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Plasma-Based Accelerator Concepts for Colliders

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APC seminar

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Workshop Goals

- Review of the status
- Build plans for the future
- Three sections
  - Beam based accelerators
  - Plasma based accelerators
  - Lasers

General Point of View of a Beam Physicist

- Three major trusts
  - Standard linacs (based on cavities with metallic walls)
  - Dielectric accelerating structures (smooth or corrugated structures)
  - Plasma based accelerators (laser and beam excited)
- From general point of view all structures have a lot common
- Longitudinal and transverse short range wakes are closely related

$$W_{\perp} \approx \frac{2}{a^2}\int_0^z W_{\parallel} dz' , \quad z << a$$

- For a single bunch acceleration it relates the energy efficiency and energy spread – BNS is required for transverse stability
  - For CLIC and ILC it is achieved by using many bunches
  - Each bunch takes only small fraction of energy stored in the cavity. Energy is replenished to the next bunch arrival

Limitations on Efficiency of Single Bunch Acceleration

- High efficiency acceleration requires the beam deceleration due to longitudinal wake being comparable to accelerating field ($QW_{\|} \sim E_0/2$) and the bunch length close to a quarter of wave length ($L_b \sim \lambda/4$)
- Then for bunch displaced by Δx the deflecting field at the bunch end is

$$E_{\perp} = QW_{\perp} \approx Q\left(\frac{W_{\|}}{a^2} L_b\right) = E_0 \frac{x}{\lambda/2}, \quad a \approx \frac{\lambda}{4}$$

- "Defocusing" due to wake has to be compensated by focusing.
  - External focusing cannot do it for short wave length and high accelerating gradients: for E_0=1 GeV/m and λ=1 mm => $E_{\perp}/x \sim 7 \cdot 10^3$ T/m – at least an order of magnitude larger than present state of the art
  - Strong focusing is supported by plasma ions in plasma accelerators
  - Can be created in a dielectric-based accelerator by corrugations with broken axial symmetry

Transverse Beam stability

- BNS is required to stabilize beam transversely
- There are two possibilities
  - Introduction of energy droop to the bunch tail to avoid betatron tune spread along the bunch
    - Requires too large momentum spread for a single bunch acceleration with high efficiency
  - Time dependent focusing
    - Potentially possible in corrugated dielectric structures

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Simple Estimate of Particle Deceleration in Plasma

- Energy transfer to an electron in the small angle approximation

$$\Delta p = \frac{Ze^2}{r^2}\Delta t = \frac{Ze^2}{r^2}\frac{2r}{\mathrm{v}} = \frac{2Ze^2}{r\mathrm{v}}$$ or exact $$\Delta p = \frac{e^2}{r^2}\Delta t = \int eE_\perp(s)dt = \frac{Ze}{\mathrm{v}}\int E_\perp(s)ds$$

$$\Delta \mathrm{E} = \frac{\Delta p^2}{2m} = \left(\frac{2e^2}{r\mathrm{v}}\right)^2 \frac{1}{2m} = \frac{2e^4}{m\mathrm{v}^2}\frac{1}{r^2}$$

- Particle deceleration

$$\frac{d\mathrm{E}}{dx} = \int \Delta \mathrm{E}\, 2\pi n_e r dr = \frac{2e^4}{m\mathrm{v}^2}\int \frac{2\pi n_e r dr}{r^2} = \frac{4\pi n_e e^4}{m\mathrm{v}^2}\int \frac{dr}{r} = \frac{4\pi n_e e^4}{m\mathrm{v}^2}\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right)$$

Electric Field of a Bunch in Plasma

(based on (1) a note by G. Stupakov, ~2013 & (2) Chapter 13 in "Non-Linear Theory and Fluctuations: Plasma Electrodynamics" (Volume 2) Jan 1, 1975 by A. I. Akhiezer)

