematics completely specified in terms of 2 invariants:

$$p^2 = m_B^2, \quad k_i^2 = 0, \quad s_{ij} \equiv \frac{(k_i + k_j)^2}{m_B^2}, \quad s_{12} + s_{13} + s_{23} = 1$$

For example s_{12} and s_{23} :



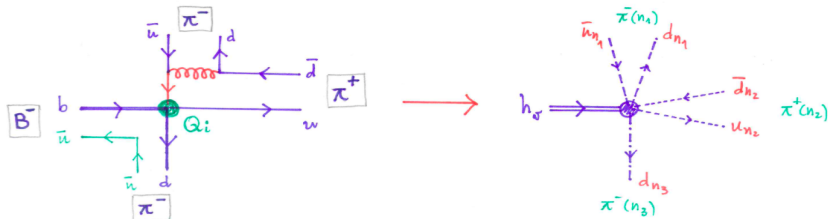
:: Regions of phase space



:: Region I: Center

Krankl, Mannel, JV '15

- ★ Three collinear directions n_1, n_2, n_3 , disconnected at the leading power.



$$\begin{aligned}
 \langle \pi_{n_1}^- \pi_{n_2}^+ \pi_{n_3}^- | \mathcal{O}_i | B \rangle &= \langle \pi_{n_3}^- | \bar{d}_{n_3} \Gamma_3 h_v | B \rangle \\
 &\times \int du dv T_i(u, v) \langle \pi_{n_1}^- | \bar{d}_{n_1}(\bar{u}) \Gamma_1 u_{n_1}(u) | 0 \rangle \langle \pi_{n_2}^+ | \bar{u}_{n_2}(\bar{v}) \Gamma_2 d_{n_2}(v) | 0 \rangle \\
 &\sim F^{B \rightarrow \pi} T_i \otimes \phi_\pi \otimes \phi_\pi
 \end{aligned}$$

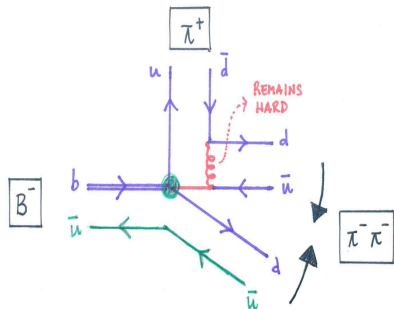
- ★ **Power** ($1/m_b^2$) & α_s suppressed with respect to two-body.
- ★ At leading order/power/twist all convolutions are finite \rightarrow **factorization** \checkmark
- ★ Proof of factorization to $\mathcal{O}(\alpha_s^2)$ required (HS spectator terms [SCET-II])

:: Region I: Extrapolating towards $(\pi^- \pi^-)$ Edge

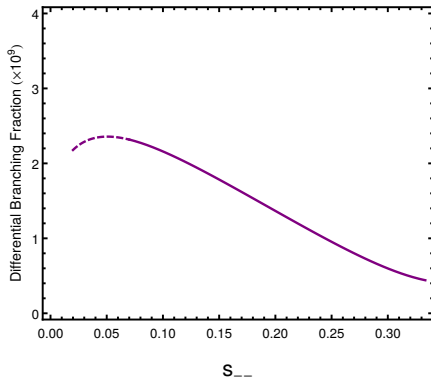
Krankl, Mannel, JV '15

★ **No resonances** \rightarrow perturbative result should be regular.

