The Minimal SUSY B-L Model: Sequential/Simultaneous Wilson Lines and String Thresholds

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Simplicity II

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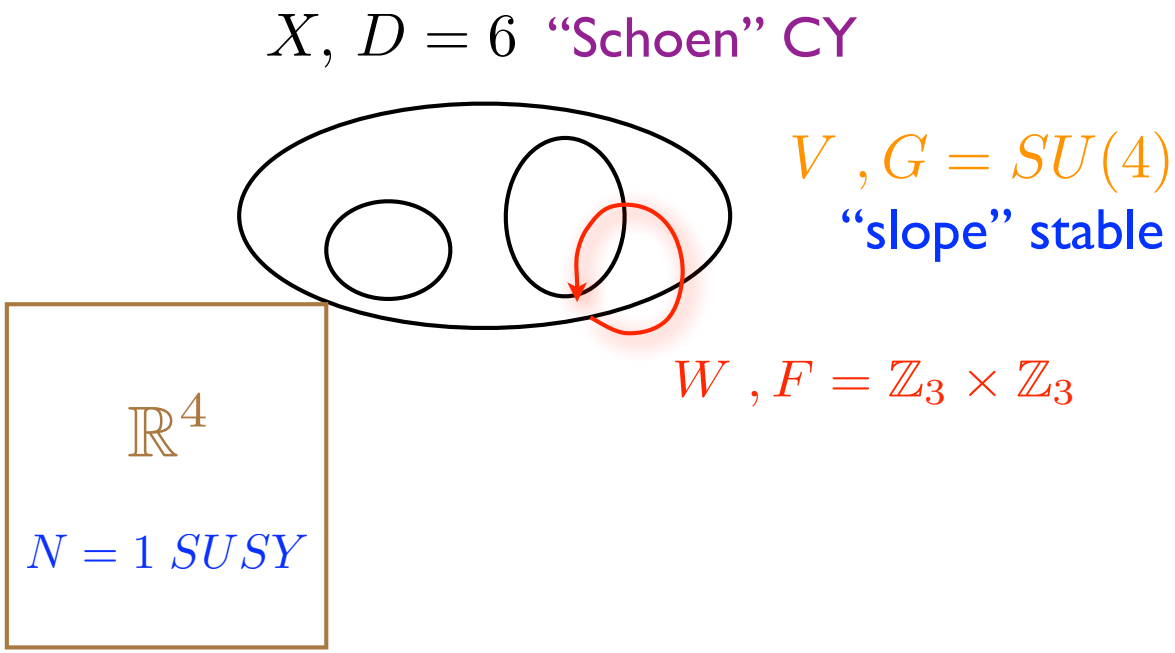
SU(4) Heterotic Compactification:



$\mathbb{R}^4$ Theory Gauge Group:

$G=SU(4)\Rightarrow \quad E_8\rightarrow\ Spin(10)$

Choose the $\mathbb{Z}_3\times\mathbb{Z}_3$ Wilson lines to be

$$\chi_{T_{3R}}=e^{iY_{T_{3R}}\frac{2\pi}{3}}\ ,\quad \chi_{B-L}=e^{iY_{B-L}\frac{2\pi}{3}}$$

where

$$Y_{B-L} = \; 2(H_1 + H_2 + H_3) = 3(B - L)$$

$$Y_{T_{3R}} = \; H_4 + H_5 = 2(Y - \frac{1}{2}(B - L)) = 2T_{3R}$$

arise "naturally" and is called the "canonical basis". $\Longrightarrow$

$$Spin(10) \to SU(3)_C \times SU(2)_L \times U(1)_{T_{3R}} \times U(1)_{B-L}$$

$\mathbb{R}^4$ Theory Spectrum:

$n_r = (h^1(X, U_R(V)) \otimes \mathbf{R})^{\mathbb{Z}_3 \times \mathbb{Z}_3} \Longrightarrow$ 3 families of quarks/leptons

$$Q = (U, D)^T = (\mathbf{3}, \mathbf{2}, 0, \frac{1}{3}), \quad u = (\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{2}, -\frac{1}{3}), \quad d = (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{2}, -\frac{1}{3})$$

$$L = (N, E)^T = (\mathbf{1}, \mathbf{2}, 0, -1), \quad \nu = (\mathbf{1}, \mathbf{1}, -\frac{1}{2}, 1), \quad e = (\mathbf{1}, \mathbf{1}, \frac{1}{2}, 1)$$

and 1 pair of Higgs-Higgs conjugate fields

$$H = (\mathbf{1}, \mathbf{2}, \frac{1}{2}, 0), \quad \bar{H} = (\mathbf{1}, \mathbf{2}, -\frac{1}{2}, 0)$$

under $SU(3)_C \times SU(2)_L \times U(1)_{T_{3R}} \times U(1)_{B-L}$.

That is

- *When the two Wilson lines corresponding to the canonical basis are turned on simultaneously, the resulting low energy spectrum is precisely that of the MSSM–that is, three families of quark/lepton chiral superfields, each family with a right-handed neutrino supermultiplet, and one pair of Higgs-Higgs conjugate chiral multiplets. There are no vector-like pairs or exotic particles.*

- *Since each quark/lepton and Higgs superfield of the low energy Lagrangian arises from a different* **16** *and* **10** *representation of Spin*(10) *respectively, the parameters of the effective theory, and specifically the Yukawa couplings and the soft supersymmetry breaking parameters, are uncorrelated by the Spin*(10) *unification. For example, the soft mass squared parameters of the right-handed sneutrinos need not be universal with the remaining slepton supersymmetry breaking parameters.*

There are many pairs of U(1)XU(1) generators with these two properties--such as Y_Y, Y_{B-L} . So why have we chosen the canonical basis? Answer--kinetic mixing.