- Beam electric field in plasma

$$E_i(k,\omega) = \frac{4\pi}{i\omega}\left\{\frac{k_i k_j}{k^2}\frac{1}{\varepsilon_l(k,\omega)} + \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\frac{1}{\varepsilon_t(k,\omega) - (ck/\omega)^2}\right\} j_{0j}(k,\omega)$$

For relativistic beam and cold plasma

$$\varepsilon_l(k,\omega) = \varepsilon_t(k,\omega) = \varepsilon_l(\omega) = 1 - \frac{\omega_p^{\ 2}}{\omega^2}, \qquad k_p = \frac{\omega_p}{c}, \quad \omega_p^{\ 2} = \frac{4\pi n e^2}{m}$$

- The field of ultra-relativistic point-like beam has two parts:
  - The field of bunch itself screened by plasma

$$E_z(\mathbf{r},t) = -2NeK_0(k_p r_\perp)\delta'(s)$$
$$B_\theta(\mathbf{r},t) = 2Nek_p K_1(k_p r_\perp)\delta(s)$$
$$s = z - ct$$

  - And the wakefield which can be described by scalar potential

$$\varphi(\mathbf{r},t) = 2Nek_p K_0(k_p r_\perp)\sin(k_p s) \quad \Rightarrow \quad E_z(\mathbf{r},t) = -2Nek_p^{\ 2} K_0(k_p r_\perp)\cos(k_p s)$$

    magnetic field is equal to zero in the wakefield

- Wakefield has logarithmic divergence at $r_\perp$=0,

$$K_0(x) = \ln(2/(\gamma x)) + \ldots, \quad x \ll 1, \quad \gamma \approx 1.78$$

Single Particle Deceleration and Wake in Plasma

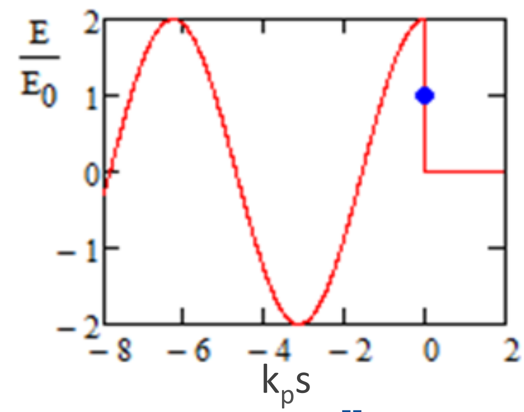
- Well-known three-step solution to find particle deceleration:
  1. **Collective plasma response at large impact parameters**
     - Solution of Maxwell equations with $\varepsilon = 1 - \omega_p{}^2/\omega^2 + i\delta\,,\quad \delta \to 0$
       - diverges at small ρ where perturbation theory does not work
  2. **Particle interaction with independent plasma electrons**
     - Energy transfer to a single electron => Deceleration rate
       $\Rightarrow$ diverges at large ρ where screening of particle field by plasma needs to be taken into account
  3. **Combining two approaches for $m >> m_e$ one obtains:**

$$\left(\frac{dE}{ds}\right)_0 \approx \frac{4\pi n_e Z^2 e^4}{m_e \mathrm{v}^2}\ln\left(\frac{2}{\gamma}\frac{\rho_{max}}{\rho_{min}}\right), \qquad \rho_{max} = \frac{\mathrm{v}}{\omega_p},\quad \rho_{min} = \frac{Ze^2}{m_e \mathrm{v}^2},\quad \frac{\mathrm{v}}{c} \ll \alpha Z,\quad \frac{2}{\gamma} \approx 1.123,\quad \alpha \approx \frac{1}{137}.$$

