



Highlight new
results since
FPCP 2014!

Lattice QCD for the CKM matrix

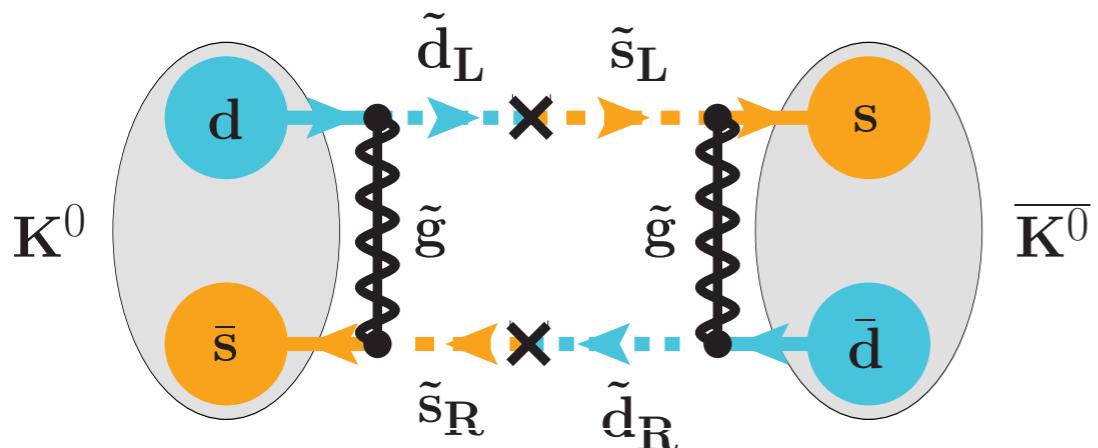
Ruth Van de Water
Fermilab

Flavor Physics & CP Violation
June 7, 2016

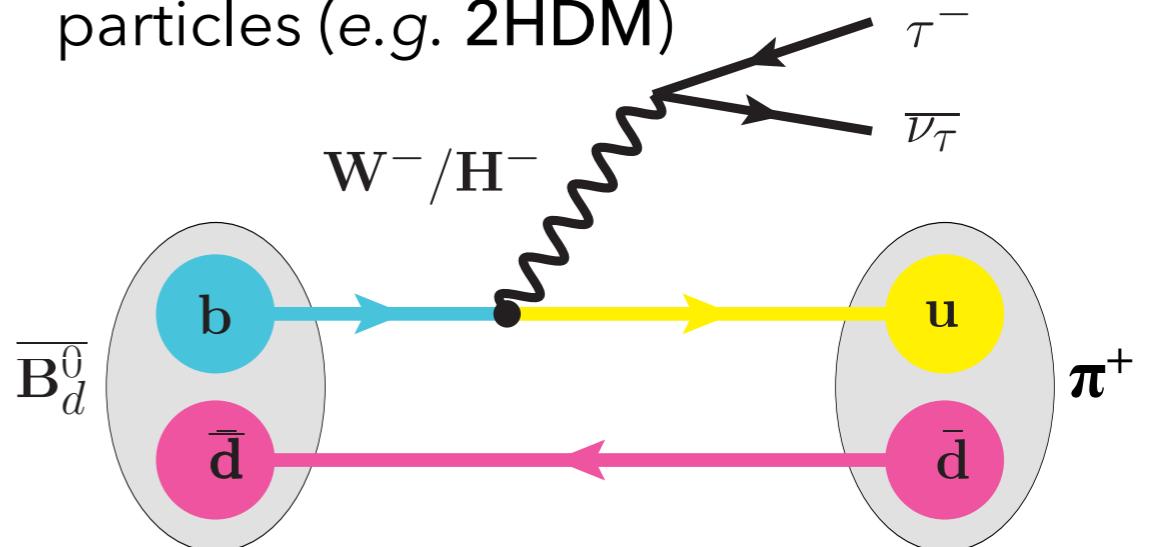
Why study quark-flavor physics?

- ♦ Most Standard-Model extensions have additional sources of flavor & CP violation
- ♦ **Quark-flavor-changing processes receive contributions from exchange of new particles that arise in wide range of models:**

s \rightarrow d & b \rightarrow d,s flavor-changing neutral currents sensitive to SUSY, flavor-changing Z', leptoquarks, 4th generation, composite Higgs, ...



tree-level b-hadron decays to τ leptons sensitive to charged scalars that couple preferentially to heavier particles (e.g. 2HDM)



- ♦ **Precision quark-flavor measurements sensitive to QM loop effects from new particles too heavy to be directly accessible at the LHC** (or foreseeable colliders)

Lattice QCD for quark-flavor physics

Broad experimental effort to study flavor-changing interactions of strange, charm, and beauty hadrons



- ◆ Quark-flavor sector provides numerous experimental observables to compare with Standard Model (differential decay rates, angular distributions & asymmetries, oscillation frequencies, ...)
- ◆ Revealing new-physics effects requires reliable Standard-Model predictions with uncertainties commensurate with experimental measurements
- ◆ Theory error often limited by QCD matrix elements → nonperturbative lattice-QCD calculations crucial to maximize scientific impact of current & future quark-flavor experiments!

Precision lattice QCD

"In the last five years lattice QCD has matured into a precision tool. Results with fully controlled errors are available for nearly 20 matrix elements. ... *The ultimate aim of lattice-QCD calculations is to reduce errors in hadronic quantities to the level at which they become subdominant either to experimental errors or other sources of error.*"

– **Snowmass 2013 Quark-flavor WG report (1311.1076)**

Quantum ChromoDynamics

$$\mathcal{L}_{\text{QCD}} = \frac{1}{2g^2} \text{tr} [F_{\mu\nu} F^{\mu\nu}] - \sum_{f=1}^{n_f} \bar{\psi}_f (\not{D} + m_f) \psi_f + \underbrace{\frac{i\bar{\theta}}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{tr} [F_{\mu\nu} F_{\rho\sigma}]}_{\text{violates } CP}$$

- ♦ QCD Lagrangian contains $1 + n_f + 1$ parameters that can be fixed from equal number of experimental inputs

FUNDAMENTAL PARAMETER

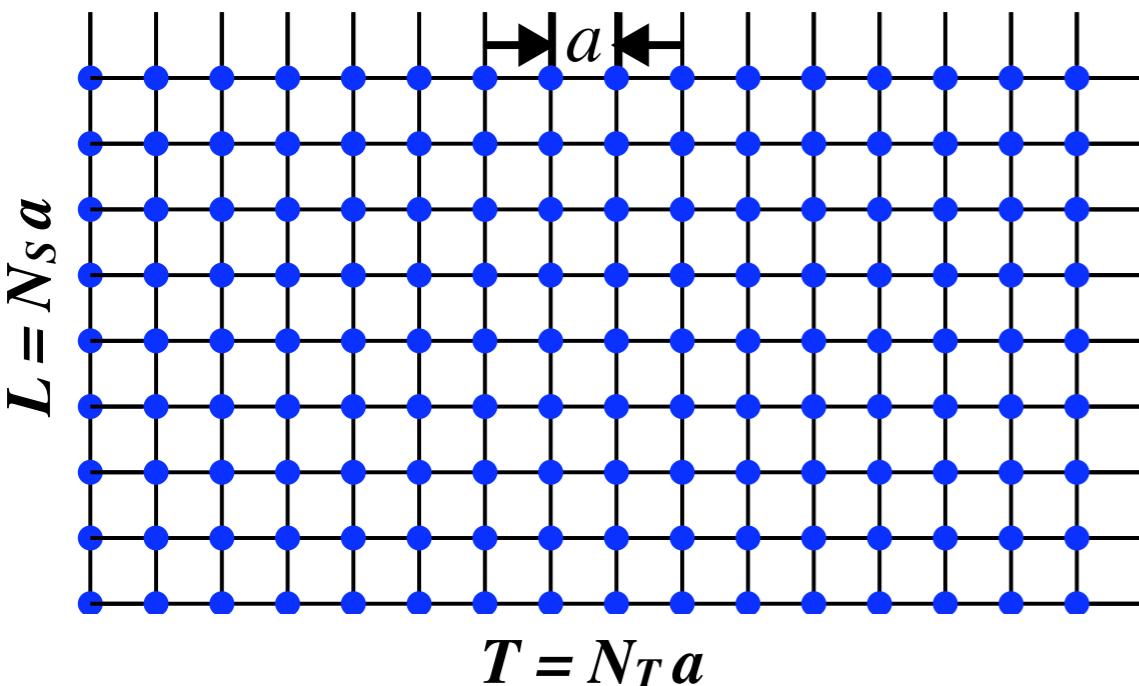
- ❖ Gauge coupling g^2
- ❖ n_f quark masses m_f
- ❖ $\theta = 0$
- ♦ Once the parameters are fixed, everything else is a prediction of the theory
- ♦ Calculations of hadronic parameters challenging in practice because low-energy QCD is nonperturbative

EXPERIMENTAL INPUT

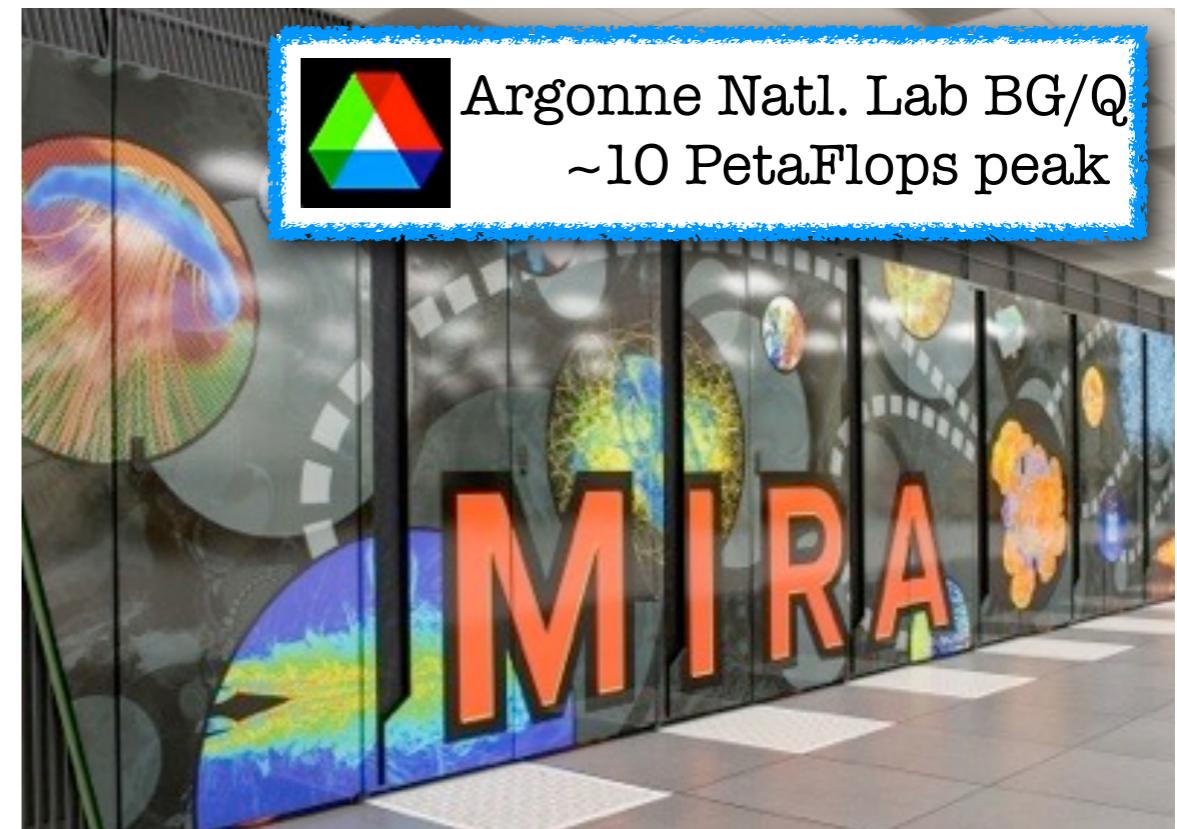
- $r_1, m_\Omega, Y(2S-1S),$ or f_π
- $m_\pi, m_K, m_{J/\psi}, m_Y, \dots$
- neutron EDM ($|\theta| < 10^{-11}$)

Numerical lattice QCD

- ◆ Systematic method for calculating hadronic parameters from QCD first principles
- ◆ Define QCD on (Euclidean) spacetime lattice and solve path integral numerically
 - ❖ Recover QCD when lattice spacing $a \rightarrow 0$ and box size $L \rightarrow \infty$

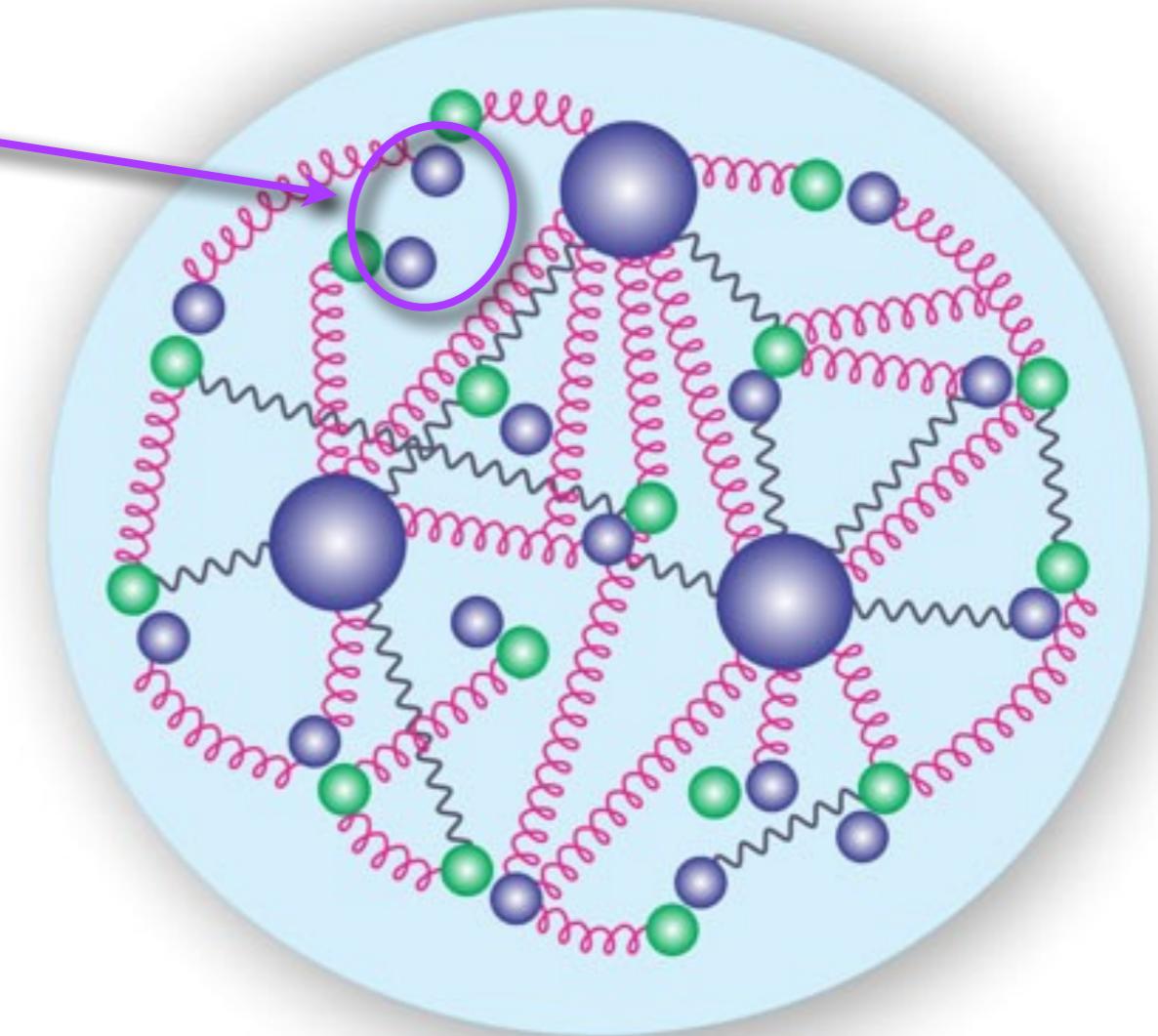


- ◆ Simulate using Monte-Carlo methods and importance sampling
 - ❖ Sample from all possible field configurations using a distribution given by $\exp(-S_{\text{QCD}})$
- ◆ Run codes upon supercomputers and dedicated clusters



Modern lattice-QCD simulations

- ◆ Standard simulations include dynamical u, d, s (& c) quarks in the vacuum
 - ❖ (Typically sea $m_u=m_d$)
- ◆ Control systematic errors using gauge-field ensembles with different parameters:
 - ❖ Multiple lattice spacings to extrapolate to continuum limit ($a \rightarrow 0$)
 - ❖ Multiple up/down-quark masses to extrapolate to physical $M_\pi = 135$ MeV
 - ❖ Multiple spatial volumes to estimate finite-size effects



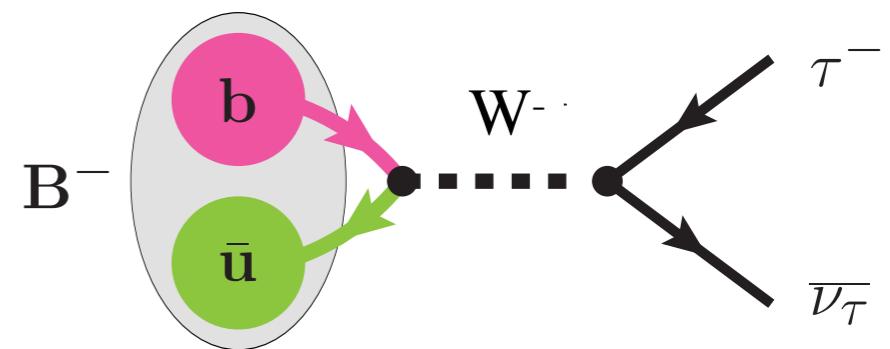
Test and validate methods by

- (1) Comparison with experiment
- (2) Independent calculations sensitive to different systematics

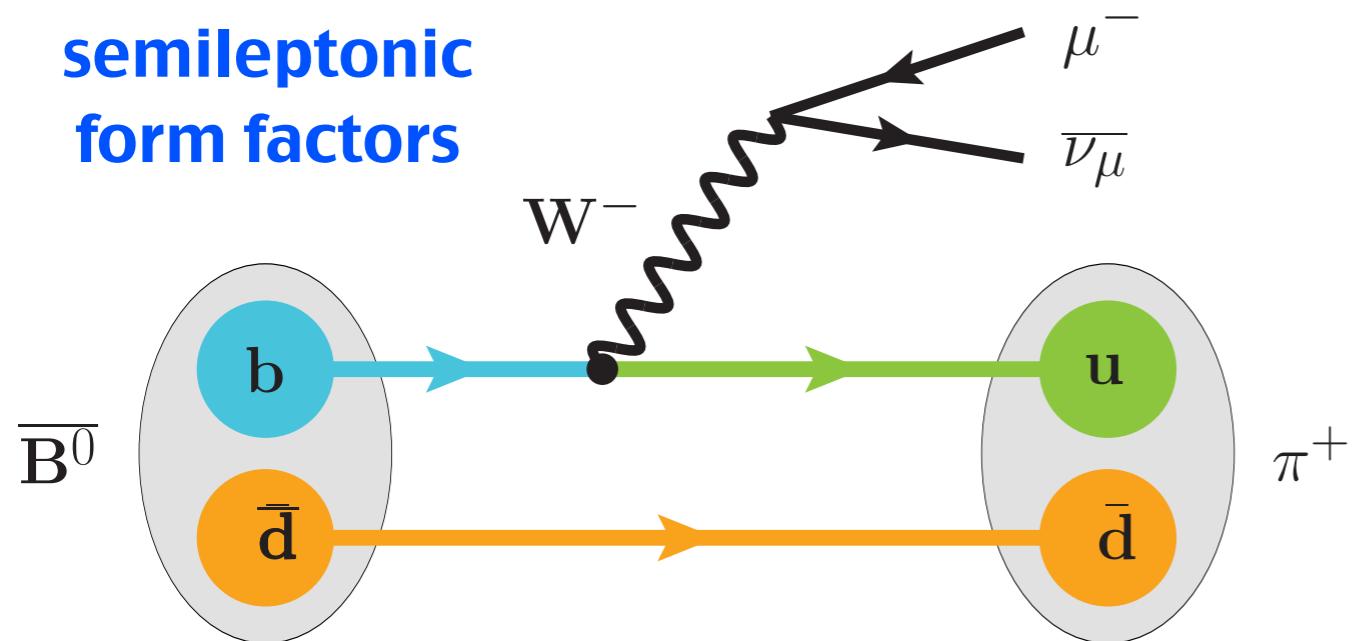
“GOLD-PLATED” lattice processes

- ◆ Most precise results for simple processes with single (stable) initial hadron & at most 1 final-state hadron
 - ❖ Includes numerous quantities needed to obtain CKM matrix elements and test Standard Model
 - ❖ Excludes QCD resonances (ρ , K^* , ...); fully hadronic decays ($K \rightarrow \pi\pi$, $B \rightarrow D\bar{K}$, ...); long-distance dominated quantities (ΔM_K , D-mixing, ...)

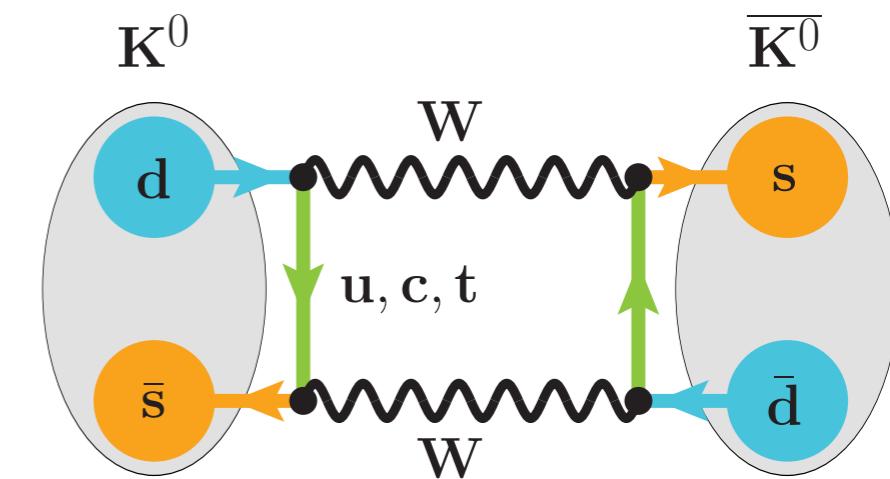
leptonic decay constants



semileptonic
form factors

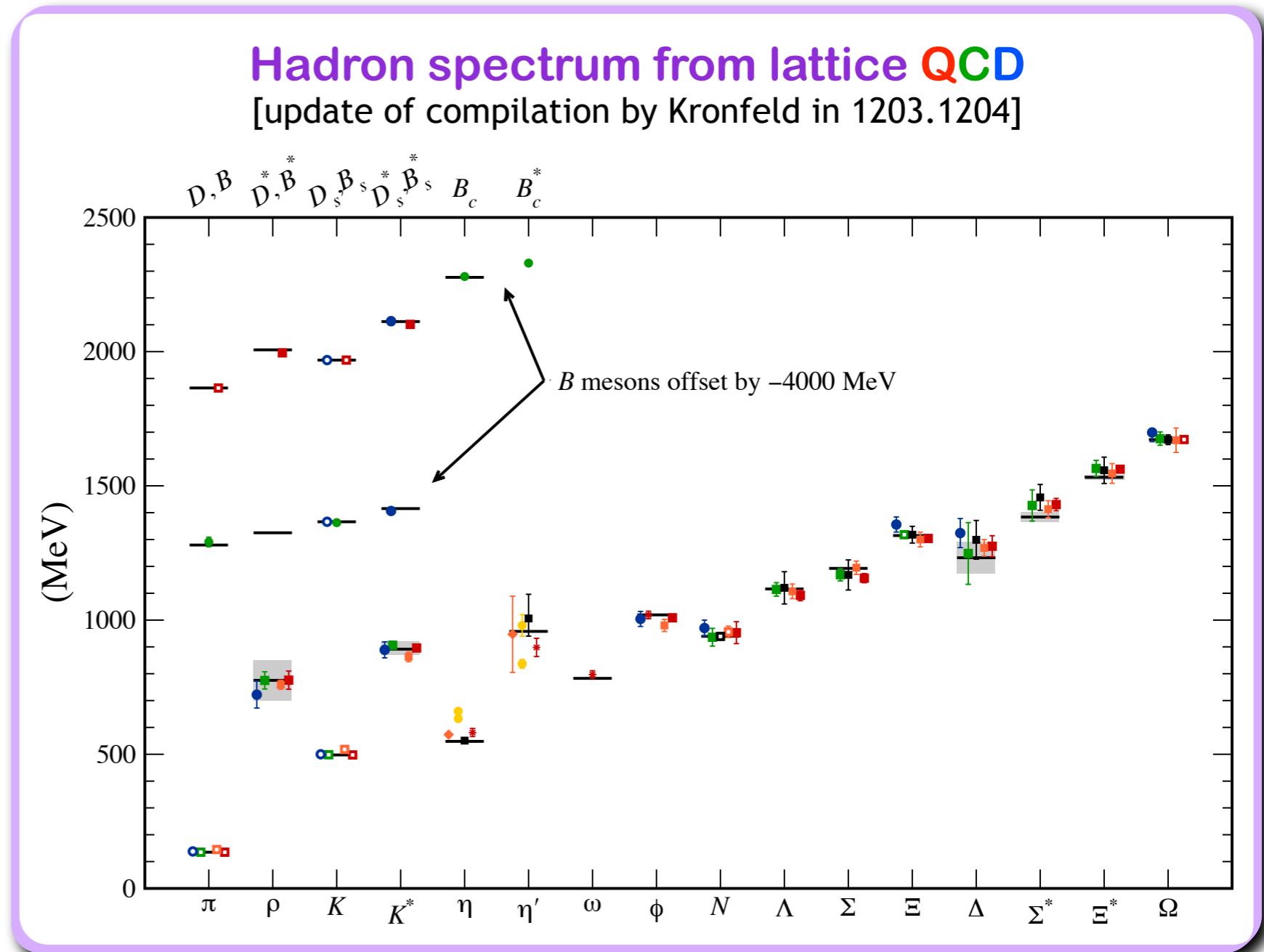


neutral meson mixing



Lattice-QCD validation

- ♦ Reproduce experimental results for wide variety of hadron properties and provide the only *ab initio* QCD calculation of others
 - ❖ Light- and heavy-hadron spectrum (including n-p mass difference)
 - ❖ Most precise α_s determination and competitive b-, c-quark masses
 - ❖ Predictions of B_c meson mass, decay constants f_D & f_{D_s} , and $D \rightarrow K\bar{K}$ form factor
- ♦ Demonstrate that lattice-QCD calculations are reliable with controlled uncertainties





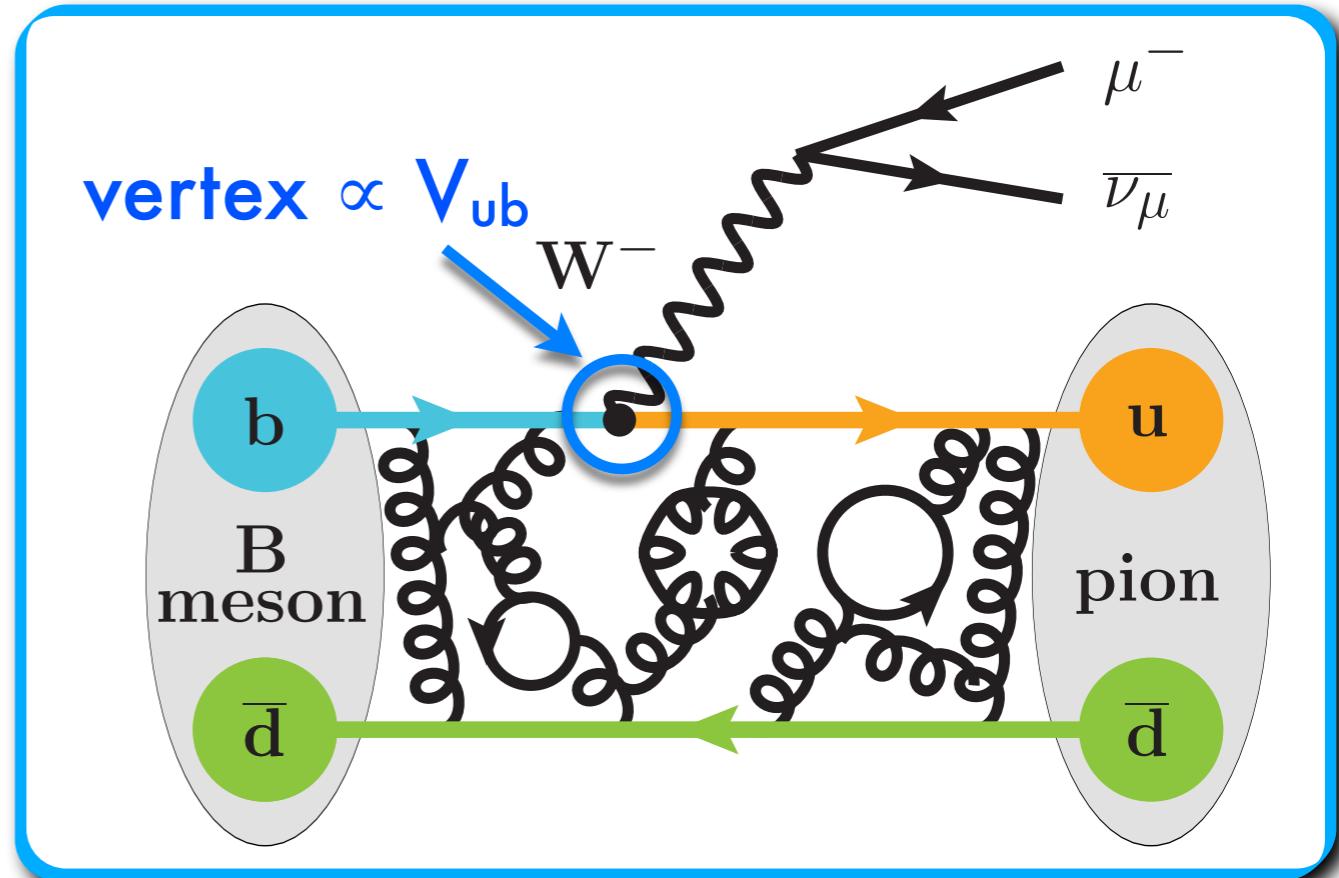
Recent lattice results for the CKM matrix

"[An] area of striking progress has been lattice gauge theory. ... It is now possible to compute the spectrum of hadrons with high accuracy, and lattice computations have been crucial in the measurement of the properties of heavy quarks. Continuing improvements in calculational methods are anticipated in coming years."

