

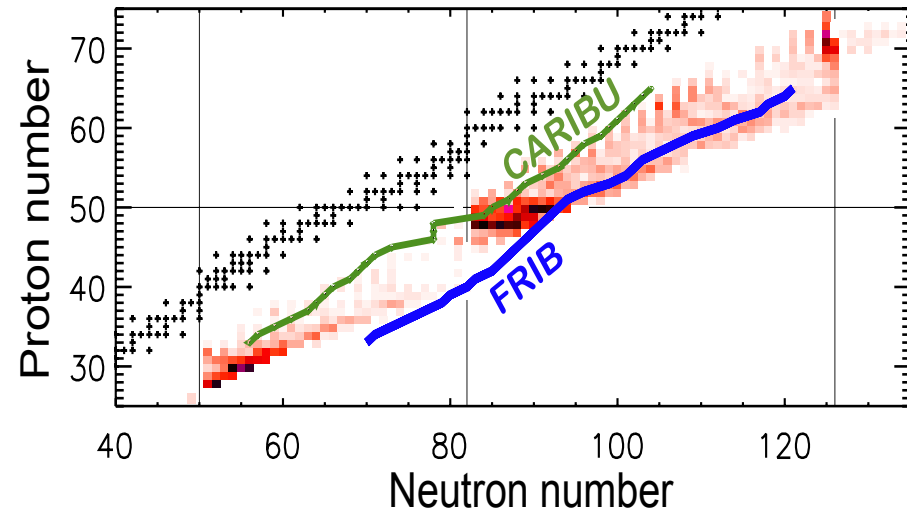
CHALLENGES IN THEORY FOR (D,P) REACTIONS

my very own perspective

Filomena Nunes, Michigan State University
International Collaborations in Nuclear Theory, 18 July 2016

Supported by: NNSA, NSF, DOE

FRIB in the near horizon



Reach into the r-process nuclei:
masses and detailed spectroscopy
of the r-process path nuclei

Reaction theory for heavy exotic nuclei

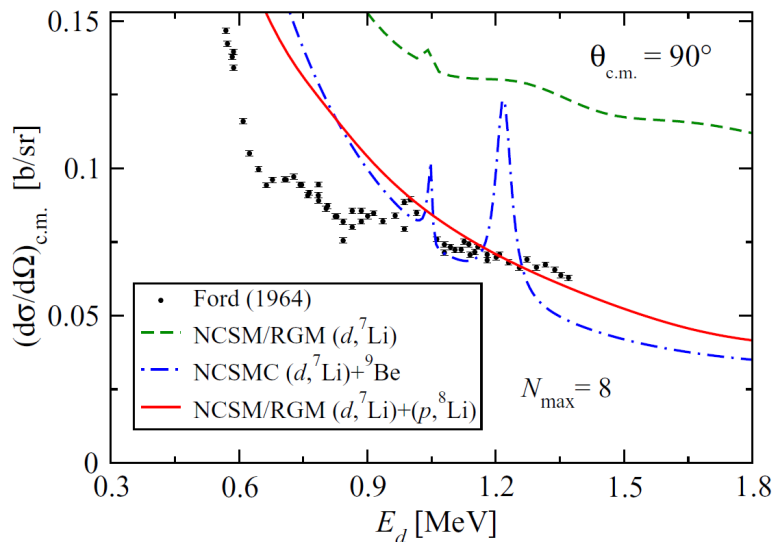
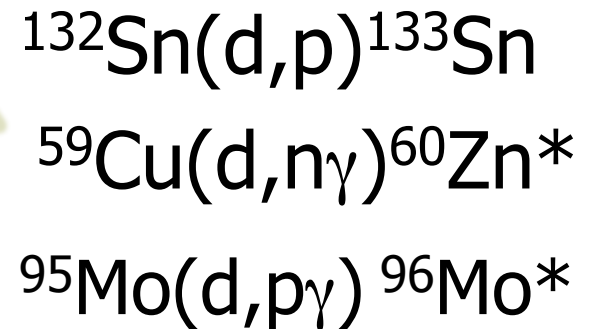


FIG. 7. Computed ${}^7\text{Li}(d,d){}^7\text{Li}$ differential cross sections in the c.m. frame at the deuteron scattering angle of 90° as function of the kinetic energy of deuterons in the laboratory system, compared to the experimental data of Ref. [39]. The three sets of theoretical curves correspond to calculations within the $(d, {}^7\text{Li})$ NCSM-RGM (green dashed line), $(d, {}^7\text{Li}) + {}^9\text{Be}$ NCSMC (blue dash-dotted line), and $(d, {}^7\text{Li}) + (p, {}^8\text{Li})$ NCSM-RGM (red solid line) model spaces.

PRC93, 054606 (2016)





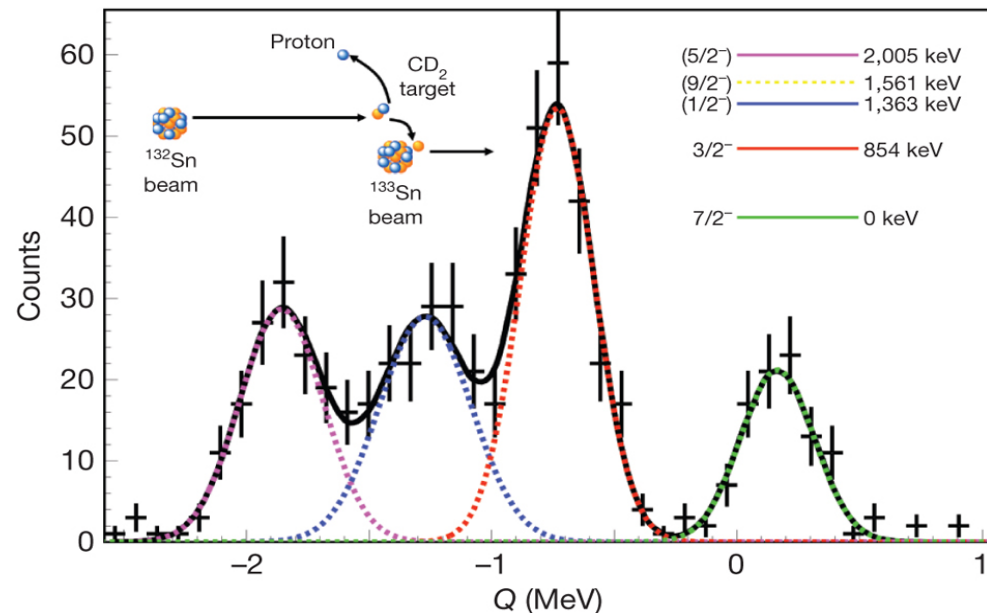
Outline

0. Starting point
1. Reduction to a few-body problem
2. Solving the few-body problem
3. Determining the effective interactions
4. Including non-locality
5. Quantifying uncertainties

Our starting point

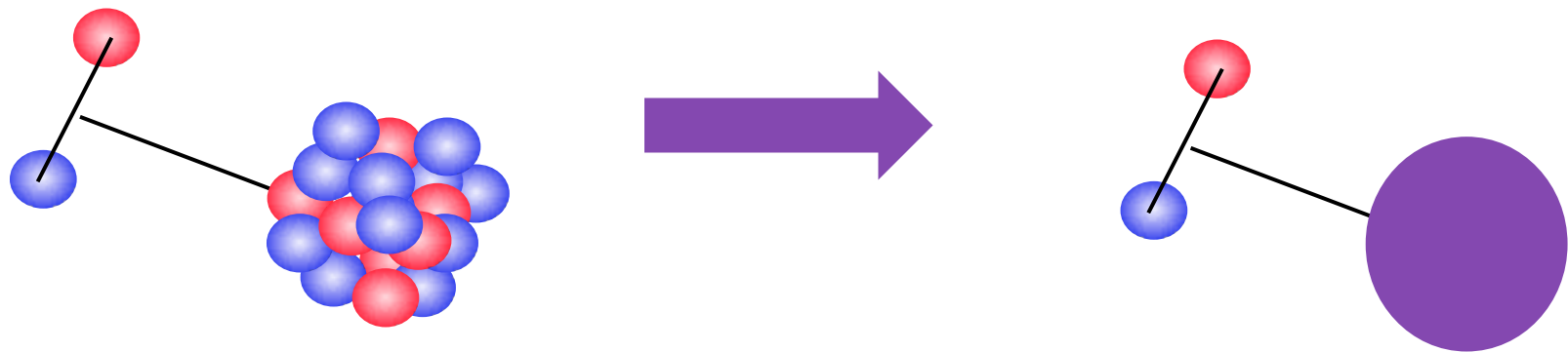
- A complex many-body problem
- Scattering boundary conditions
- Importance of thresholds
- Large Coulomb interactions
- Specific clustering