Logarithmic approximation : it works well as long as $\rho_{max} \gg \rho_{min}$

- Longitudinal wake is

$$\frac{dE}{ds} = \left(\frac{dE}{ds}\right)_0 2\cos(k_p s)$$

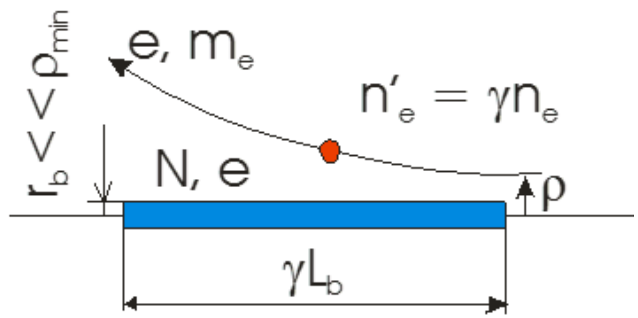


- At $\rho > \rho_{min}$ (plasma pert. theory works)

$$\mathbf{B} = 0,\quad \mathbf{E} = -\nabla\phi,\quad \delta n \equiv n_i - n_e = 0$$

Deceleration of Ultra-relativistic Bunch

- For sufficiently small number of particles the bunch deceleration is not much different from single particle
  - Classical approximation ($v/c \ll \alpha Z$) is always valid due to large N
  - Field screening and ρ_{max} will stay the same if the bunch is short, $L_b \ll k_p^{-1}$
  - ρ_{min} will be modified because of finite bunch length
  - Deceleration will be growing from bunch head to its tail
- For small impact parameters we can neglect collective electric field of plasma on plasma electron motion
  - In the BF the plasma electric field is:



$$eE' = -e\frac{d\varphi'}{ds'} = 4\pi n'_e e^2 \frac{d}{ds'}\left(\int_0^{\rho_{max}} \rho \ln\left(\frac{r(s',\rho)}{\rho}\right) d\rho\right)$$

where $r(s',\rho)$ is particle trajectory for impact parameter ρ

- We assume zero ⊥ beam size (actually any size < ρ_{min} is small enough)

  - In logarithmic approximation an integration yields:

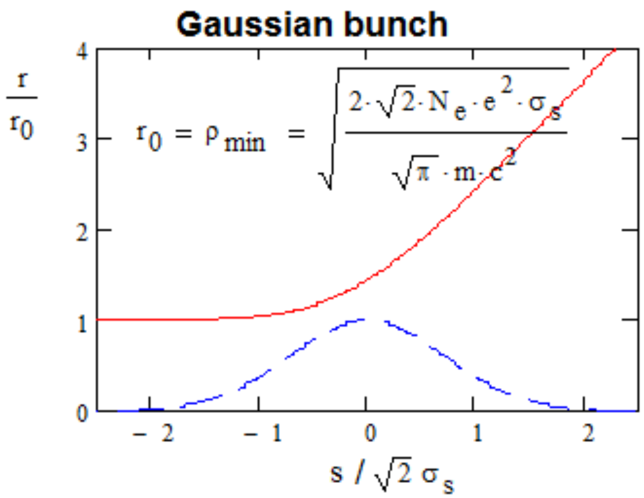
$$\frac{d\mathrm{E}}{ds} \approx \frac{4\pi n_e N_e e^4}{mc^2} \mathrm{L}_c F_g(s), \quad F_g(s) = 2\int_s^{\infty} \cos\left(k_p(s-s')\right) f(s')\, ds', \quad \int_{-\infty}^{\infty} f(s)\, ds = 1$$

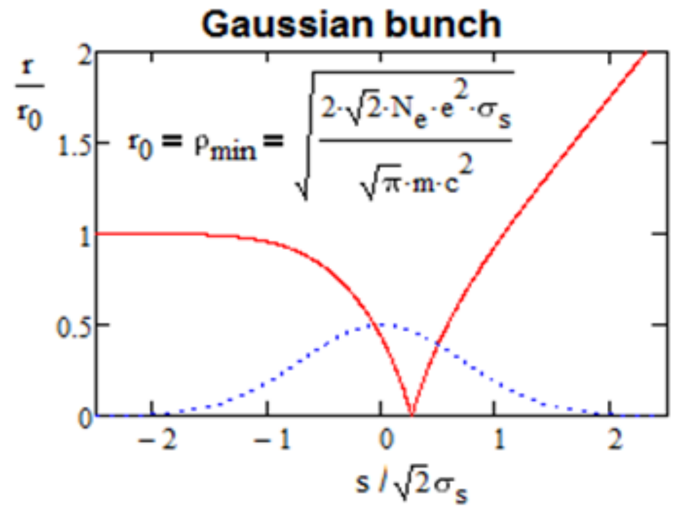
Coulomb Logarithm and Minimum Impact Parameter