– Snowmass 2013 Executive Summary (1401.6075)

Determination of CKM elements

- ◆ Measure flavor-changing processes involving hadrons
- ◆ Infer CKM elements within Standard Model by comparing experimental measurements of flavor-changing interactions with theory predictions
 - ❖ Absorb nonperturbative QCD dynamics into hadronic parameters



$$\Delta m_{(d,s)}, \frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2}, \frac{d\Gamma(B \rightarrow D^{(*)} \ell \nu)}{dw}, \dots$$

Compute nonperturbative QCD parameters (decay constants, form factors, B-parameters,...) numerically with LATTICE QCD

(Experiment) = (known) \times (CKM factors) \times (Hadronic Matrix Element)

Lattice QCD & the CKM matrix

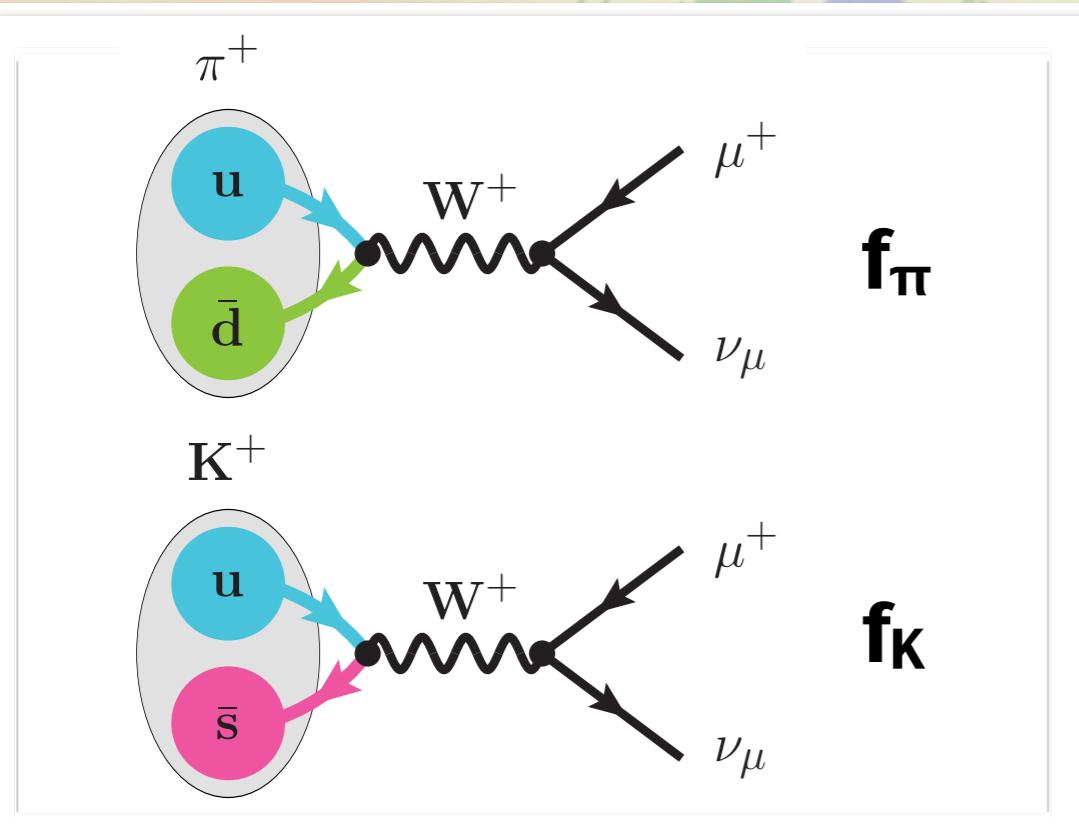
- ♦ Lattice-QCD community has mature & successful program to calculate weak matrix elements needed to obtain CKM elements & phase (see [recent lattice reviews](#))
- ♦ **Here focus on new results since FPCP 2014:** appendices present additional results for light flavors (u,d,s) and heavy flavors (c,b)

♦ Simple lattice processes enable determinations of all CKM elements except $|V_{tb}|$ 

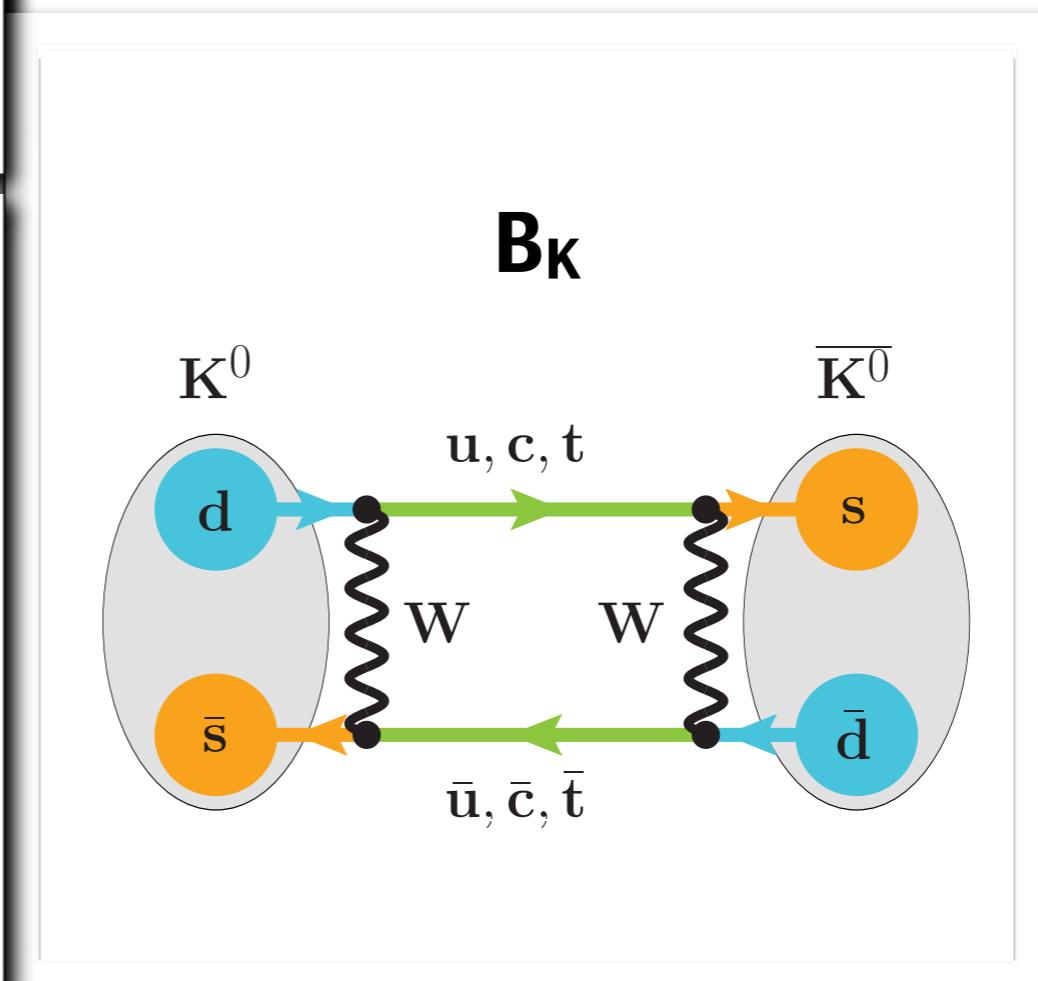
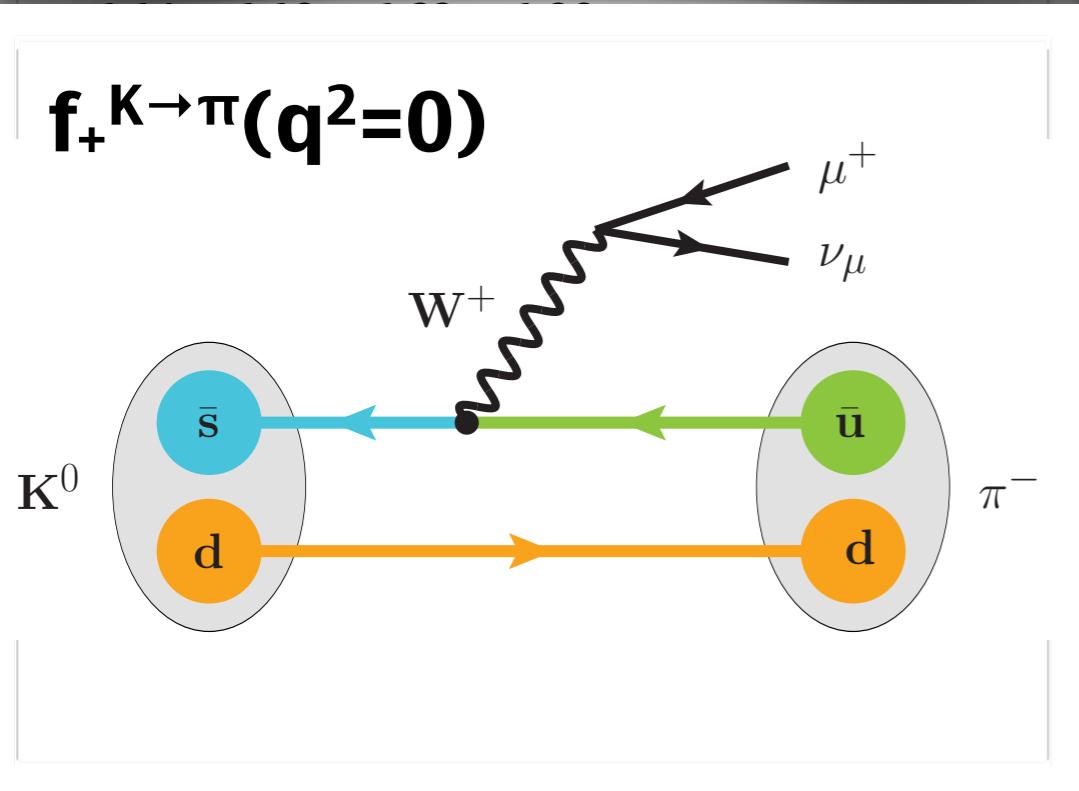
 Neutral kaon mixing also gold-plated & can be used to obtain phase $(\bar{\rho}, \bar{\eta})$

V_{ud} $\pi \rightarrow \ell\nu$	V_{us} $K \rightarrow \ell\nu$ $K \rightarrow \pi\ell\nu$	V_{ub} $B \rightarrow \ell\nu$ $B \rightarrow \pi\ell\nu$
V_{cd} $D \rightarrow \ell\nu$ $D \rightarrow \pi\ell\nu$	V_{cs} $D_s \rightarrow \ell\nu$ $D \rightarrow K\ell\nu$	V_{cb} $B \rightarrow D\ell\nu$ $B \rightarrow D^*\ell\nu$
V_{td} $\langle B_d \bar{B}_d \rangle$ $B \rightarrow \pi\ell\ell$	V_{ts} $\langle B_s \bar{B}_s \rangle$ $B \rightarrow K\ell\ell$	V_{tb}

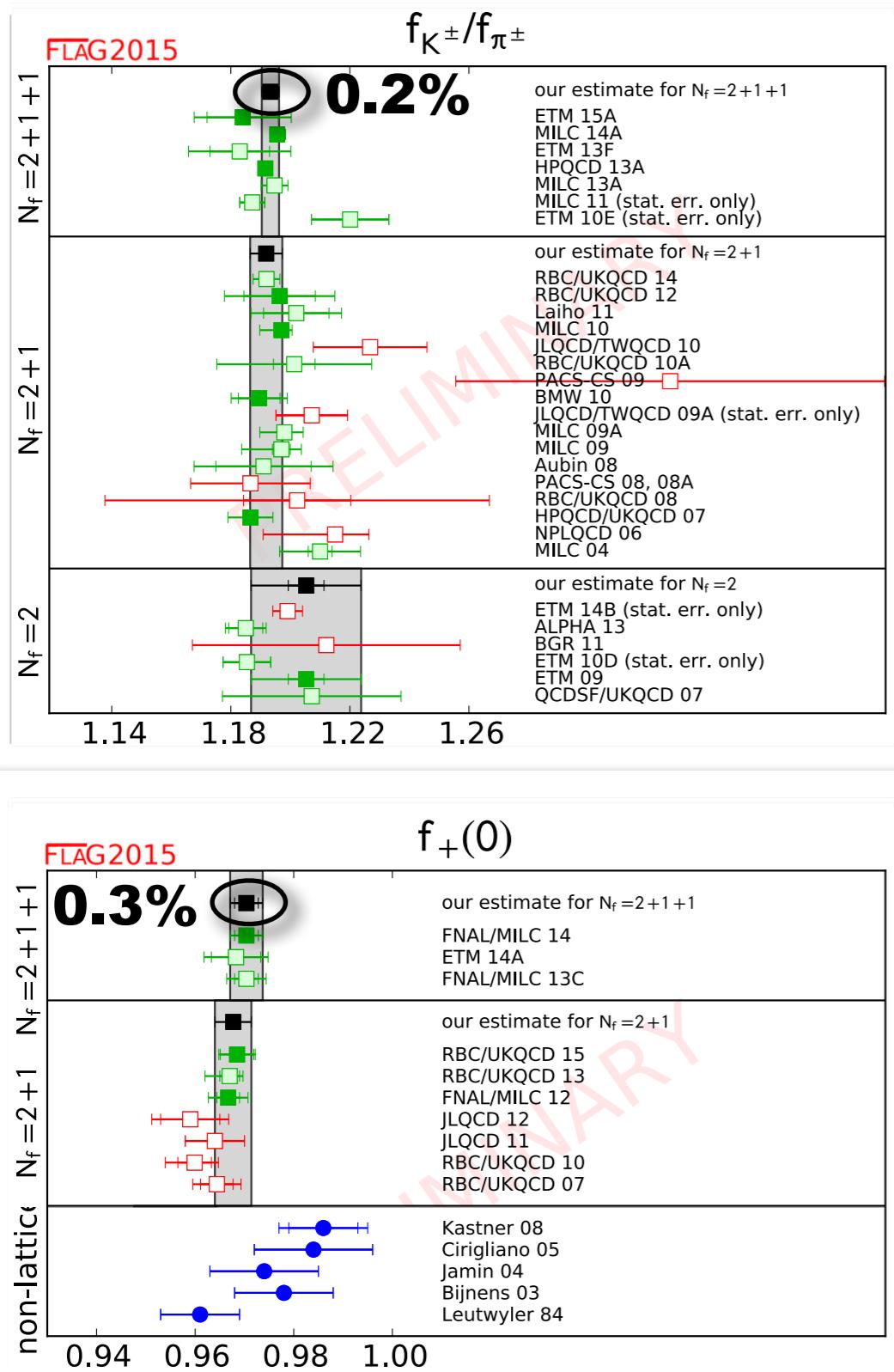
Kaon physics summary



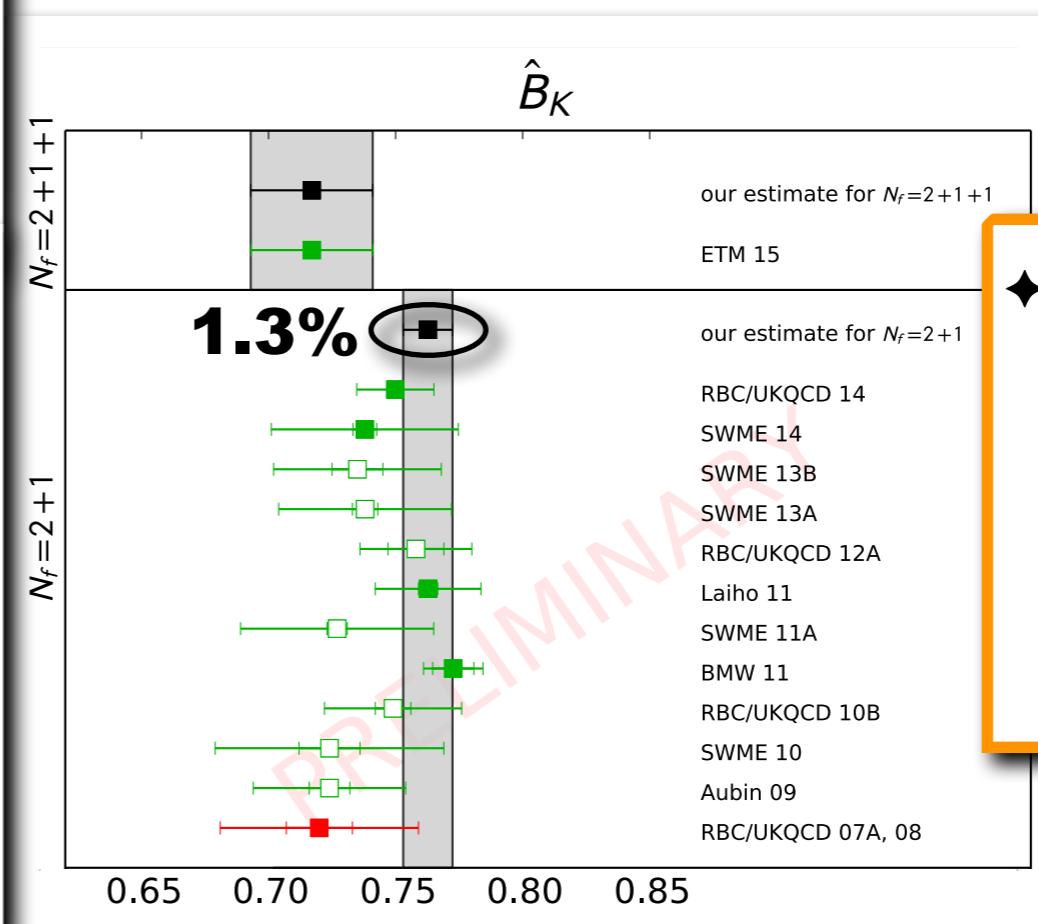
- ◆ For all simple quantities
 - ❖ Physical light-quark masses
 - ❖ Nonperturbative or no renormalization
 - ❖ Confirmation from independent results
- ◆ (Sub-)percent precision → EM & isospin-breaking becoming relevant



Kaon physics summary

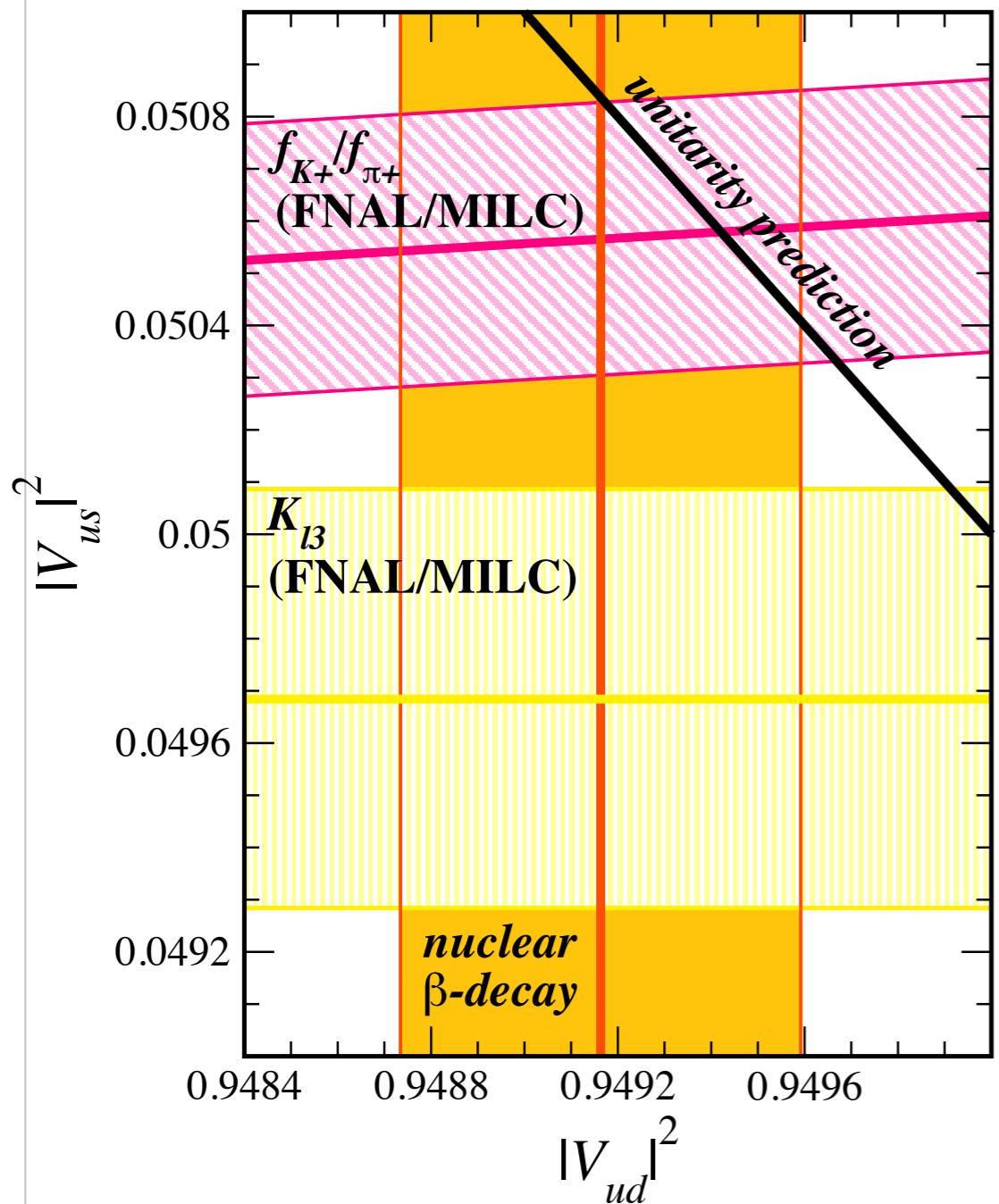


- ♦ For all simple quantities
 - ❖ Physical light-quark masses
 - ❖ Nonperturbative or no renormalization
 - ❖ Confirmation from independent results
- ♦ (Sub-)percent precision → EM & isospin-breaking becoming relevant



- ♦ Kaon-mixing matrix elements of all possible $\Delta S=2$ BSM operators also available

Implications for 1st-row CKM unitarity



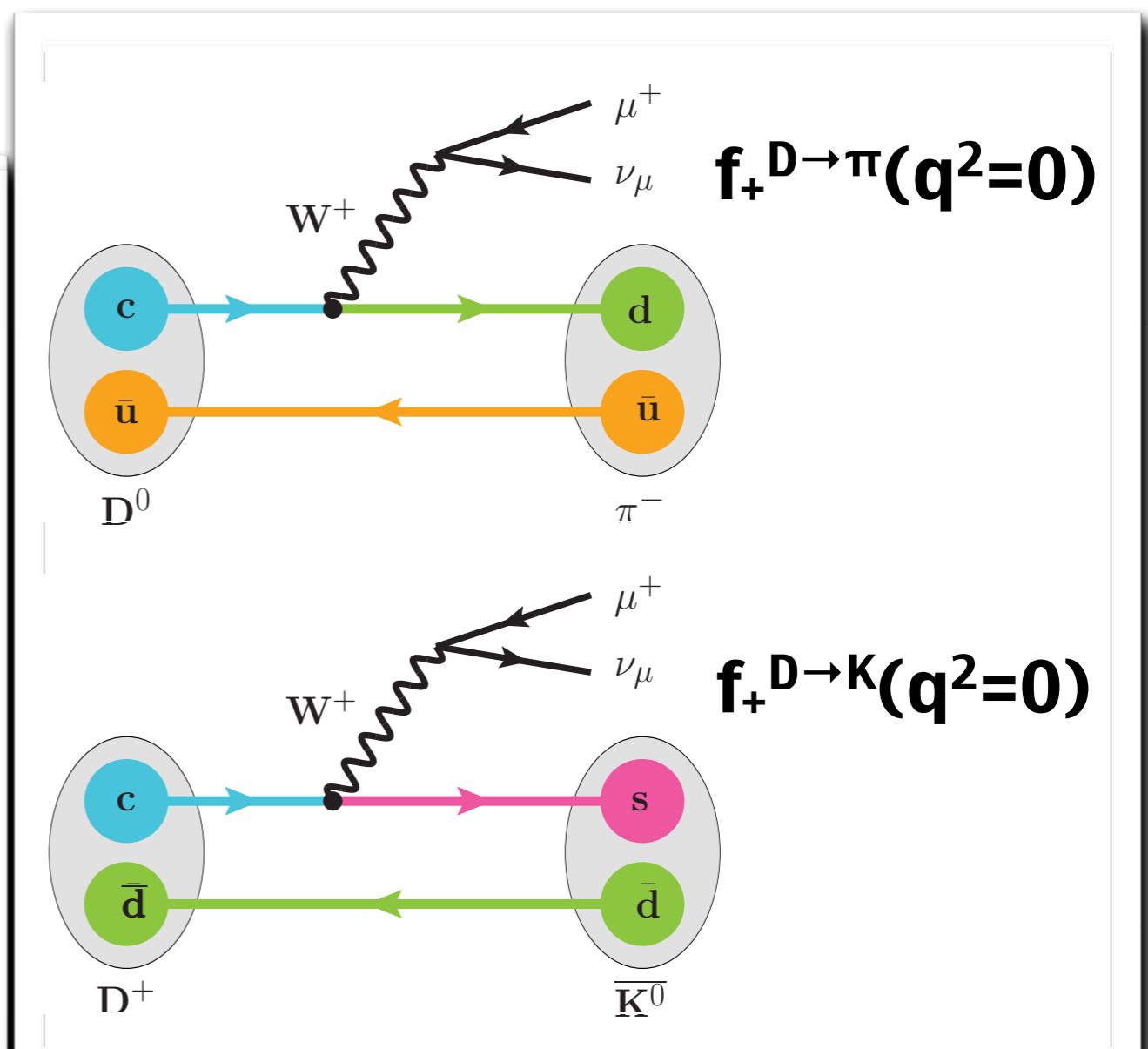
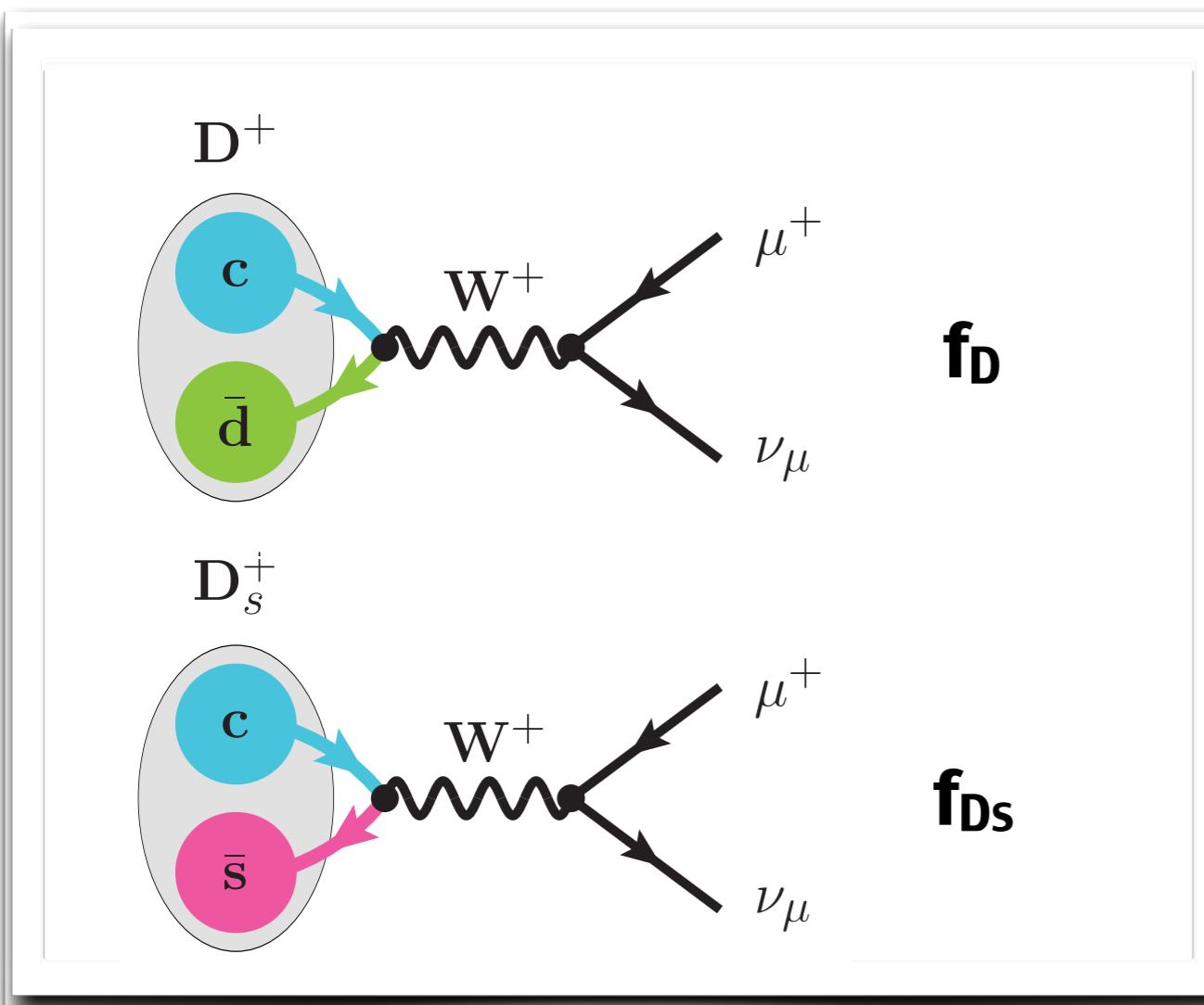
- ◆ ~2 σ tension between leptonic π & K decays, semileptonic K decays, β -decay, and CKM unitarity [see e.g. Rosner, Stone, & RV (PDG), 1509.02220]
- ◆ Unitarity test using $|V_{us}|$ from K_{l3} decay [FLAG 2015 prel. + Moulson, 1411.5252] and $|V_{ud}|$ from superallowed β -decay [Hardy & Towner, 1411.5987] limited by error on: $|V_{ud}|^2$:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = \\ -0.0021(29)_{V_{us}}(41)_{V_{ud}}$$

→ Can $|V_{ud}|$ from β -decay be improved?

D-meson physics summary

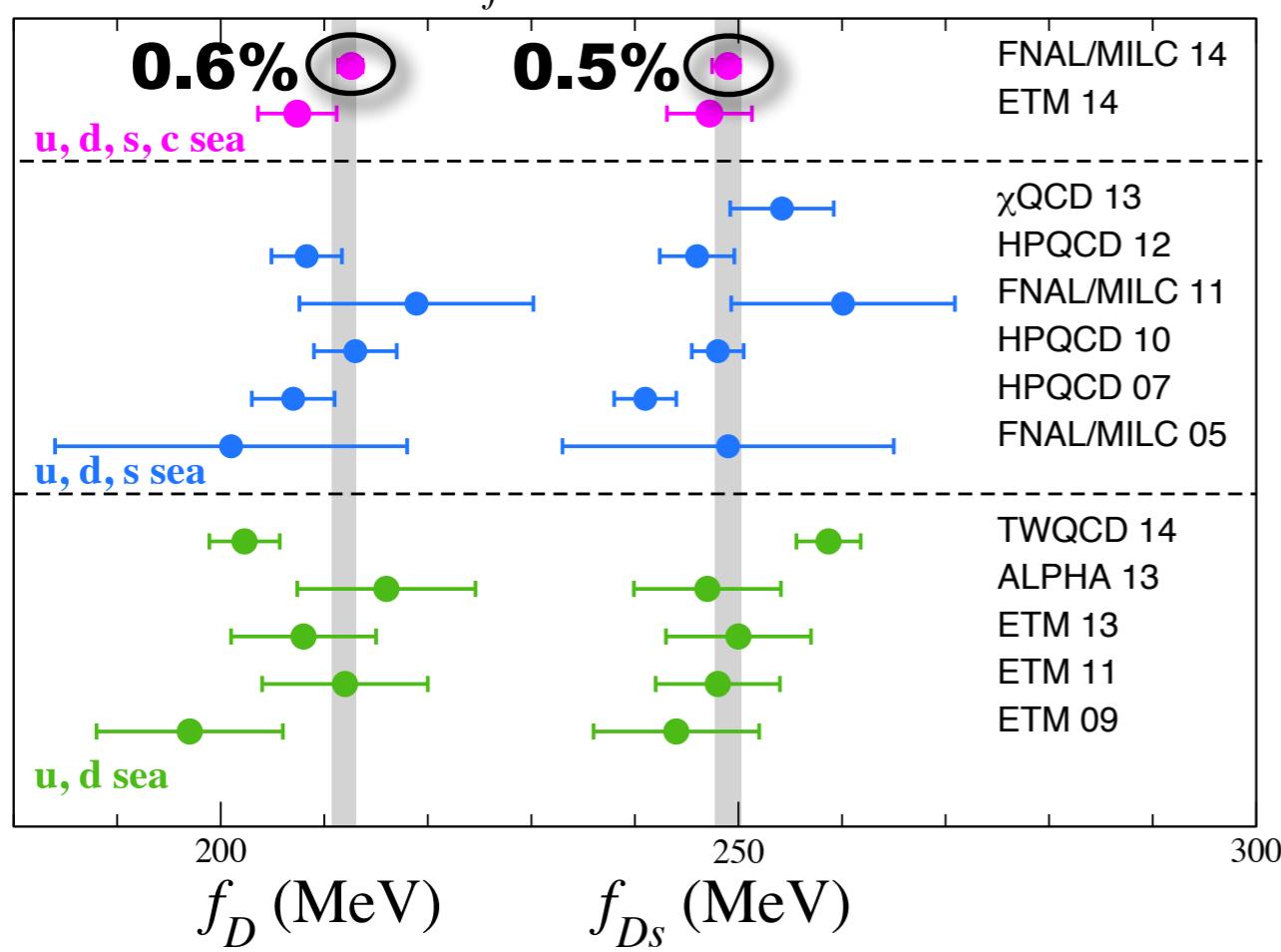
- ◆ Small errors due to
 - ❖ Physical light-quark masses
 - ❖ Improved charm-quark actions + fine lattice spacings
 - ❖ No renormalization (PCAC)



D-meson physics summary

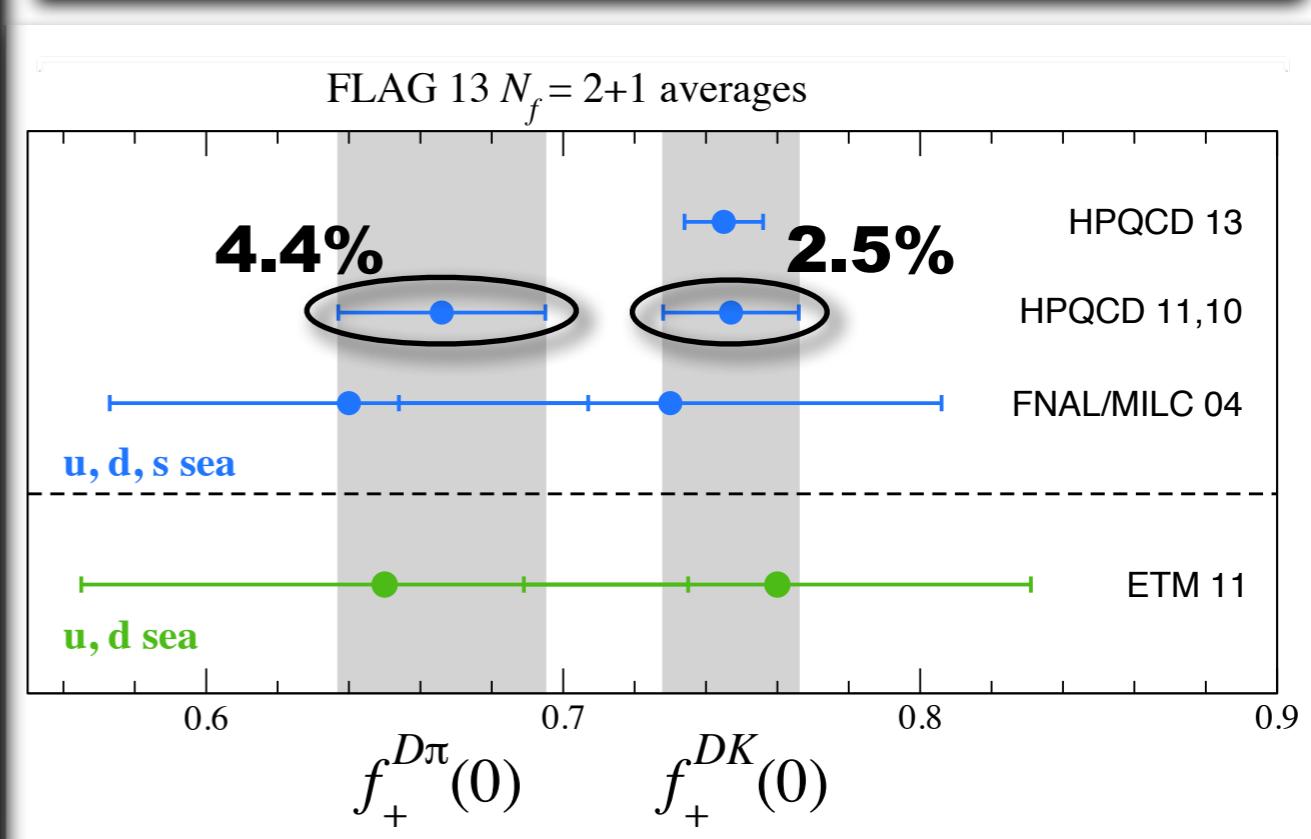
- ◆ Small errors due to
 - ❖ Physical light-quark masses
 - ❖ Improved charm-quark actions + fine lattice spacings
 - ❖ No renormalization (PCAC)

PDG 2015 $N_f \geq 3$ averages

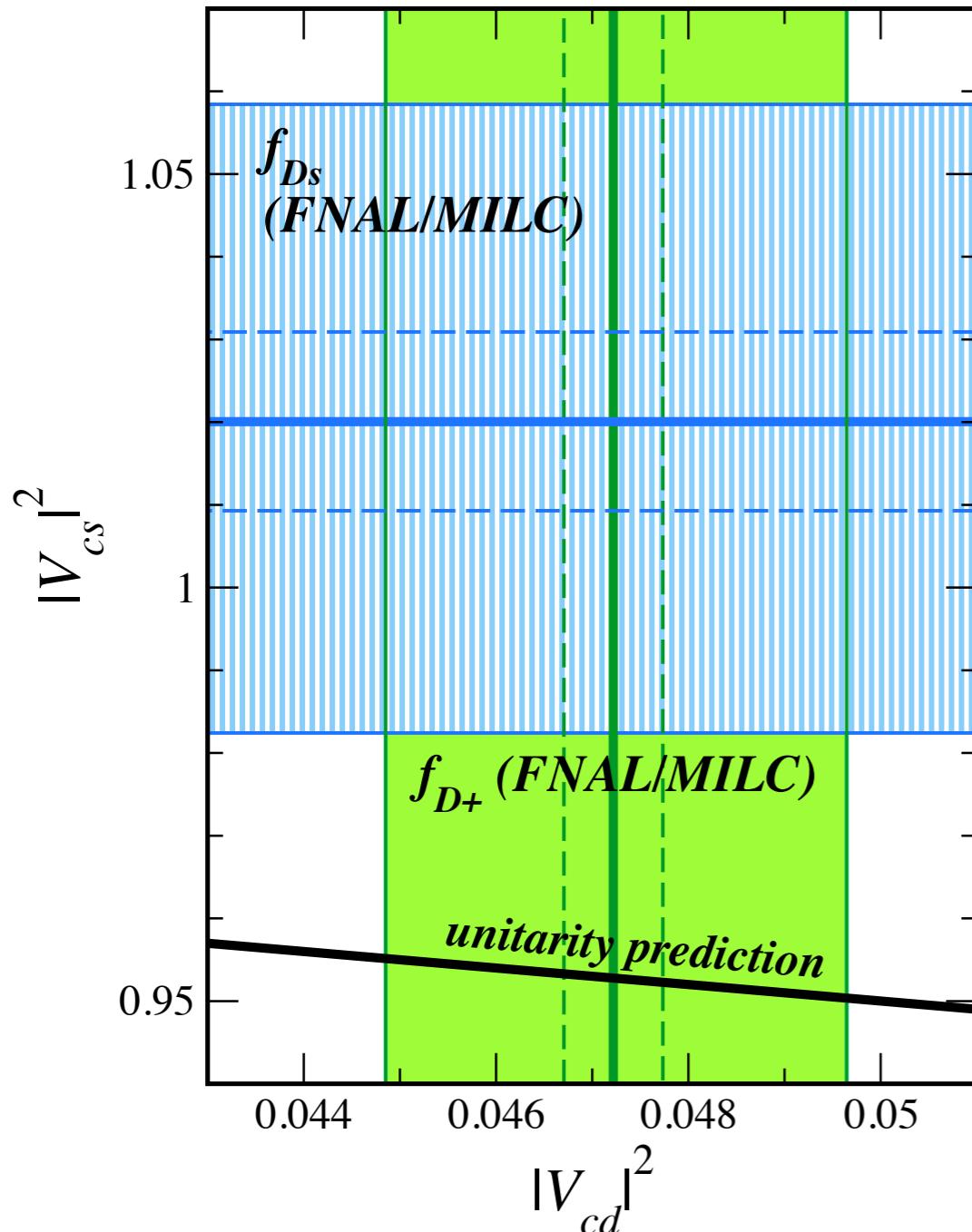


Recent highlights:

