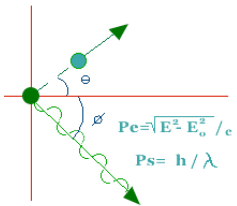


# Linking nuclear reactions and nuclear structure



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Bob Charity

Lee Sobotka

Arturo Polls

Carlo Barbieri

Helber Dussan

MSU collaboration

Hossein Mahzoon (Ph.D. 2015)

Mack Atkinson

- Motivation
- Green's functions method
  - ab initio
  - as a framework to analyze experimental data (and extrapolate and predict properties of exotic nuclei)
- > dispersive optical model (DOM)
- Mahaux implementation
- Focus on recent DOM → DSM developments
- Can do more than expected!
- Conclusions

# Motivation

- Rare isotope physics requires a **much** stronger link between nuclear reactions and nuclear structure descriptions
- We need an ab initio approach for optical potential → optical potentials must therefore become **nonlocal** and **dispersive**
- Current status to extract structure information from nuclear reactions involving strongly interacting probes **unsatisfactory**
- Intermediate step: dispersive optical model as originally proposed by Claude Mahaux → in need of **extensions** some discussed here

# Propagator / Green's function

- Lehmann representation
 
$$G_{\ell j}(k, k'; E) = \sum_m \frac{\langle \Psi_0^A | a_{k\ell j} | \Psi_m^{A+1} \rangle \langle \Psi_m^{A+1} | a_{k'\ell j}^\dagger | \Psi_0^A \rangle}{E - (E_m^{A+1} - E_0^A) + i\eta} + \sum_n \frac{\langle \Psi_0^A | a_{k'\ell j}^\dagger | \Psi_n^{A-1} \rangle \langle \Psi_n^{A-1} | a_{k\ell j} | \Psi_0^A \rangle}{E - (E_0^A - E_n^{A-1}) - i\eta}$$

- Any other single-particle basis can be used

- Overlap functions --> numerator

- Corresponding eigenvalues --> denominator

- Spectral function
 
$$S_{\ell j}(k; E) = \frac{1}{\pi} \text{Im} G_{\ell j}(k, k; E) \quad E \leq \varepsilon_F^-$$

$$= \sum_n \left| \langle \Psi_n^{A-1} | a_{k\ell j} | \Psi_0^A \rangle \right|^2 \delta(E - (E_0^A - E_n^{A-1}))$$

- Spectral strength in the continuum

$$S_{\ell j}(E) = \int_0^\infty dk k^2 S_{\ell j}(k; E)$$

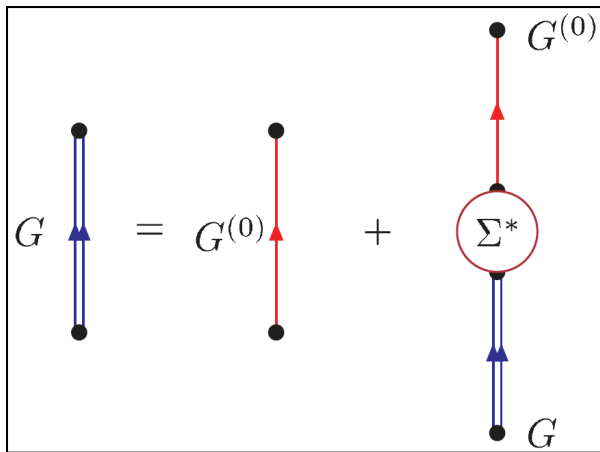
- Discrete transitions  $\sqrt{S_{\ell j}^n} \phi_{\ell j}^n(k) = \langle \Psi_n^{A-1} | a_{k\ell j} | \Psi_0^A \rangle$

- Positive energy → see later

# Propagator in principle generates

- Elastic scattering cross sections for p and n
- Including all polarization observables
- Total cross sections for n
- Reaction cross sections for p and n
- Overlap functions for adding p or n to bound states in Z+1 or N+1
- Plus normalization --> spectroscopic factor
- Overlap function for removing p or n with normalization
- Hole spectral function including high-momentum description
- One-body density matrix; occupation numbers; natural orbits
- Charge density
- Neutron distribution
- p and n distorted waves
- Contribution to the energy of the ground state from  $V_{NN}$

# Propagator from Dyson Equation and "experiment"



Equivalent to ...

Schrödinger-like equation with:  $E_n^- = E_0^A - E_n^{A-1}$

**Self-energy:** non-local, energy-dependent potential

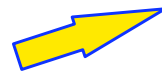
With energy dependence: spectroscopic factors  $< 1$

$\Rightarrow$  as extracted from (e,e'p) reaction

$$\frac{k^2}{2m} \phi_{lj}^n(k) + \int dq q^2 \Sigma_{lj}^*(k, q; E_n^-) \phi_{lj}^n(q) = E_n^- \phi_{lj}^n(k)$$

Spectroscopic factor  $S_{lj}^n = \int dk k^2 |\langle \Psi_n^{A-1} | a_{klj} | \Psi_0^A \rangle|^2 < 1$

Dyson equation also yields  $[\chi_{lj}^{elE}(r)]^* = \langle \Psi_{elE}^{A+1} | a_{rlj}^\dagger | \Psi_0^A \rangle$  for positive energies



Elastic scattering wave function for protons or neutrons

Dyson equation therefore provides:

Link between scattering and structure data from **dispersion relations**

reactions and structure

# Optical potential $\leftrightarrow$ nucleon self-energy

- e.g. Bell and Squires  $\rightarrow$  elastic T-matrix = reducible self-energy
- Mahaux and Sartor Adv. Nucl. Phys. **20**, 1 (1991)
  - relate dynamic (energy-dependent) real part to imaginary part
  - employ subtracted dispersion relation

General dispersion relation for self-energy:

$$\text{Re } \Sigma(E) = \Sigma^{HF} - \frac{1}{\pi} \mathcal{P} \int_{E_T^+}^{\infty} dE' \frac{\text{Im } \Sigma(E')}{E - E'} + \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{E_T^-} dE' \frac{\text{Im } \Sigma(E')}{E - E'}$$

Calculated at the Fermi energy  $\varepsilon_F = \frac{1}{2} \{ (E_0^{A+1} - E_0^A) + (E_0^A - E_0^{A-1}) \}$

$$\text{Re } \Sigma(\varepsilon_F) = \Sigma^{HF} - \frac{1}{\pi} \mathcal{P} \int_{E_T^+}^{\infty} dE' \frac{\text{Im } \Sigma(E')}{\varepsilon_F - E'} + \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{E_T^-} dE' \frac{\text{Im } \Sigma(E')}{\varepsilon_F - E'}$$

Subtract

$$\text{Re } \Sigma(E) = \text{Re } \widetilde{\Sigma}^{HF}(\varepsilon_F)$$

$$- \frac{1}{\pi} (\varepsilon_F - E) \mathcal{P} \int_{E_T^+}^{\infty} dE' \frac{\text{Im } \Sigma(E')}{(E - E')(\varepsilon_F - E')} + \frac{1}{\pi} (\varepsilon_F - E) \mathcal{P} \int_{-\infty}^{E_T^-} dE' \frac{\text{Im } \Sigma(E')}{(E - E')(\varepsilon_F - E')}$$

# DOM = Dispersive Optical Model (standard version)

C. Mahaux and R. Sartor, *Adv. Nucl. Phys.* **20**, 1 (1991)

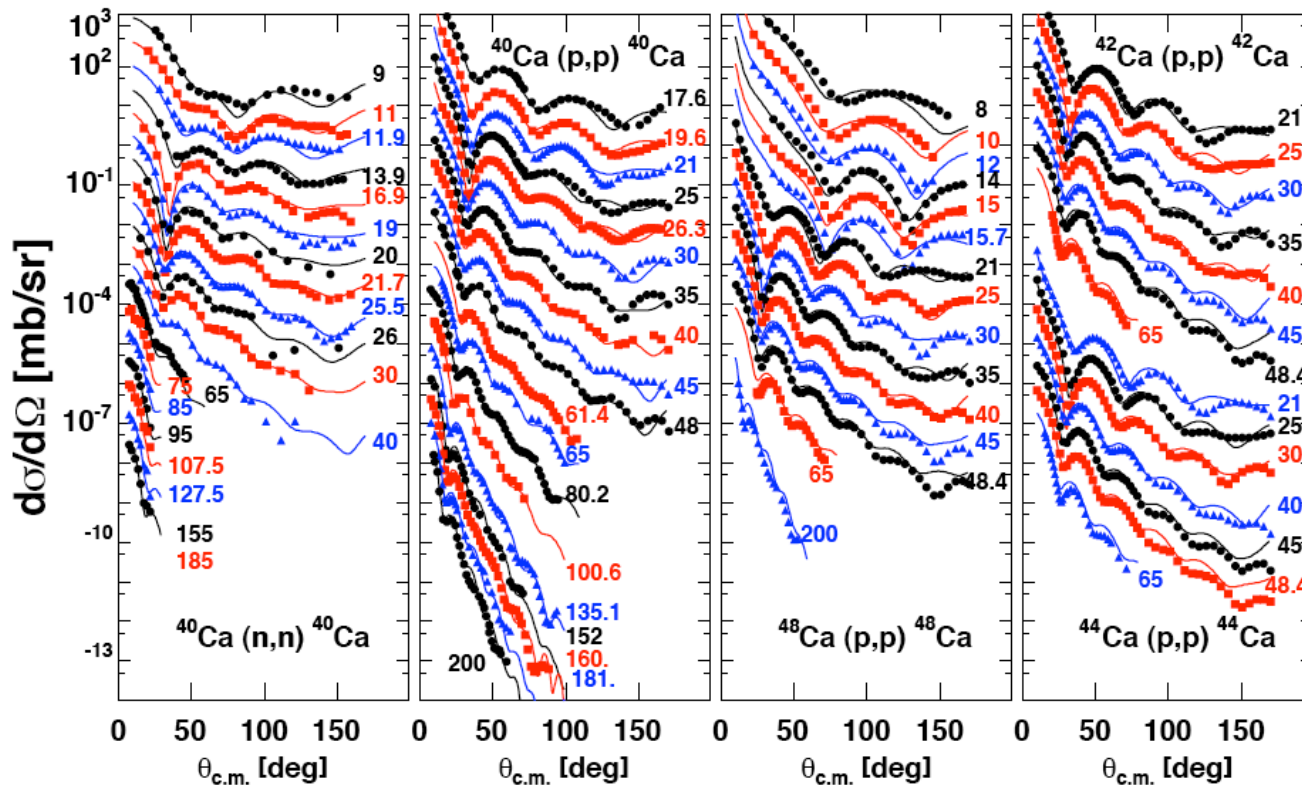
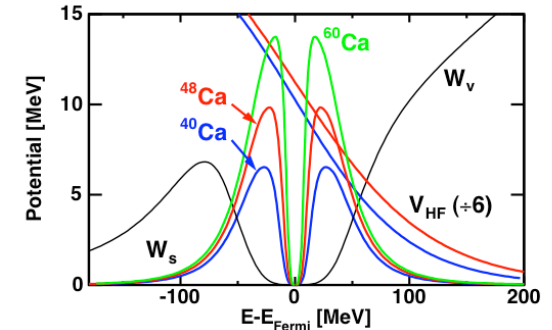
Goal: extract "propagator"/"self-energy" from data

Vehicle for data-driven extrapolations / predictions to the drip lines

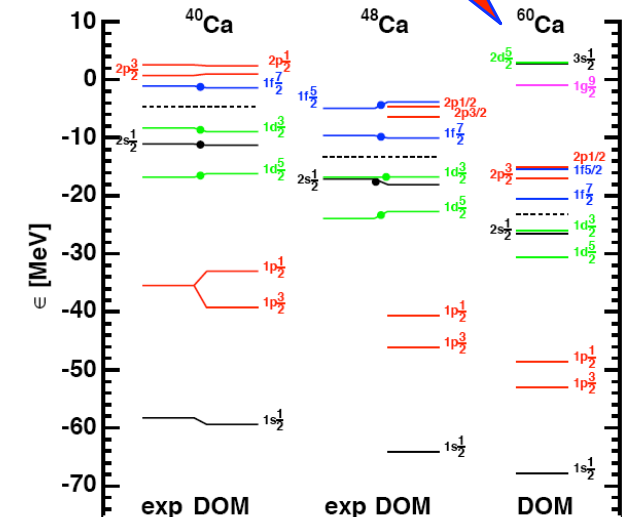
Combined analysis of protons/neutrons in  $^{40}\text{Ca}$  and  $^{48}\text{Ca}$

Charity, Sobotka, & WD, *PRL* **97**, 162503 (2006)

Charity, Mueller, Sobotka, & WD, *PRC* **76**, 044314 (2007)



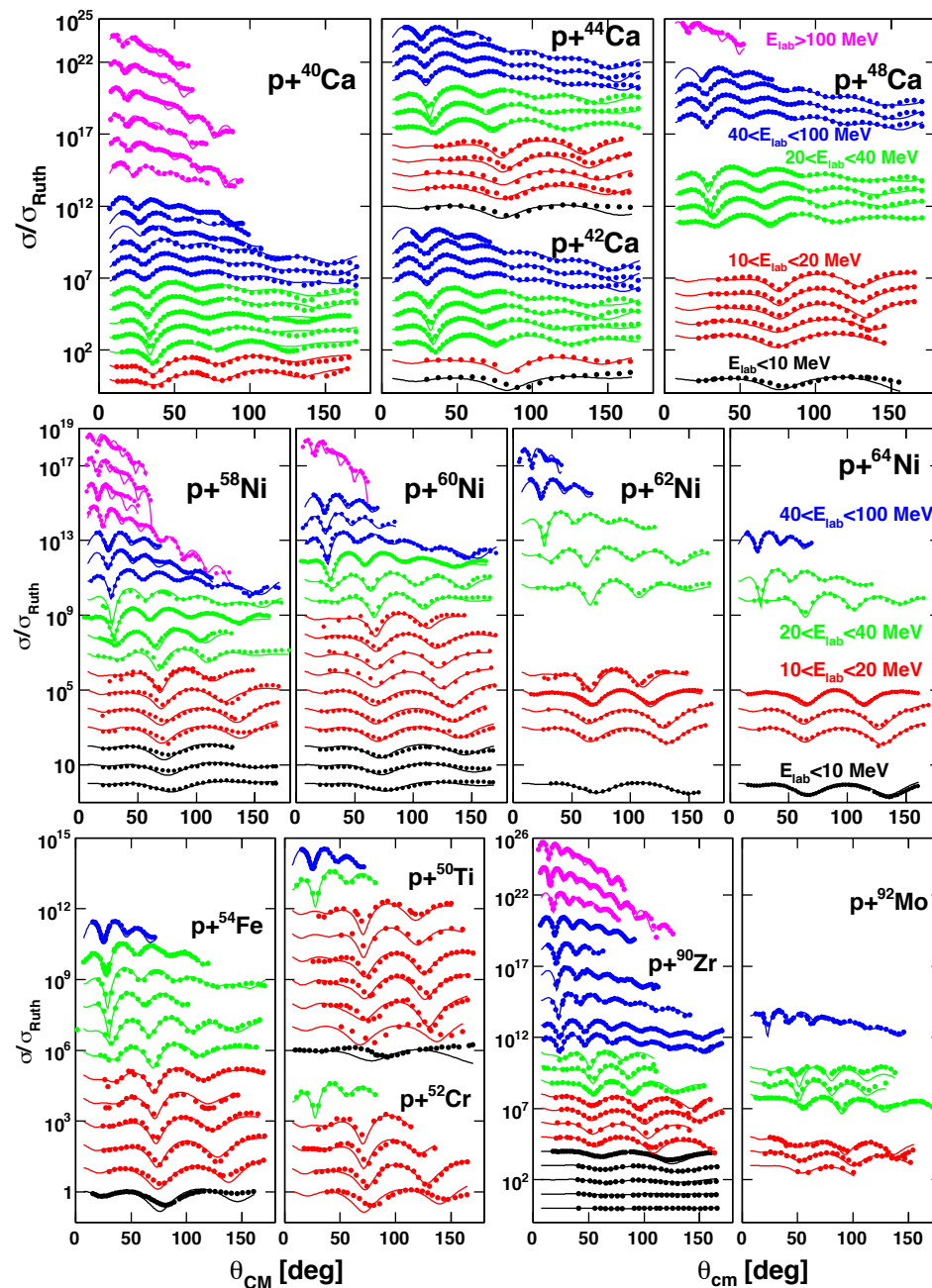
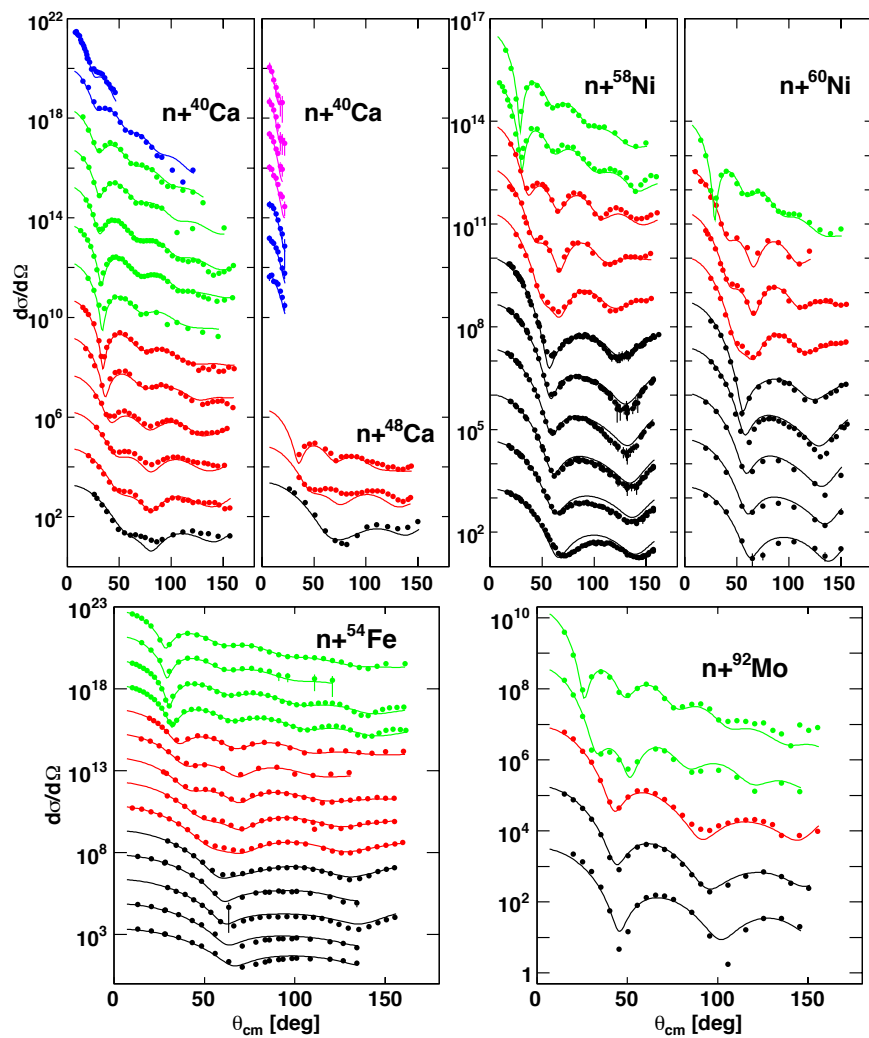
Predict