$$L_c = \begin{cases} \ln\left(1 + \dfrac{\sqrt{2}c}{\omega_p \rho_{\min}}\right), & \text{electrons}, \\ \ln\left(1 + \dfrac{2.15\sqrt{2}c}{\omega_p \rho_{\min}}\right), & \text{positrons}, \end{cases} \qquad \rho_{\min} = \sqrt{\frac{2\sqrt{2} N_e e^2 \sigma_s}{\sqrt{\pi} m_e c^2}}$$

1 is added to logarithm to make equation working for $\rho_{min} \sim \rho_{max}$. Good coincidence with numerical simulations[*]!

Electrons | **Positrons**





Plasma electron scattering on the bunch with impact parameter ρ_{min}

- Trajectory shape ρ/ρ_{min} depends only on ρ_0/ρ_{min}
  - Definition of ρ_{min}: $\delta\rho \equiv (\rho - \rho_0) \approx \rho_0$ for $\rho_0 = \rho_{min}$
- The contribution for impact parameters $\rho < \rho_{min}$ is small

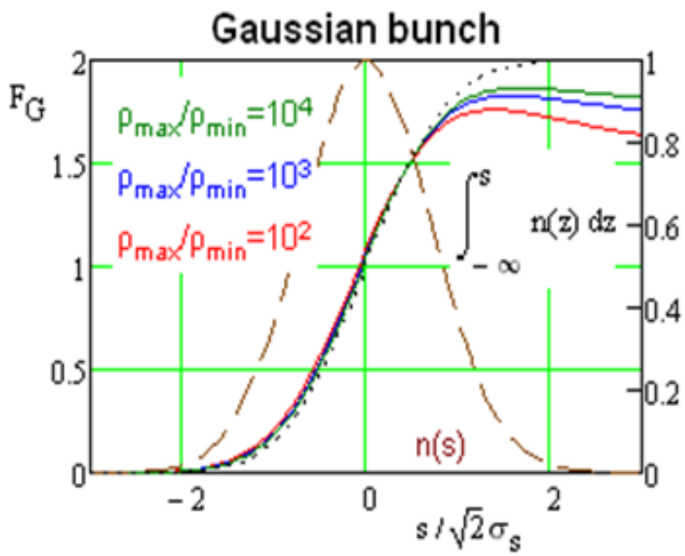
[*] W. Lu, et.al.,"Limits of linear plasma wakefield theory for electron or positron beams" Physics of plasmas 12, 063101 (2005)

Longitudinal Wake-Function

- Using wake-function definition

$$\frac{dE}{ds} \equiv N_e e^2 \int_s^{\infty} W(s-s') f(s') ds' \approx \frac{4\pi n_e N_e e^4}{mc^2} L_c \, 2\int_s^{\infty} \cos\left(k_p (s-s')\right) f(s') ds'$$

- One obtains $W_{\|}(s) = 2k_p{}^2 L_c \cos(k_p s)$
- Strictly speaking the plasma response is not completely linear
  - Consequently, the wake-function depends on longitudinal particle distribution and bunch intensity
  - Still, it is greatly helpful for initial analysis