$d(^{132}\text{Sn}, ^{133}\text{Sn})p@5 \text{ MeV/u}$



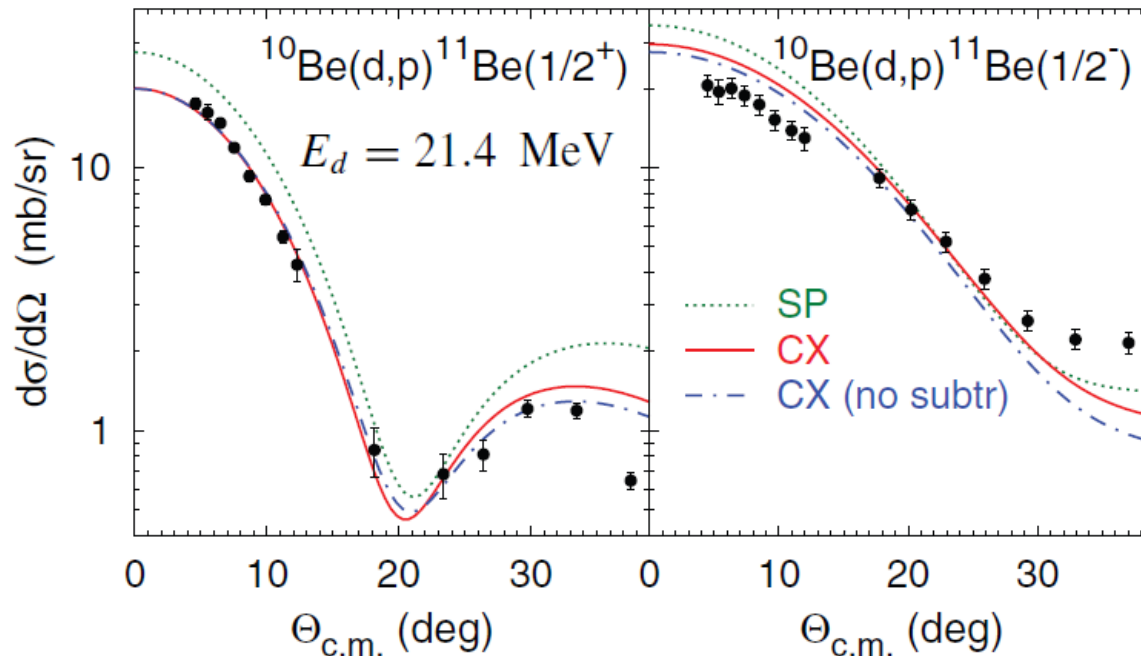
1. reduction to few-body

- Reducing the many-body problem to a few-body problem introduces effective interactions.
- How does the original many-body Hamiltonian relate to the few-body Hamiltonian?
- We assume that $\mathcal{H}_{3B} = T_{\mathbf{r}} + T_{\mathbf{R}} + U_{nA} + U_{pA} + V_{np}$
- What are these U_{nA} ? R.C. Johnson, ECT*, November 2014



1. reduction to few-body

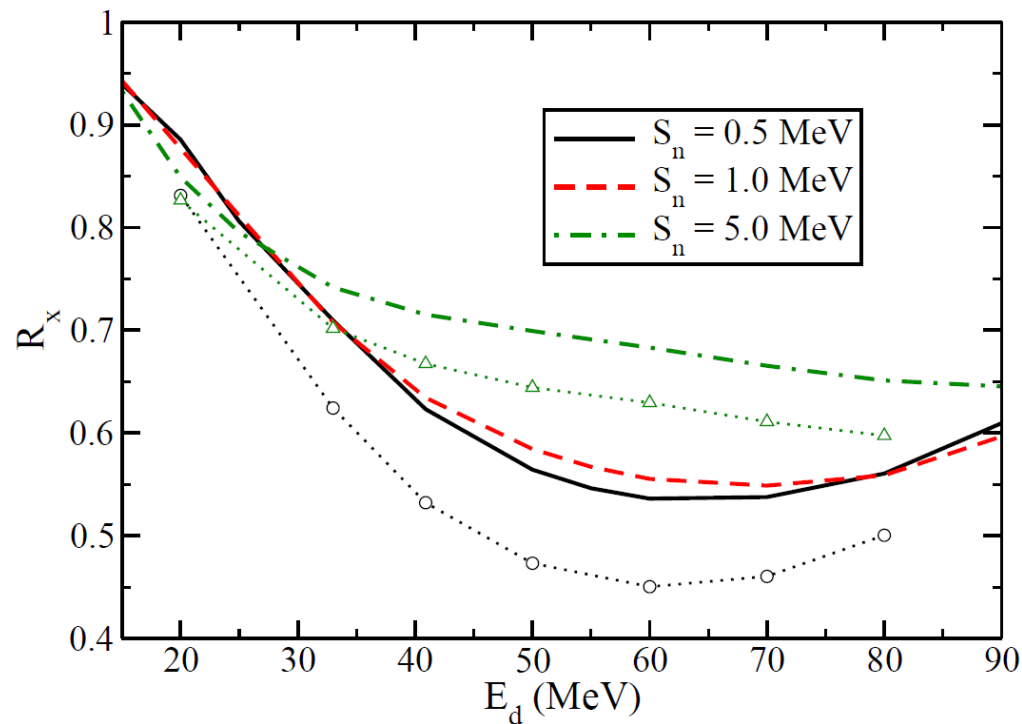
- The role of core excitation? (Summers, Nunes, Moro, Deltuva, etc)
- Seems to be important for (d,p) on loosely bound nuclei...



Deltuva, PRC91, 024607 (2015)

1. reduction to few-body

- The role of core excitation?

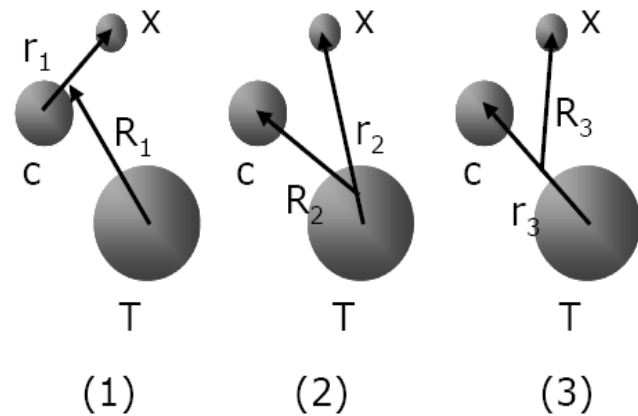


Ross, Deltuva, Nunes, in preparation

2. solving the few-body

Faddeev Formalism

$$\begin{aligned} (E - T_1 - V_{xc})\Psi^{(1)} &= V_{xc}(\Psi^{(2)} + \Psi^{(3)}) \\ (E - T_2 - V_{ct})\Psi^{(2)} &= V_{ct}(\Psi^{(3)} + \Psi^{(1)}) \\ (E - T_3 - V_{tx})\Psi^{(3)} &= V_{tx}(\Psi^{(1)} + \Psi^{(2)}) \end{aligned}$$



