Muon Physics

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Citation: W.-M. Yao et al. (Particle Data Group), J. Phys. G 33, 1 (2006) (URL: http://pdg.lbl.gov)

“Who Ordered That?”

The muon is the best known unstable fundamental particle.

The muon is also the heaviest fundamental particle we can directly work with. It is a unique, priceless resource for physicists.

ANS: “We did!”

μ $\qquad J = \frac{1}{2}$

μ MASS (atomic mass units u)

The primary determination of a muon's mass comes from measuring the ratio of the mass to that of a nucleus, so that the result is obtained in u (atomic mass units). The conversion factor to MeV is more uncertain than the mass of the muon in u. In this datablock we give the result in u, and in the following datablock in MeV.

VALUE (u)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.1134289264±0.0000000030	MOHR	05	RVUE	2002 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.1134289168±0.0000000034	[1] MOHR	99	RVUE	1998 CODATA value
0.113428913 ±0.000000017	[2] COHEN	87	RVUE	1986 CODATA value

[1] MOHR 99 make use of other 1998 CODATA entries below.
[2] COHEN 87 make use of other 1986 CODATA entries below.

μ MASS

2002 CODATA gives the conversion factor from u (atomic mass units, see the above datablock) as 931.494 043 (80). Earlier values use the then-current conversion factor. The conversion error dominates the masses given below.

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
105.6583692±0.0000094	MOHR	05	RVUE		2002 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •					
105.6583568±0.0000052	MOHR	99	RVUE		1998 CODATA va
105.658353 ±0.000016	[3] COHEN	87	RVUE		1986 CODATA va
105.658386 ±0.000044	[4] MARIAM	82	CNTR	+	
105.65836 ±0.00026	[5] CROWE	72	CNTR		
105.65865 ±0.00044	[6] CRANE	71	CNTR		

[3] Converted to MeV using the 1998 CODATA value of the conversion const: 931.494013 ± 0.0000037 MeV/u.
[4] MARIAM 82 give m_μ/m_e = 206.768259(62).
[5] CROWE 72 give m_μ/m_e = 206.7682(5).
[6] CRANE 71 give m_μ/m_e = 206.76878(85).

μ MEAN LIFE τ

Measurements with an error > 0.001×10^{-6} s have been omitted.

VALUE (10^{-6} s)	*DOCUMENT ID*		*TECN*	*CHG*
2.19703 ±0.00004 OUR AVERAGE				
2.197078±0.000073	BARDIN	84	CNTR	+
2.197025±0.000155	BARDIN	84	CNTR	−
2.19695 ±0.00006	GIOVANETTI	84	CNTR	+
2.19711 ±0.00008	BALANDIN	74	CNTR	+
2.1973 ±0.0003	DUCLOS	73	CNTR	+

“Ordinary” Muon Decay

Virtually 100% of the time the muon decays into an electron and two invisible states (neutrinos).

$$\mu^- \to e^- \nu_\mu \bar{\nu}_e$$

Given its small mass (compared to that of the W-boson), muon decay can be parameterized by the effective Lagrangian

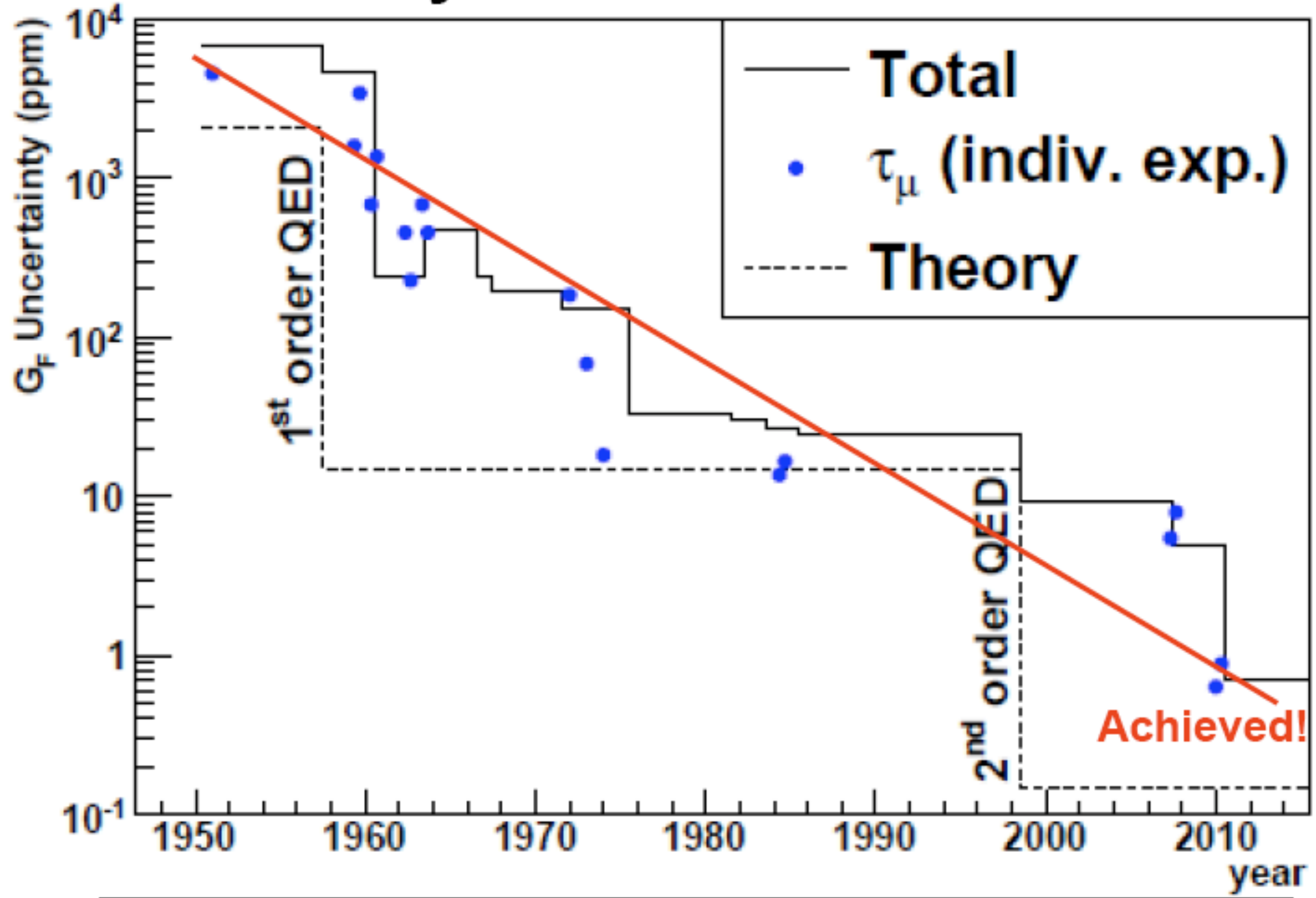
$$-\frac{4G_F}{\sqrt{2}} \sum_{\gamma,\alpha,\beta} g^\gamma_{\alpha\beta} \left(\bar{e}_\alpha \Gamma^\gamma \nu\right) \left(\bar{\nu} \Gamma_\gamma \mu_\beta\right),$$

where $\alpha, \beta = L, R$, and $\gamma = S, V, T$ ($\Gamma_S = 1$, $\Gamma_V = \gamma_\mu$ and $\Gamma_T = \sigma_{\mu\nu}/\sqrt{2}$).

In the Standard Model, $g^V_{LL} = 1$, while all other $g^\gamma_{\alpha\beta}$ vanish. ($V - A$).

G_F & τ_μ precision has improved by ~4 orders of magnitude over 60 years.

[David Hertzog, 2010 DNP Meeting]



Oct. 2010

Measurement of the Positive Muon Lifetime and Determination of the Fermi Constant to Part-per-Million Precision

D.M. Webber,[1] V. Tishchenko,[2] Q. Peng,[3] S. Battu,[2] R.M. Carey,[3] D.B. Chitwood,[1] J. Crnkovic,[1] P.T. Debevec,[1] S. Dhamija,[2] W. Earle,[3] A. Gafarov,[3] K. Giovanetti,[4] T.P. Gorringe,[2] F.E. Gray,[5] Z. Hartwig,[3] D.W. Hertzog,[1] B. Johnson,[6] P. Kammel,[1] B. Kiburg,[1] S. Kizilgul,[1] J. Kunkle,[1] B. Lauss,[7] I. Logashenko,[3] K.R. Lynch,[3] R. McNabb,[1] J.P. Miller,[3] F. Mulhauser,[1,7] C.J.G. Onderwater,[1,8] J. Phillips,[3] S. Rath,[2] B.L. Roberts,[3] P. Winter,[1] and B. Wolfe[1]
(MuLan Collaboration)

The $g^{\gamma}_{\alpha\beta}$ coefficients can be measured by precision measurements of the electron energy spectrum. For example, if one ignores the mass of the neutrino and the electron, and does not measure the electron polarization,

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}x\mathrm{d}\cos\theta} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left\{3(1-x) + \frac{2\rho}{3}(4x-3) \pm P_\mu \xi \cos\theta \left[1 - x + \frac{2\delta}{3}(4x-3)\right]\right\} 2x^2$$

ρ, δ, ξ are (some of) the Michel Parameters;

P_μ = μ-polarization; θ = angle between P_μ and the e-momentum;

$x = 2E_e/m_\mu$.

The Michel parameters are functions of the $g^{\gamma}_{\alpha\beta}$, and are sensitive to New Physics. For example, in a left-right model

$$\Delta\rho \simeq -\frac{3}{2}\vartheta_{LR}^2, \quad \Delta\xi = -2\vartheta_{LR}^2 - 2\left(\frac{M_W}{M_{W_R}}\right)^4$$

.

ϑ_{LR} = mixing between SM ("left-handed") W-boson and "right-handed" W_R-boson. Current constraints competitive with collider searches.

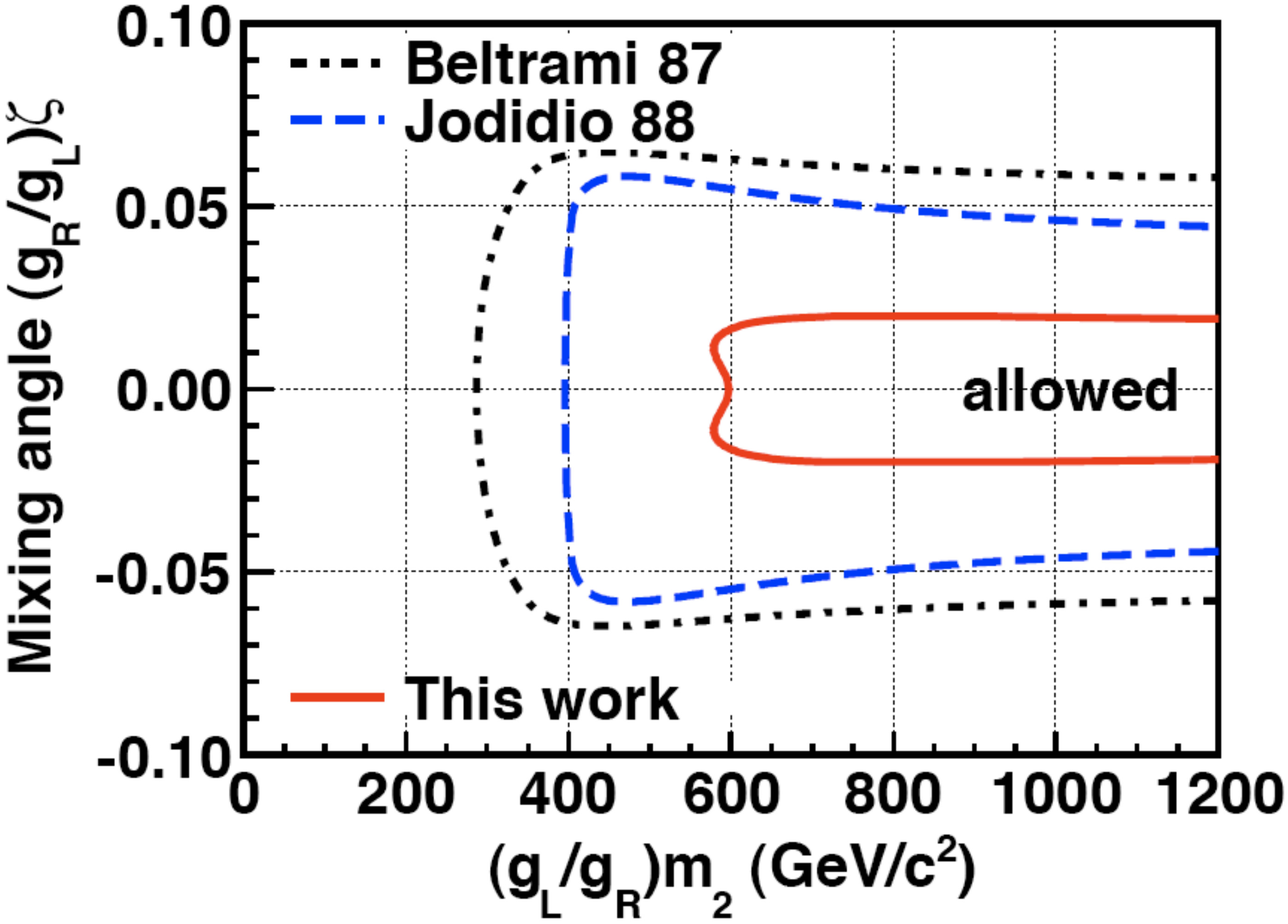


FIG. 3. Allowed region (90% C.L.) of mixing angle (ζ) and heavy W mass (m_2) for the general LRS model.