- ◆ First 3-flavor $D_s \rightarrow \phi l \bar{\nu}$ form factors [HPQCD, PRD90 (2014) 7, 074506]
- ◆ First 4-flavor D-mixing matrix elements (all five $\Delta C=2$ SM & BSM operators, only short-distance contributions) [ETM, 1505.06639]



Implications for 2nd-row CKM unitarity



- ♦ Errors on $|V_{cd}|$ & $|V_{cs}|$ from leptonic $D_{(s)}$ decays limited by experimental branching fractions [Rosner, Stone, & RV (PDG), 1509.02220]

$$|V_{cd}| = 0.217(1)_{\text{LQCD}}(5)_{\text{expt}}$$
$$|V_{cs}| = 1.007(4)_{\text{LQCD}}(16)_{\text{expt}}$$

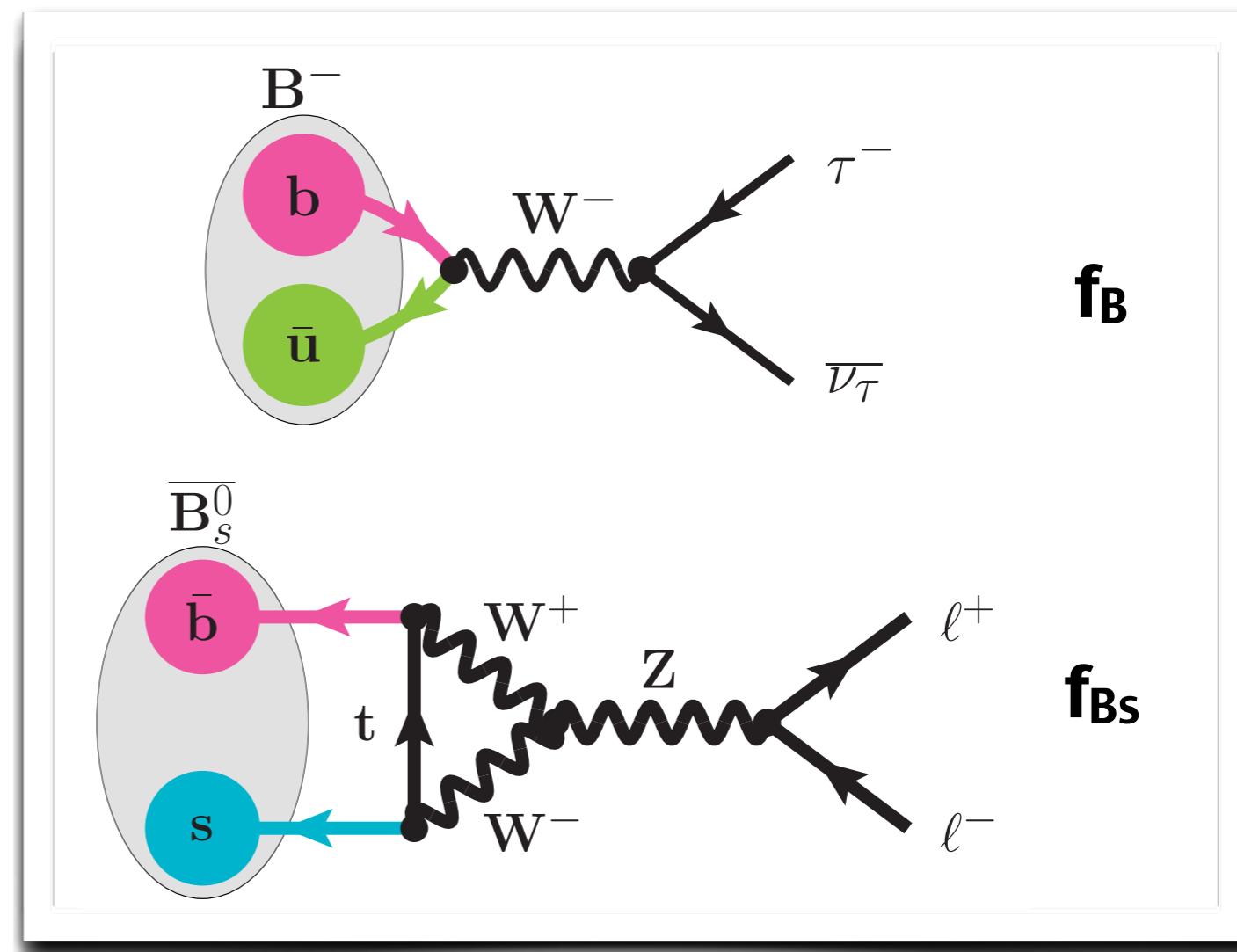
- ♦ Some tension with CKM unitarity

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.064(36)$$

→ Reaching precision where need estimate of hadronic structure-dependent EM contributions to $D_{(s)}$ leptonic decay rates

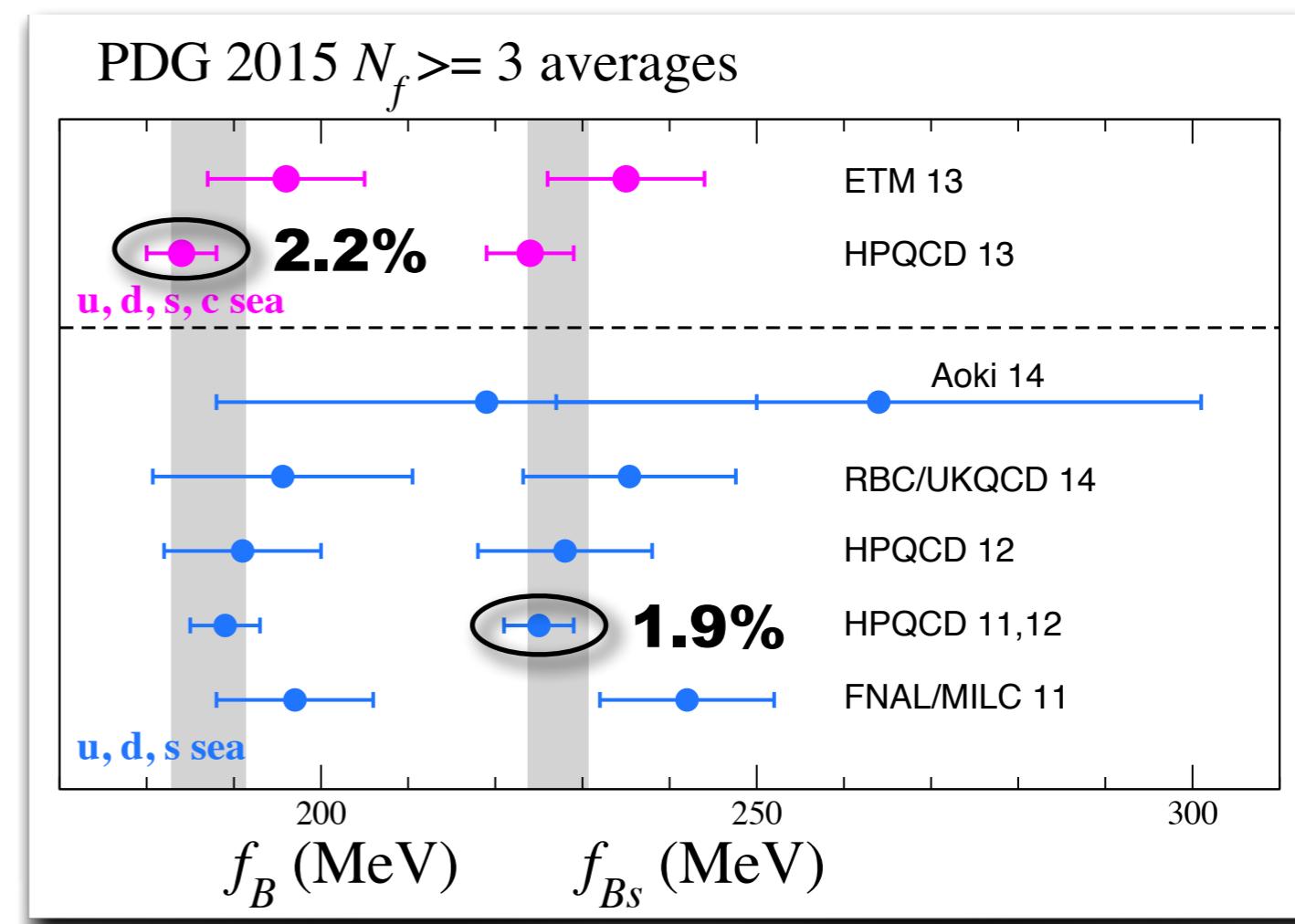
B-meson physics overview

- ◆ Lattice B-physics calculations rapidly maturing to the level of pion/kaon calculations
- ◆ For leptonic decay constants, confirmation from several independent calculations using different gauge-field configurations, light-, and b-quark actions [see Rosner, Stone, & RV (PDG 2016), 1509.02220]
- ◆ f_{B+} obtained with physical-mass pions [HPQCD, PRL110, 222003 (2013)]
→ No chiral-extrapolation error
- ◆ f_{Bs} obtained with highly-improved staggered (HISQ) b-quark action [HPQCD, PRD85, 031503 (2012)]
 - ❖ Lattice axial current absolutely normalized → no renormalization error



B-meson physics overview

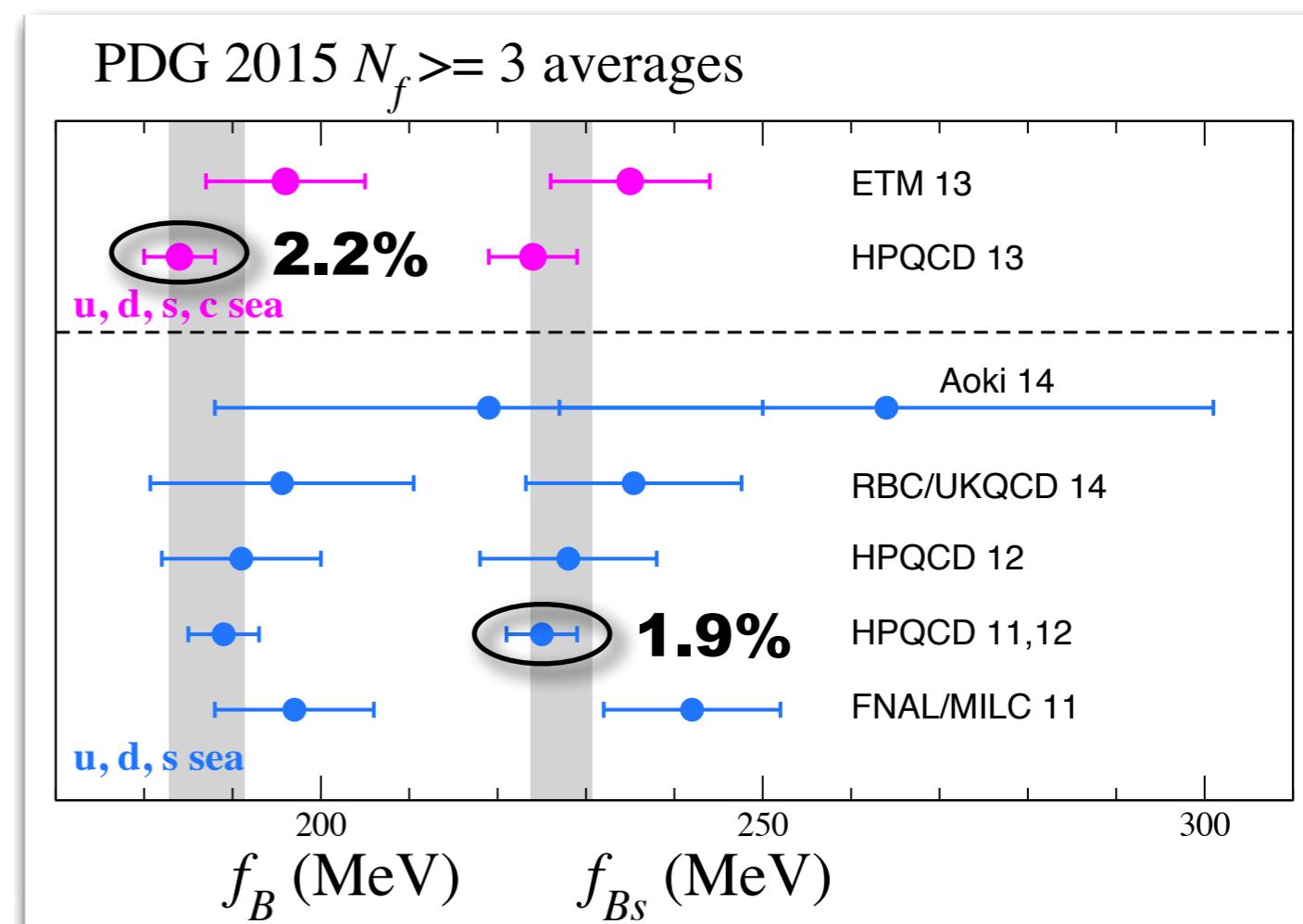
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B-meson physics overview

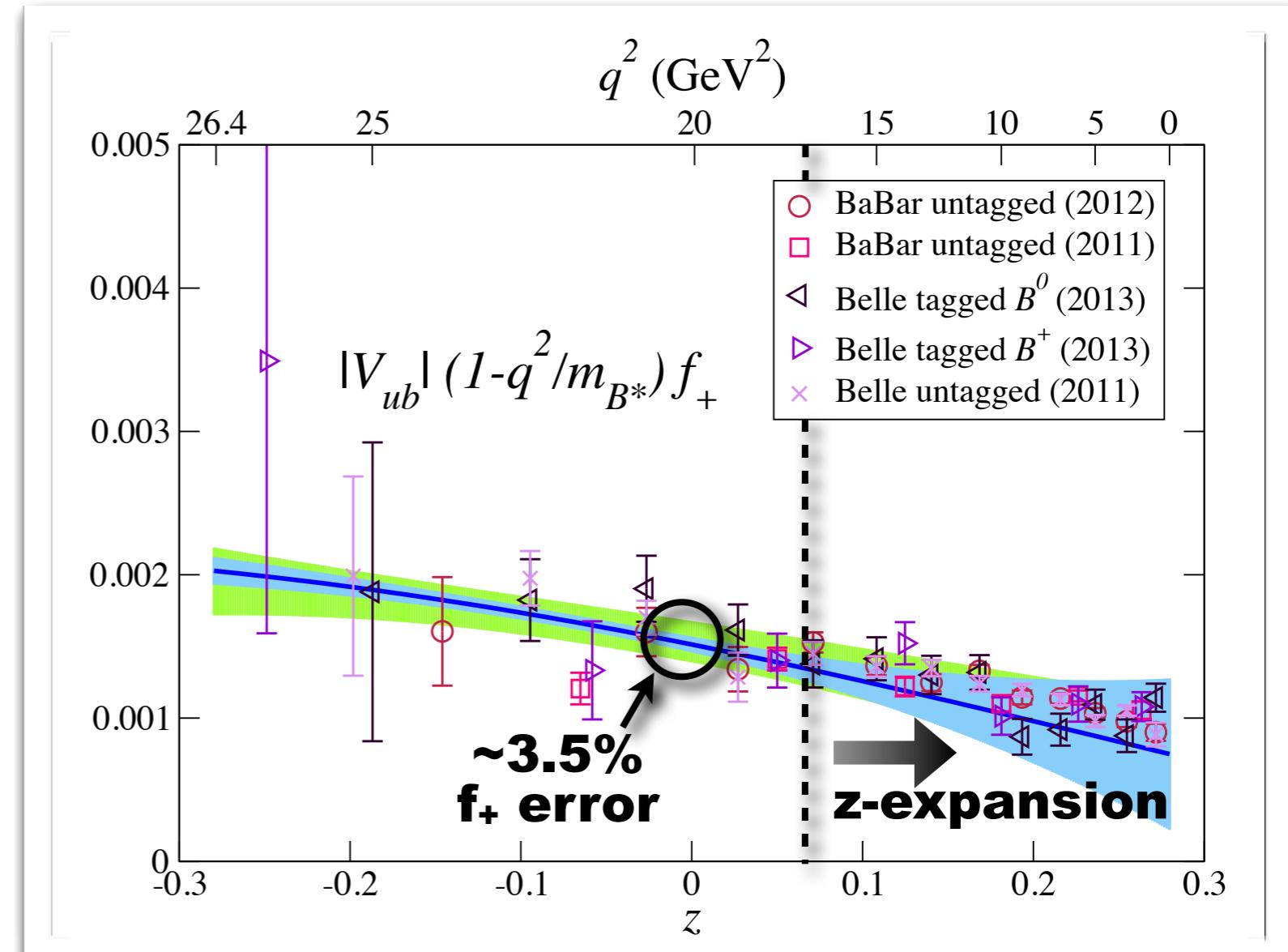
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For many quantities, still only 1 or 2 lattice calculations with ≥ 3 dynamical quarks...



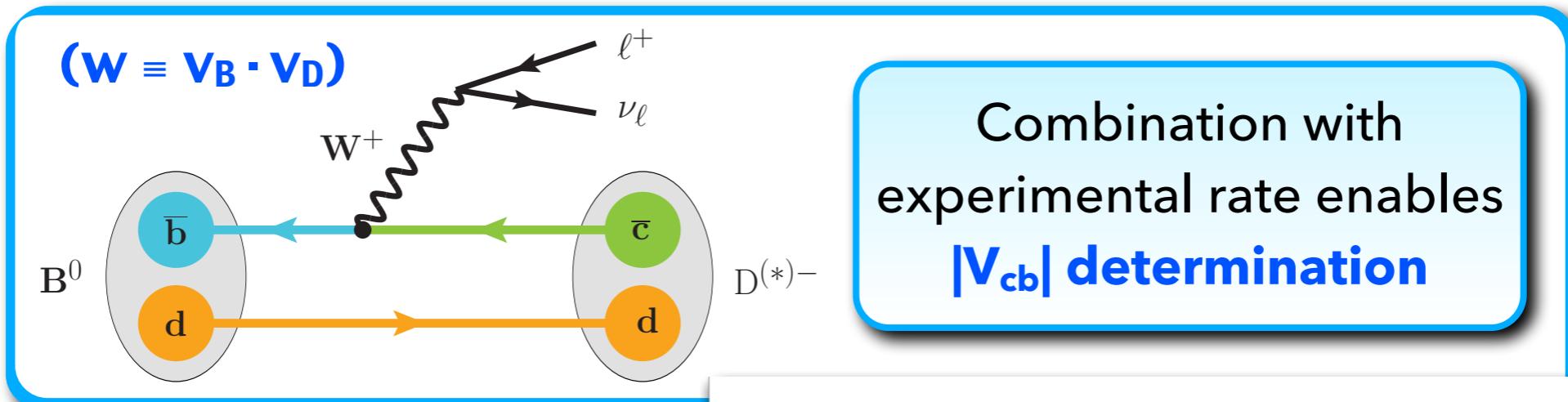
$B \rightarrow \pi \ell \nu$ form factors (2015)

- ♦ Two independent calculations last year!
 - ❖ First **RBC/UKQCD** f_+ & f_0 [**PRDD91 (2015) 7, 074510**]
 - ❖ New **FNAL/MILC** f_+ with more statistics & finer lattice spacings, and first f_0 [**PRDD92 (2015) 1, 014024**]
 - ❖ Extend lattice results to $q^2 \leq 16 \text{ GeV}^2$ using model-independent z-expansion
- ♦ $f_+(q^2)$ shape consistent with measured $B \rightarrow \pi \ell \nu$ dB/dq^2
- ♦ Obtain $|V_{ub}|$ from joint z-fit to lattice + experimental data

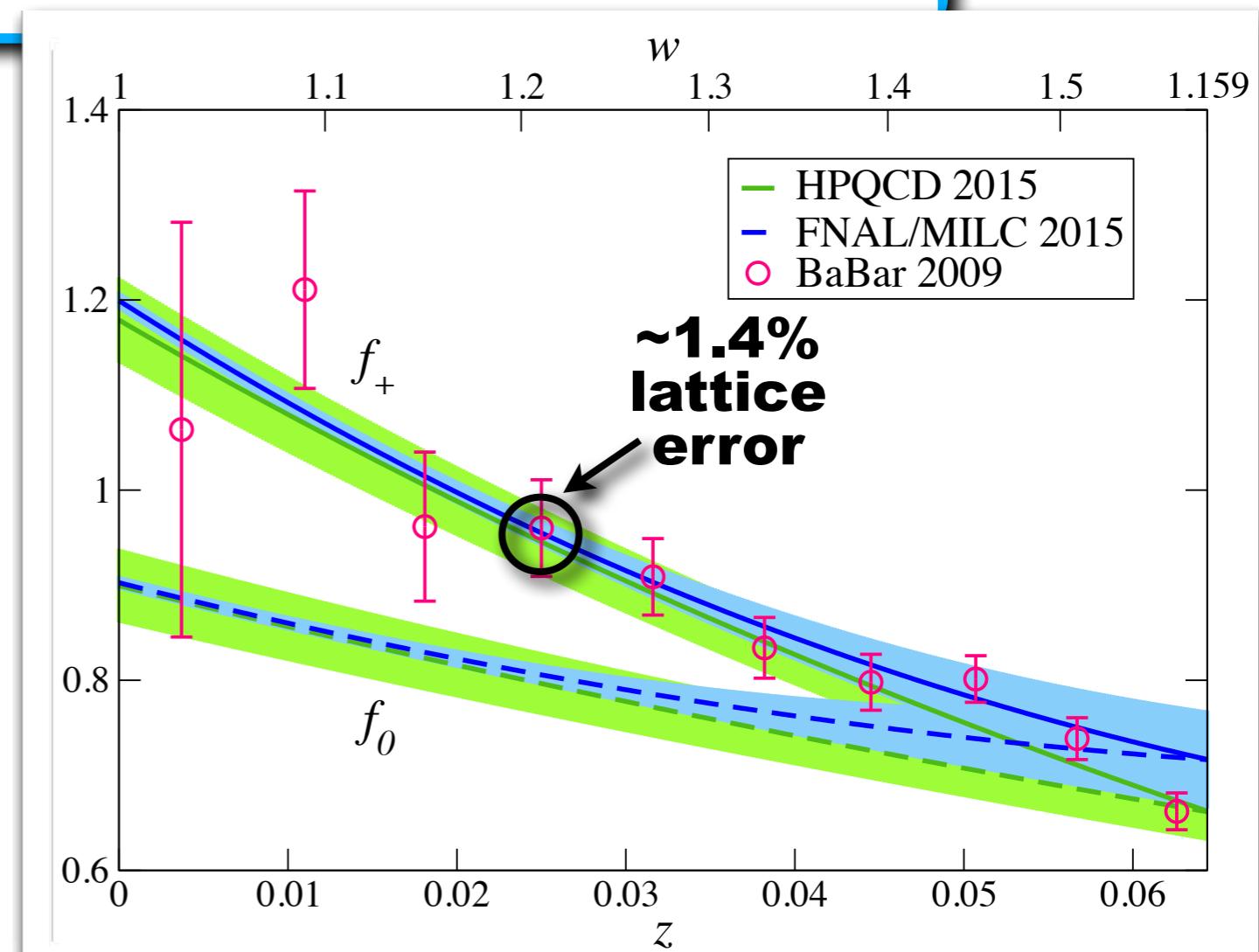


Use of all experimental & theoretical information minimizes error on $|V_{ub}|$

B \rightarrow Dlv form factors @ nonzero recoil (2015)



- ◆ Comparing theory & experiment at $w=1 \rightarrow$ large experimental errors in $|V_{cb}|$ because decay rate suppressed
- ◆ **First three-flavor form-factor results over full kinematic range** [Fermilab/MILC, PRD92, 034506 (2015); HPQCD, PRD92, 054510 (2015)]
 - ❖ Independent calculations agree
 - ❖ Shapes consistent with experiment
- ◆ **Joint lattice + experiment fit using $w>1$ data reduces error on $|V_{cb}|$**



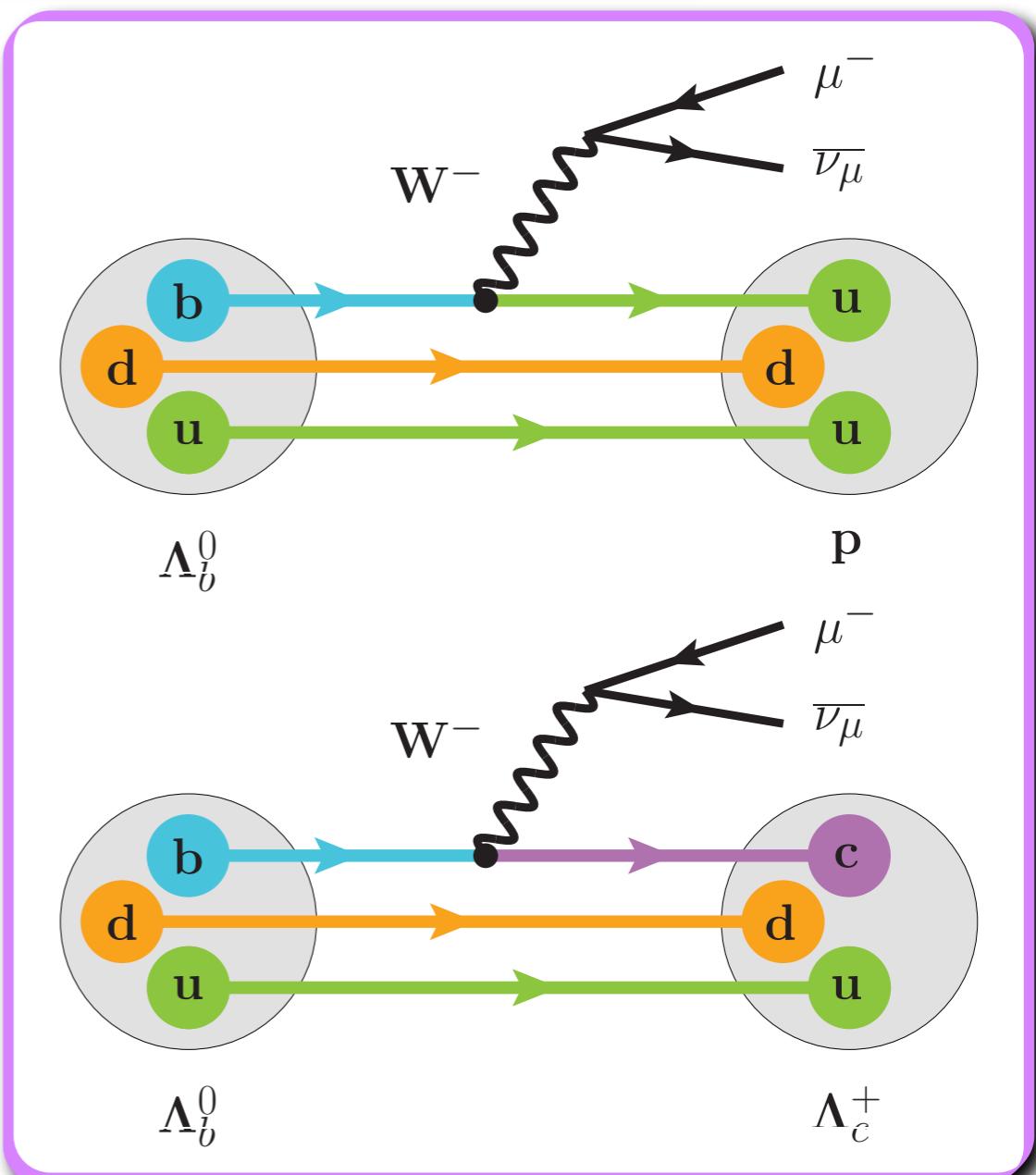
$\Lambda_b \rightarrow p \bar{\nu}$ and $\Lambda_b \rightarrow \Lambda_c \bar{\nu}$ form factors (2015)

Combination with ratio of experimental rates **enables determination of $|V_{ub}| / |V_{cb}|$** → first from baryon decay!

- ♦ First three-flavor $\Lambda_b \rightarrow p$ form factors with relativistic b-quark at physical mass; first three-flavor $\Lambda_b \rightarrow \Lambda_c$ form factors
[Detmold, Lehner, & Meinel, PRD92, 034503 (2015)]
- ♦ Combine chiral-continuum extrapolation with q^2 fit via modified z-expansion

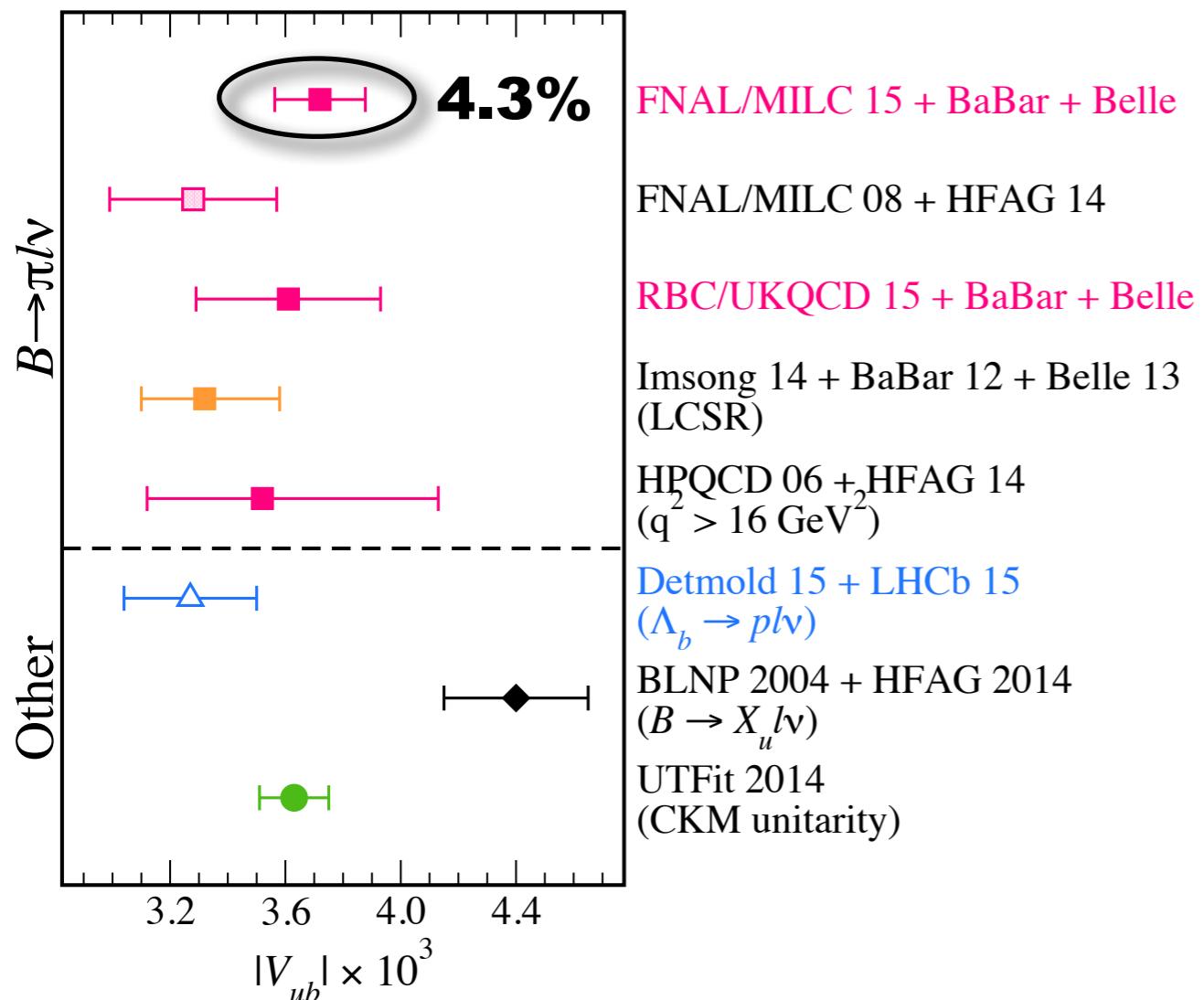
$$\frac{|V_{cb}|^2}{|V_{ub}|^2} \frac{\int_{15 \text{ GeV}^2}^{q_{\max}^2} \frac{d\Gamma(\Lambda_b \rightarrow p \mu^- \bar{\nu}_\mu)}{dq^2} dq^2}{\int_{7 \text{ GeV}^2}^{q_{\max}^2} \frac{d\Gamma(\Lambda_b \rightarrow \Lambda_c \mu^- \bar{\nu}_\mu)}{dq^2} dq^2} = 1.470 \pm 0.115 \pm 0.104$$

→ 5.3% LQCD error on $|V_{ub}| / |V_{cb}|$



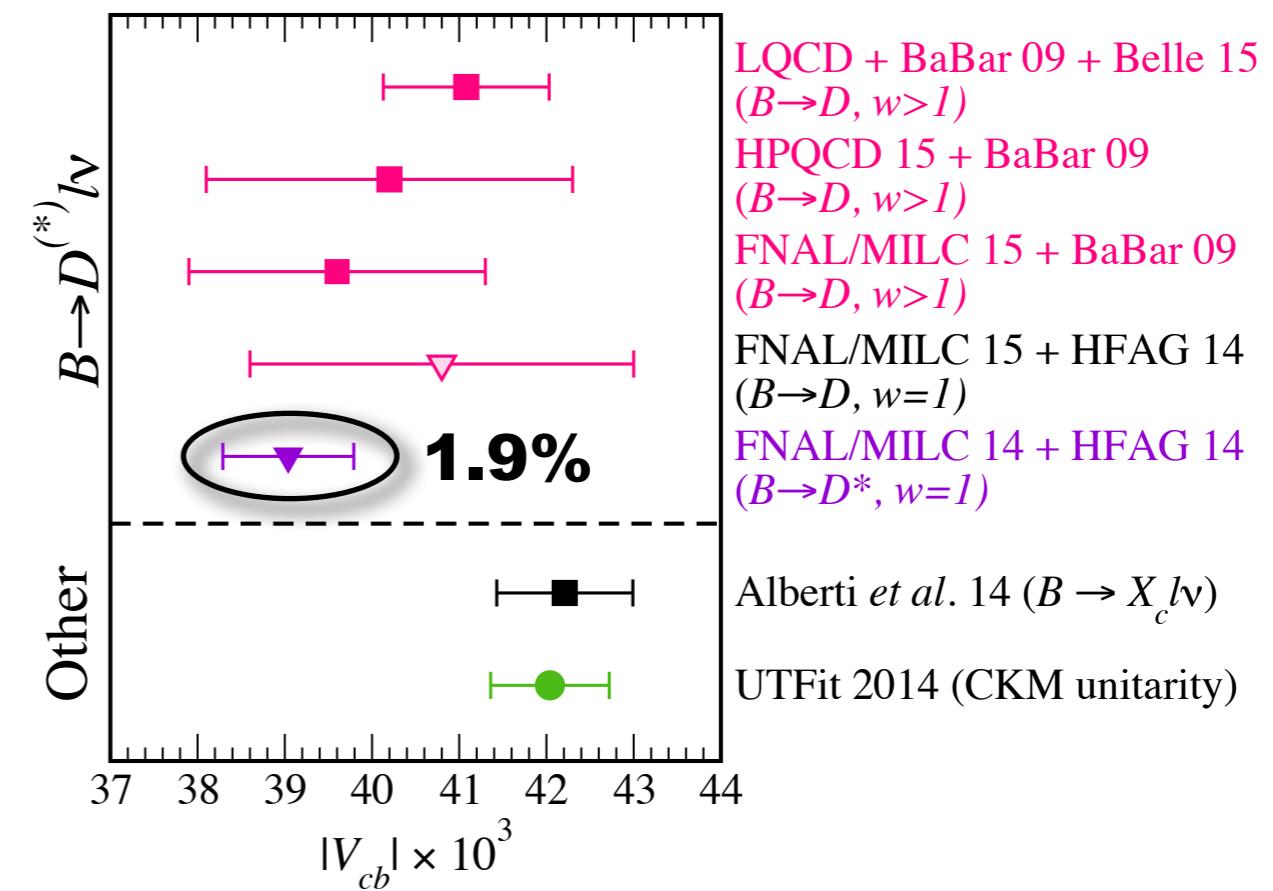
Implications for $|V_{ub}|$ & $|V_{cb}|$

$|V_{ub}|$



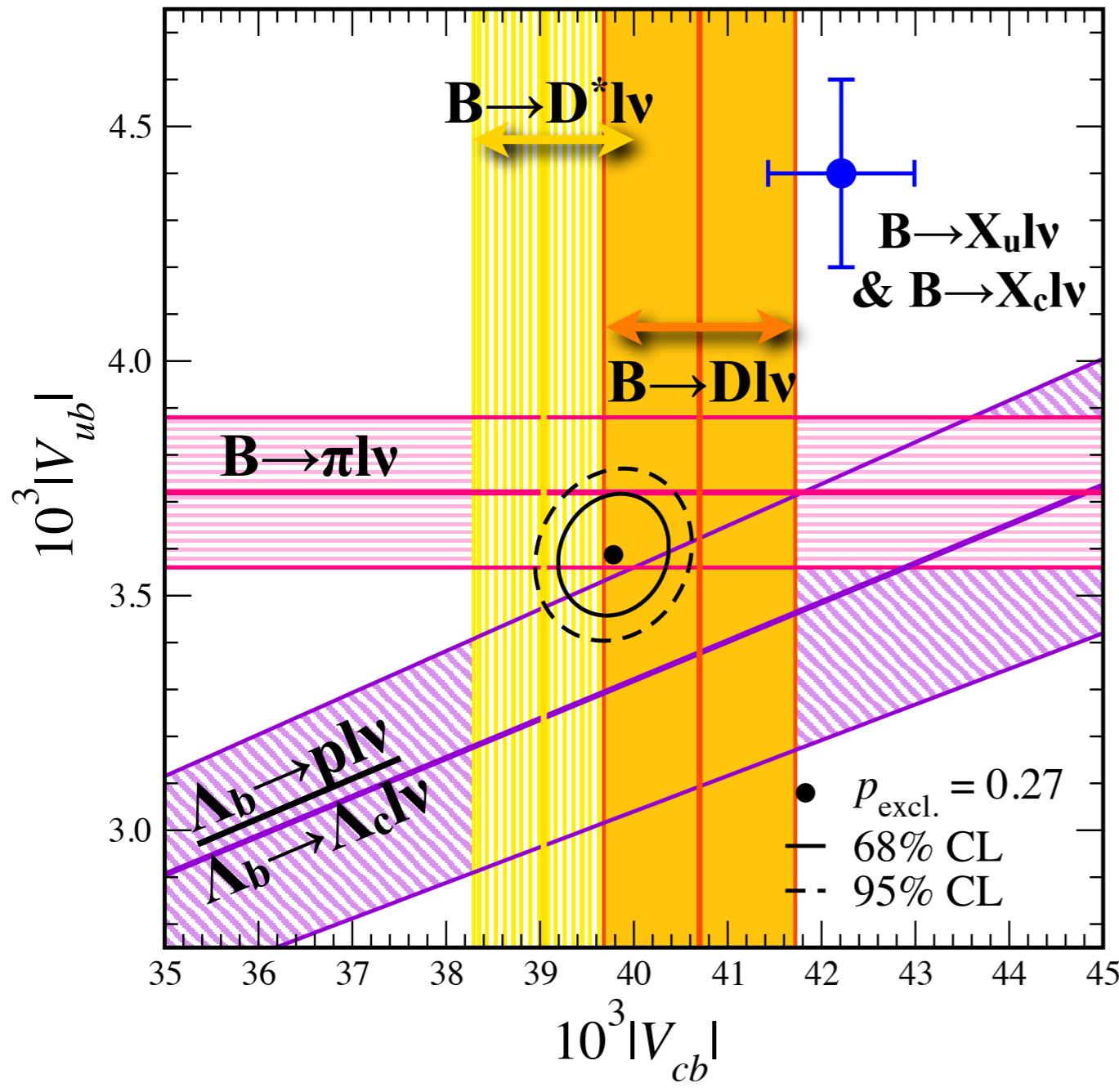
- ◆ Error on $|V_{ub}|$ from $B \rightarrow \pi l \nu$ reduced by factor of 2!

