Revisiting the problem of inclusive breakup within the IAV model

Antonio M. Moro

University of Seville (Spain)

Workshop on "Deuteron-induced reactions and beyond: inclusive breakup fragment cross sections, MSU, July 2016"

Outline

In 2012:

- Understanding (and quantification) of large inclusive breakup cross sections observed in reactions with neutron-halo nuclei (^{6}He, ^{11}Li, ^{11}Be)

In 2016:

- Understanding of larger inclusive yields for other (non-halo) weakly-bound nuclei (^{6,7}Li, ^{7}Be, ...).
- Possible extension to incomplete fusion.
- Surrogate reactions.

Ph.D thesis of Jin Lei:

"*Study of inclusive breakup reactions induced by weakly bound nuclei*" (Univ. of Seville, July 2016)

In 2012:

- Understanding (and quantification) of large inclusive breakup cross sections observed in reactions with neutron-halo nuclei (^{6}He, ^{11}Li, ^{11}Be)

In 2016:

- Understanding of larger inclusive yields for other (non-halo) weakly-bound nuclei (^{6,7}Li, ^{7}Be, . . .).
- Possible extension to incomplete fusion.
- Surrogate reactions.

Ph.D thesis of Jin Lei:

"*Study of inclusive breakup reactions induced by weakly bound nuclei*" (Univ. of Seville, July 2016)

In 2012:

- Understanding (and quantification) of large inclusive breakup cross sections observed in reactions with neutron-halo nuclei (^{6}He, ^{11}Li, ^{11}Be)

In 2016:

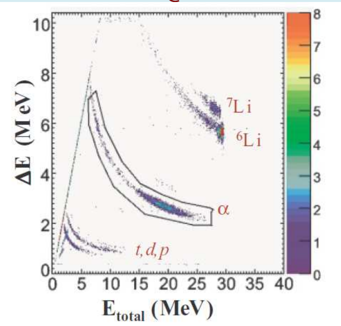
- Understanding of larger inclusive yields for other (non-halo) weakly-bound nuclei (^{6,7}Li, ^{7}Be, . . .).
- Possible extension to incomplete fusion.
- Surrogate reactions.

Ph.D thesis of Jin Lei:

"*Study of inclusive breakup reactions induced by weakly bound nuclei*" (Univ. of Seville, July 2016)

Evidence of NEB contributions in inclusive ^{209}Bi(^{6}Li,α)X

^{6}Li+^{209}Bi @ 32 MeV

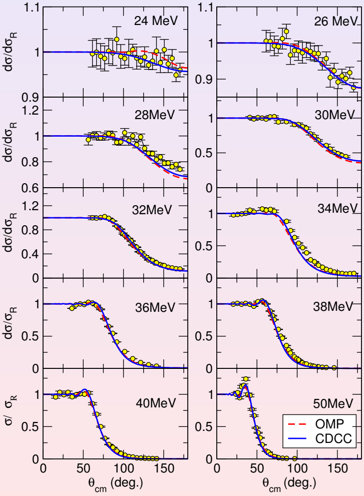


Santra *et al*, PRC85,014612(2008)

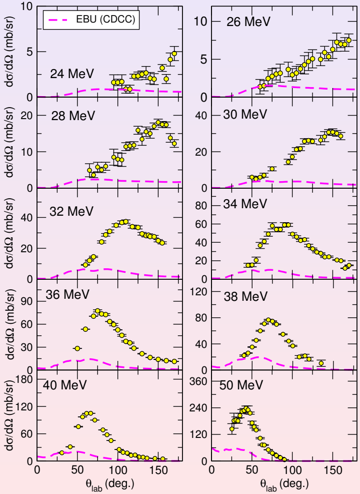
α yields much larger than *d* yields
$\Rightarrow$ expect other breakup modes rather than EBU