Benchmarking few-body methods

4N bound state

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	-128.33(10)	-25.94(5)	1.485(3)
CRCGV	102.30	-128.20	-25.90	1.482
SVM	102.35	-128.27	-25.92	1.486
HH	102.44	-128.34	-25.90(1)	1.483
GFMC	102.3(1.0)	-128.25(1.0)	-25.93(2)	1.490(5)
NCSM	103.35	-129.45	-25.80(20)	1.485
EIHH	100.8(9)	-126.7(9)	-25.944(10)	1.486

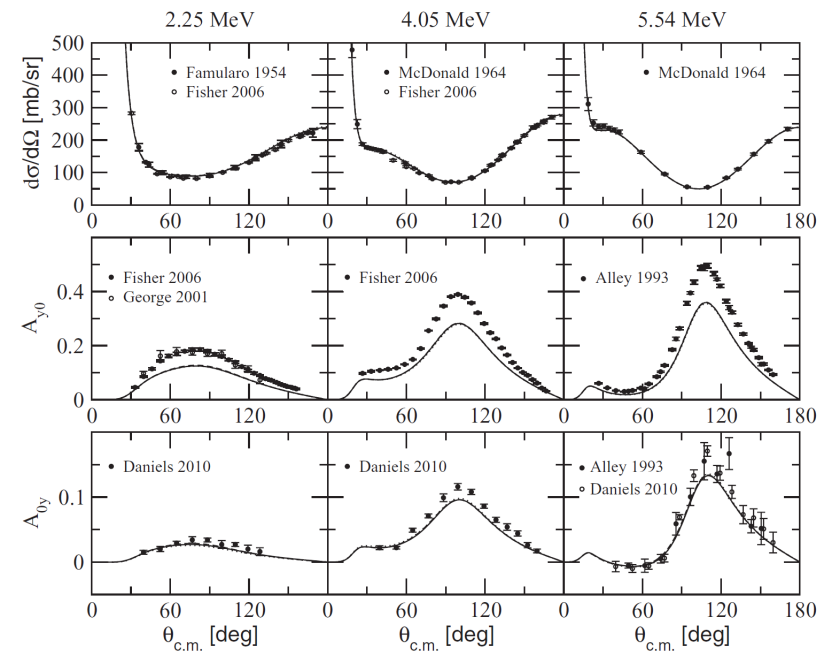
Method	S wave	P wave	D wave
FY	85.71	0.38	13.91
CRCGV	85.73	0.37	13.90
SVM	85.72	0.368	13.91
HH	85.72	0.369	13.91
NCSM	86.73	0.29	12.98
EIHH	85.73(2)	0.370(1)	13.89(1)

Benchmarking few-body methods

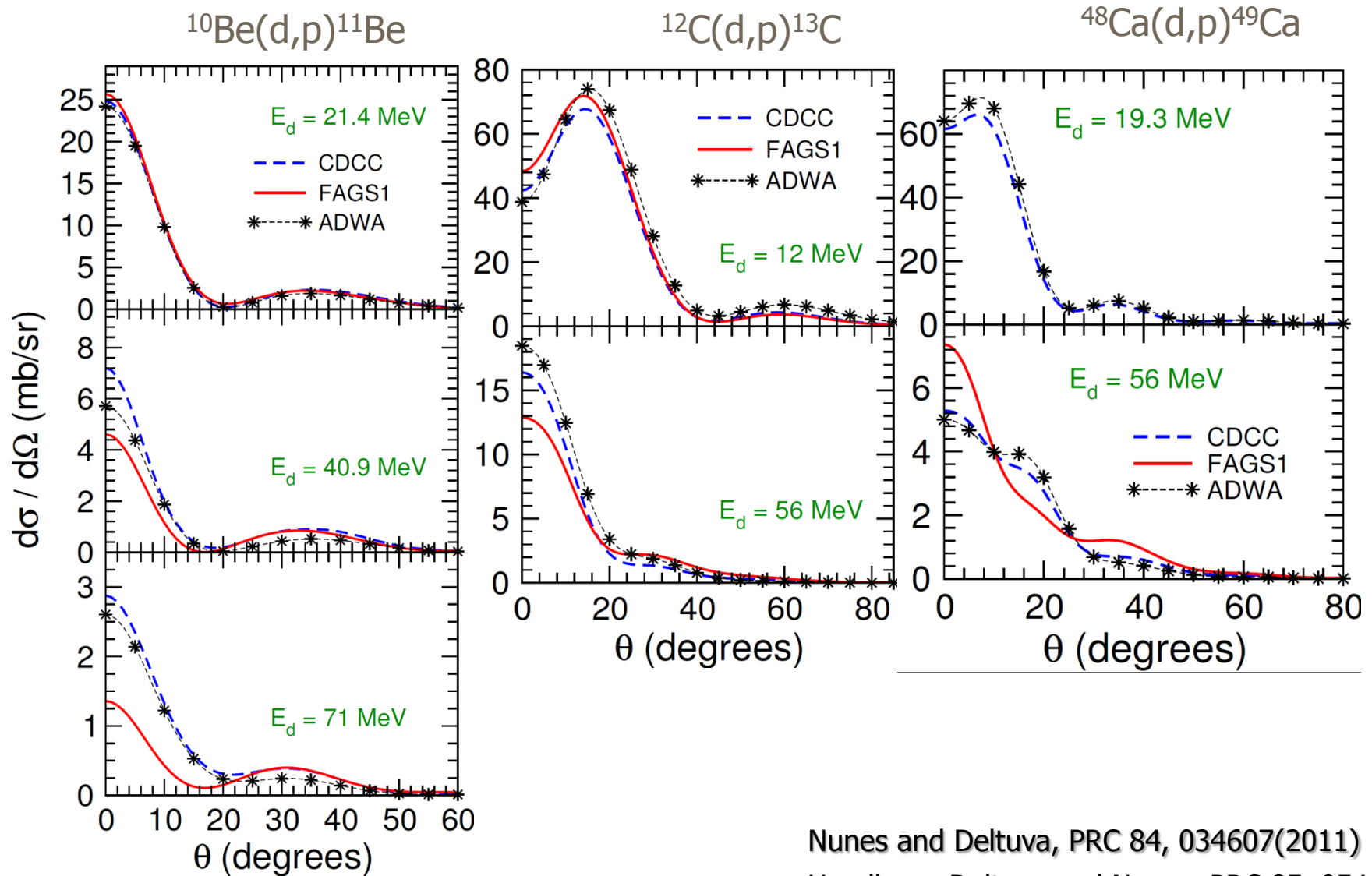
n-³He scattering

E_n	I-N3LO		Method
	σ_t	σ_t	
1.0	1.77	1.80	AGS
	1.77	1.78	HH
	1.81	1.81	FY
2.0	2.13	2.12	AGS
	2.13	2.10	HH
	2.19	2.13	FY
3.5	2.38	2.33	AGS
	2.38	2.32	HH
	2.41	2.33	FY
6.0	1.97	1.93	AGS
	1.97	1.93	HH
	1.97	1.92	FY

p-³He scattering (N3LO)