$|V_{cb}|$



- ◆ QCD error in $|V_{cb}|$ from $B \rightarrow D^* l \nu$ now commensurate with experimental error
- ◆ New Belle nonzero-recoil $B \rightarrow D$ measurement raises $|V_{cb}|$

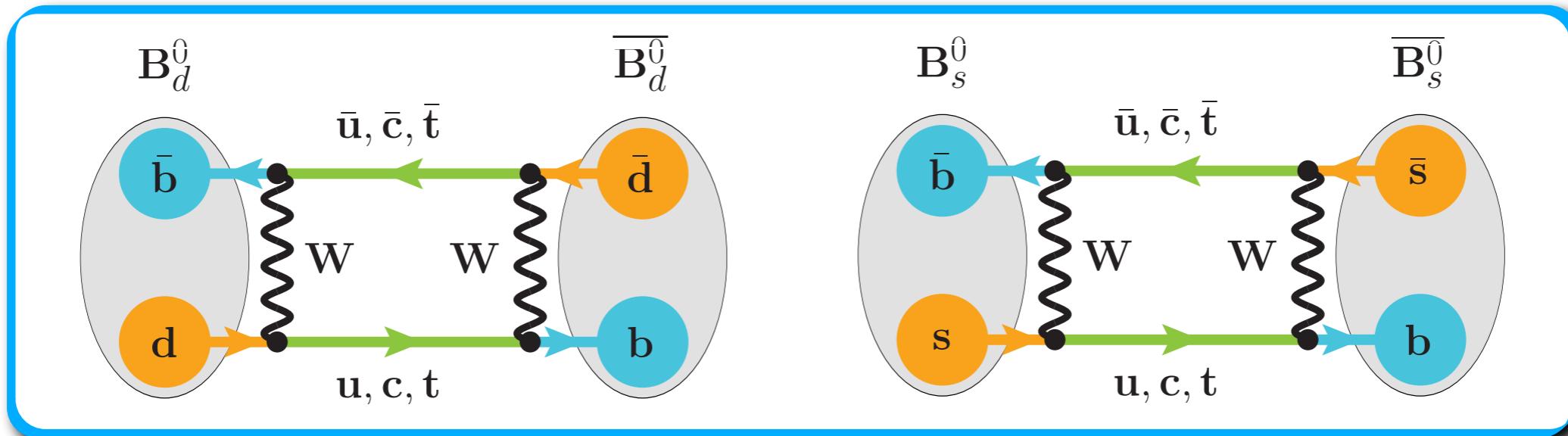
Implications for the “ $|V_{xb}|$ puzzle”



Increased lattice-QCD precision + new measurements sharpening picture of inclusive-exclusive tensions

- ◆ LQCD will continue to address “ $|V_{xb}|$ ” puzzle through:
 - ❖ New $b \rightarrow u$ decays (e.g. $B_s \rightarrow K l \bar{\nu}$)
 - ❖ Independent $\Lambda_b \rightarrow p$ & $\Lambda_b \rightarrow \Lambda_c$ form factors
 - ❖ $B \rightarrow D^* l \bar{\nu}$ form factors at $w > 1$
- ◆ $|V_{cb}|$ from $B \rightarrow D^* l \bar{\nu}$ extrapolates measurement to zero recoil using CLN parameterization → time for model-independent analysis!

Neutral B-meson mixing

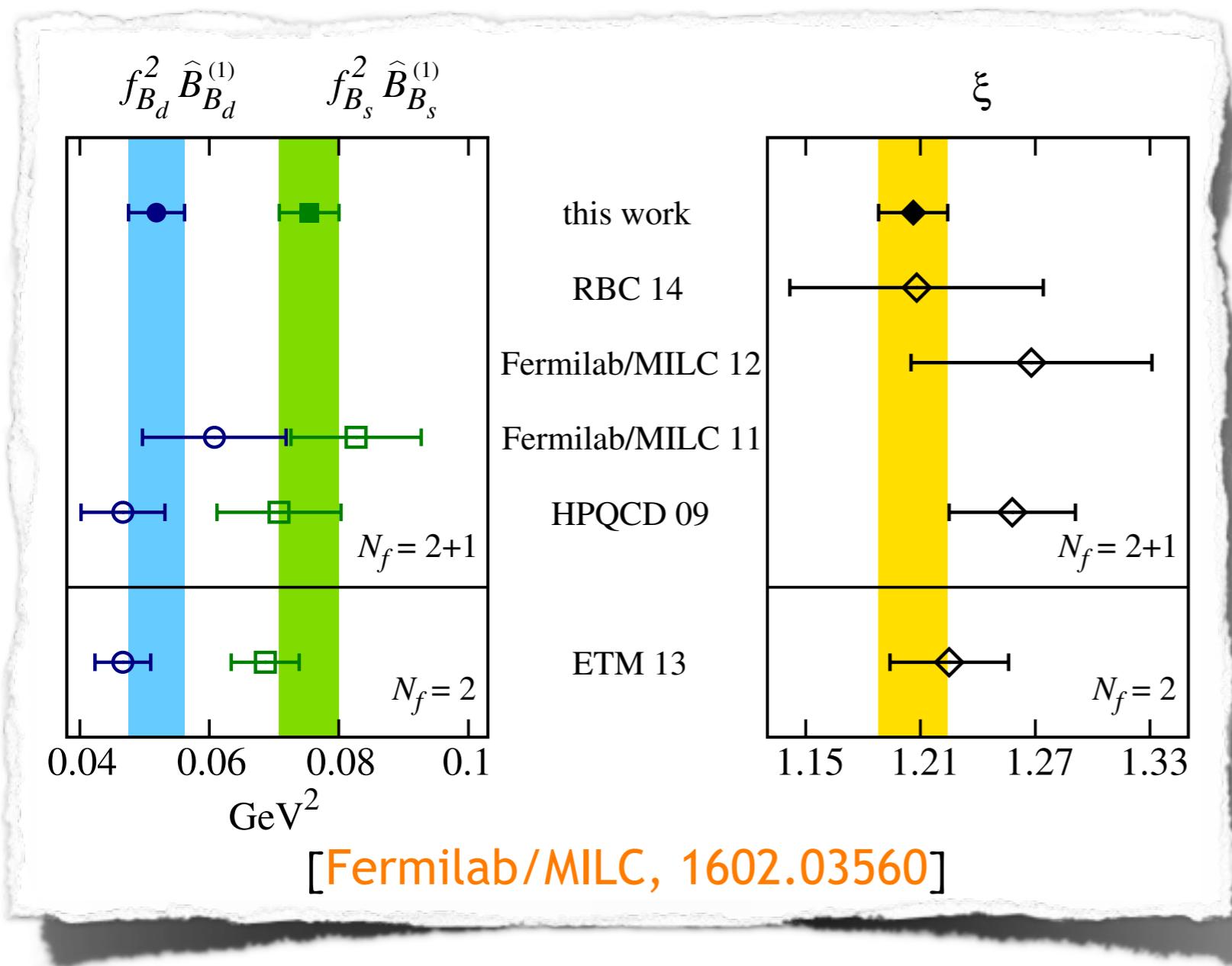


- ♦ Ratio of B_d to B_s oscillation frequencies (Δm_q) determines apex of UT ($\bar{\rho}, \bar{\eta}$) via

$$\begin{aligned} \frac{\Delta m_d}{\Delta m_s} &= \left(\frac{f_{B_d} \sqrt{\hat{B}_{B_d}}}{f_{B_s} \sqrt{\hat{B}_{B_s}}} \right)^2 \frac{m_{B_d}}{m_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2} \\ &= \xi^2 \frac{m_{B_d}}{m_{B_s}} \left(\frac{\lambda}{1 - \lambda^2/2} \right)^2 \frac{((1 - \bar{\rho})^2 + \bar{\eta}^2)}{\left(1 + \frac{\lambda^2}{1 - \lambda^2/2} \bar{\rho} \right) + \lambda^4 \bar{\eta}^2} \end{aligned}$$

- ♦ **Ratio of B_d - to B_s -mixing hadronic matrix elements can be computed precisely in lattice QCD because:**
 - ❖ Statistical fluctuations correlated
 - ❖ Matrix elements equal in SU(3) limit $m_s = m_{ud}$, so some systematic errors suppressed by $(m_s - m_{ud})/\Lambda_{QCD}$

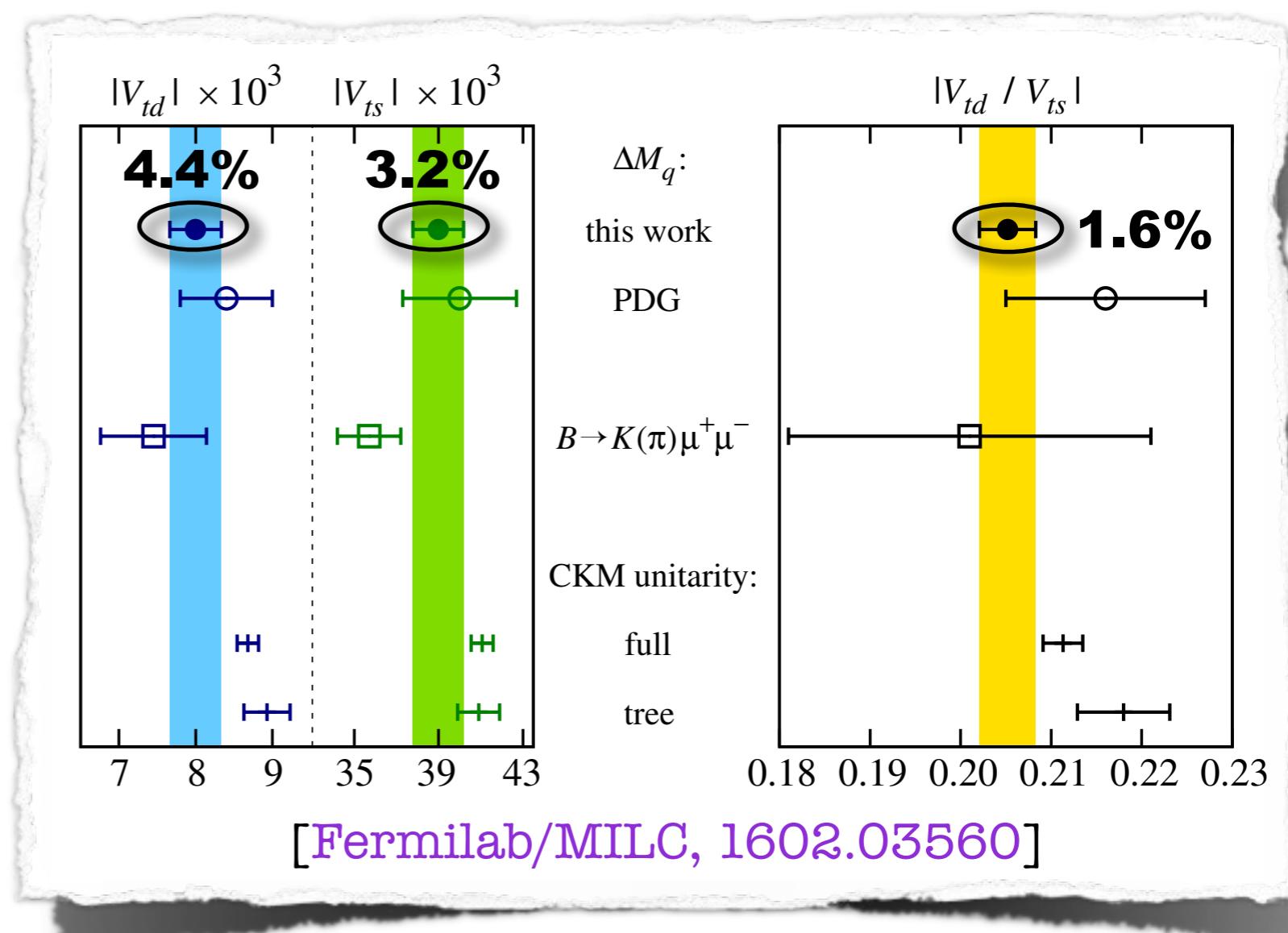
New! B-mixing matrix elements



- ◆ Significant Fermilab/MILC update of 2012 ξ calculation with increased statistics, finer lattice spacings, better treatment of chiral extrapolation
→ Error on ξ reduced from ~5% to 1.6%!
- ◆ Also present individual B_d - and B_s -mixing matrix elements
- ◆ ... and first three-flavor results for all five local operators that contribute to neutral B-meson mixing in and beyond the Standard Model

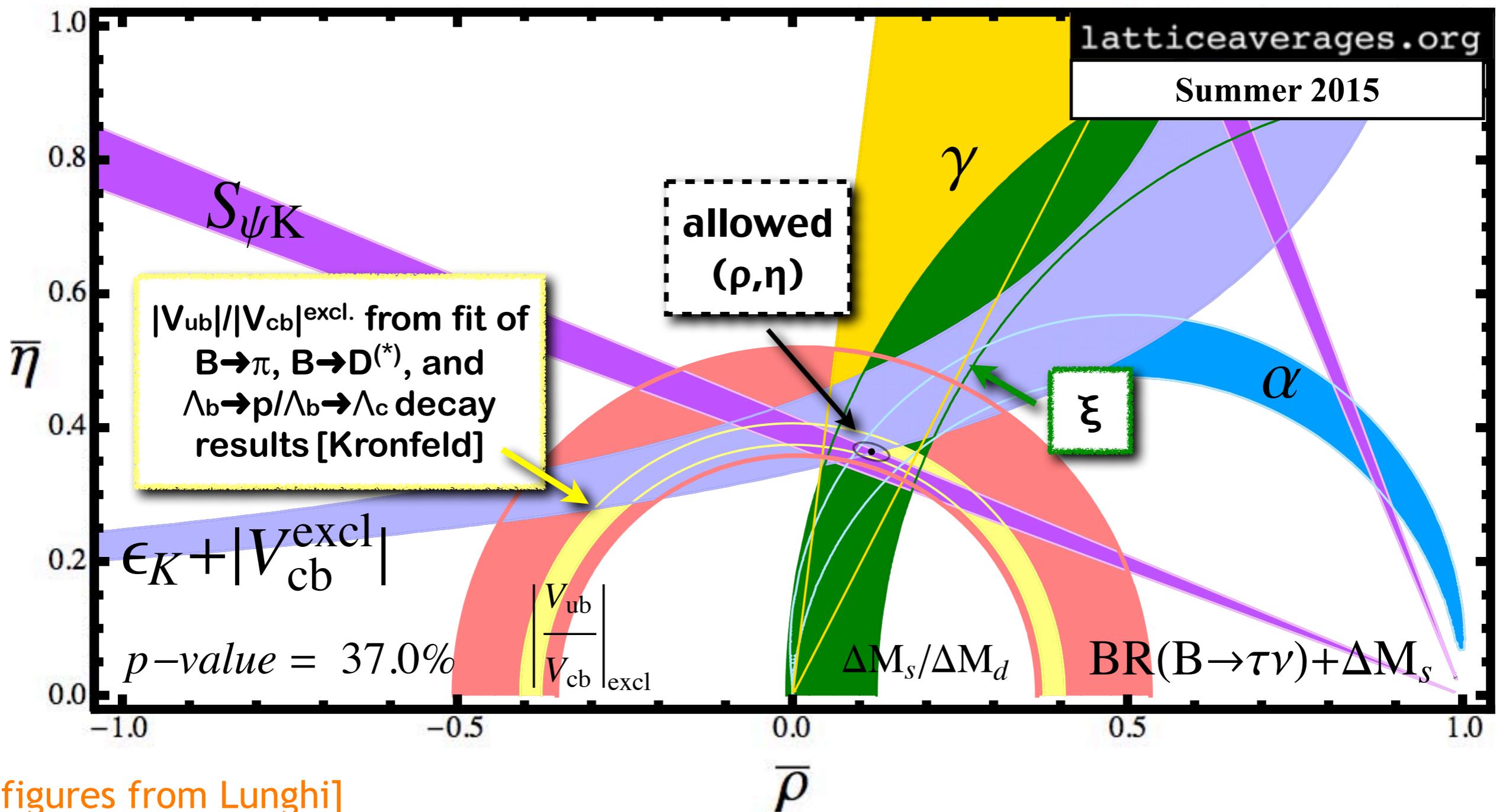
Implications for $|V_{td}|$ & $|V_{ts}|$

- ♦ Standard-Model rates for $B_{d,s}$ -meson mixing and $B \rightarrow \pi \mu^+ \mu^-$ & $B \rightarrow K \mu^+ \mu^-$ decays proportional to $|V_{tb} V_{td}^*|$ & $|V_{tb} V_{ts}^*|$ → enable independent determinations of $|V_{td}|$, $|V_{ts}|$, and $|V_{td}/V_{ts}|$
- ♦ **Errors on $|V_{tq}|$ from B-mixing**
~2-3 x smaller with new lattice matrix elements, but still limited by theory
- ♦ $|V_{ts}|$ & $|V_{td}/V_{ts}|$ from semileptonic decays >2x more precise using new lattice $B \rightarrow \pi$ & $B \rightarrow K$ form factors, with commensurate theory & experimental errors [c.f. LHCb, JHEP 1406, 133 (2014); JHEP 1510 (2015) 034]
- ♦ Both display $\sim 2\sigma$ tensions with CKM unitarity!



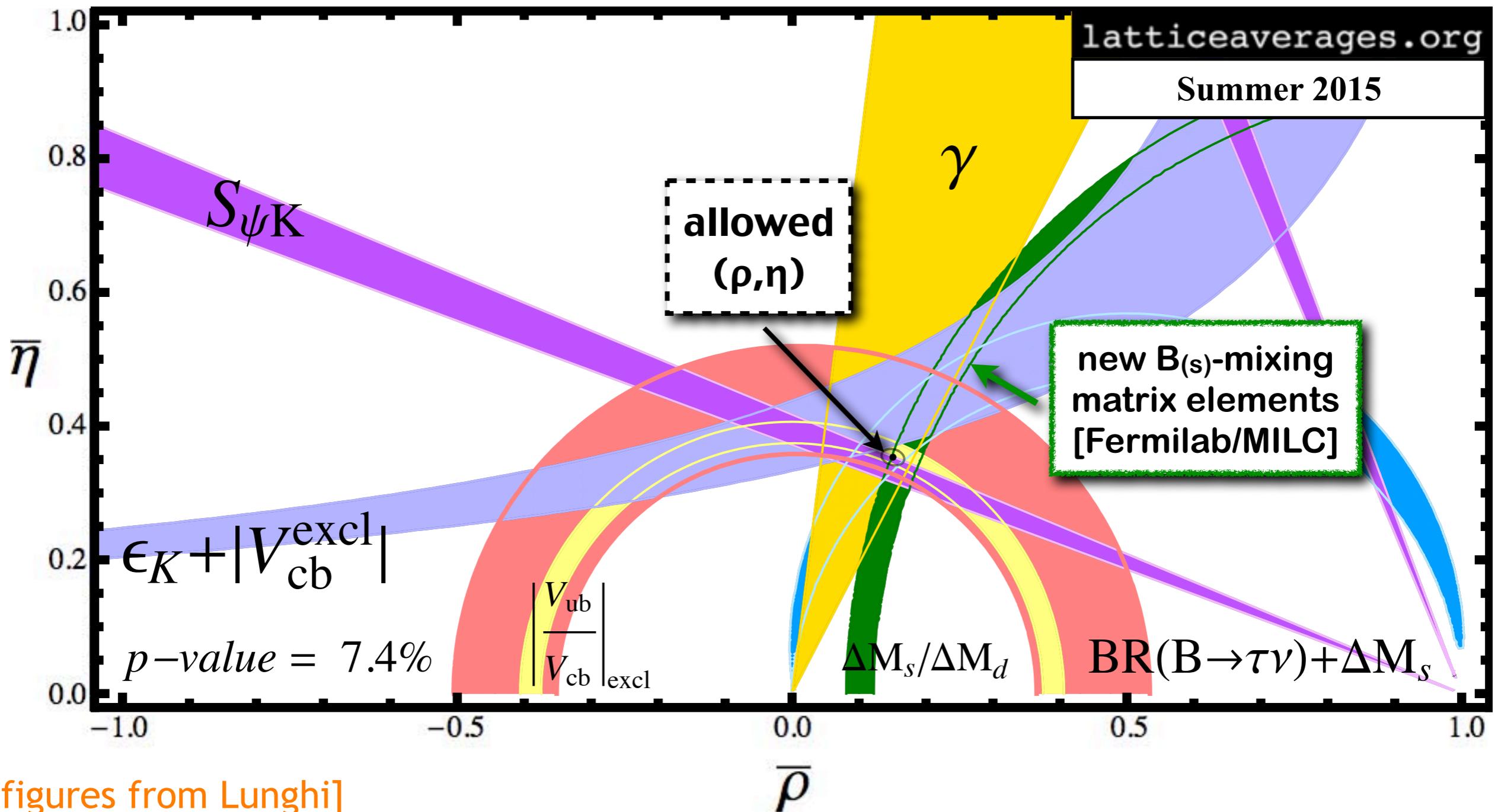
Impact on CKM unitarity-triangle fit

Summer 2015



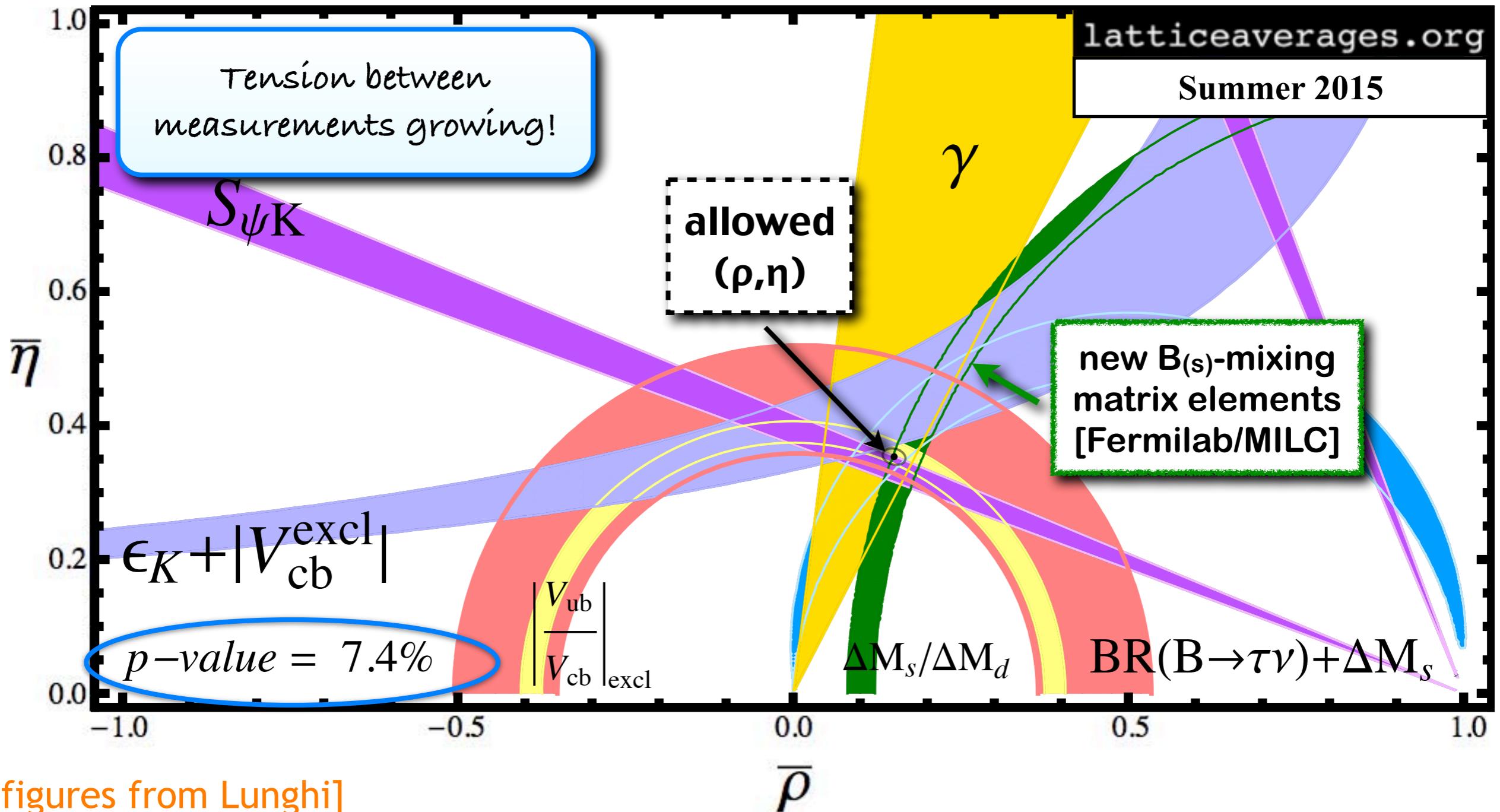
Impact on CKM unitarity-triangle fit

February 2016



Impact on CKM unitarity-triangle fit

February 2016



Conclusions

"Lattice QCD has become an important tool in flavor physics. ... Matrix elements which contain at most one hadron in the final state [such as (semi)leptonic decays and meson mixing] should soon be calculable with percent level uncertainties. ...

The full exploitation of the experimental program requires continued support of theoretical developments." – **Snowmass 2013 Quark-flavor WG report (1311.1076)**

★ New since
FPCP 2014!

CKM summary

CKM element	process	LQCD input	$ V_{q_1 q_2} $	% error
$ V_{ud} $	$\pi \rightarrow \ell\nu$	f_π	0.9764	(1.3) _{LQCD} (0.0) _{exp} (0.1) _{EM}
$ V_{us} $	$K \rightarrow \ell\nu$	f_K	0.2255	(0.3) _{LQCD} (0.1) _{exp} (0.1) _{EM}
	$K \rightarrow \pi\ell\nu$	$f_+^{K\pi}(0)$	0.22310	(0.3) _{LQCD} (0.2) _{exp}
$ V_{us} / V_{ud} $	$K \rightarrow \ell\nu/\pi \rightarrow \ell\nu$	f_K/f_π	0.2314	(0.2) _{LQCD} (0.1) _{exp} (0.1) _{EM}
$ \mathbf{V}_{ub} $	$B \rightarrow \pi\ell\nu$	$f_+^{\mathbf{B}\pi}(\mathbf{q}^2)^\star$	3.72×10^{-3}	$(\sim 3.4)_{\text{LQCD}} (\sim 2.8)_{\text{exp}}$
	$B \rightarrow \ell\nu$	f_B	4.12×10^{-3}	(2.2) _{LQCD} (9.0) _{exp}
$ V_{cd} $	$D \rightarrow \ell\nu$	f_D	0.217	(0.5) _{LQCD} (2.3) _{exp}
	$D \rightarrow \pi\ell\nu$	$f_+^{D\pi}(0)$	0.2140	(4.4) _{LQCD} (1.3) _{exp}
$ V_{cs} $	$D_s \rightarrow \ell\nu$	f_{D_s}	1.007	(0.5) _{LQCD} (1.6) _{exp}
	$D_s \rightarrow K\ell\nu$	$f_+^{DK}(0)$	0.9746	(2.5) _{LQCD} (0.7) _{exp}
$ \mathbf{V}_{cb} $	$B \rightarrow D^*\ell\nu$	$F(1)$	38.9×10^{-3}	(1.3) _{LQCD} (1.3) _{exp} (0.5) _{QED}
	$B \rightarrow D\ell\nu$	$f_+^{\mathbf{BD}}(\mathbf{w})^\star$	40.7×10^{-3}	$(\sim 1.2)_{\text{LQCD}} (\sim 2.1)_{\text{exp}} (0.5)_{\text{QED}}$
$ \mathbf{V}_{td} $	ΔM_d	$f_{B_d}(\hat{B}_{B_d})^{-1/2}^\star$	8.00×10^{-3}	(4.1) _{LQCD} (0.3) _{exp} (0.4) _{other}
$ \mathbf{V}_{ts} $	ΔM_s	$f_{B_s}(\hat{B}_{B_s})^{-1/2}^\star$	39.0×10^{-3}	(3.1) _{LQCD} (0.0) _{exp} (0.5) _{other}
$ \mathbf{V}_{td} / \mathbf{V}_{ts} $	$\Delta M_d/\Delta M_s$	ξ^\star	0.2052	(1.5) _{LQCD} (0.2) _{exp} (0.0) _{other}
$(\bar{\rho}, \bar{\eta})$	ϵ	\hat{B}_K	—	(1.3) _{B_K} [cf. (0.7) _{ϵ, exp}]

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Hints of new physics?

- ♦ Lattice-QCD results + experimental measurements imply several 2σ “tensions” with Standard-Model CKM framework:
 - ❖ $|V_{us}|$ from semileptonic K decay inconsistent with leptonic π & K decays + superallowed β -decay
 - ❖ $|V_{cd}|$ & $|V_{cs}|$ from leptonic decays differ from second-row unitarity expectations
 - ❖ $|V_{ub}|$ & $|V_{cb}|$ from inclusive and exclusive B decays differ (*although new $B \rightarrow D/\bar{D}$ nonzero-recoil measurements and form factors yield exclusive $|V_{cb}|$ in better agreement with inclusive value*)
 - ❖ $|V_{td}|$ & $|V_{ts}|$ from $B_{(d,s)}$ -meson mixing and rare $B \rightarrow \pi(K)\ell\bar{\nu}$ decays differ from global CKM unitarity-triangle fit
- ♦ Global CKM unitarity-triangle fit still compatible with Standard Model, but tension is growing...

Outlook

- ♦ Quark-flavor physics program needs lattice-QCD calculations on same time scale as experiments with reliable uncertainties and commensurate precision
- ♦ Lattice flavor-physics community devoting considerable human effort & computing resources to meet theory needs of current & upcoming experiments
 - ❖ Petascale computing enabling simulations with physical-mass pions, very fine lattice spacings, very large volumes, & dynamical charm quarks → *increased precision*
 - ❖ Precision on simple quantities approaching level where **strong isospin** (straightforward) & **EM corrections** (methods being developed) **are becoming relevant**
 - ❖ Working towards first **QCD calculations with controlled uncertainties of more challenging quantities** such as long-distance amplitudes, decays to QCD resonances or multi-hadron final states, baryon form factors, ...
- ♦ Future quark-flavor measurements + anticipated lattice-QCD improvements will increase precision on CKM elements & sharpen Standard-Model tests
 - ❖ If deviations are seen, pattern of measurements will provide information on new particle masses and couplings and helps distinguish between high-scale models!

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 - ❖ Petascale computing enabling simulations with physical-mass pions, very fine lattice spacings, very large volumes, & dynamical charm quarks → *increased precision*
 - ❖ Precision measurements of CKM elements & other parameters
 - ❖ Continued support for lattice-QCD hardware & software essential to achieve scientific goals and fully capitalize on enormous investments in experimental quark-flavor physics program
 - ❖ Working on challenging problems such as long-distance amplitudes, decay, and QCD resonances or multi-hadron final states, baryon form factors, ...
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 - ❖ If deviations are seen, pattern of measurements will provide information on new particle masses and couplings and helps distinguish between high-scale models!

The lattice community is working hard, so stay tuned!