Evidence of NEB contributions in inclusive $^{209}Bi(^{6}Li,\alpha)X$

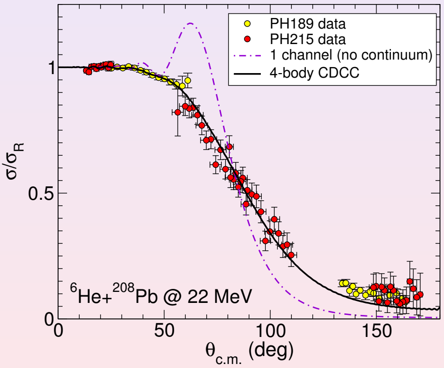
Elastic scattering



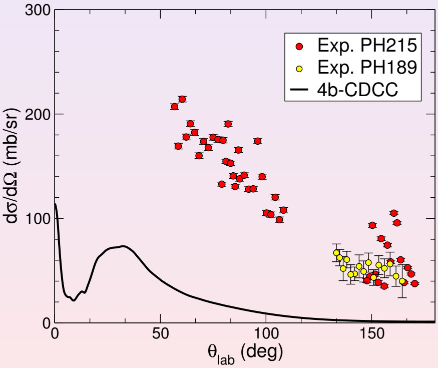
Inclusive α's



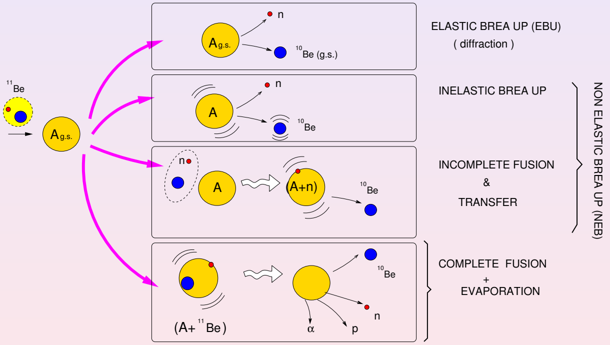
Elastic scattering



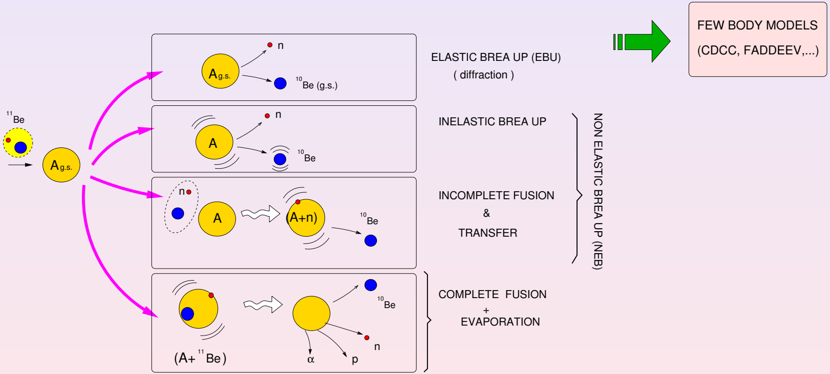
Inclusive α's



Breakup modes: ^{11}Be+A $\to$ ^{10}Be+n+ A example



Breakup modes: ^{11}Be+A $\rightarrow$ ^{10}Be+n+ A example



- Inclusive breakup:

$$\underbrace{(b+x)}_{a} + A \rightarrow b + \underbrace{(x+A)^*}_{c}$$

- Inclusive differential cross section: $\sigma_b^{\mathrm{BU}} = \sigma_b^{\mathrm{EBU}} + \sigma_b^{\mathrm{NEB}}$
- Post-form expression for inclusive breakup:

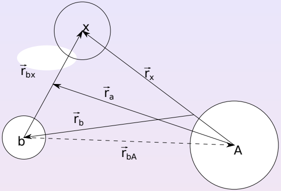
$$\frac{d^2\sigma}{d\Omega_b E_b} = \frac{2\pi}{\hbar v_a}\rho(E_b)\sum_c |\langle \chi_b^{(-)} \Psi_{xA}^{c,(-)} | V_{bx} | \Psi^{(+)}\rangle|^2 \delta(E - E_b - E^c)$$

⇨ $\Psi_{xA}^{c,(-)}$ wavefunctions for $c \equiv x + A$ states

⇨ $\Psi^{(+)}$ exact scattering wavefunction

Inclusion of all relevant $c = x + A$ channels is not feasible in general $\Rightarrow$ use closed-form models

Ichimura, Austern, Vincent model for NEB (IAV, PRC32, 431 (1985))



- Treat b particle as spectator $\Rightarrow \chi_b^{(-)}(\vec{k}_b, \vec{r}_b)$
- Model Hamiltonian (still many-body for $x + A$):

$$H = K_b + V_{bx} + U_{bA}(\mathbf{r}_{bA}) + \underbrace{K_x + H_A(\xi) + V_{xA}(\xi, \mathbf{r}_x)}_{H_B},$$