★ Regularity also expected from absence of soft propagators in QCD:



★ We confirm this expectation:

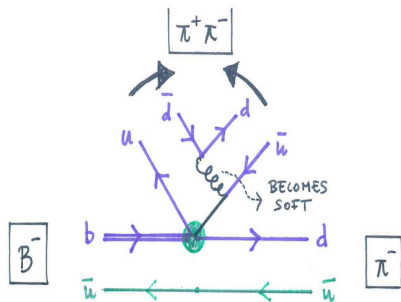


★ Asymptotic result:
$$\frac{d\Gamma}{ds_{--} ds_{+-}} \simeq 0.84 \Gamma_0 f_+ (m_B^2/2)^2 + \mathcal{O}(s_{--})$$

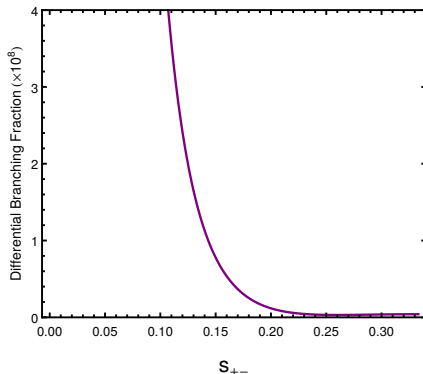
:: Region I: Extrapolating towards $(\pi^+\pi^-)$ Edge

Krankl, Mannel, JV '15

- ★ Resonances ($\rho, \omega, \rho', \dots$) \rightarrow perturbative result should break down.
- ★ Non-regularity also expected from presence of soft propagators in QCD:



- ★ We confirm this expectation:

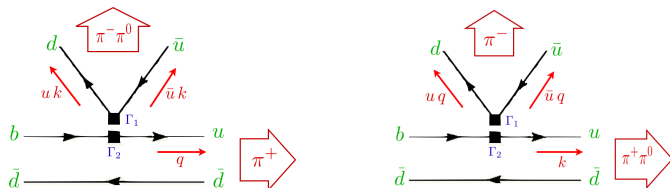


- ★ Asymptotic result:
$$\frac{d\Gamma}{ds_{+-} ds_{--}} \simeq \frac{0.38}{s_{+-}} \Gamma_0 f_+(0)^2 + \text{regular}$$

:: Region IIb: New non-perturbative input

Krankl, Mannel, JV '15

- Breakdown of factorization at resonant edges requires **new NP functions**.
- 3-body decay resembles 2-body, but with new $(\pi\pi)$ “compound object”:



- Operators are the same as in 2-body, but final states different:

$$\begin{aligned}
 \langle \pi_{\bar{n}}^- \pi_{\bar{n}}^+ \pi_{\bar{n}}^- | \mathcal{O} | B \rangle &= \langle \pi_{\bar{n}}^- | \bar{h}_V \Gamma \xi_n | B \rangle \times \int dz T_1(z) \langle \pi_{\bar{n}}^- \pi_{\bar{n}}^+ | \bar{\chi}_{\bar{n}}(z\bar{n}) \Gamma' \chi_{\bar{n}}(0) | 0 \rangle \\
 &+ \langle \pi_{\bar{n}}^- \pi_{\bar{n}}^+ | \bar{h}_V \Gamma \xi_{\bar{n}} | B \rangle \times \int dz T_2(z) \langle \pi_{\bar{n}}^- | \bar{\chi}_{\bar{n}}(zn) \Gamma' \chi_n(0) | 0 \rangle \\
 &\sim F^{B \rightarrow \pi} T_1 \otimes \phi_{\pi\pi} + F^{B \rightarrow \pi\pi} T_2 \otimes \phi_{\pi}
 \end{aligned}$$

- New non-perturbative input:

- **Generalized Distribution Amplitudes (GDAs)** [Diehl, Polyakov, Gousset, Pire...]
- **Generalized Form Factors (GFFs)** [Faller, Feldmann, Khodjamirian, Mannel, van Dyk...]