Comparison to Numeric Simulations

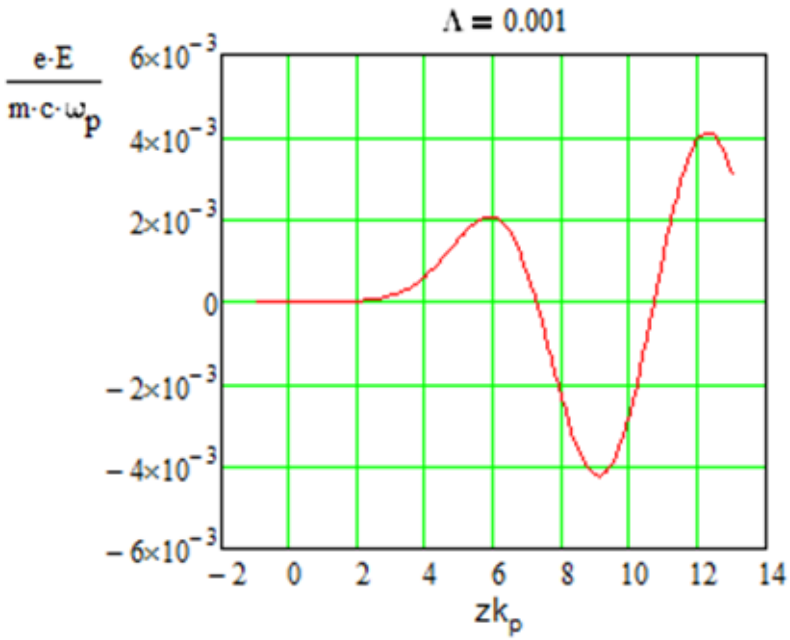
- Paper presents numerical simulation of Gaussian bunch wakefield in uniform plasma for optimal bunch length of $\sigma_s = \sqrt{2}/k_p$. It parameterizes deceleration force through parameter

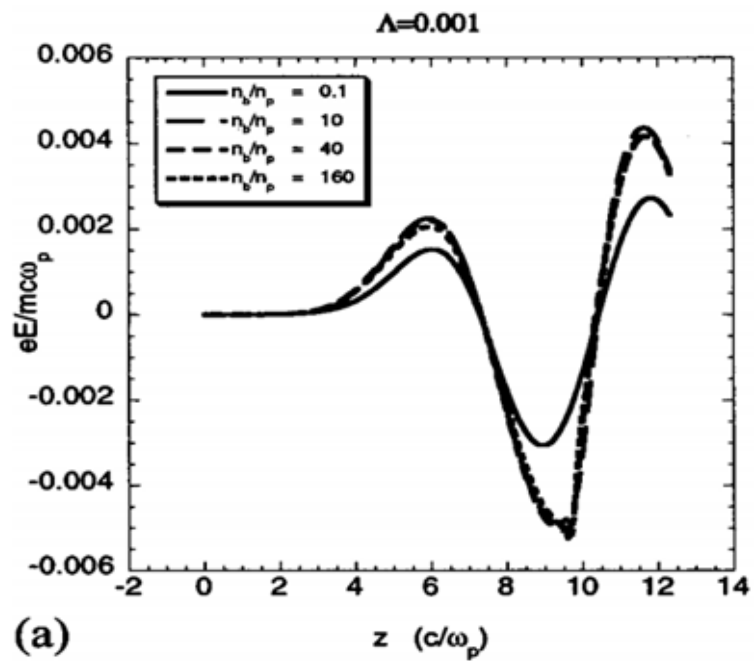
$$\Lambda = \sqrt{\frac{2}{\pi}}\frac{r_e N_e}{\sigma_s} \quad \text{=> for } e^-: \quad \frac{eE}{m_e c\omega_p} = \Lambda L_c \int_s^\infty \cos\left(k_p(s-z)\right) e^{-z^2/(2\sigma_s)}, \quad L_c = \ln\left(1+\frac{1}{\sigma_s k_p \sqrt{\Lambda}}\right)$$

Limits of linear plasma wakefield theory for electron or positron beams

W. Lu[1], C. Huang[1], M. M. Zhou[1], W. B. Mori[2] and T. Katsouleas[3]

Phys. Plasmas 12, 063101 (2005); http://dx.doi.org/10.1063/1.1905587
http://www.slac.stanford.edu/grp/arb/tn/arbvol5/AARD454.pdf





Wakefield for e-bunch, $\Lambda=10^{-3}$ (L_c=3.1). Bunch center at z=6. Good coincedence for dence e-beam.