We can prove a theorem that

- *The only basis of $\mathfrak{h}_{3\oplus 2} \subset \mathfrak{h}$ for which $U(1)_{Y_1} \times U(1)_{Y_2}$ kinetic mixing vanishes at all values of energy-momentum is the canonical basis $Y_{T_{3R}}$, Y_{B-L} and appropriate multiples of this basis.*

Wilson Line Breaking:

$\pi_1\left(X/(\mathbb{Z}_3 \times \mathbb{Z}_3)\right) = \mathbb{Z}_3 \times \mathbb{Z}_3 \Longrightarrow$ 2 independent classes of non-contractible curves. $\Longrightarrow$ each Wilson line has a mass scale $M_{\chi_{T_{3R}}}$, $M_{\chi_{B-L}}$ .Three possibilities

$$M_{\chi_{T_{3R}}} \simeq M_{\chi_{B-L}} \ , \ M_{\chi_{B-L}} > M_{\chi_{T_{3R}}} \ , \ M_{\chi_{T_{3R}}} > M_{\chi_{B-L}}$$

We begin by considering the two sequential breaking patterns.

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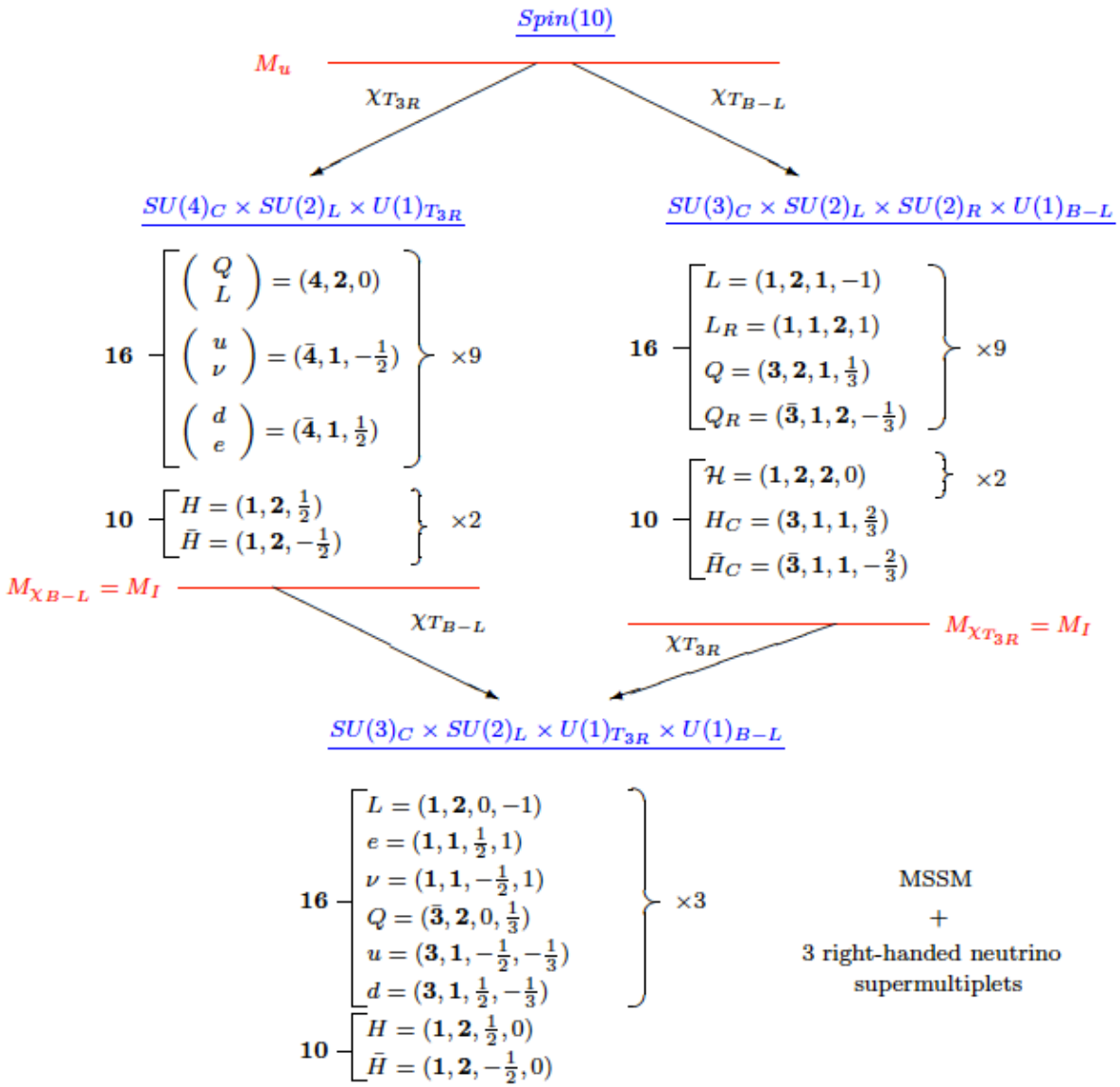


- The two sequential Wilson line breaking patterns of $Spin(10)$.

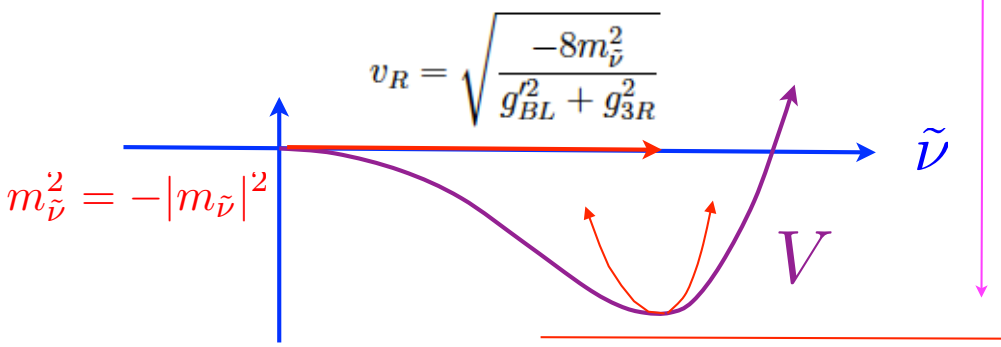
For specificity, we examine the left-right model sequence.

