# Introduction to the SM (1)

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# General remarks

- Please ask questions!
- Email: yg73@cornell.edu
- The plan:
  - Intro to model building and the SM
  - The gauge sector and SSB
  - Flavors: quarks and leptons

# Why are we here?

# What is HEP?

**Find the basic laws of Nature**

Under some assumption this translated into

$$\mathcal{L} = ?$$

- We have quite a good answer
- It is very elegant, it is based on simple axioms and symmetries
- The generalized coordinates are fields
- We use particles to answer this question

# QFT: a reminder

# What is a field?

- In math: something that has a value in each point in space
- For physics we care about $\phi(x,t)$
  - Temperature (scalar field)
  - Wind (vector field)
  - Density of people (?)
  - Electric and magnetic fields (vector fields)
- How good is the field description of each of these?
- In physics a field used to be associated with a source, but now we know that fields are fundamental

# How to solve a field theory?

Recall mechanics: We need to find $x(t)$

- $x(t)$ minimizes the action, $S$. This is an axiom
- There is one action for the whole system

$$S = \int_{t_1}^{t_2} L(x, \dot{x}) dt$$

- The solution is given by the E-L equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$$

- Once we know $L$ we can find $x(t)$ up to initial conditions
- Mechanics is reduced to the question “what is $L$?”

# Solving field theories

A field theory: mechanics with many time dimensions

- We need to find the $\phi(x,t) \equiv \phi(t_\mu)$
- $\phi$ is the generalized coordinate, while $x$ and $t$ are treated the same
- We still need to minimize $S$

$$S = \int \mathcal{L}[\phi(t_\mu), \partial_\mu \phi(t_\mu)] d^4 t \quad \Rightarrow \quad \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial \left( \partial_\mu \phi \right)} \right) = \frac{\partial \mathcal{L}}{\partial \phi}$$

- We have a way to solve field theories!
- Field theory is reduced to the question "what is $\mathcal{L}$?"

# The harmonic oscillator

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- Because we really care about pendulums?

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- Because we really care about springs?
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Because almost any function around its minimum can be approximated as a harmonic function!

# Coupled oscillators

- There are normal modes, and they are are not “local”
- The energy of each mode is conserved
- Once we keep non-harmonic terms energy moves between modes

$$V(x,y) = \frac{k_1 x^2}{2} + \frac{k_2 y^2}{2} + \alpha x^2 y$$

- What determines the rate of energy transfer?
- Assuming small oscillations, fields are coupled oscillators!

# Quantum mechanics

# The quantum SHO

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} \qquad E_n = (n + 1/2)\hbar\omega$$

Consider a system with 2 DOFs and same mass with

$$V(x, y) = \frac{kx^2}{2} + \frac{ky^2}{2} + \alpha xy$$

The normal modes are

$$q_\pm = \frac{1}{\sqrt{2}}(x \pm y) \qquad \omega_\pm^2 = \frac{k \pm \alpha}{m}$$

# Quantum coupled oscillators

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2) What we will have when we add a $\lambda x^2 y$ term

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Energy can transfer between the modes!

# SHO and photons

I have two questions:

- What is the energy that it takes to excite a SHO by one level?
- What is the energy of the photon?

**Excitations of SHOs are particles**

- Feynman diagrams are a tool that use perturbation theory to calculate probably of transition

# A short summary

- QFT is our framework to understand the basic laws of Nature
- Given $\mathcal{L}$ we use perturbation theory and Feynman diagrams to calculate
- Our aim is to find $\mathcal{L}$

# Symmetries

# Symmetries and representations

Example: 3d real space in classical mechanics with several particles with coordinated $a$, $b$, $c$,...

- We require that $L$ is invariant under rotation
- We construct invariants from the DOFs. They are called singlets or scalars

$$C_s = a_i b_i, \qquad C_a = \epsilon_{ijk} a_i b_j c_k$$

- We then require that $L$ is a function of the $C_s$ and $C_a$ and their time derivative

# Generalizations

- In mechanics, $\vec{r}$ lives in 3d real space and is a vector
- Fields do not live in real space. They live in some mathematical space
- They also do not have to be vectors, but can be scalars or tensors (representation)
- The idea is similar to what we did in mechanics
  - We require $\mathcal{L}$ to be invariant under rotation in that mathematical space
  - Thus $\mathcal{L}$ depends only on combinations of fields that form singlets
- All this is related to a subject called Lie groups
- We usually care about $SO(N)$, $SU(N)$ and $U(1)$

# Local symmetries

# Local symmetry

Basic idea: rotations depend on $x_\mu$

- It is kind of logical and we think that all imposed symmetries in Nature are local
- The kinetic term $|\partial_\mu \phi|^2$ in not invariant
- We want a kinetic term (why?)
- We can save the kinetic term if we add a field that is
  - Massless
  - Spin 1

**Local symmetries $\Rightarrow$ force fields**

# Accidental symmetries

- We only impose local symmetries
- Yet, because we truncate the expansion, we can get symmetries as output
- They are global, and are called accidental
- Example: $U(1)$ with $X(q=1)$ and $Y(q=-4)$

$$V(XX^*, YY^*) \Rightarrow U(1)_X \times U(1)_Y$$

  - $X^4Y$ breaks this symmetry
- In the SM baryon and lepton numbers are accidental symmetries

# The SM

# How to “built” Lagrangians

- $\mathcal{L}$ is:
  - The most general one that is invariant under some symmetries (democratic principle)
  - We work up to some order (usually 4)
- We need the following input:
  - What are the symmetries we impose
  - What DOFs we have and how they transform under the symmetry
- The output is
  - A Lagrangian with $N$ parameters
  - We need to measure its parameters and test it

# The SM

Input: Symmetries and fields

- Symmetry: 4d Poincare and

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

- Fields:
  - 3 copies of QUDLE fermions

$$Q_L(3,2)_{1/6} \quad U_R(3,1)_{2/3} \quad D_R(3,1)_{-1/3}$$
$$L_L(1,2)_{-1/2} \quad E_R(1,1)_{-1}$$

  - One scalar

$$\phi(1,2)_{+1/2}$$

# Then Nature is described by

- Output: the most general $\mathcal{L}$ up to dim 4

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{\phi} + \mathcal{L}_{\psi} + \mathcal{L}_{Yukawa}$$

- This model has a $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$ accidental symmetry
- Initial set of measurements to find the parameters
  - SSB: $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$
  - Fermion masses, gauge couplings and mixing angles

**The SM pass (almost) all of it tests**

# Some summary

- We have rules to built Lagrangians based on symmetries
- We use them to get the SM
- Tomorrow we will discuss the SM in more details