reactions and structure

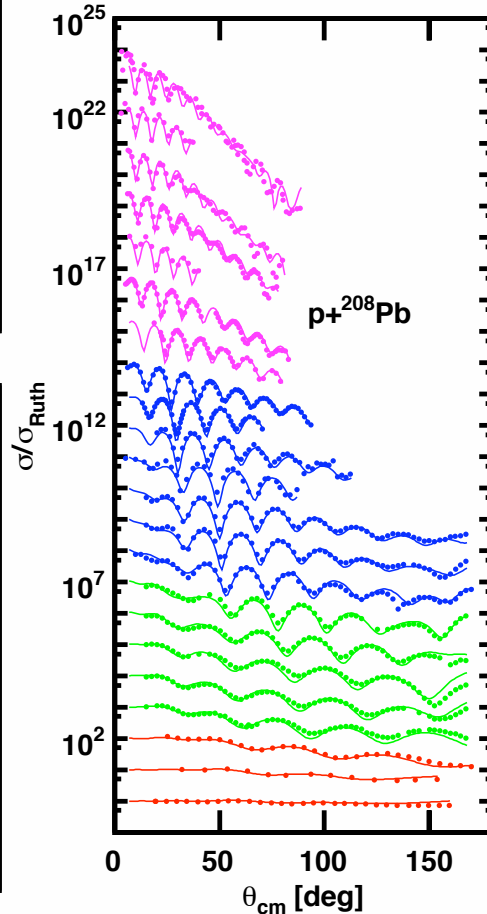
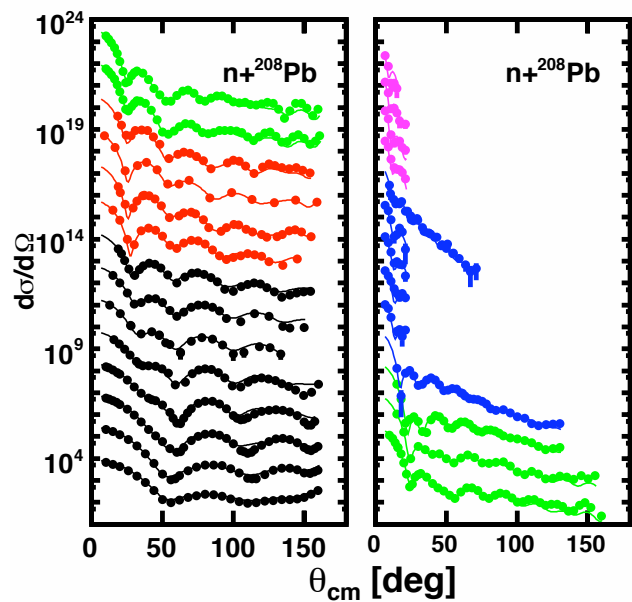
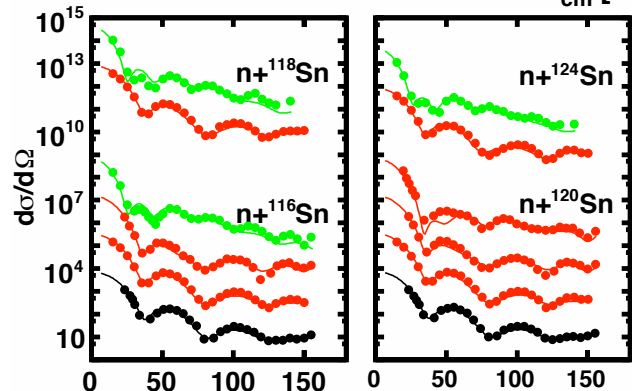
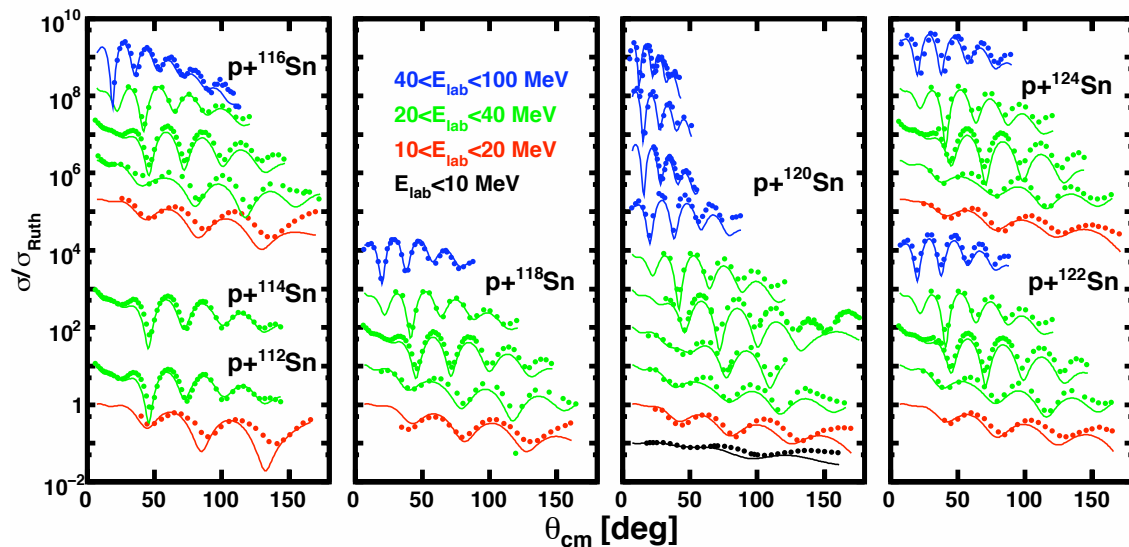
# Elastic scattering data for protons and neutrons

- Local DOM implementation



J. Mueller et al.  
 PRC83,064605 (2011), 1-32





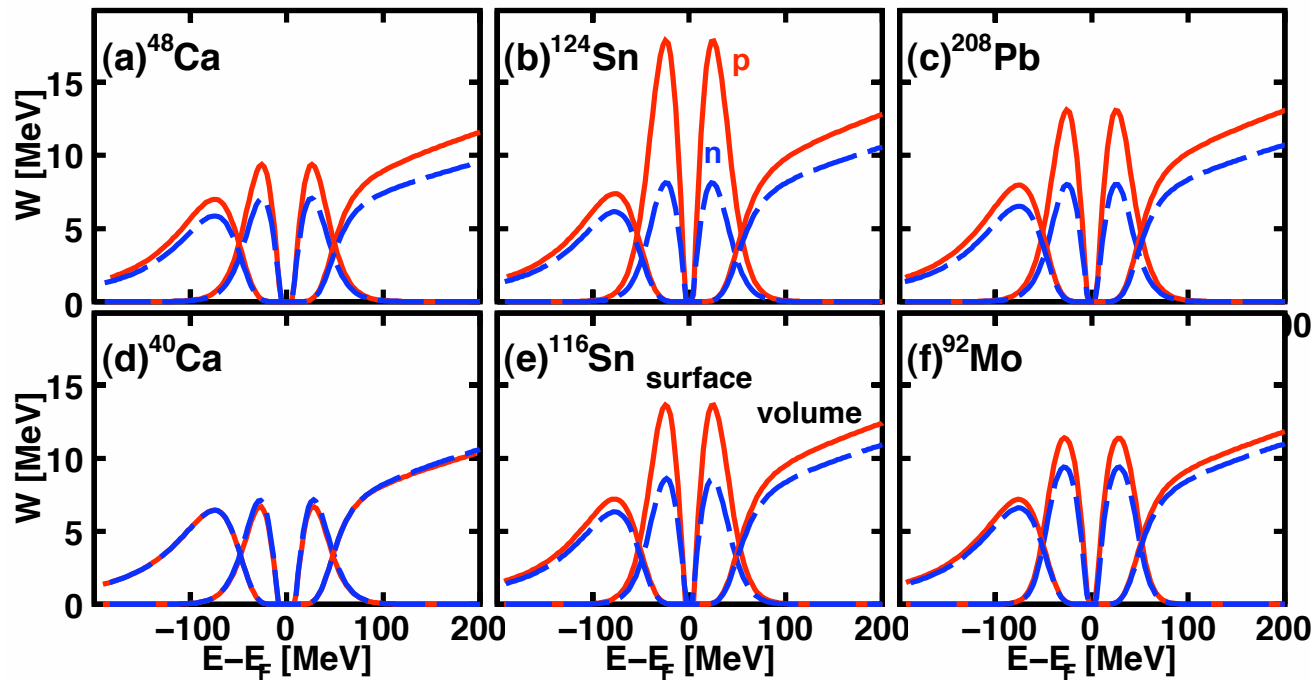
Recent local  
DOM analysis  
--> towards  
global

J. Mueller et al.  
PRC83,064605 (2011), 1-32

# DOM predictions

- Use non-local "HF" potential and dispersive DOM potential to extrapolate to unstable Sn isotopes and predict (e.g.) properties of the last proton (based on the analysis of elastic scattering data on **STABLE** Sn nuclei)

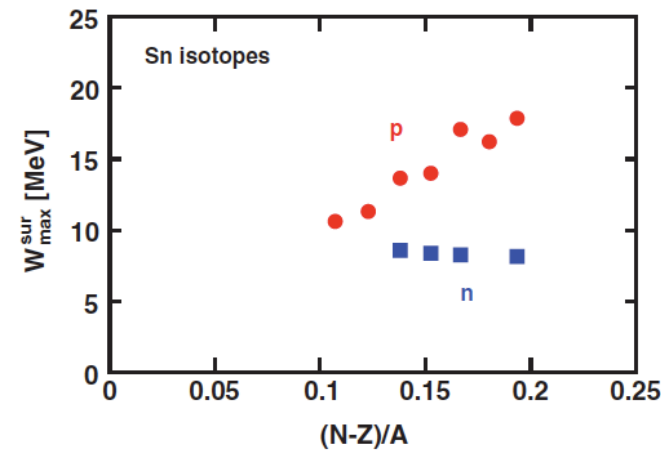
# Asymmetry dependence of imaginary potentials



- Volume  $\rightarrow$  small asymmetry dependence determined

in  $^{208}\text{Pb}$  
$$W_{\text{volume}} = W_{\text{volume}}^0 \pm \frac{N-Z}{A} W_{\text{volume}}^1$$

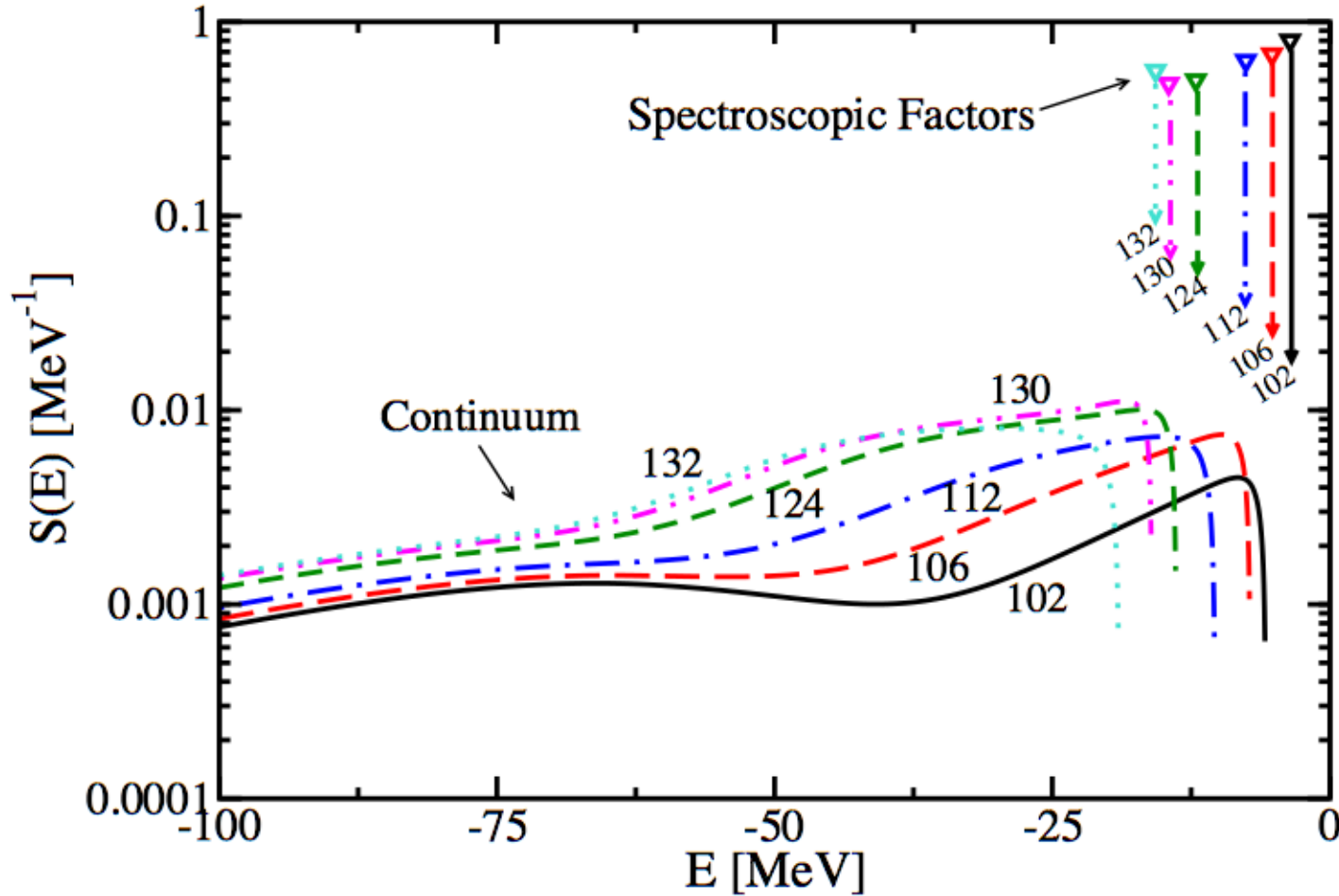
- Neutron surface  $\rightarrow$  no strong dependencies on  $A$  or  $(N-Z)/A$
- Proton surface absorption  $\rightarrow$  increases with increasing neutron number



reactions and structure

# Last proton in Sn nuclei ( $g_{9/2}$ )

Spectral function for different Sn isotopes



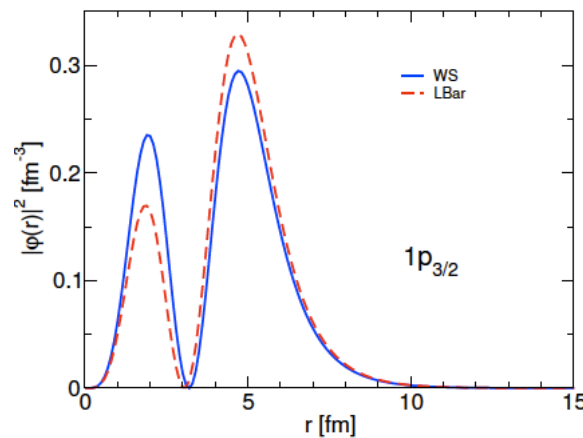
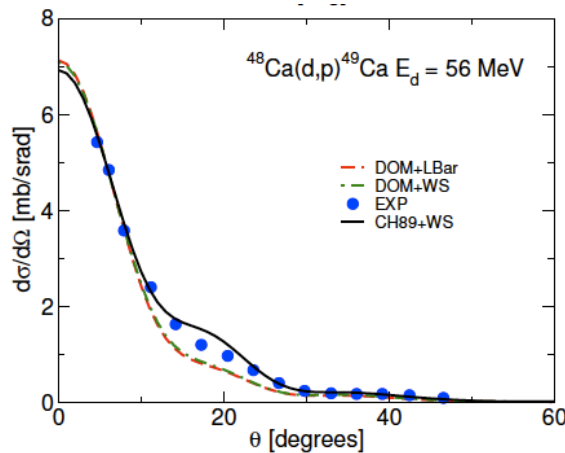
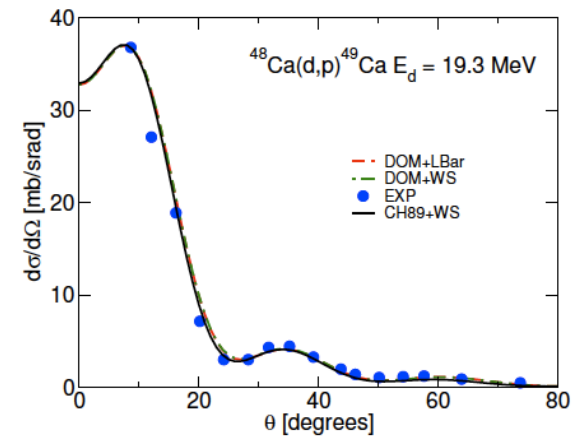
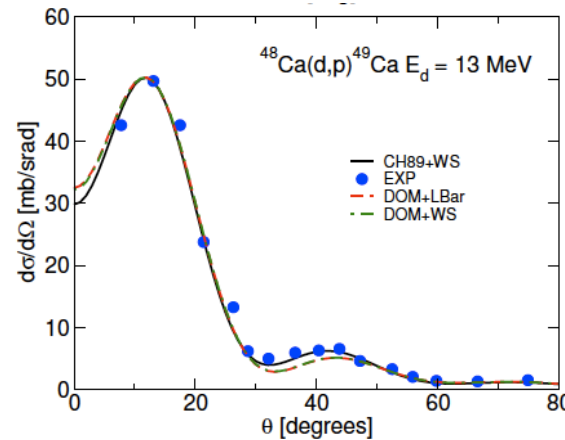
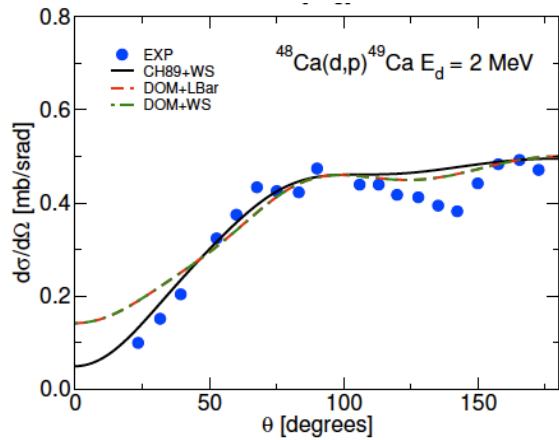
Sn	S	n
102	0.80	0.91
106	0.68	0.85
112	0.63	0.83
124	0.50	0.78
130	0.48	0.78
132	0.56	0.81