Local Organising Committee
Gert Aarts (Swansea)
Jonathan Flynn (Southampton)
Andreas Jüttner (Southampton)
Kurt Langfeld (Plymouth, Liverpool)
Biagio Lucini (Swansea)
Antonio Rago (Plymouth)
Chris Sachrajda (Southampton)
Mike Teper (Oxford)
Christopher Thomas (Cambridge)
Matthew Wingate (Cambridge)

24-30 July 2016

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Algorithms and Machines
Applications Beyond QCD
Chiral Symmetry
Hadron Spectroscopy and Interactions
Hadron Structure
Nonzero Temperature and Density
Physics Beyond the Standard Model
Standard Model Parameters and Renormalization
Theoretical Developments
Vacuum Structure and Confinement
Weak Decays and Matrix Elements

UNIVERSITY OF Southampton SHEP Southampton High Energy Physics
Science & Technology Facilities Council EMMI
intel IP³ sgi

<http://www.southampton.ac.uk/lattice2016/lattice2016@soton.ac.uk>

34th International Symposium on Lattice Field Theory

Additional reading

General reviews:

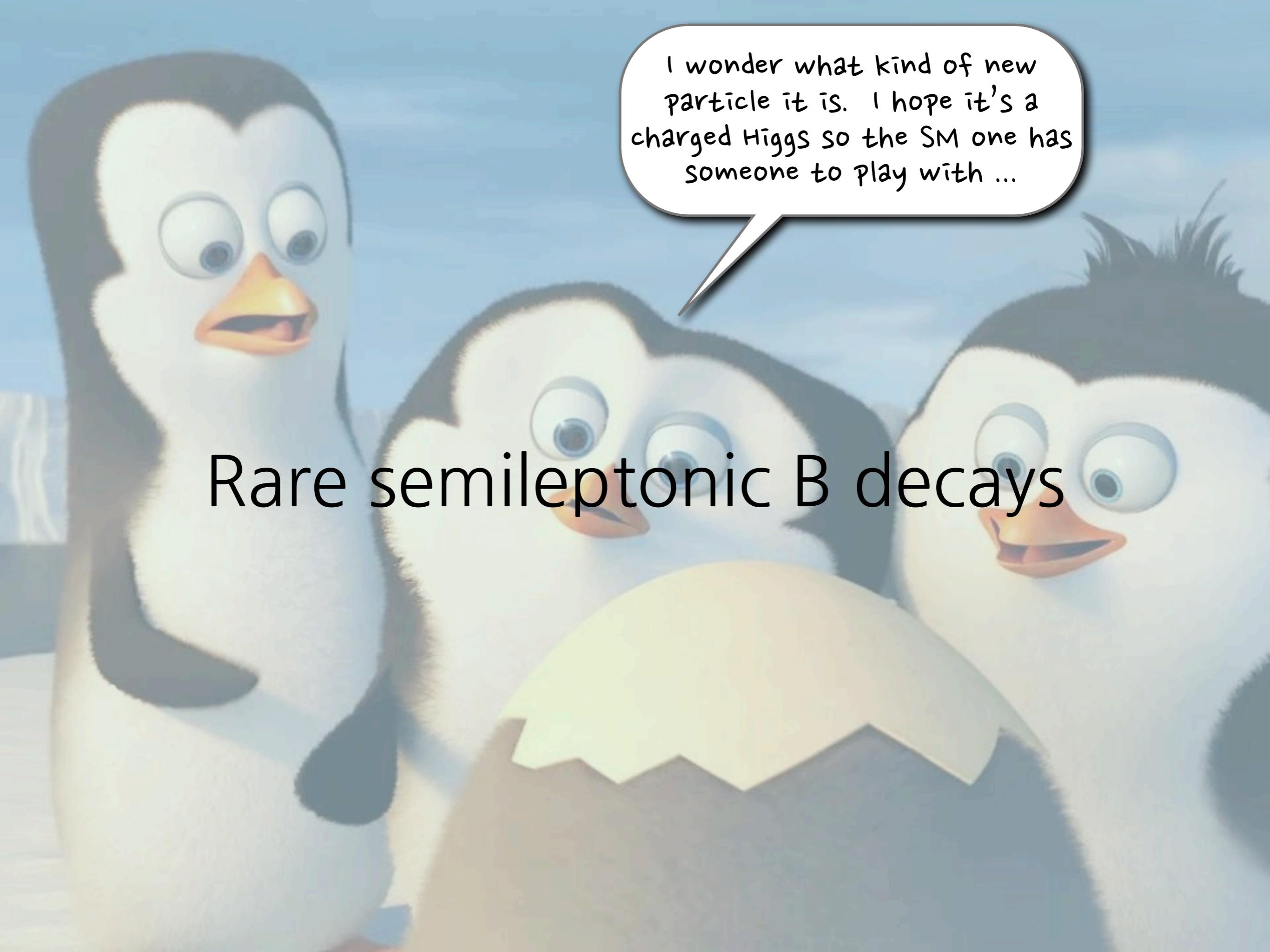
- ✿ “[Lattice Quantum Chromodynamics](#)” (Laiho, Sharpe, Hashimoto, PDG 2015)
- ✿ “[Review of lattice results concerning low-energy particle physics](#)” (FLAG 2013)
- ✿ “[Leptonic Decays of Charged Pseudoscalar Mesons](#)” (Rosner, Stone, RV, PDG 2015)

Pion & kaon physics:

- ✿ “[Light flavour physics](#)” (Jüttner, Lattice 2015)
- ✿ “[CP violation & Kaon weak matrix elements from LQCD](#)” (Garron, Chiral Dynamics 2015)

Heavy flavor physics:

- ✿ “[Progress and prospects for heavy flavour physics on the lattice](#)” (Pena, Lat. 2015)
- ✿ “[Testing the Standard Model under the weight of heavy flavors](#)” (Bouchard, Lat. 2015)

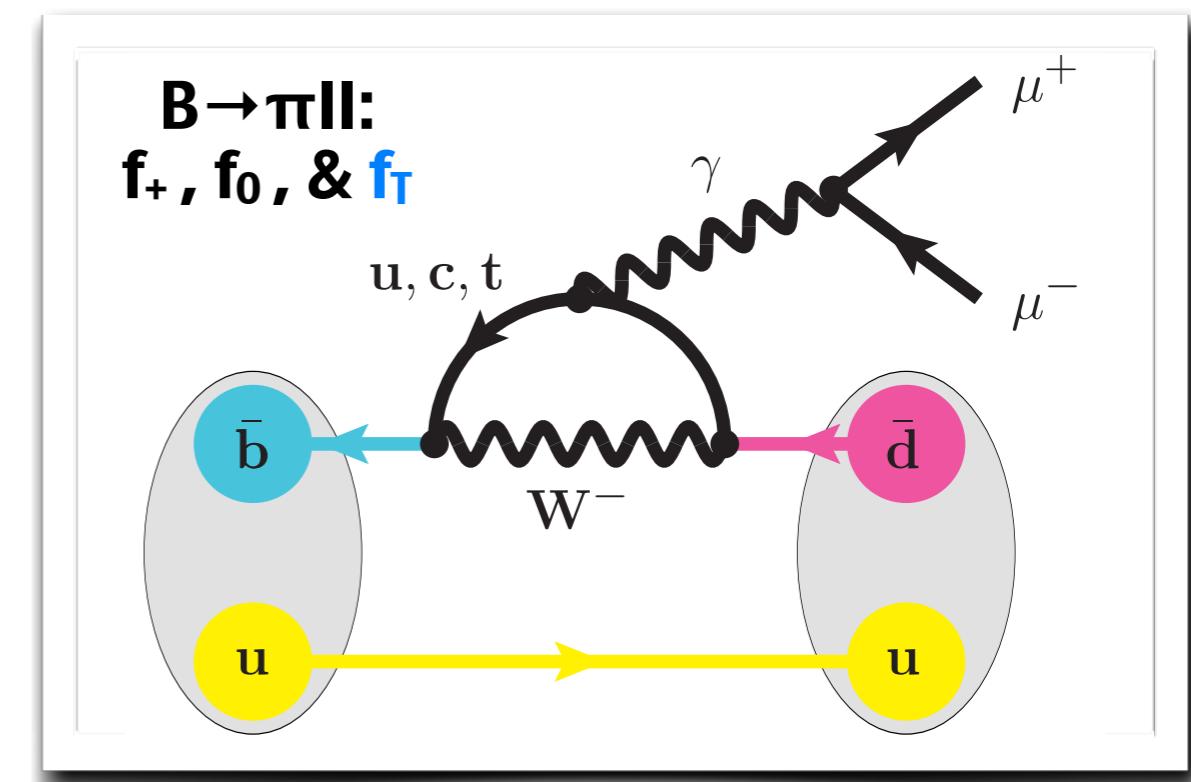
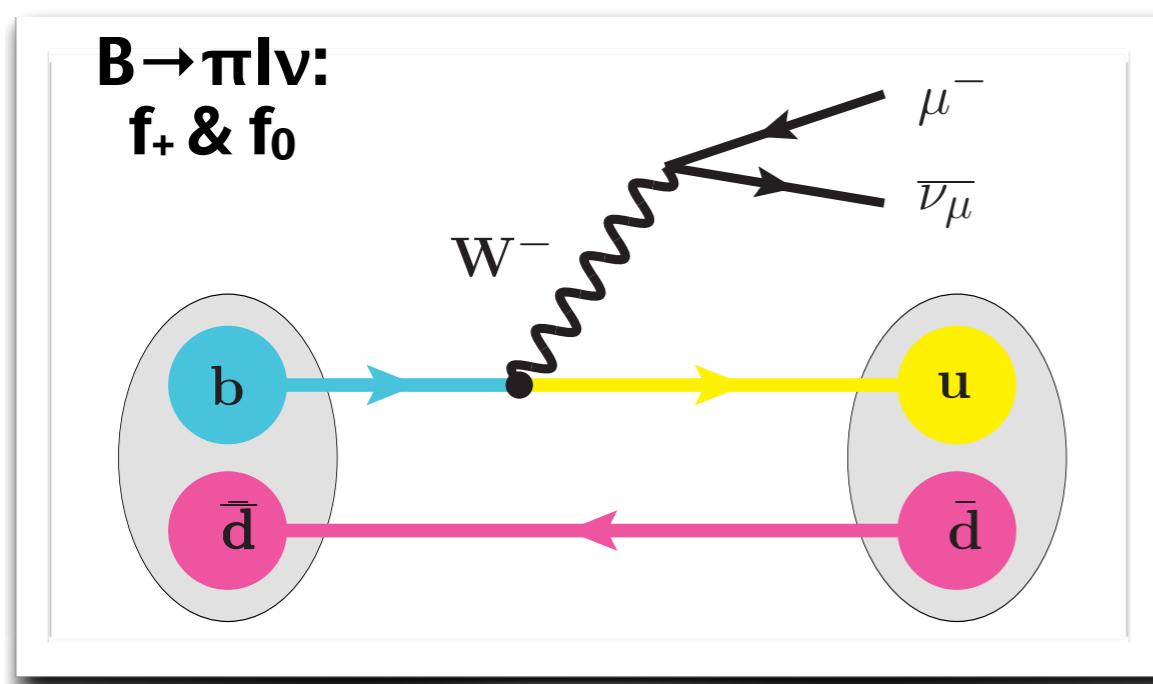
A cartoon illustration of three penguins standing on a sandy beach. They have white bodies, dark grey heads, and orange beaks. In the background, there are palm trees and a clear blue sky.

I wonder what kind of new
particle it is. I hope it's a
charged Higgs so the SM one has
someone to play with ...

Rare semileptonic B decays

Semileptonic $B \rightarrow \pi$ and $B \rightarrow K$ decays

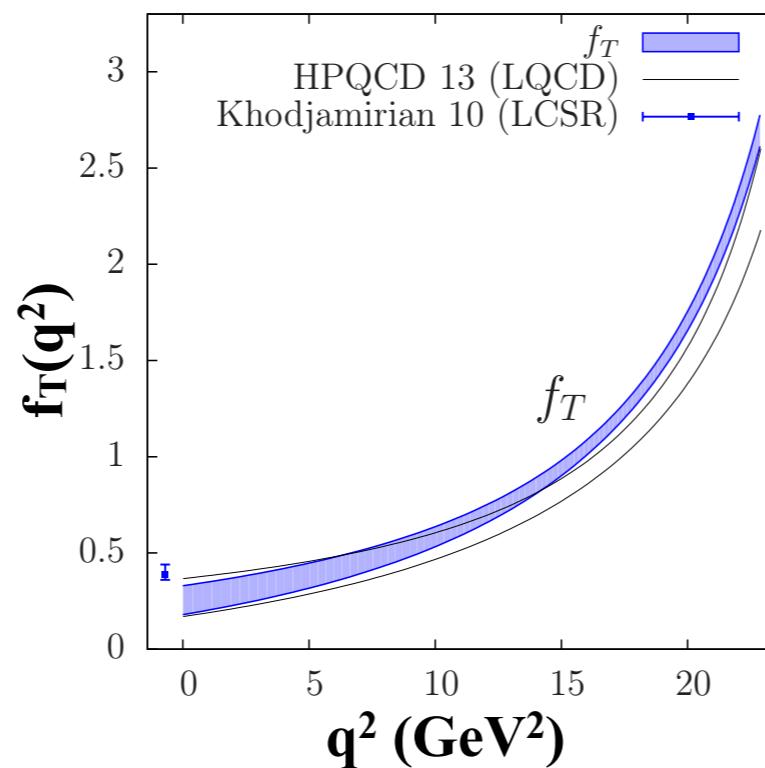
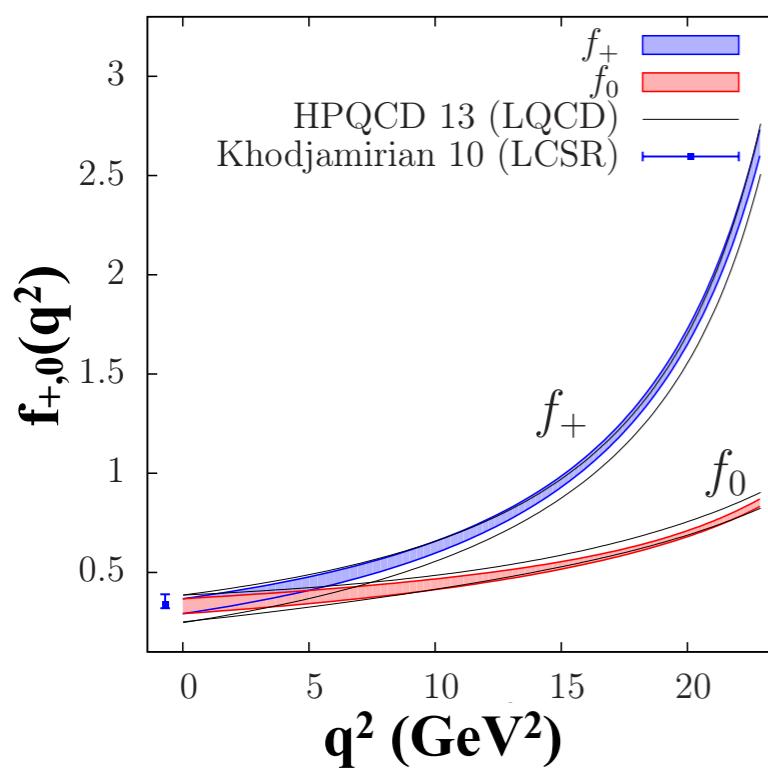
- ◆ Predictions for semileptonic decay rates and other observables depend upon hadronic form factors
 - ❖ Standard-Model tree-level $B \rightarrow \pi l \bar{\nu}$ decays mediated by **vector current**
 - ❖ **Flavor-changing-neutral-current decays** ($B \rightarrow \pi l^+ l^-$, $B \rightarrow K l^+ l^-$, $B \rightarrow \pi \bar{v} \bar{v}$, & $B \rightarrow K \bar{v} \bar{v}$) **also involve tensor current**
 - ❖ **Three form factors f_+ , f_0 , & f_T suffice to parameterize (factorizable) hadronic contributions to ALL Standard Model and new-physics $B \rightarrow \pi(K)$ processes**



$B \rightarrow \pi \parallel$ & $B \rightarrow K \parallel$ form factors (2015)

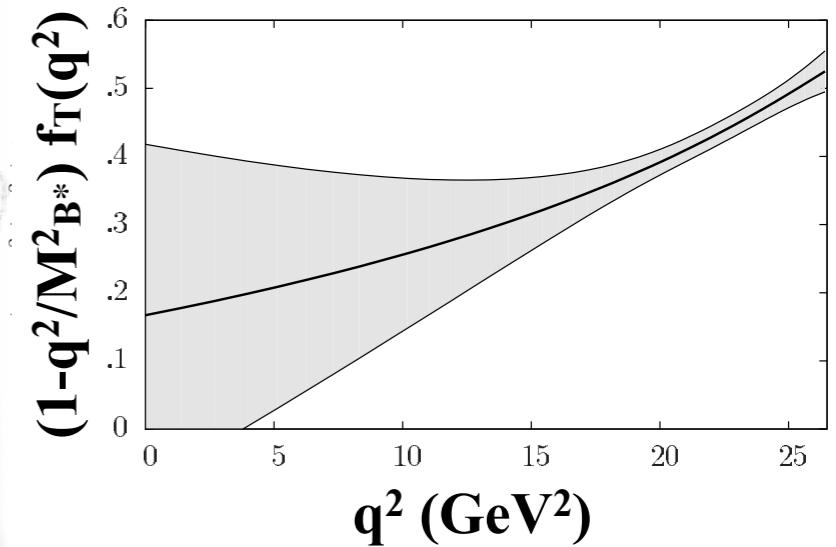
$B \rightarrow K$ form factors

[Fermilab/MILC, PRD 93, 025026 (2016)]



$B \rightarrow \pi$ tensor form factor

[PRL 115, 152002 (2015)]

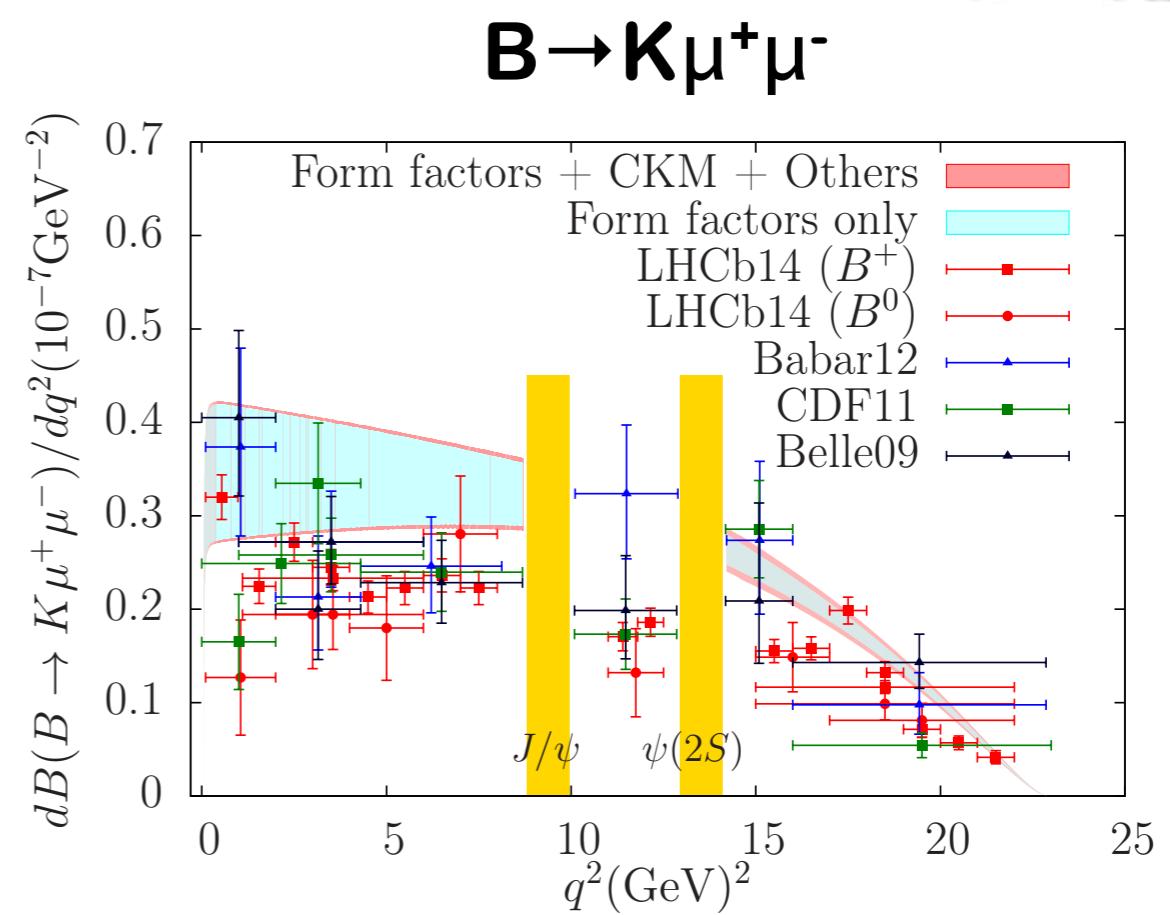
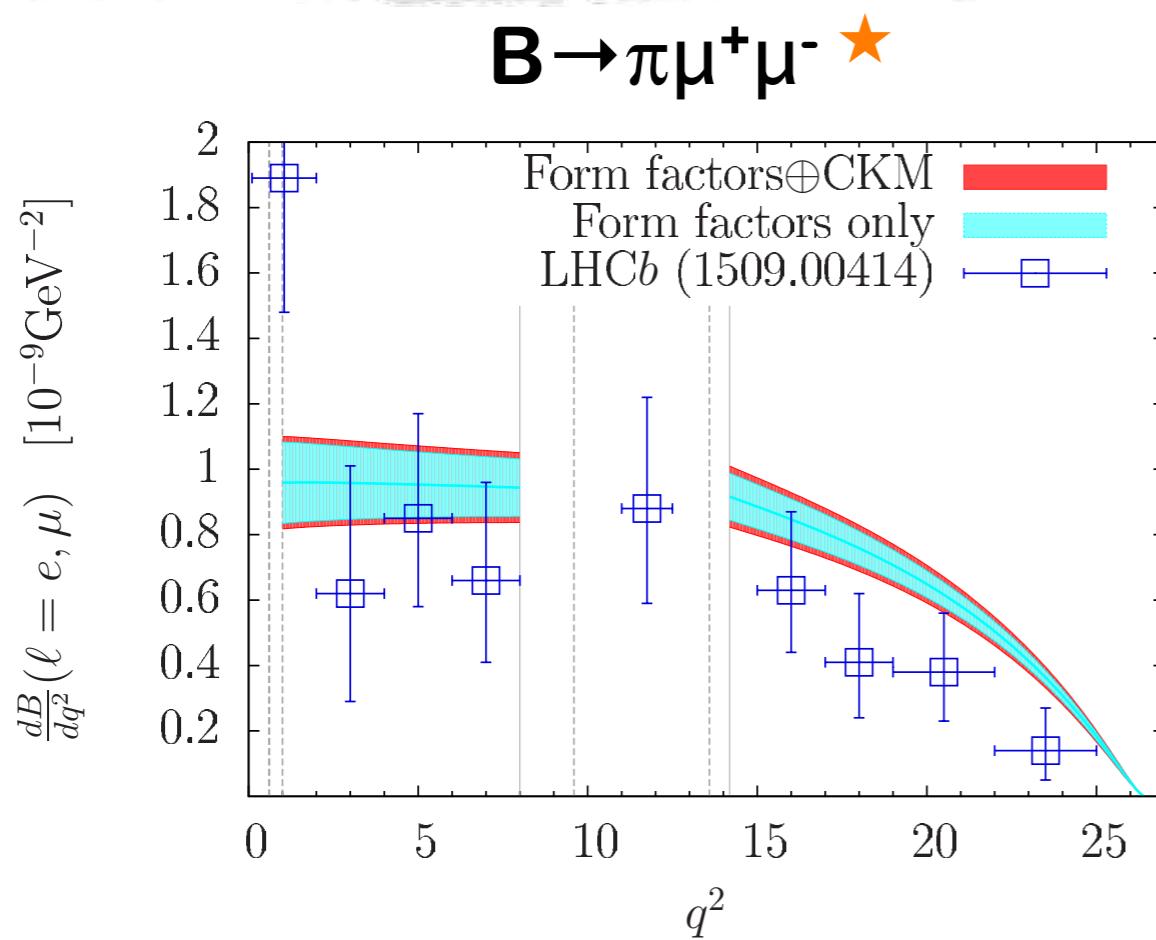


First lattice result for $B \rightarrow \pi$ tensor form factor!

- ◆ $B \rightarrow K$ form factors consistent with HPQCD results obtained with different b-quark action [PRD 88, 054509 (2013)] and LCSR at $q^2=0$ [JHEP 1009, 089 (2010)]
- ★ New lattice-QCD $B \rightarrow \pi$ and $B \rightarrow K$ form factors enable calculations of $B \rightarrow \pi(K)l^+l^-$, $B \rightarrow \pi(K)v\bar{v}$, and $B \rightarrow \pi \zeta v$ observables with fewer assumptions than previously possible!

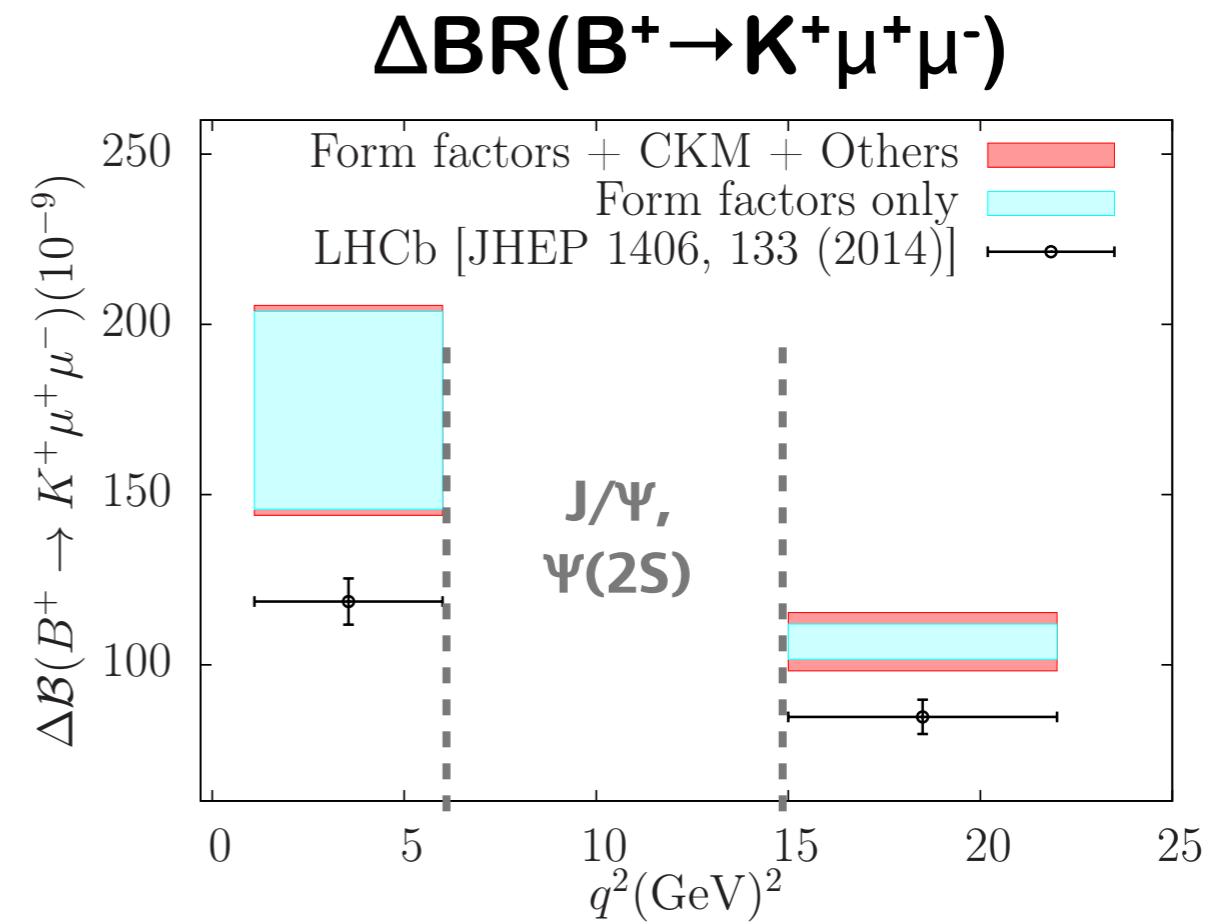
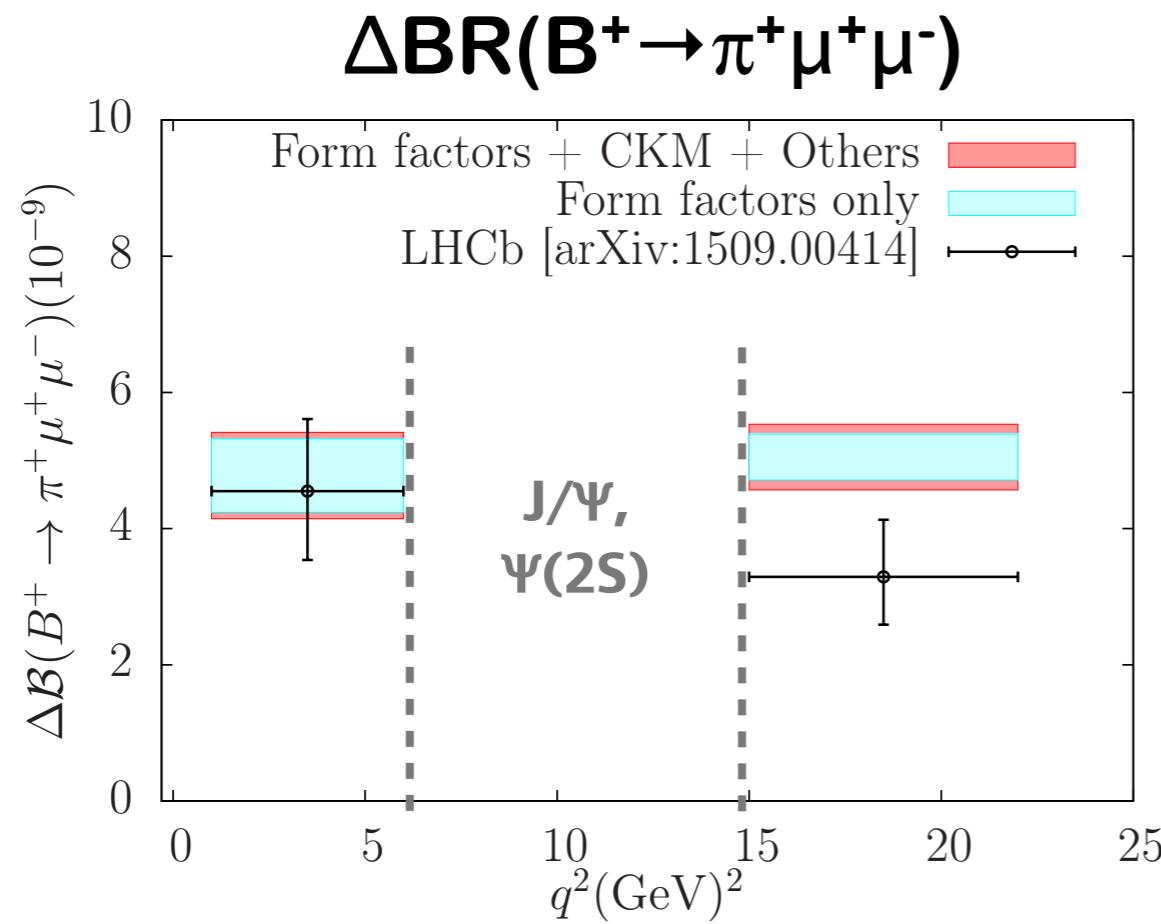
$B \rightarrow \pi \mu^+ \mu^-$ and $B \rightarrow K \mu^+ \mu^-$ decay rates

- ♦ Shapes consistent, but measurements slightly below Standard-Model expectations
- ★ Lattice prediction for $d\mathcal{B}(B \rightarrow \pi \mu^+ \mu^-)/dq^2$ appeared before LHCb measurement!
- ❖ Large difference for $B \rightarrow \pi \mu^+ \mu^-$ in lowest q^2 bin from effects of light (ρ, ω, φ) resonances, which are difficult to estimate in model-independent manner



$B \rightarrow \pi \mu^+ \mu^-$ and $B \rightarrow K \mu^+ \mu^-$ decay rates

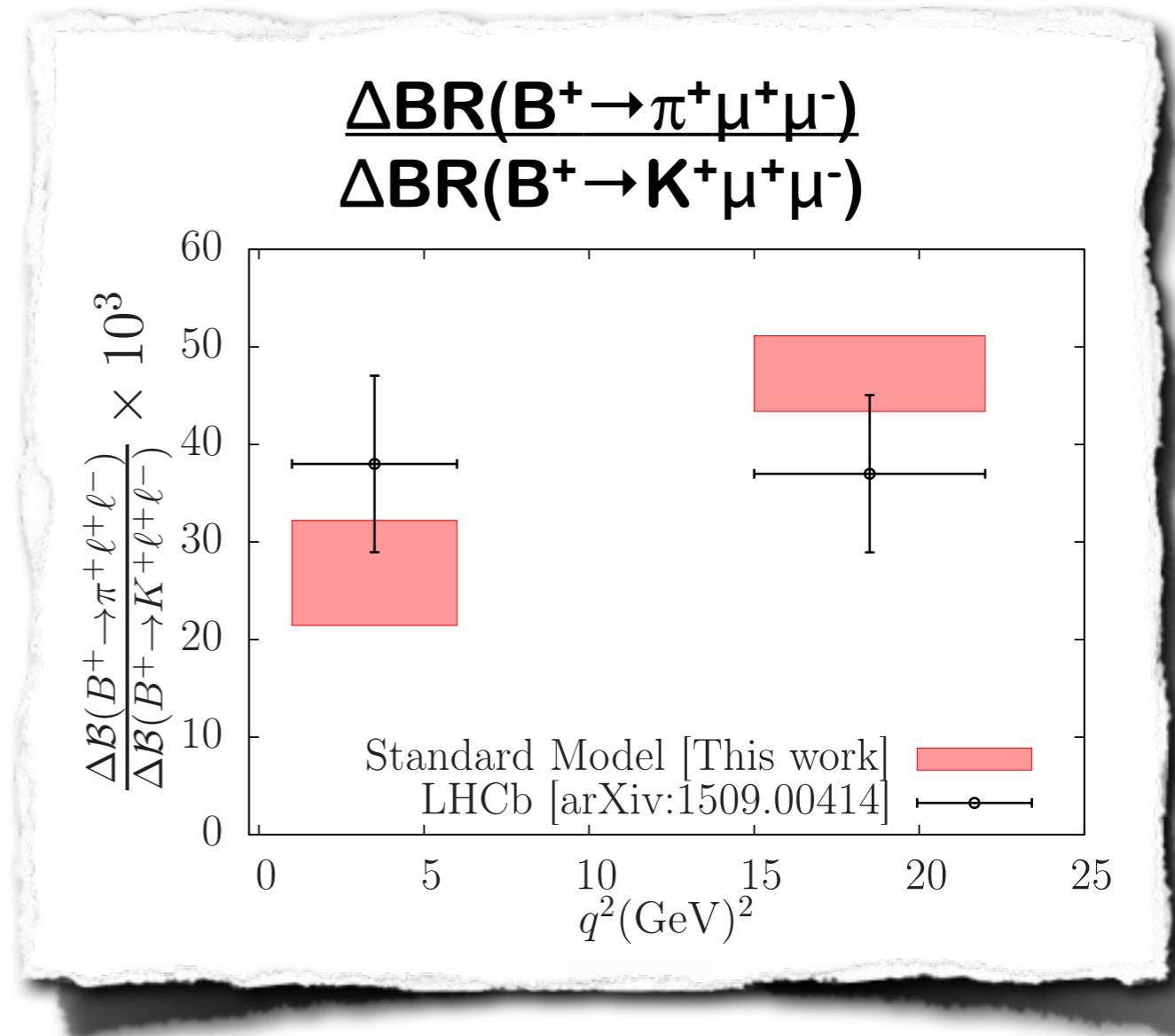
- ♦ For quantitative comparison, use measurements of $B^+ \rightarrow \pi^+ \mu^+ \mu^-$ and $B^+ \rightarrow K^+ \mu^+ \mu^-$ partially integrated branching ratios in wide q^2 bins away from charmonium resonances
 - ❖ Theory errors commensurate with experiment!
 - ❖ Measurements in 1.7σ (combined) tension with Standard Model



$(B \rightarrow \pi)$ -to- $(B \rightarrow K)$ ratios

- ◆ Probes new-physics scenarios that would alter shape of q^2 distribution differently for $B \rightarrow \pi l^+ l^-$ and $B \rightarrow K l^+ l^-$
 - ❖ (...but insensitive to new physics that would affect overall rates)
- ◆ Standard Model and experiment compatible within 1.1σ for both individual bins and combination

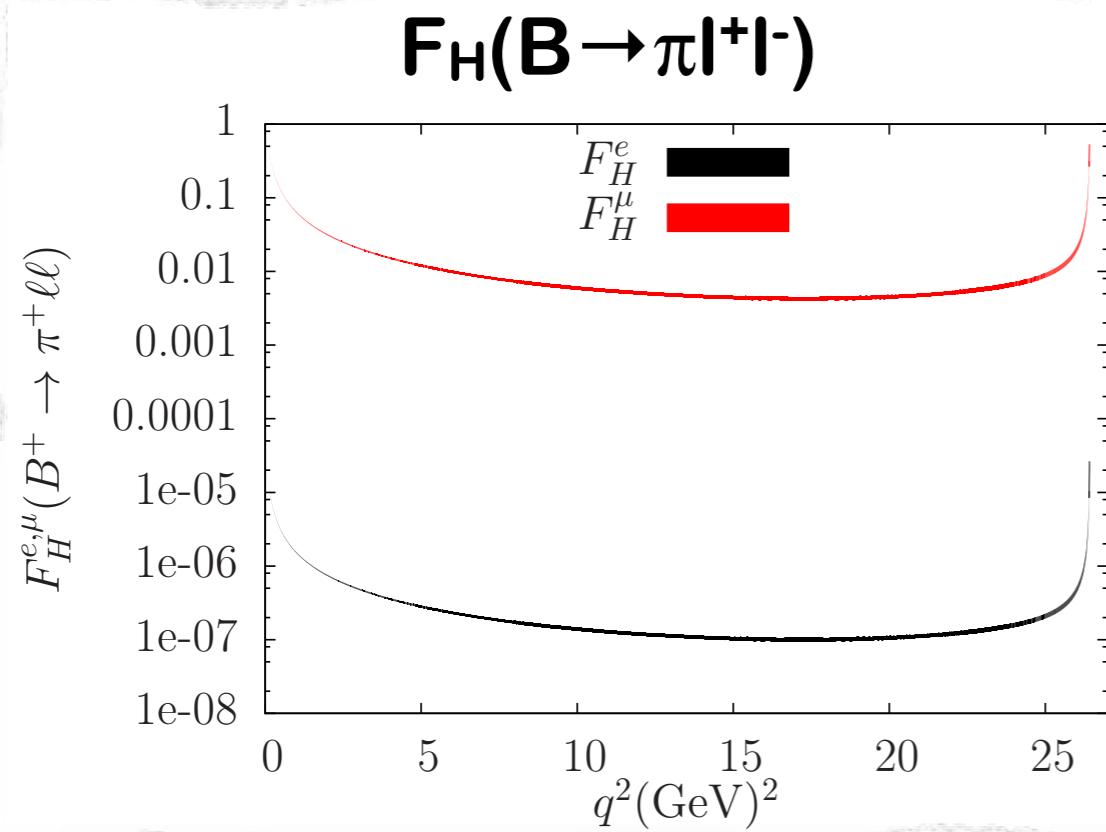
$\chi^2/\text{dof} = 2.7/2$
 $p = 0.26$



More sensitive $B \rightarrow \pi$ & $B \rightarrow K$ observables?