Deceleration in the Blowup Regime

- To get a rough estimate we assume

$$L_c \approx \ln\left(\frac{r_b + k_p^{-1}}{r_b}\right) \approx \frac{1}{r_b k_p}, \quad \rho_b \ge k_p^{-1}$$

where r_b is the bubble radius

- Then we obtain

$$\frac{dE}{ds} \approx \frac{4\pi n_e N_e e^4}{mc^2} L_c F_g(s) \rightarrow \frac{N_e e^2 k_p}{r_b} F_g(s) \quad \Rightarrow \quad \boxed{W_{\parallel}(s) \approx \frac{2k_p}{r_b}}$$

Here we neglect that the electron density near bubble boundary is somewhat higher. That should increase the wake by a factor $\approx \sqrt{1 + r_b k_p / 2}$, Keeping it would exceed an accuracy in such rough estimate. We also assume non-relativistic motion of electrons.

Transverse Wake

- To make an estimate we assume linear regime with following potential and density perturbations:

$$\varphi = 2\pi\sigma_\perp^2 e n_e \frac{\Delta n}{n}\cos(kz - \omega t)\,e^{-r^2/2\sigma_\perp^2}$$

$$n = n_e \frac{\Delta n}{n}\cos(kz - \omega t)\,e^{-r^2/2\sigma_\perp^2}\left(1 - \frac{r^2}{2\sigma_\perp^2}\right)$$

- Let the bunch fraction with length ΔL be displaced by Δx, then in the beam frame it creates additional asymmetric kick to the plasma electrons:

$$\Delta\theta = \frac{2e^2 N_e}{\gamma^2 m_e c^2}\frac{\Delta s}{L_b}\frac{\Delta x}{r^2}\cos\phi$$

- $\cos(\phi)$ displacement of plasma electrons located in cylinder with radii [r,r+dr] creates electric field:

$$dE(s) = 2\pi e \frac{dn_e}{dr}\Delta X(s)\,dr\,, \quad \Delta X(s) = \Delta\theta\, s$$

- Integrating we obtain:

$$\frac{\Delta E}{\Delta x} = 2\pi e \frac{2e^2 N_e}{\gamma^2 m_e c^2} s \frac{\Delta s}{L_b}\int_{\rho_{min}}^{\infty}\frac{dn_e}{dr}\frac{1}{r^2}dr$$

- Returning to the lab frame, performing integration and taking that $\omega = \omega_p/\sqrt{2}$ we finally obtain the transverse wake function in the logarithmic approximation:

$$W_\perp(s) \approx 2\sqrt{2}\,\frac{\Delta n}{n}\frac{\mathrm{k}_p \sin(\mathrm{k}_p s/\sqrt{2})}{\sigma_\perp^2}\ln\left(\frac{1}{k_p\rho_{min}}\right), \quad \rho_{min} = \sqrt{\frac{N_e e^2 L_b}{m_e c^2}}$$

Transverse Wake (continue)

- Comparing it with focusing strength of plasma channel we obtain at the bunch end

$$\frac{E_{wake}(s=L_b)}{E_{plasma\_foc}} \approx \frac{2L_b}{\sigma_\perp} \frac{(dE/ds)_{loss}}{(dE/ds)_{\max}} \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right)$$