M_I

$$W = Y_u Q H_u u^c - Y_d Q H_d d^c - Y_e L H_d e^c + Y_\nu L H_u \nu^c + \mu H_u H_d$$

$$-\mathcal{L}_{\rm soft} = m^2_{\tilde\nu^c}|\tilde\nu^c|^2 + m^2_{\tilde L}|\tilde L|^2 + m^2_{H_u}|H_u|^2 + m^2_{H_d}|H_d|^2 + \left(M_R \tilde W_R^2 + M_2 \tilde W^2 + M_{BL}\tilde B'^2 + M_3 \tilde g^2 + a_\nu \tilde L H_u \tilde\nu^c + b H_u H_d + \text{h.c.}\right) + \cdots$$

Third family sneutrino:



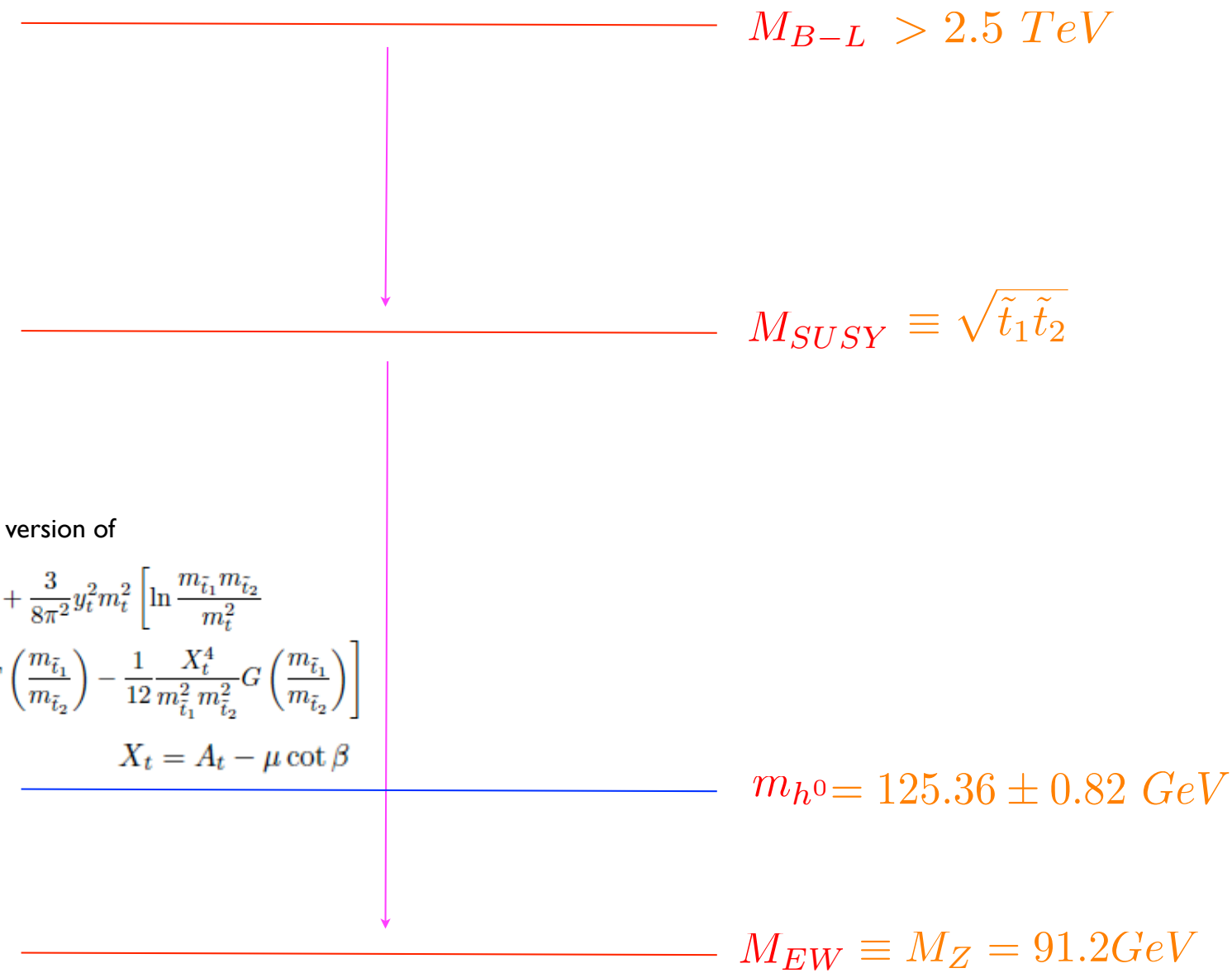
$\Rightarrow \quad U(1)_{T_{3R}} \times U(1)_{B-L} \to U(1)_Y$

$M_{B-L}\,(= M_{Z'} = \sqrt{2}\,|m_{\tilde\nu}|)$

$R = (-1)^{3(B-L)+2s} \Rightarrow R|_{\tilde\nu} = -1 \Rightarrow \langle\tilde\nu\rangle$ *spontaneously breaks* $R-$*parity*

$\Rightarrow$ $SU(3)_C \times SU(2)_L \times U(1)_Y$ MSSM *with*

$$W \supset \epsilon_i L_i H_u \quad , \quad \mathcal{L} \supset -\frac{1}{2} v_R \left[-g_R \nu_3^c \tilde W_R + g_{BL} \nu_3^c \tilde B'\right] + \text{h.c.} \quad , \quad \epsilon_i \equiv \frac{1}{\sqrt{2}} Y_{\nu i 3} v_R$$



leading log improved version of

$$m_{h^0}^2 = M_Z^2 \cos^2 2\beta + \frac{3}{8\pi^2} y_t^2 m_t^2 \left[\ln \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} + \frac{X_t^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}} F\left(\frac{m_{\tilde{t}_1}}{m_{\tilde{t}_2}}\right) - \frac{1}{12} \frac{X_t^4}{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2} G\left(\frac{m_{\tilde{t}_1}}{m_{\tilde{t}_2}}\right) \right]$$

$$X_t = A_t - \mu \cot \beta$$

$$|\mu|^2 = \frac{m_{H_u}^2 \tan^2 \beta - m_{H_d}^2}{1 - \tan^2 \beta} - \frac{1}{2} M_Z^2$$

$$\frac{2b}{\sin 2\beta} = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2$$

$$\Rightarrow \quad v_{Li} = \frac{\frac{v_R}{\sqrt{2}} (Y_{\nu i3}^* \mu v_d - a_{\nu i3}^* v_u)}{m_{\tilde{L}_i}^2 - \frac{g_2^2}{8}(v_u^2 - v_d^2) - \frac{g_{BL}^2}{8} v_R^2}$$