# Local DOM ingredients and transfer reactions

- Overlap function
- p and n optical potential
- ADWA (developed by Ron Johnson)
- **MSU-WashU**:-->
- $^{40,48}\text{Ca}, ^{132}\text{Sn}, ^{208}\text{Pb}(d,p)$

N. B. Nguyen, S. J. Waldecker, F. M. Nuñez, R. J. Charity, and W. H. Dickhoff

[Phys. Rev. C84, 044611 \(2011\), 1-9](#)



E	CH+ws	DOM
2	0.94	0.72
13	0.82	0.67
19.3	0.77	0.68
56	1.1	0.70

reactions and structure

# $^{132}\text{Sn}(d,p)$

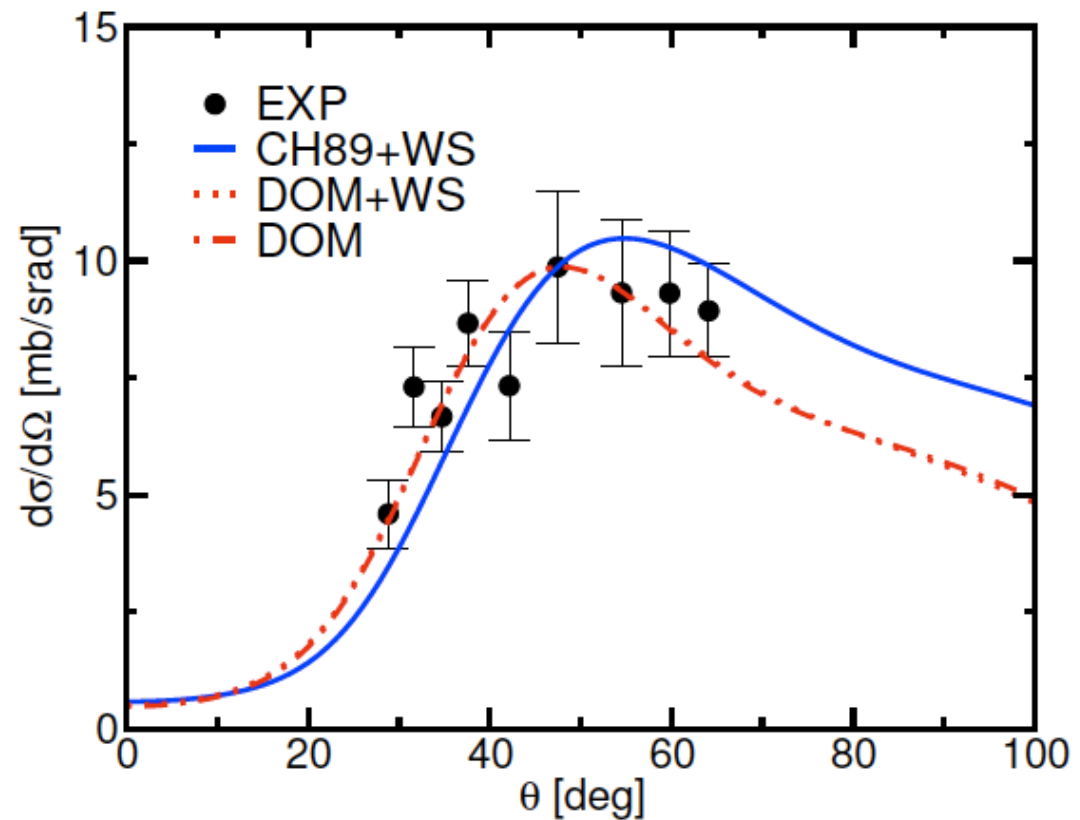
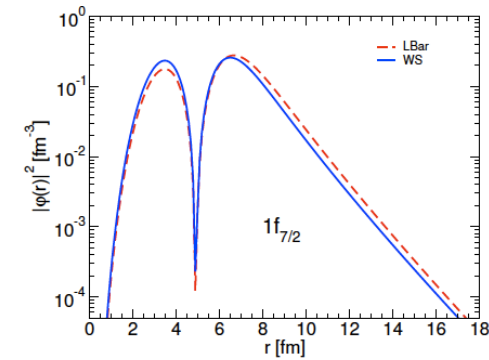
• Does it work when the potentials are extrapolated?

• Data: K.L. Jones et al., Nature 465, 454 (2010)

•  $E_d = 9.46 \text{ MeV}$   $^{132}\text{Sn}(d,p)^{133}\text{Sn}$

• CH89+ws  $\rightarrow S_{1f_{7/2}} = 1.1$

• DOM  $\rightarrow S_{1f_{7/2}} = 0.72$



# Nonlocal DOM implementation PRL112,162503(2014)

- Particle number --> **nonlocal** imaginary part
- Microscopic FRPA & SRC --> different nonlocal properties above and below the Fermi energy Phys. Rev. C84, 034616 (2011) & Phys. Rev. C84, 044319 (2011)
- Include charge density in fit
- Describe high-momentum nucleons <--> (e,e'p) data from JLab

## Implications

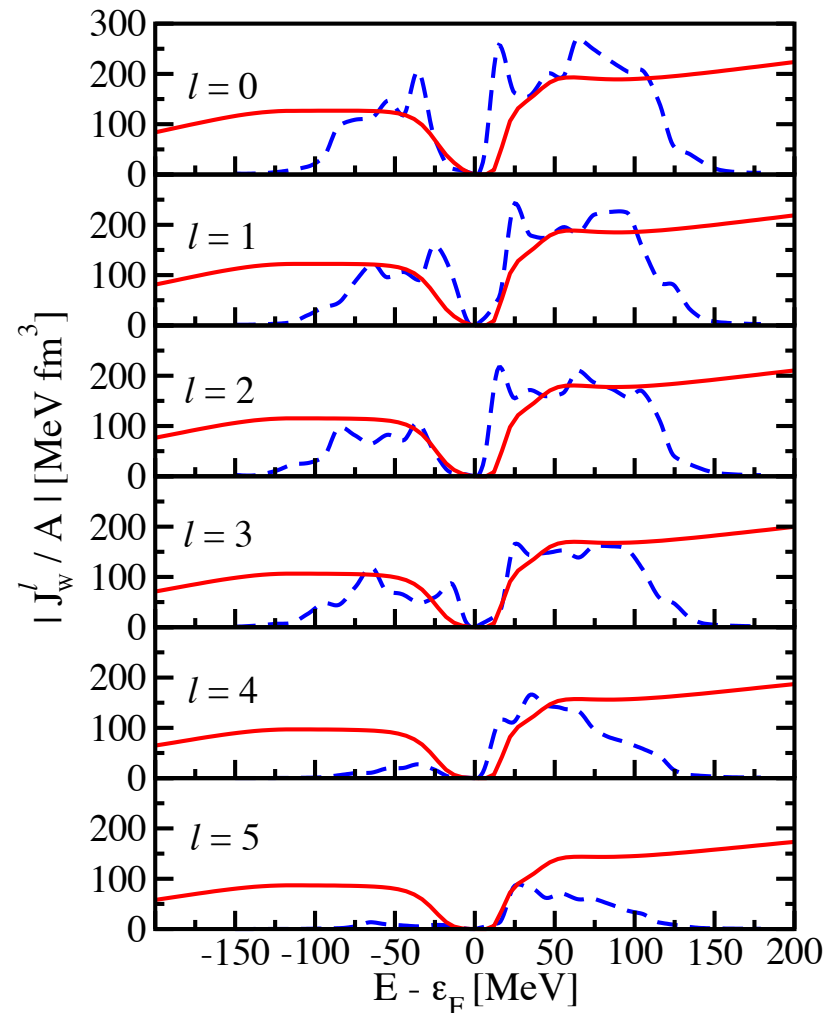
- Changes the description of hadronic reactions because interior nucleon wave functions depend on non-locality
- Consistency test of interpretation (e,e'p) reaction (see later)
- Independent "experimental" statement on size of three-body contribution to the energy of the ground state--> two-body only:

$$E/A = \frac{1}{2A} \sum_{\ell j} (2j+1) \int_0^{\infty} dk k^2 \frac{k^2}{2m} n_{\ell j}(k) + \frac{1}{2A} \sum_{\ell j} (2j+1) \int_0^{\infty} dk k^2 \int_{-\infty}^{\epsilon_F} dE E S_{\ell j}(k; E)$$

reactions and structure

# Comparison with ab initio FRPA calculation

- Volume integrals of imaginary part of nonlocal ab initio (FRPA) self-energy compared with DOM result for  $^{40}\text{Ca}$



- Ab initio

S. J. Waldecker, C. Barbieri and W. H. Dickhoff

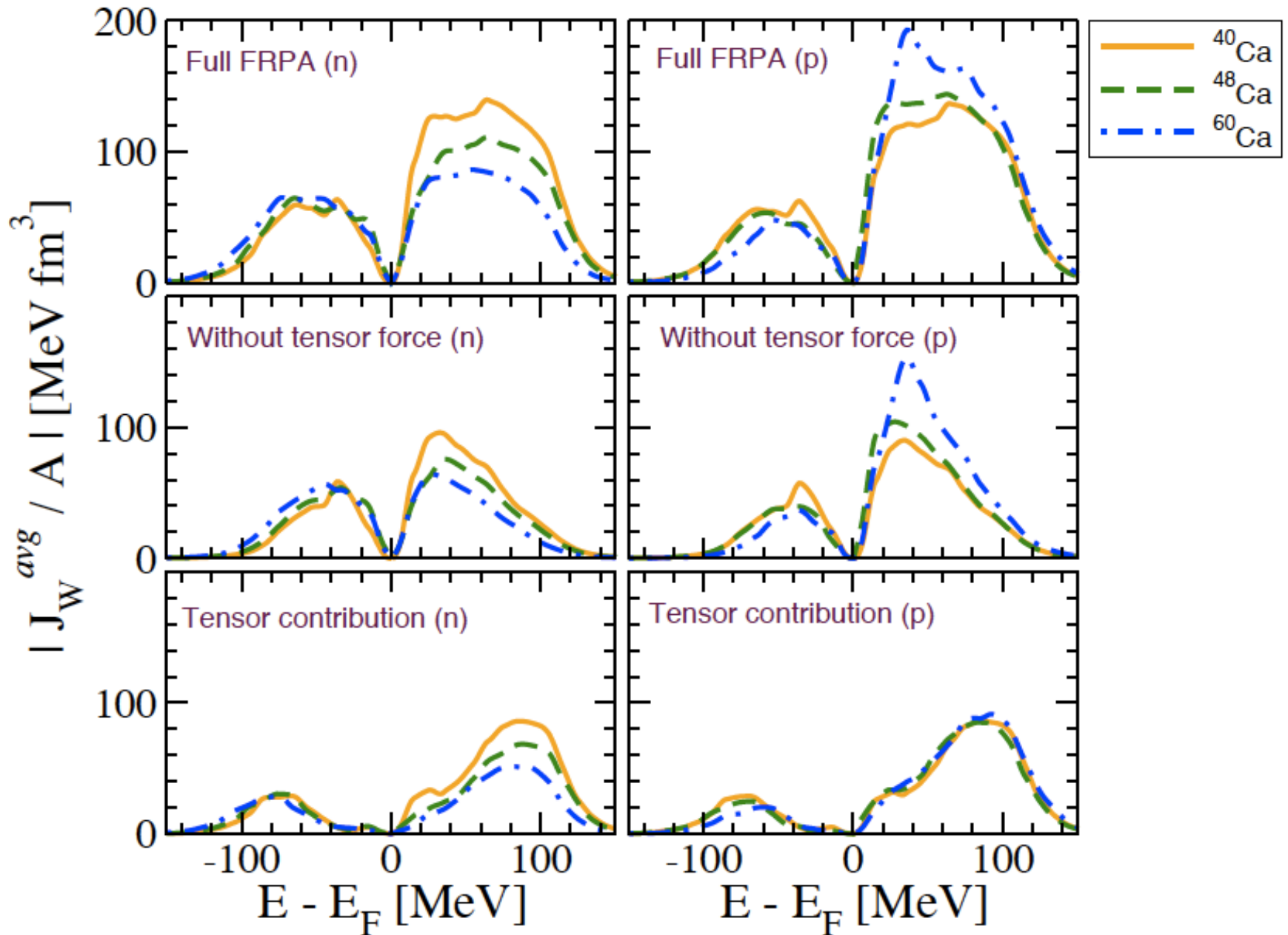
Microscopic self-energy calculations and dispersive-optical-model potentials.

[Phys. Rev. C84, 034616 \(2011\), 1-11.](#)

reactions and structure



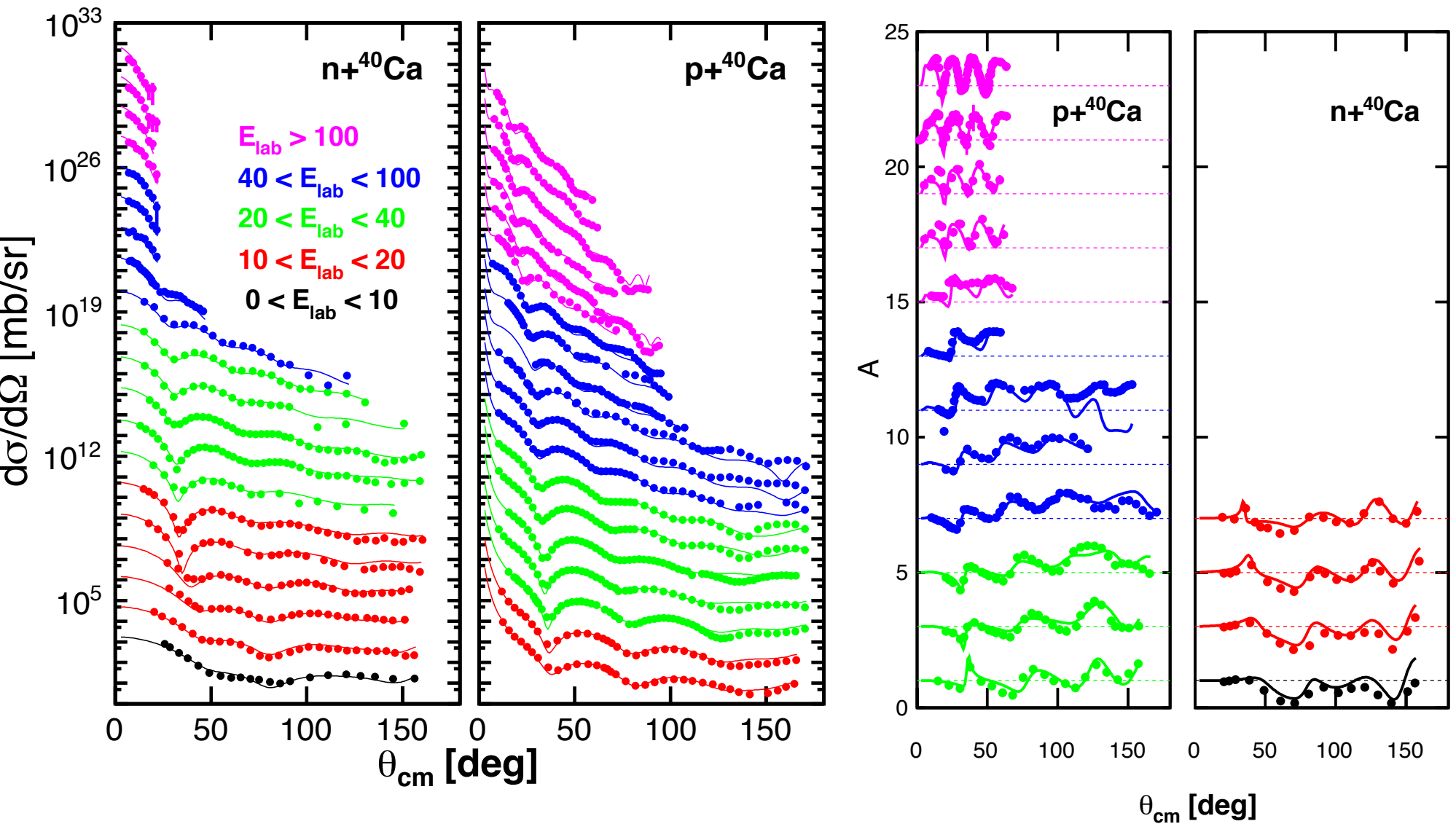
# Tensor force



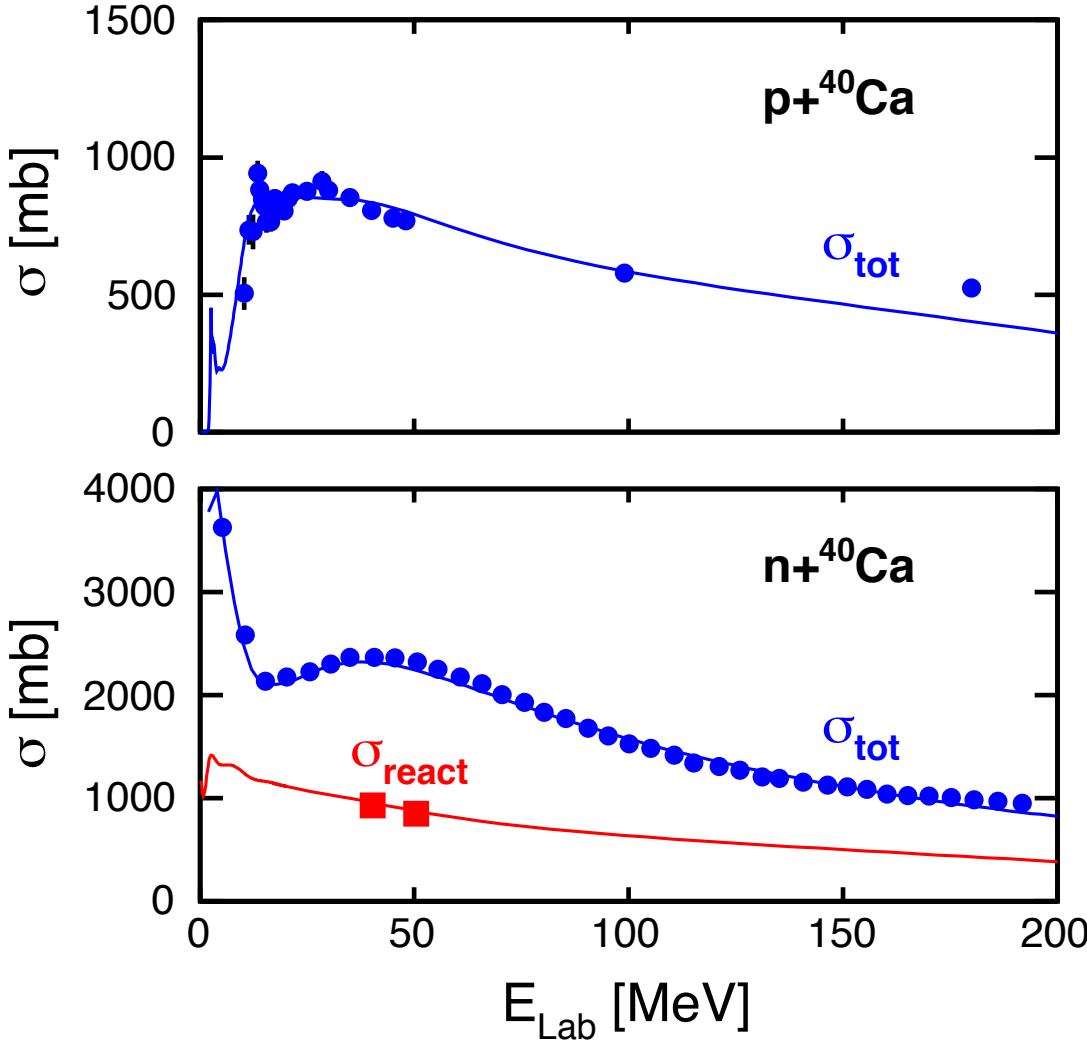
# Non-local DOM fit for $^{40}\text{Ca}$