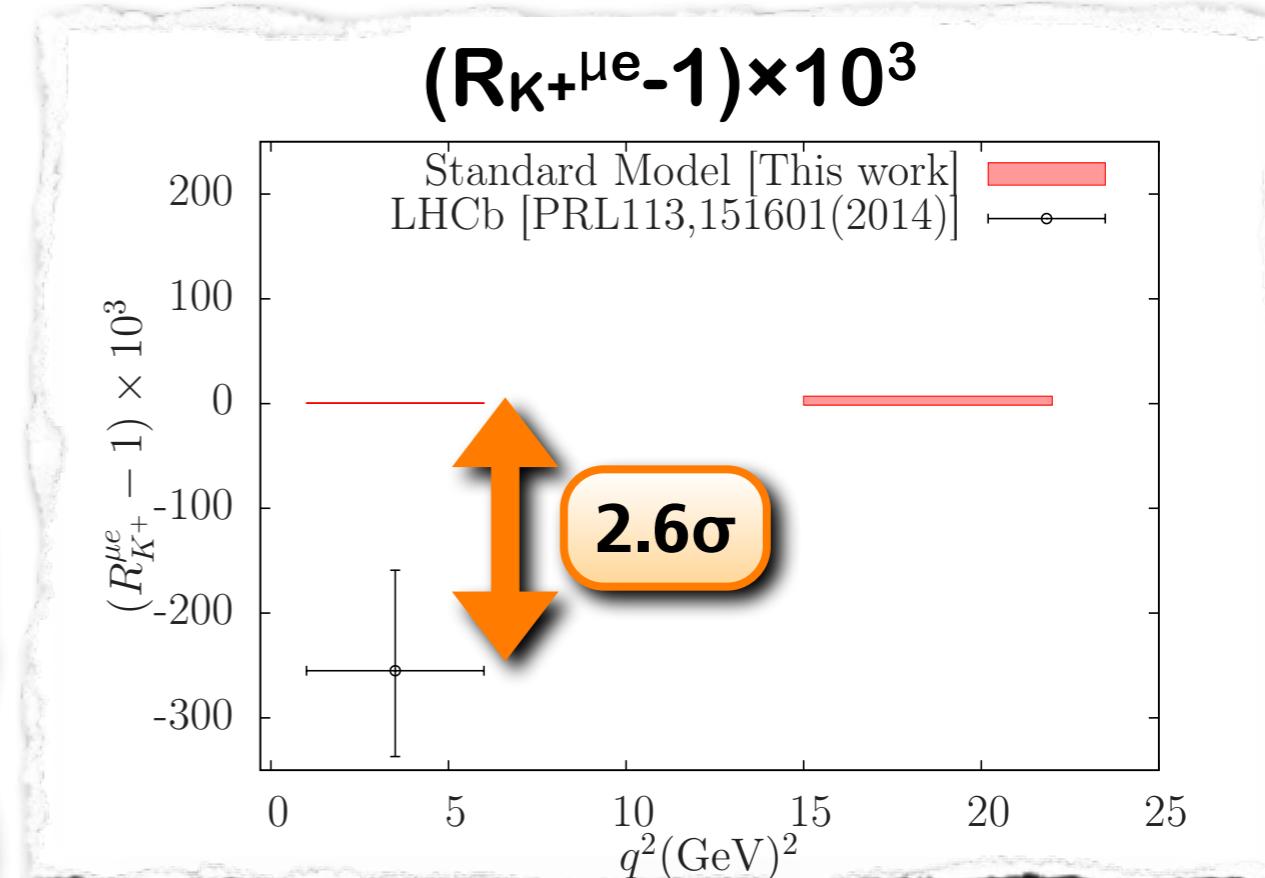
ANGULAR OBSERVABLES

- ◆ Flat term in angular distribution F_H of $O(m_l^2/m_B^2)$ in Standard Model
- ◆ $B \rightarrow \pi(K)\mu^+\mu^- F_H$ large enough to measure in future experiments
- ◆ $B \rightarrow \pi(K)e^+e^- F_H$ so small that nonzero measurement would suggest new physics



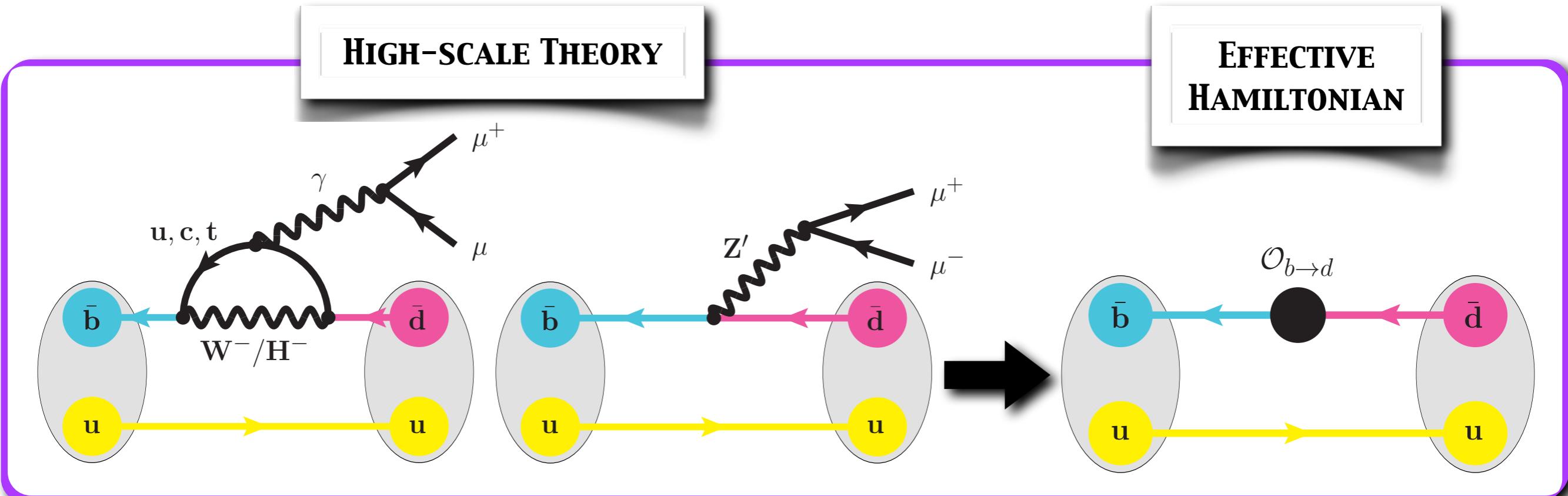
LEPTON-UNIVERSALITY TESTS

- ◆ Ratio of partially-integrated decay rates to different charged-lepton final states $R_P^{l_1 l_2}$ unity in Standard Model up to $O(m_l^2/m_B^2, m_l^4/q^4)$ corrections
- ◆ Explicit calculation of $R_{K^+\mu e}$ confirms intuitively significant deviation observed



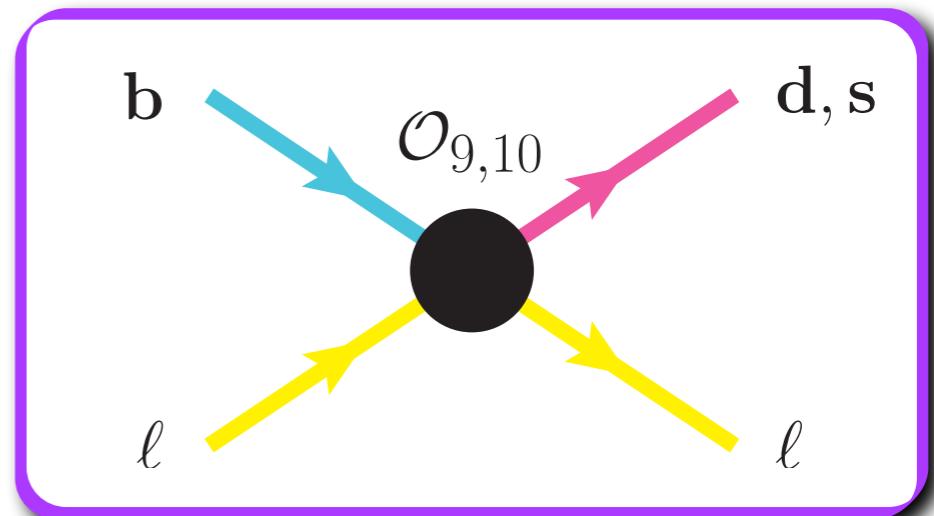
The effective EW Hamiltonian

- ♦ Because masses of W, Z, top, Higgs \gg bottom quark, can integrate out to yield effective Hamiltonian
 - ❖ Contributions of particles at or above electroweak scale parameterized by short-distance Wilson coefficients
 - ❖ QCD contribution parameterized by matrix elements of effective operators
- ♦ New heavy particles can alter Wilson coefficients or introduce new operator structures



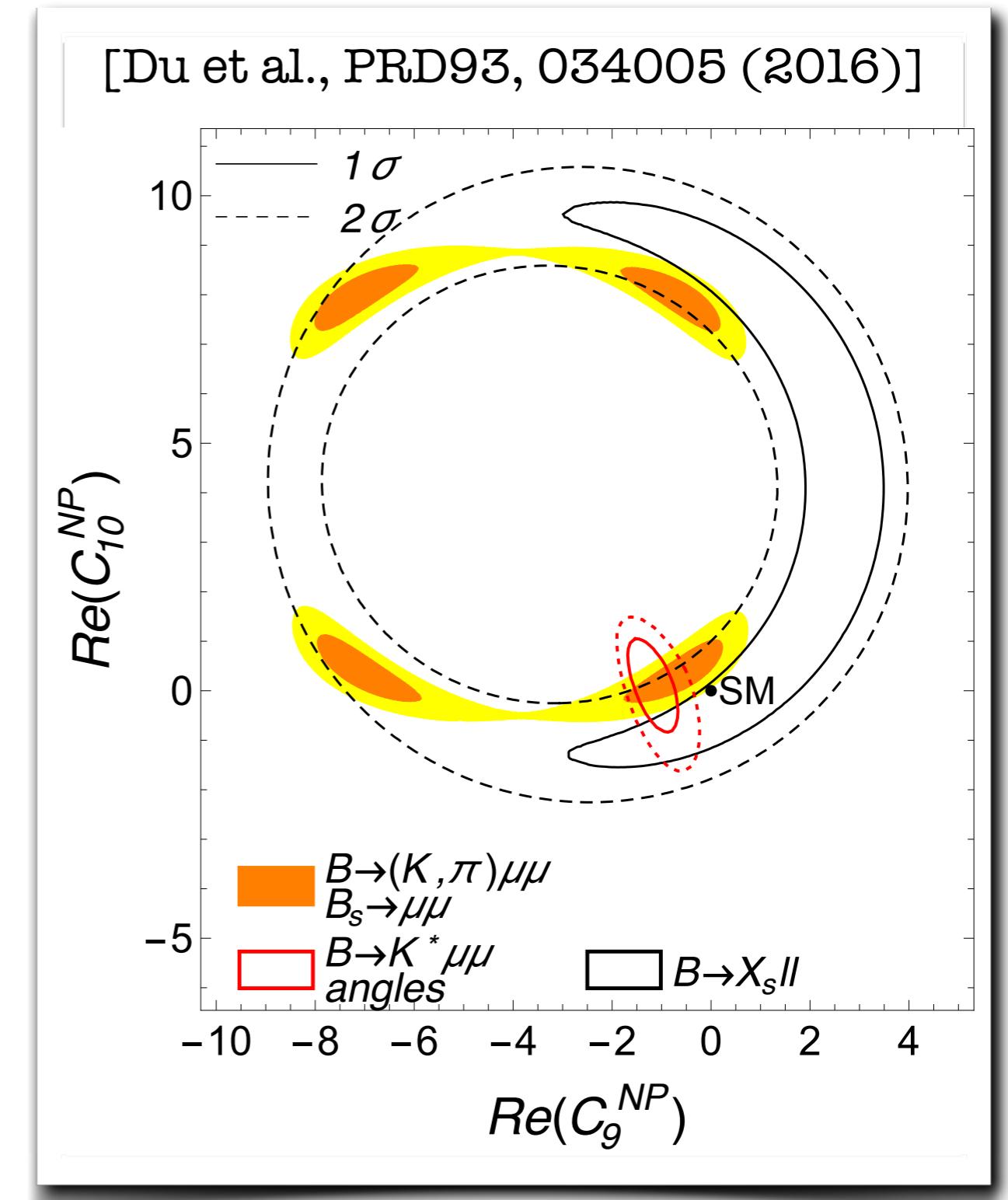
New-physics constraints from $B \rightarrow \pi(K)\mu^+\mu^-$

- ♦ Use measured decay rates + lattice-QCD form factors to **constrain new-physics contributions to coefficients of effective operators \mathcal{O}_9 and \mathcal{O}_{10}**
- ❖ Also include $BR(B_s \rightarrow \mu^+\mu^-)$ because lattice QCD provides reliable f_{B_s}



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- ❖ Also include $BR(B_s \rightarrow \mu^+\mu^-)$ because lattice QCD provides reliable f_{B_s}
- ◆ **Best-fit values for C_9 and C_{10} 2.1σ away from Standard Model**
- ◆ Region favored by exclusive (semi)leptonic decays compatible with those from **$B \rightarrow X_s l^+l^-$ inclusive observables**
[Huber et al., JHEP 06, 176 (2015)];
 $B \rightarrow K^*\mu^+\mu^-$ angular observables
[Altmannshofer & Straub, 1503.06199]
- ◆ **Constraints from theoretically clean B decays now competitive with $B \rightarrow K^*$!**



P.S. New-physics assumptions

- ◆ Allow new-physics contributions to the coefficients of the **four-fermion effective operators \mathcal{O}_9 and \mathcal{O}_{10}**

$$\mathcal{O}_9 \propto (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \ell)$$
$$\mathcal{O}_{10} \propto (\bar{s}_L \gamma_\mu b_L) (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

- ◆ Define the new-physics contributions to C_9 and C_{10} as:

$$C_i(\mu_0) = C_i^{\text{SM}}(\mu_0) + \mathbf{C}_i^{\text{NP}}(\mu_0)$$

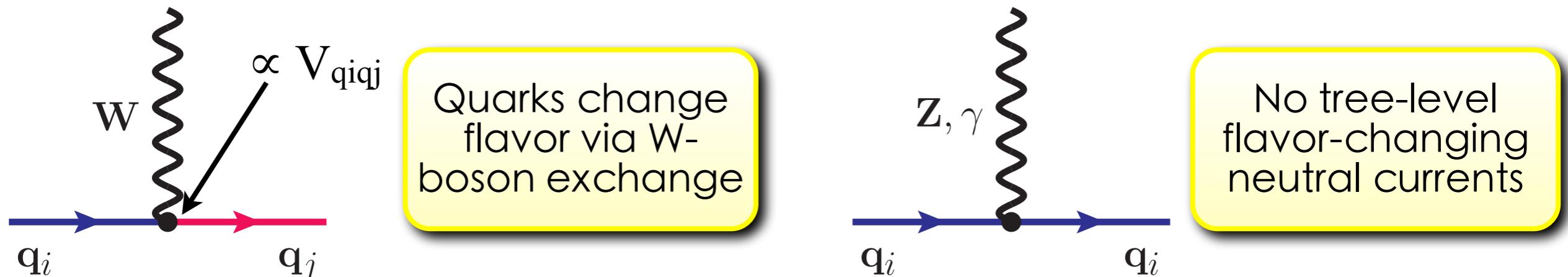
with $\mu_0 \approx 120$ GeV, and assume for the analysis:

- ❖ Coefficients identical for $b \rightarrow d l^+ l^-$ and $b \rightarrow s l^+ l^-$ transitions (as in minimal flavor violation)
- ❖ No new CP-violating phases $\rightarrow C_9^{\text{NP}}, C_{10}^{\text{NP}}$ real
- ◆ Measurements of $B \rightarrow X_s \gamma$ suggest that **any new-physics contributions to the coefficients of the photon penguin operators \mathcal{O}_7 and \mathcal{O}_8 must be small**, so take $C_7^{\text{NP}} = C_8^{\text{NP}} = 0$

Appendix: Many more lattice results!

- ◆ Light flavor physics
- ◆ Heavy flavor physics
- ◆ Strong coupling constant

The CKM quark-mixing matrix



- ♦ Mixing between quark flavors under charged weak interactions parameterized by **Cabibbo-Kobayashi-Maskawa (CKM) matrix:**

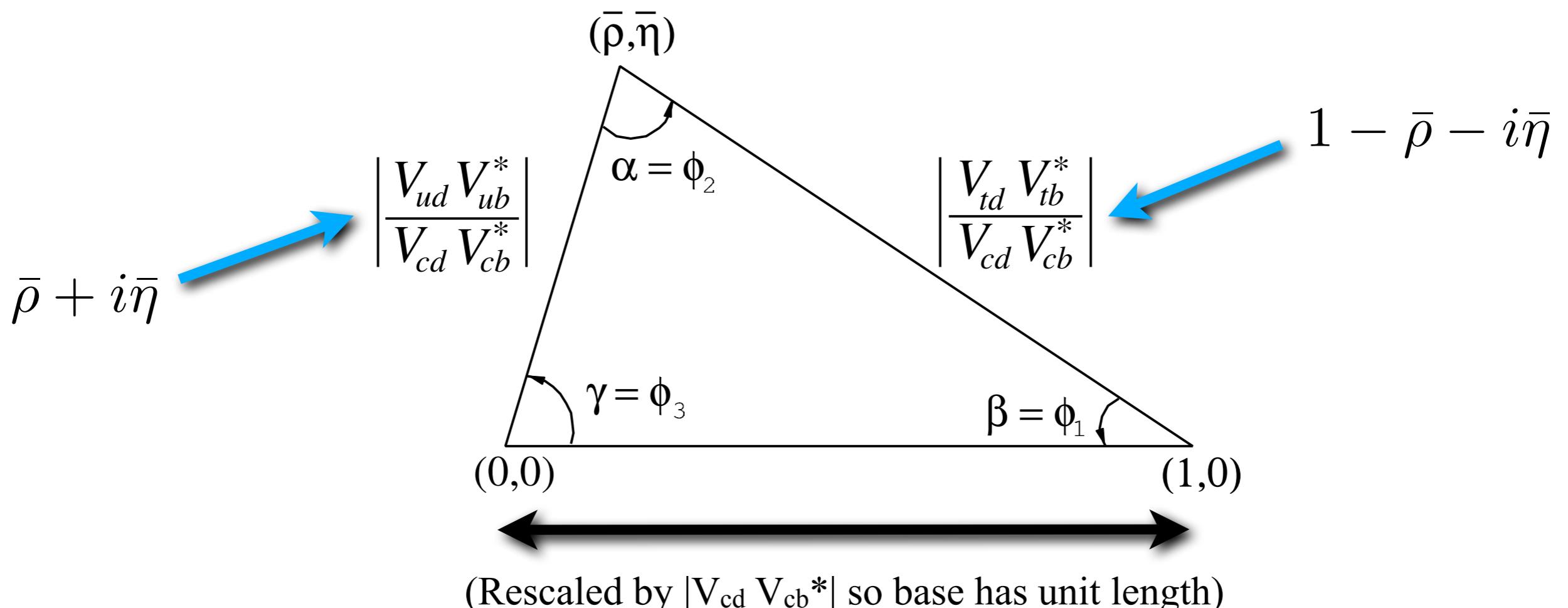
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} d & s & b \\ 0.9742 & 0.2257 & 3.59 \times 10^{-3} \\ 0.2256 & 0.9733 & 41.5 \times 10^{-3} \\ 8.74 \times 10^{-3} & 40.7 \times 10^{-3} & 0.9991 \end{pmatrix}$$

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- ♦ CKM elements & phase are **parametric inputs to Standard Model predictions for many flavor-changing processes** such as neutral kaon mixing and rare kaon decays

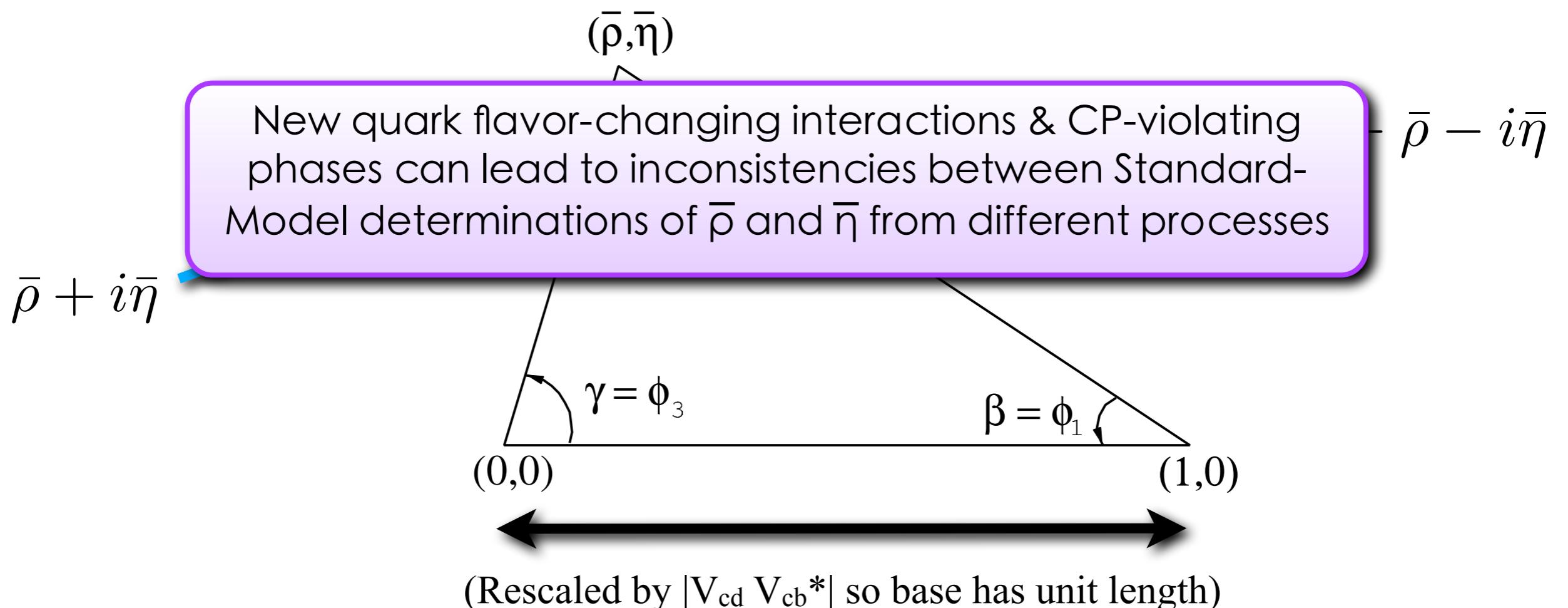
The CKM unitarity triangle

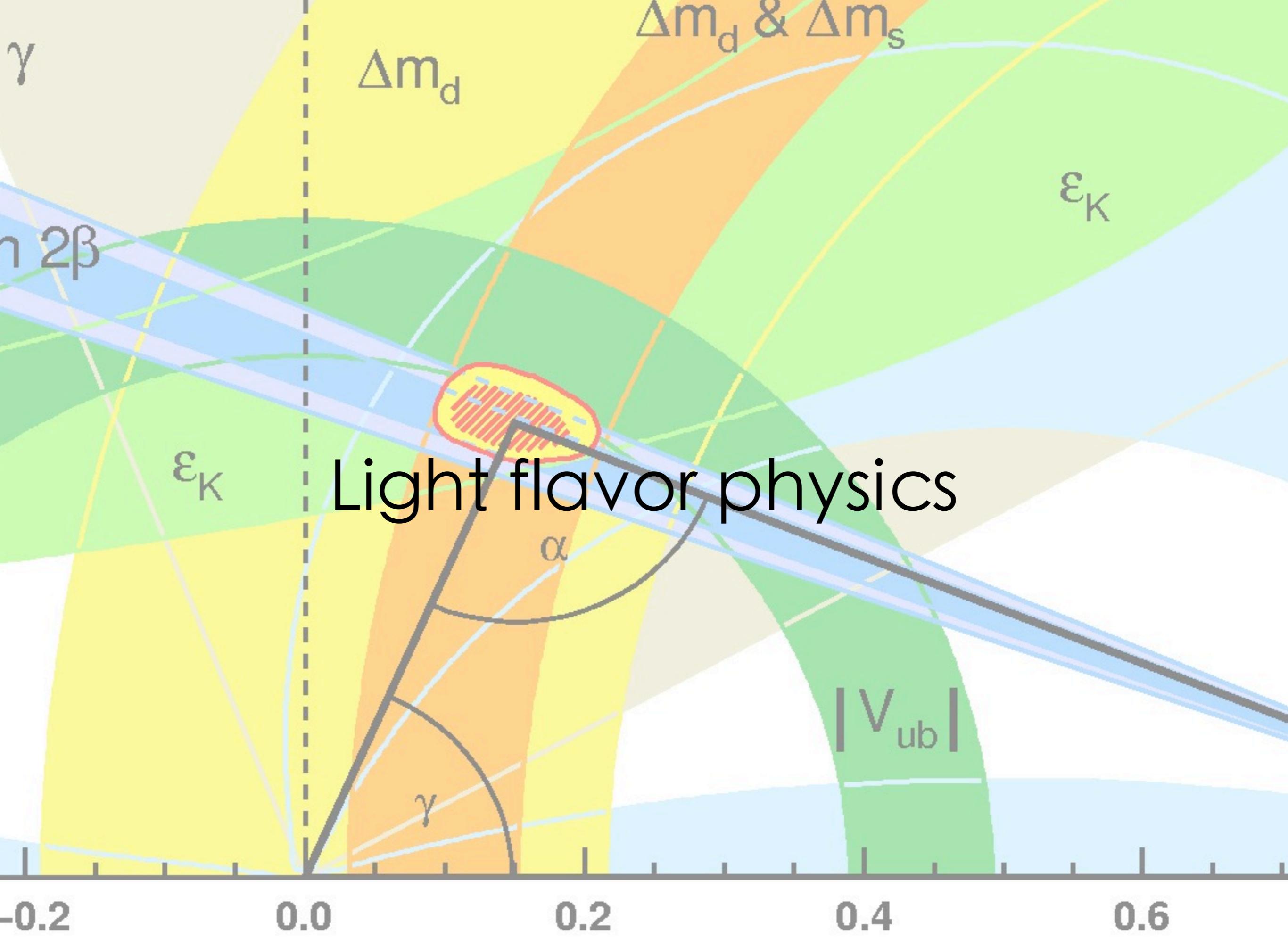
- ◆ Standard-Model CKM matrix is unitary → **elements are not all independent**
- ◆ One of the relationships between CKM matrix elements is:
- ◆ **Can express as triangle in complex plane known as the CKM unitarity triangle**



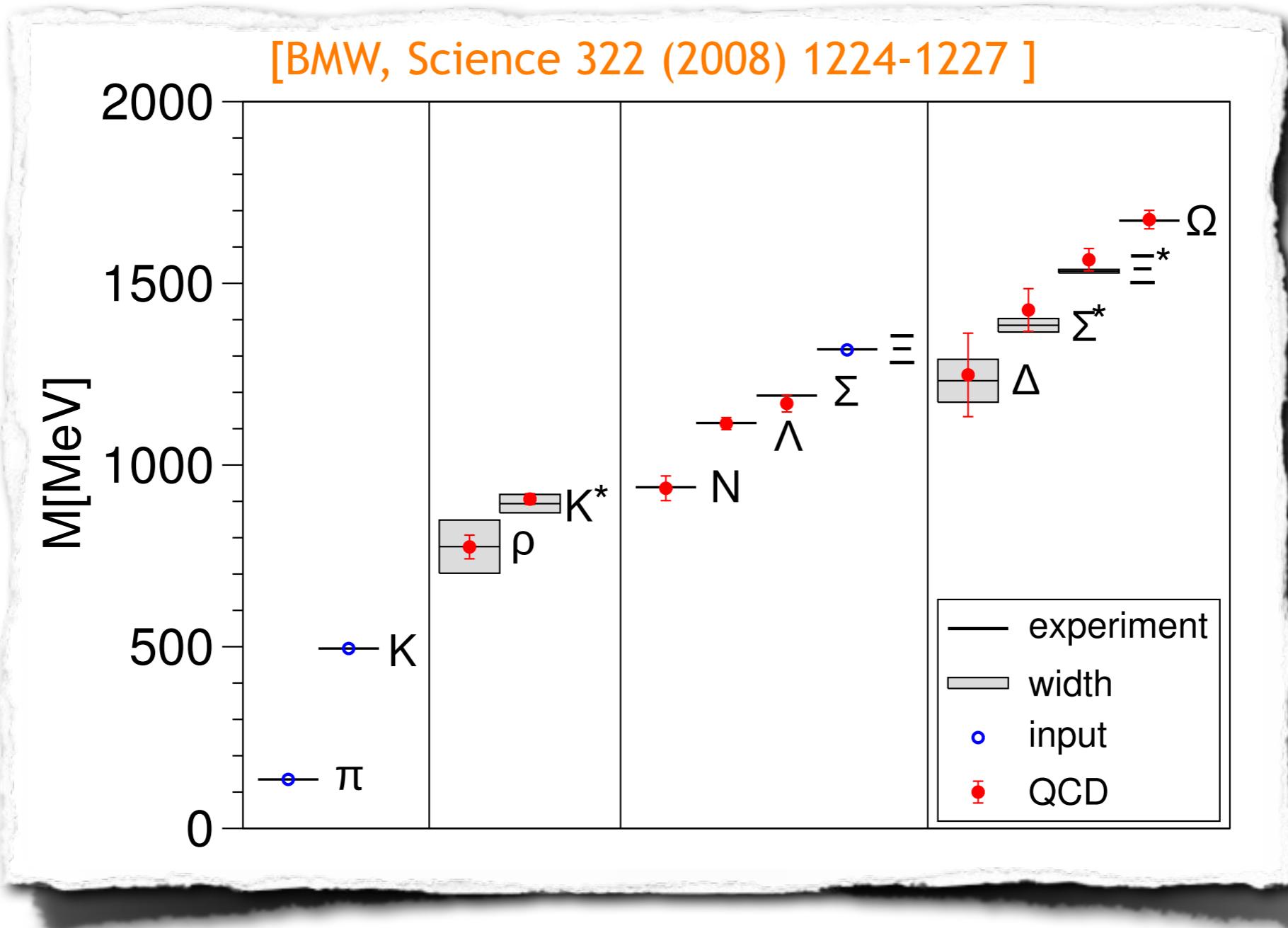
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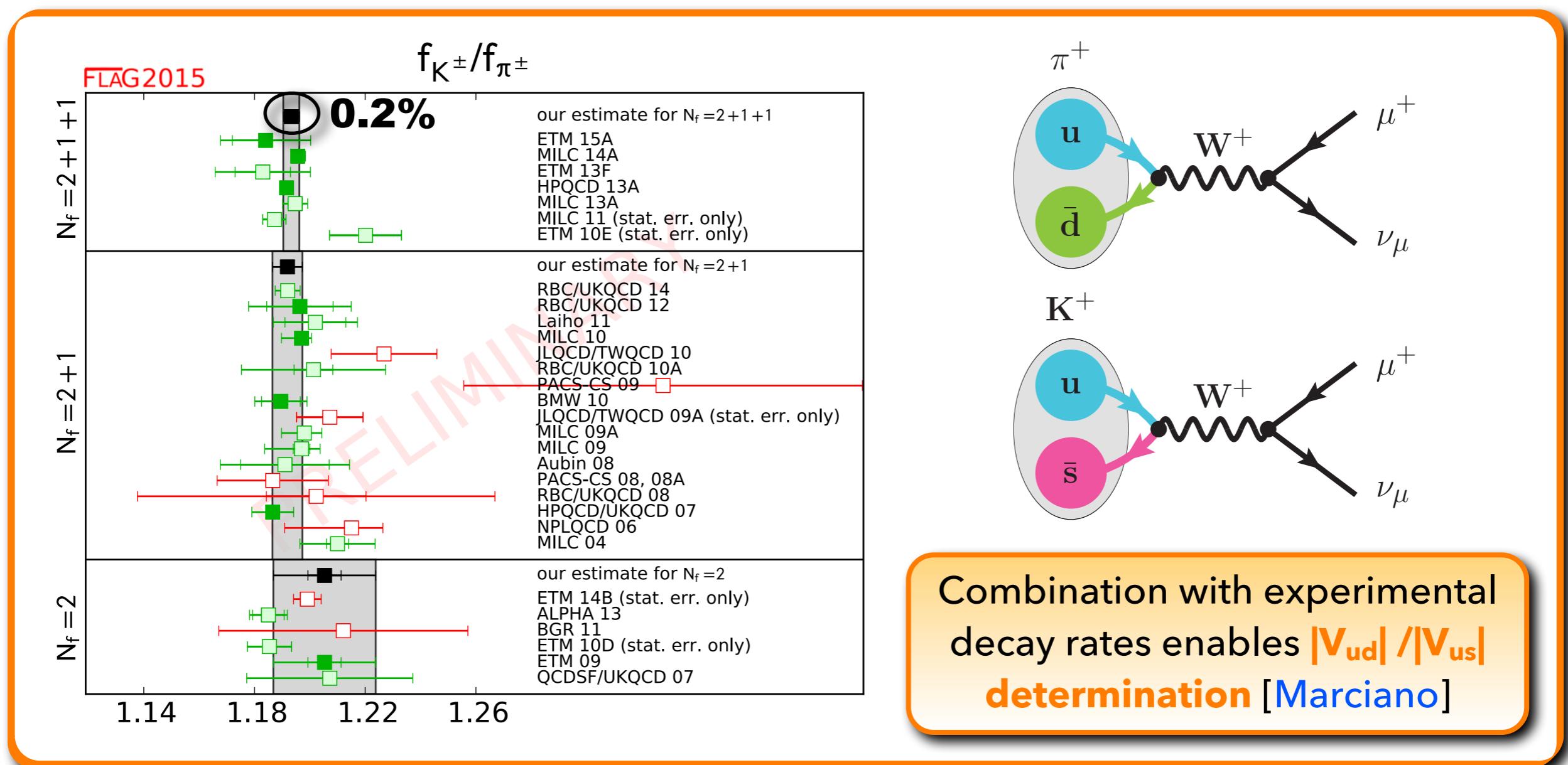
Light hadron spectrum