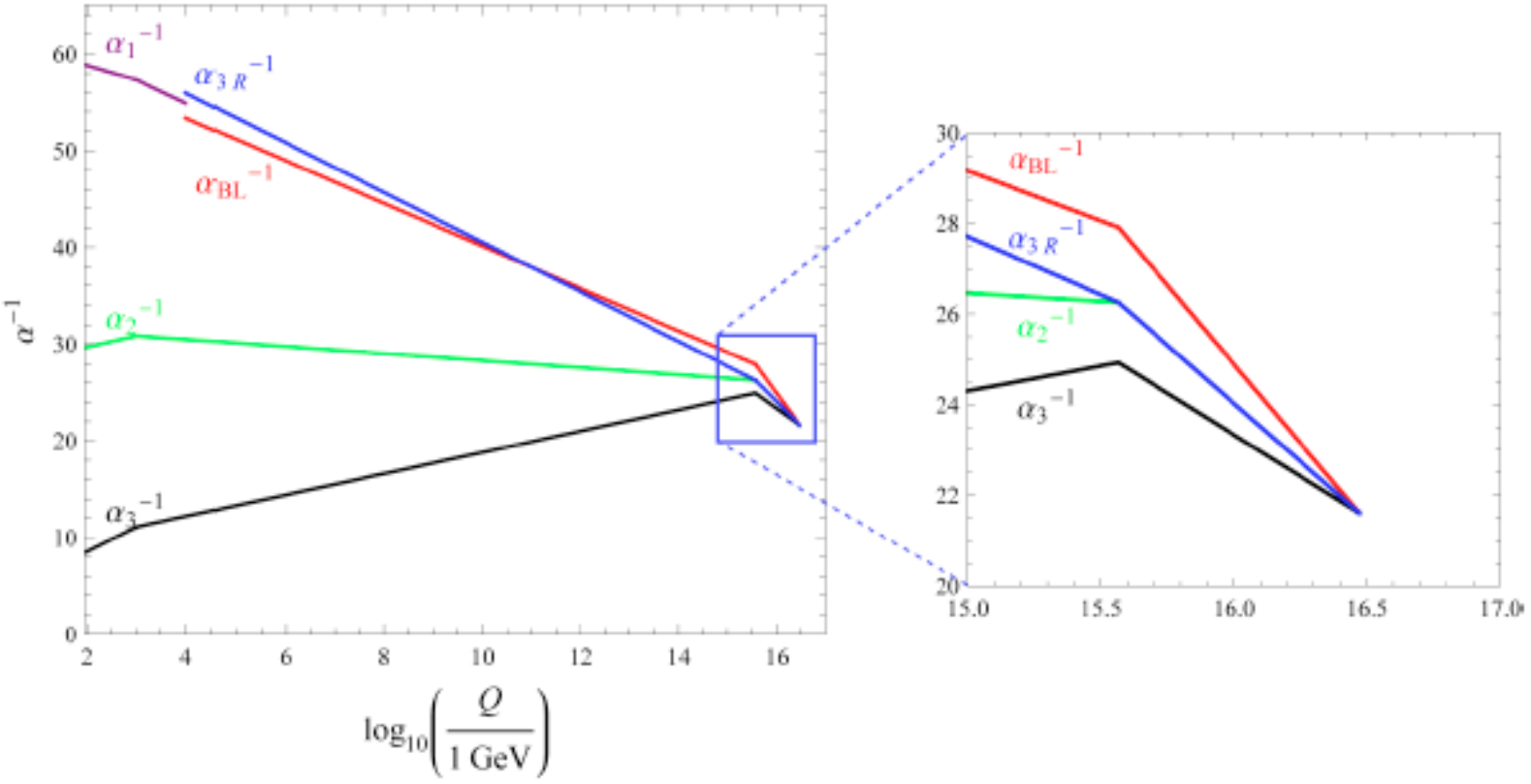
We will enforce gauge coupling unification using the experimental values $\alpha_1 = 0.017, \quad \alpha_2 = 0.034, \quad \alpha_3 = 0.118$ at M_{EW} .This allows us to determine both M_u , α_u and M_I in terms of M_{SUSY} and M_{B-L} .

For example, in the left-right case taking

$$M_{SUSY} = 1\ TeV, \quad M_{B-L} = 10\ TeV$$

$\Rightarrow$

$$M_u = 3.0 \times 10^{16}\ GeV, \quad \alpha_u = 0.046, \quad M_I = 3.7 \times 10^{15}\ GeV$$



In addition, we will enforce that all sparticle masses exceed their present experimental bounds. These are given by

Particle(s)	Lower Bound
Left-handed sneutrinos	45.6 GeV
Charginos, sleptons	100 GeV
Squarks, except for stop or sbottom LSP's	1000 GeV
Stop LSP (admixture)	450 GeV
Stop LSP (right-handed)	400 GeV
Sbottom LSP	500 GeV
Gluino	1300 GeV
Z_R	2500 GeV

Finally, we will require that the physical Higgs mass be within 2σ of the ATLAS measured value. That is,

$$m_{h^0} = 125.36 \pm 0.82\ GeV$$

Of more than 100 soft SUSY breaking dimensionful parameters, experimental constraints, such as flavor changing neutral currents, reduce the number to 24. We will statistically scatter all 24 initial massive parameters at M_I around a chosen "average" mass M.