- Standard ingredients -> Gaussian non-locality
- Different non-locality for surface and volume absorption
- Different non-locality above and below the Fermi energy
  
- Critical new constraints:
  - particle number
  - charge density
  - ~10% high-momentum protons
  
- Thesis Hossein Mahzoon, Washington U St. Louis, 2015  
 $^{40}\text{Ca}$  and  $^{48}\text{Ca}$

# Differential cross sections and analyzing powers from non-local fit



# Reaction (p&n) and total (n) cross sections



Below  $\epsilon_F$

# $^{40}\text{Ca}$ spectral function

Nonlocal imaginary self-energy:

proton number  $\rightarrow 19.88$

neutron number  $\rightarrow 19.79$

$$l \leq 5$$

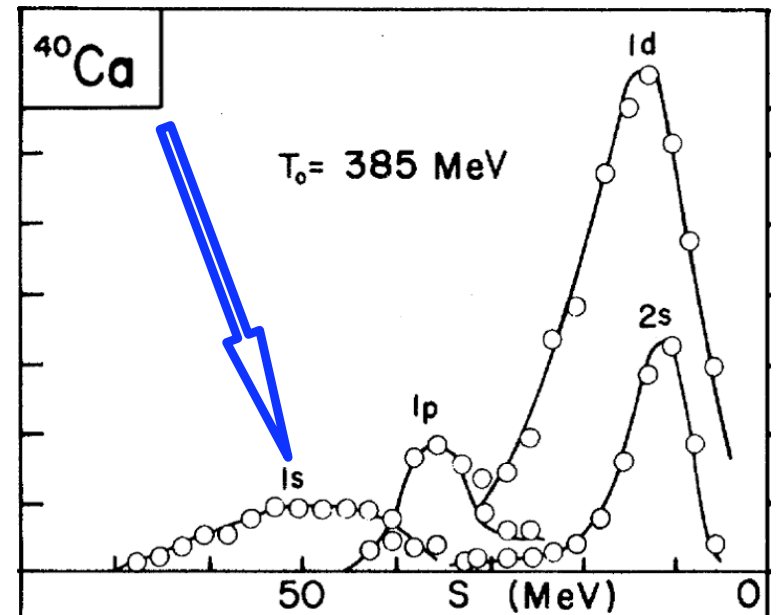
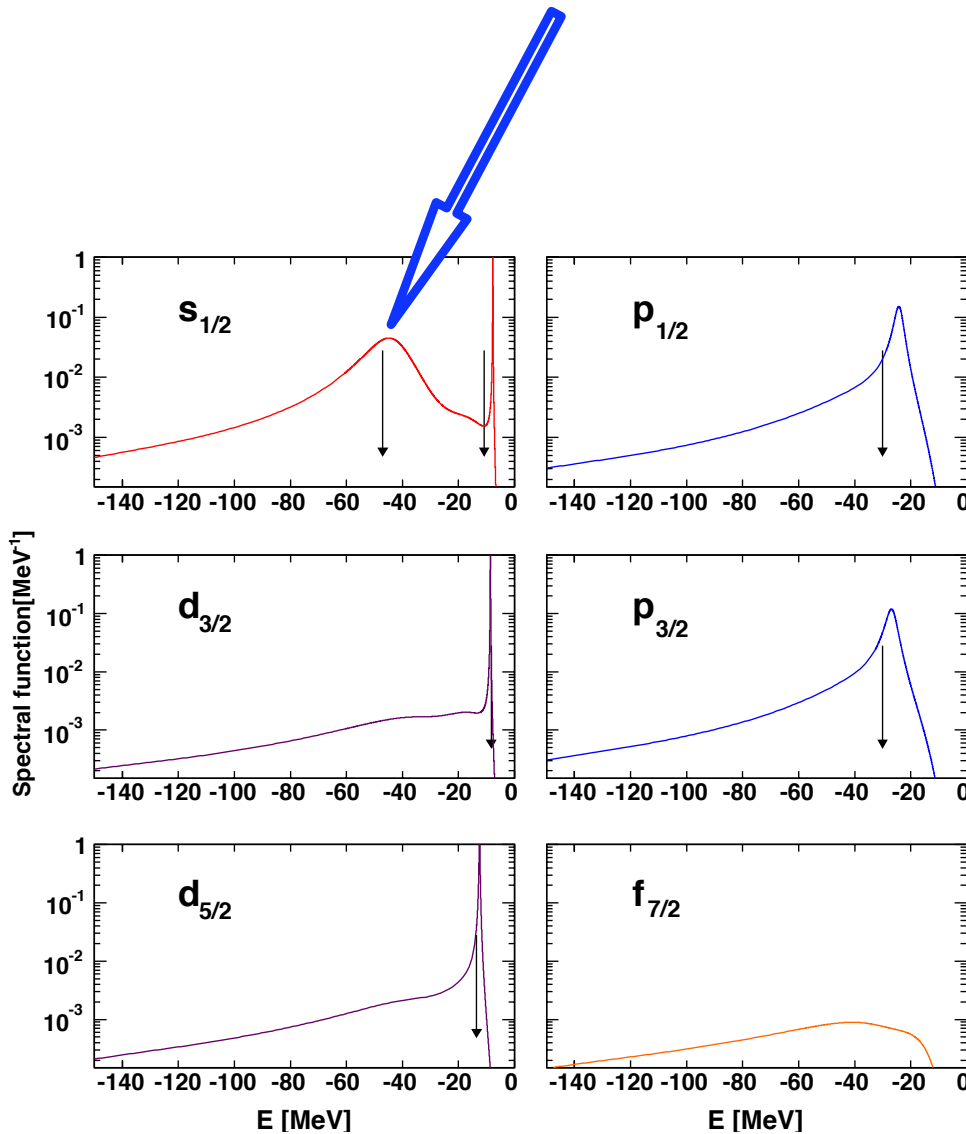
$S_{0d_{3/2}} = 0.76$

$S_{1s_{1/2}} = 0.78$

**Not part of fit!!**

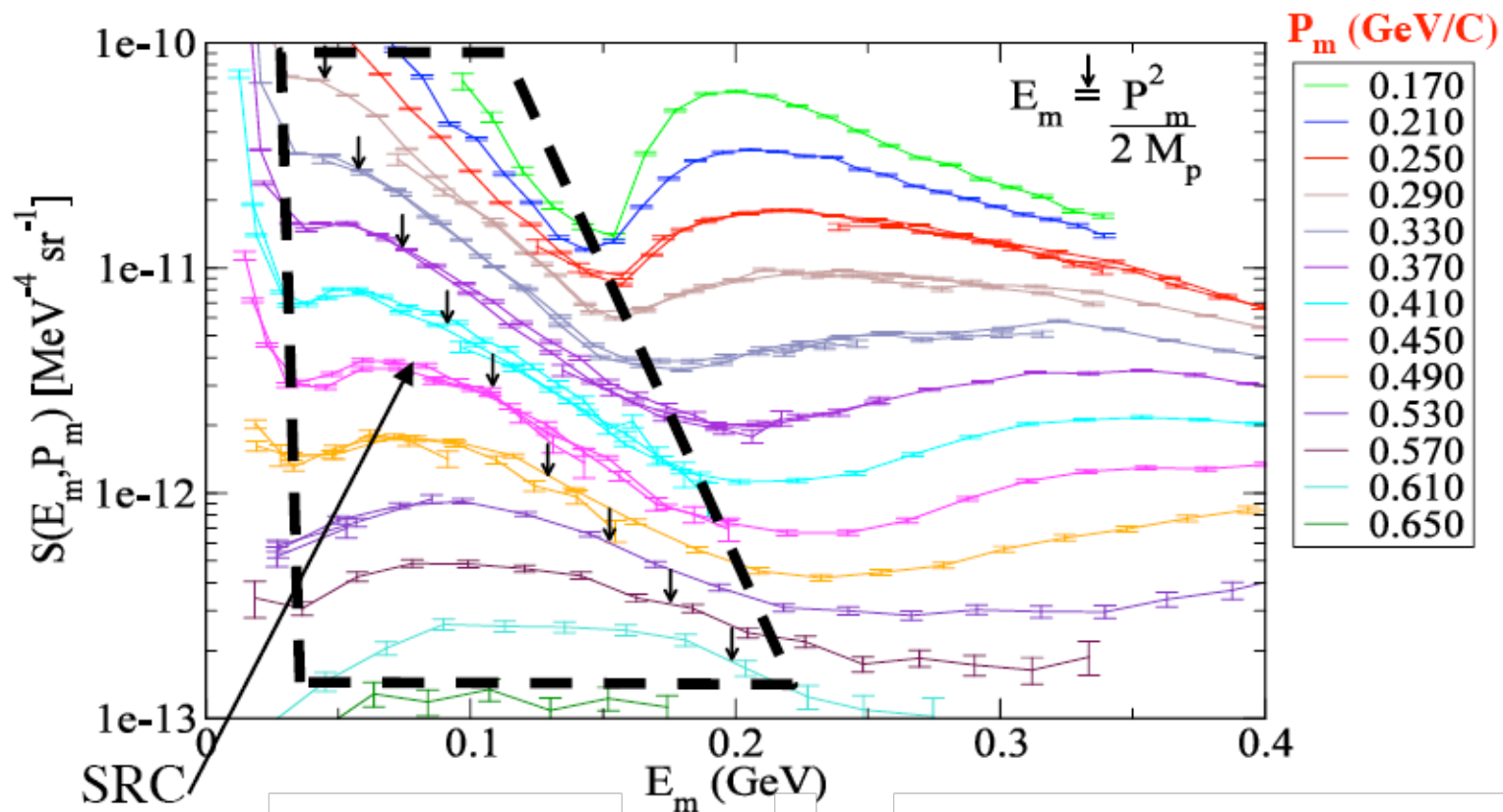
0.15 larger than NIKHEF analysis!

Old (p,2p) data from Liverpool  
or (e,e'p) from Saclay



# High-momentum protons have been seen in nuclei!

Jlab E97-006 Phys. Rev. Lett. 93, 182501 (2004) D. Rohe et al.



$^{12}\text{C}$

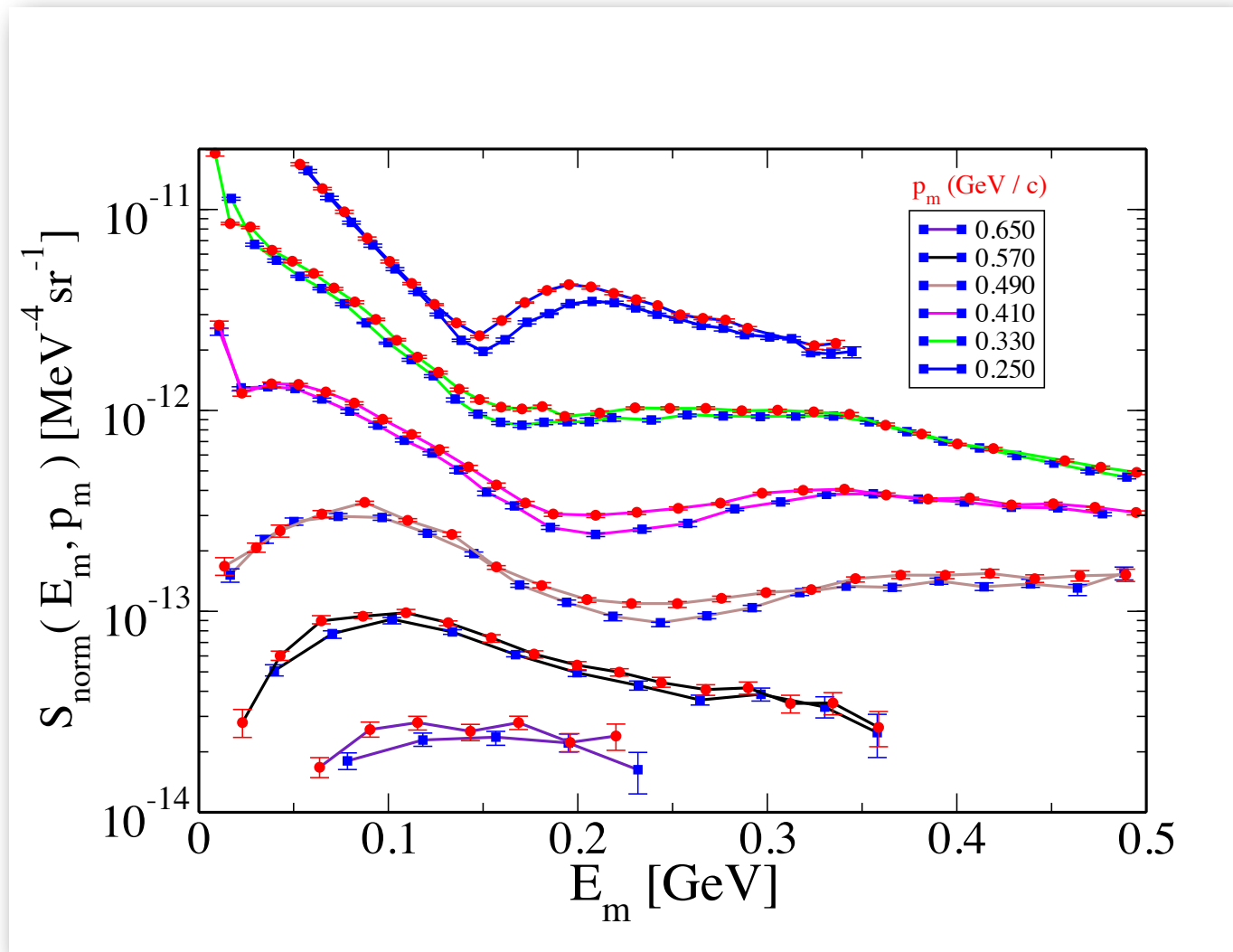
- Location of high-momentum components
- Integrated strength agrees with theoretical prediction Phys. Rev. C49, R17 (1994)

$\Rightarrow \sim 0.6$  protons for  $^{12}\text{C} \Rightarrow \sim 10\%$

reactions and structure

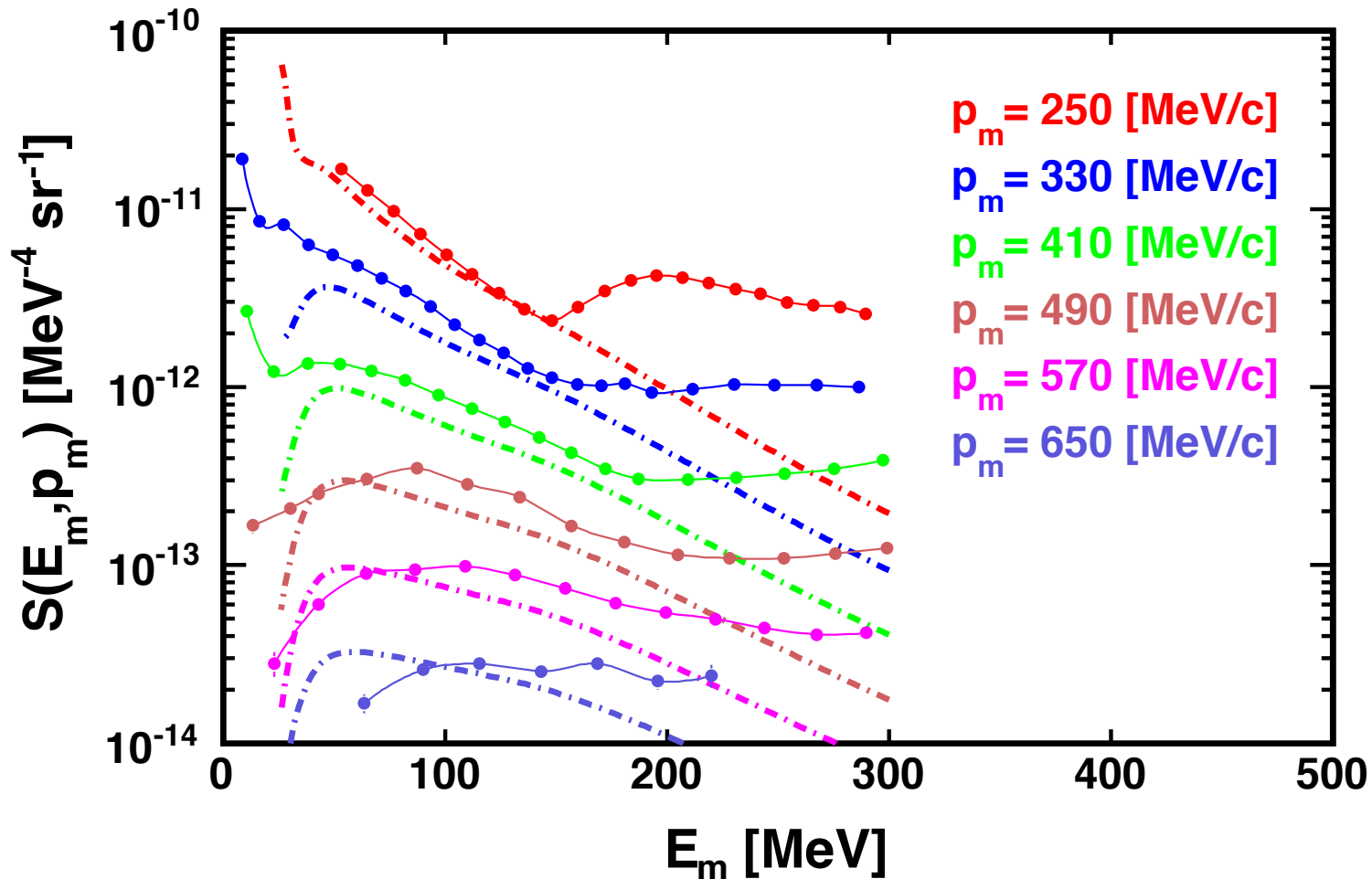
# High-momentum components

Rohe, Sick et al. JLab data for Al and Fe ( $e, e'p$ )  
per proton



# Jefferson Lab data per proton

- Pion/isobar contributions cannot be described
- Rescattering contributes some cross section (Barbieri, Lapikas)