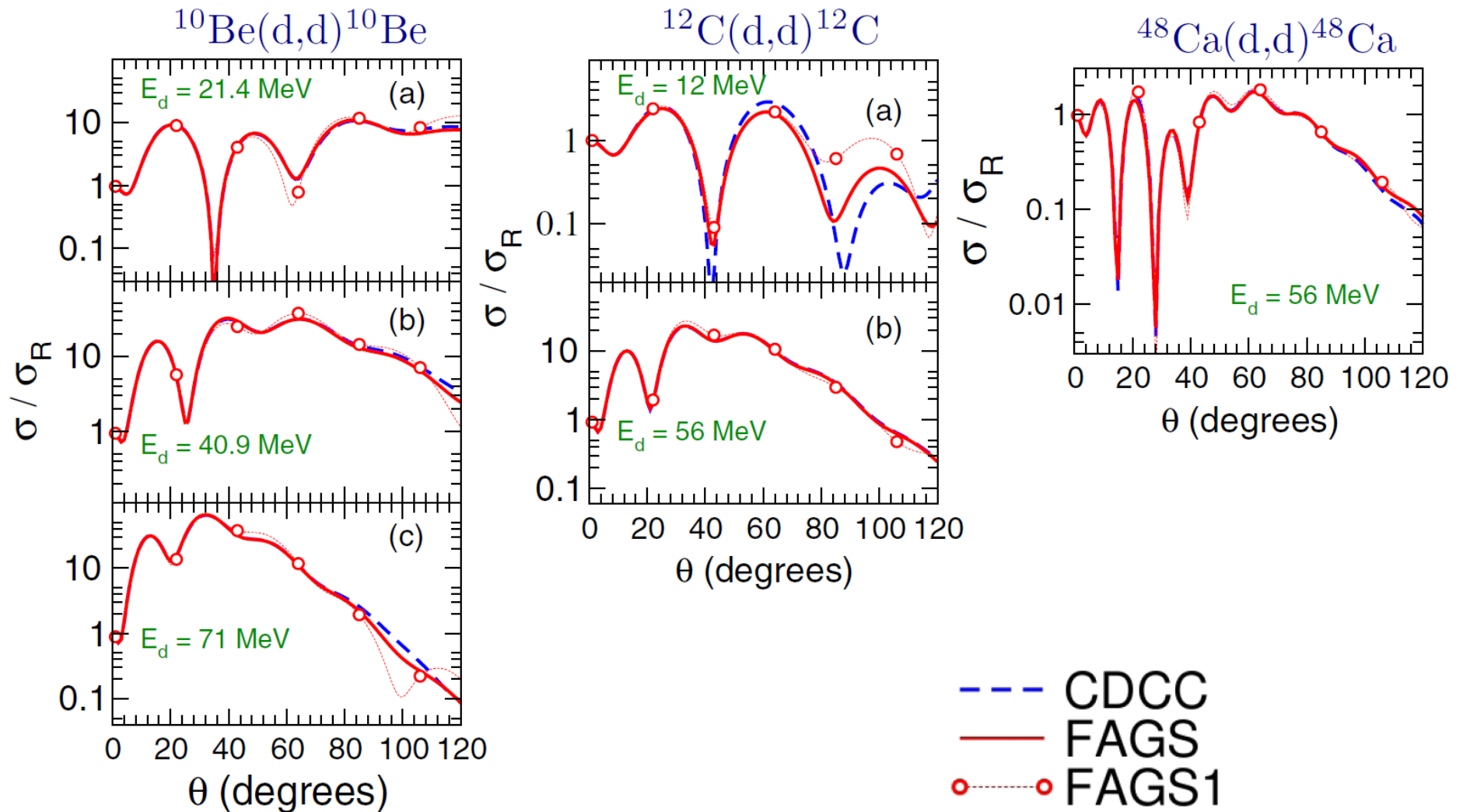
Benchmarking few-body methods



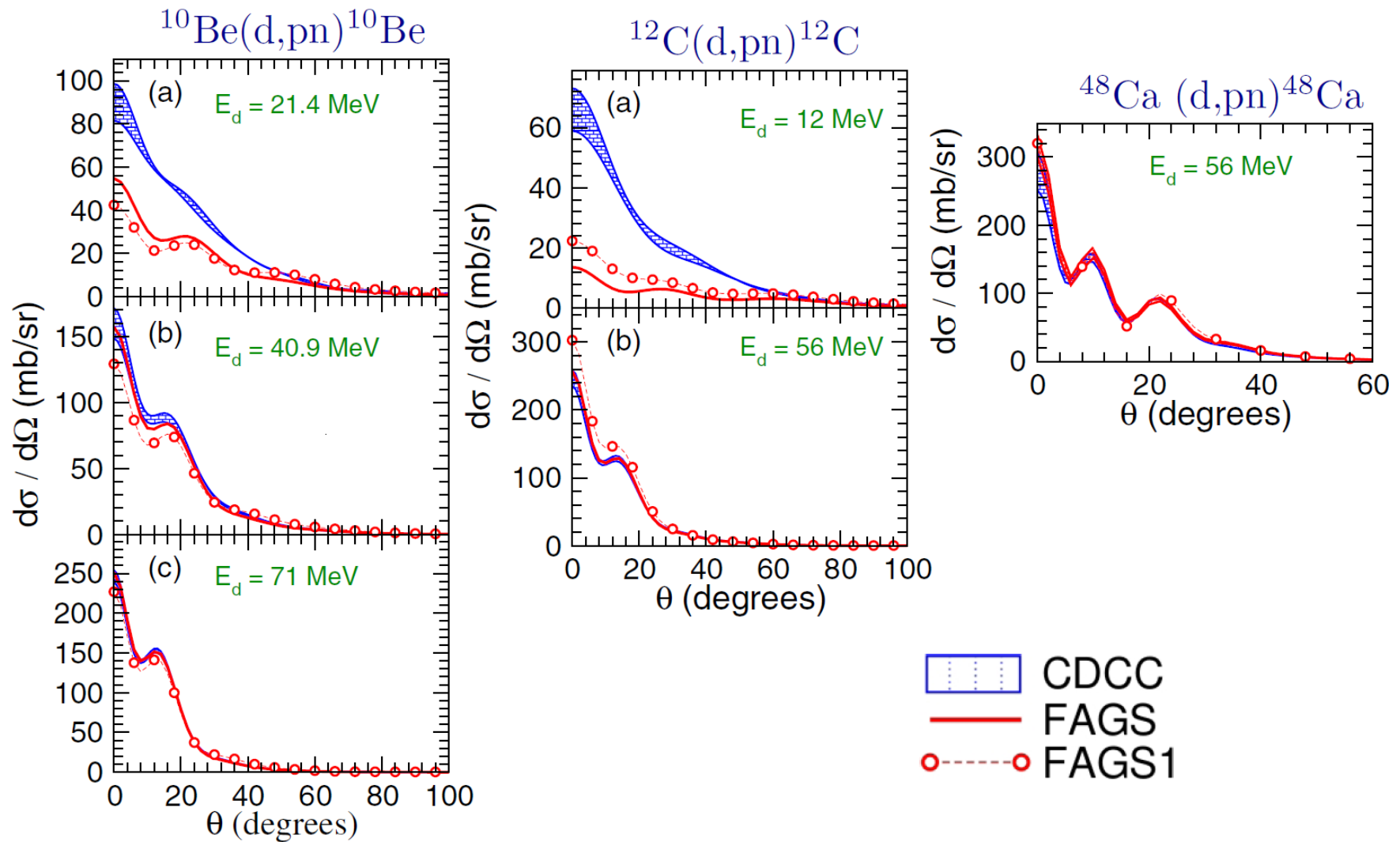
Nunes and Deltuva, PRC 84, 034607(2011)

Upadhyay, Deltuva and Nunes, PRC 85, 054621

Benchmarking few-body methods



Benchmarking few-body methods



Faddeev in the Coulomb distorted basis

Faddeev AGS with screened Coulomb (Deltuva et al., PRC71, 054004)

- equations written in the plane wave basis
- screening radius increases with increasing Z target
- larger number of partial waves needed for convergence
- integral equation solvers break down

Faddeev AGS including unscreened Coulomb

- equations written in the momentum space Coulomb distorted basis
Mukhamedzhanov et al., PRC86, 034001 (2012)
- assumes interactions are separable
Hlophe et al., PRC88, 06408(2013); PRC 90, 061602 (2014)
- no screening of interactions – Coulomb included in the basis
Eremenko et al., CPC 187, 195 (2015)
- challenge to calculate the Coulomb distorted nuclear form factors
Upadhyay et al. PRC 90, 014615 (2014)
- implementation of the equations ongoing

What about transfer to the continuum?