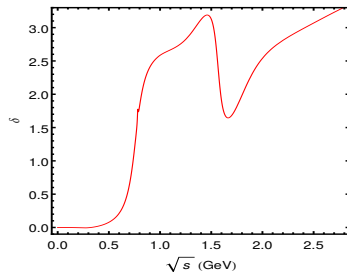
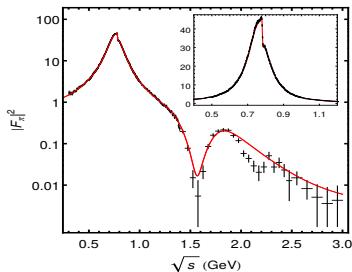
- Definition: $[s = (k_1 + k_2)^2, k_1 = \zeta k_{12}, k_2 = \bar{\zeta} k_{12}]$

$$\phi_{\pi\pi}^q(z, \zeta, s) = \int \frac{dx^-}{2\pi} e^{iz(k_{12}^+ x^-)} \langle \pi^+(k_1) \pi^-(k_2) | \bar{q}(x^- n_-) \not{n}_+ q(0) | 0 \rangle$$

- Normalization (local correlator):

$$\int dz \phi_{\pi\pi}(z, \zeta, s) = (2\zeta - 1) F_\pi(s) \quad (\text{pion time-like FF})$$

- $F_\pi(s)$: Data ($e^+e^- \rightarrow \pi\pi(\gamma)$ [BaBar]) + Theory ((R) χPT , Asymptotics...)



:: $B \rightarrow \pi\pi$ form factors from LCSRs

Hambrock, Khodjamirian 2015

→ Use Light-cone sum rule with 2π distribution amplitudes:

▷ Correlation function

$$\Pi^{(5)}(k^2, q, q \cdot \bar{k}) = -im_b^2 \int d^4x e^{iq \cdot x} \langle \pi(k_1) \pi(k_2) | T \{ \bar{d}(x) \gamma_5 b(x), \bar{b}(0) \gamma_5 u(0) \} | 0 \rangle$$

▷ Unitarity relation

$$\Pi^{(5)}(k^2, q, q \cdot \bar{k}) = \frac{\langle \pi \pi | \bar{d} i m_b \gamma_5 b | B \rangle \langle B | \bar{b} i m_b \gamma_5 u | 0 \rangle}{m_B^2 - p^2} + \dots$$

▷ Dispersion relation + OPE + Borel + duality

$$F_{\perp}(k^2, q^2, \zeta) = \frac{m_b^2 \sqrt{q^2}}{\sqrt{2} f_B m_B^2} \int_{u_0}^1 \frac{du}{u^2} (m_b^2 - q^2 + u^2 k^2) \phi_{\parallel}(u, \zeta, k^2) e^{\frac{m_b^2}{M^2} - \frac{m_b^2 - \bar{u}q^2 + u\bar{u}k^2}{uM^2}}$$

▷ Also other LCSRs for F_{\perp} and F_{\parallel}

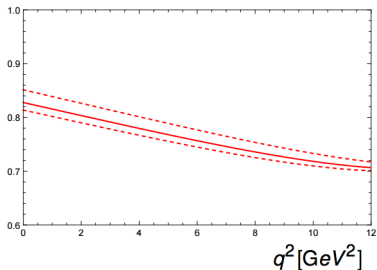
:: $B \rightarrow \pi\pi$ form factors from LCSRs

Hambrock, Khodjamirian 2015

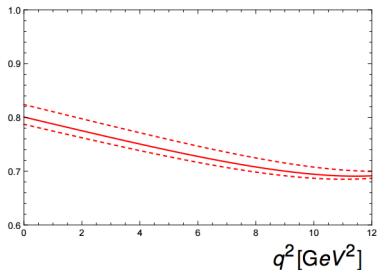
→ Use Light-cone sum rule with 2π distribution amplitudes:

▷ **Sample result:** ρ contribution to the total vector form factor

$$\frac{[F_{\perp}^{(\ell=1)}(q^2, k_{min}^2)](\rho)}{[F_{\perp}^{(\ell=1)}(q^2, k_{min}^2)](LCSR)}$$



$$\frac{[F_{\parallel}^{(\ell=1)}(q^2, k_{min}^2)](\rho)}{[F_{\parallel}^{(\ell=1)}(q^2, k_{min}^2)](LCSR)}$$



:: $B \rightarrow \pi\pi$ form factors from LCSRs

Cheng, Khodjamirian, JV w.i.p

→ Use Light-cone sum rule with B -meson distribution amplitudes:

