Charmless non-leptonic $B$ decays - Theory

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:: Motivations

- Huge multiplicity of final states (2-body + multi-body), large data sets
- Important input in CKM studies (mostly angles)
- CP violation (SM and new physics)
- Non-trivial hadronic dynamics $\Rightarrow$ Perturbative and non-perturbative QCD methods
Non-leptonic $B$-decay Amplitudes

- Effective Hamiltonian at the hadronic scale $\mu \sim m_B$

$$\mathcal{H}_{\text{eff}} = -\mathcal{L}_{\text{QED+QCD}} + \sum_i C_i(\mu) \mathcal{O}_i(\mu)$$

- $C_i$ – Wilson coefficients (UV physics) → perturbation theory

Known to NNLL: Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06.

- $\mathcal{O}_i$ – Effective operators (IR physics) [e.g. $\mathcal{O} = (\bar{b}\gamma^\mu u)(\bar{u}\gamma_\mu d)$]

- Amplitudes:

$$\mathcal{A}(B \rightarrow M_1 M_2) = \sum_i C_i \langle M_1 M_2 | \mathcal{O}_i | B \rangle$$

The problem is to compute the operator matrix elements

→ non-perturbative, process dependent (non-universal)
:: OUTLINE

QCD FACTORIZATION

TWO-BODY DECAYS
- Perturbative calculation
- Tree and penguin decays
- Power corrections

THREE-BODY DECAYS
- Kinematics
- Factorization properties
- Hadronic input
- Quasi-two-body decays

CHALLENGES
Multiscale problem

- 3 scales: $m_b$, $\sqrt{m_b\Lambda_{QCD}}$, $\Lambda_{QCD}$.

- 4 modes: hard ($p_h^2 \sim m_b^2$)
  - hard-collinear ($p_{hc}^2 \sim m_b\Lambda_{QCD}$)
  - collinear and soft ($p_{c,\bar{c},s}^2 \sim \Lambda_{QCD}^2$)

1. **QCD → SCET-1**: Integrate out hard modes

   - $O = \int dt \tilde{F}^I(t)O^I(t) + \int dt ds \tilde{H}^{I\!I}(t,s)O^{I\!I}(t,s)$
   - $O^I(t) = [(\bar{\chi}W\bar{c})(tn-)...(W^\dagger_{\bar{c}}\chi)(0)] [(\bar{\xi}W_c)(0)...h_V(0)]$
   - $O^{I\!I}(t,s) = [(\bar{\chi}W\bar{c})(tn-)...(W^\dagger_{\bar{c}}\chi)(0)] [(\bar{\xi}W_c)(0)...(W^\dagger_{\bar{c}}iD_{\perp c}W_c)(sn_+)...h_V(0)]$

   - Decoupling of anti-collinear modes. $\langle M_2 | [(\bar{\chi}W\bar{c})(tn-)...(W^\dagger_{\bar{c}}\chi)(0)] | 0 \rangle \sim \phi_{M_2}$

2. **SCET-1 → SCET-2**: Integrate out hard-collinear modes

   - $\langle M_1 | [(\bar{\xi}W_c)(0)...(W^\dagger_{\bar{c}}iD_{\perp c}W_c)(sn_+)...h_V(0)] | B \rangle \sim J(s) \otimes \phi_B \otimes \phi_{M_1}$

   - Hard-collinear factorization fails for $O^I(t)$.

   - End-point divergences can be absorbed into form factor $F^{BM_1}$. 

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:: Factorization formula for $B \to M_1 M_2$

To leading power in the heavy-quark expansion

$$\langle M_1 M_2 | O | B \rangle = F^{BM_1} \int du \, T^I(u) \phi_{M_2}(u) + \int d\omega \, du \, dv \, T^{II}(\omega, u, v) \phi_B(\omega) \phi_{M_1}(u) \phi_{M_2}(v)$$

- Vertex corrections: $T^I(u) = 1 + O(\alpha_s)$
- Spectator scattering: $T^{II}(\omega, u, v) = O(\alpha_s)$ – (power suppressed if $M_1$ is heavy)
- Strong phases are perturbative [$O(\alpha_s)$] or power suppressed [$O(\Lambda/m_b)$].
:: Perturbative calculation

Two hard-scattering kernels for each operator insertion: $T^I$ (vertex), $T^\text{II}$ (spectator)

$$\langle M_1M_2|O_i|B\rangle \simeq F^{BM_1} T^I_i \otimes \phi_{M_2} + T^\text{II}_i \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

and two classes of topological amplitudes: “Tree”, “Penguin”.