- $x - A$ wave function for a given final b state:

$$\boxed{Z_x^{(b)}(\xi, \mathbf{r}_x) \equiv (\chi_b^{(-)}(\vec{k}_b)|\Psi\rangle = \left[E^+ - E_b - H_B\right]^{-1} (\chi_b^{(-)}|V_{\rm post}|\Psi\rangle} \quad V_{\rm post} \approx V_{bx}(\mathbf{r}_x)$$

- Define projector onto target g.s. ($A_{\mathrm{g.s.}}$): $\mathcal{P} = |\phi_A^{(0)}\rangle\langle\phi_A^{(0)}|$

- 3-body reduction: $\mathcal{P} Z_x^{(b)}(\xi, \mathbf{r}_x) = \varphi_x^{(0)}(\mathbf{r}_x)\phi_A^{(0)}(\xi)$

$$[K_x + U_{xA} - E_x]\varphi_x^{(0)}(\mathbf{r}_x) = (\chi_b^{(-)}|V_{bx}|\Psi_{3b}^{(+)}\rangle$$

  - ⇨ $\Psi_{3b}^{(+)}$=three-body scattering wave function (DWBA, CDCC, Faddeev...)
  - ⇨ U_{xA}=optical model potential operator.
  - ⇨ The asymptotics of $\varphi_x^{(0)}(\mathbf{r}_x)$ gives ONLY the EBU part.

- σ_b^{NEB} is the flux loss (“absorption”) in the $x + A_{gs}$ channel:

$$\frac{d\sigma^{\mathrm{NEB}}}{d\Omega_b dE_b} = -\frac{2}{\hbar v_a}\rho_b(E_b)\langle\varphi_x|W_{xA}|\varphi_x\rangle \quad \text{(optical theorem)}$$

- Ignore internal spins $\Rightarrow \vec{l}_a + \vec{l}_{bx} = \vec{l}_b + \vec{l}_x$
- DWBA: $\Psi_{3b}^{(+)} \approx \chi_d^{(+)}(\mathbf{R})\phi_d(\mathbf{r}_{bx})$
- $\chi_b^{(-)}(\vec{k}_b, \vec{r}_b)$ distorted waves averaged in energy *bins*.

Details in:

J. Lei and A.M.M., PRC 92, 044616 (2015); PRC 92, 061602 (2015)

Two regularization procedures tested and compared:

1. Damping factor method (Huby and Mines):

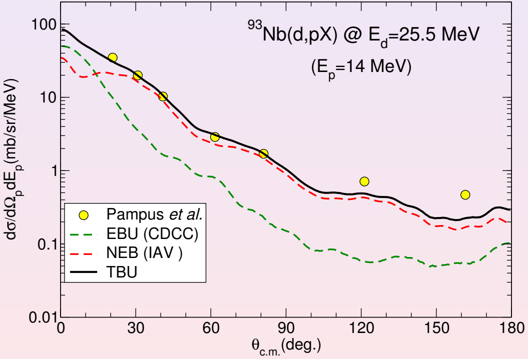
$$\rho(\vec{k}_b, \vec{r}_x) \to \lim_{\alpha \to 0} e^{-\alpha r_x} \rho(\vec{k}_b, \vec{r}_x)$$

2. Binning method (I.J. Thompson): For $b - B$ distorted waves:

$$\bar{R}_{\ell_b}(r_b, k_b^i) = N \int_{k_b^i - \Delta k_b/2}^{k_b^i + \Delta k_b/2} dk_b \, R_{\ell_b}(r_b, k_b),$$

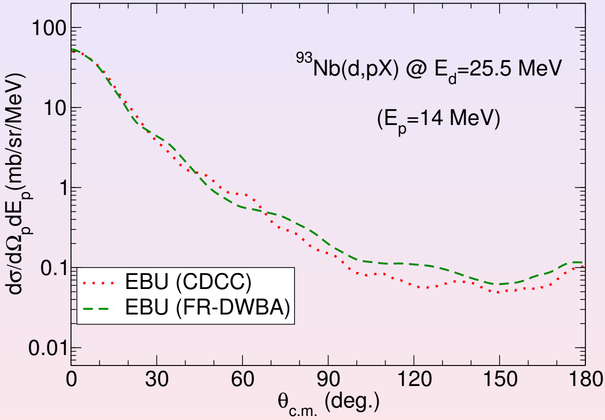
Application of IAV model to deuteron inclusive breakup

- EBU → CDCC (Fresco).
- NBU → post-form IAV model.