TWIST Coll. [1010.4998]

The Muon Magnetic Dipole Moment

The magnetic moment of the muon is defined by $\vec{M} = g_\mu \frac{e}{2m_\mu}\vec{S}$.

The Dirac equation predicts $g_\mu = 2$, so that the anomalous magnetic moment is defined as (note: dimensionless)

$$a_\mu \equiv \frac{g_\mu - 2}{2}$$

In the standard model, the (by far) largest contribution to a_μ comes from the one-loop QED vertex diagram, first computed by Schwinger:

$$a_\mu^{QED}(1-\text{loop}) = \frac{\alpha}{2\pi} = 116,140,973.5 \times 10^{-11}$$

The theoretical estimate has been improved significantly since then, mostly to keep up with the impressive experimental reach of measurements of the $g-2$ of the muon. (Tenth order (!!) QED completed recently, arXiv:1205.5368. The answer is $9.16(58)(\alpha/\pi)^5$, if you are curious.)

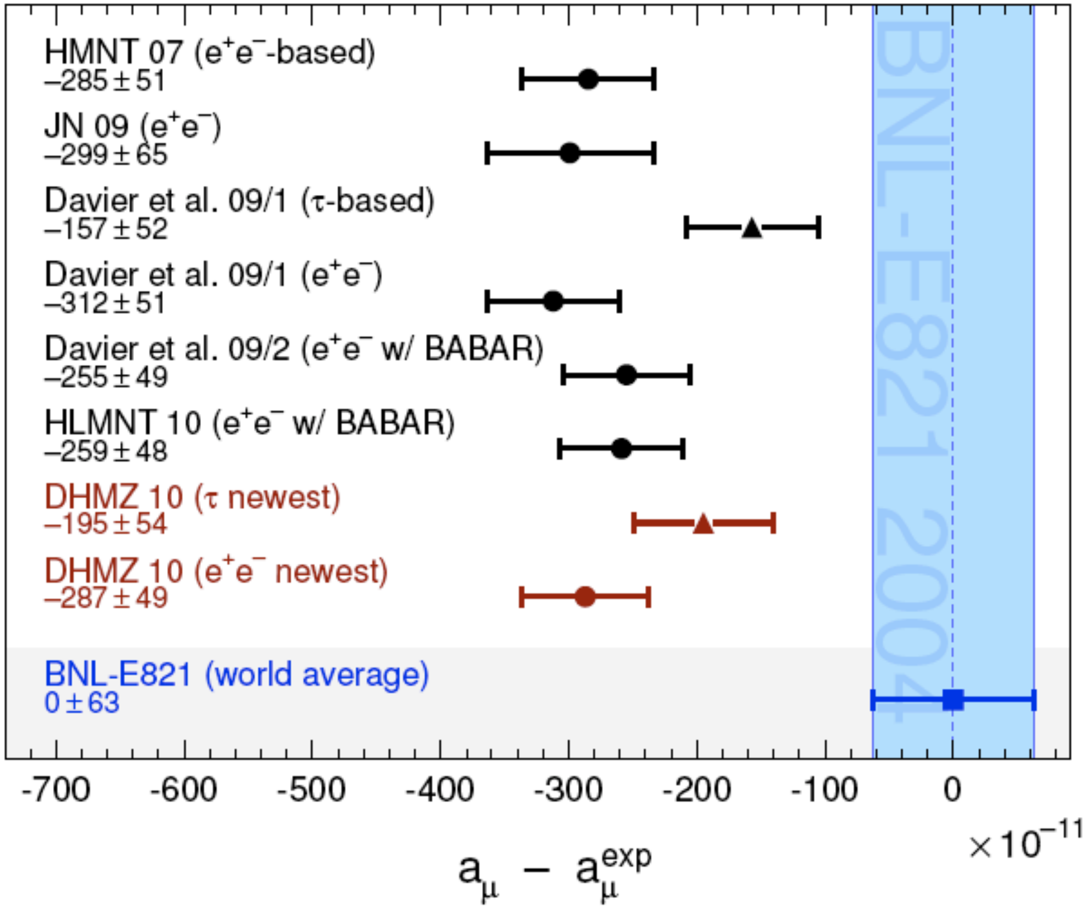


FIG. 9: Compilation of recent results for $a_\mu^{\rm SM}$ (in units of 10^{-11}), subtracted by the central value of the experimental average [12, 57]. The shaded vertical band indicates the experimental error. The SM predictions are taken from: this work (DHMZ 10), HLMNT (unpublished) [58] (e^+e^- based, including BABAR and KLOE 2010 $\pi^+\pi^-$ data), Davier *et al.* 09/1 [15] (τ-based), Davier *et al.* 09/1 [15] (e^+e^--based, not including BABAR $\pi^+\pi^-$ data), Davier *et al.* 09/2 [10] (e^+e^--based including BABAR $\pi^+\pi^-$ data), HMNT 07 [59] and JN 09 [60] (not including BABAR $\pi^+\pi^-$ data).

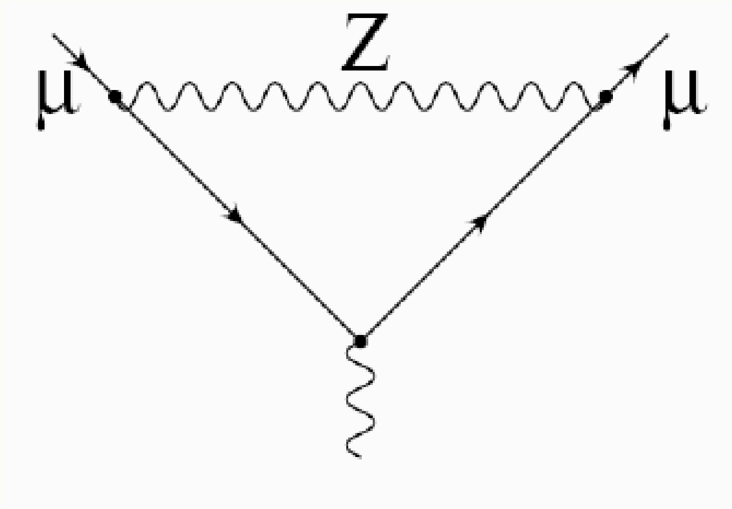
NOTE: $a_\mu^{LbL} = 105 \pm 26 \times 10^{-11}$

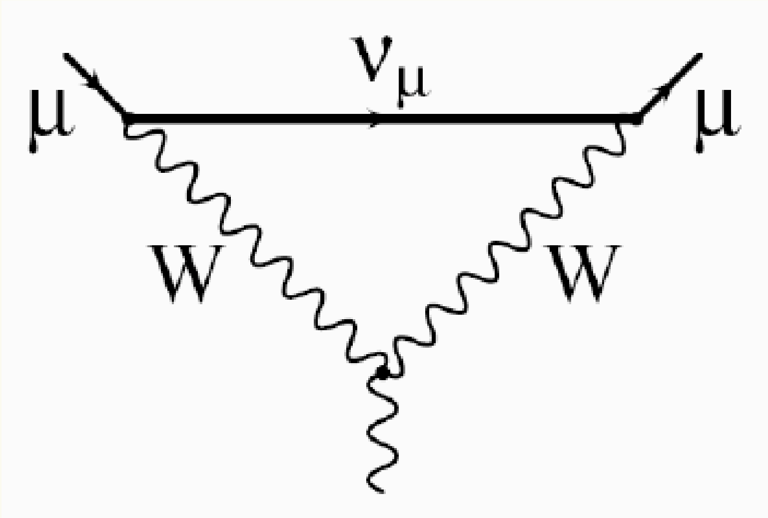
[**Davier** *et al*, **1010.4180**]

[talk by A. Czarnecki at CIPANP 2006]

Electroweak effects

Small part of the total g-2: $154(3)\times 10^{-11}$





$$\left(-1+2\right)\cdot\frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\simeq 195\cdot 10^{-11}$$

very similar to New Physics!

(more on this later)

Dependence on muon mass;
that's why muons so much more sensitive
to New Physics than the electron

Sensitivity to New Physics

If there is new ultra-violet physics, it will manifest itself, as far as a_μ is concerned, via the following effective operator (dimension 6):

$$\frac{\lambda H}{\Lambda^2}\bar{\mu}\sigma_{\mu\nu}\mu F^{\mu\nu} \to \frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma_{\mu\nu}\mu F^{\mu\nu},$$

where Λ is an estimate for the new physics scale. (dependency on muon mass is characteristic of several (almost all?) models. It is NOT guaranteed)

Contribution to a_μ from operator above is

$$\delta a_\mu = \frac{4m_\mu^2}{e\Lambda^2}$$

Current experimental sensitivity: $\Lambda \sim 10$ TeV.

Note that, usually, new physics scale can be much lower due to loop-factors, gauge couplings, etc. In the SM the heavy gauge boson contribution yields

$$\frac{1}{\Lambda^2} \sim \frac{eg^2}{16\pi^2 M_W^2} \quad \longrightarrow \quad \delta a_\mu \sim \frac{m_\mu^2 G_F}{4\pi^2}$$

Not A Bad Estimate!

Some Examples:

- Low energy supersymmetry:

$$\delta a_\mu \simeq \pm \frac{5\alpha_2 + \alpha_Y}{48\pi} \frac{m_\mu^2}{m_{\rm SUSY}^2} \tan\beta \sim \pm 100 \times 10^{-11} \left(\frac{100~{\rm GeV}}{m_{\rm SUSY}}\right)^2 \tan\beta,$$

  where all SUSY particles weigh the same ($m_{\rm SUSY}$). A nonzero δa_μ translates into an upper bound for $m_{\rm SUSY}$.

- Theory with large extra-dimensions where the right-handed neutrinos propagate on the bulk:

$$\delta a_\mu = -\epsilon \frac{g^2}{32\pi^2} \frac{m_\mu^2}{M_W^2} \sum_j |U_{j\mu}|^2 \frac{m_{\nu_j}^2}{\Delta m^2_{\rm atm}} \sim -10^{-9}\epsilon,$$

  where ϵ is a small parameter which depends on the extra-dimensional physics (how many extra-dimensions, how large, etc). Note the “wrong” sign. [AdG, Giudice, Strumia, Tobe, hep-ph/0107156]

- In general, need $\Lambda \sim 10$ TeV – as large as the electroweak one. New physics must couples strongly to the muon (or be lighter than the W-boson).