- ◆ Light hadron masses primarily due to energy stored in gluon field and to quarks' kinetic energy → **tests nonperturbative QCD dynamics**
- ◆ Also calculation of neutron-proton mass difference [BMW, Science 347, 1452, 2015])

Pion & kaon decay constants

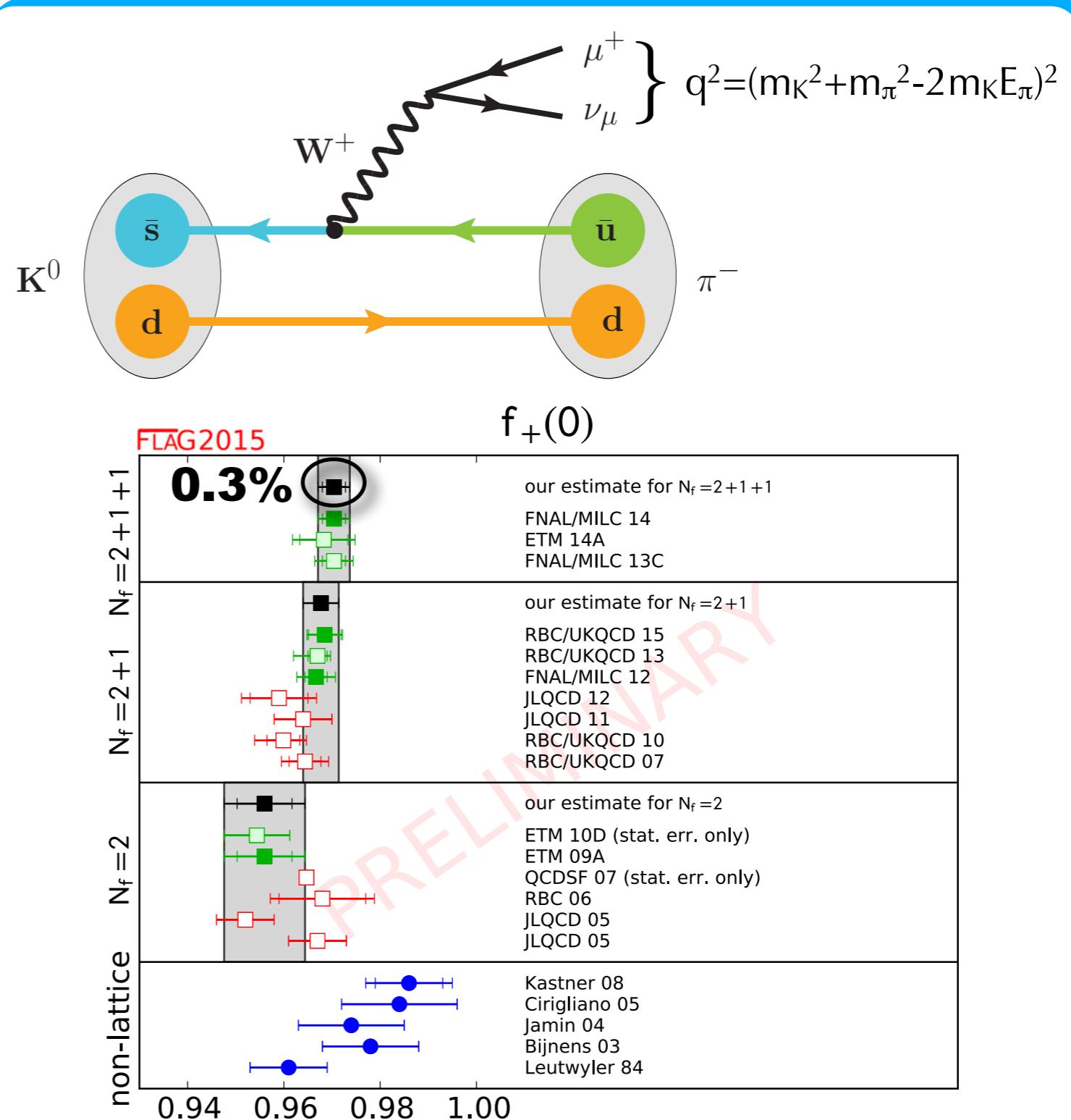
- ♦ Decay-constant ratio f_K/f_π can be computed to sub-% precision with lattice QCD:
 - ❖ Statistical fluctuations correlated between numerator & denominator
 - ❖ $f_K=f_\pi$ in SU(3) limit $m_s=m_{ud}$, so some systematics suppressed by $(m_s-m_{ud})/\Lambda_{\text{QCD}}$



$K \rightarrow \pi \ell \bar{\nu}$ semileptonic form factor

- ♦ **Zero-recoil form factor $f_+(q^2=0)$ highly constrained by SU(3)_f and chiral symmetries:**
 - ❖ $f_+(0) = 1$ in SU(3) limit $m_s=m_{ud}$
 - ❖ Leading-order correction to unity is known function of $\{m_\pi, m_K, f_\pi\}$ [Leutwyler & Roos]
 - ❖ $f_2 = -0.023$ numerically small because second-order in $(m_K^2 - m_\pi^2)$ [Ademollo-Gatto]
- ♦ **Lattice-QCD calculation does not require renormalization**

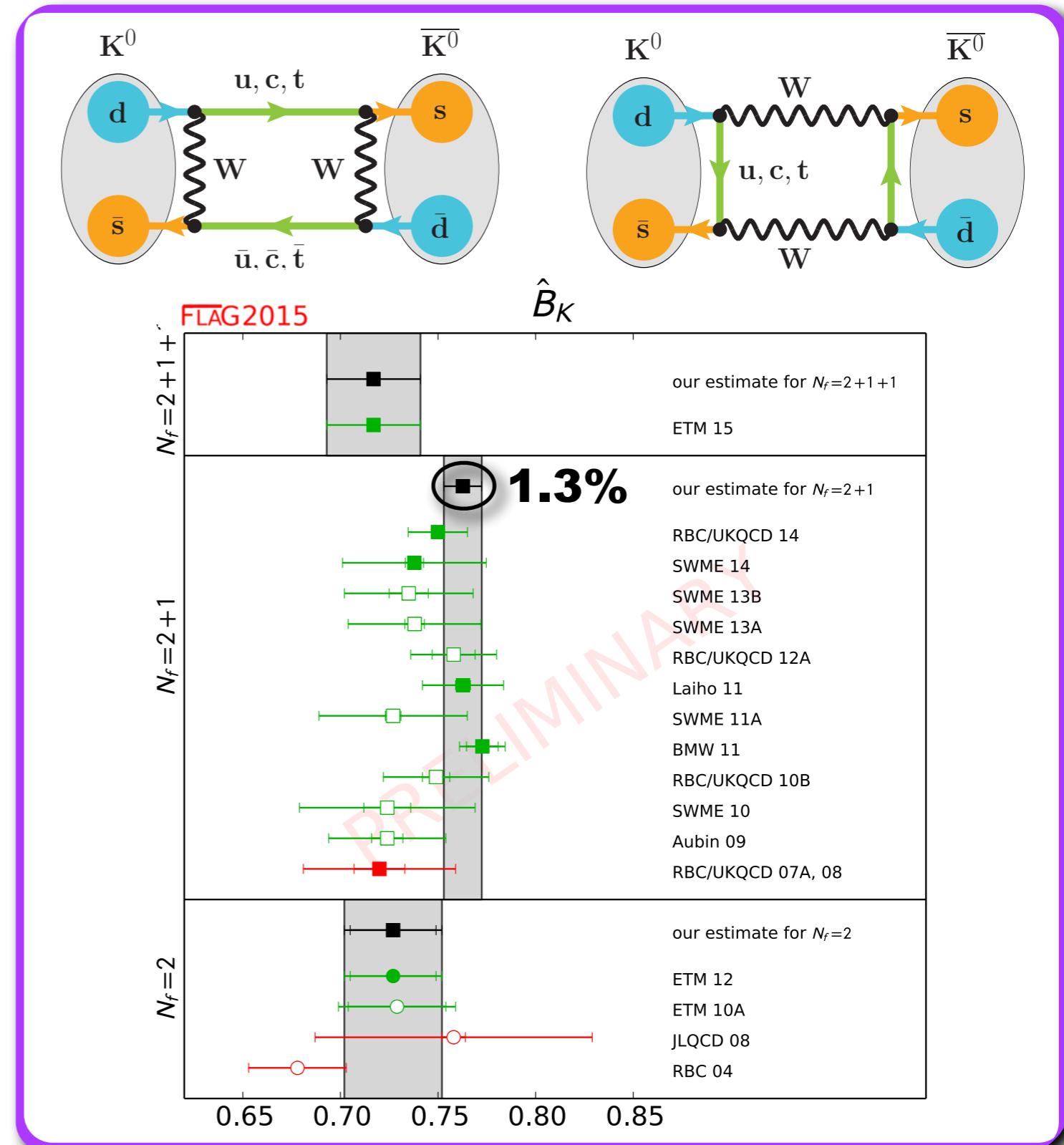
Combination with experimental rate enables $|V_{us}|$ determination



Neutral kaon mixing parameter

- ♦ Percent-level lattice-QCD calculation of B_K enabled by:
 - ❖ Chiral fermion actions
 - ❖ Nonperturbative renormalization
- ♦ Kaon mixing constrains scale of new physics with generic $O(1)$ flavor couplings to $\gtrsim 10,000$ TeV
[Isidori, Nir, Perez (2010)]
 - ❖ Lattice-QCD results for matrix elements of all possible $\Delta S=2$ BSM operators available for model building

Combination with measurement of indirect CP violation in kaon system (ε_K) **constrains CKM phase**



Status of the $|\epsilon_K|$ band

- ◆ Brod & Gorbahn [PRL108 (2012) 121801] give following error breakdown for $|\epsilon_K|$ in the Standard Model:

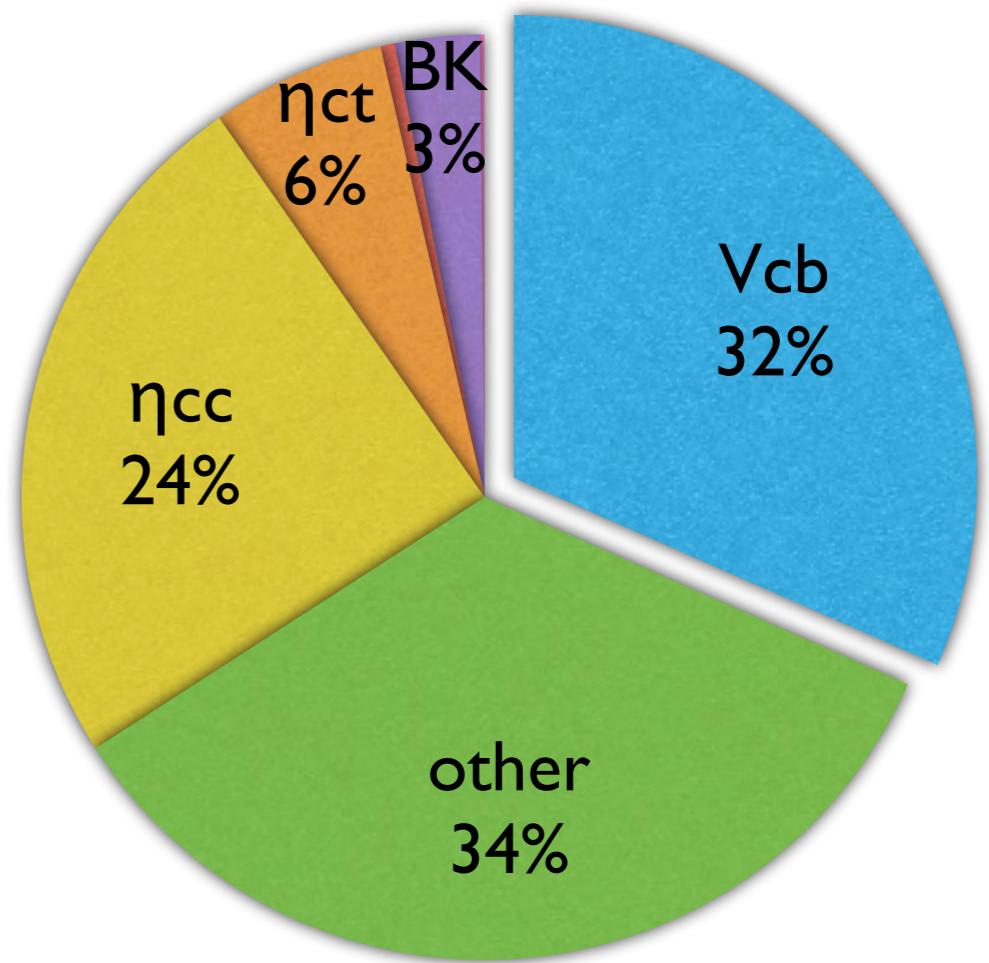
$$|\epsilon_K| = (1.81 \pm 0.14_{\eta_{cc}} \pm 0.02_{\eta_{tt}} \pm 0.07_{\eta_{ct}} \pm 0.05_{\text{LD}} \pm 0.23_{\text{parametric}}) \times 10^{-3}$$

(1) Largest individual uncertainty is from ~10% parametric error in $\propto |V_{cb}|^4$

(2) η_{cc} and η_{ct} are both known to 3-loops (NNLO)

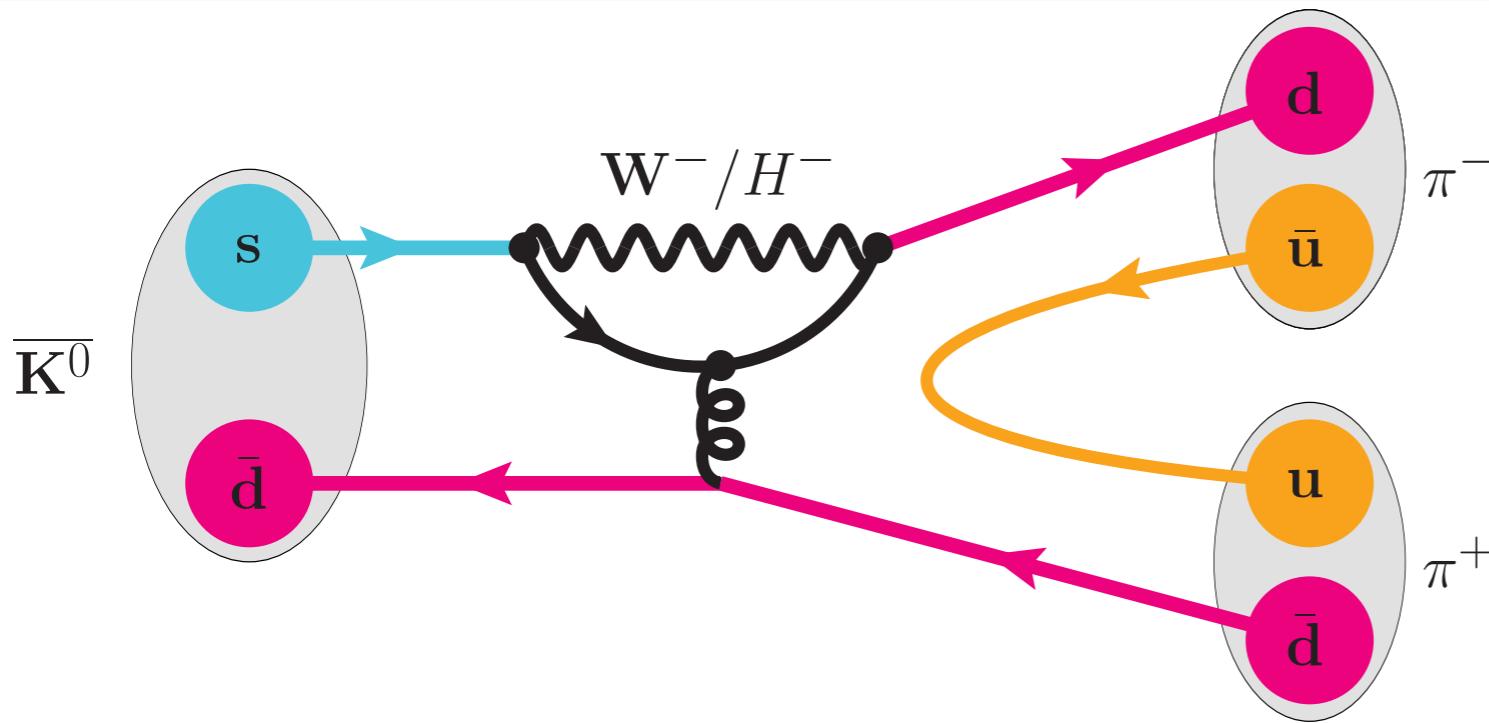
(3) Error from B_K only fourth-largest individual contribution

- ◆ Lattice community moving on to other more challenging kaon-physics quantities such as $K \rightarrow \pi\pi$ and $\Delta(M_K)$...



$K \rightarrow \pi\pi$ decays

See [Kelly](#)
 @ FPCP 2016



- Sensitive to new particles, interactions, sources of CP violation
- “ $\Delta l=1/2$ rule”: empirically observe enhancement $\text{Re}A_0/\text{Re}A_2 \approx 22.5$

- ♦ Describe $\Delta S=1$ FCNC transitions with **effective Hamiltonian**
 - ❖ Short-distance effects factorized in Wilson coefficients \rightarrow continuum perturbation theory
 - ❖ Long-distance effects factorized in **matrix elements** $\langle \pi\pi | Q_i | K \rangle \rightarrow$ **lattice QCD**
- ♦ New physics above EW scale modifies Wilson coefficients, but hadronic matrix elements remain the same

$$\mathcal{H}_{\text{eff}}(\Delta S = 1) = \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} \left(V_{us}^* V_{ud} z_i(\mu) - V_{ts}^* V_{td} y_i(\mu) \right) Q_i(\mu)$$

$K \rightarrow \pi\pi$ matrix elements (2015)

* **Lattice complication:** additional Lüscher formalism needed to relate amplitudes calculated in Euclidean box to physical observables in Minkowski space

[Briceño review, PoS LATTICE2014 (2015) 008]

- ◆ **First complete three-flavor $K \rightarrow \pi\pi$ amplitudes with controlled errors using domain-wall (chiral) fermions**
- ◆ Also first Wilson-fermion results from Ishizuka et al. [arXiv:1505.05289] with heavy, zero-momentum pions

Emerging explanation of $\Delta I=1/2$ rule:
Significant cancellation between dominant contributions to $\text{Re}(A_2)$ which does not occur for $\text{Re}(A_0)$

$\Delta I=3/2$ amplitude (A_2)

[RBC/UKQCD, PRD 91 (2015) 7, 074502]

- ◆ Physical-mass pions, continuum limit, and approximately physical kinematics
- ◆ → ~10% errors on $\text{Re}(A_2)$, $\text{Im}(A_2)$
- ◆ Dominant uncertainty from perturbative truncation error in continuum Wilson coefficients

$\Delta I=1/2$ amplitude (A_0)

[RBC/UKQCD, arXiv:1505.07863]

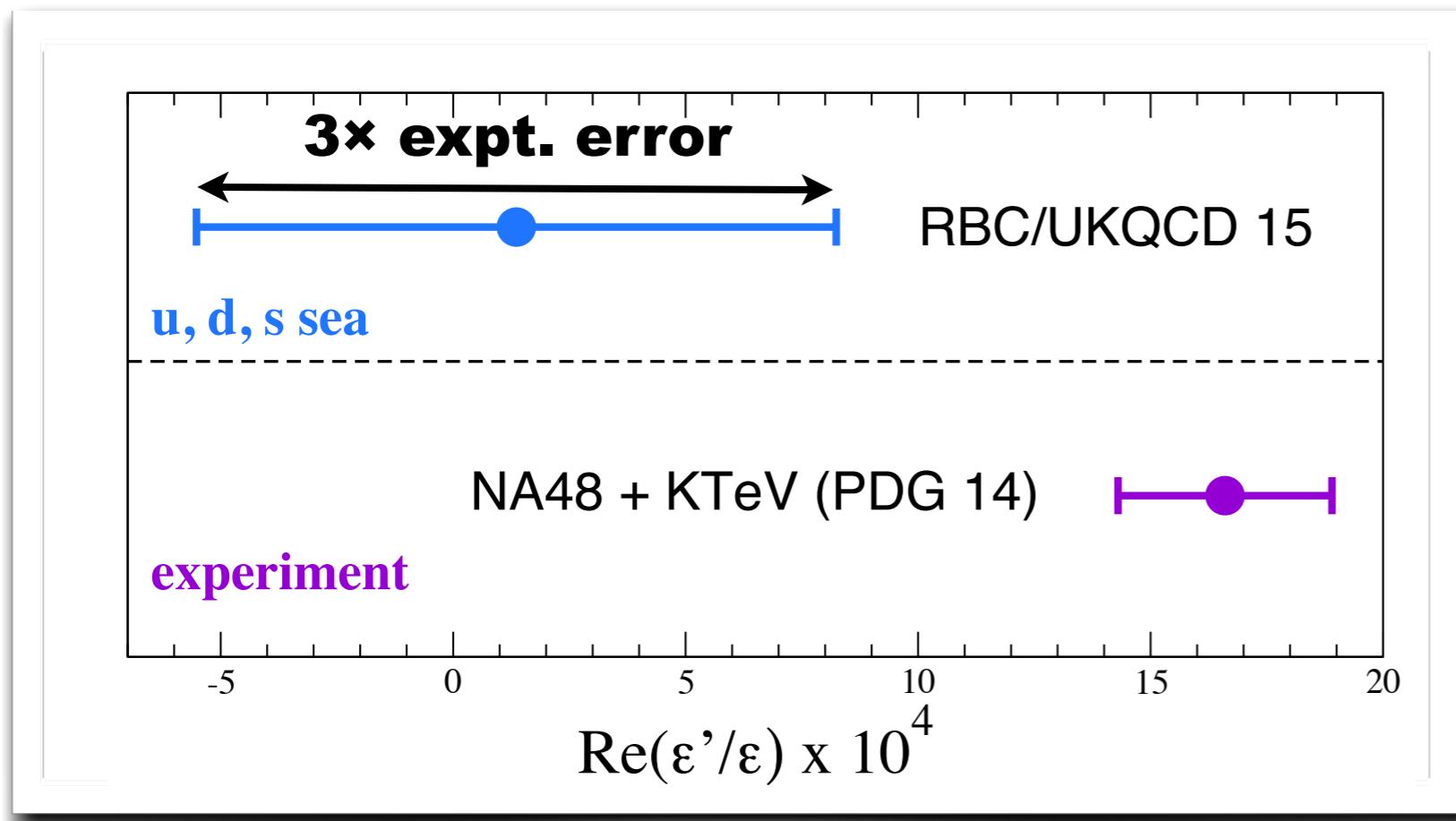
- ◆ 170 MeV pions with physical kinematics
- ◆ Small spatial volume $m_\pi L \approx 3.2$ and single lattice spacing $a \sim 0.14$ fm
- ◆ → ~35% error on $\text{Re}(A_0)$
- ◆ Will be reduced with higher statistics, larger volumes, continuum limit, ...

$\text{Re}(\varepsilon'/\varepsilon)$ in the Standard Model (2015)

[RBC/UKQCD, PRL 115, 212001 (2015)]

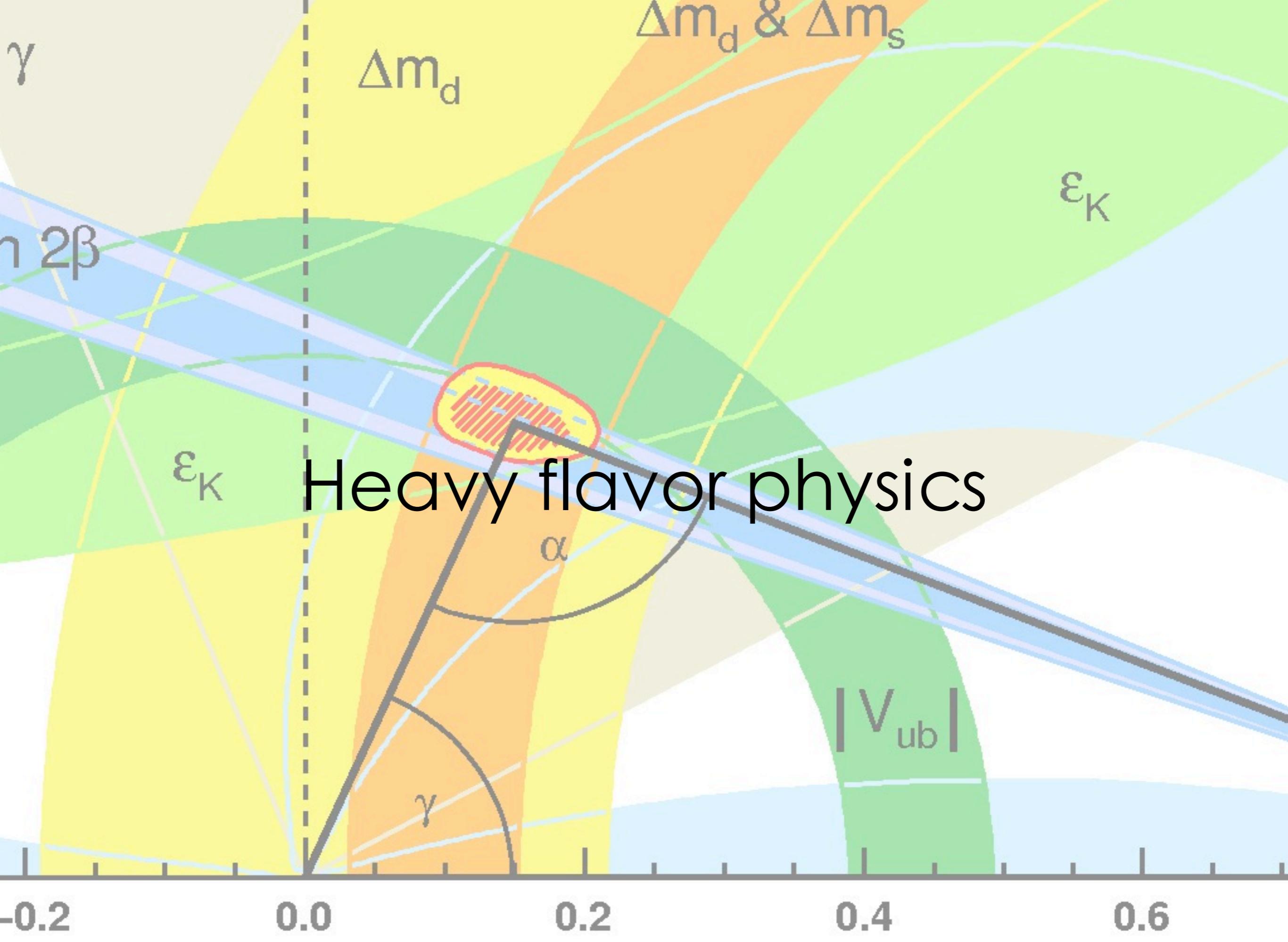
- ◆ Measures direct CP violation in $K \rightarrow \pi\pi$ decays

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) \approx \frac{1}{6} \left[\frac{\Gamma(K_L \rightarrow \pi^+ \pi^-)/\Gamma(K_S \rightarrow \pi^+ \pi^-)}{\Gamma(K_L \rightarrow \pi^0 \pi^0)/\Gamma(K_S \rightarrow \pi^0 \pi^0)} - 1 \right]$$



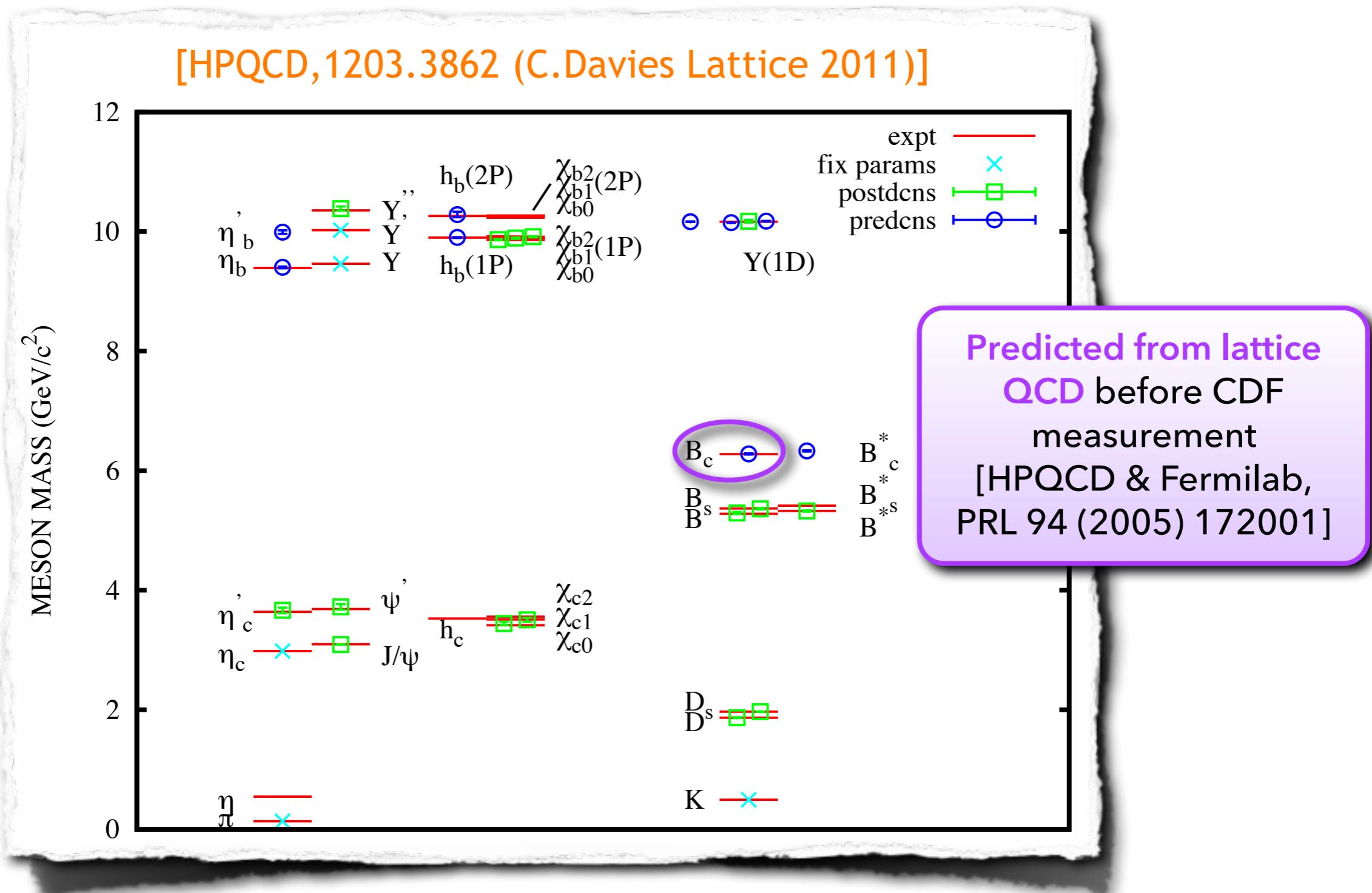
- ◆ Lattice $\text{Im}(A_0, A_2) +$ experimental $\text{Re}(A_0, A_2)$
→ first $\text{Re}(\varepsilon'/\varepsilon)$ in
Standard Model with
controlled errors

Lattice-QCD calculation of
 $\text{Re}(\varepsilon'/\varepsilon)$ with ~10%
uncertainty achievable
in the foreseeable future!



