Inclusive deuteron–induced reactions

Grégory Potel Aguilar (NSCL/FRIB)
Filomena Nunes (NSCL)
Ian Thompson (LLNL)

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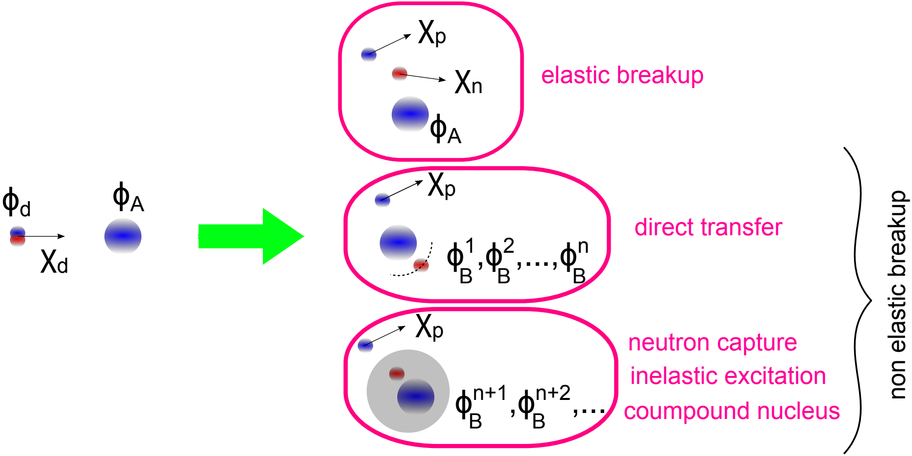
let's concentrate in the reaction A+d→ B(=A+n)+p



we are interested in the inclusive cross section, *i.e.*, we will sum over all final states ϕ_B^c.

Derivation of the differential cross section

the double differential cross section with respect to the proton energy and angle for the population of a specific final ϕ_B^c

$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d}\rho(E_p)\left|\left\langle \chi_p \phi_B^c | V \,|\, \Psi^{(+)} \right\rangle\right|^2 .$$

Sum over all channels, with the approximation $\Psi^{(+)} \approx \chi_d \phi_d \phi_A$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2\pi}{\hbar v_d}\rho(E_p)$$
$$\times \sum_c \langle\, \chi_d \phi_d \phi_A |\, V \,|\chi_p \phi_B^c\rangle\, \delta(E - E_p - E_B^c)\, \langle \phi_B^c \chi_p |\, V \,|\, \phi_A \chi_d \phi_d \rangle$$

χ_d $\rightarrow$ deuteron incoming wave, ϕ_d $\rightarrow$ deuteron wavefunction,
χ_p $\rightarrow$ proton outgoing wave ϕ_A $\rightarrow$ target core ground state.

Sum over final states

the imaginary part of the Green's function G is an operator representation of the δ–function,

$$\pi\delta(E - E_p - E_B^c) = \lim_{\epsilon\to 0} \Im \sum_c \frac{|\phi_B^c\rangle\langle\phi_B^c|}{E - E_p - H_B + i\epsilon} = \Im G$$

$$\frac{d^2\sigma}{d\Omega_p dE_p} = -\frac{2}{\hbar v_d}\rho(E_p)\Im\langle\chi_d\phi_d\phi_A|\, V\, |\chi_p\rangle\, G\, \langle\chi_p|\, V\, |\phi_A\chi_d\phi_d\rangle$$

- We got rid of the (infinite) sum over final states,
- but G is an extremely complex object!
- We still need to deal with that.

Optical reduction of G

If the interaction V do not act on ϕ_A

$$
\begin{aligned}
\langle \chi_d \phi_d \phi_A | \, V \, | \chi_p \rangle G \langle \chi_p | \, V \, | \phi_A \chi_d \phi_d \rangle & \\
&= \langle \chi_d \phi_d | \, V \, | \chi_p \rangle \langle \phi_A | G | \phi_A \rangle \langle \chi_p | \, V \, | \chi_d \phi_d \rangle \\
&= \langle \chi_d \phi_d | \, V \, | \chi_p \rangle \, G_{opt} \, \langle \chi_p | \, V \, | \chi_d \phi_d \rangle \, ,
\end{aligned}
$$

where G_{opt} is the optical reduction of G

$$
G_{opt} = \lim_{\epsilon \to 0} \frac{1}{E - E_p - T_n - U_{An}(r_{An}) + i\epsilon},
$$

now $U_{An}(r_{An}) = V_{An}(r_{An}) + iW_{An}(r_{An})$ and thus G_{opt} are single–particle, tractable operators.