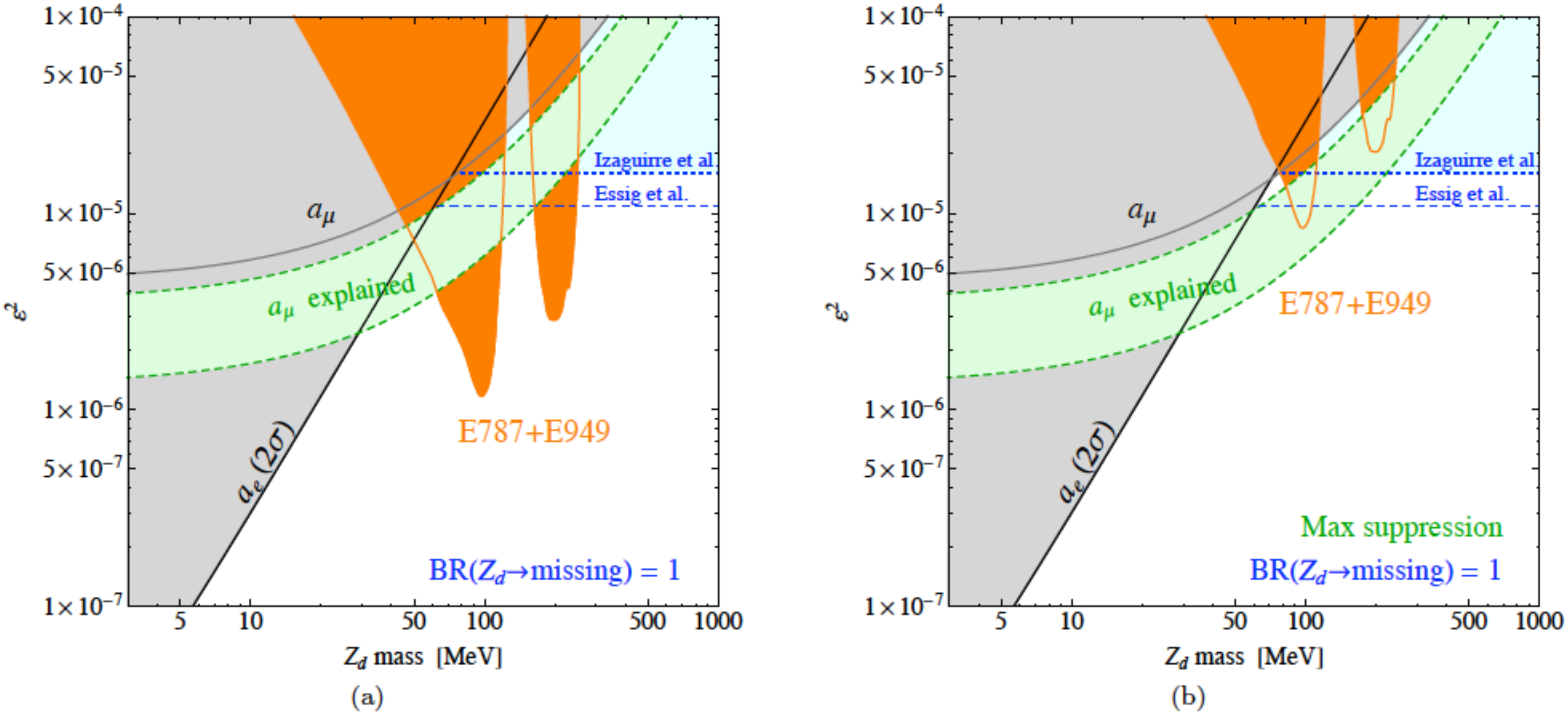
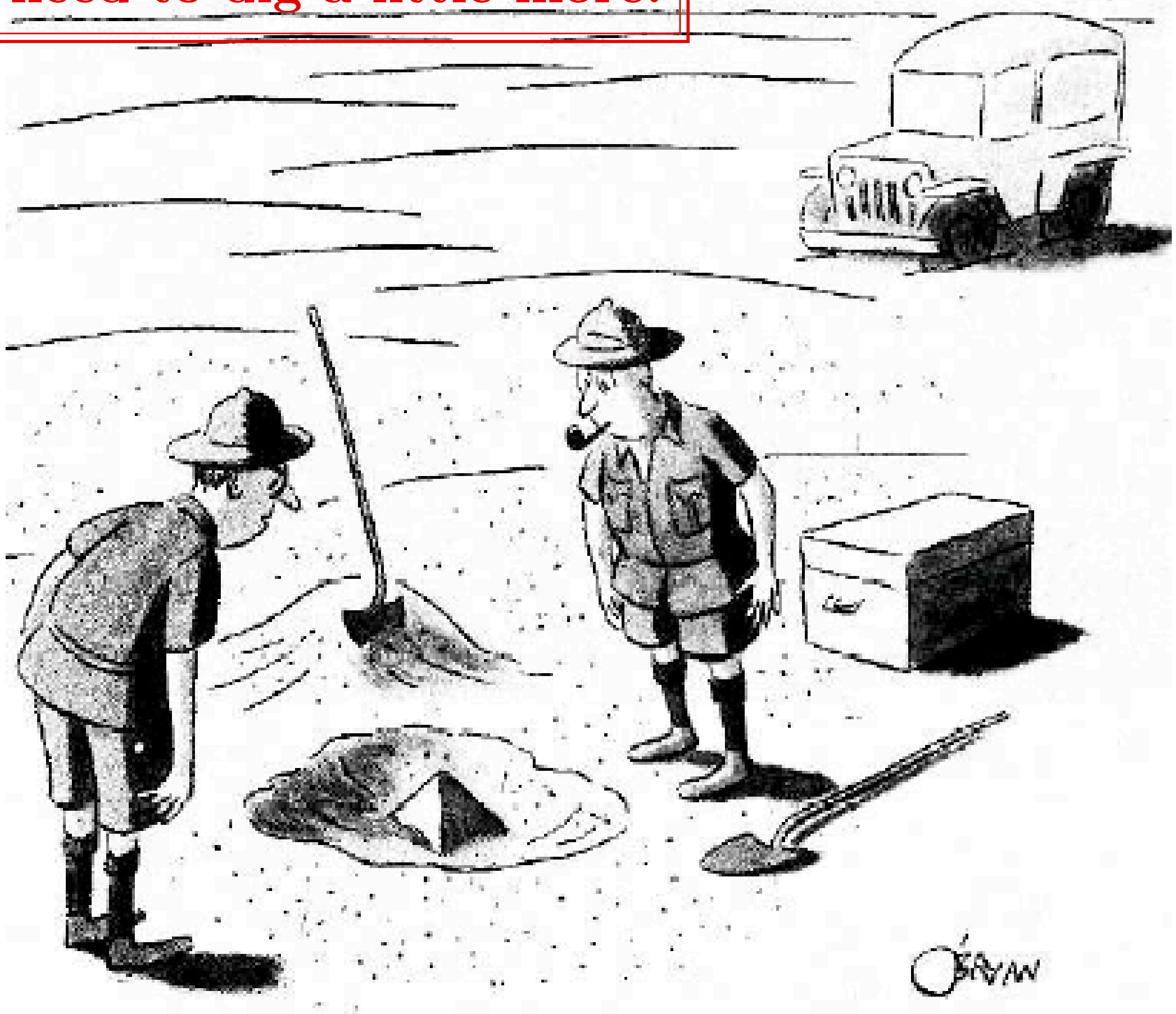


FIG. 4: Constraints from BNL E787+E949 experiments ($K \to \pi$ + nothing), at 95% C.L., on the dark photon parameter space (orange area) for BR($Z_d \to$ missing) = 1 for (a) dark photon and (b) dark Z with maximum suppression. Also illustrated there are constraints from $e^+e^- \to \gamma$ + 'invisible' based on BaBar data as given in Ref. [41] by Izaguirre et al. and Ref. [53] by Essig et al.

"Dark Photons and Z's," Davoudiasl, Lee, and Marciano, arXiv:1402.3620

Δa_μ: we need to dig a little more!



This could be the greatest discovery of the century. Depending, of course, on how far down it goes.

Very quick comments on the muon electric-dipole moment, d_μ

- CP-violating observable;
- Predicted to be non-zero-but-tiny in the SM: $d_\mu < 10^{-36}$ e-cm. Great place to look for new physics!
- Current bounds: $d_\mu < 1.8 \times 10^{-19}$e-cm. Compare to $d_e < 10^{-27}$ e-cm.
- In general, $d_\ell \propto m_\ell$, so $d_\mu \sim d_e \times (m_\mu/m_e)$.
- New $g-2$ experiment at FNAL would be sensitive to $d_\mu > 10^{-21}$e-cm. Dedicated effort could reach $d_\mu > 10^{-24}$e-cm. Is it worth it? [yes!]
- Same effective operator contributes to a_μ and d_μ

$$\frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma_{\mu\nu}\mu F^{\mu\nu} \quad \text{versus} \quad \epsilon_{\rm CP}\frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma_{\mu\nu}\gamma_5\mu F^{\mu\nu}.$$

$\epsilon_{\rm CP}$ measures how much the new physics violates CP.

If $\Lambda \sim 10$ TeV, $\epsilon_{\rm CP} \ll 1$.

Charged-Lepton Flavor Violation

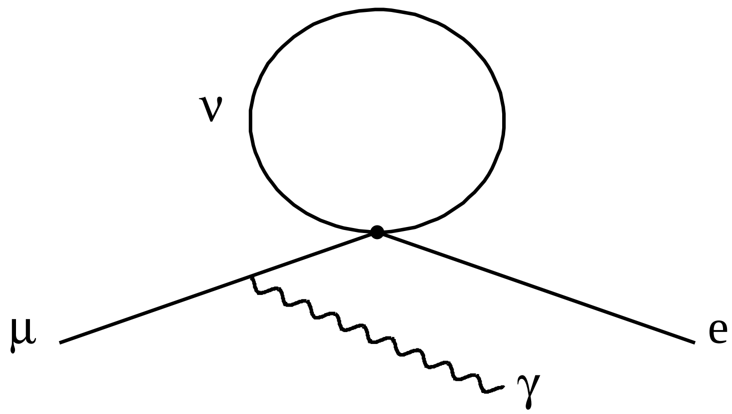
Concentrating on rare muon processes, like

$$\mu \rightarrow e\gamma$$

$$\mu \rightarrow ee^+e^-$$

$$\mu \rightarrow e\text{−conversion in nuclei}$$

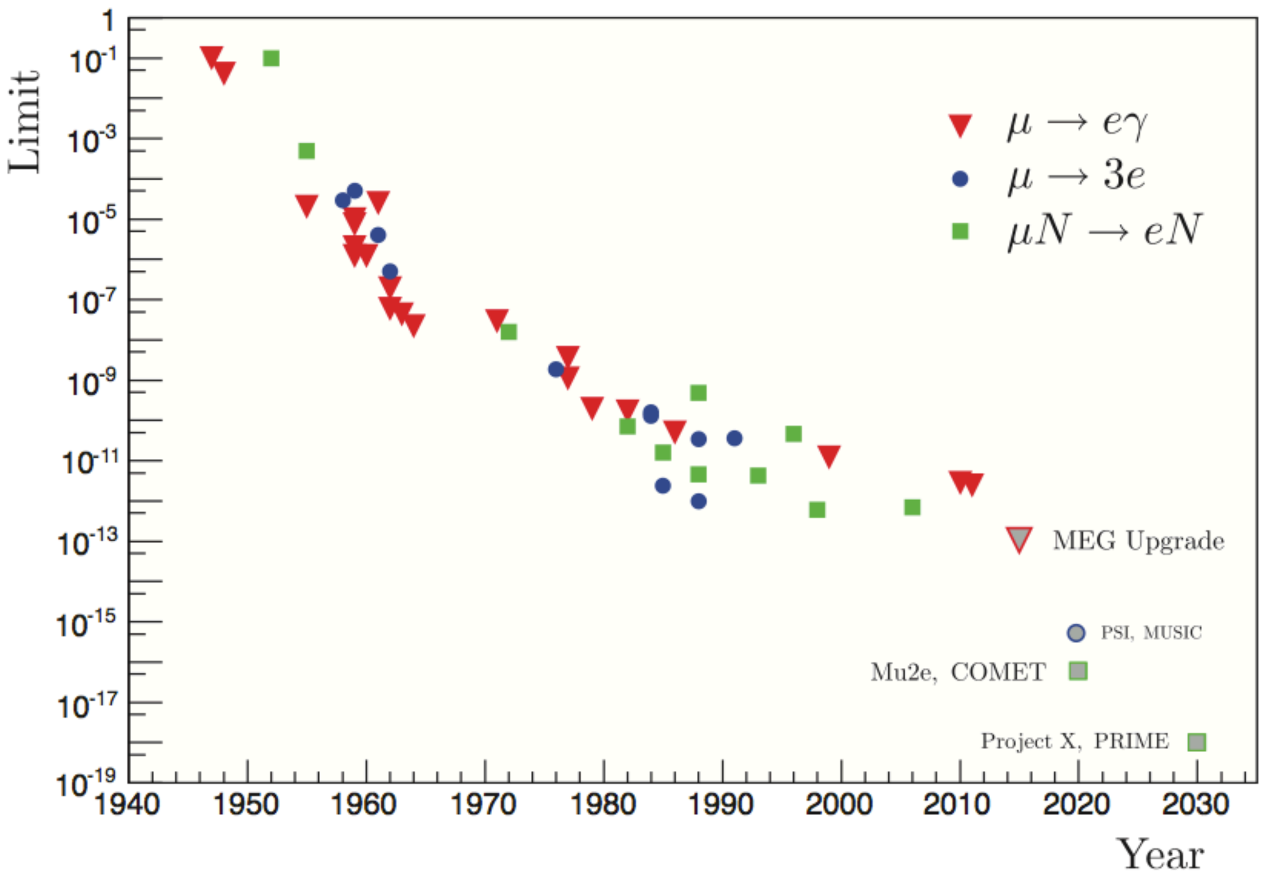
Ever since it was established that $\mu \to e\nu\bar{\nu}$, people have searched for $\mu \to e\gamma$, which was thought to arise at one-loop, like this:



The fact that $\mu \to e\gamma$ did not happen, led one to postulate that the two neutrino states produced in muon decay were distinct, and that $\mu \to e\gamma$, and other similar processes, were forbidden due to symmetries.

To this date, these so-called individual lepton-flavor numbers seem to be conserved in the case of charged lepton processes, in spite of many decades of (so far) fruitless searching...