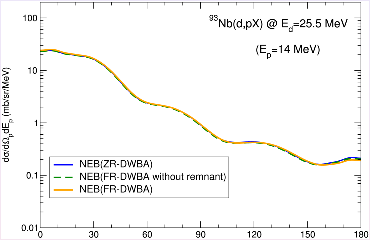
Data: Pampus et al, NPA311 (1978)141

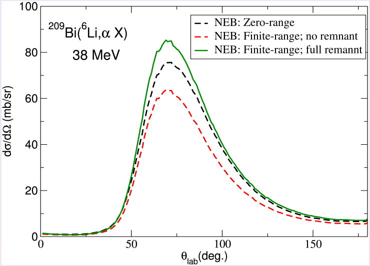
CDCC vs post-DWBA for EBU



⇨ Good agreement in this case, but needs to be tested for other cases

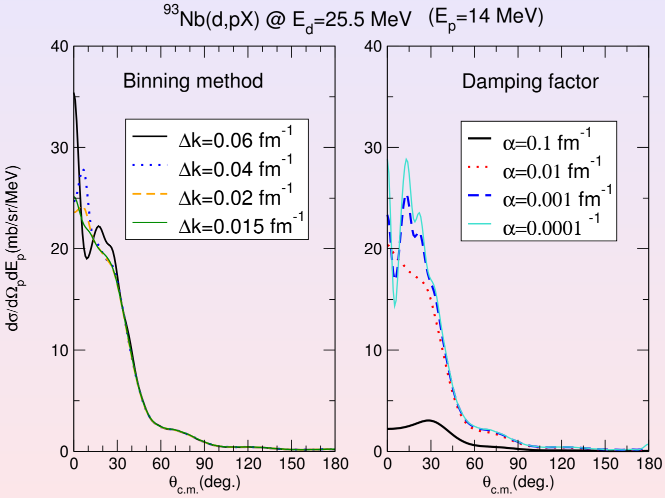
Importance of finite-range and remnant term



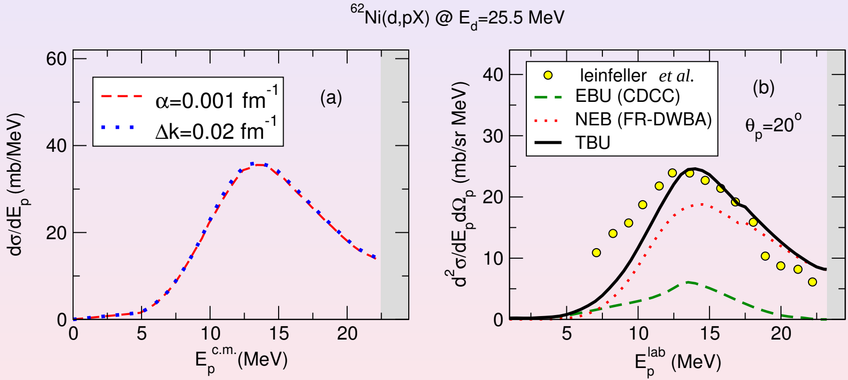


- ⇨ Zero-range accurate for deuterons, but not for other projectiles like ^{6}Li
- ⇨ Remnant term also important for ^{6}Li