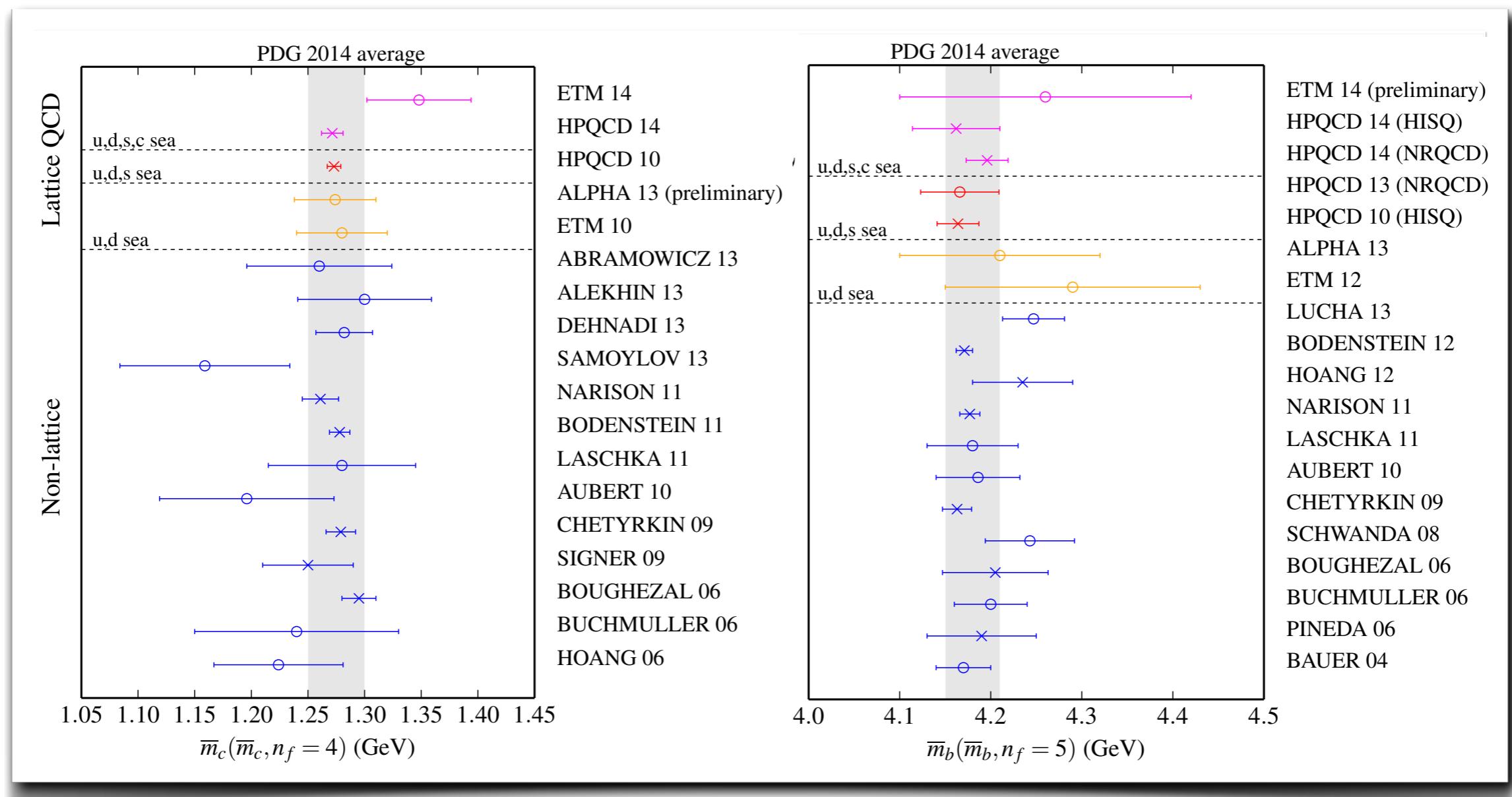
Heavy hadron spectrum

- ◆ Tests lattice methods for charm & bottom quarks, which often rely on effective theories



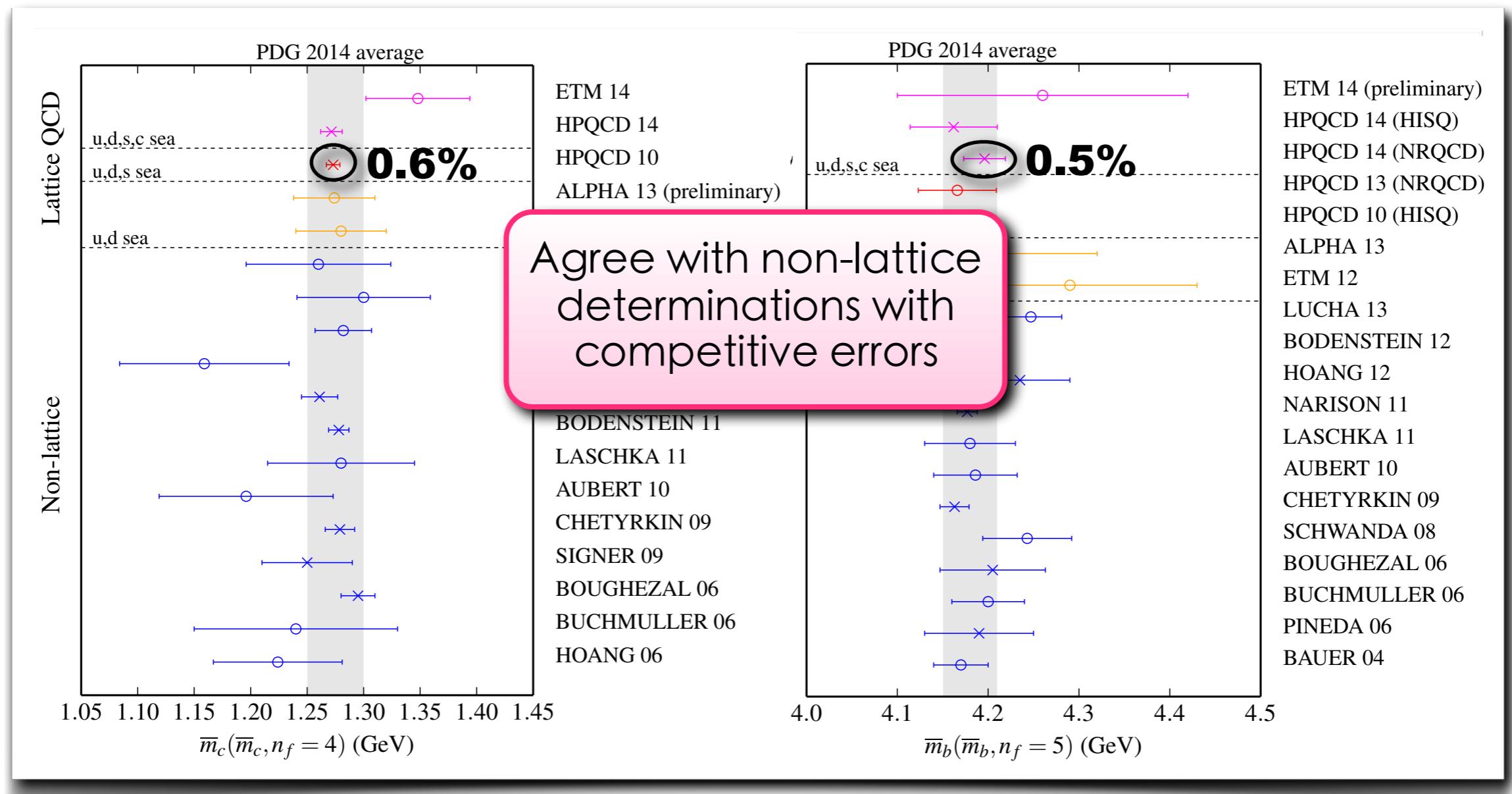
Heavy-quark masses

- ♦ Most precise m_c and m_b obtained by fitting moments of correlation functions of the quarks' electromagnetic current to $O(\alpha_s^3)$ perturbative expressions
- ♦ Moments can be obtained from experimental e^+e^- annihilation data, and also **computed numerically with lattice-QCD simulations with negligible statistical uncertainties**



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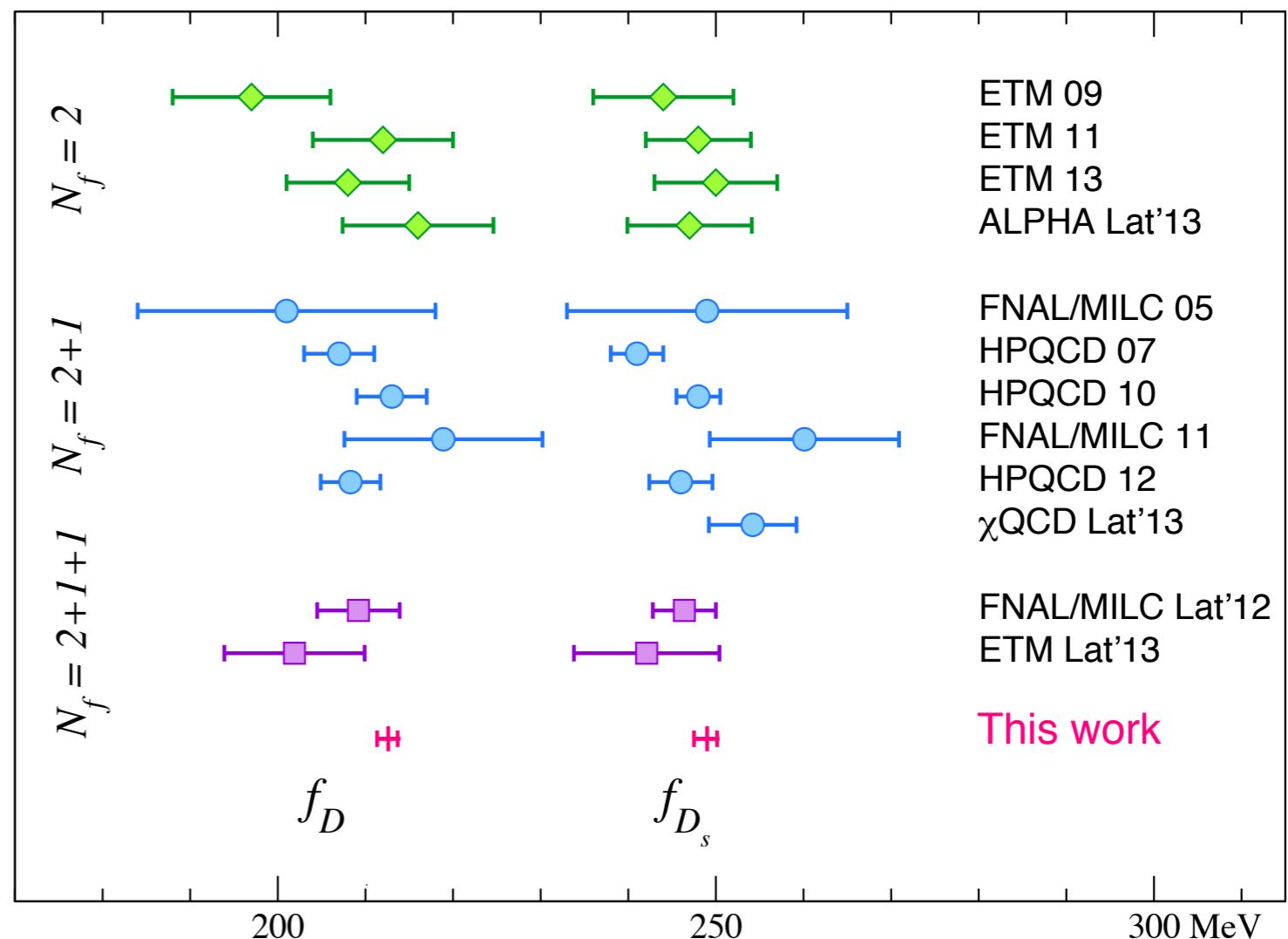
D_(s)-meson decay constants (2014)

- ♦ f_D & f_{D_s} can be used to obtain $|V_{cd}|$ and $|V_{cs}|$ via:

$$\Gamma(D_{(s)} \rightarrow \ell\nu) = \frac{G_F^2}{8\pi} \mathbf{f}_{D_{(s)}}^2 M_{D_{(s)}} \left(1 - \frac{m_\ell^2}{M_{D_{(s)}}^2}\right)^2 |\mathbf{V}_{cd(s)}|^2$$

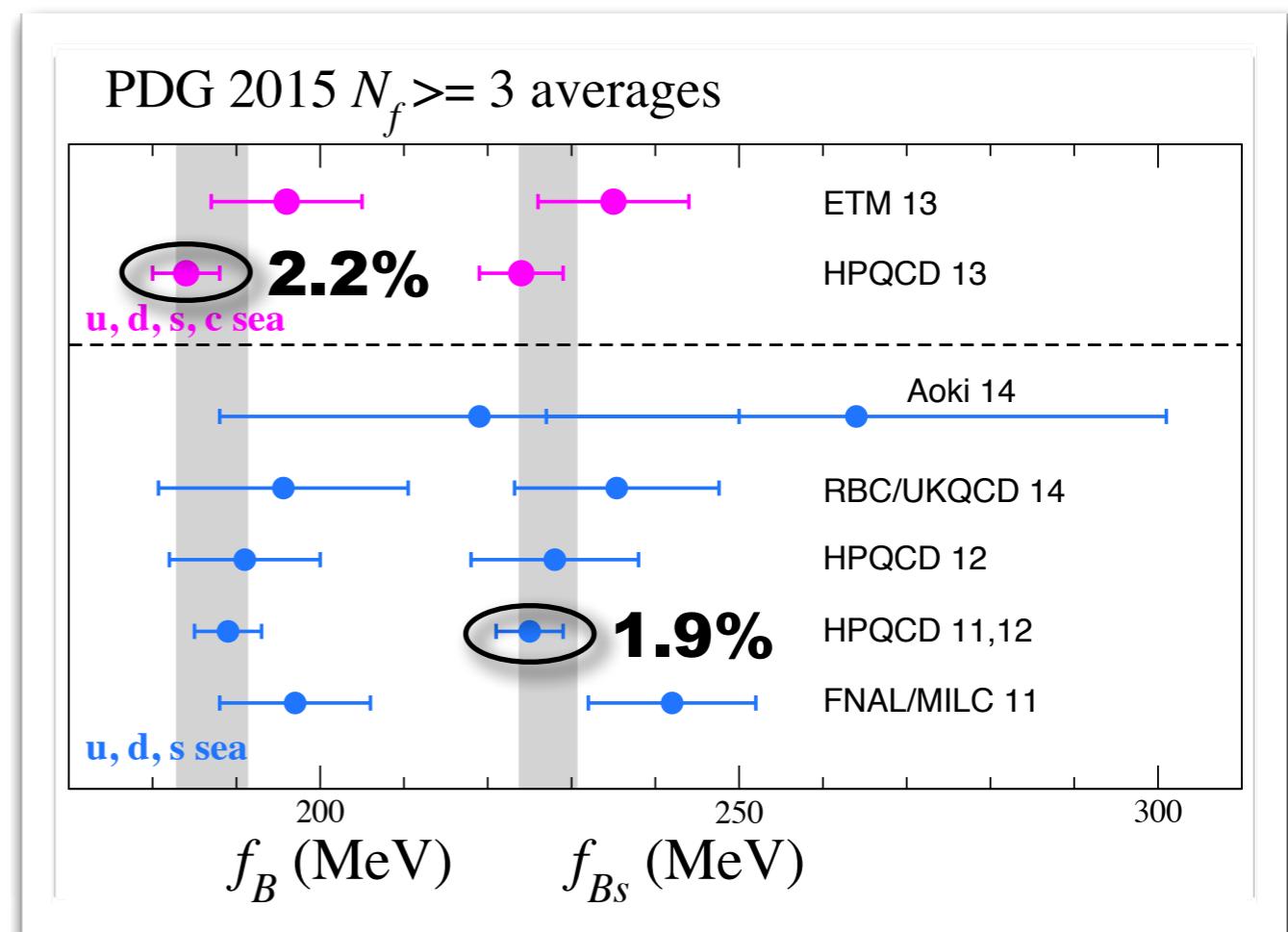
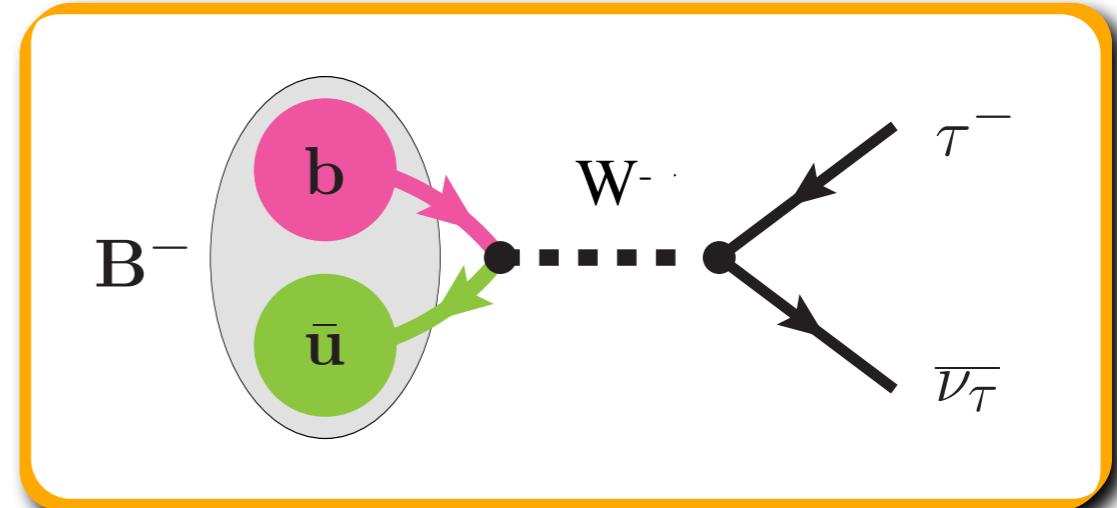
[Phys. Rev. D90 (2014) 7, 074509]

- ♦ Fermilab/MILC recently obtained first four-flavor results for f_D & f_{D_s} with physical pions
- ♦ HISQ action for u,d,s, and c quarks and fine lattice spacings eliminates renormalization error and leads to small discretization errors
- ♦ 0.5% errors on f_D & f_{D_s} and 0.3% error on f_{D_s}/f_D 2–4x more precise than previous best results



B-meson decay constants

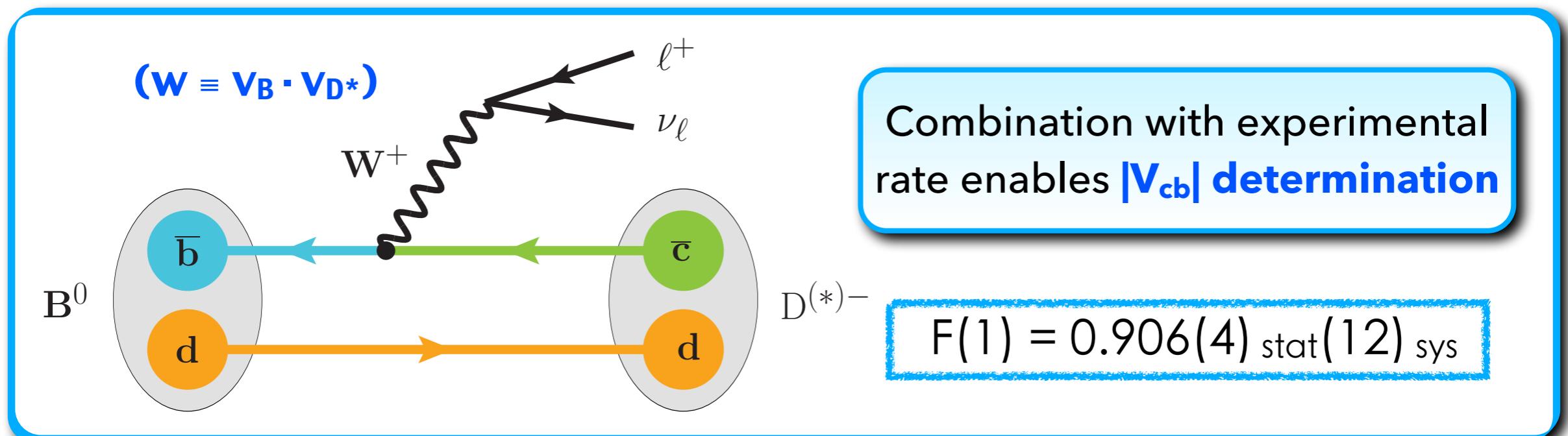
- ◆ Enter rates for leptonic decays $B \rightarrow \tau \bar{\nu}$ and $B_{d,s} \rightarrow \mu^+ \mu^-$
- ◆ Most precise f_{B_s} calculation employs **physical-mass pions** [HPQCD, PRL 110, 222003 (2013)]
→ **No chiral-extrapolation error**
- ◆ Most precise f_{B_s} uses **highly-improved staggered (HISQ) b-quark action** [HPQCD, PRD 85, 031503 (2012)]
 - ❖ Lattice axial current absolutely normalized → **no renormalization error**
- ◆ **Confirmation from several independent calculations** using different gauge-field configurations, light-, and b-quark actions [see PDG review by Rosner, Stone, & RV, arXiv:1509.02220]



$B \rightarrow D^* \ell \nu$ form factor @ zero recoil (2015)

[Bailey *et al.* [FNAL/MILC], PRD89 (2014) 11, 114504]

- ◆ Only need one normalization point from lattice QCD → **choose zero recoil ($w=1$) where it can be computed most precisely**
 - ❖ $F(1) \rightarrow 1$ in the static limit ($m_b = m_c \rightarrow \infty$) [**Isgur & Wise**], and **Luke's theorem** ensures that the leading heavy-quark corrections to $F(1)$ are of $\mathcal{O}(1/m_b^2, 1/m_c^2)$
 - ❖ Can compute form factor using double ratio of lattice three-point correlation functions in which statistical and systematic errors largely cancel
- ◆ New FNAL/MILC calculation with increased statistics, lighter quark masses, & finer lattice spacings → **1.4% precision on $F(1)$**

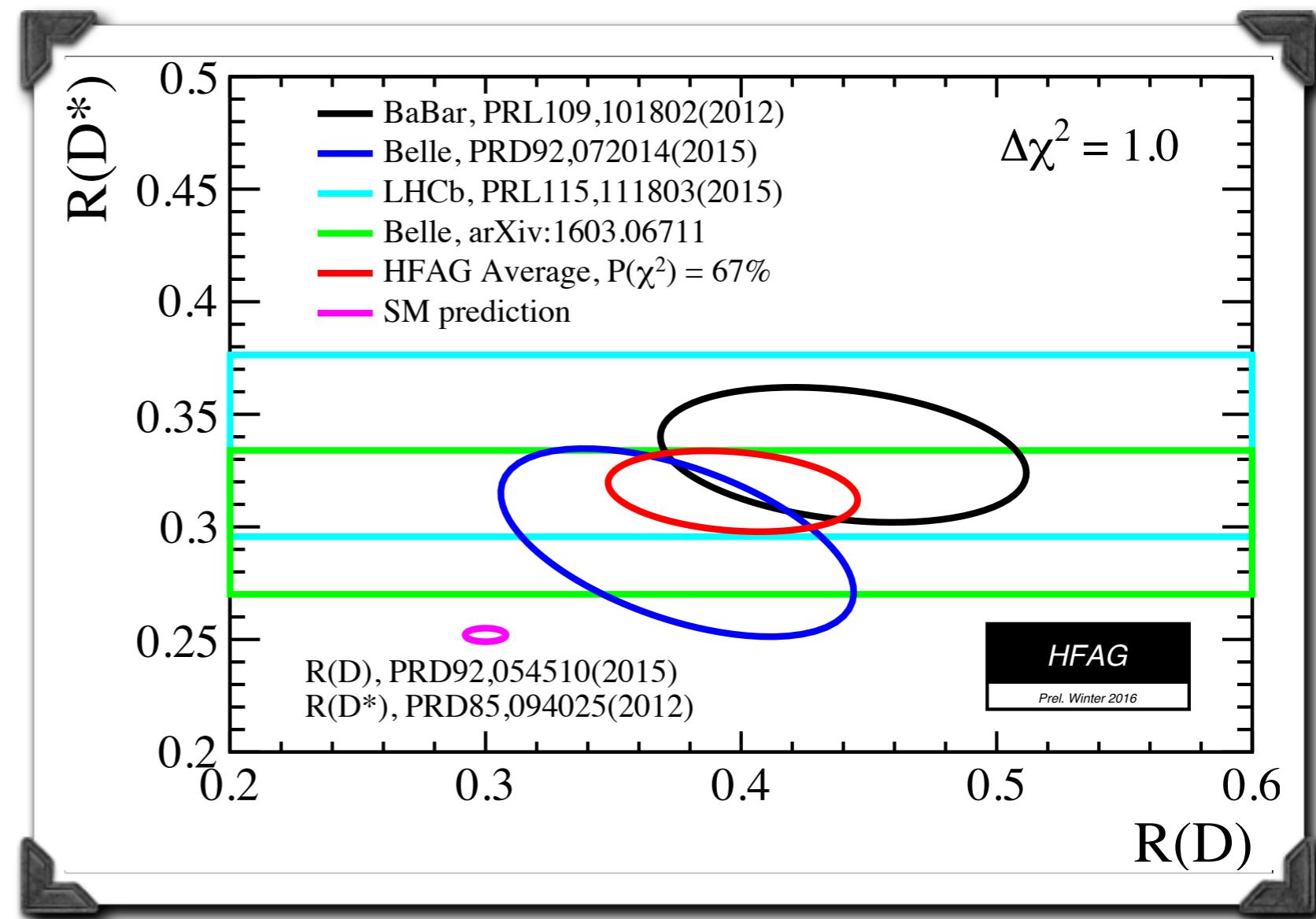


B \rightarrow D $\tau\nu$ and B \rightarrow D $^*\tau\nu$ excesses

- ◆ Ratios of B \rightarrow D $(^*)\ell\nu$ decays to τ over light lepton final states provide **especially clean new-physics probe**
 - ❖ Form-factor errors partially cancel in ratio
 - ❖ $|V_{cb}|$ cancels exactly

$$R(D) \equiv \frac{\mathcal{B}(B \rightarrow D\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D\ell\nu_\ell)}$$

$$R(D^*) \equiv \frac{\mathcal{B}(B \rightarrow D^*\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D\ell\nu_\ell)}$$

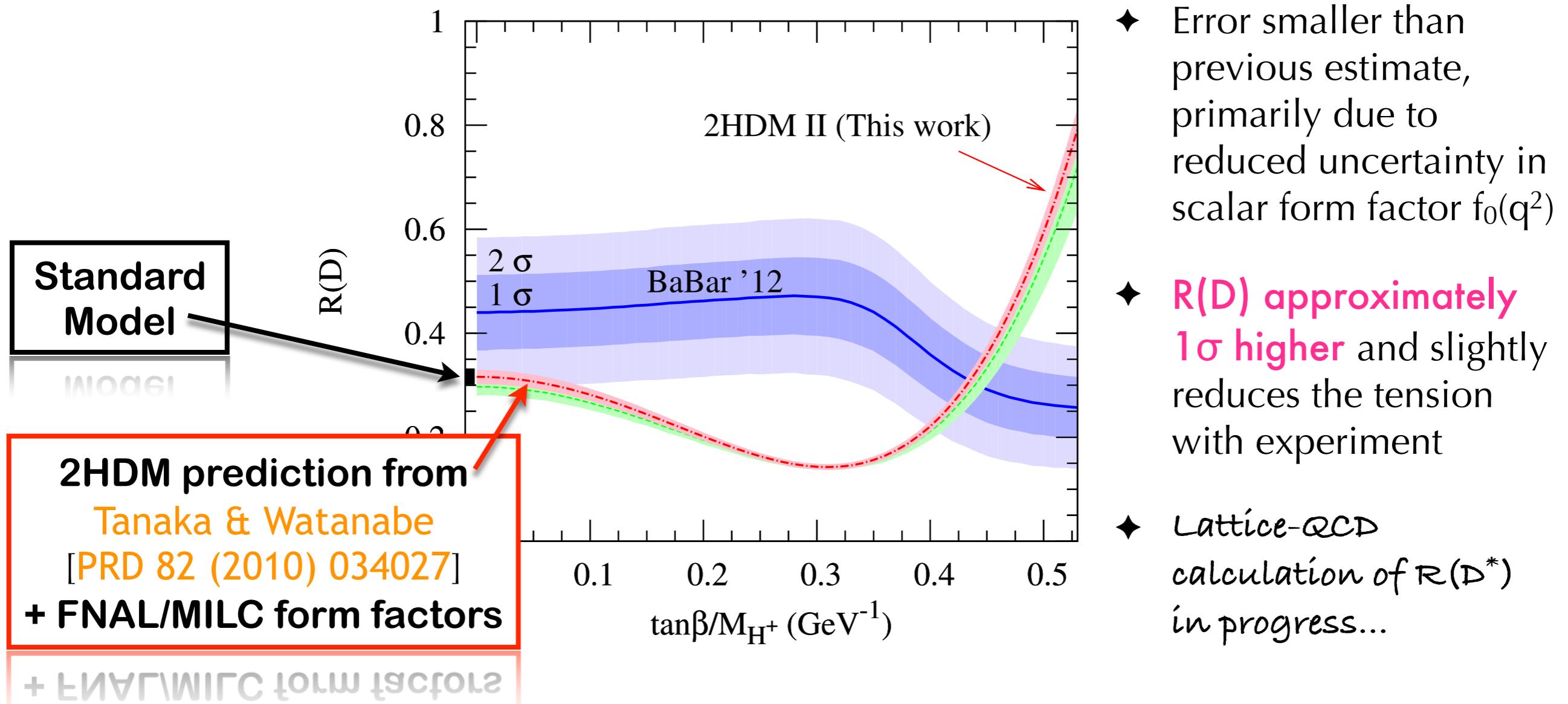


- ◆ In 2012, BaBar measured **R(D)** and **R(D *)** 3.4σ (combined) **above Standard-Model expectations**

confirmed recently by Belle and LHCb

Lattice-QCD calculation of R(D)

- ◆ FNAL/MILC Collaboration [PRD 85 (2012) 114502] responded quickly to the BABAR result and obtained the first Standard-Model calculation of R(D) from *ab initio* lattice-QCD using results for the form factors $f_+(q^2)$ and $f_0(q^2)$ at nonzero recoil from Phys. Rev. D85 (2012) 114502

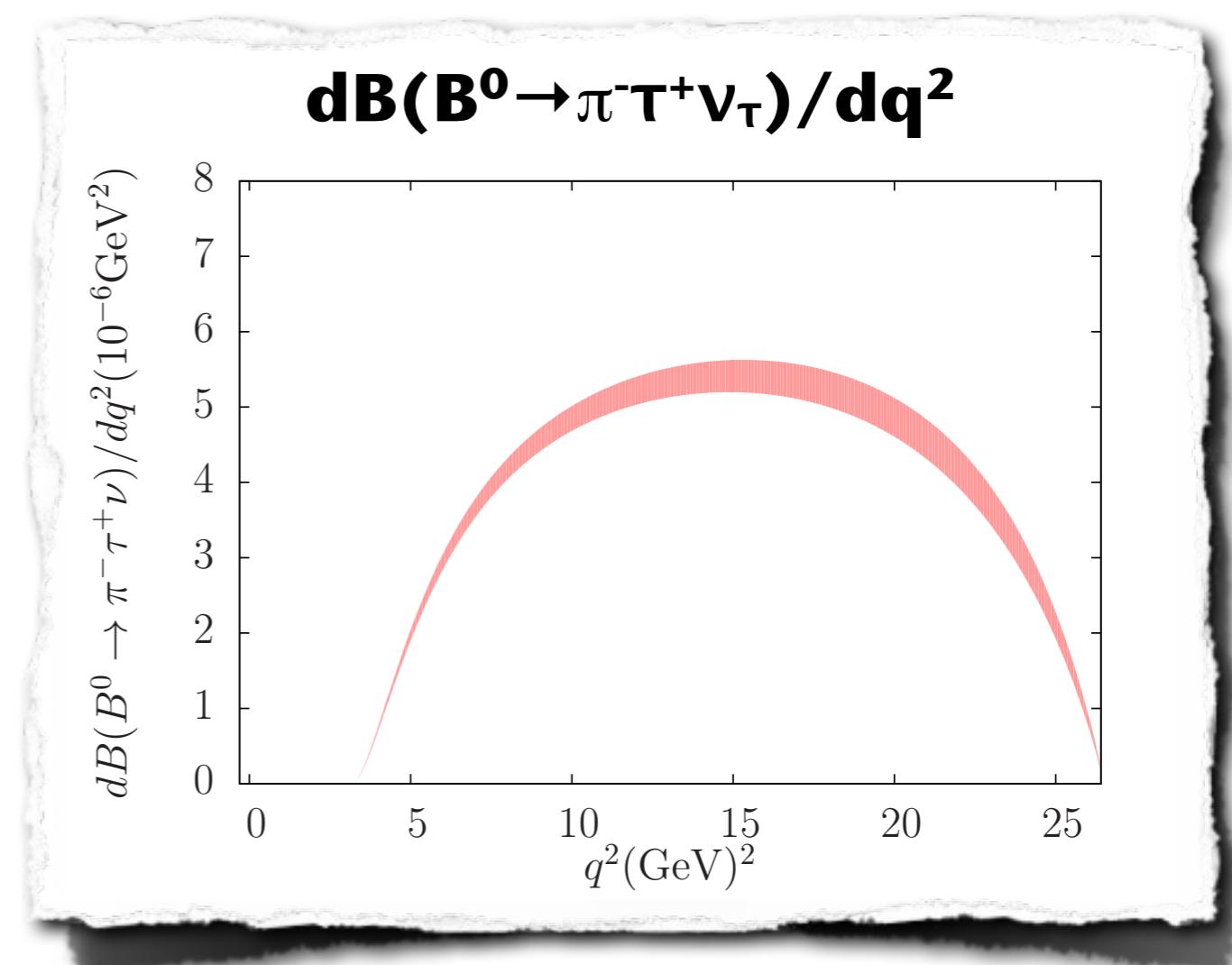


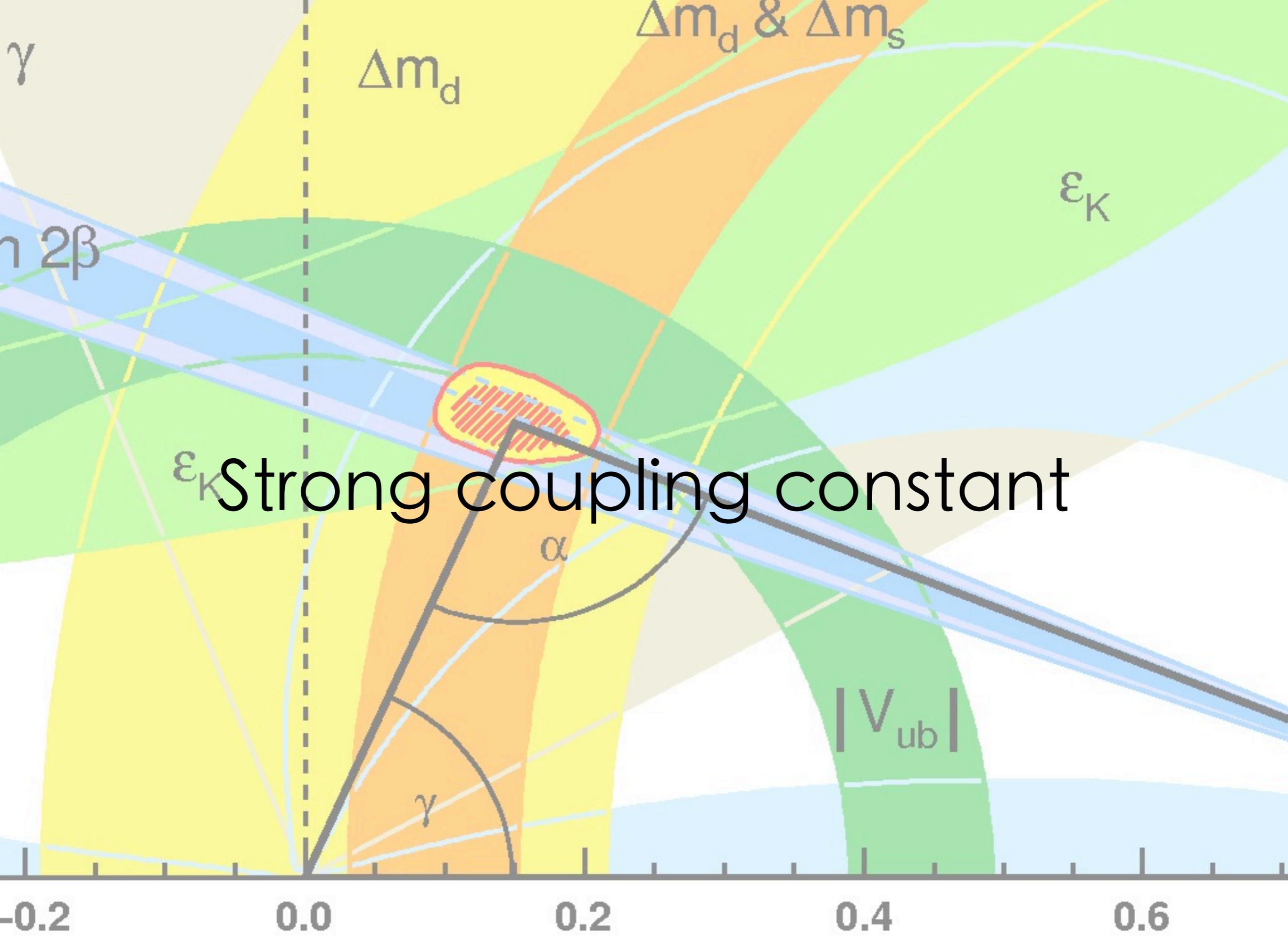
Prospects for $B \rightarrow \pi \tau \bar{\nu}_\tau$

- ♦ Experimental upper limit close to Standard-Model result → **Belle II well poised to measure total rate and test predicted q^2 dependence**
- ♦ Form-factor errors partially cancel in ratio of decay rate to light leptons, while $|V_{ub}|$ cancels exactly → **especially clean new-physics probe** insensitive to inclusive-exclusive tension

$$R(\pi) \equiv \frac{\mathcal{B}(B \rightarrow \pi \tau \bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow \pi \ell \bar{\nu}_\ell)} = 0.641(17)$$

$\text{BR}(B^0 \rightarrow \pi^- \tau^+ \bar{\nu}_\tau) \times 10^5$	
Standard Model	Belle [1509.06521]
9.35(38)	< 25 (90% CL)





Strong coupling constant

- ♦ Lattice average of $\alpha_s(M_Z)$ (RED) agrees with experimental determinations, and has smaller uncertainties

[Particle Data Group]