The effective neutron–target interaction $U_{An}(r_{An})$, a.k.a. optical potential, a.k.a. self–energy can be provided by structure calculations

Capture and elastic breakup cross sections

the imaginary part of G_{opt} splits in two terms

$$\Im G_{opt} = \overbrace{-\pi \sum_{k_n} |\chi_n\rangle \delta\left(E - E_p - \frac{k_n^2}{2m_n}\right) \langle\chi_n|}^{\text{elastic breakup}} + \overbrace{{G_{opt}}^{\dagger} W_{An}\, G_{opt}}^{\text{non elastic breakup}},$$

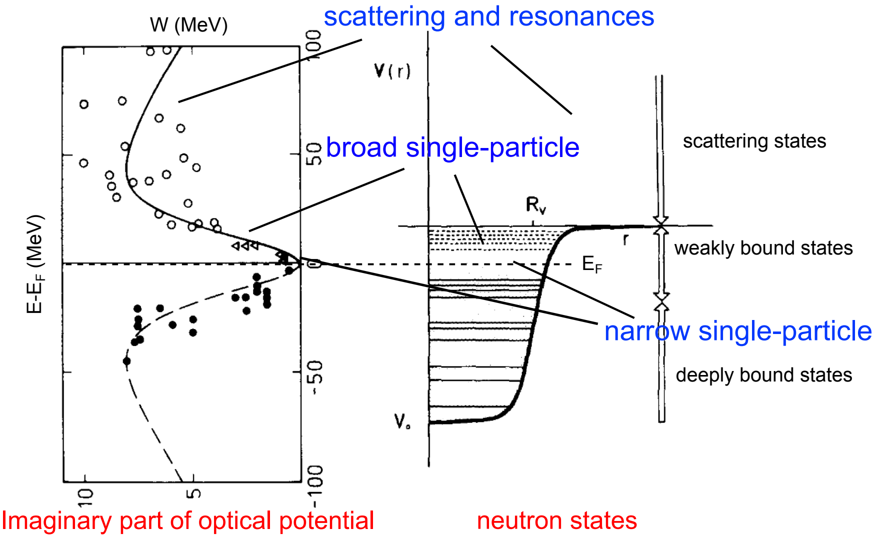
we define the neutron wavefunction $|\psi_n\rangle = G_{opt} \langle\chi_p| V |\chi_d\phi_d\rangle$

cross sections for non elastic breakup (NEB) and elastic breakup (EB)

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{NEB} = -\frac{2}{\hbar v_d}\rho(E_p) \langle\psi_n| W_{An} |\psi_n\rangle \,,$$

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{EB} = -\frac{2}{\hbar v_d}\rho(E_p)\rho(E_n) \left|\langle\chi_n\chi_p| V |\chi_d\phi_d\rangle\right|^2 \,,$$

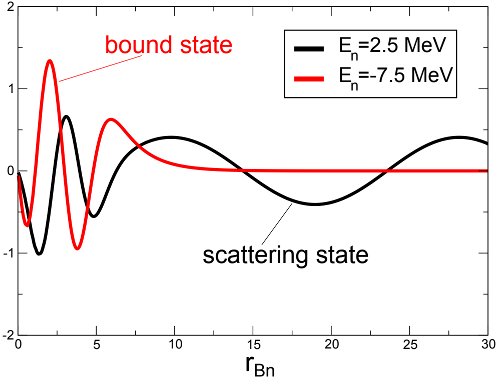
Neutron states in nuclei



Mahaux, Bortignon, Broglia and Dasso Phys. Rep. **120** (1985) 1 the neutron wavefunctions