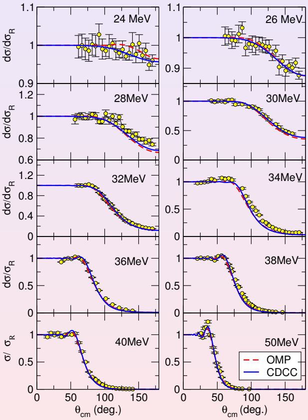
Comparison of regularization procedures



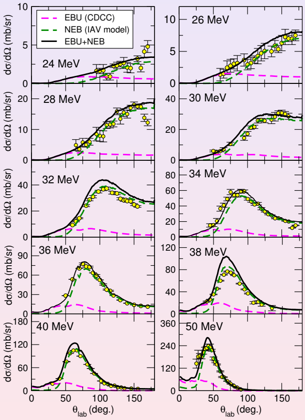
Comparison of regularization procedures



Elastic scattering

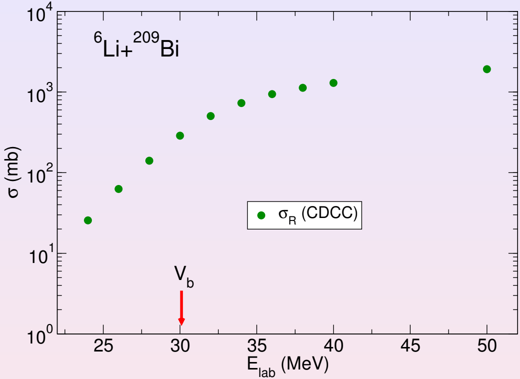


Inclusive α's

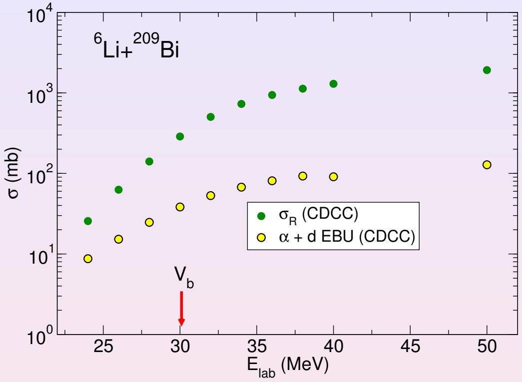


Good agreement but... W_{dA} has to be readjusted to give good elastic scattering.

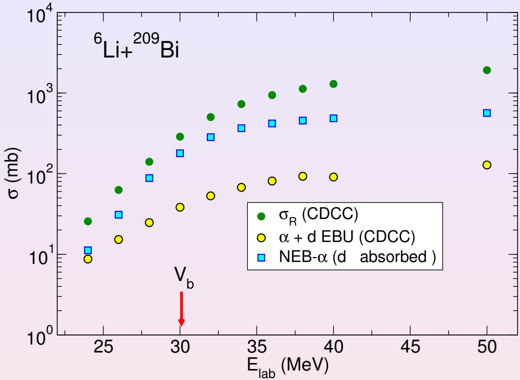
$^6Li+^{209}Bi$: incident energy dependence of cross sections



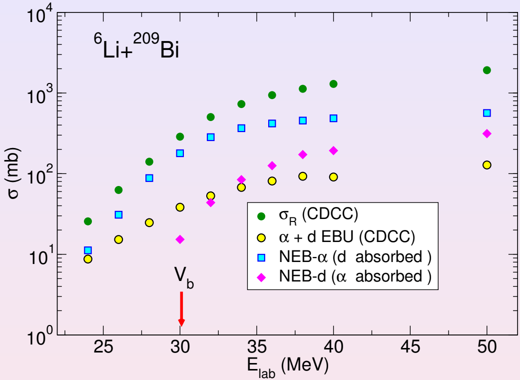
$^{6}Li+^{209}Bi$: incident energy dependence of cross sections



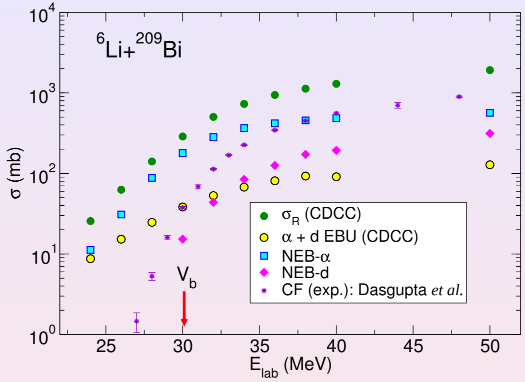
$^{6}Li+^{209}Bi$: incident energy dependence of cross sections



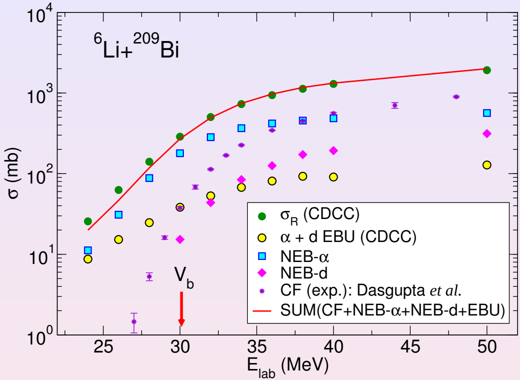
$^{6}Li+^{209}Bi$: incident energy dependence of cross sections



$^{6}Li+^{209}Bi$: incident energy dependence of cross sections



^{6}Li+^{209}Bi: incident energy dependence of cross sections



$$\sigma_{reac} \approx \sigma_{\alpha+d}(EBU) + \sigma_{\alpha}(NBU) + \sigma_{d}(NBU) + \sigma(CF)$$