<table>
<thead>
<tr>
<th></th>
<th>$T^I$, tree</th>
<th>$T^I$, penguin</th>
<th>$T^\text{II}$, tree</th>
<th>$T^\text{II}$, penguin</th>
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<td><strong>LO: $\mathcal{O}(1)$</strong></td>
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<td><strong>NLO: $\mathcal{O}(\alpha_s)$</strong></td>
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<td>BBNS ’99-’04</td>
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<td><strong>NNLO: $\mathcal{O}(\alpha_s^2)$</strong></td>
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<td>Bell ’07,’09 Beneke, Huber, Li ’09</td>
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<td>Kim, Yoon ’11, Bell Beneke, Huber, Li ’15</td>
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<td>Beneke, Jager ’05 Kivel ’06, Pilipp ’07</td>
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<td>Beneke, Jager ’06 Jain, Rothstein, Stewart ’07</td>
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:: Perturbative calculation

Motivation for NNLO: first correction to CP asymmetries

NNLO: non-trivial calculation

- $O(70)$ diagrams
- 2 loops, 3 scales ($m_b$, $u m_b$, $m_c$), 4 legs
- charm contribution has non-trivial threshold at $\bar{u} m_b^2 \gtrsim 4 m_c^2$

Missing NNLO pieces:

- 2-loop tree insertions of penguin operators $O_{3-6}$
  Similar to $O_{1,2}^{u'}$ calculation, easier than $O_{1,2}^c$
- 2-loop penguin insertions of penguin operators $O_{3-6}$
  Additional topology with “closed” quark loop.
Individual NNLO corrections large, but cancellations between FF and sp. terms.

Perturbative expansion well behaved (remember color suppression).

Color suppressed $a_2(\pi\pi)$ dominated by spectator scattering [larger uncertainty]. Can be large if $\lambda_B$ is small.

Relative phase $\text{arg}(C/T)$ remains small.
<table>
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<tr>
<th>&amp; $B^{-} \rightarrow \pi^{-} \pi^{0}$</th>
<th>$B^{0}_{d} \rightarrow \pi^{+} \pi^{-}$</th>
<th>$B^{0}_{d} \rightarrow \pi^{0} \pi^{0}$</th>
<th>$B^{-} \rightarrow \pi^{-} \rho^{0}$</th>
<th>$B^{-} \rightarrow \pi^{0} \rho^{-}$</th>
<th>$B^{0} \rightarrow \pi^{+} \rho^{-}$</th>
<th>$B^{0} \rightarrow \pi^{-} \rho^{+}$</th>
<th>$B^{0} \rightarrow \pi^{\pm} \rho^{\mp}$</th>
<th>$B^{0} \rightarrow \pi^{0} \rho^{0}$</th>
<th>$B^{-} \rightarrow \rho^{-}<em>{L} \rho</em>{L}^{0}$</th>
<th>$B^{0}<em>{d} \rightarrow \rho^{+}</em>{L} \rho_{L}^{-}$</th>
<th>$B^{0}<em>{d} \rightarrow \rho</em>{L}^{0} \rho_{L}^{0}$</th>
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<tbody>
<tr>
<td>Theory I</td>
<td>5.43(1.45) &amp; 7.37(1.22) &amp; 0.33(0.42) &amp; 8.68(2.71) &amp; 12.38(2.18) &amp; 17.80(2.10) &amp; 10.28(1.42) &amp; 28.08(3.82) &amp; 0.52(0.43) &amp; 18.42(3.92) &amp; 25.98(3.93) &amp; 0.39(0.36)</td>
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<td>Theory II</td>
<td>5.82(1.42) &amp; 5.70(1.16) &amp; 0.63(0.42) &amp; 9.84(2.54) &amp; 12.13(2.23) &amp; 13.76(2.18) &amp; 8.14(1.49) &amp; 21.90(3.06) &amp; 1.49(1.77) &amp; 19.06(4.59) &amp; 20.66(3.75) &amp; 1.05(1.62)</td>
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<td>Experiment</td>
<td>5.59(0.41) &amp; 5.16(0.22) &amp; 1.55(0.19) &amp; 8.3(1.2) &amp; 10.9(1.4) &amp; 15.7(1.8) &amp; 7.3(1.2) &amp; 23.0(2.3) &amp; 2.0(0.5) &amp; 22.8(1.8) &amp; 23.7(3.2) &amp; 0.55(0.22)</td>
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BELLE CKM 14: $0.90 \pm 0.16$