History of $\mu \to e\gamma$, $\mu N \to eN$, and $\mu \to 3e$



[R. Bernstein, P. Cooper, arXiv 1307.5787]

Figure 3: The history of CLFV searches in muons (not including muonium.) One sees a steady improvement in all modes and then a flattening of the rate improvement throughout the 1990s. MEG has upgrade plans for the $\mu \to e\gamma$ search. The two next generations of $\mu N \to eN$, Mu2e/COMET at FNAL and J-PARC are labeled, and possible extensions at Project X and PRIME are shown. Letters-of-intent are in process for $\mu \to 3e$ experiments at PSI and Osaka's MUSIC facility. Individual experiments are discussed in the text.

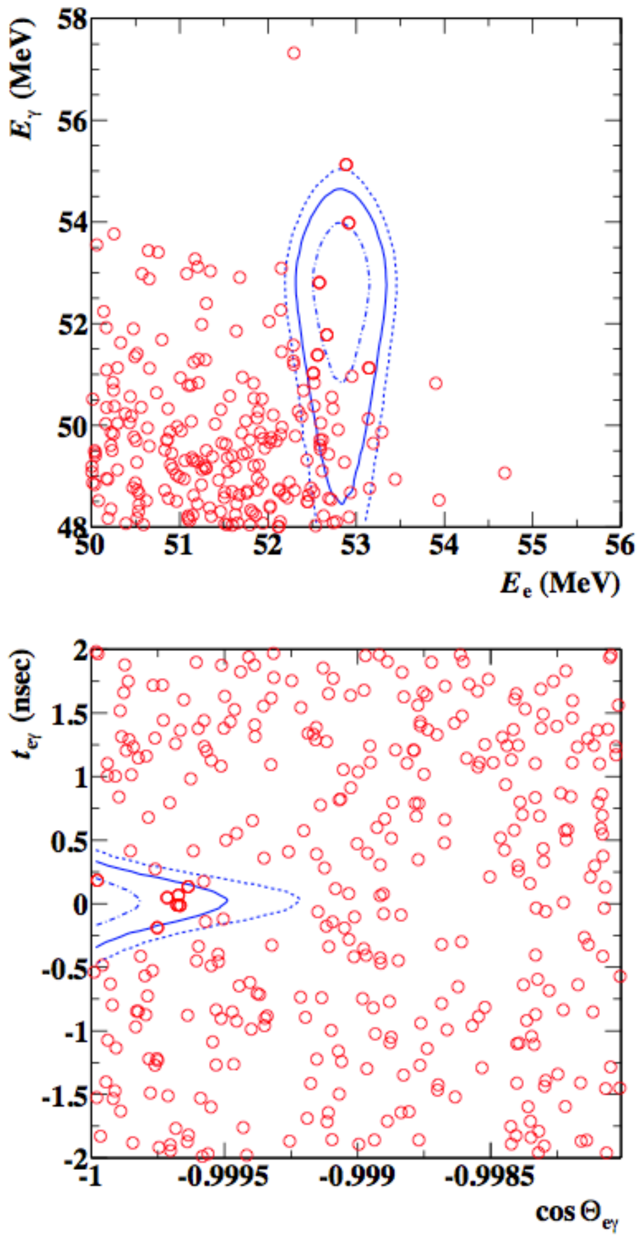
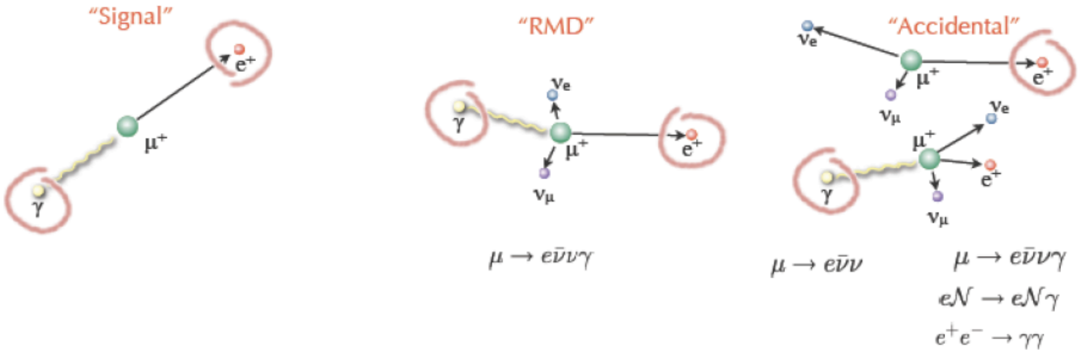


FIG. 2: Event distributions for the combined 2009–2011 dataset in the (E_e, E_γ)- and $(\cos\Theta_{e\gamma}, t_{e\gamma})$-planes. In the top (bottom) panel, a selection of $|t_{e\gamma}| < 0.244$ ns and $\cos\Theta_{e\gamma} < -0.9996$ with 90% efficiency for each variable ($52.4 < E_e < 55$ MeV and $51 < E_\gamma < 55.5$ MeV with 90% and 74% efficiencies for E_e and E_γ, respectively) is applied. The signal PDF contours (1, 1.64 and 2 σ) are also shown.

$$\mathbf{Br}(\mu \to e\gamma) < 5.7 \times 10^{-13}\ \mathbf{(90\%\ CL)}$$



[MEG Coll. arXiv:1303.0754]

SM Expectations?

In the old SM, the rate for charged lepton flavor violating processes is trivial to predict. It vanishes because individual lepton-flavor number is conserved:

- $N_\alpha(\text{in}) = N_\alpha(\text{out})$, for $\alpha = e, \mu, \tau$.

But individual lepton-flavor number are NOT conserved– ν oscillations!

Hence, in the νSM (the old Standard Model plus operators that lead to neutrino masses) $\mu \to e\gamma$ is allowed (along with all other charged lepton flavor violating processes).

These are Flavor Changing Neutral Current processes, observed in the quark sector ($b \to s\gamma$, $K^0 \leftrightarrow \bar{K}^0$, etc).

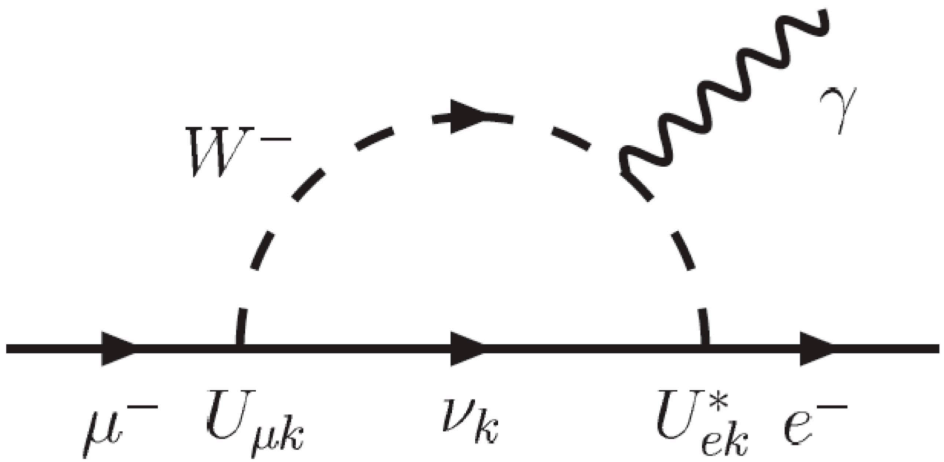
Unfortunately, we do not know the νSM expectation for charged lepton flavor violating processes $\to$ **we don't know the νSM Lagrangian !**

One contribution known to be there: active neutrino loops (same as quark sector).

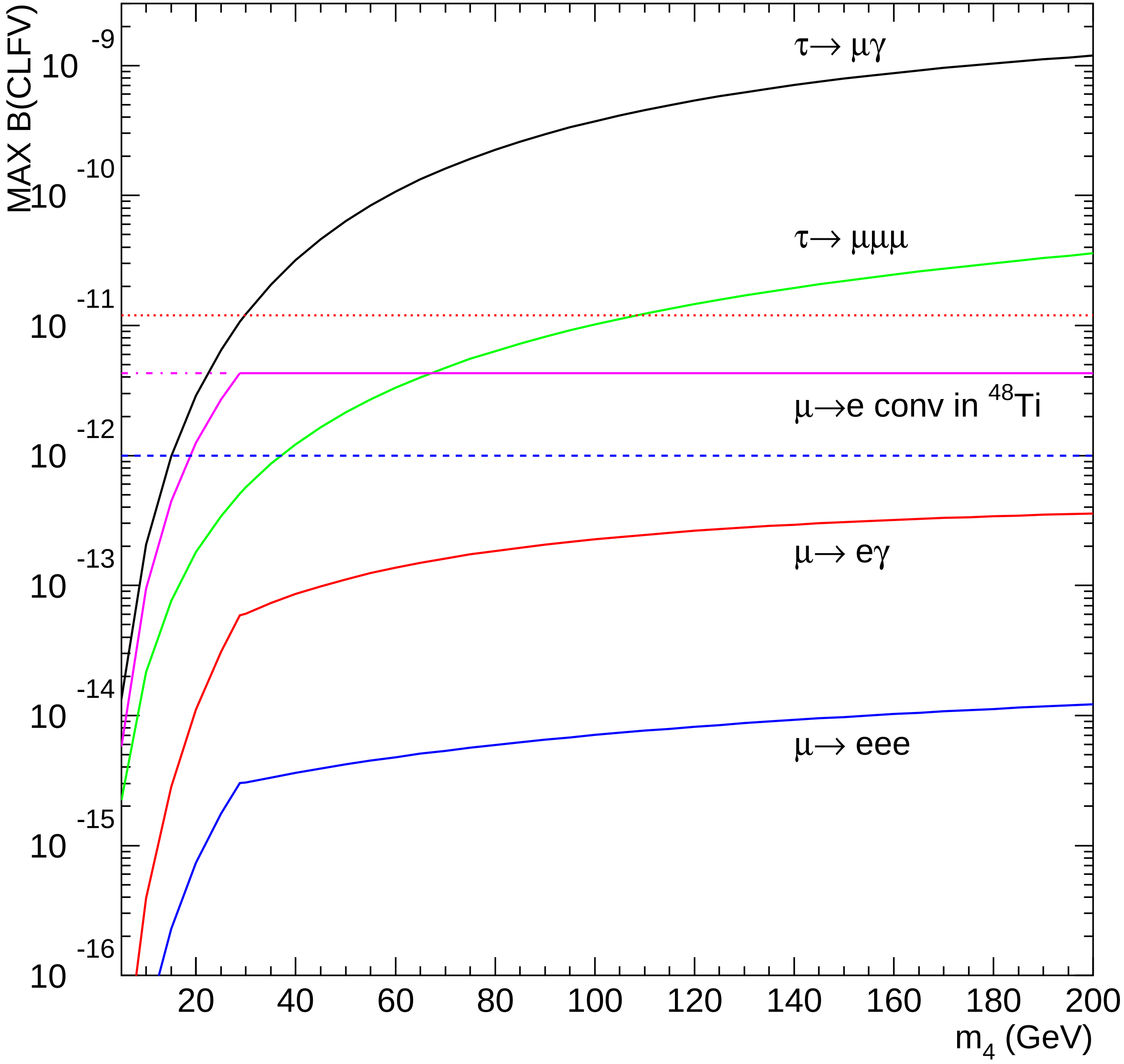
In the case of charged leptons, the **GIM suppression is very efficient**...

$$\text{e.g.: } Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U_{\mu i}^* U_{ei} \frac{\Delta m_{1i}^2}{M_W^2}\right|^2 < 10^{-54}$$

[$U_{\alpha i}$ are the elements of the leptonic mixing matrix,
$\Delta m_{1i}^2 \equiv m_i^2 - m_1^2$, $i = 2, 3$ are the neutrino mass-squared differences]



e.g.: SeeSaw Mechanism [minus "Theoretical Prejudice"]



arXiv:0706.1732 [hep-ph]

Independent from neutrino masses, there are strong theoretical reasons to believe that the expected rate for flavor changing violating processes is much, much larger than naive νSM predictions and that discovery is just around the corner.

Due to the lack of SM "backgrounds," searches for rare muon processes, including $\mu \to e\gamma$, $\mu \to e^+e^-e$ and $\mu + N \to e + N$ (μ-e–conversion in nuclei) are considered ideal laboratories to probe effects of new physics at or even above the electroweak scale.

Indeed, if there is new physics at the electroweak scale (as many theorists will have you believe) and if mixing in the lepton sector is large "everywhere" the question we need to address is quite different:

Why haven't we seen charged lepton flavor violation yet?