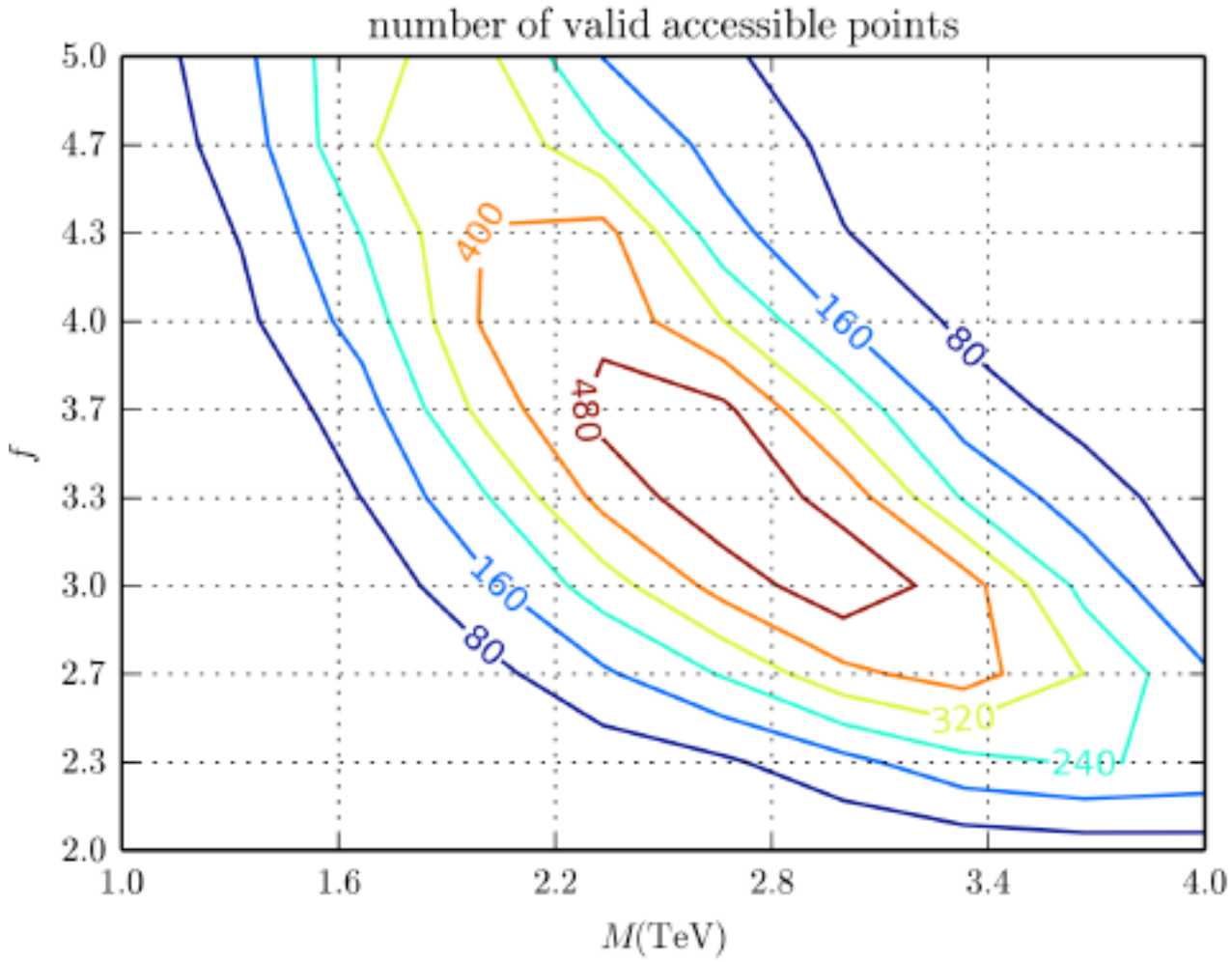
That is, for some dimensionless number f

$$\frac{M}{f} < m < Mf \qquad \text{for} \qquad m = m_{soft},\ M_{gaugino},\ A_{cubic}$$

M and f are chosen as follows.

We are interested in the low energy spectra being accessible at the LHC or a next generation collider. Therefore, in addition to the experimental constraints mentioned in the previous section, we further demand that all sparticle masses be lighter than 10 TeV. We call any point that satisfies this, as well as all previous criteria, a "valid accessible" point. The parameters M and f are chosen in such a way so as to maximize the number of such points. To determine the values of M and f which yield the greatest number of valid accessible points, we begin by making a ten by ten grid in the $M - f$ plane. At each of these hundred points, we randomly generate one hundred thousand initial points in the 24-dimensional parameter space discussed above, RG scale them to low energy, and count the subset that satisfies the experimental checks discussed above. We then plot curves corresponding to a constant number of valid accessible points.

The result is



The number a valid accessible points is maximized approximately at

$$M = 2.7\ TeV\ ,\ \ f = 3.3$$

which we use henceforth.

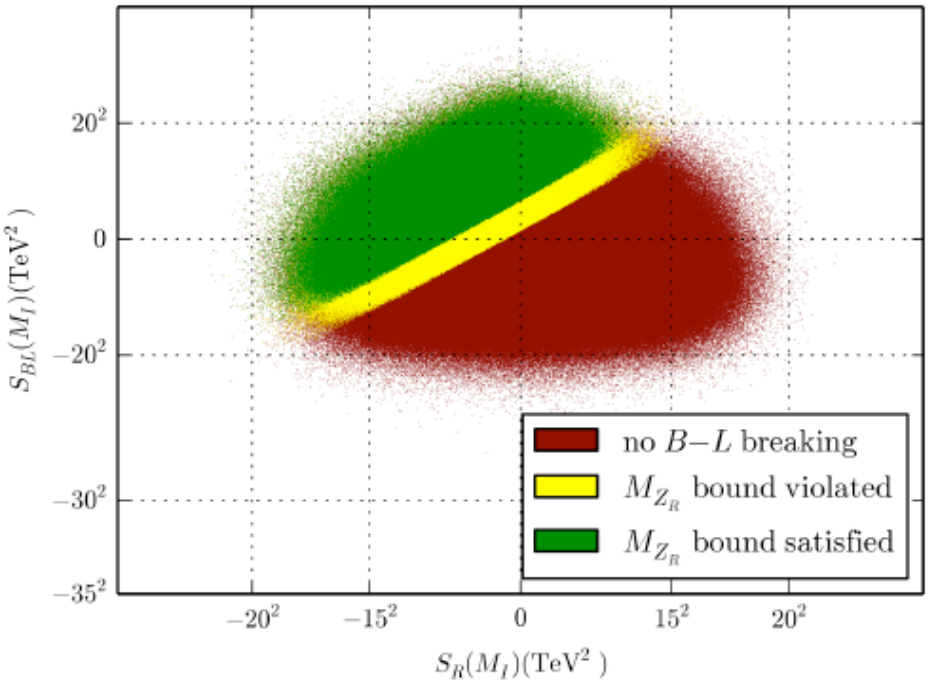
We first consider the prevalence of B-L symmetry breaking.