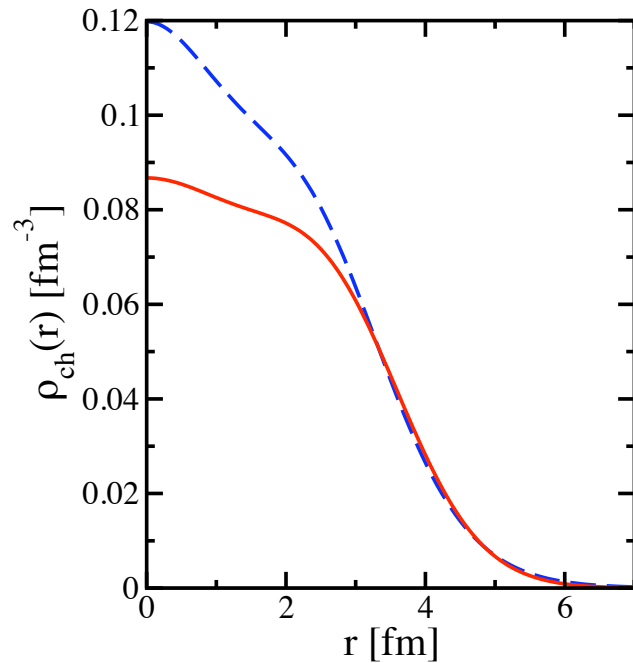


# Critical experimental data → charge density

Local version

radius correct...

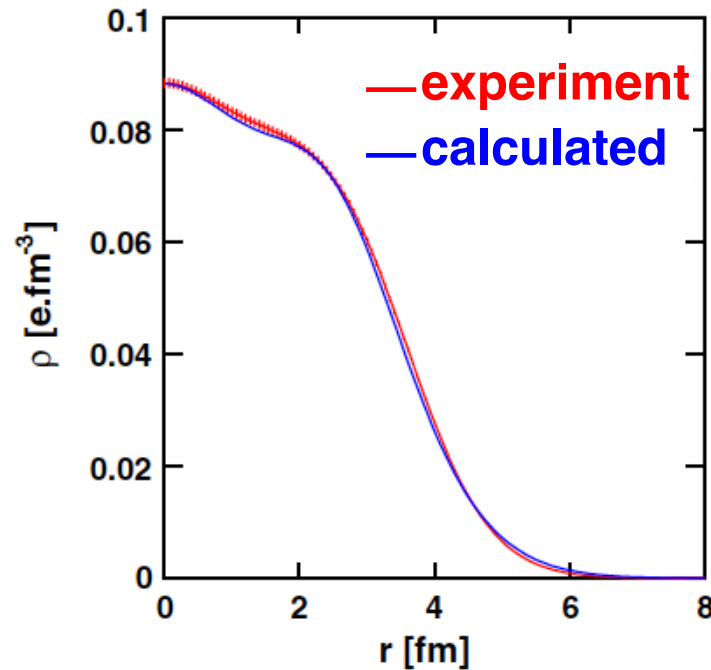
PRC82, 054306 (2010)



Charge density  $^{40}\text{Ca}$

Non-locality essential

PRL 112,162503(2014)



High-momentum nucleons → JLab can also be described → E/A

# Do elastic scattering data tell us about correlations?

- Scattering T-matrix (neutrons)

$$\Sigma_{\ell j}(k, k'; E) = \Sigma_{\ell j}^*(k, k'; E) + \int dq q^2 \Sigma_{\ell j}^*(k, q; E) G^{(0)}(q; E) \Sigma_{\ell j}(q, k'; E)$$

- Free propagator  $G^{(0)}(q; E) = \frac{1}{E - \hbar^2 q^2 / 2m + i\eta}$

- Propagator

$$G_{\ell j}(k, k'; E) = \frac{\delta(k - k')}{k^2} G^{(0)}(k; E) + G^{(0)}(k; E) \Sigma_{\ell j}(k, k'; E) G^{(0)}(k; E)$$

- Spectral representation

$$G_{\ell j}^p(k, k'; E) = \sum_n \frac{\phi_{\ell j}^{n+}(k) [\phi_{\ell j}^{n+}(k')]^*}{E - E_n^{*A+1} + i\eta} + \sum_c \int_{T_c}^{\infty} dE' \frac{\chi_{\ell j}^{cE'}(k) [\chi_{\ell j}^{cE'}(k')]^*}{E - E' + i\eta}$$

- Spectral density for  $E > 0$

$$S_{\ell j}^p(k, k'; E) = \frac{i}{2\pi} \left[ G_{\ell j}^p(k, k'; E^+) - G_{\ell j}^p(k, k'; E^-) \right] = \sum_c \chi_{\ell j}^{cE}(k) [\chi_{\ell j}^{cE}(k')]^*$$

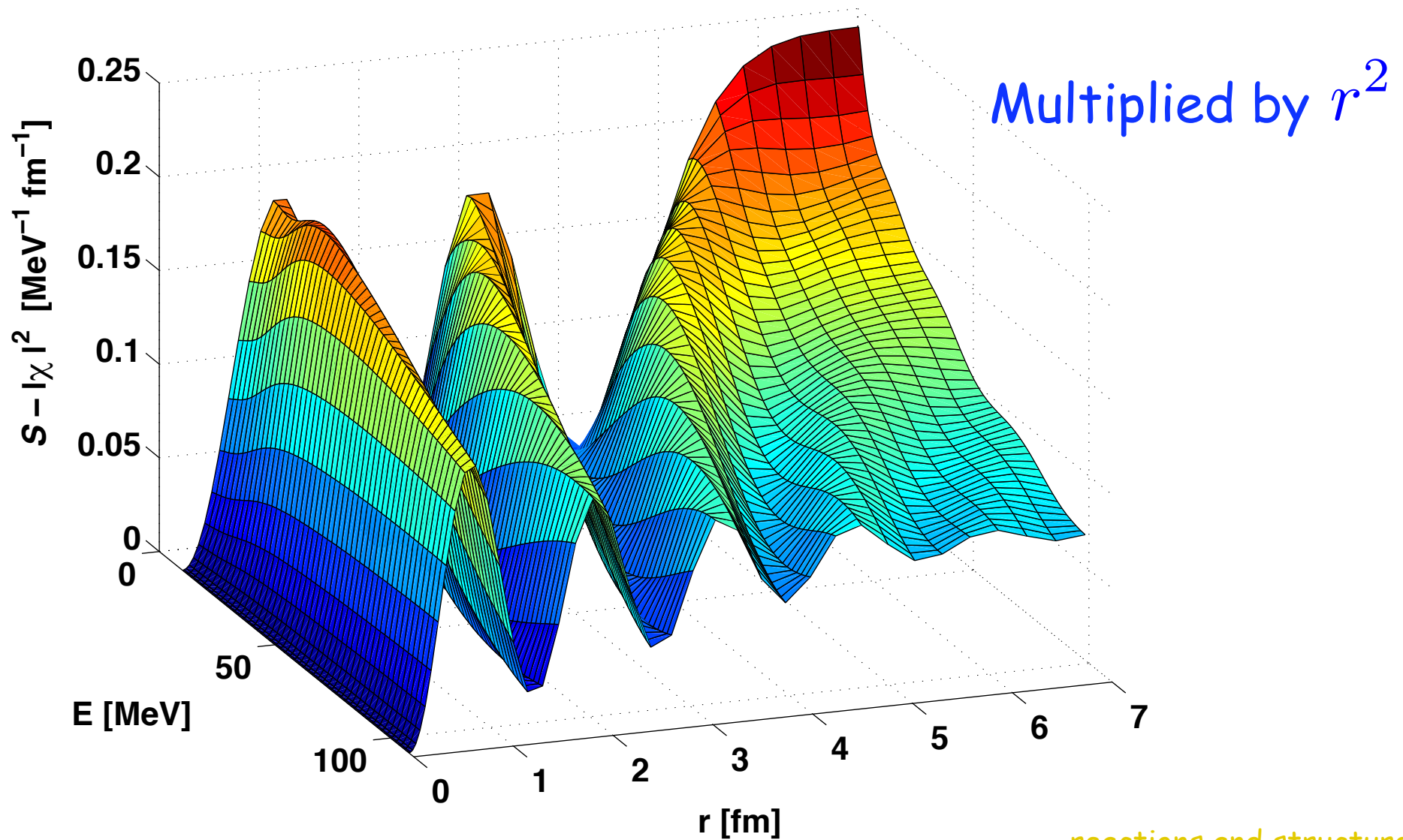
- Coordinate space  $S_{\ell j}^p(r, r'; E) = \sum_c \chi_{\ell j}^{cE}(r) [\chi_{\ell j}^{cE}(r')]^*$

- Elastic scattering also explicitly available

$$\chi_{\ell j}^{elE}(r) = \left[ \frac{2mk_0}{\pi\hbar^2} \right]^{1/2} \left\{ j_{\ell}(k_0 r) + \int dk k^2 j_{\ell}(kr) G^{(0)}(k; E) \Sigma_{\ell j}(k, k_0; E) \right\}$$

# Adding an $s_{1/2}$ neutron to $^{40}\text{Ca}$

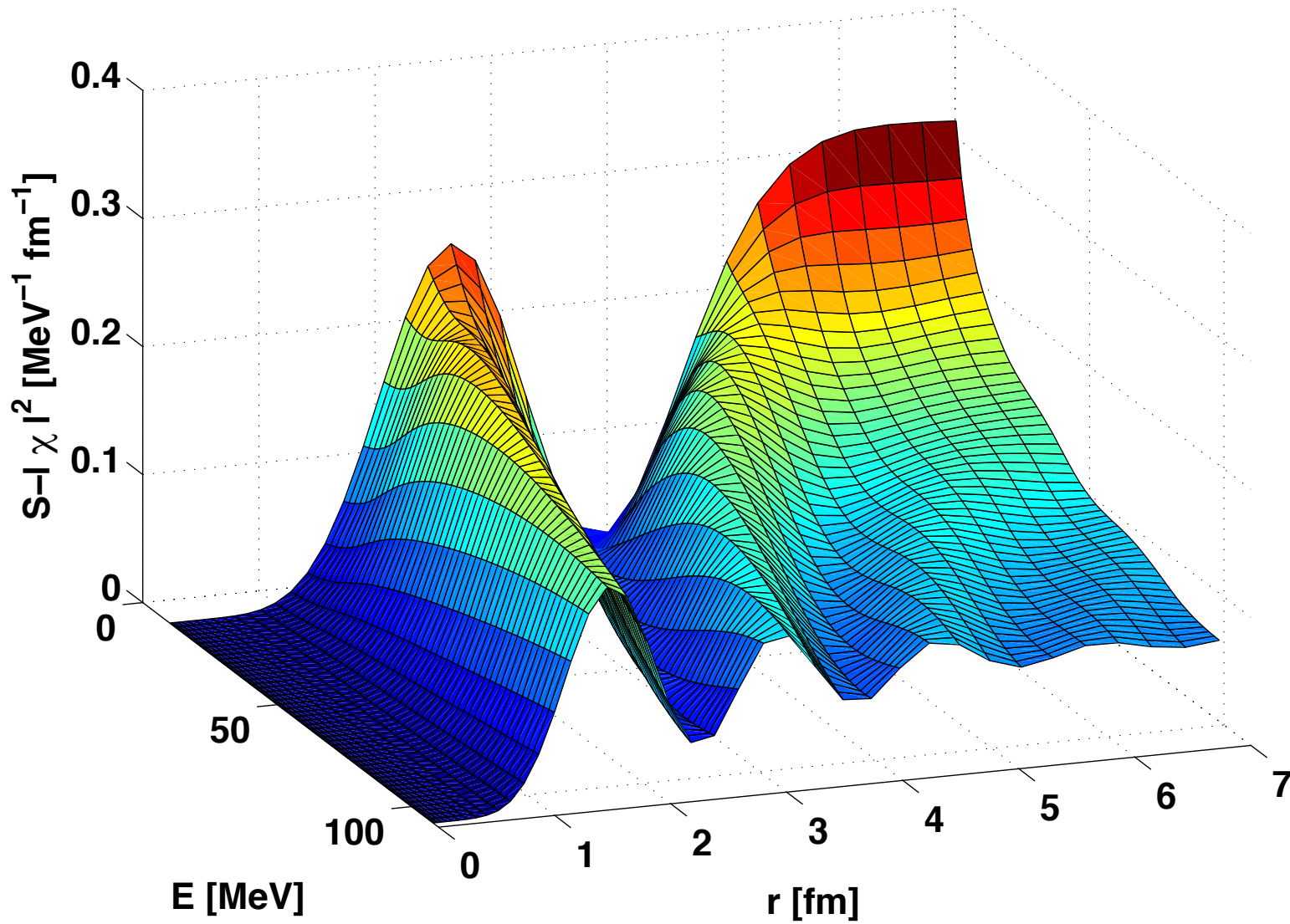
- Inelastically!
- Zero when there is no absorption!



$d_{3/2}$

$l=2$

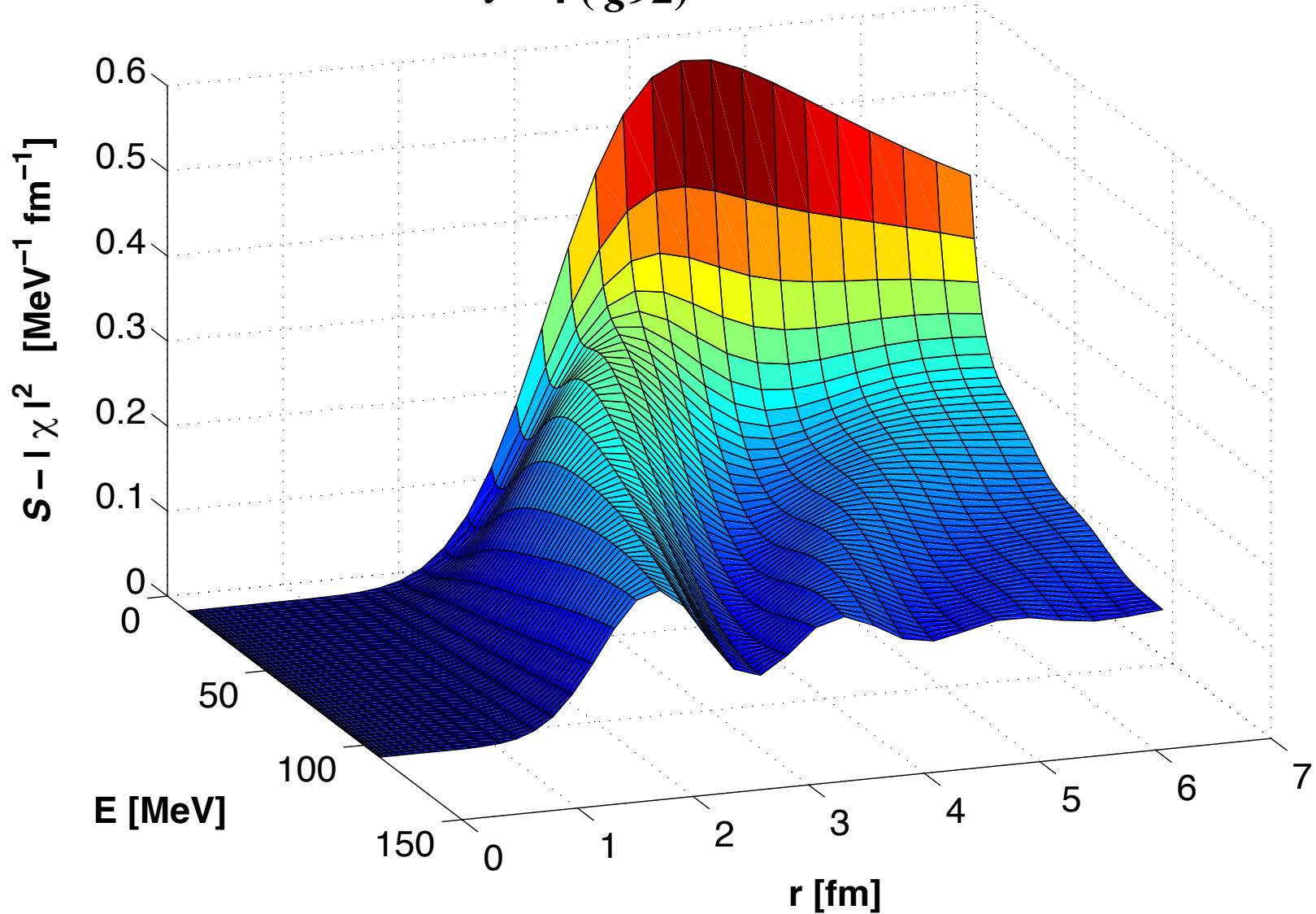
- One node now



# No nodes

- Asymptotically determined by inelasticity

$l = 4$  (g92)