Model Independent Approach

As far as charged lepton flavor violating processes are concern, new physics effects can be parameterized via a handful of higher dimensional operators. For example, say that the following effective Lagrangian dominates CLFV phenomena:

$$\mathcal{L}_{\rm CLFV} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R\sigma_{\mu\nu}e_L F^{\mu\nu} + \frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L\gamma_\mu e_L\left(\bar{u}_L\gamma^\mu u_L + \bar{d}_L\gamma^\mu d_L\right)$$

First term: mediates $\mu\to e\gamma$ and, at order α, $\mu\to eee$ and $\mu+Z\to e+Z$

Second term: mediates $\mu+Z\to e+Z$ and, at one-loop, $\mu\to e\gamma$ and $\mu\to eee$

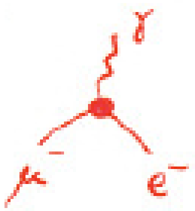
Λ is the "scale of new physics". κ interpolates between transition dipole moment and four-fermion operators.

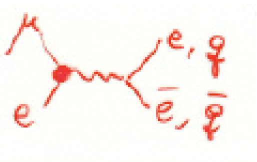
Which term wins? $\to$ Model Dependent

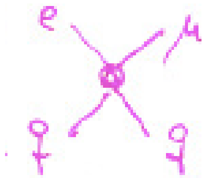
- $\mu \to e$-conv at 10^{-17} "guaranteed" deeper probe than $\mu \to e\gamma$ at 10^{-14}.

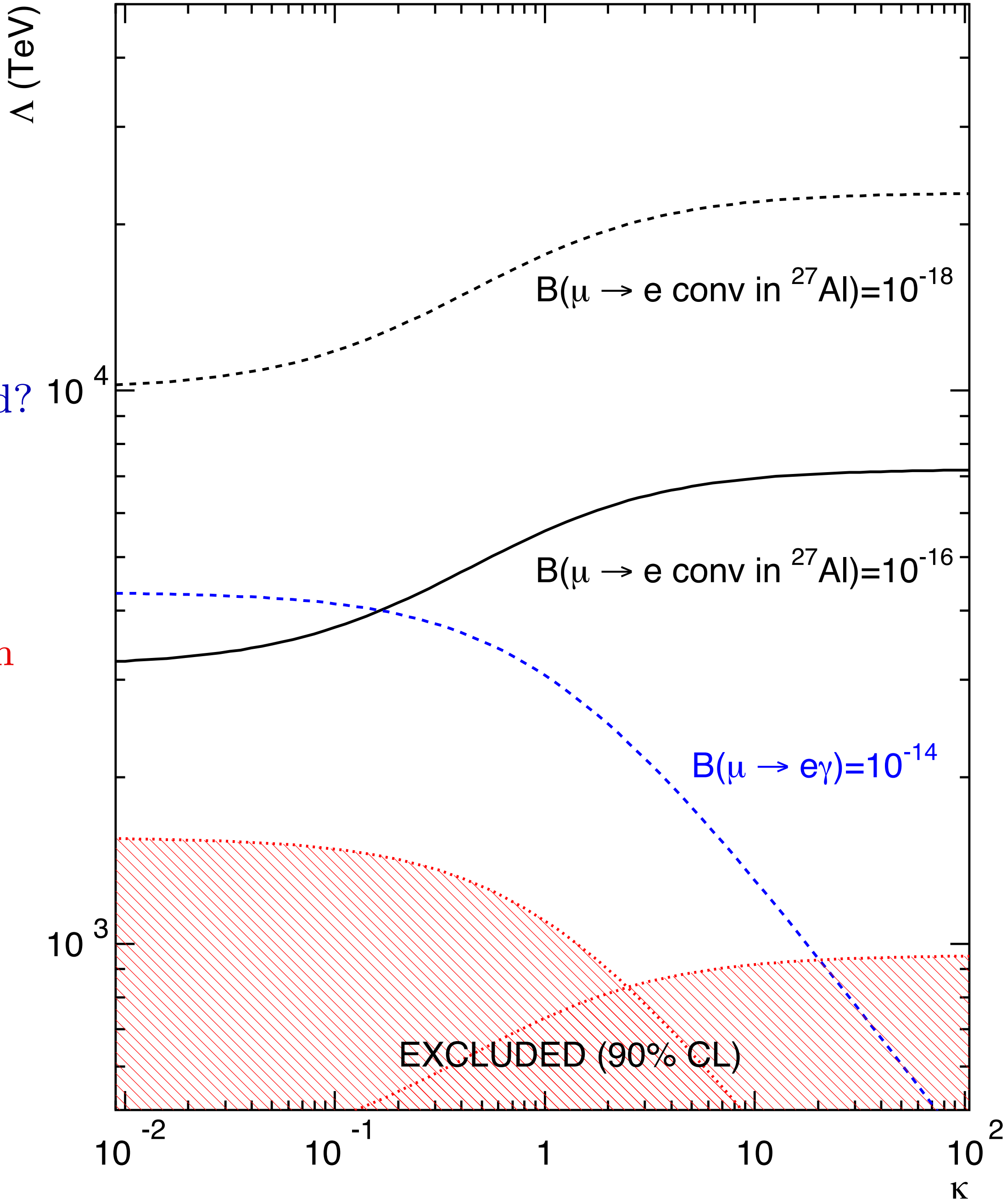
- It is really hard to do $\mu \to e\gamma$ much better than 10^{-14}. $\mu \to e$–conv "best" way forward?

- If the LHC does not discover new states $\mu \to e$-conv among very few process that can access 10,000+ TeV new physics scale: tree-level new physics: $\kappa \gg 1$, $\frac{1}{\Lambda^2} \sim \frac{g^2\theta_{e\mu}}{M^2_{\rm new}}$.









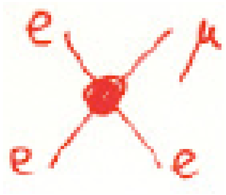
Other Example: $\mu \to e e^+ e^-$

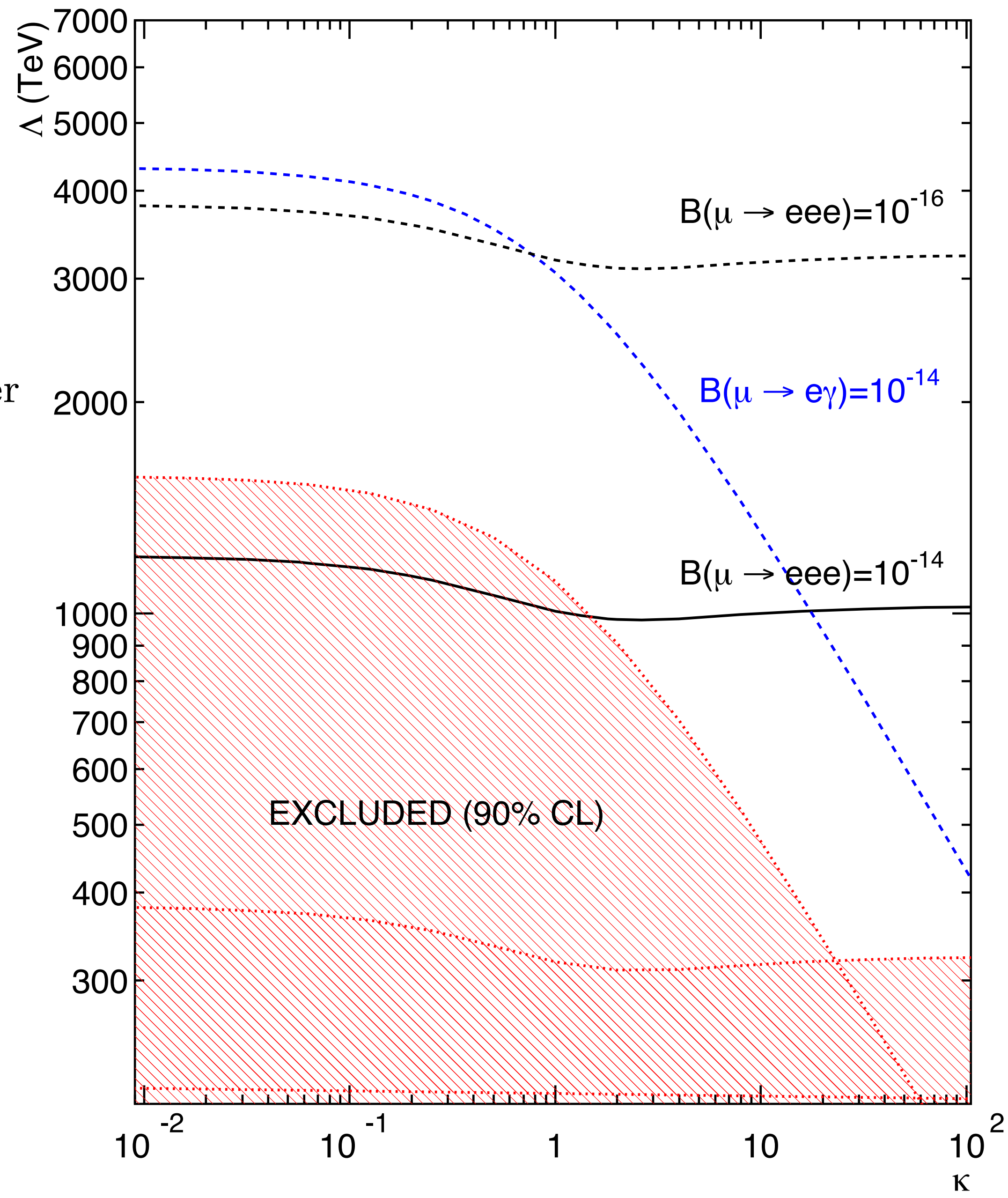
$$\mathcal{L}_{\rm CLFV} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R\sigma_{\mu\nu}e_L F^{\mu\nu} + $$
$$+ \frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L\gamma_\mu e_L \bar{e}\gamma^\mu e$$

- $\mu \to eee$-conv at 10^{-16} "guaranteed" deeper probe than $\mu \to e\gamma$ at 10^{-14}.

- $\mu \to eee$ another way forward after MEG?

- If the LHC does not discover new states $\mu \to eee$ among very few process that can access 1,000+ TeV new physics scale: tree-level new physics: $\kappa \gg 1$, $\frac{1}{\Lambda^2} \sim \frac{g^2\theta_{e\mu}}{M^2_{\rm new}}$.





What does "Λ" mean?

This is clearly model dependent! However, some general issues are easy to identify…

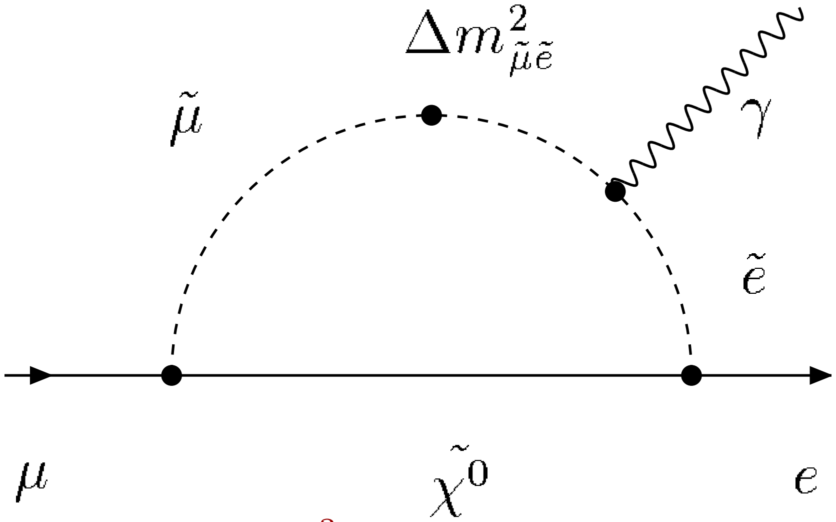
- $\mu \to e\gamma$ always occurs at the loop level, and is suppressed by E&M coupling e. Also chiral suppression (potential for "$\tan\beta$" enhancement).

$$\frac{1}{\Lambda^2} \sim \frac{e}{16\pi^2}\frac{\tan\beta}{M_{\rm new}^2}$$

- $\mu \to eee$ and $\mu \to e$-conversion in nuclei can happen at the tree-level

$$\frac{1}{\Lambda^2} \sim \frac{y_{\rm new}^2}{M_{\rm new}^2}$$

"Bread and Butter" SUSY plus High Energy Seesaw



$\rightarrow \ \theta_{\tilde{e}\tilde{\mu}} \sim \frac{\Delta m^2_{\tilde{e}\tilde{\mu}}}{\tilde{m}}$

$Br(\mu \to e\gamma) \simeq \frac{\alpha^3\pi}{G_F^2 \tilde{m}^4}\theta^2_{\tilde{e}\tilde{\mu}}$, $\tilde{m}^2$ is a typical supersymmetric mass.

$\theta_{\tilde{e}\tilde{\mu}}$ measures the "amount" of flavor violation.