Theory I: $f_{+}^{B_{2}}(0) = 0.25 \pm 0.05$, $A_{0}^{B_{2}}(0) = 0.30 \pm 0.05$, $\lambda_{B}(1 \text{ GeV}) = 0.35 \pm 0.15 \text{ GeV}$

Theory II: $f_{+}^{B_{2}}(0) = 0.23 \pm 0.03$, $A_{0}^{B_{2}}(0) = 0.28 \pm 0.03$, $\lambda_{B}(1 \text{ GeV}) = 0.20^{+0.05}_{-0.00} \text{ GeV}$

First error $\gamma, |V_{cb}|, |V_{ub}|$ uncertainty not included. Second error from hadronic inputs. Brackets: form factor uncertainty not included.
Impact of $\lambda_B$

B-meson LCDA inverse moment: $\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega} \phi_B(\omega, \mu)$

**Dominant parametric uncertainty in QCDF**

- QCD sum rule estimate $\lambda_B(1\text{GeV}) \sim (460 \pm 110) \text{ MeV}$ [Braun, Ivanov, Korchemsky 03]
- $\pi\pi/\pi\rho/\rho\rho$ data seems to prefer $\sim 200 \text{ MeV}$?

$\lambda_B$ can be measured in $B \rightarrow \gamma\ell\nu$ decays

- state-of-the-art analysis (NLL, tree-level $1/m_b$) [Beneke, Rohrwild 11; Braun, Khodjamirian 12]
- Babar 09 data ($E_\gamma > 1\text{GeV}$) $\Rightarrow \lambda_B(1\text{GeV}) > 115 \text{ MeV}$
- Belle 15 data ($E_\gamma > 1\text{GeV}$) $\Rightarrow \lambda_B(1\text{GeV}) > 238 \text{ MeV}$
- good prospects to measure $\lambda_B$ at Belle-II
\begin{align*}
    a_4^u(\pi K)/10^{-2} &= -2.87 - [0.09 + 0.09i]v_t + [0.49 - 1.32i]p_1 - [0.32 + 0.71i]p_2 \\
    &+ \left[ \frac{r_{sp}}{0.434} \right] \left\{ [0.13]_{LO} + [0.14 + 0.12i]_{HV} - [0.01 - 0.05i]_{HP} + [0.07]_{tw3} \right\} \\
    &= (-2.46^{+0.49}_{-0.24}) + (-1.94^{+0.32}_{-0.20})i
\end{align*}

\begin{align*}
    a_4^c(\pi K)/10^{-2} &= -2.87 - [0.09 + 0.09i]v_t + [0.05 - 0.62i]p_1 - [0.77 + 0.50i]p_2 \\
    &+ \left[ \frac{r_{sp}}{0.434} \right] \left\{ [0.13]_{LO} + [0.14 + 0.12i]_{HV} + [0.01 + 0.03i]_{HP} + [0.07]_{tw3} \right\} \\
    &= (-3.34^{+0.43}_{-0.27}) + (-1.05^{+0.45}_{-0.36})i
\end{align*}

- Two-loop is 40% (15%) of the imaginary (real) part of $a_4^u(\pi K)$, and 50% (25%) in the case of $a_4^c(\pi K)$.
- Spectator-scattering not relevant.