# Determine location of bound-state strength

- Fold spectral function with bound state wave function

$$S_{\ell j}^{n+}(E) = \int dr r^2 \int dr' r'^2 \phi_{\ell j}^{n-}(r) S_{\ell j}^p(r, r'; E) \phi_{\ell j}^{n-}(r')$$

- → Addition probability of bound orbit
- Also removal probability

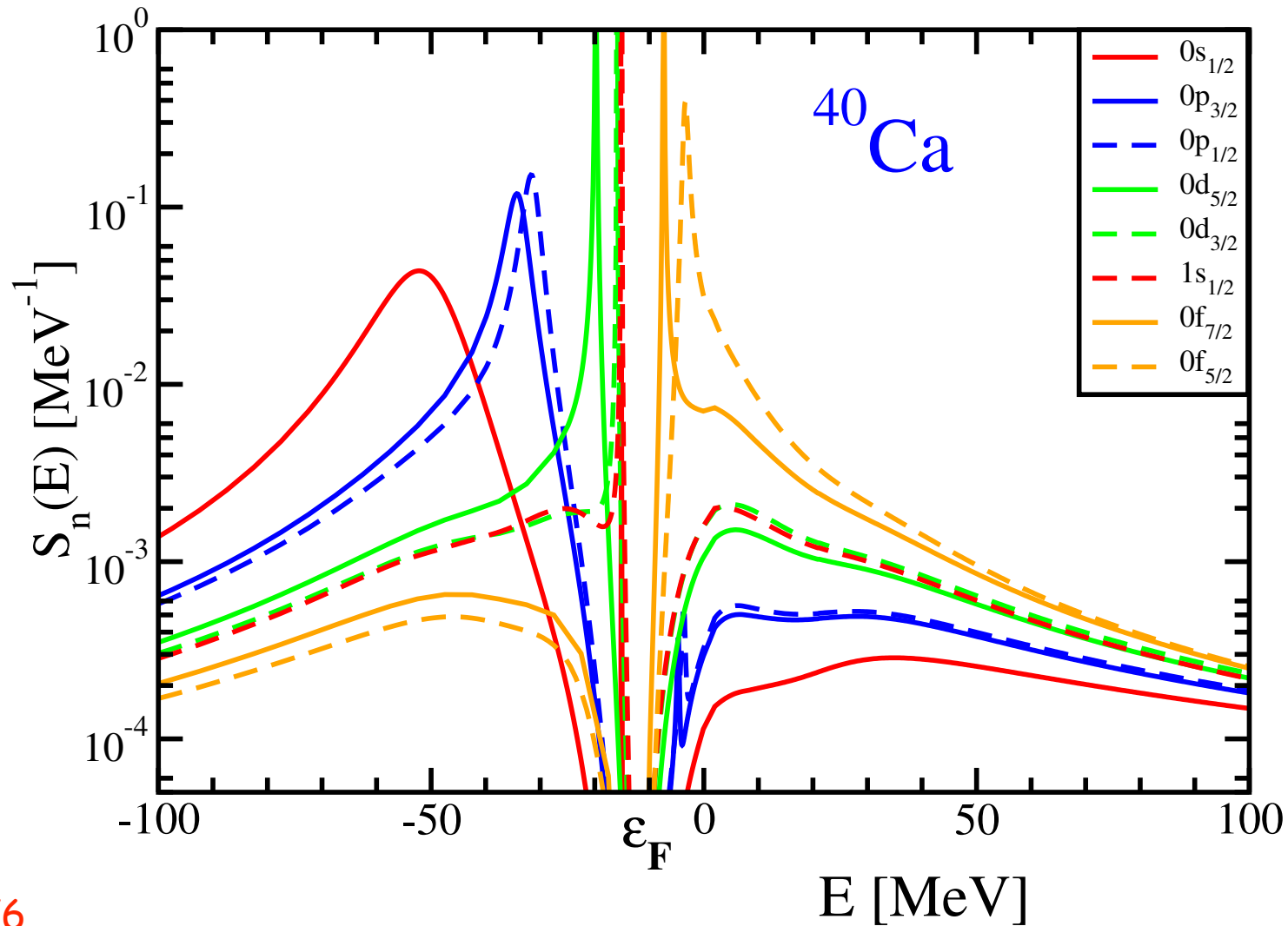
$$S_{\ell j}^{n-}(E) = \int dr r^2 \int dr' r'^2 \phi_{\ell j}^{n-}(r) S_{\ell j}^h(r, r'; E) \phi_{\ell j}^{n-}(r')$$

- Overlap function  $\sqrt{S_{\ell j}^n} \phi_{\ell j}^{n-}(r) = \langle \Psi_n^{A-1} | a_{r\ell j} | \Psi_0^A \rangle$

- Sum rule  $1 = n_{n\ell j} + d_{n\ell j} = \int_{-\infty}^{\epsilon_F} dE S_{\ell j}^{n-}(E) + \int_{\epsilon_F}^{\infty} dE S_{\ell j}^{n+}(E)$

# Spectral function for bound states

- [0,200] MeV → constrained by elastic scattering data



$$S_{0d_{3/2}} = 0.76$$

$$S_{1s_{1/2}} = 0.78$$

0.15 larger than NIKHEF analysis!

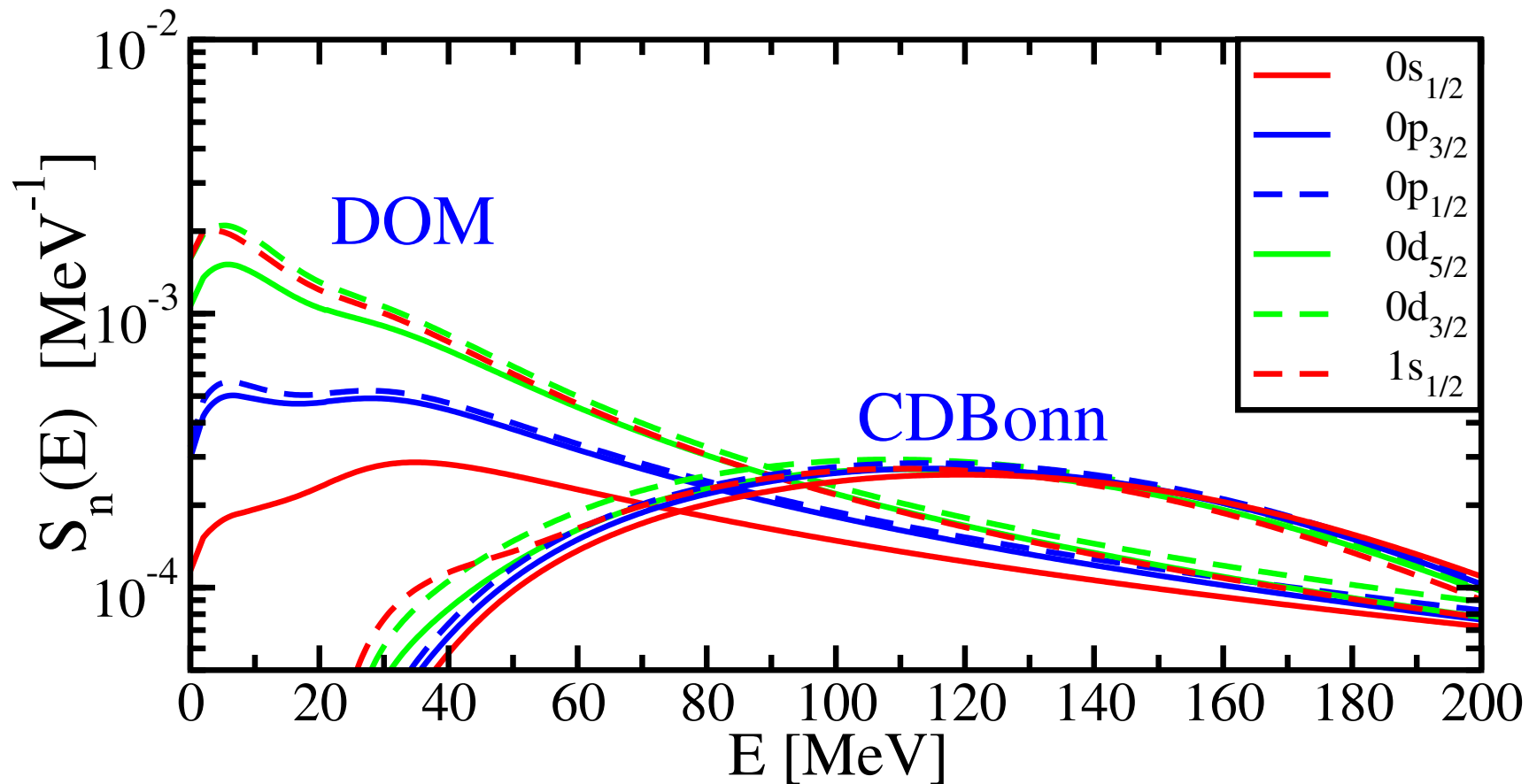
PRC90, 061603(R) (2014)

reactions and structure



# Compared with ab initio $\rightarrow$ SRC only

- CDBonn probably too soft
- SRC relevant at higher energy





# Quantitatively

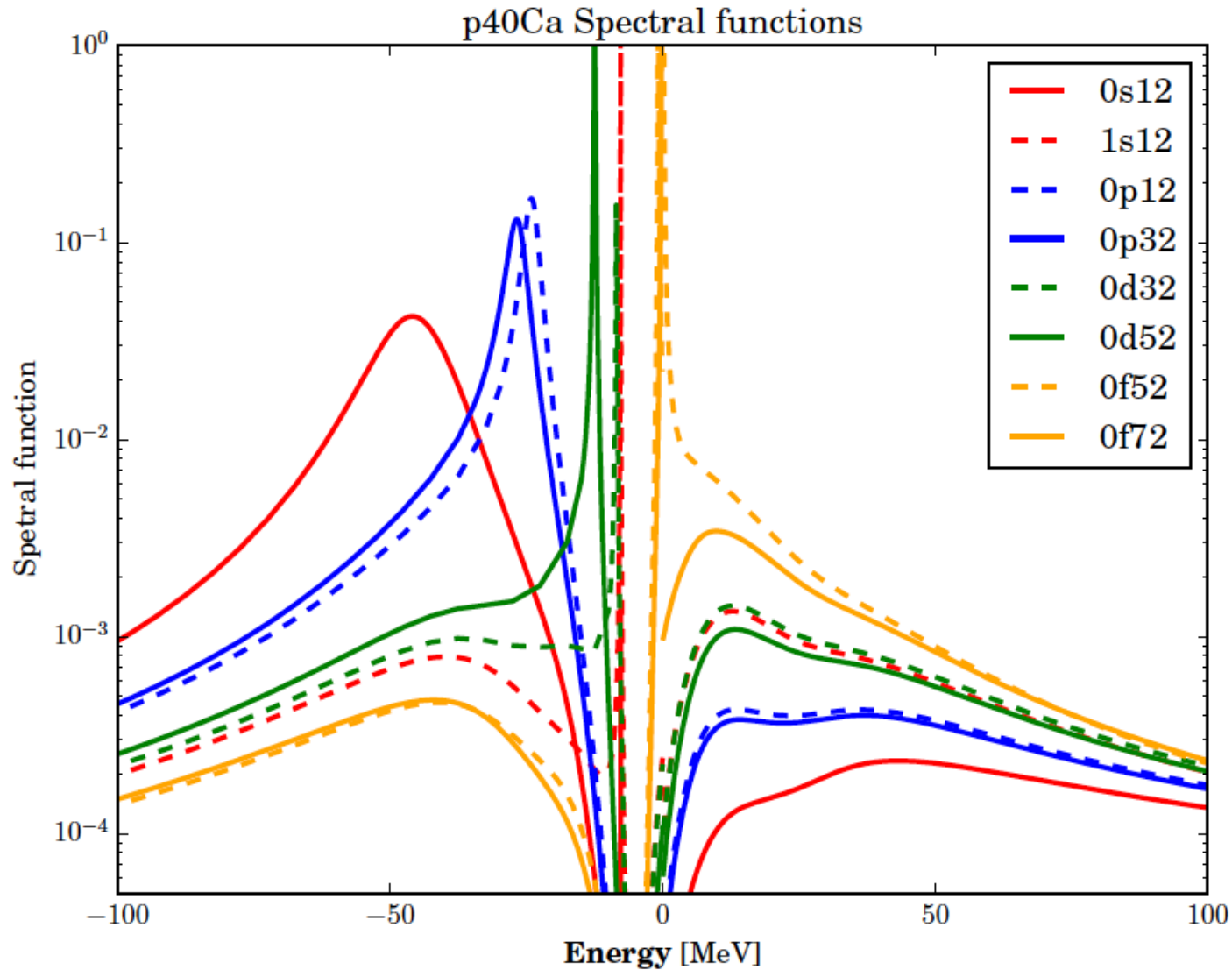
- Orbit closer to the continuum  $\rightarrow$  more strength in the continuum
- Note “particle” orbits
- Drip-line nuclei have valence orbits very near the continuum

Table 1: Occupation and depletion numbers for bound orbits in  $^{40}\text{Ca}$ .  $d_{nlj}[0, 200]$  depletion numbers have been integrated from 0 to 200 MeV. The fraction of the sum rule that is exhausted, is illustrated by  $n_{nlj} + d_{nlj}[\varepsilon_F, 200]$ . Last column  $d_{nlj}[0, 200]$  depletion numbers for the CDBonn calculation.

orbit	$n_{nlj}$ DOM	$d_{nlj}[0, 200]$ DOM	$n_{nlj} + d_{nlj}[\varepsilon_F, 200]$ DOM	$d_{nlj}[0, 200]$ CDBonn
$0s_{1/2}$	0.926	0.032	0.958	0.035
$0p_{3/2}$	0.914	0.047	0.961	0.036
$1p_{1/2}$	0.906	0.051	0.957	0.038
$0d_{5/2}$	0.883	0.081	0.964	0.040
$1s_{1/2}$	0.871	0.091	0.962	0.038
$0d_{3/2}$	0.859	0.097	0.966	0.041
$0f_{7/2}$	0.046	0.202	0.970	0.034
$0f_{5/2}$	0.036	0.320	0.947	0.036

# Proton spectral function in $^{40}\text{Ca}$

- Learning how to deal with Coulomb in momentum space



- So far material reviewed in

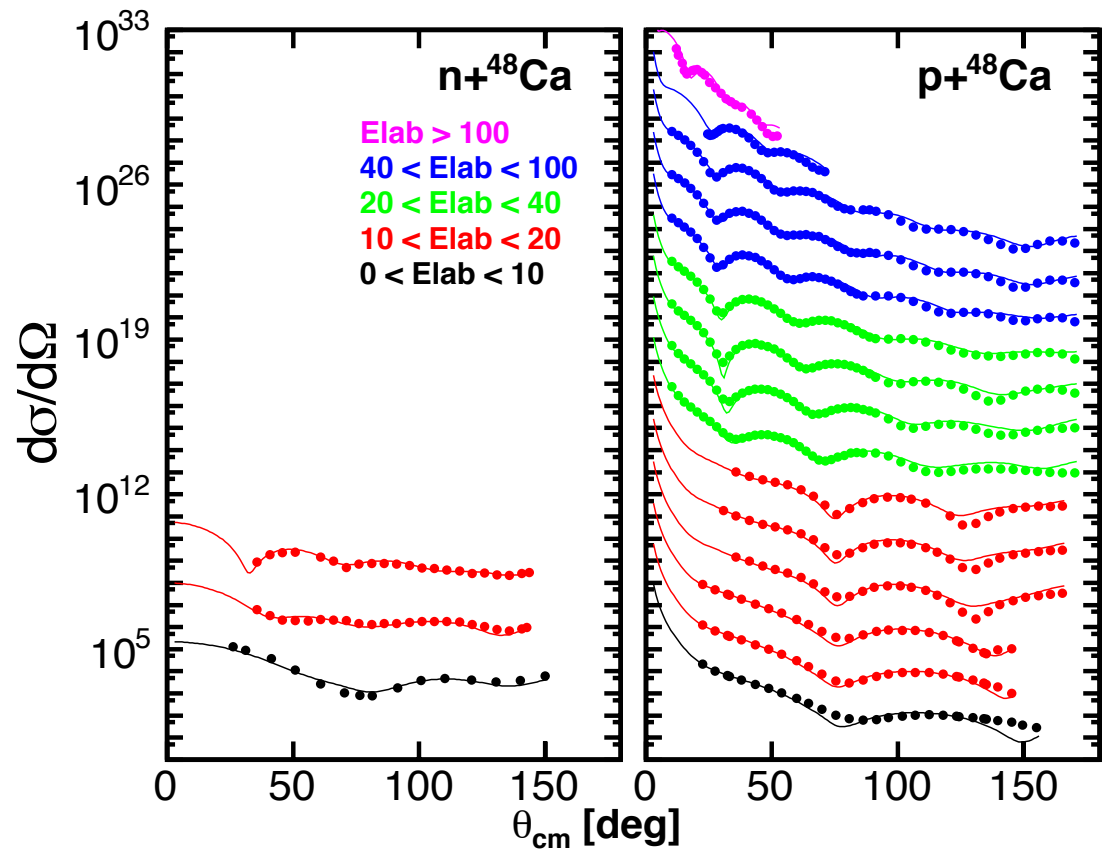
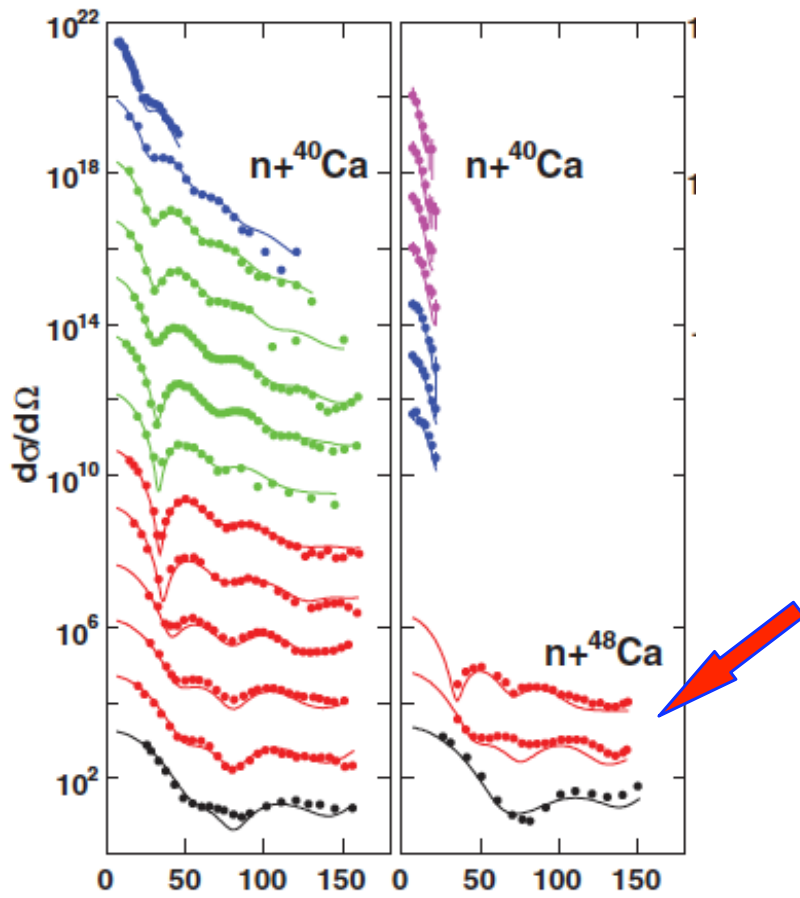
- W. H. Dickhoff, R. J. Charity, and M. H. Mahzoon
- Novel applications of the dispersive optical model.  
J. Phys. G, (2016), invited topical review, submitted.
- <http://arxiv.org/abs/1606.08822v1>

## New DOM results for $^{48}\text{Ca}$

- Change of proton properties when 8 neutrons are added to  $^{40}\text{Ca}$ ?
- Change of neutron properties?
- Can hard to measure quantities be indirectly constrained?