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THE EUROPEAN
PHYSICAL JOURNAL A

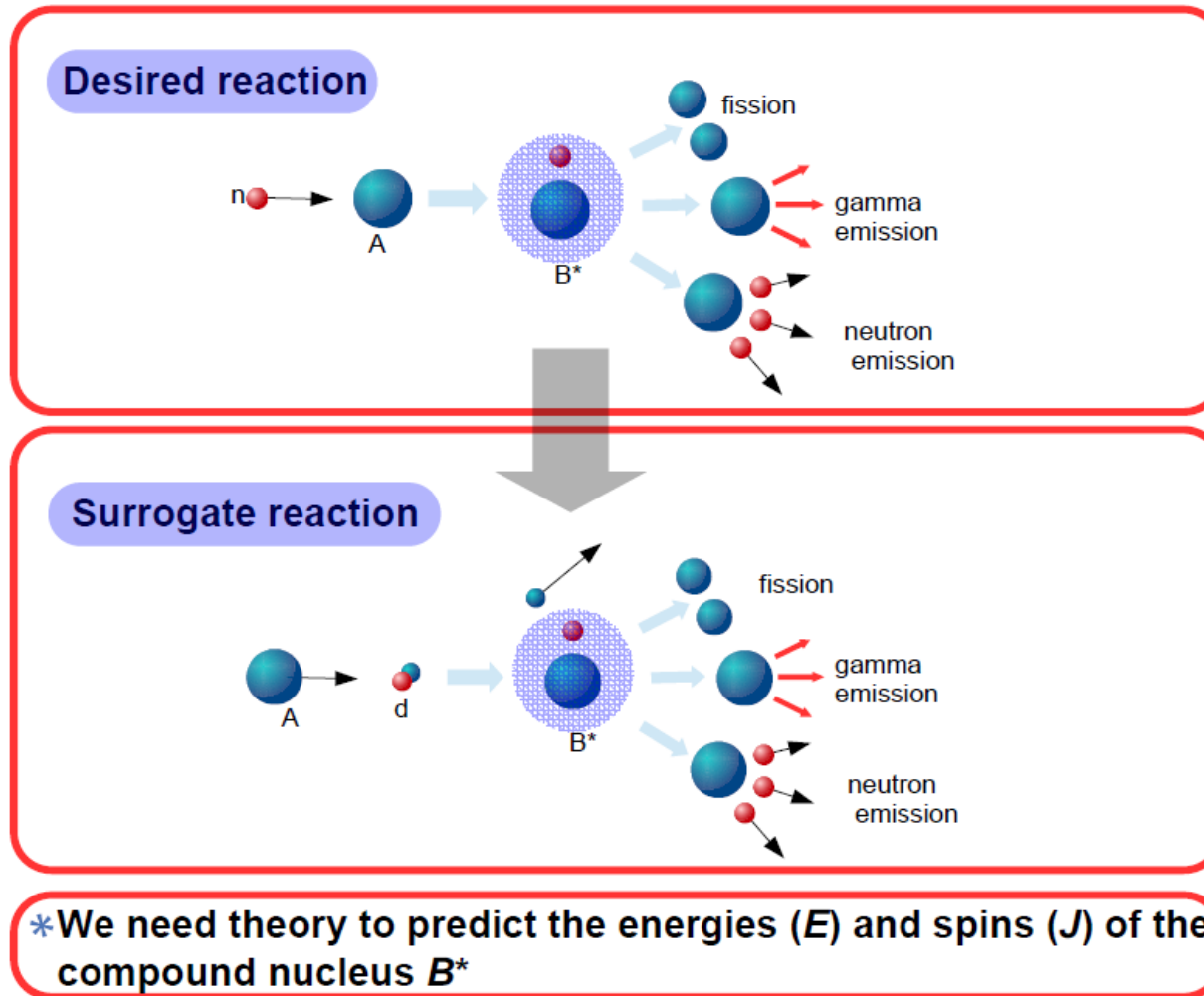
Regular Article – Experimental Physics

Angle-integrated measurements of the $^{26}\text{Al}(d, n)^{27}\text{Si}$ reaction cross section: a probe of spectroscopic factors and astrophysical resonance strengths

A. Kankainen^{1,a}, P.J. Woods¹, F. Nunes^{2,3,4}, C. Langer^{2,4,b}, H. Schatz^{2,3,4}, V. Bader^{2,3}, T. Baugher^{2,c}, D. Bazin², B.A. Brown^{2,3,4}, J. Browne^{2,3,4}, D.T. Doherty^{1,d}, A. Estrade^{1,e}, A. Gade^{2,3}, A. Kontos², G. Lotay^{1,f}, Z. Meisel^{2,3,4,g}, F. Montes^{2,4}, S. Noji^{2,h}, G. Perdikakis^{4,5}, J. Pereira^{2,4}, F. Recchia^{2,i}, T. Redpath⁵, R. Stroberg^{2,3}, M. Scott^{2,3}, D. Seweryniak⁴, J. Stevens^{2,4}, D. Weisshaar², K. Wimmer⁵, and R. Zegers^{2,3,4}

E_x (keV)	E_{res} (keV)	J^π	l	σ_{exp} (μb)	σ_{theor} (μb)	$C^2S(d, n)$	$C^2S(^3\text{He}, d)$	$C^2S(d, p)$	C^2S_{SM}
5547.3(1)		$9/2^+$	2	520(110)	850	0.61(13)			0.44/0.42
6734.0(2)		$11/2^+$	2	390(90)	1104	0.35(8)			0.50/0.50
7129.0(2)		$13/2^+$	2	630(130)	1262	0.5(1)			0.77/0.74
7590.1(9)	126.9(9)	$9/2^+$	0	≤ 37	375	≤ 0.10	≤ 0.002	0.0093(17)	0.011/0.017
			2	≤ 37	757	≤ 0.05	–	0.068(14)	0.053/0.052
7651.9(6)	188.7(6)	$11/2^-$	1	280(70)	1260	0.22(5)	0.16	0.14(3)	0.067
			3	280(70)	2517	0.11(3)	0.49	–	0.480
7739.3(4)	276.1(4)	$9/2^+$	0	70(30)	370	0.19(9)	0.087	–	0.019/0.011
			2	70(30)	746	0.10(5)	0.124	–	0.0092/0.011
		$9/2^-$	1	70(30)	982	0.07(4)	0.064	–	0.038
			3	70(30)	2070	0.035(16)	0.199	–	0.11

(d,pg) as surrogate for (n,g)



Carlson, Moro and Potel will report on this

2. solving the few-body

The problem for a few nucleons (weak Coulomb!)

- Scattering is harder than bound states: there are small discrepancies...

When the problem involves intermediate mass systems:

- Approximate methods are often used
- Depending on the observables, different energy region of validity

When the problem involves heavy mass systems:

- No exact methods currently available
 - Cannot determine whether approximate methods are suitable
 - The goal: to benchmark various methods for reactions with heavy nuclei and at low energy!!
-
- Transfer to the continuum: recent revival but still approximate methods. Need to benchmark...

3. determining V_{eff}

Currently our thinking:

- V_{eff} is effective interaction between N-A and should describe elastic scattering (global optical potential)
- V_{eff} is self energy of N+A system and can be extracted from many-body theories (microscopic optical potential)

How do these two approaches compare?