where $(dE/ds)_{loss}$ – average energy loss in plasma

$(dE/ds)_{max}$ - maximum accelerating field for given plasma density

- for 50% beam loading and blowup regime

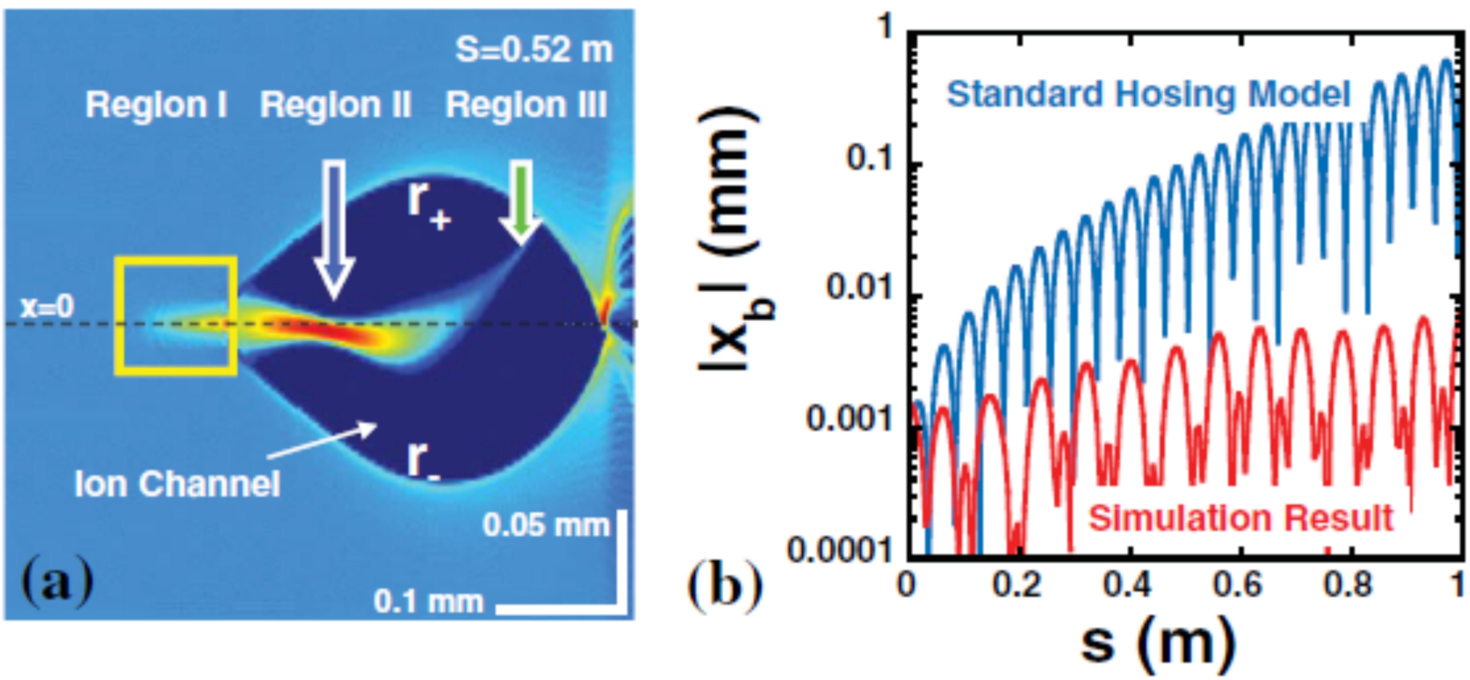
$$\frac{E_{wake}(s=L_b)}{E_{plasma\_foc}} \approx 1$$

View of Enthusiastic Plasma Collider Folks

PRL 99, 255001 (2007) PHYSICAL REVIEW LETTERS week ending 21 DECEMBER 2007

Hosing Instability in the Blow-Out Regime for Plasma-Wakefield Acceleration

C. Huang,[1] W. Lu,[1] M. Zhou,[1] C. E. Clayton,[1] C. Joshi,[1] W. B. Mori,[1] P. Muggli,[2] S. Deng,[2] E. Oz,[2] T. Katsouleas,[2] M. J. Hogan,[3] I. Blumenfeld,[3] F. J. Decker,[3] R. Ischebeck,[3] R. H. Iverson,[3] N. A. Kirby,[3] and D. Walz[3]



Deduced parameters: $\gamma=6\cdot10^4$, E=32 GeV, ΔE=[-2.5, +5]GeV

View of Enthusiastic Plasma Collider Folks (continue)

Input parameters (Gaussian beam)