$$|\psi_n\rangle = G_{opt} \langle \chi_p | V | \chi_d \phi_d \rangle$$

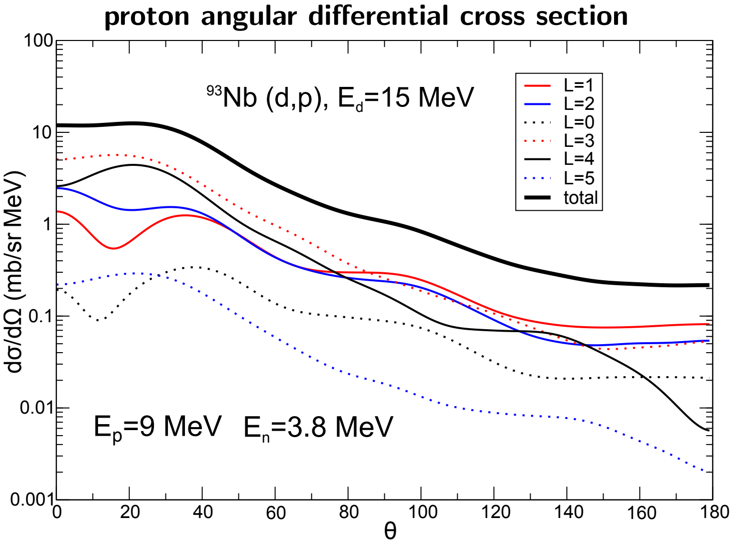
can be computed for **ANY** neutron energy, positive or negative



transfer to resonant and non-resonant continuum well described

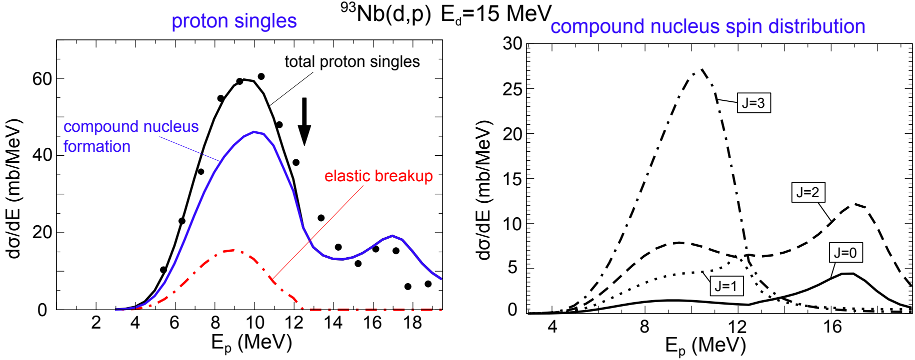
$|\psi_n\rangle$ are the solutions of an inhomogeneous Schrödinger equation

$$(H_{An} - E_{An}) |\psi_n\rangle = \langle \chi_p | V | \chi_d \phi_d \rangle$$


proton angular differential cross section
93Nb (d,p), Ed=15 MeV
Ep=9 MeV En=3.8 MeV
L=1
L=2
L=0
L=3
L=4
L=5
total
dσ/dΩ (mb/sr MeV)
θ
100
10
1
0.1
0.01
0.001
0
20
40
60
80
100
120
140
160
180

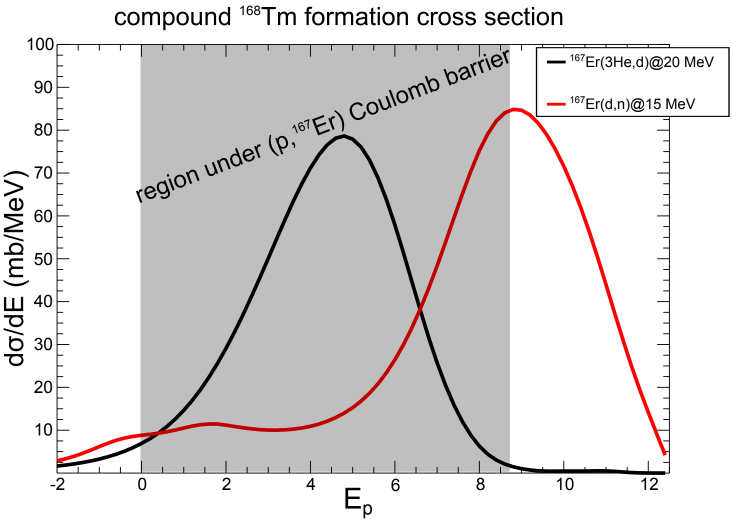
Disentangling elastic and non elastic breakup