- Study optical potentials for known systems
- Study extrapolations to unknown regions of nuclear chart

Dickhoff and Rotureau will report on this

4. dealing with non-locality

- The optical potential derived from many-body theories is inherently non-local
- The optical potential extracted from data is usually made local

- Effect of non-locality?
- How to deal with non-locality?
- How to pin down non-locality?
- Is this a relevant question?

non-locality effect in transfer reactions

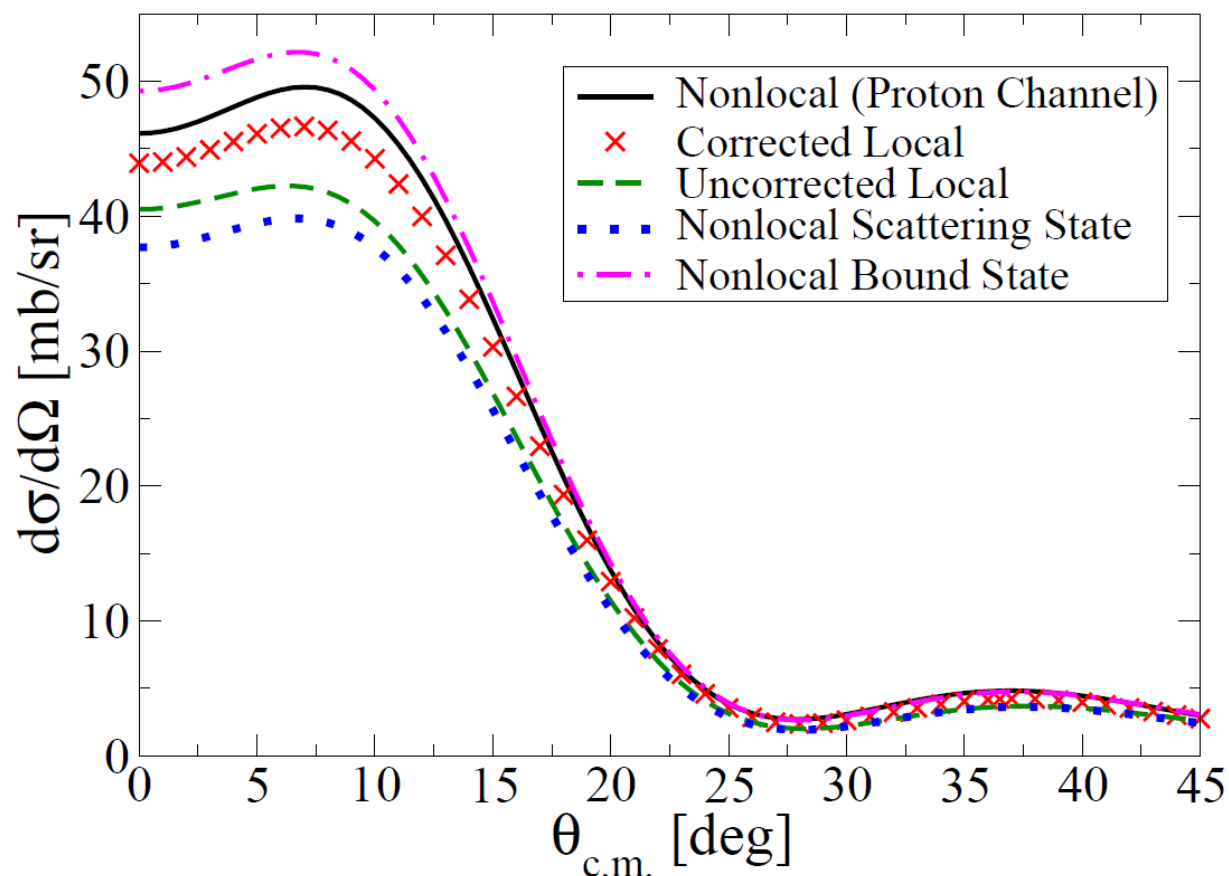
- Systematic study of effect of nonlocality in (d,p)
 - Titus et al., PRC89, 034609
- Similar study with DOM interaction
 - Ross et al., PRC92, 044607
- Inclusion of non-locality in adiabatic theories implemented
 - Titus et al. PRC 93, 014604
- New reaction code NLAT
 - Titus et al., CPC accepted
- Systematic study of effect of nonlocality in (d,n)
 - Ross et al., PRC 94, 014607 (2016)

non-locality effect in transfer reactions

$^{49}\text{Ca}(p,d)^{48}\text{Ca}$ at 20 MeV

(p,d) Transfer Cross Sections

- Nonlocality only added to proton channel.
- Bound state nonlocality enhances cross section.
- Scattering state nonlocality decreases cross section.
- Correction factor not sufficient.



non-locality effect in (d,p) with ADWA

Transfer cross sections: Nonlocal relative to local at first peak

	$E_{lab} = 10 \text{ MeV}$	$E_{lab} = 20 \text{ MeV}$	$E_{lab} = 50 \text{ MeV}$
$^{16}\text{O}(1d_{5/2})(d, p)$	27.2%	24.9%	22.3%
$^{16}\text{O}(2s_{1/2})(d, p)$	15.5%	7.1%	20.7%
$^{40}\text{Ca}(d, p)$	48.5%	43.3%	4.8%
$^{48}\text{Ca}(d, p)$	19.4%	14.9%	41.9%
$^{126}\text{Sn}(d, p)$	36.9%	33.6%	6.9%
$^{132}\text{Sn}(d, p)$	25.7%	3.2%	-10.9%
$^{208}\text{Pb}(d, p)$	52.5%	35.0%	64.8%

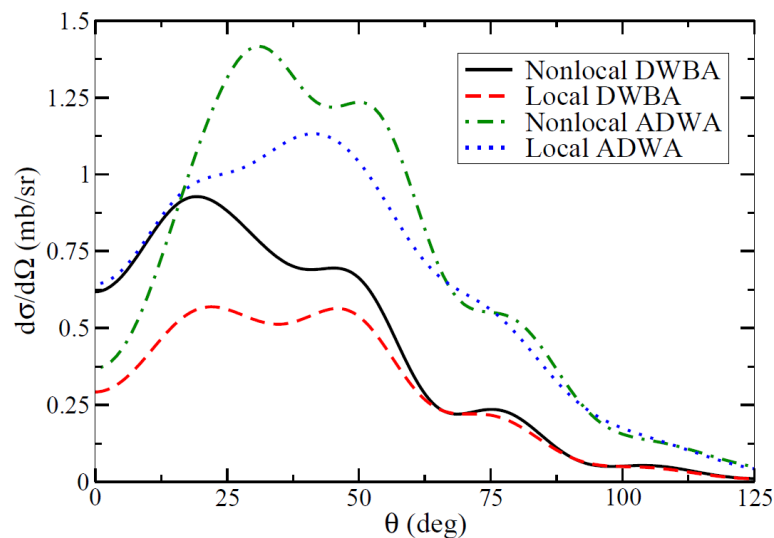
Low Energy

- General enhancement of cross section
- Proton channel most important
- Deuteron channel had a modest impact

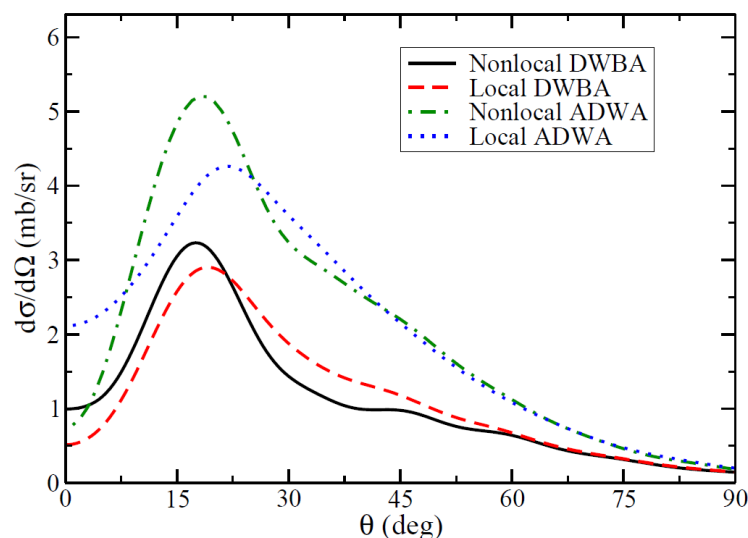
High Energy

- Deuteron channel more important, specially for heavy targets
- Competition between effects of bound and scattering effects in proton channel.