Penguin decays (CPAs)

<table>
<thead>
<tr>
<th>$f$</th>
<th>NLO</th>
<th>NNLO</th>
<th>NNLO + LD</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^- K^0$</td>
<td>0.71 ±0.13±0.21</td>
<td>0.77 ±0.14±0.23</td>
<td>0.10 ±0.02±1.24</td>
<td>-1.7 ± 1.6</td>
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<tr>
<td>$\pi^0 K^-$</td>
<td>9.42 ±1.77±1.87</td>
<td>10.18 ±1.91±2.03</td>
<td>-1.17 ±0.22±20.00</td>
<td>4.0 ± 2.1</td>
</tr>
<tr>
<td>$\pi^+ K^-$</td>
<td>7.25 ±1.36±2.13</td>
<td>8.08 ±1.52±2.52</td>
<td>-3.23 ±0.61±19.17</td>
<td>-8.2 ± 0.6</td>
</tr>
<tr>
<td>$\pi^0 K^0$</td>
<td>-4.27 ±0.83±1.48</td>
<td>-4.33 ±0.84±3.29</td>
<td>-1.41 ±0.27±5.54</td>
<td>1 ± 10</td>
</tr>
<tr>
<td>$\delta(\pi K)$</td>
<td>2.17 ±0.40±1.39</td>
<td>2.10 ±0.39±1.40</td>
<td>2.07 ±0.39±2.76</td>
<td>12.2 ± 2.2</td>
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<tr>
<td>$\Delta(\pi K)$</td>
<td>-1.15 ±0.21±0.55</td>
<td>-0.88 ±0.16±1.31</td>
<td>-0.48 ±0.09±1.09</td>
<td>-14 ± 11</td>
</tr>
<tr>
<td>$\pi^- K^{*0}$</td>
<td>1.36 ±0.25±0.60</td>
<td>1.49 ±0.27±0.69</td>
<td>0.27 ±0.05±3.18</td>
<td>-3.8 ± 4.2</td>
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<tr>
<td>$\pi^0 K^{*-}$</td>
<td>13.85 ±2.40±5.84</td>
<td>18.16 ±3.11±7.79</td>
<td>-15.81 ±3.01±69.35</td>
<td>-6 ± 24</td>
</tr>
<tr>
<td>$\pi^+ K^{*-}$</td>
<td>11.18 ±2.00±9.75</td>
<td>19.70 ±3.37±10.54</td>
<td>-23.07 ±4.35±86.20</td>
<td>-23 ± 6</td>
</tr>
<tr>
<td>$\pi^0 K^{*0}$</td>
<td>-17.23 ±3.33±7.59</td>
<td>-15.11 ±2.93±12.34</td>
<td>2.16 ±0.39±17.53</td>
<td>-15 ± 13</td>
</tr>
<tr>
<td>$\delta(\pi K^*)$</td>
<td>2.68 ±0.72±5.44</td>
<td>-1.54 ±0.45±4.60</td>
<td>7.26 ±1.21±12.78</td>
<td>17 ± 25</td>
</tr>
<tr>
<td>$\Delta(\pi K^*)$</td>
<td>-7.18 ±1.38±3.38</td>
<td>-3.45 ±0.67±9.48</td>
<td>-1.02 ±0.19±4.32</td>
<td>-5 ± 45</td>
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</table>

- Overall, large experimental and/or theory uncertainties
- $\delta(\pi K)$ remains a puzzle.
Main limitation of QCDF approach, e.g. weak annihilation

\[ \sim \int d\omega \, du \, dv \, T(\omega, u, v) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u) \]

- convolutions diverge at endpoints \( \Rightarrow \) non-factorisation in SCET-2
- currently modelled with arbitrary soft rescattering phase

Pure annihilation decays

\[
\begin{align*}
10^6 \text{Br}(B_d \to K^+ K^-) &= 0.13 \pm 0.05 \\
10^6 \text{Br}(B_s \to \pi^+ \pi^-) &= 0.76 \pm 0.13
\end{align*}
\]

\( \Rightarrow \) extract weak annihilation amplitudes from data

- Or use “clean” combinations, e.g. \( \Delta = T - P \) in penguin mediated decays

[Descotes-Genon, Matias, JV '06,'07,'11]