^{93}Nb(d,p) (Mastroleo *et al.*, Phys. Rev. C **42** (1990) 683)



- We obtain spin–parity distributions for the compound nucleus.
- Contributions from elastic and non elastic breakup disentangled.

Dropping a proton



We can also transfer charged clusters

neutron transfer limit (isolated–resonance, first–order approximation)

Let's consider the limit $W_{An} \to 0$ (single–particle width $\Gamma \to 0$). For an energy E such that $|E - E_n| \ll D$, (isolated resonance)

$$G_{opt} \approx \lim_{W_{An}\to 0} \frac{|\phi_n\rangle\langle\phi_n|}{E - E_p - E_n - i\langle\phi_n|W_{An}|\phi_n\rangle};$$

with $|\phi_n\rangle$ eigenstate of $H_{An} = T_n + \Re(U_{An})$

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim \lim_{W_{An}\to 0} \langle\, \chi_d\phi_d|\, V\, |\chi_p\rangle \\ \times \frac{|\phi_n\rangle\langle\phi_n|W_{An}|\phi_n\rangle\langle\phi_n|}{(E - E_p - E_n)^2 + \langle\phi_n|W_{An}|\phi_n\rangle^2} \langle\chi_p|\, V\, |\, \chi_d\phi_d\rangle\,,$$

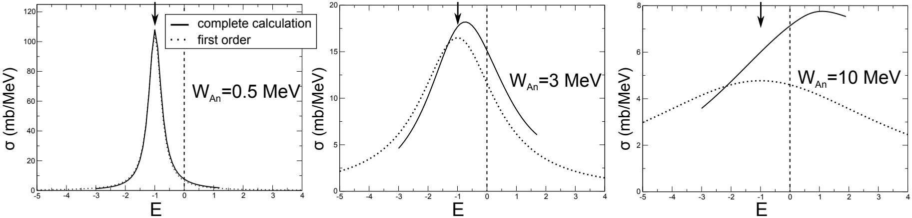
we get the direct transfer cross section:

$$\frac{d^2\sigma}{d\Omega_p dE_p} \sim |\,\langle\chi_p\phi_n|\, V\, |\,\chi_d\phi_d\rangle\,|^2\delta(E - E_p - E_n)$$

Validity of first order approximation

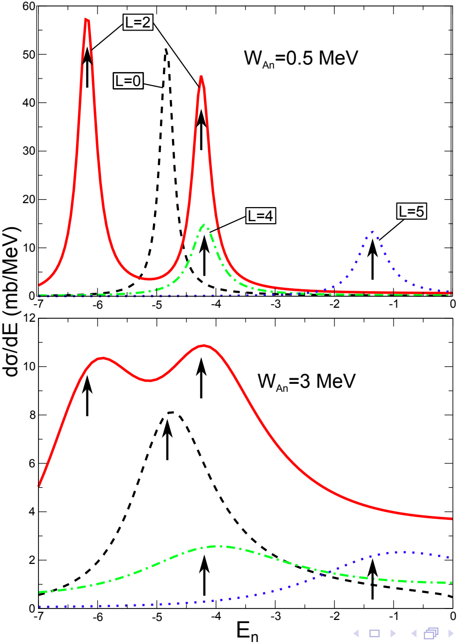
For W_{An} small, we can apply first order perturbation theory,