non-locality effect in transfer reactions



$^{208}\text{Pb}(d,n)^{209}\text{Bi}$ @ 20 MeV

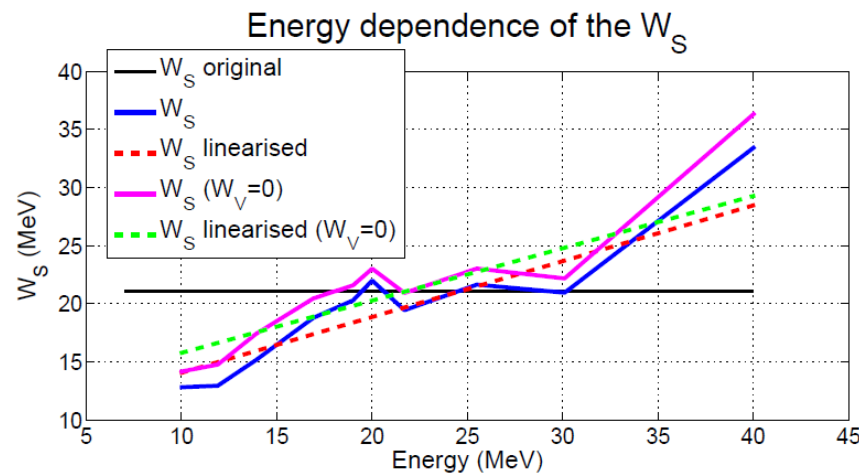


$^{208}\text{Pb}(d,n)^{209}\text{Bi}$ @ 50 MeV

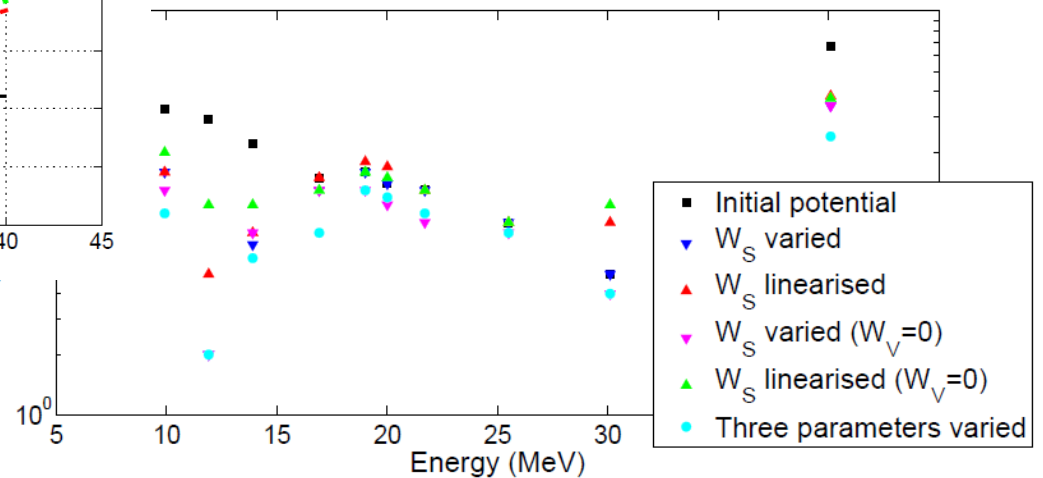
- In general there are very few examples of (d,n) data out there
- Non-locality in optical potential can produce large differences in the angular distribution
- Neutron angular distributions can provide constrains
- Important to get the most forward angles!!!

a new phenomenological non-local optical potential

- Comparing two nonlocal interactions PB (NP 1964) and TPM (IJMP 2015)
 - PB worked best at lower energy and TPM at intermediate energy
- Fitting a new interaction including energy dependent explicitly
- Summer project of Pierre-Loic Bacq
 - (became a Master thesis of ULB Brussels, advisor Pierre Capel)



$^{40}\text{Ca}(p,p)$



5. uncertainty quantification

- All these challenges introduce uncertainties in our model predictions
- The additional challenge is:
 - to determine those uncertainties
 - to propagate those uncertainties to the observables of interest



5. uncertainty quantification

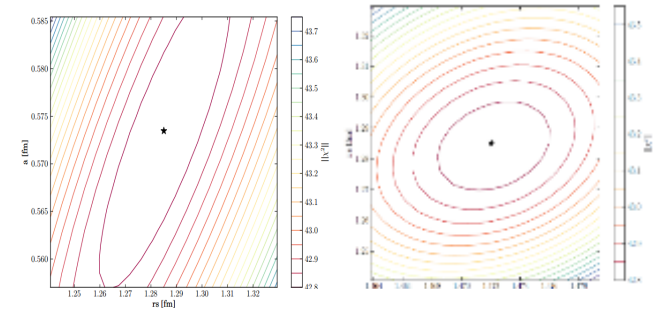
- Determine best fit by minimizing: $\chi^2 = \sum_{i=1}^n (R_i)^2$ $R_i = \frac{\sigma_i^{\text{th}} - \sigma_i^{\text{exp}}}{\Delta\sigma_i}$

Creating 95% confidence bands:

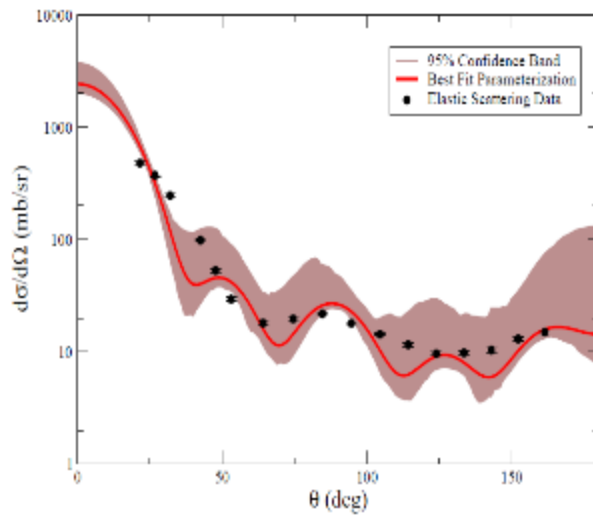
- Assume a Gaussian distribution around the minimum (verifiable)
- Draw 200 parameter sets from the Gaussian distribution
 - (if not Gaussian, draw from the real distribution – more time consuming)
- For each angle, remove the 5 highest and 5 lowest calculations (2.5%) to define 95% confidence bands
- Thesis work of Amy Lovell in collaboration with Wild and Sarich (ANL)

Conventional χ^2 approach to fitting

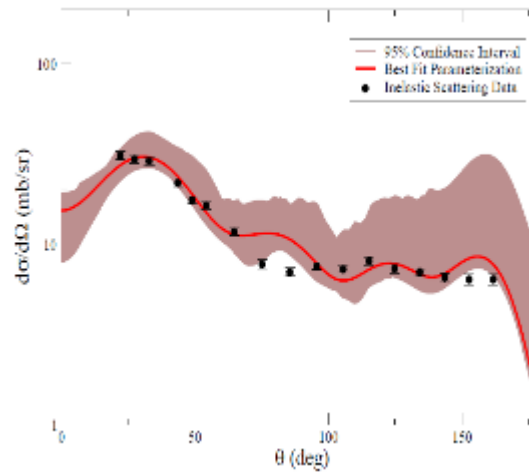
Contours are elliptical (check Gaussian distribution of parameter space)