Defining

$$S_{B-L} = \mathrm{Tr}\,(2m_Q^2 - m_U^2 - m_D^2 - 2m_L^2 + m_N^2 + m_E^2)$$

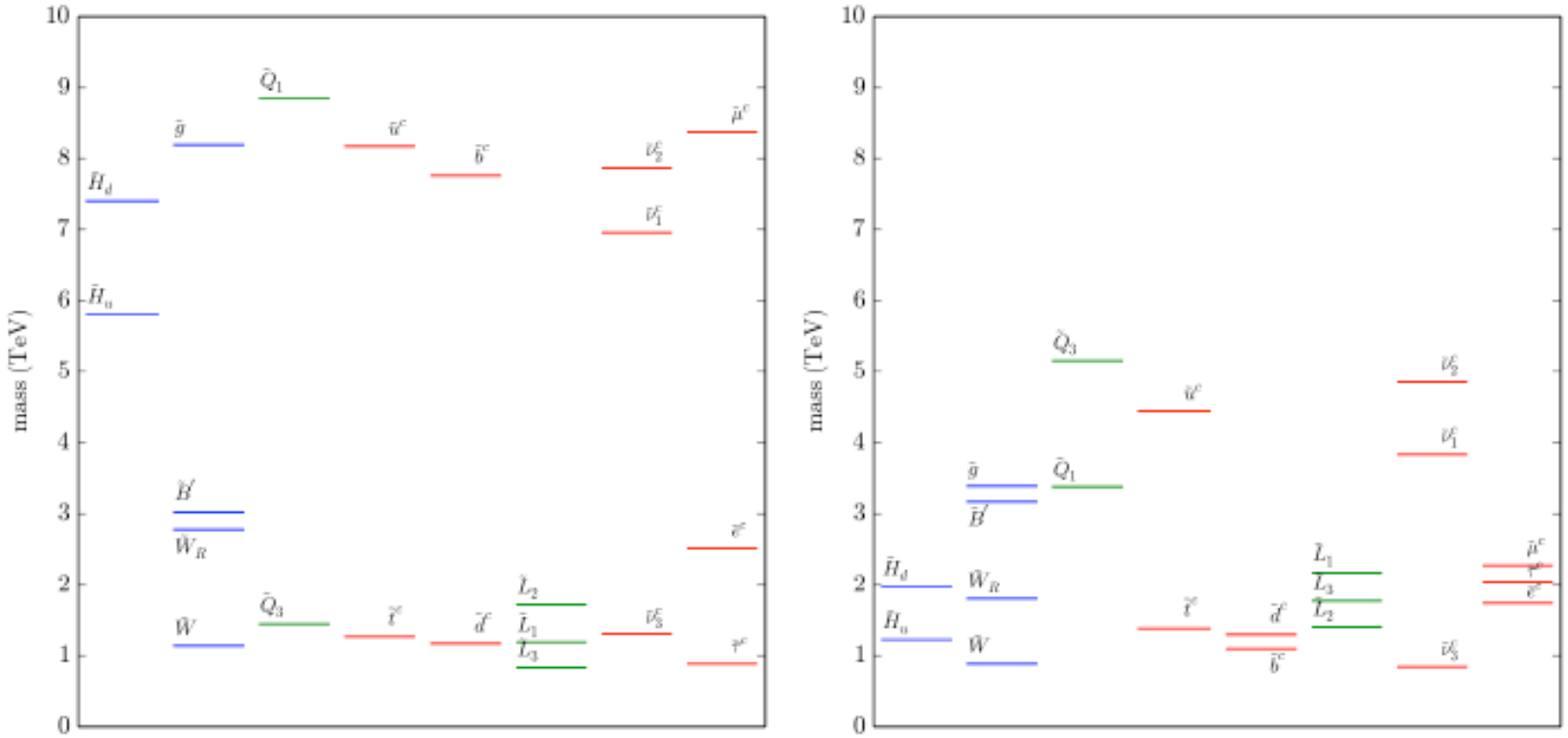
$$S_R = m_{H_u}^2 - m_{H_d}^2 + \mathrm{Tr}\left(-\frac{3}{2}m_U^2 + \frac{3}{2}m_D^2 - \frac{1}{2}m_N^2 + \frac{1}{2}m_E^2\right)$$

which determine this breaking, we find



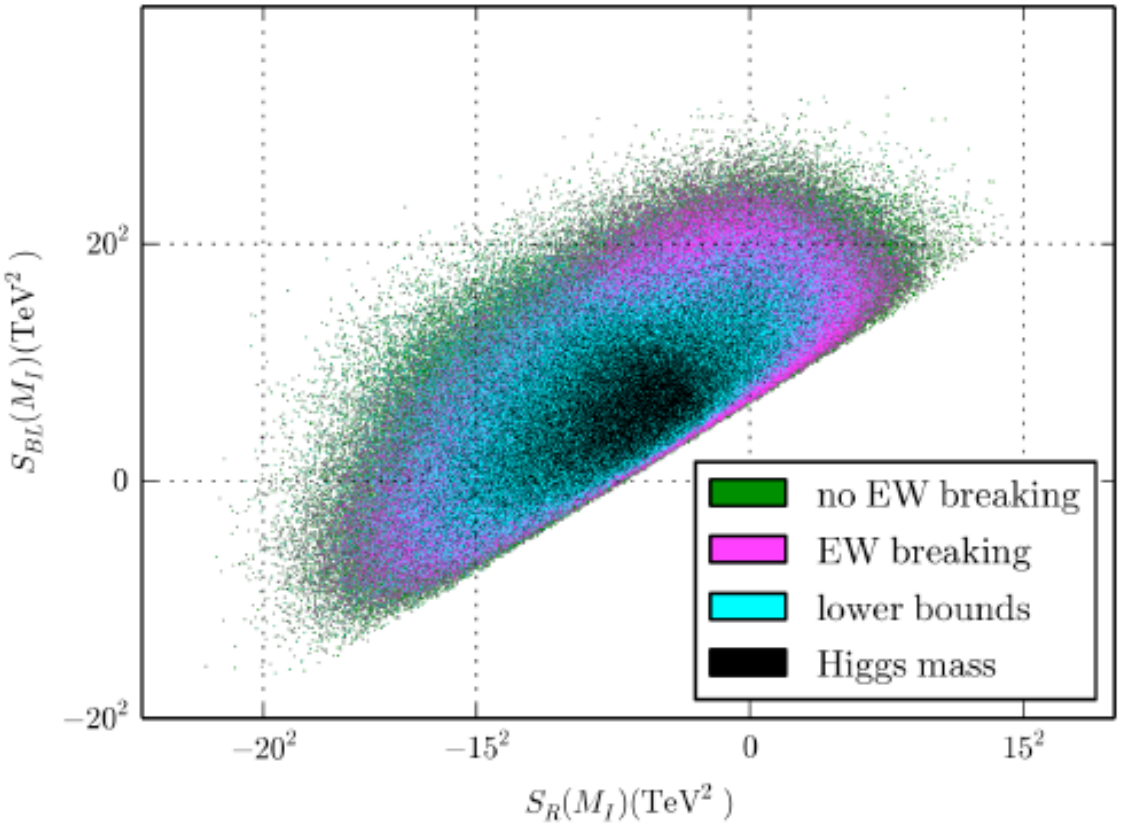
Points from the main scan in the $S_{BL}(M_\mathrm{I})$ - $S_R(M_\mathrm{I})$ plane. Red indicates no $B-L$ breaking, in the yellow region $B-L$ is broken but the Z_R mass is not above its 2.5 TeV lower bound, while green points have M_{Z_R} above this bound. The figure expresses the fact that, despite there being 24 parameters at the UV scale scanned in our work, $B-L$ physics is essentially dependent on only two combinations of them–the two S-terms.