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}(E,\Omega)\right]^{NEB} \approx \frac{1}{\pi}\frac{\langle\phi_n|W_{An}|\phi_n\rangle}{(E_n-E)^2+\langle\phi_n|W_{An}|\phi_n\rangle^2}\left.\frac{d\sigma_n}{d\Omega}(\Omega)\right]^{transfer}$$



we compare the complete calculation with the isolated–resonance, first–order approximation for $W_{An} = 0.5$ MeV, $W_{An} = 3$ MeV and $W_{An} = 10$ MeV

Spectral function and absorption cross section



Austern (post)–Udagawa (prior) formalisms

The interaction V can be taken either in the *prior* or the *post* representation,

- Austern (post)$\rightarrow V \equiv V_{post} \sim V_{pn}(r_{pn})$ (recently revived by Moro and Lei, from Sevilla and Carlson from São Paulo)
- Udagawa (prior) $\rightarrow V \equiv V_{prior} \sim V_{An}(r_{An}, \xi_{An})$ (used in calculations showed here)

in the prior representation, V can act on $\phi_A \rightarrow$ the optical reduction gives rise to new terms:

$$\left.\frac{d^2\sigma}{d\Omega_p dE_p}\right]^{post} = -\frac{2}{\hbar v_d}\rho(E_p)\left[\Im\left\langle \psi_n^{prior}\right| W_{An}\left|\psi_n^{prior}\right\rangle + 2\Re\left\langle \psi_n^{NON}\right|W_{An}\left|\psi_n^{prior}\right\rangle + \left\langle \psi_n^{NON}\right| W_{An}\left|\psi_n^{NON}\right\rangle\right],$$

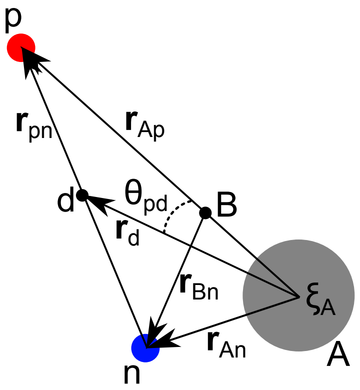
where $\psi_n^{NON} = \langle\chi_p| \; \chi_d\phi_d\rangle$.

The nature of the 2–step process depends on the representation

Summary, conclusions and some prospectives

- We have presented a reaction formalism for inclusive deuteron–induced reactions.
- Valid for final neutron states from Fermi energy $\rightarrow$ to scattering states
- Disentangles elastic and non elastic breakup contributions to the proton singles.
- Probe of nuclear structure in the continuum.
- Provides spin–parity distributions.
- Useful for surrogate reactions.
- Need for optical potentials.
- Need to address non–locality.
- Can be generalized to other three–body problems.
- Can be extended for (p, d) reactions (hole states).

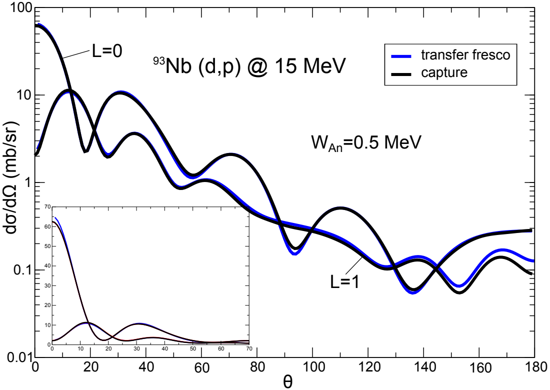
The 3–body model



From H to H_{3B}

- $H = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + V_{An}(r_{An}, \xi_A) + V_{Ap}(r_{Ap}, \xi_A)$
- $H_{3B} = T_p + T_n + H_A(\xi_A) + V_{pn}(r_{pn}) + U_{An}(r_{An}) + U_{Ap}(r_{Ap})$

Observables: angular differential cross sections (neutron bound states)