Post-prior equivalence

$$\left.\frac{d^2\sigma}{dE_b d\Omega_b}\right|_{\text{NEB}} = -\frac{2}{\hbar v_a}\rho_b(E_b)\langle\psi_x|W_x|\psi_x\rangle,$$

$$(E_x^+ - K_x - U_x)\psi_x^{\text{post}}(\mathbf{r}_x) = (\chi_b^{(-)}|V_{\text{post}}|\chi_a^{(+)}\phi_a\rangle; \quad V_{\text{post}} \equiv V_{bx} + U_{bA} - U_{bB}$$

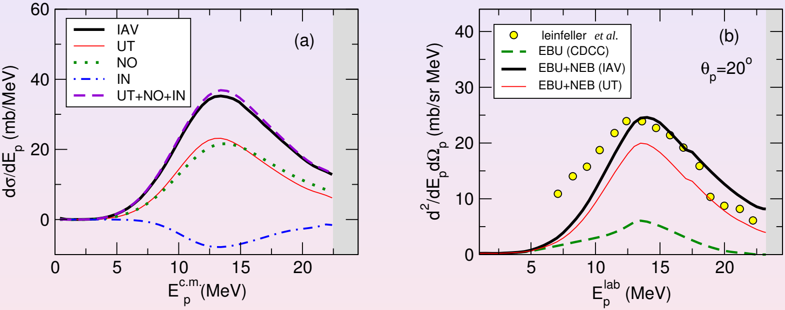
$$(E_x^+ - K_x - U_x)\psi_x^{\text{prior}}(\mathbf{r}_x) = (\chi_b^{(-)}|V_{\text{prior}}|\chi_a^{(+)}\phi_a\rangle; \quad V_{\text{prior}} \equiv U_{xA} + U_{bA} - U_{aA}$$

$$\psi_x^{\text{post}} = \psi_x^{\text{prior}} + \psi_x^{\text{NO}} \qquad \psi_x^{\text{NO}}(\vec{r}_x) \equiv \langle\chi_b^{(-)}|\chi_a^{(+)}\phi_a\rangle$$

$$\left.\frac{d^2\sigma}{dE_b d\Omega_b}\right|_{\text{NEB}}^{\text{IAV}} = \left.\frac{d^2\sigma}{dE_b d\Omega_b}\right|_{\text{NEB}}^{\text{UT}} + \left.\frac{d^2\sigma}{dE_b d\Omega_b}\right|_{\text{NEB}}^{\text{NO}} + \left.\frac{d^2\sigma}{dE_b d\Omega_b}\right|_{\text{NEB}}^{\text{IN}}$$

Numerical assessment of post-prior equivalence

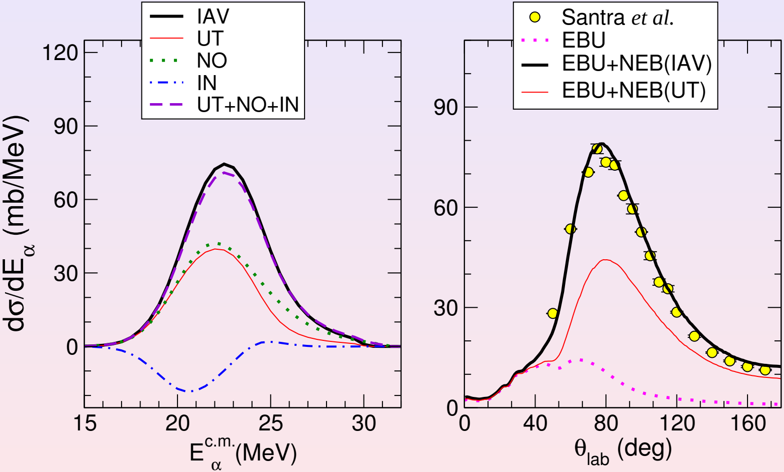
^{62}Ni(d,pX) @ E_d=25.5 MeV



J. Lei,A.M.M.,PRC92, 061602(R)(2015)

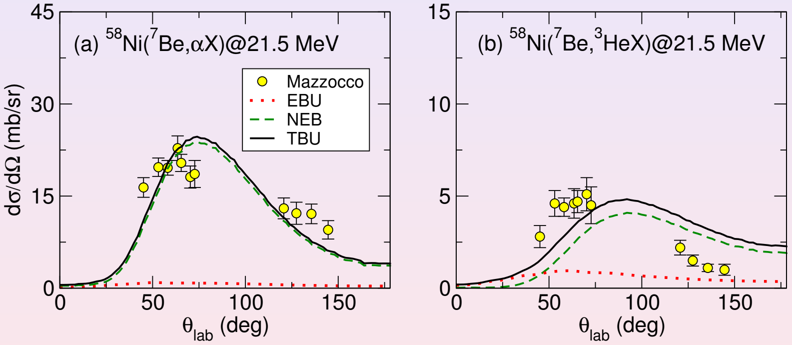
Numerical assessment of post-prior equivalence