For $\tilde{m}$ around 1 TeV, $\theta_{\tilde{e}\tilde{\mu}}$ is severely constrained. Very big problem.

"Natural" solution: $\boxed{\theta_{\tilde{e}\tilde{\mu}} = 0}$ $\rightarrow$ modified by quantum corrections.

The Seesaw Mechanism

$\mathcal{L} \supset -y_{i\alpha} L^i H N^\alpha - \frac{M_N^{\alpha\beta}}{2} N_\alpha N_\beta + H.c.,$ $\Rightarrow$ N^α gauge singlet fermions, $y_{i\alpha}$ dimensionless Yukawa couplings, $M_N^{\alpha\beta}$ (very large) mass parameters.

At low energies, integrate out the "right-handed neutrinos" N_α:

$$\mathcal{L} \supset \left(y M_N^{-1} y^t\right)_{ij} L^i H L^j H + \mathcal{O}\left(\frac{1}{M_N^2}\right) + H.c.$$

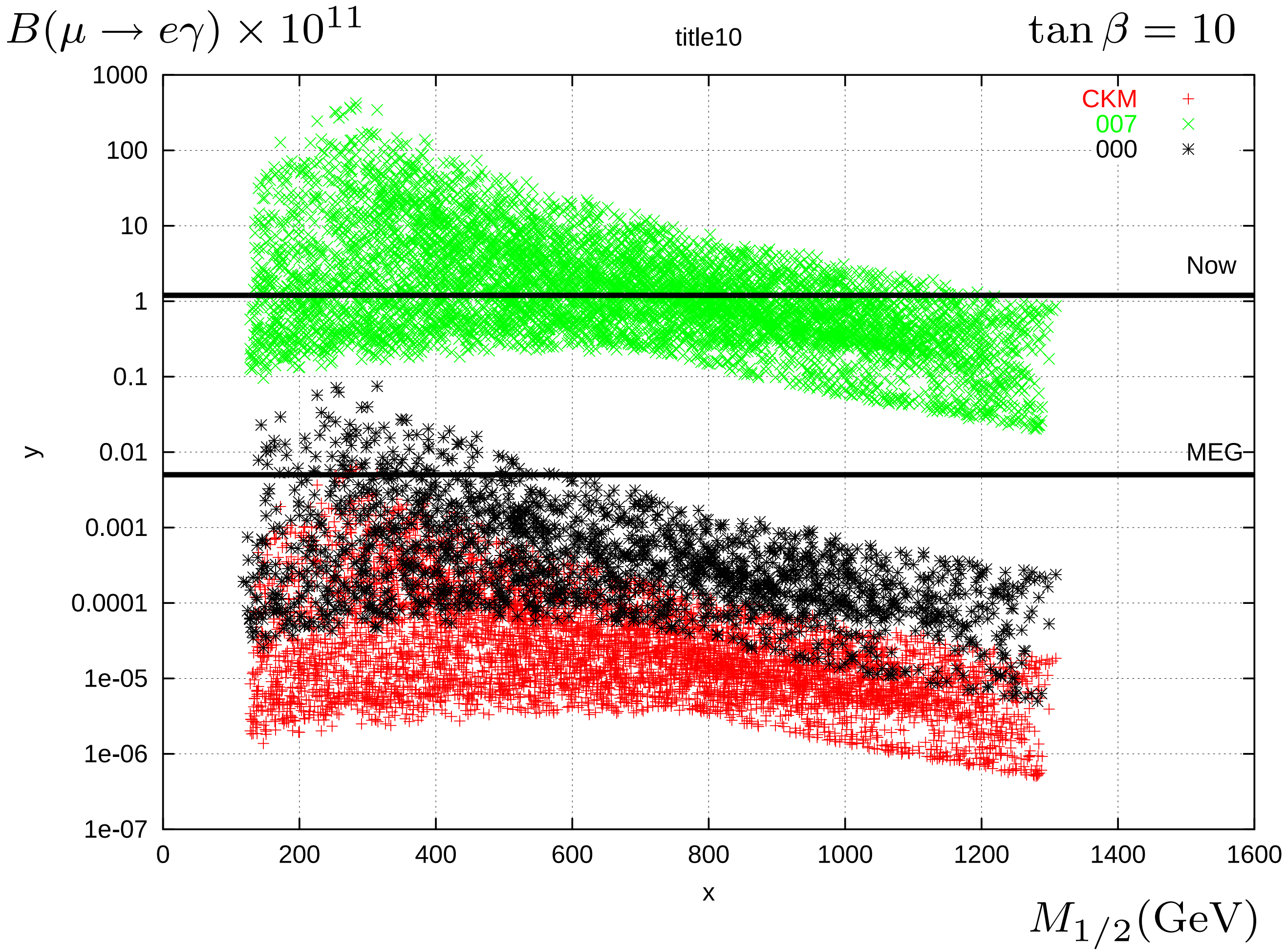
y are not diagonal $\rightarrow$ right-handed neutrino loops generate non-zero $\Delta m^2_{\tilde{e}\tilde{\mu}}$

$$\left(m^2_{\tilde{\ell}_L}\right)_{ij} \simeq -\frac{3m_0^2 + A_0^2}{8\pi^2} \sum_k (y)^*_{ki} (y)_{kj} \ln \frac{M_X}{M_{N_k}} , \qquad X = \text{Planck}, GUT, \text{etc}$$

If this is indeed the case, CLFV would serve as another channel to probe neutrino Yukawa couplings, which are not directly accessible experimentally.

Fundamentally important for "testing" the seesaw, leptogenesis, GUTs, etc

What are the neutrino Yukawa couplings → ansatz needed!



$SO(10)$ inspired model.

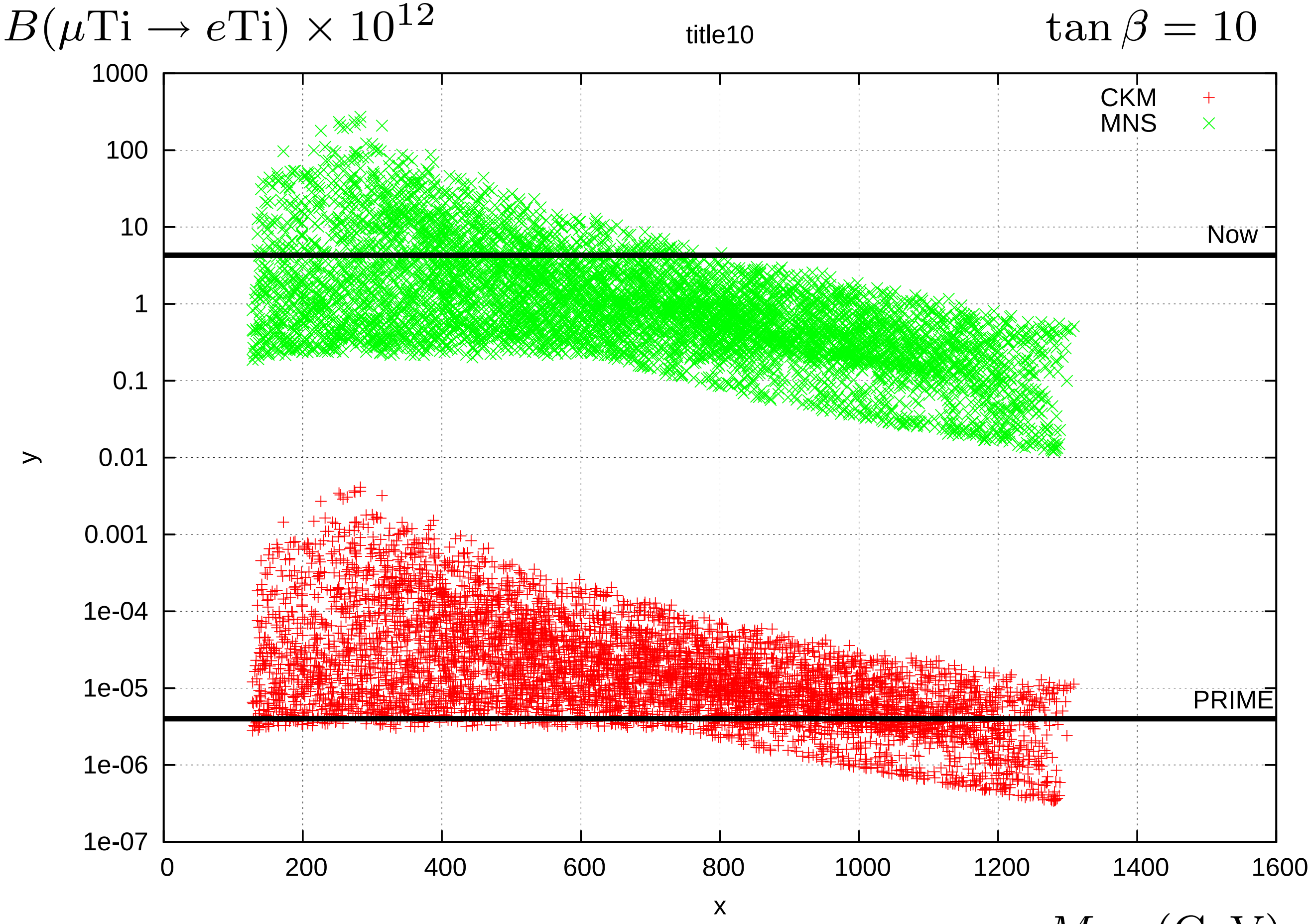
remember B scales with y^2.

$B(\mu \to e\gamma) \propto M_R^2[\ln(M_{Pl}/M_R)]^2$

[Calibbi, Faccia, Masiero, Vempati, hep-ph/0605139]

$\mu \to e$ conversion is at least as sensitive as $\mu \to e\gamma$

$B(\mu\text{Ti} \to e\text{Ti}) \times 10^{12}$ $\tan\beta = 10$



$M_{1/2}$(GeV)

$SO(10)$ inspired model.

remember B scales with y^2.

$B(\mu \to e\gamma) \propto M_R^2[\ln(M_{Pl}/M_R)]^2$

[Calibbi, Faccia, Masiero, Vempati, hep-ph/0605139]

Input From/To Leptogenesis

In the case of the seesaw mechanism, the matter-antimatter asymmetry generated via leptogenesis is (yet another) function of the neutrino Yukawa couplings:

If one is to hope to ever reconstruct the seesaw Lagrangian and test leptogenesis, LFV needs to be measured.

Note that this is VERY ambitious, and we need to get lucky a few times:

- Weak scale SUSY has to exist;
- "Precision" measurement of $\mu \to e$, $\tau \to \mu$, $\tau \to e$;
- "Precision" measurement of SUSY masses;
- Very good understanding of mechanism of SUSY breaking;
- There are no other relevant degrees of freedom between the weak scale and $> 10^9$ GeV;
- etc

Other ways to do this would be much appreciated!

SUSY with R-parity Violation

The MSSM Lagrangian contains several marginal operators which are allowed by all gauge interactions but violate baryon and lepton number.

A subset of these (set λ'' to zero to prevent proton decay, and ignore bi-linear terms, which do not contribute as much to CLFV) is:

$$\begin{aligned}\mathcal{L} &= \lambda_{ijk}\left(\bar{\nu}^c_{Li}e_{Lj}\tilde{e}^*_{Rk} + \bar{e}_{Rk}\nu_{Li}\tilde{e}_{Lj} + \bar{e}_{Rk}e_{Lj}\tilde{\nu}_{Li}\right) \\ &+ \lambda'_{ijk}V^{j\alpha}_{KM}\left(\bar{\nu}^c_{Li}d_{L\alpha}\tilde{d}^*_{Rk} + \bar{d}_{Rk}\nu_{Li}\tilde{d}_{L\alpha} + \bar{d}_{Rk}d_{L\alpha}\tilde{\nu}_{Li}\right) \\ &- \lambda'_{ijk}\left(\bar{u}^c_j e_{Li}\tilde{d}^*_{Rk} + \bar{d}_{Rk}e_{Li}\tilde{u}_{Lj} + \bar{d}_{Rk}u_{Lj}\tilde{e}_{Li}\right) + \text{h.c.},\end{aligned}$$

The presence of different combinations of these terms leads to very distinct patterns for CLFV. Proves to be an excellent laboratory for probing all different possibilities.