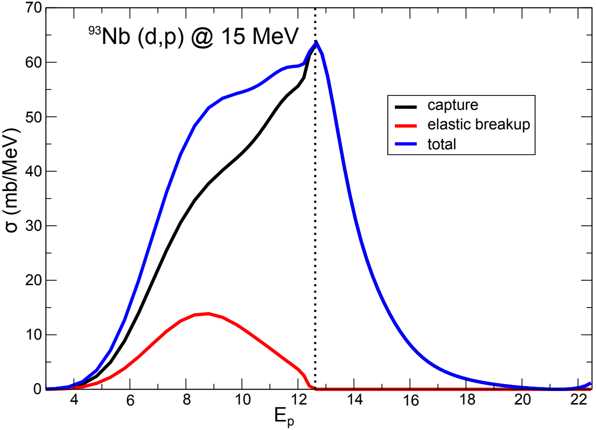


- capture at resonant energies compared with
- direct transfer (FRESCO) calculations,
- capture cross sections rescaled by a factor $\langle \phi_n | W_{An} | \phi_n \rangle \pi$.

double proton differential cross section

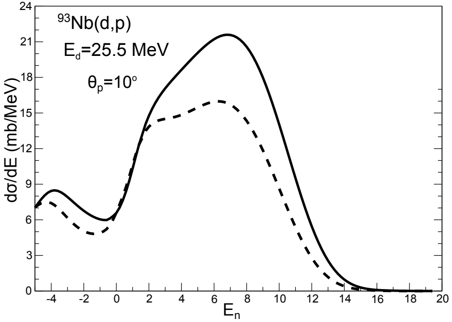
$$\frac{d^2\sigma}{d\Omega_p dE_p} = \frac{2\pi}{\hbar v_d} \rho(E_p) \sum_{l,m,l_p} \int \left| \varphi_{lml_p}(r_{Bn}; k_p) Y^{l_p}_{-m}(\theta_p) \right|^2 W(r_{An})\ dr_{Bn}.$$

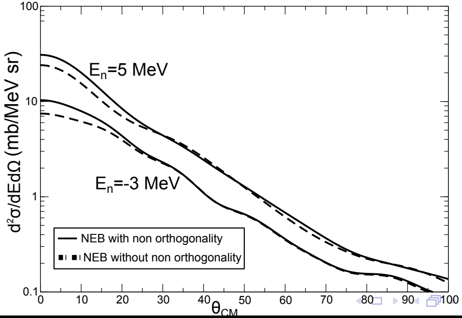
Observables: elastic breakup and capture cross sections



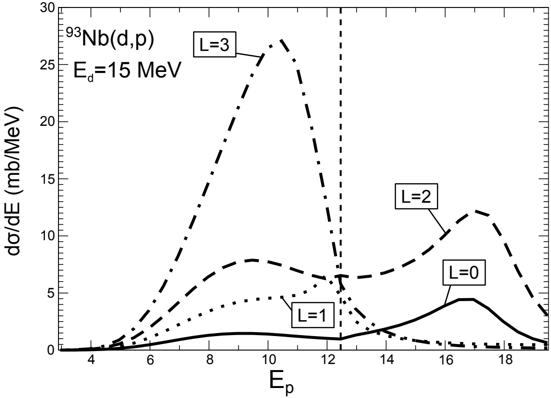
elastic breakup and capture cross sections as a function of the proton energy. The Koning–Delaroche global optical potential has been used as the U_{An} interaction (Koning and Delaroche, Nucl. Phys. A **713** (2003) 231).

Non–orthogonality term





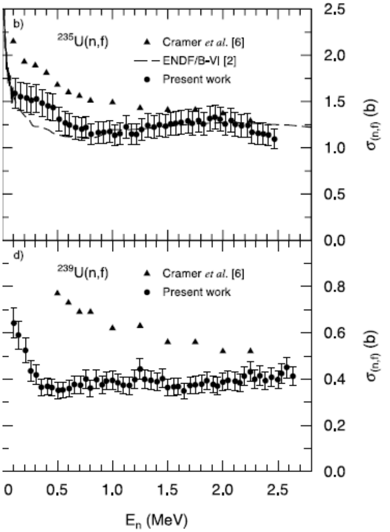
Obtaining spin distributions



spin distribution of compound nucleus

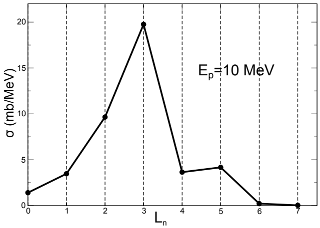
$$\frac{d\sigma_l}{dE_p} = \frac{2\pi}{\hbar v_d}\rho(E_p)\sum_{l_p,m}\int \left|\varphi_{lml_p}(r_{Bn};k_p)\right|^2 W(r_{An})\, dr_{Bn}.$$