[Wang, Zhu 13; Bobeth, Gorbahn, Vickers 14; Chang, Sun, Yang, Li 14]
:: OUTLINE

QCD FACTORIZATION

TWO-BODY DECAYS
  Perturbative calculation
  Tree and penguin decays
  Power corrections

THREE-BODY DECAYS
  Kinematics
  Factorization properties
  Hadronic input
  Quasi-two-body decays

CHALLENGES
Three-body $B$ decays

- Model-independent treatment of vector resonances:
  - $B \to \rho \ell \nu \quad \rightarrow \quad B \to [\pi \pi] \ell \nu$
  - $B \to K^* \ell \ell \quad \rightarrow \quad B \to [K \pi] \ell \ell$
  - Finite-width effects, interference (S-wave pollution, etc.)

- More complicated kinematics $\rightarrow$ more observables

- Larger phase space: different kinematic regimes, different theory descriptions

- Kinematic distributions $\rightarrow$ tests of EFT expansions & Factorization

- $E$-dependent rescattering effects $\rightarrow$ large strong phases
  $\quad \rightarrow$ Large localized CP asymmetries

- Huge data sets

- Many applications: CKM parameters, spectroscopy, etc.
:: Kinematics

\[ B^-(p) \to \pi^-(k_1)\pi^+(k_2)\pi^-(k_3) \]

Kinematics completely specified in terms of 2 invariants:

\[ p^2 = m_B^2, \quad k_i^2 = 0, \quad s_{ij} \equiv \frac{(k_i + k_j)^2}{m_B^2}, \quad s_{12} + s_{13} + s_{23} = 1 \]

For example \( s_{12} \) and \( s_{23} \):
:: Regions of phase space

Region III

Region IIa

Region I

$S_{+/-}^{high}$

$S_{+/-}^{low}$
Three collinear directions $n_1$, $n_2$, $n_3$, disconnected at the leading power.

\[
\langle \pi^- n_1 \pi^- n_2 \pi^- n_3 | O_i | B \rangle = \langle \pi^- n_3 | \bar{d} n_3 \Gamma h \nu | B \rangle 
\times \int dudv \ T_i(u, v) \langle \pi^- n_1 | \bar{d} n_1 (\bar{u}) \Gamma_1 u n_1 (u) | 0 \rangle \langle \pi^+ n_2 | \bar{u} n_2 (\bar{v}) \Gamma_2 d n_2 (v) | 0 \rangle
\sim F^{B \to \pi} \ T_i \otimes \phi_\pi \otimes \phi_\pi
\]

Power $(1/m_b^2)$ & $\alpha_s$ suppressed with respect to two-body.

At leading order/power/twist all convolutions are finite $\Rightarrow$ factorization $\checkmark$

Proof of factorization to $O(\alpha_s^2)$ required (HS spectator terms [SCET-II])
:: Region I: Extrapolating towards \((\pi^- \pi^-)\) Edge

* No resonances \(\rightarrow\) perturbative result should be regular.

* Regularity also expected from absence of soft propagators in QCD:

* We confirm this expectation:

\[
\frac{d\Gamma}{ds_{--} ds_{+-}} \simeq 0.84 \Gamma_0 f_+ \left(\frac{m_B^2}{2}\right)^2 + \mathcal{O}(s_{--})
\]
:: Region I: Extrapolating towards \((\pi^+ \pi^-)\) Edge

* Resonances \((\rho, \omega, \rho', \ldots)\) → perturbative result should break down.

* Non-regularity also expected from presence of soft propagators in QCD:

\[
\begin{align*}
\text{\B}^- &\rightarrow \pi^+ \pi^- + \text{soft terms} \\
\text{\B}^- &\rightarrow \text{hadrons} + \text{soft terms}
\end{align*}
\]

* We confirm this expectation:

\[
\frac{d\Gamma}{ds_{++} ds_{--}} \simeq \frac{0.38}{s_{+-}} \Gamma_0 f_+(0)^2 + \text{regular}
\]
Breakdown of factorization at resonant edges requires new NP functions.