[AdG, Lola, Tobe, hep-ph/0008085]

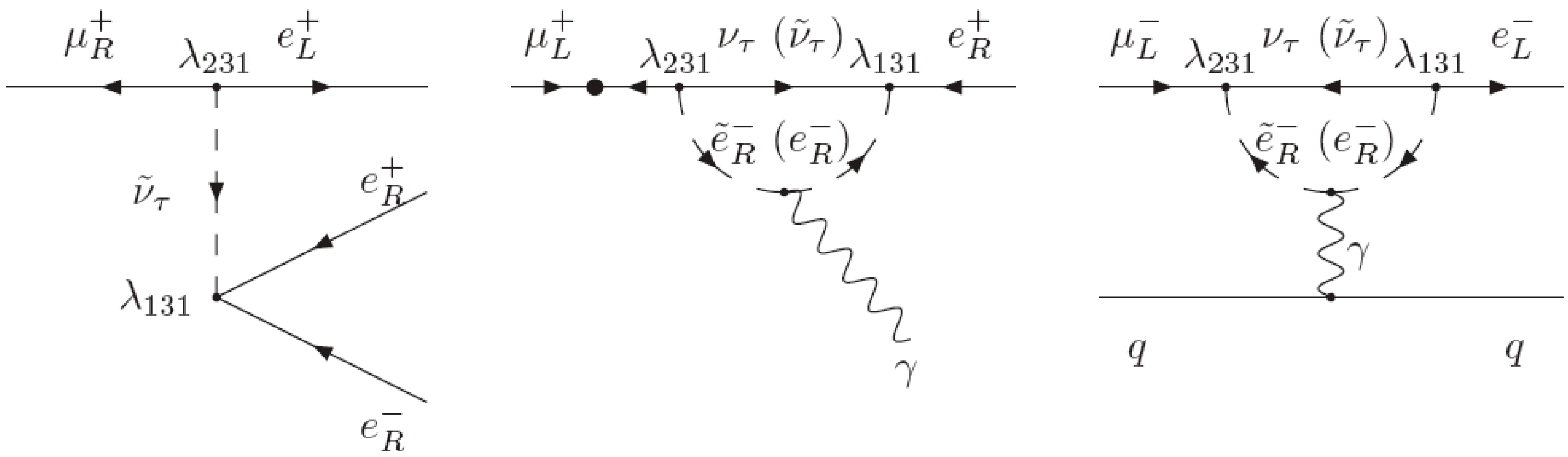


Figure 1: *Lowest order Feynman diagrams for lepton flavour violating processes induced by $\lambda_{131}\lambda_{231}$ couplings (see Eq. (2.1)).*

$$\frac{\mathrm{Br}(\mu^+\to e^+\gamma)}{\mathrm{Br}(\mu^+\to e^+e^-e^+)} = \frac{4\times 10^{-4}\left(1-\frac{m^2_{\tilde{\nu}_\tau}}{2m^2_{\tilde{e}_R}}\right)^2}{\beta} \simeq 1\times 10^{-4} \qquad (\beta\sim 1)$$

$$\frac{\mathrm{R}(\mu^-\to e^- \text{ in Ti (Al)})}{\mathrm{Br}(\mu^+\to e^+e^-e^+)} = \frac{2\ (1)\times 10^{-5}}{\beta}\left(\frac{5}{6}+\frac{m^2_{\tilde{\nu}_\tau}}{12m^2_{\tilde{e}_R}}+\log\frac{m^2_e}{m^2_{\tilde{\nu}_\tau}}+\delta\right)^2 \simeq 2\ (1)\times 10^{-3},$$

$\mu^+\to e^+e^-e^+$ most promising channel!

[AdG, Lola, Tobe, hep-ph/0008085]

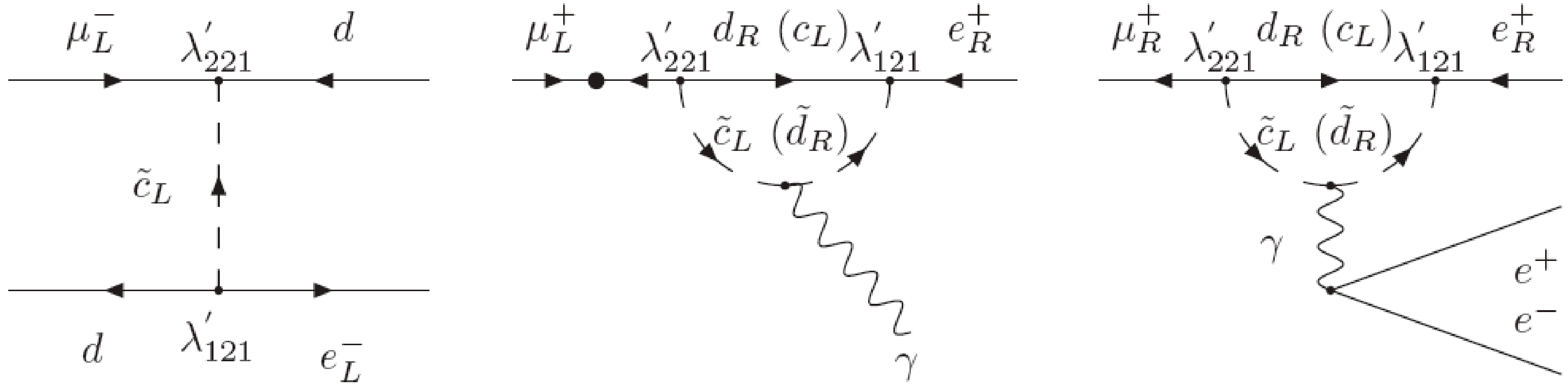


Figure 4: *Lowest order Feynman diagrams of lepton flavour violating processes induced by $f'_{121}f'_{221}$ couplings (see Eq. (2.1)).*

$$\frac{\mathrm{Br}(\mu^+\to e^+\gamma)}{\mathrm{Br}(\mu^+\to e^+e^-e^+)} = 1.1$$

$$(m_{\tilde{d}_R} = m_{\tilde{c}_L} = 300\ \mathrm{GeV})$$

$$\boxed{\frac{\mathrm{R}(\mu^-\to e^-\ \text{in Ti (Al)})}{\mathrm{Br}(\mu^+\to e^+e^-e^+)} = 2\ (1)\times 10^5}$$

$\mu - e$-conversion “only hope”!

[AdG, Lola, Tobe, hep-ph/0008085]

Type-II Seesaw: SM plus $SU(2)$ Triplet Higgs, $Y_T = 1$

$$\mathcal{L} \in \frac{\lambda_{\alpha\beta}}{2} L^\alpha L^\beta T.$$

Neutrino Majorana masses if T develops a vev …

$$m_{\alpha\beta} = \lambda_{\alpha\beta} v_T$$

$\mu \to e\gamma$, $\mu \to e$-conversion at the loop-level. However, $\mu \to eee$ at the tree level (note direct connection to neutrino mass-matrix flavor sctructure)…

$$\frac{1}{\Lambda^2} = \frac{m_{ee} m_{\mu e}}{v_T^2 M_T^2}$$

Key issue: are neutrino masses small because λ are small or because v_T is small (or both)? EWPD already push v_T below ~ 1 GeV…

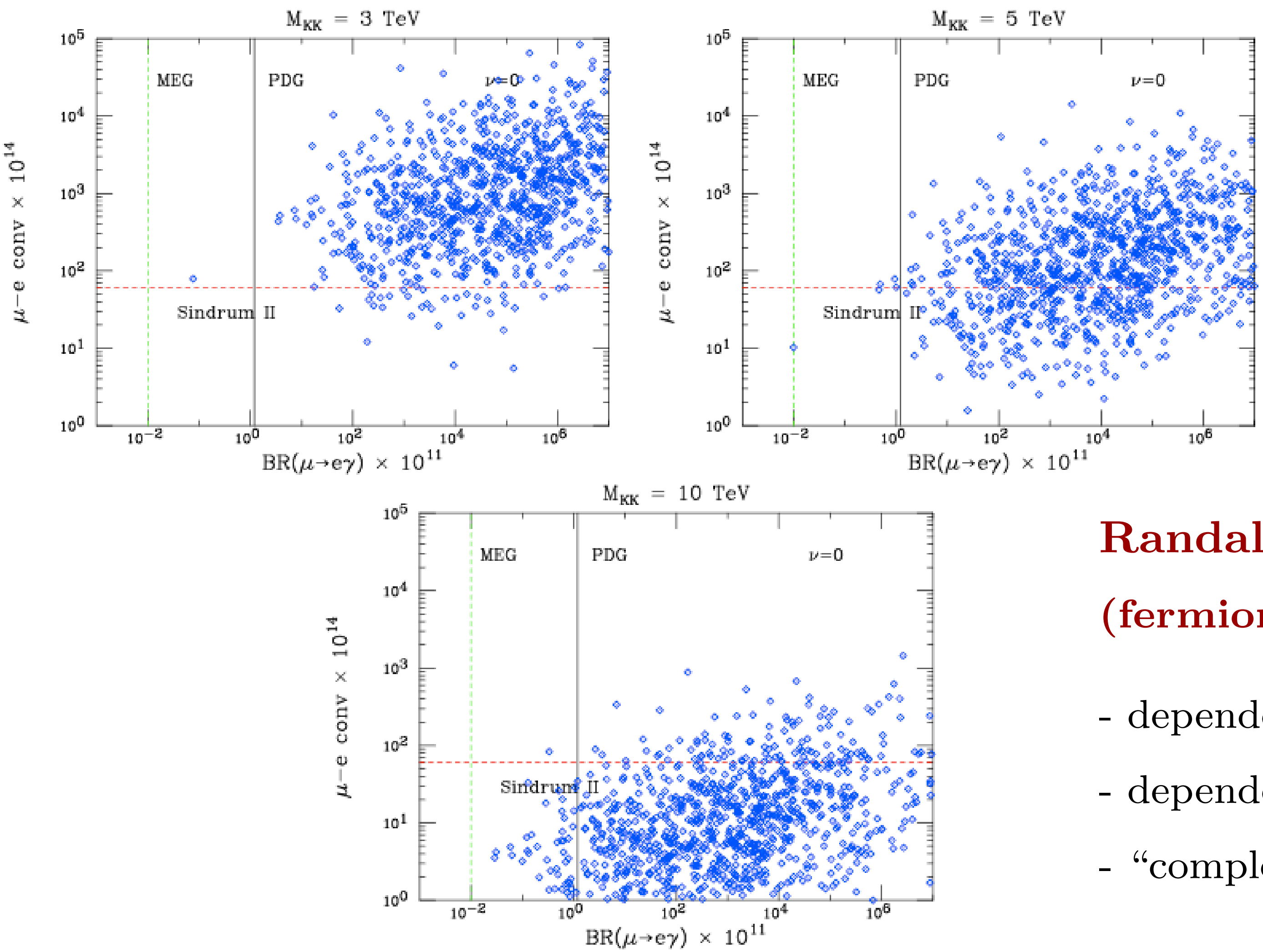


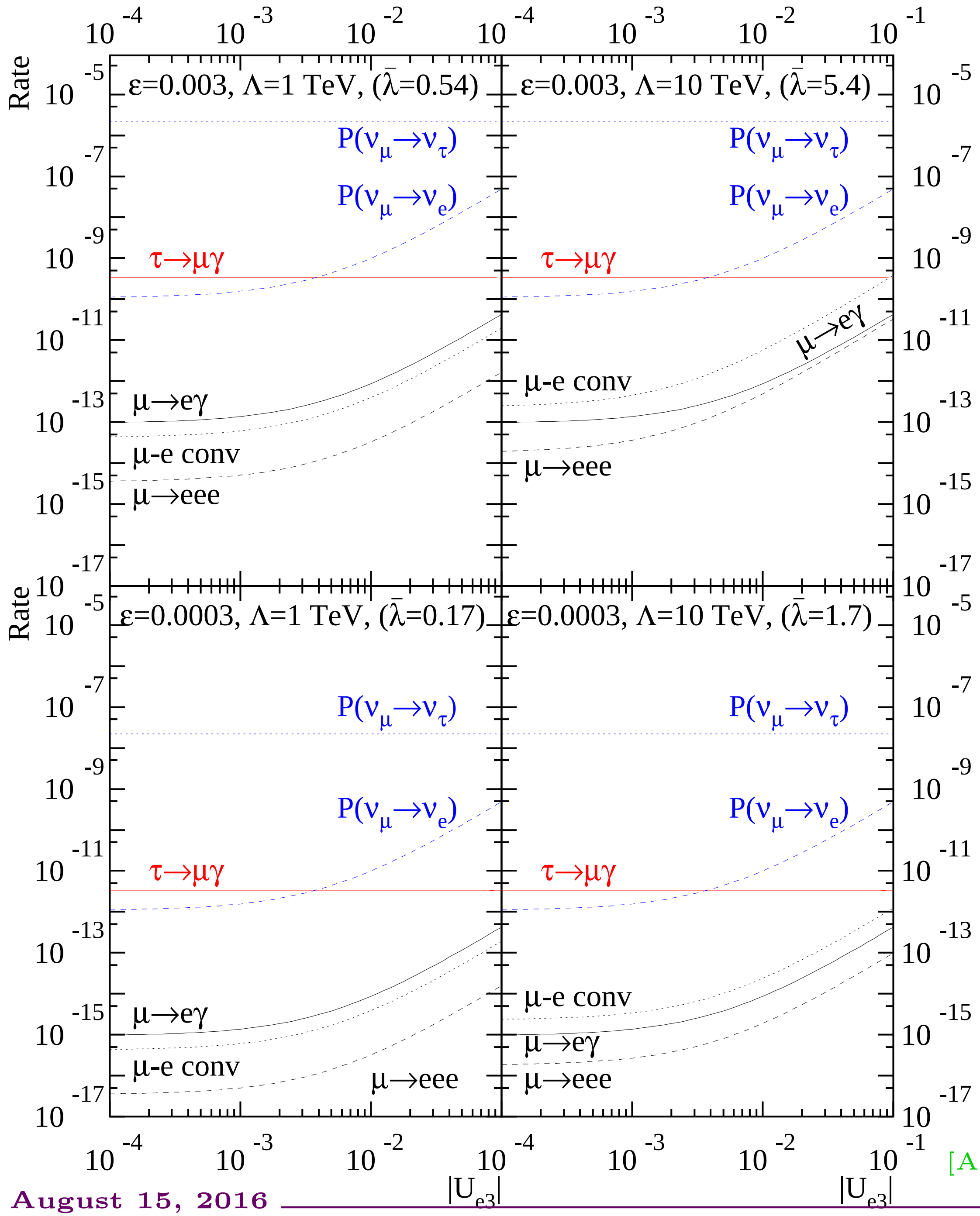
FIG. 6: Scan of the $\mu \to e\gamma$ and $\mu - e$ conversion predictions for $M_{KK} = 3, 5, 10$ TeV and $\nu = 0$. The solid line denotes the PDG bound on $BR(\mu \to e\gamma)$, while the dashed lines indicate the SINDRUM II limit on $\mu - e$ conversion and the projected MEG sensitivity to $BR(\mu \to e\gamma)$.