Getting rid of Weisskopf–Ewing approximation

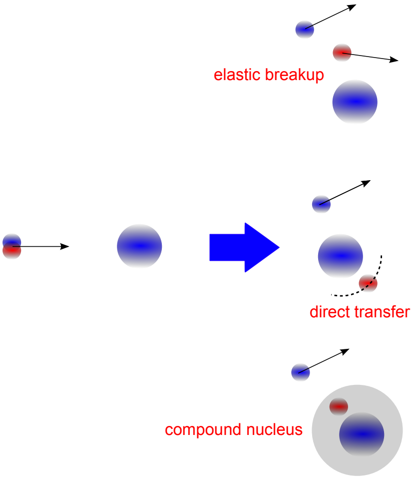


Younes and Britt, PRC **68**(2003)034610

- Weisskopf–Ewing approximation: $P(d,nx) = \sigma(E)G(E,x)$
- inaccurate for $x=\gamma$ and for $x=f$ in the low–energy regime
- can be replaced by $P(d,nx) = \sum_{J,\pi} \sigma(E,J,\pi)G(E,J,\pi,x)$ if $\sigma(E,J,\pi)$ can be predicted.



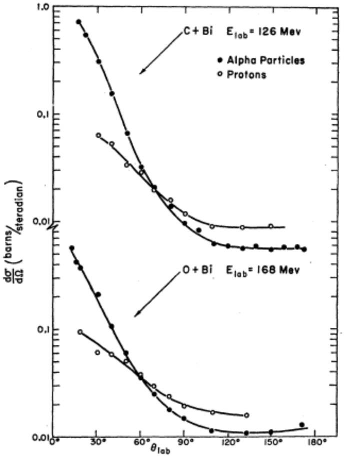
We present a formalism for inclusive deuteron–induced reactions. We thus want to describe within the same framework:



- Direct neutron transfer: should be compatible with existing theories.
- Elastic deuteron breakup: **"transfer" to continuum states**.
- Non elastic breakup (direct transfer, inelastic excitation and compound nucleus formation): **absorption above and below neutron emission threshold**.
- Important application in surrogate reactions: **obtain spin–parity distributions, get rid of Weisskopf–Ewing approximation**.

Historical background

breakup-fusion reactions

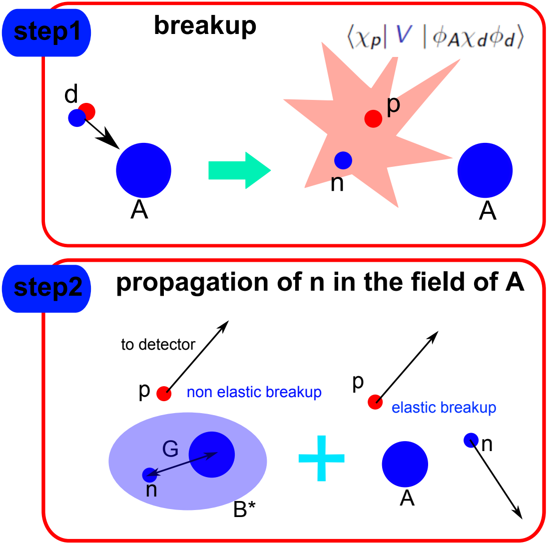


Britt and Quinton, Phys. Rev. **124** (1961) 877

protons and α yields bombarding ^{209}Bi with ^{12}C and ^{16}O

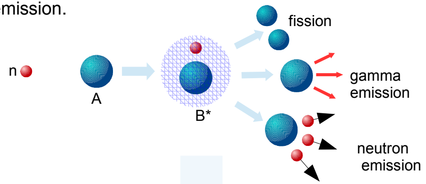
- Kerman and McVoy, Ann. Phys. **122** (1979)197
- Austern and Vincent, Phys. Rev. C**23** (1981) 1847
- Udagawa and Tamura, Phys. Rev. C**24**(1981) 1348
- Last paper: Mastroleo, Udagawa, Mustafa Phys. Rev. C**42** (1990) 683
- Controversy between Udagawa and Austern formalism left somehow unresolved.