3-body decay resembles 2-body, but with new \((\pi\pi)\) “compound object”:

Operators are the same as in 2-body, but final states different:

\[
\langle \pi_n^- \pi_n^+ \pi_n^- | O | B \rangle = \langle \pi_n^- | h_v \Gamma \xi_n | B \rangle \times \int dz \ T_1(z) \langle \pi_n^- \pi_n^+ | \bar{\chi}_n(z \bar{n}) \Gamma' \chi_n(0) | 0 \rangle
\]

\[
+ \langle \pi_n^- \pi_n^+ | h_v \Gamma \xi_n | B \rangle \times \int dz \ T_2(z) \langle \pi_n^- | \bar{\chi}_n(zn) \Gamma' \chi_n(0) | 0 \rangle
\]

\[
\sim F^{B\to\pi} \ T_1 \otimes \phi_{\pi\pi} + F^{B\to\pi\pi} \ T_2 \otimes \phi_{\pi}
\]

New non-perturbative input:

- Generalized Distribution Amplitudes (GDAs) [Diehl, Polyakov, Gousset, Pire...]
- Generalized Form Factors (GFFs) [Faller, Feldmann, Khodjamirian, Mannel, van Dyk...]
**GDAs from data**

- **Definition:** \[ s = (k_1 + k_2)^2, \quad k_1 = \zeta k_{12}, \quad k_2 = \bar{\zeta} k_{12} \]

\[
\phi_{\pi\pi}^q(z, \zeta, s) = \int \frac{dx^-}{2\pi} e^{iz(k_{12}^+ x^-)} \langle \pi^+ (k_1) \pi^- (k_2) | \bar{q}(x^- n^-) \gamma_+ q(0) | 0 \rangle
\]

- **Normalization (local correlator):**

\[
\int dz \phi_{\pi\pi}(z, \zeta, s) = (2\zeta - 1) F_\pi(s) \quad \text{(pion time-like FF)}
\]

- **\( F_\pi(s) \):** Data \((e^+ e^- \rightarrow \pi\pi(\gamma) \ [BaBar]) + \) Theory \(((R)\chi PT, \text{Asymptotics}...\)

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**Figure:**

- A graph showing \( F_\pi^R \) versus \( \sqrt{s} \) (GeV) with data points and a fitted curve.
- Another graph showing \( \delta \) versus \( \sqrt{s} \) (GeV) with a rise and slight oscillation.

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**References:**

1. Krankl, Mannel, J.V. 2015

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**Notes:**

- Javier Virto (Uni Bern)
- Charmless non-leptonic B decays
- June 6, 2016
Use Light-cone sum rule with $2\pi$ distribution amplitudes:

**Correlation function**

$$\Pi^{(5)}(k^2, q, q \cdot \bar{k}) = -im_b^2 \int d^4x e^{iq \cdot x} \langle \pi(k_1)\pi(k_2) | T(\bar{d}(x)\gamma_5 b(x), \bar{b}(0)\gamma_5 u(0)) | 0 \rangle$$

**Unitarity relation**

$$\Pi^{(5)}(k^2, q, q \cdot \bar{k}) = \frac{\langle \pi\pi | \bar{d}im_b\gamma_5 b | B \rangle \langle B | \bar{b}im_b\gamma_5 u | 0 \rangle}{m_B^2 - p^2} + \ldots$$

**Dispersion relation + OPE + Borel + duality**

$$F_t(k^2, q^2, \zeta) = \frac{m_b^2 \sqrt{q^2}}{\sqrt{2} f_B m_B^2} \int_{u_0}^{1} \frac{du}{u^2} \left( m_b^2 - u^2 k^2 \right) \phi_{\parallel}(u, \zeta, k^2) e^{\frac{m_b^2}{M^2} - \frac{m_b^2 - \bar{u}q^2 + u\bar{u}k^2}{uM^2}}$$

**Also other LCSRs for $F_\perp$ and $F_\parallel$**
\[ B \rightarrow \pi\pi \text{ form factors from LCSRs} \]

Hambrock, Khodjamirian 2015

→ Use Light-cone sum rule with 2π distribution amplitudes:

▶ **Sample result:** \( \rho \) contribution to the total vector form factor

\[
\frac{[F^{(\ell=1)}(q^2, k_{min}^2)](\rho)}{[F_{LCSR}^{(\ell=1)}(q^2, k_{min}^2)]} \]

\[
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\[ q^2 [\text{GeV}^2] \]

\[ q^2 [\text{GeV}^2] \]
\[ B \rightarrow \pi\pi \text{ form factors from LCSRs} \]