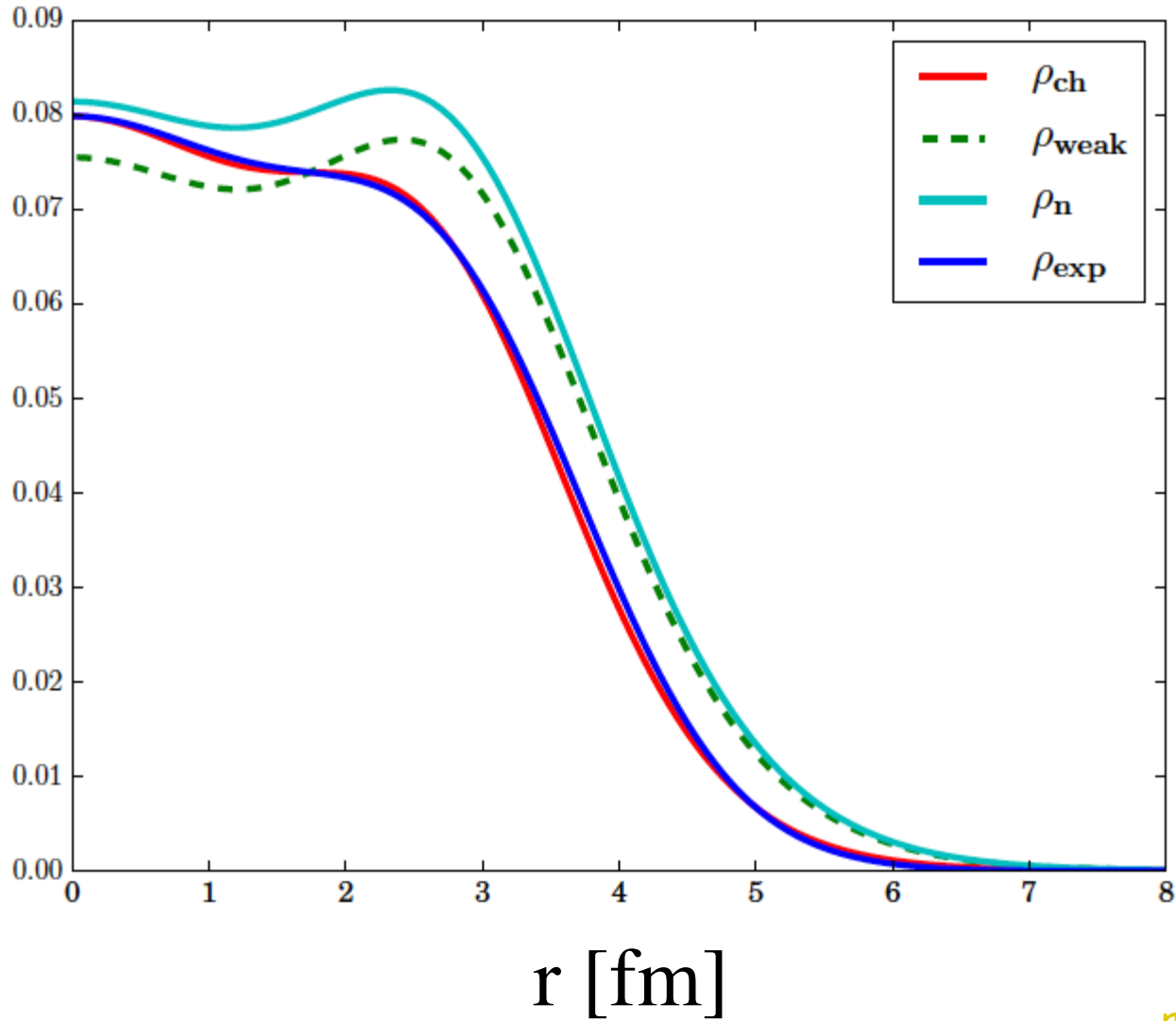
# What about neutrons?

- $^{48}\text{Ca}$   $\rightarrow$  charge density has been measured
- Recent neutron elastic scattering **data**  $\rightarrow$  PRC83,064605(2011)
- Local DOM **OLD** Nonlocal DOM **NEW**



# Results $^{48}\text{Ca}$

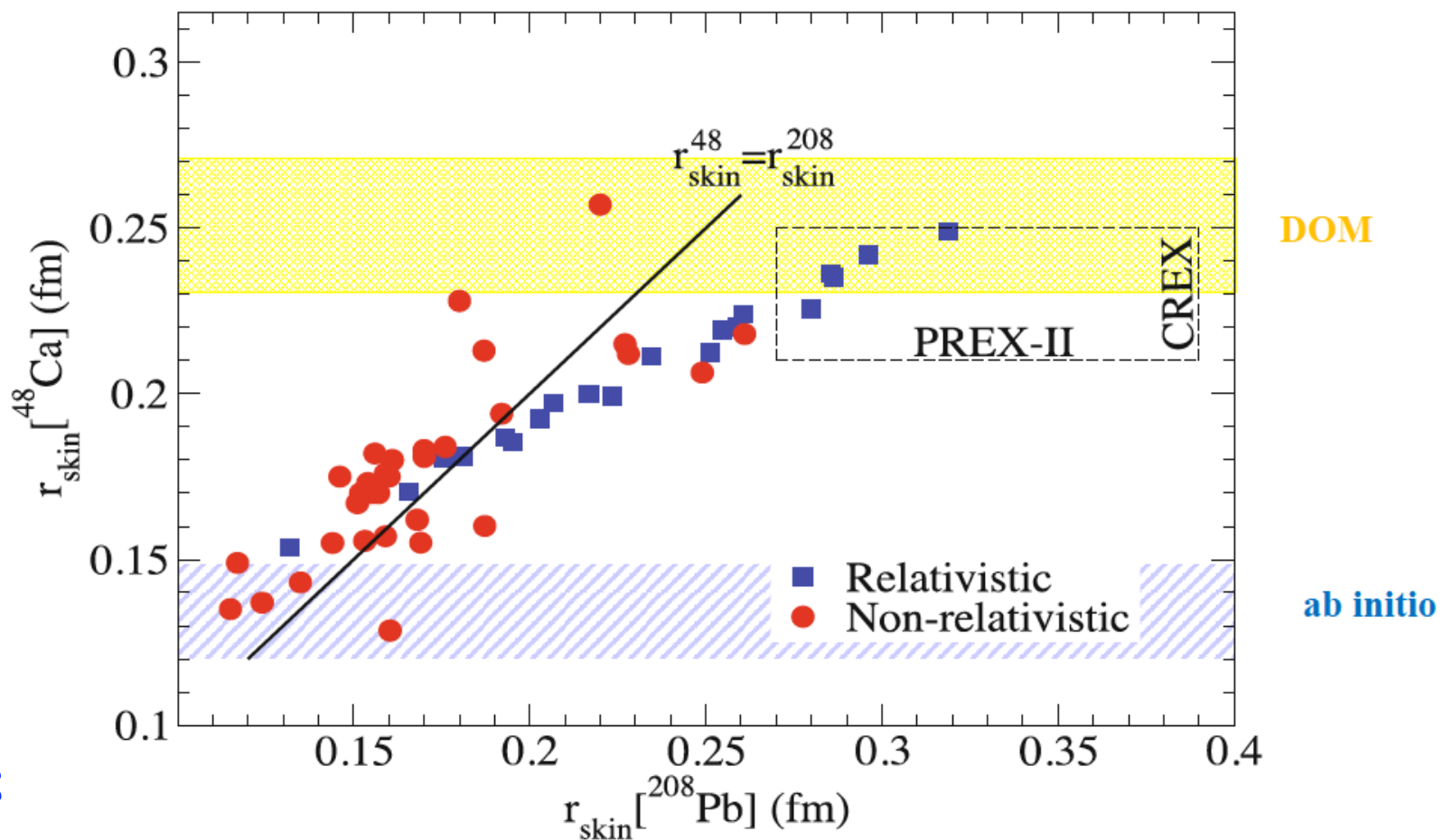
- Density distributions
- DOM  $\rightarrow$  neutron distribution  $\rightarrow R_n - R_p$



# Comparison of neutron skin with other calculations and future experiments...

- Figure adapted from

C.J. Horowitz, K.S. Kumar, and R. Michaels, Eur. Phys. J. A (2014)



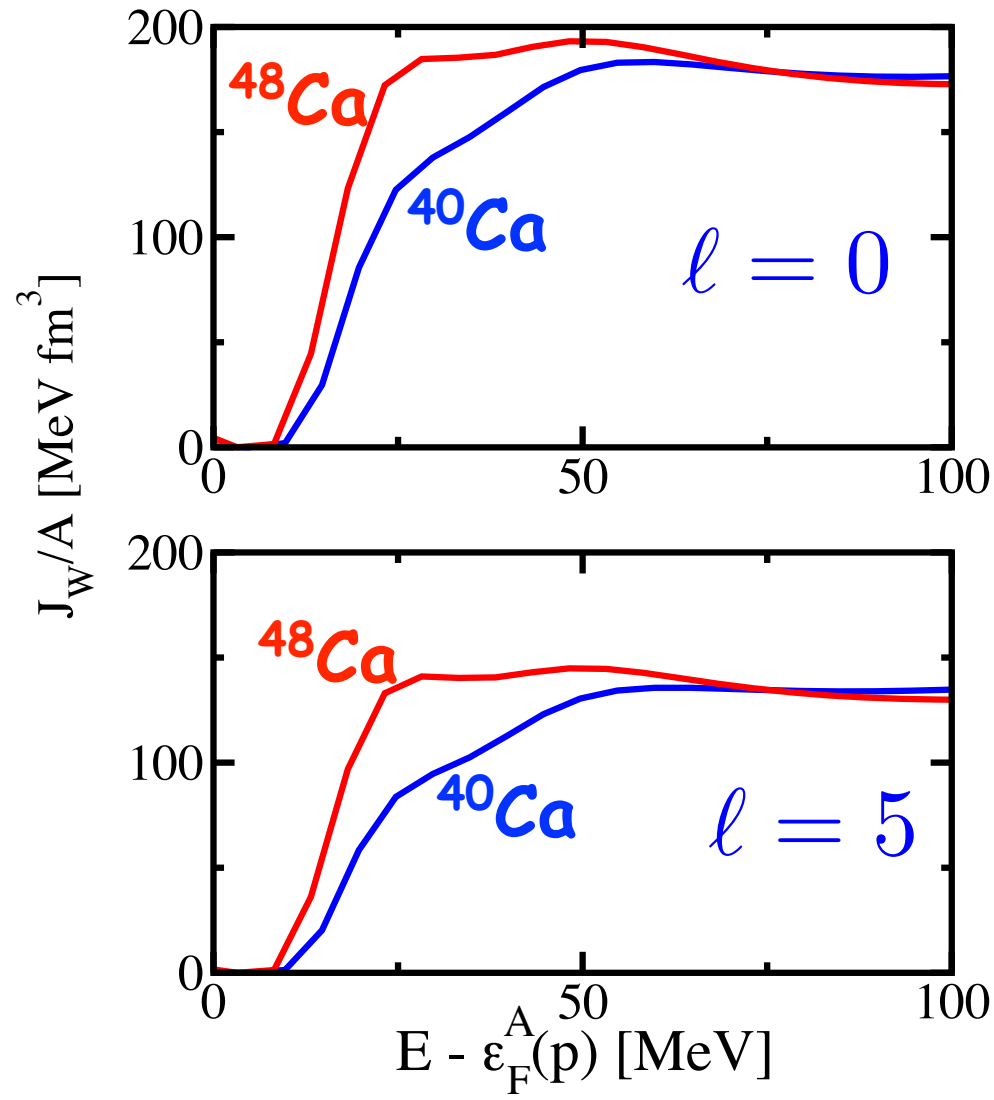
- "Ab initio":

G. Hagen et al., Nature Phys. 12, 186 (2016)

--> drip line

# Volume integrals for $^{40-48}\text{Ca}$

- Protons see the same interior but a different surface!



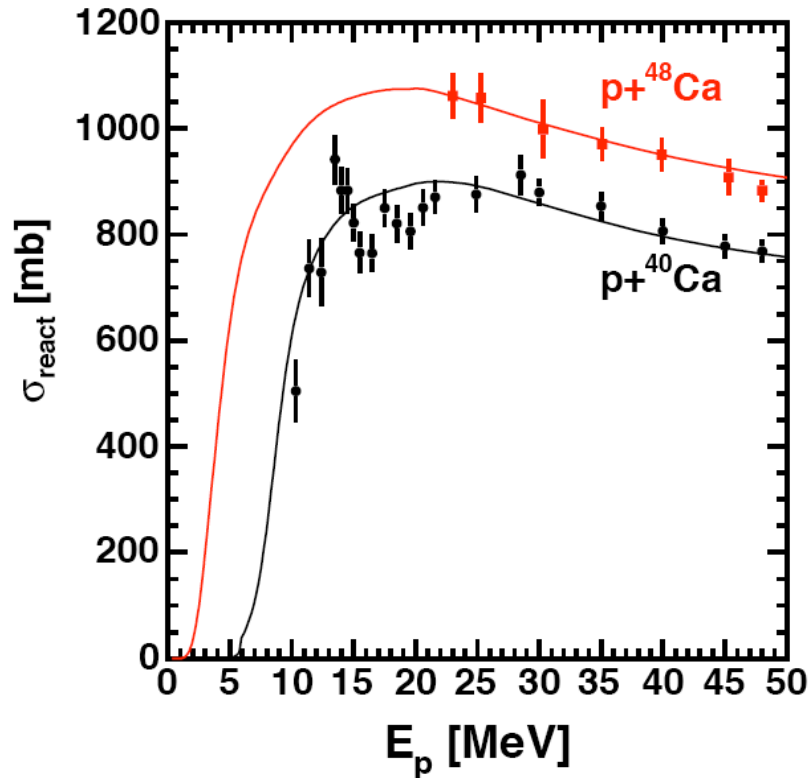
--> drip line

# Quantitative comparison of $^{40}\text{Ca}$ and $^{48}\text{Ca}$

Spectroscopic factors	$^{40}\text{Ca}$	p $^{48}\text{Ca}$	n $^{48}\text{Ca}$
$0d_{3/2}$	0.76	0.65 ↓	0.80 ↑
$1s_{1/2}$	0.78	0.71 ↓	0.83 ↑
$0f_{7/2}$	0.73	0.59 ↓	0.84 ↑



# Why are protons in $^{48}\text{Ca}$ more correlated than in $^{40}\text{Ca}$ ?

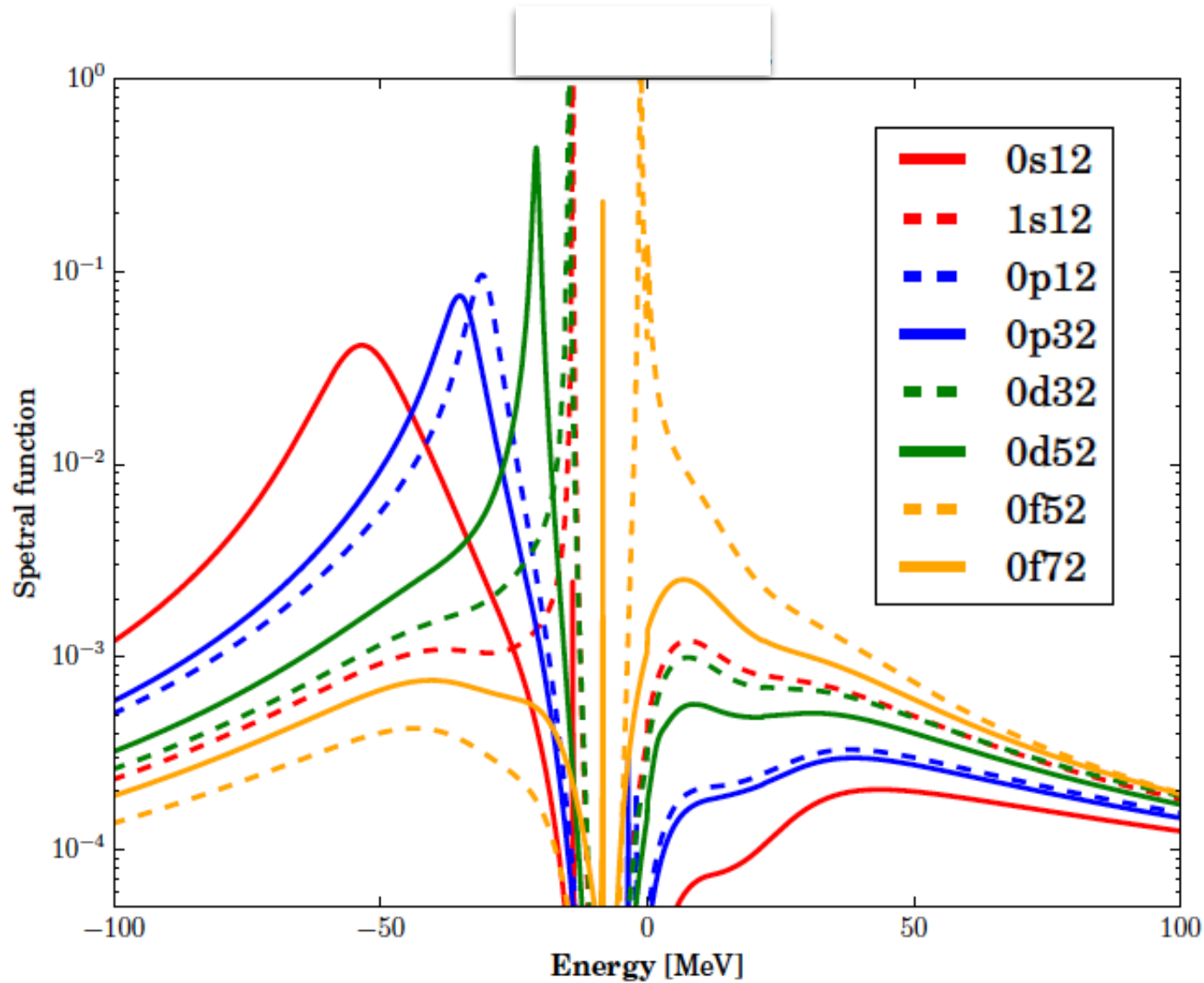


Loss of flux in the elastic channel

Answer: data require more surface absorption in  $^{48}\text{Ca}$  than in  $^{40}\text{Ca}$