$^{209}Bi(^{6}Li,\alpha X)$ @ E=36 MeV

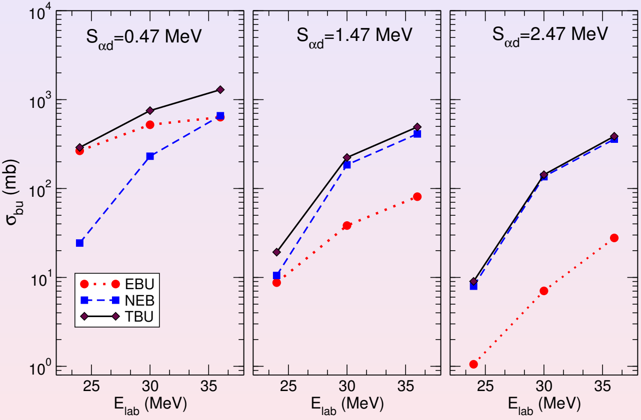


J. Lei,A.M.M.,PRC92, 061602(R)(2015)

Data: Mazzoco *et al*: ☞ $\sigma_\alpha \approx 5\sigma_{3\mathrm{He}}$



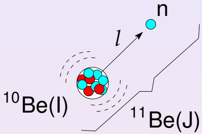
Effect of binding energy and incident energy: ^{209}Bi(^{6}Li+,α)X



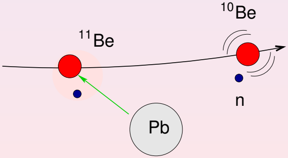
Core excitations will affect:

1. the structure of the projectile ➩ core-excited admixtures

$$\Psi_{JM}(\vec{r},\xi) = \sum_{\ell,j,I} \left[\varphi^{J}_{\ell,j,I}(\vec{r}) \otimes \Phi_I(\xi)\right]_{JM}$$



2. the dynamics ➩ collective excitations of the ^{10}Be during the collision compete with halo (single-particle) excitations.

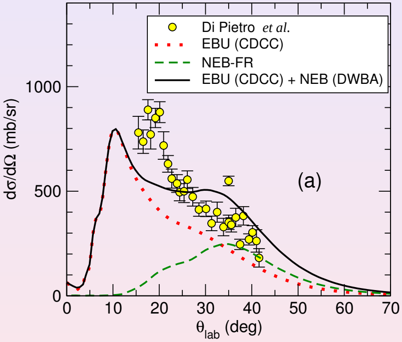


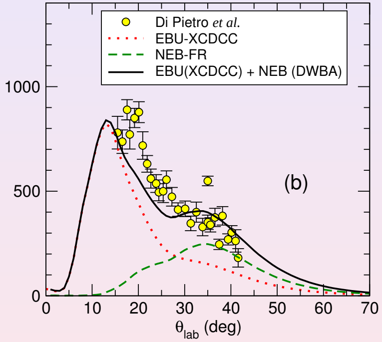
➩ Both effects have been recently implemented in an extended version of of the CDCC formalism (CDCC): Summers *et al*, PRC74 (2006) 014606, R. de Diego *et al*, PRC 89, 064609 (2014)

Application to halo nuclei: ^{11}Be+^{64}Zn $\rightarrow$ ^{10}Be + X @ E=28 MeV

Standard CDCC + NEB

Extended CDCC + NEB





⇨ Core deformation/excitations very important for EBU. What about for NEB?