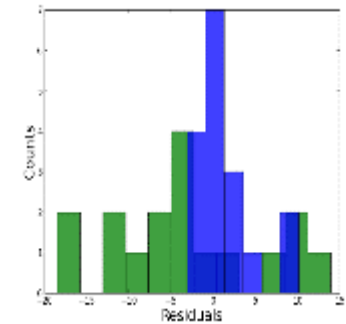
Simultaneous fit



$^{12}\text{C}(n,n)^{12}\text{C}$ @ 17 MeV



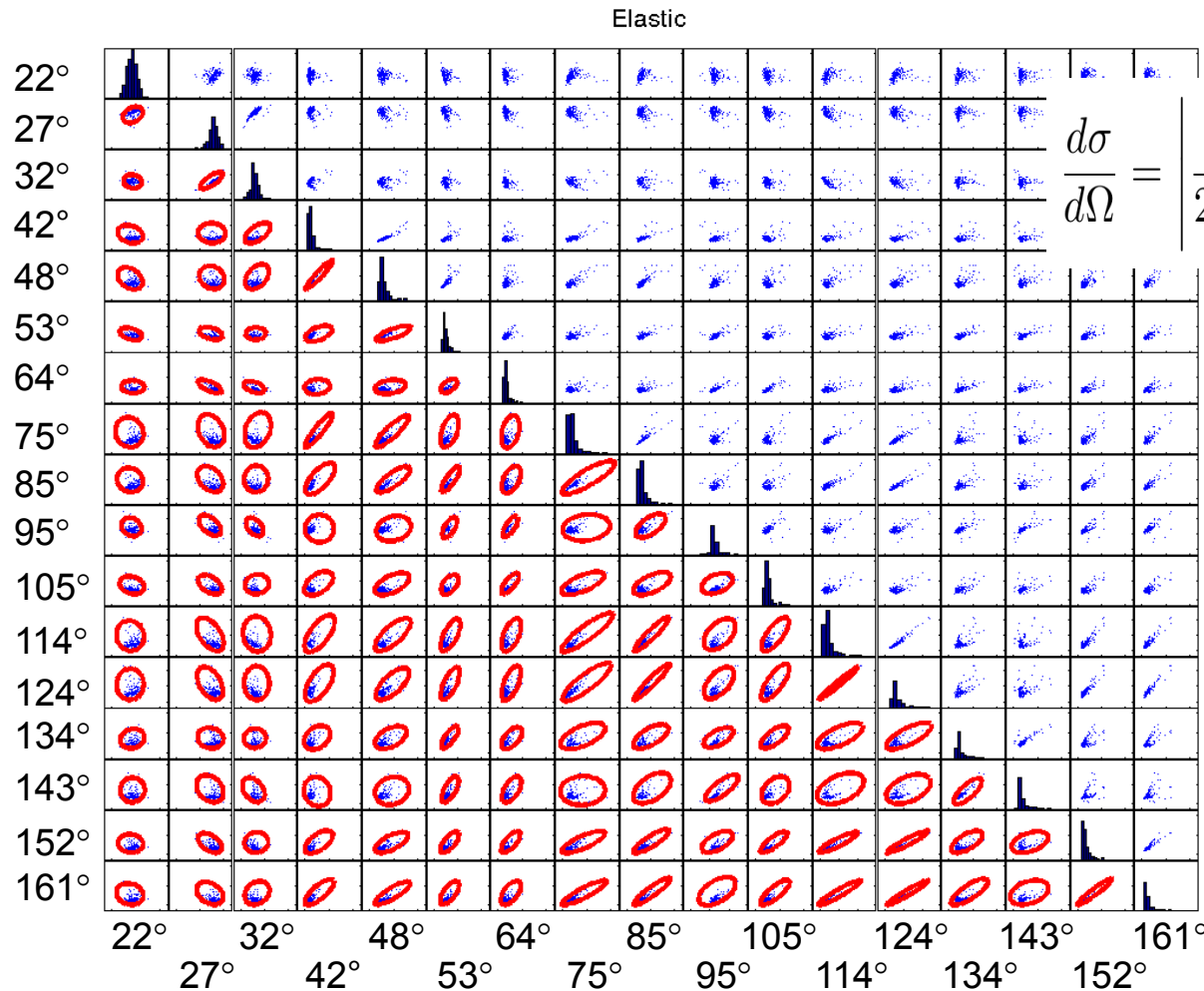
$^{12}\text{C}(n,n')^{12}\text{C}(2^+_1)$ @ 17 MeV



Residuals overlap
Elastic residuals
Inelastic residuals

Understanding model correlations

Differential Cross



$$\frac{d\sigma}{d\Omega} = \left| \frac{1}{2ik} \sum_{L=0}^{\infty} (2L+1) P_L(\cos\theta) (\mathbf{S}_L - 1) \right|^2$$

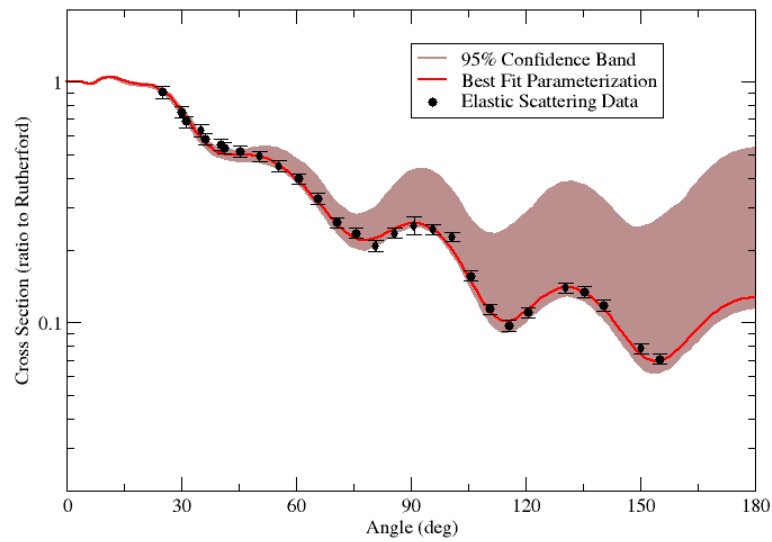
Angular correlations of 200 parameter pulls through the model

Distribution of cross section values at the experimental angles

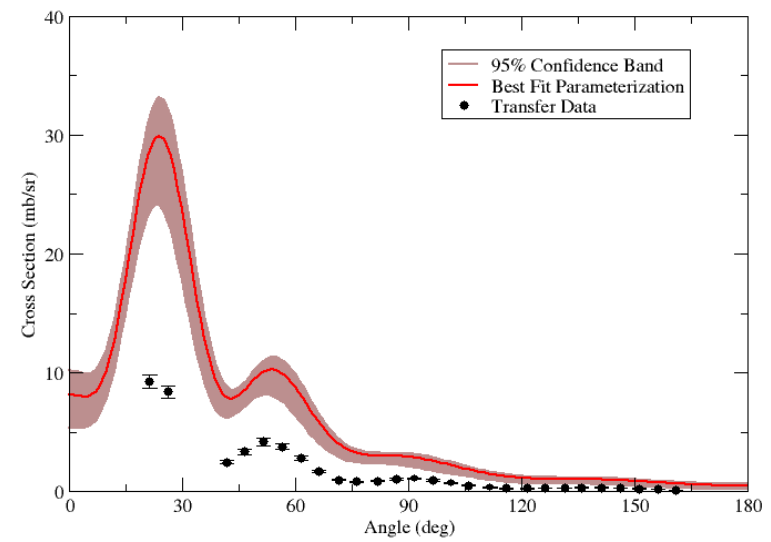
Including correlations in the fit

- Constructed a correlated χ^2 function
- Repeated minimization procedure and confidence band extraction
- Example below: fit to elastic, prediction of transfer
- Error in the extraction of a SF is around 30%

$^{90}\text{Zr}(d,d)^{90}\text{Zr}$ @ 12 MeV



$^{90}\text{Zr}(d,p)^{91}\text{Zr}$ @ 12 MeV



Concluding remarks

1. Reduction to a few-body problem

We are beginning to understanding better the role of core excitation and the conditions under which we can neglect it

2. Solving the few-body problem

A lot of progress has been made and more developments are ongoing for (d,p) on heavy targets. Particular challenge on transfer to the continuum. Much more on this tomorrow.

3. Determining the effective interactions

We will hear a lot more about this later this week

4. Including non-locality

We understand non-locality affects transfer observables and know how to include it. How do we constrain it? Need guidance from microscopic theory

5. Quantifying uncertainties

An area in its infancy but very much needed in our field.
How to quantify model uncertainties? Bayesian approach?