Randall-Sundrum Model

(fermions in the bulk)

- dependency on UV-completion(?)
- dependency on Yukawa couplings
- "complementarity" between $\mu \to e\gamma$, $\mu - e$ conv

[Agashe, Blechman, Petriello, hep-ph/0606021]



Large Extra-Dimensions

-no ambiguity in y (neutrinos Dirac)

-dependency on UV-completion

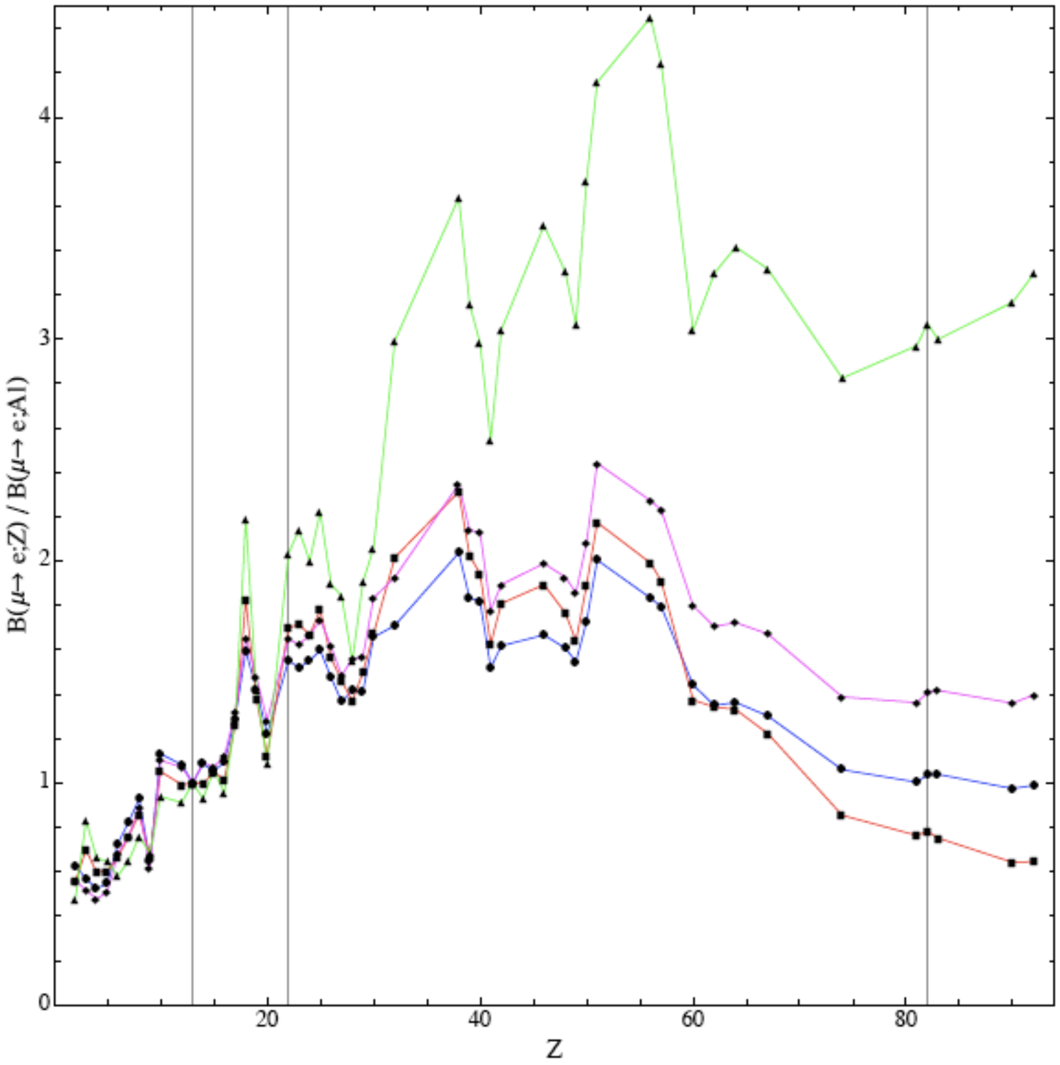
[AdG, Giudice, Strumia, Tobe, hep-ph/0107156]

What is This Really Good For?

While specific models (see last slides) provide estimates for the rates for CLFV processes, the observation of one specific CLFV process cannot determine the underlying physics mechanism (this is always true when all you measure is the coefficient of an effective operator).

Real strength lies in combinations of different measurements, including:

- kinematical observables (e.g. angular distributions in $\mu \to eee$);
- other CLFV channels;
- neutrino oscillations;
- measurements of $g-2$ and EDMs;
- collider searches for new, heavy states;
- etc.



Vector 4-Fermion Interaction (Z)

$\propto (\bar{\mu}\gamma_\alpha e)(\bar{q}\gamma^\alpha q)$

Vector 4-Fermion Interaction (γ)

Dipole ($\propto \bar{\mu}\sigma_{\alpha\beta} e F^{\alpha\beta}$)

Scalar 4-Fermion Interaction

$\propto (\bar{\mu}e)(\bar{q}q)$

Figure 3: Target dependence of the $\mu \to e$ conversion rate in different single-operator dominance models. We plot the conversion rates normalized to the rate in Aluminum ($Z = 13$) versus the atomic number Z for the four theoretical models described in the text: D (blue), S (red), $V^{(\gamma)}$ (magenta), $V^{(Z)}$ (green). The vertical lines correspond to $Z = 13$ (Al), $Z = 22$ (Ti), and $Z = 83$ (Pb).

[Cirigliano, Kitano, Okada, Tuzon, 0904.0957]

Model Independent Comparison Between $g-2$ and CLFV:

The dipole effective operators that mediate $\mu \to e\gamma$ and contribute to a_μ are virtually the same:

$$\frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma^{\mu\nu}\mu F_{\mu\nu} \qquad \times \qquad \theta_{e\mu}\frac{m_\mu}{\Lambda^2}\bar{\mu}\sigma^{\mu\nu}eF_{\mu\nu}$$

$\theta_{e\mu}$ measures how much flavor is violated. $\theta_{e\mu}=1$ in a flavor indifferent theory, $\theta_{e\mu}=0$ in a theory where indiviadual lepton flavor number is exactly conserved.

If $\theta_{e\mu}\sim 1$, $\mu \to e\gamma$ is a much more stringent probe of Λ.

On the other hand, if the current discrepancy in a_μ is due to new physics, $\theta_{e\mu} \ll 1$ ($\theta_{e\mu} < 10^{-4}$). [Hisano, Tobe, hep-ph/0102315]

e.g., in SUSY models, $Br(\mu \to e\gamma) \simeq 3 \times 10^{-5}\left(\frac{10^{-9}}{\delta a_\mu}\right)\left(\frac{\Delta m^2_{\tilde{e}\tilde{\mu}}}{\tilde{m}^2}\right)^2$

Comparison restricted to dipole operator. If four-fermion operators are relevant, they will "only" enhance rate for CLFV with respect to expectations from $g-2$.

On CLFV Processes Involving τ Leptons (Brief Comment)

Current Bound On Selected τ CLFV Processes (All from the B-Factories):

- $B(\tau \to e\gamma) < 1.1 \times 10^{-7}$; $B(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$. $(\mu \to e\gamma)$
- $B(\tau \to e\pi) < 8.0 \times 10^{-8}$; $B(\tau \to \mu\pi) < 1.1 \times 10^{-7}$. $(\mu \to e\text{–conversion})$
- $B(\tau \to eee) < 3.6 \times 10^{-8}$; $B(\tau \to ee\mu) < 2.0 \times 10^{-8}$, $(\mu \to eee)$
- $B(\tau \to e\mu\mu) < 2.3 \times 10^{-8}$; $B(\tau \to \mu\mu\mu) < 3.2 \times 10^{-8}$. $(\mu \to eee)$

Relation to $\mu \to e$ violating processes is model dependent. Typical enhancements, at the amplitude-level, include:

- Chirality flipping: $m_\tau \gg m_\mu$;
- Lepton mixing effects: $U_{\tau 3} \gg U_{e3}$;
- Mass-Squared Difference effects: $\Delta m^2_{13} \gg \Delta m^2_{12}$;
- etc

Future: SuperB-factories will get to 10^{-9} level. (similar to LHCb (?))

What we can learn from CLFV and other searches for new physics at the TeV scale (a_μ and Colliders):

$g-2$	CLFV	What Does it Mean?
YES	YES	New Physics at the TeV Scale; Some Flavor Violation
YES	NO	New Physics at the TeV Scale; Tiny Flavor Violation
NO	YES	New Physics Above TeV Scale; Some Flavor Violation – How Large?
NO	NO	No New Physics at the TeV Scale; CLFV only way forward?

Colliders	CLFV	What Does it Mean?
YES	YES	New Physics at the TeV Scale; Info on Flavor Sector!
YES	NO	New Physics at the TeV Scale; New Physics Very Flavor Blind. Why?
NO	YES	New Physics "Leptonic" or Above TeV Scale; Which one?
NO	NO	No New Physics at the TeV Scale; CLFV only way forward?

What Will Happen in the Near Future (my Optimistic Impression)

- MEG: $\mu \to e\gamma$ at 10^{-13}.
- $g-2$ measurement a factor of 3–4 more precise.
- *Mu2e* (Fermilab) and COMET (J-PARC): $\mu \to e$-conversion at 10^{-16}.
- PSI: $\mu \to eee$ at 10^{-15}.
- SuperB: Rare τ processes at 10^{-10}.
- Next-generation *Mu2e*: $\mu \to e$-conversion at 10^{-18} (or precision studies).
- Muon Beams/ Storage Rings: $\mu \to e$-conversion at 10^{-20}? Revisit rare muon decays ($\mu \to e\gamma$, $\mu \to eee$) with new idea?

Summary and Conclusions

- Low-energy muon processes constitute a powerful (often unique) probe of new physics around the electroweak scale, not unlike high-energy collider experiments (similar sensitivity to new physics energy scale).

- Muon decay is the cleanest weak decay process (not as "messy" as nuclear beta decay...). It provides one of the "input" constants of the Standard Model (G_F), which is used as input for computing other electroweak observables. Precision studies of polarized muon decay are still very sensitive to New Physics.

- Precision measurements of the anomalous magnetic moment of the muon are among the most stringent tests of the Standard Model. Understanding of the Standard Model expectations has settled somewhat, and an intriguing discrepancy ($> 3\ \sigma$) remains? First evidence of new physics at the electroweak physics? Time will tell.

- We know that charged lepton flavor violation must occur. Effects are, however, really tiny in the νSM (neutrino masses too small).
- If there is new physics at the electroweak scale, there is every reason to believe that CLFV is well within the reach of next generation experiments. Indeed, it is fair to ask: 'Why haven't we seen it yet?'
- It is fundamental to probe all CLFV channels. While in many scenarios $\mu \to e\gamma$ is the "largest" channel, there is no theorem that guarantees this (and many exceptions).
- CLFV may be intimately related to new physics unveiled with the discovery of non-zero neutrino masses. It may play a fundamental role in our understanding of the seesaw mechanism, GUTs, the baryon-antibaryon asymmetry of the Universe. We won't know for sure until we see it!