▷ Correlation function

$$F_\mu(k, q) = i \int d^4x e^{ik \cdot x} \langle 0 | T \{ \bar{d}(x) \gamma_\mu u(x), im_b \bar{u}(0) \gamma_5 b(0) \} | \bar{B}^0(q+k) \rangle$$

▷ Unitarity relation

$$2\text{Im}F_\mu(k, q) = im_b \int d\tau_{2\pi} \langle 0 | \bar{d} \gamma_\mu | \pi(k_1) \pi(k_2) \rangle \langle \pi(k_1) \pi(k_2) | \bar{u} \gamma_5 b | \bar{B}^0(q+k) \rangle + \dots$$

▷ Dispersion relation + OPE + Borel + duality

$$\int_{4m_\pi^2}^{s_0^{2\pi}} ds e^{-s/M^2} \frac{s \sqrt{q^2} [\beta_\pi(s)]^2}{4\sqrt{6}\pi^2 \sqrt{\lambda}} F_\pi^*(s) F_t^{(1)}(s, q^2) = -f_B m_B^2 m_b \left\{ \int_0^{\sigma_0^{2\pi}} d\sigma e^{-s(\sigma, q^2)/M^2} \times \right. \\ \left. \times \left[\frac{\sigma}{\bar{\sigma}} \phi_+^B(\sigma m_B) - \frac{\sigma}{\bar{\sigma}} \left[\phi_+^B(\sigma m_B) - \phi_-^B(\sigma m_B) \right] - \frac{1}{\bar{\sigma} m_B} \bar{\Phi}_\pm^B(\sigma m_B) \right] + \Delta A_0^{BV}(q^2, \sigma_0^{2\pi}, M^2) \right\}$$

:: $B \rightarrow \pi\pi$ form factors from LCSRs

Cheng, Khodjamirian, JV w.i.p

→ Use Light-cone sum rule with B -meson distribution amplitudes

- ▷ Also other LCSRs for vector F_{\perp} and axial F_0, F_{\parallel} form factors.
- ▷ In the **single-pole approximation** (ρ -dominance in the narrow width approximation)
⇒ **one recovers analytically** the results for the $B \rightarrow \rho$ form factors V, A_0, A_1, A_2 .
- ▷ Include **finite-width effects** and **contributions from ρ', ρ'' resonances**.

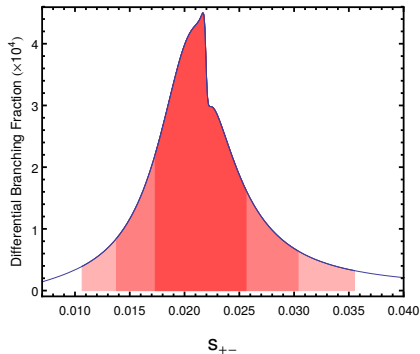
$$B \rightarrow (\pi\pi)_\rho \pi$$

★ Leading order amplitude:

$$\mathcal{A}|_{s_{+-} \ll 1} = \frac{G_F}{\sqrt{2}} [4m_B^2 f_0(s_{+-})(2\zeta - 1)F_\pi(s_{+-})(a_2 + a_4) + f_\pi m_\pi (a_1 - a_4)F_t(\zeta, s_{+-})]$$

★ Integrating around the ρ :

$$BR(B^- \rightarrow \rho \pi^-) \simeq \int_0^1 ds_{++} \int_{s_\rho^-}^{s_\rho^+} ds_{+-} \frac{\tau_B m_B |\mathcal{A}|^2}{32(2\pi)^3}$$