$n_e := 2\cdot10^{16}$ cm^{-3}

$n_{eb} := 25.9\cdot n_e$

$\sigma_s := \dfrac{1.2}{k_p}$

$\omega_p := \sqrt{\dfrac{4\cdot\pi\cdot n_e\cdot e_{SGS}^{\;2}}{M_e}}$

$\sigma_r := \dfrac{0.19}{k_p}$

$k_p := \dfrac{\omega_p}{c}$

$N_e := (2\cdot\pi)^{1.5}\cdot n_{eb}\cdot\sigma_s\cdot\sigma_r^{\;2}$

$\rho_{min} := \sqrt{\dfrac{2\cdot\sqrt{2}\cdot N_e\cdot e_{SGS}^{\;2}\sigma_s}{\sqrt{\pi}\cdot M_e\cdot c^2}}$

$L_c := \ln\left(1 + \dfrac{\sqrt{2}}{\rho_{min}\cdot k_p}\right)$

$10^4\cdot\dfrac{2\cdot\pi}{k_p} = 234.688$ μm

$\sigma_s\cdot10^4 = 44.822$ μm

$\sigma_r\cdot10^4 = 7.097$ μm

$N_e = 1.842\times10^{10}$

$k_p^{-1}\cdot10^4 = 37.352$ μm

$\rho_{min}\cdot10^4 = 61.293$ μm

$L_c = 0.622$

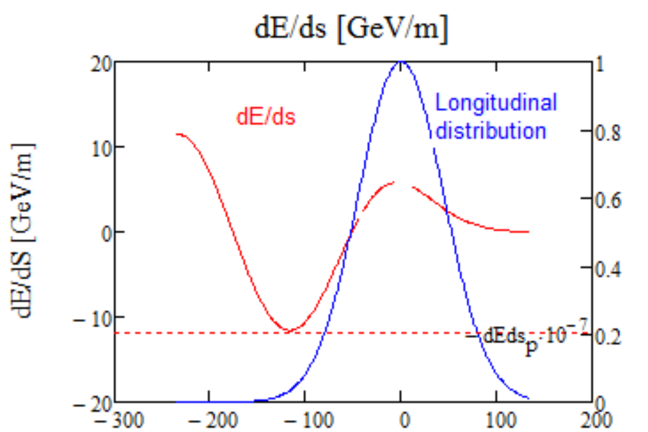
Decelerating rate

$$dEds(s) := \frac{8\pi\cdot n_e\cdot e_{SGS}^{\;3}\cdot e_{conv}\cdot N_e}{M_e\cdot c^2}\cdot\ln\left(1 + \frac{\sqrt{2}}{k_p\cdot\rho_{min}}\right)\cdot\frac{1}{\sqrt{2\cdot\pi}\cdot\sigma_s}\cdot\int_s^{\infty}\cos\left[k_p\cdot(s-z)\right]\cdot\exp\left(-\frac{z^2}{2\cdot\sigma_s^{\;2}}\right)dz$$

$s := -\dfrac{2\cdot\pi}{k_p}, -\dfrac{2\cdot\pi}{k_p} + 10^{-4} .. 3\cdot\sigma_s$

$$dEds_p := \frac{4\cdot\pi\cdot n_e\cdot N_e\cdot e_{SGS}^{\;2}\cdot e_{conv}^{\;2}}{m_e}L_c$$

$dEds_p\cdot10^{-7} = 11.673$ GeV/m



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Conclusions

- Presently presented single bunch colliders cannot support high efficiency acceleration and are not competitive to CLIC or ILC technology
- Colliders based on the dielectric based strictures with multiple bunches look as a possibility
  - However in practical conditions they do not promise accelerating gradients much higher than the already achieved gradients in a metallic structure (250 MeV/m - SLAC)
- Plasma based linac cannot accelerate bright positron bunches => e^+e^- collider does not look feasible
  - Plasma acceleration can support $\gamma\gamma$ collider with moderate luminosity and efficiency
- Interesting workshop, a lot if interesting physics
  - but it did not change our understanding of collider feasibility