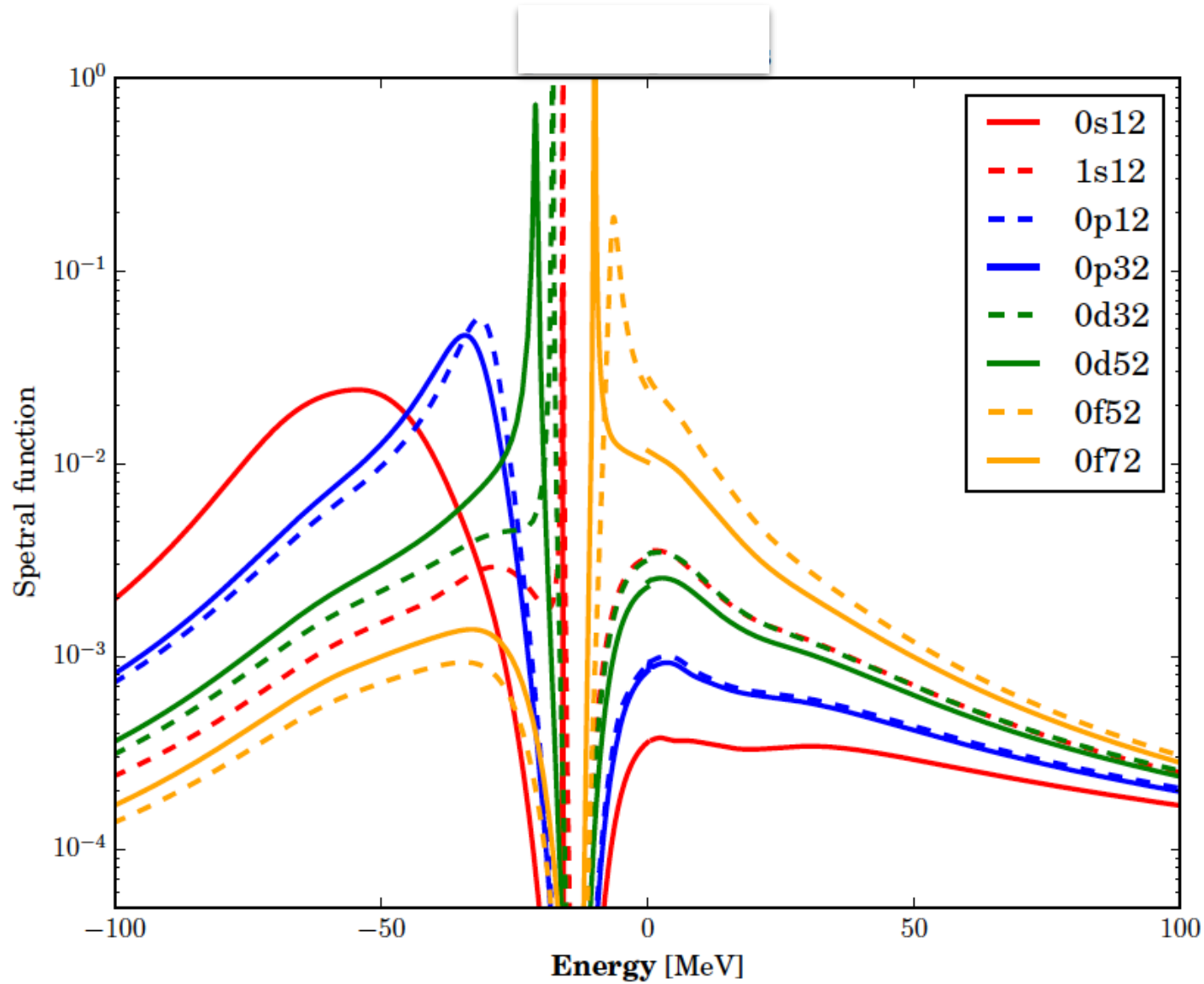
# Neutron spectral function in $^{48}\text{Ca}$

- New results for  $^{48}\text{Ca}$ : Neutrons in  $^{48}\text{Ca}$  less correlated  $\leftrightarrow$   $^{40}\text{Ca}$



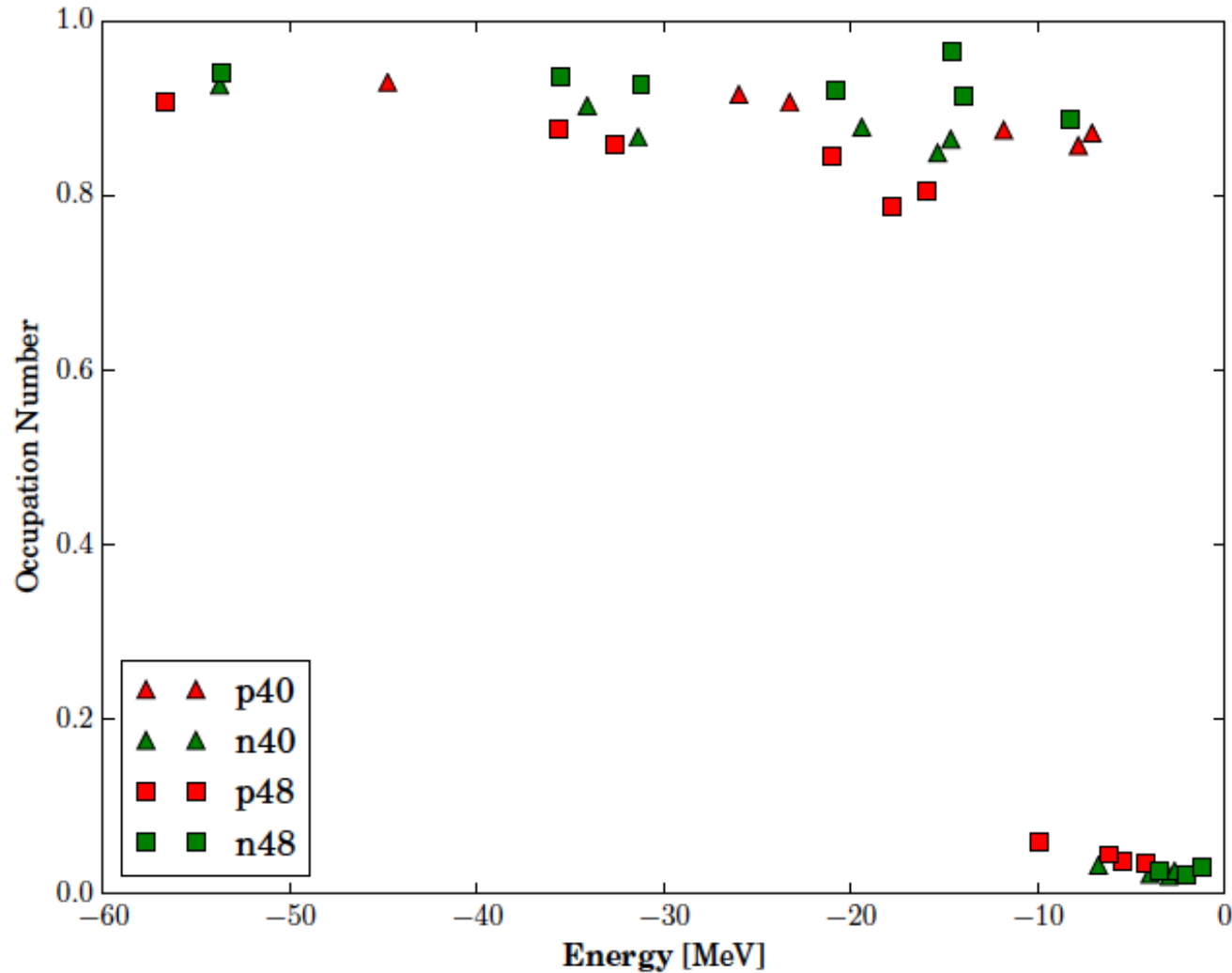
# Protons in $^{48}\text{Ca}$

- Protons in  $^{48}\text{Ca}$  more correlated than in  $^{40}\text{Ca}$



# Occupation numbers

- Modest decrease for protons and increase for neutrons going from  $^{40}\text{Ca}$  to  $^{48}\text{Ca}$



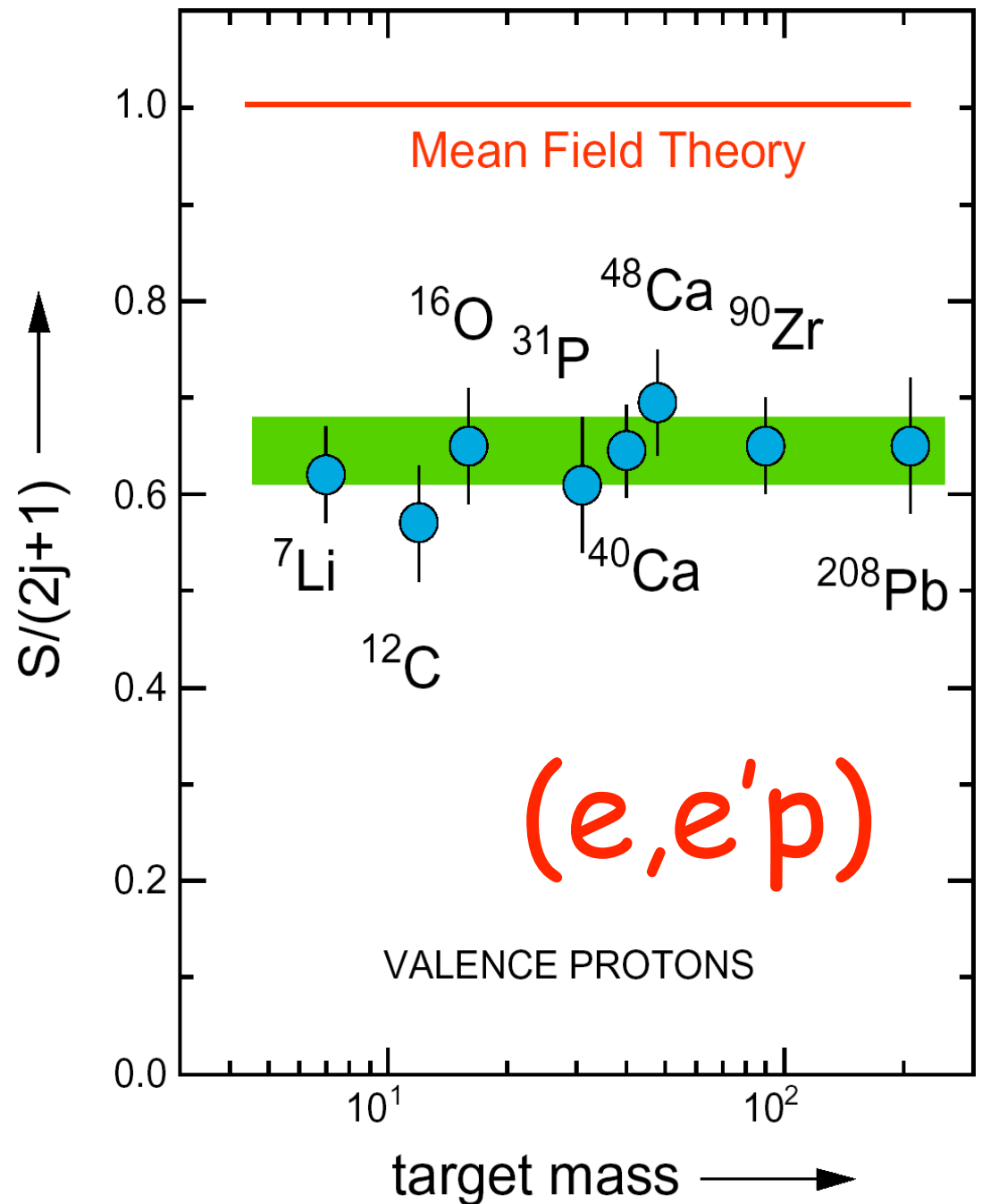
# Removal probability for valence protons from NIKHEF data

L. Lapikás, Nucl. Phys. A553,297c (1993)

$S \approx 0.65$  for valence protons  
Reduction  $\Rightarrow$  both SRC and LRC

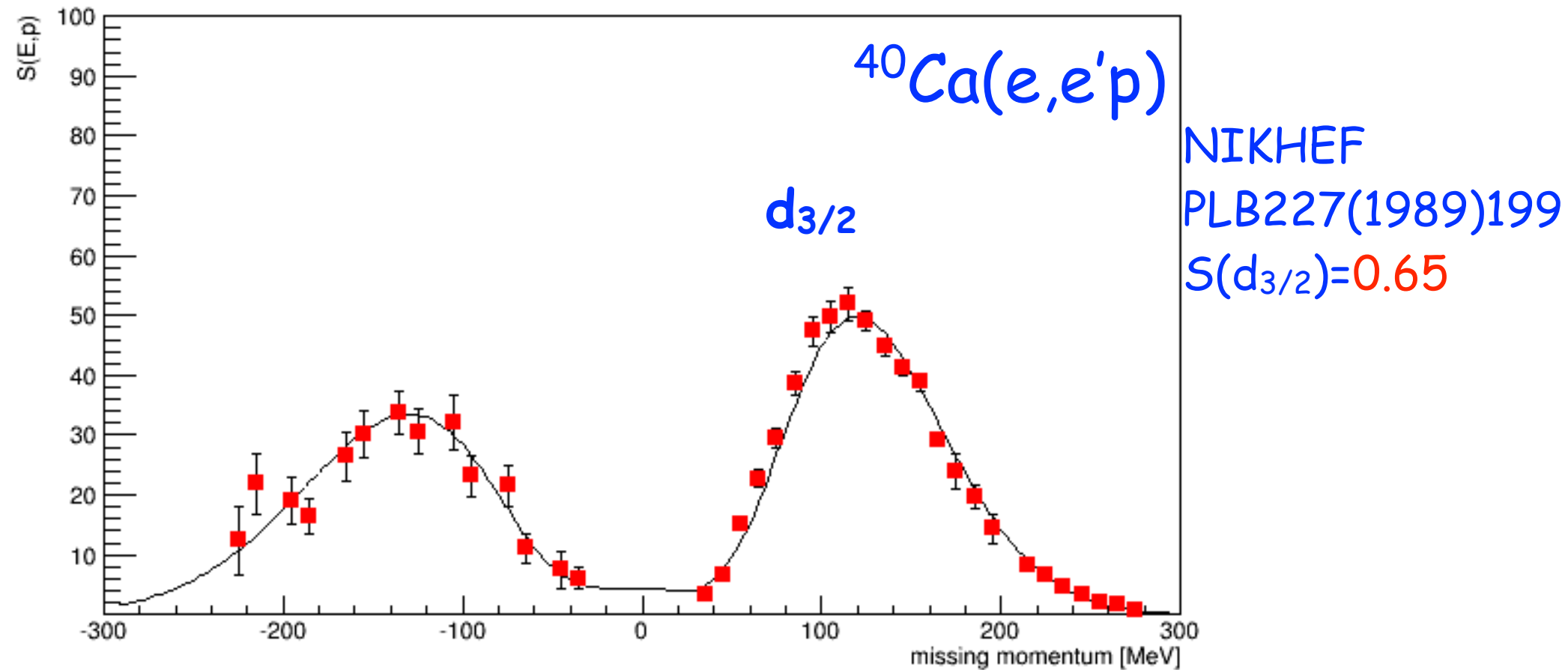
Weak probe but propagation in the nucleus of removed proton using standard optical potentials to generate distorted wave  $\rightarrow$  associated uncertainty  $\sim 5-15\%$

Why: details of the interior scattering wave function uncertain since non-locality is not constrained (so far.....)



# Very recent analysis (preliminary)

- NIKHEF (e,e'p) data with only DOM input (Atkinson in progress)
- Confirms larger spectroscopic factors  $\sim 0.8$
- $\rightarrow$  Consequences for analysis of ALL nuclear reactions



# Conclusions

- It **is** possible to link nuclear reactions and nuclear structure
- Vehicle: **nonlocal** version of **Dispersive Optical Model** (Green's function method) as developed by Mahaux → **DSM**
- Can be used as input for analyzing nuclear reactions
- Can predict properties of exotic nuclei
- "Benchmark" for ab initio calculations: e.g.  $V_{NNN}$  → binding
- Can describe ground-state properties
  - charge density & momentum distribution
  - spectral properties including high-momentum Jefferson Lab data
- **Elastic scattering** determines depletion of bound orbitals
- **Outlook:** reanalyze many reactions with nonlocal potentials...
- For  $N \geq Z$  sensitive to properties of neutrons → weak charge prediction, **large neutron skin**, perhaps more... reactions and structure