2–step process (post representation)

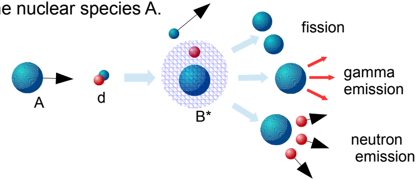


Surrogate for neutron capture

* Desired reaction: neutron induced fission, gamma emission and neutron emission.



* The surrogate method consists in producing the same compound nucleus B* by bombarding a deuteron target with a radio active beam of the nuclear species A.

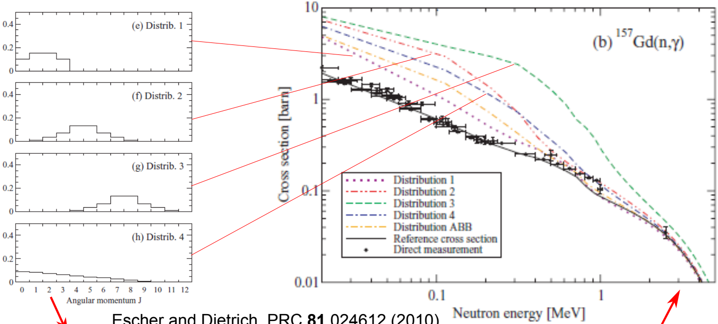


* **A theoretical reaction formalism that describes the production of all open channels B* is needed.**

Weisskopf–Ewing approximation

$$\sigma_{\alpha\chi}(E_a) = \sum_{J,\pi} \sigma_\alpha^{\mathrm{CN}}(E_{\mathrm{ex}}, J, \pi) G_\chi^{\mathrm{CN}}(E_{\mathrm{ex}}, J, \pi) \xrightarrow{\text{W-E approximation}} \sigma_{\alpha\chi}^{\mathrm{WE}}(E_a) = \sigma_\alpha^{\mathrm{CN}}(E_{\mathrm{ex}}) \mathcal{G}_\chi^{\mathrm{CN}}(E_{\mathrm{ex}})$$

Weisskopf-Ewing approximation: probability of γ decay independent of J,π



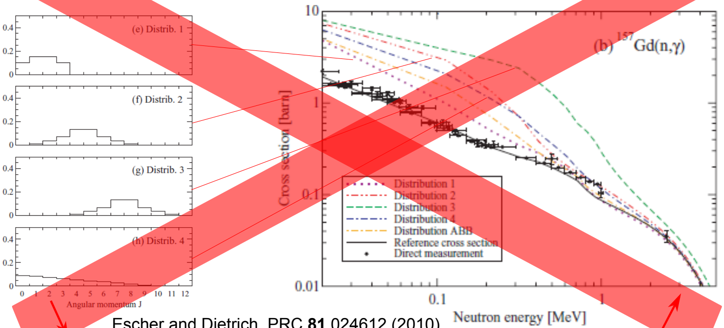
Escher and Dietrich, PRC **81** 024612 (2010)

Different J,π → Different cross section for γ emission

Weisskopf–Ewing is inaccurate for (n, γ)

$$\sigma_{\alpha\chi}(E_a) = \sum_{J,\pi} \sigma_\alpha^{CN}(E_{ex}, J, \pi) G_\chi^{CN}(E_{ex}, J, \pi) \xrightarrow{\text{W-E approximation}} \sigma_{\alpha\chi}^{WE}(E_a) = \sigma_\alpha^{CN}(E_{ex}) \mathcal{G}_\chi^{CN}(E_{ex})$$

Weisskopf-Ewing approximation: probability of γ decay independent of J,π



Escher and Dietrich, PRC **81** 024612 (2010)

Different J,π → Different cross section for γ emission

We need theory to predict J, π distributions