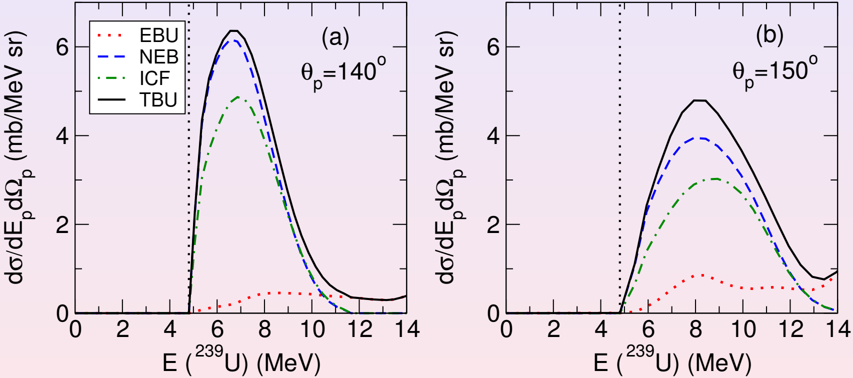
The IAV model provides the total NEB cross section, but for some applications $\sigma^{\rm ICF}$ is needed:

$$\boxed{\sigma^{\rm NEB} = \sigma^{\rm ICF} + \sigma^{\rm DR}}$$

Some ideas to evaluate $\sigma^{\rm ICF}$:

1. $W_{xA} = W_{xA}^{\rm DR} + W_{xA}^{\rm CN}$, with $W_{xA}^{\rm CN}$ constrained by CC calculations or $x + A$ fusion.
$$\boxed{\frac{d\sigma^{\rm ICF}}{d\Omega_b dE_b} = -\frac{2}{\hbar v_a}\rho_b(E_b)\langle\varphi_x|W_{xA}^{\rm CN}|\varphi_x\rangle}$$
2. Extended CDCC with target excitation (should provide EBU + INBU consistently).
3. Extended IAV model.

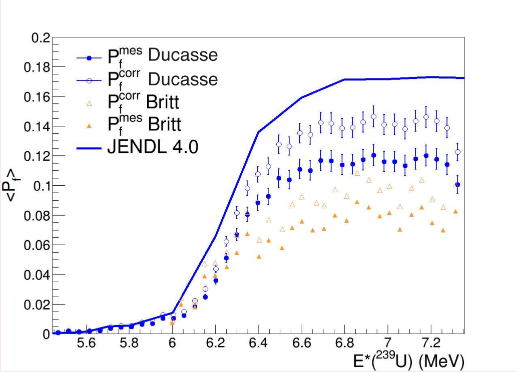
Application to (d, p) surrogate reactions: the ^{238}U(d,pf) case at E_d=15 MeV



Application to (d,p) surrogate reactions: the ^{238}U(d,pf) case at E_d=15 MeV

$$P_f^{\mathrm{corr}}(E_n) = P_f^{\mathrm{meas}}(E_n)\frac{\sigma_{\mathrm{TBU}}(E_n)}{\sigma_{\mathrm{ICF}}(E_n)},$$

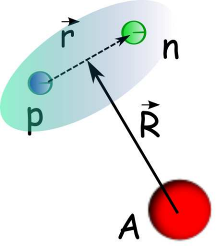
Q. Ducasse et al, arXiv:1512.06334

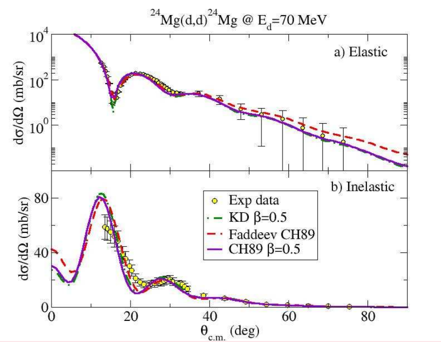


Effective model 3-body Hamiltonian with target excitation:

$$H = H_{\mathrm{proj}}(\mathbf{r}) + H_{\mathrm{tar}}(\xi_t) + \hat{T}_{\mathbf{R}} + U_{cA}(\mathbf{r}_{cA}, \xi_t) + U_{vA}(\mathbf{r}_{vA}, \xi_t),$$

Benchmark with Faddeev formalism:





M. Gomez-Ramos and A.M.M., in preparation

Possible extensions and improvements

- Inclusion of intrinsic spins of clusters.
- Explicit inclusion of target excitation, either prior to breakup (in $a + A$ interaction) and/or in the $x - A$ channel WF.
- Calculations for "stripping" part in nucleon removal cross sections (knockout) at intermediate energies may provide a benchmark for standard semiclassical approaches.
- Extension to 3-body projectiles (^{9}Be, ^{6}He, ^{6}Li, etc).
- Proper application to more weakly bound (e.g. halo) nuclei might require going beyond (DWBA) in the calculation of φ_x (eg. CDCC):

$$[K_x + U_{xA} - E_x]\varphi_x(\mathbf{r}_x) = (\chi_b^{(-)}|V_{bx}|\Psi_{3b}^{(+)}\rangle$$