$\Rightarrow$ B-L symmetry breaking with $M_{Z'} > 2.5\ TeV$ is abundant and does not require universal soft masses or other special choices of the intital parameters. For example



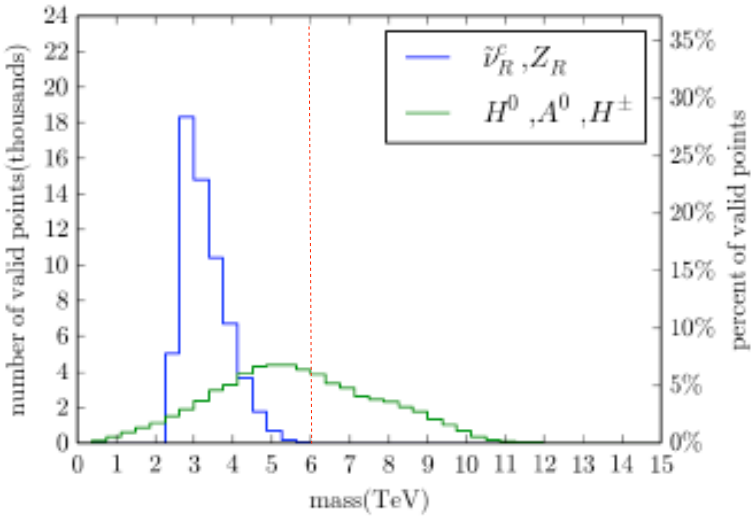
show the largest and smallest amount of splitting of the initial soft masses leading to physically acceptable B-L breaking (as well as satisfying all other physical constraints).

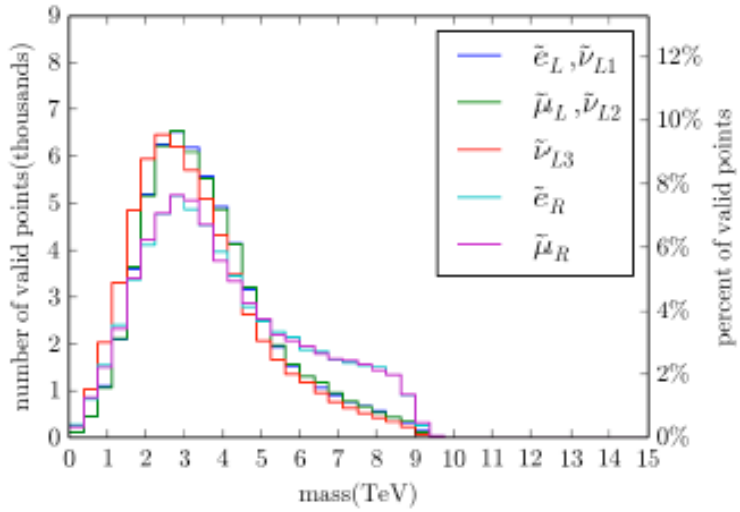
"Main Scan": Choose $M = 2.7\ TeV$, $f = 3.3$ and scan 10,000,000 points $\Rightarrow$

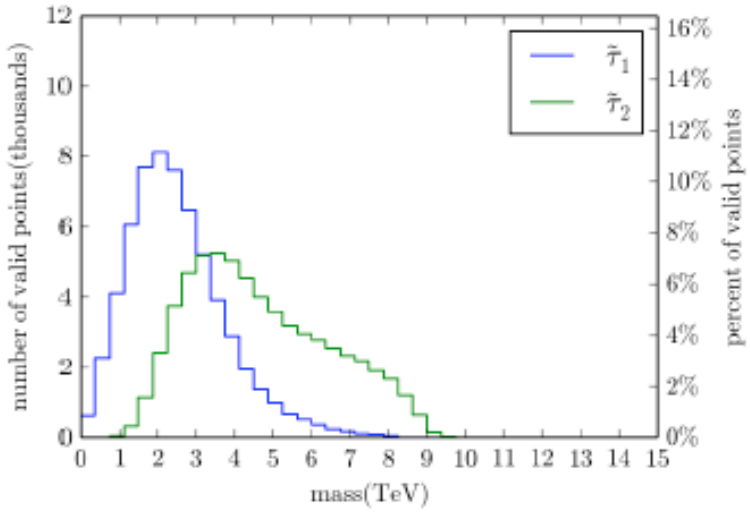


- break $U(1)_{3R} \times U(1)_{B-L} \to U(1)_Y$ with $M_{Z'} > 2.5\ TeV$ ← 919,117 points
- break $SU(2)_L \times U(1)_Y \to U(1)_{EM}$ with $M_Z = 91.2\ GeV$ ← 722,750 points
- satisfy all sparticle lower mass bounds ← 276,676 points
- $m_{h^0} = 125.36 \pm 0.82\ GeV$ ← 58,096 points