$$\text{with } s_\rho^\pm = (m_\rho \pm n\Gamma_\rho)^2 / m_B^2$$

$$BR(B^+ \rightarrow \rho \pi^+) \simeq 9.4 \cdot 10^{-6} \quad (n = 0.5)$$

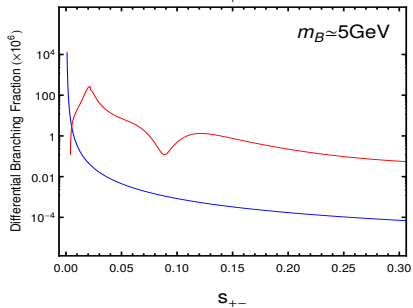
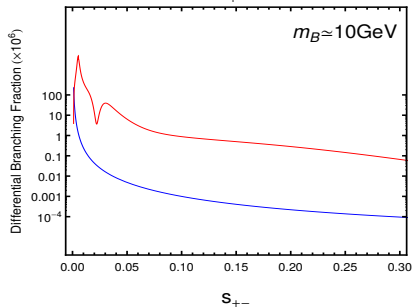
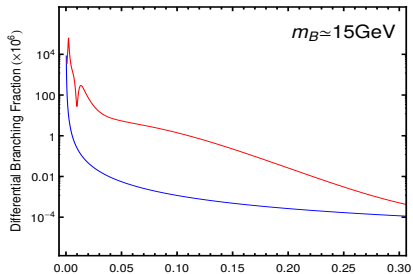
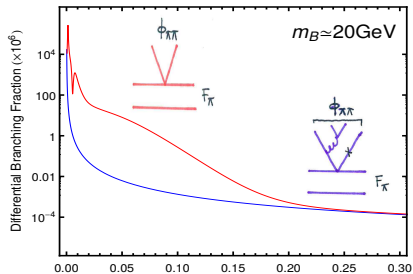
$$BR(B^+ \rightarrow \rho \pi^+) \simeq 12.8 \cdot 10^{-6} \quad (n = 1)$$

$$BR(B^+ \rightarrow \rho \pi^+) \simeq 14.1 \cdot 10^{-6} \quad (n = 1.5)$$

$$BR(B^+ \rightarrow \rho \pi^+)_{\text{EXP}} = (8.3 \pm 1.2) \cdot 10^{-6}$$

$$BR(B^+ \rightarrow \rho \pi^+)_{\text{QCDF}} = (11.9_{-6.1}^{+7.8}) \cdot 10^{-6}$$

Merging Regions: How large should m_B be? ($\phi_{\pi\pi}$ term)



:: OUTLINE

QCD FACTORIZATION

TWO-BODY DECAYS

- Perturbative calculation

- Tree and penguin decays

- Power corrections

THREE-BODY DECAYS

- Kinematics

- Factorization properties

- Hadronic input

- Quasi-two-body decays

CHALLENGES

:: Summary and Challenges

Two body decays

- ▷ NNLO: End of the road for perturbative calculations
- ▷ Mostly ok, except for a few cases.
Large uncertainty from λ_B and power corrections.
- ▷ Challenge: Precise determination of λ_B from $B \rightarrow \gamma \ell \nu$ (Belle-II).
- ▷ Challenge: Power corrections. Factorization in SCET-2.

Three body decays

- ▷ Lots of data, great potential.
- ▷ Can be studied within QCDF. Need 2π LCDA's and $B \rightarrow MM$ form factors.
- ▷ $B \rightarrow VP$: include finite-width effects and contributions from excited resonances.
- ▷ Challenge: Full analysis at NLO, including CPV.
- ▷ Challenge: Soft corners need alternative treatment. These regions include interferences from “crossed” resonances, potentially interesting for localized CP asymmetries.

Backup Slides

Kinematics: $B^-(p) \rightarrow \pi^+(k_1)\pi^-(k_2)\ell^-(q_1)\bar{\nu}(q_2)$

$$k^2 = (k_1 + k_2)^2, \quad q^2 = (q_1 + q_2)^2, \quad 2q \cdot (k_1 - k_2) = \beta_\pi \sqrt{\lambda} \cos \theta_\pi$$

$$\mathcal{L} = \mathcal{L}_{QED+QCD} + C_{LL} [\bar{u}\gamma^\mu P_L b] [\bar{\ell}\gamma_\mu P_L \nu_\ell] + \dots$$

$$\mathcal{A} = C_{LL} \langle \ell\bar{\nu} | \bar{\ell}\gamma_\mu P_L \nu_\ell | 0 \rangle \langle \pi^+\pi^- | \bar{u}\gamma^\mu P_L b | B^- \rangle = C_{LL} \mathcal{F}^\mu \bar{u}_\ell \gamma_\mu \nu_\ell$$