Cheng, Khodjamirian, JV w.i.p

→ Use Light-cone sum rule with \( B \)-meson distribution amplitudes:

\[ F_\mu(k, q) = i \int d^4x e^{ik\cdot x} \langle 0 | \{ \bar{d}(x)\gamma_\mu u(x), im_b\bar{u}(0)\gamma_5 b(0) \} | \bar{B}^0(q + k) \rangle \]

\[ 2\text{Im}F_\mu(k, q) = im_b \int d\tau_2 \tau_1 \langle 0 | \bar{d} \gamma_\mu | \pi(k_1)\pi(k_2) \rangle \langle \pi(k_1)\pi(k_2) | \bar{u} \gamma_5 b | \bar{B}^0(q + k) \rangle + \cdots \]

\[ \int_{s_0^2/\pi^2}^{2\pi} ds e^{-s/M^2} s \sqrt{q^2} \left[ \beta_\pi(s) \right]^2 F_\pi^*(s) F_t^{(1)}(s, q^2) = -f_B m_B^2 m_b \left\{ \int_0^{\sigma_0^2/\pi^2} d\sigma e^{-s(\sigma, q^2)/M^2} \times \right. \]

\[ \left. \times \left[ \frac{\sigma}{\bar{\sigma}} \phi_+^B(\sigma m_B) - \frac{\sigma}{\bar{\sigma}} \left[ \phi_+^B(\sigma m_B) - \phi_-^B(\sigma m_B) \right] - \frac{1}{\bar{\sigma} m_B} \Phi_\pm^B(\sigma m_B) \right] + \Delta A_0^{BV}(q^2, \sigma_0^2/\pi^2, M^2) \right\} \]
Use Light-cone sum rule with $B$-meson distribution amplitudes

- Also other LCSRs for vector $F_\perp$ and axial $F_0, F_\parallel$ form factors.

- In the single-pole approximation ($\rho$-dominance in the narrow width approximation)
  \[ \Rightarrow \] one recovers analytically the results for the $B \to \rho$ form factors $V, A_0, A_1, A_2$.

- Include finite-width effects and contributions from $\rho', \rho''$ resonances.
\[ B \to (\pi\pi)_{\rho}\pi \]

\* Leading order amplitude:

\[ A|_{s_{+-} \ll 1} = \frac{G_F}{\sqrt{2}} \left[ 4m_B^2 f_0(s_{+-})(2\zeta - 1)F_\pi(s_{+-})(a_2 + a_4) + f_\pi m_\pi (a_1 - a_4)F_t(\zeta, s_{+-}) \right] \]

\* Integrating around the \( \rho \):

\[ BR(B^- \to \rho \pi^-) \simeq \int_0^1 ds_{++} \int_{s_{\rho}^-}^{s_{\rho}^+} ds_{+-} \frac{\tau_B m_B |A|^2}{32(2\pi)^3} \]

with \( s_{\rho}^{\pm} = (m_\rho \pm n\Gamma_\rho)^2 / m_B^2 \)

\[ BR(B^+ \to \rho \pi^+) \simeq 9.4 \cdot 10^{-6} \quad (n = 0.5) \]
\[ BR(B^+ \to \rho \pi^+) \simeq 12.8 \cdot 10^{-6} \quad (n = 1) \]
\[ BR(B^+ \to \rho \pi^+) \simeq 14.1 \cdot 10^{-6} \quad (n = 1.5) \]

\[ BR(B^+ \to \rho \pi^+)_{\text{EXP}} = (8.3 \pm 1.2) \cdot 10^{-6} \]
\[ BR(B^+ \to \rho \pi^+)_{\text{QCDF}} = (11.9^{+7.8}_{-6.1}) \cdot 10^{-6} \]
Merging Regions: How large should $m_B$ be? ($\phi_{\pi\pi}$ term)

- $m_B \approx 20\text{GeV}$
- $m_B \approx 15\text{GeV}$
- $m_B \approx 10\text{GeV}$
- $m_B \approx 5\text{GeV}$
OUTLINE

QCD FACTORIZATION

TWO-BODY DECAYS
- Perturbative calculation
- Tree and penguin decays
- Power corrections

THREE-BODY DECAYS
- Kinematics
- Factorization properties
- Hadronic input
- Quasi-two-body decays

CHALLENGES
:: Summary and Challenges

Two body decays

▶ NNLO: End of the road for perturbative calculations
▶ Mostly ok, except for a few cases.
    Large uncertainty from $\lambda_B$ and power corrections.
▶ Challenge: Precise determination of $\lambda_B$ from $B \to \gamma\ell\nu$ (Belle-II).

Three body decays

▶ Lots of data, great potential.
▶ Can be studied within QCDF. Need $2\pi$ LCDA’s and $B \to MM$ form factors.
▶ $B \to VP$ : include finite-width effects and contributions from excited resonances.
▶ Challenge: Full analysis at NLO, including CPV.
▶ Challenge: Soft corners need alternative treatment. These regions include interferences from “crossed” resonances, potentially interesting for localized CP asymmetries.
Backup Slides
Kinematics: $B^- (p) \to \pi^+ (k_1) \pi^- (k_2) \ell^- (q_1) \bar{\nu} (q_2)$

\[
k^2 = (k_1 + k_2)^2, \quad q^2 = (q_1 + q_2)^2, \quad 2q \cdot (k_1 - k_2) = \beta_\pi \sqrt{\lambda} \cos \theta_\pi
\]

\[\mathcal{L} = \mathcal{L}_{QED + QCD} + C_{LL} [\bar{u} \gamma^\mu P_L b] [\bar{\ell} \gamma^\mu P_L \nu_\ell] + \cdots \]

\[\mathcal{A} = C_{LL} \langle \ell \bar{\nu} | \bar{\ell} \gamma_\mu P_L \nu_\ell | 0 \rangle \langle \pi^+ \pi^- | \bar{u} \gamma^\mu P_L b | B^- \rangle = C_{LL} \mathcal{F}_\mu \bar{u} \ell \gamma_\mu \nu_\nu \]

$\to \mathcal{F}_\mu: \ B \to \pi\pi$ form factor (one axial, three vector invariant FFs):

\[
\varepsilon(q, 0)_\mu^* \langle \pi \pi | \bar{u} \gamma^\mu P_L b | B \rangle = F_0
\]

\[
\varepsilon(q, t)_\mu^* \langle \pi \pi | \bar{u} \gamma^\mu P_L b | B \rangle = F_t
\]

\[
\varepsilon(q, \pm)_\mu^* \langle \pi \pi | \bar{u} \gamma^\mu P_L b | B \rangle = \beta_\pi \sin \theta_\pi e^{\pm i \phi} (F_\perp + F_\parallel) / \sqrt{2}
\]

where $F_i = F_i (q^2, k^2, \theta_\pi) = \sum_\ell F_i^{(\ell)} (q^2, k^2) P_\ell (\cos \theta_\pi)$ [partial waves in $\pi\pi$]
Factorization at large $k^2$:

$$F_i = f_{B\pi} \, T_{i}^I \otimes \phi_{\pi} + T_{i}^{II} \otimes \phi_{\pi} \otimes \phi_{\pi} \otimes \phi_{B}$$

$$\langle \pi^+(k_1)\pi^-(k_2)|\bar{\psi}_u \Gamma \psi_b|B^-(p)\rangle$$

$$= \frac{2\pi f_{\pi} \xi_{\pi}(E_2; \mu)}{k^2} \int_0^1 du \phi_{\pi}(u, \mu) T_{I}^I(u, \ldots; \mu)$$

$$+ \int_0^1 du \int_0^1 dv \int_0^\infty \frac{d\omega}{\omega} \phi_{\pi}(u; \mu) \phi_{\pi}(v; \mu) \phi_B^+(\omega; \mu) T_{II}^I(u, v, \omega, \ldots; \mu)$$

+ power corrections.

$\xi_{\pi}$ denotes the universal non-factorizable $B \rightarrow \pi$ form factor in SCET