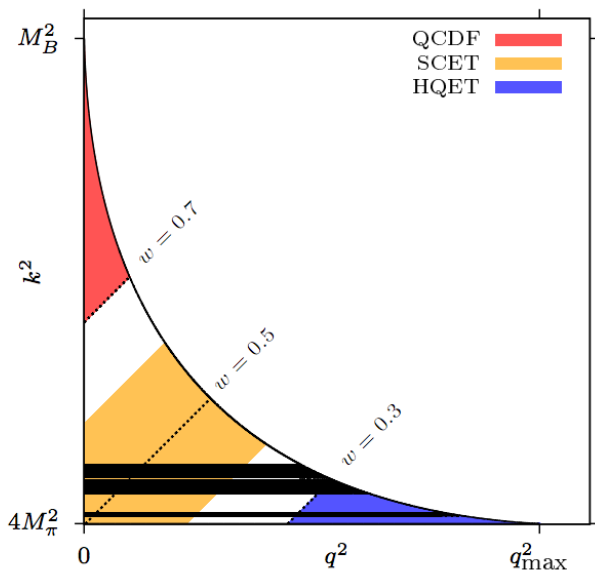
$\rightarrow \mathcal{F}^\mu$: $B \rightarrow \pi\pi$ **form factor** (one axial, three vector invariant FFs):

$$\varepsilon(\mathbf{q}, \mathbf{0})_\mu^* \langle \pi\pi | \bar{u}\gamma^\mu P_L b | B \rangle = F_0$$

$$\varepsilon(\mathbf{q}, \mathbf{t})_\mu^* \langle \pi\pi | \bar{u}\gamma^\mu P_L b | B \rangle = F_t$$

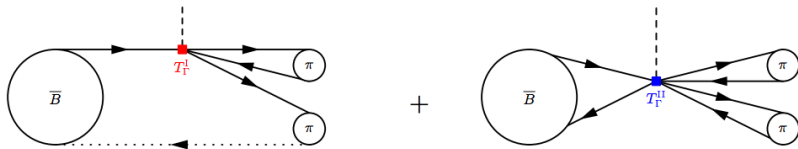
$$\varepsilon(\mathbf{q}, \pm)_\mu^* \langle \pi\pi | \bar{u}\gamma^\mu P_L b | B \rangle = \beta_\pi \sin \theta_\pi e^{\pm i\phi} (F_\perp + F_\parallel) / \sqrt{2}$$

where $F_i = F_i(q^2, k^2, \theta_\pi) = \sum_\ell F_i^{(\ell)}(q^2, k^2) P_\ell(\cos \theta_\pi)$ [partial waves in $\pi\pi$]



Factorization at large k^2 : $F_i = f_{B\pi} T_i^I \otimes \phi_\pi + T_i^{II} \otimes \phi_\pi \otimes \phi_\pi \otimes \phi_B$

$$\begin{aligned} & \langle \pi^+(k_1)\pi^-(k_2) | \bar{\psi}_u \Gamma \psi_b | B^-(p) \rangle \\ &= \frac{2\pi f_\pi \xi_\pi(E_2; \mu)}{k^2} \int_0^1 du \phi_\pi(u, \mu) T_\Gamma^I(u, \dots; \mu) \\ &+ \int_0^1 du \int_0^1 dv \int_0^\infty \frac{d\omega}{\omega} \phi_\pi(u; \mu) \phi_\pi(v; \mu) \phi_B^+(\omega; \mu) T_\Gamma^{II}(u, v, \omega, \dots; \mu) \\ &+ \text{power corrections.} \end{aligned}$$



ξ_π denotes the universal non-factorizable $B \rightarrow \pi$ form factor in SCET