One can analyze the mass spectrum over the 58,096 acceptable (black) points. For example

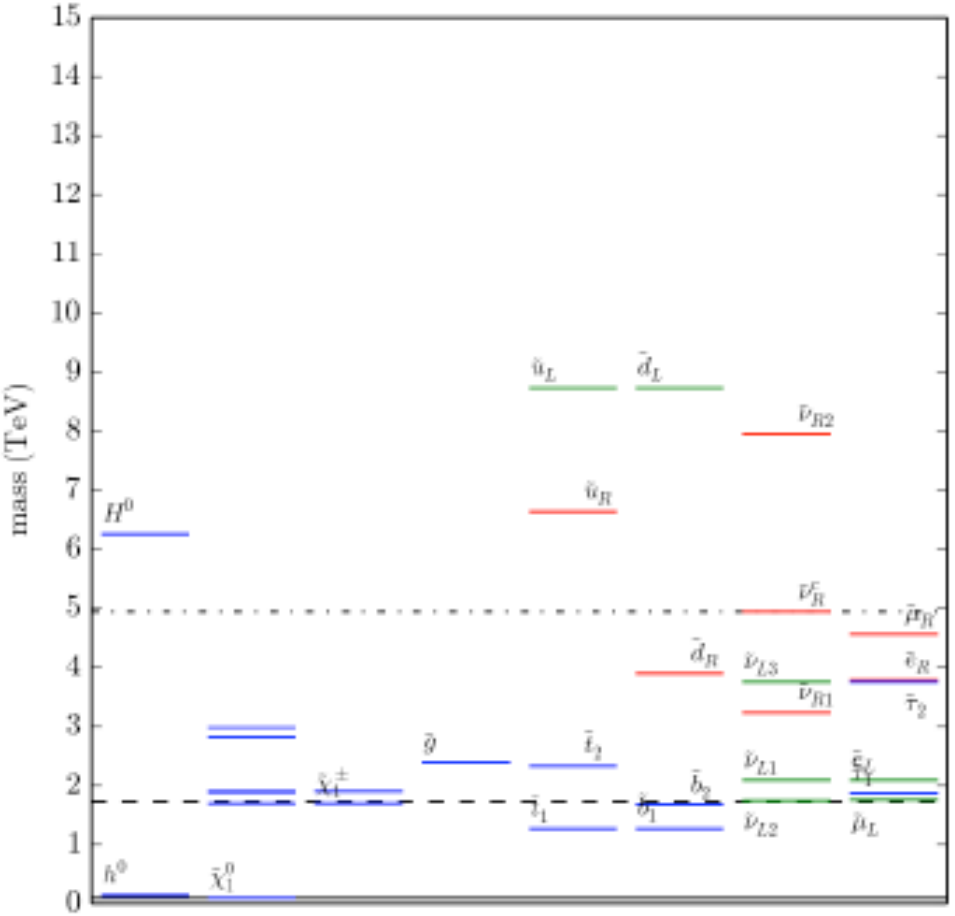




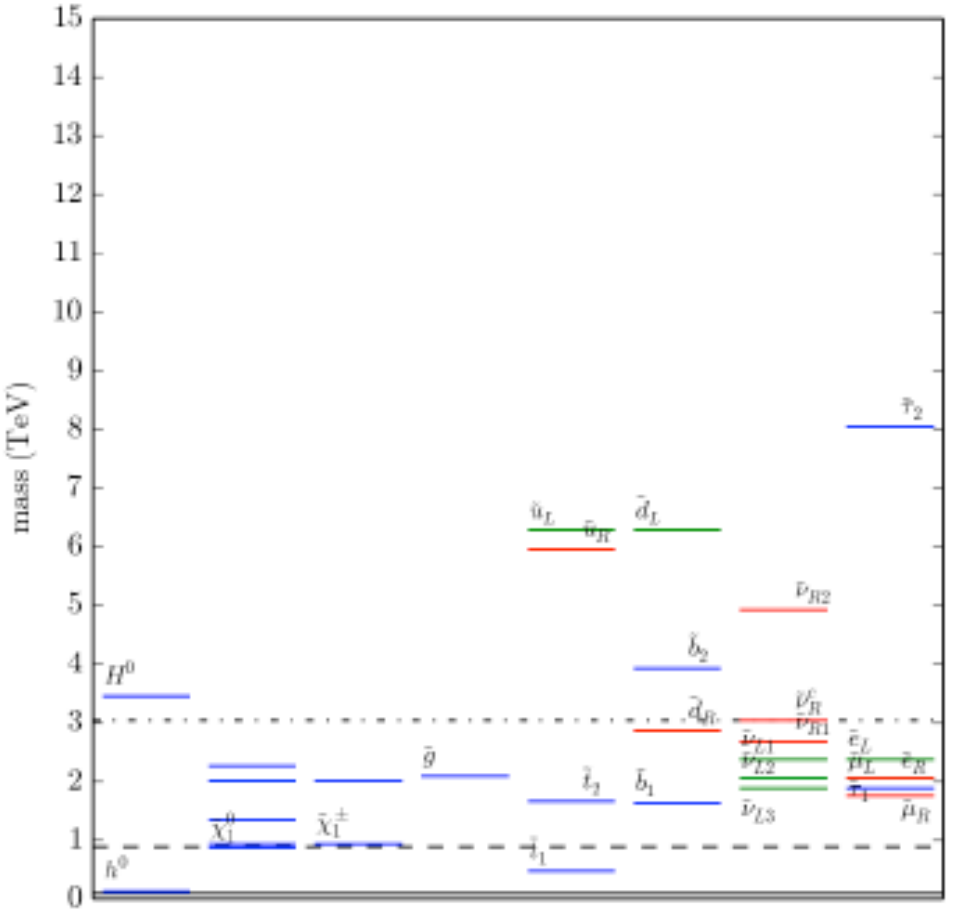


Note that $2.5\ TeV < M_{Z'} < 6\ TeV \Rightarrow Z'$ is potentially observable at the LHC. Although statistically the largest number of left-handed sleptons have mass of order 2.5 TeV, they can be < 500GeV.

For a given acceptable point, one can calculate and plot the sparticle spectrum. For example



(a)

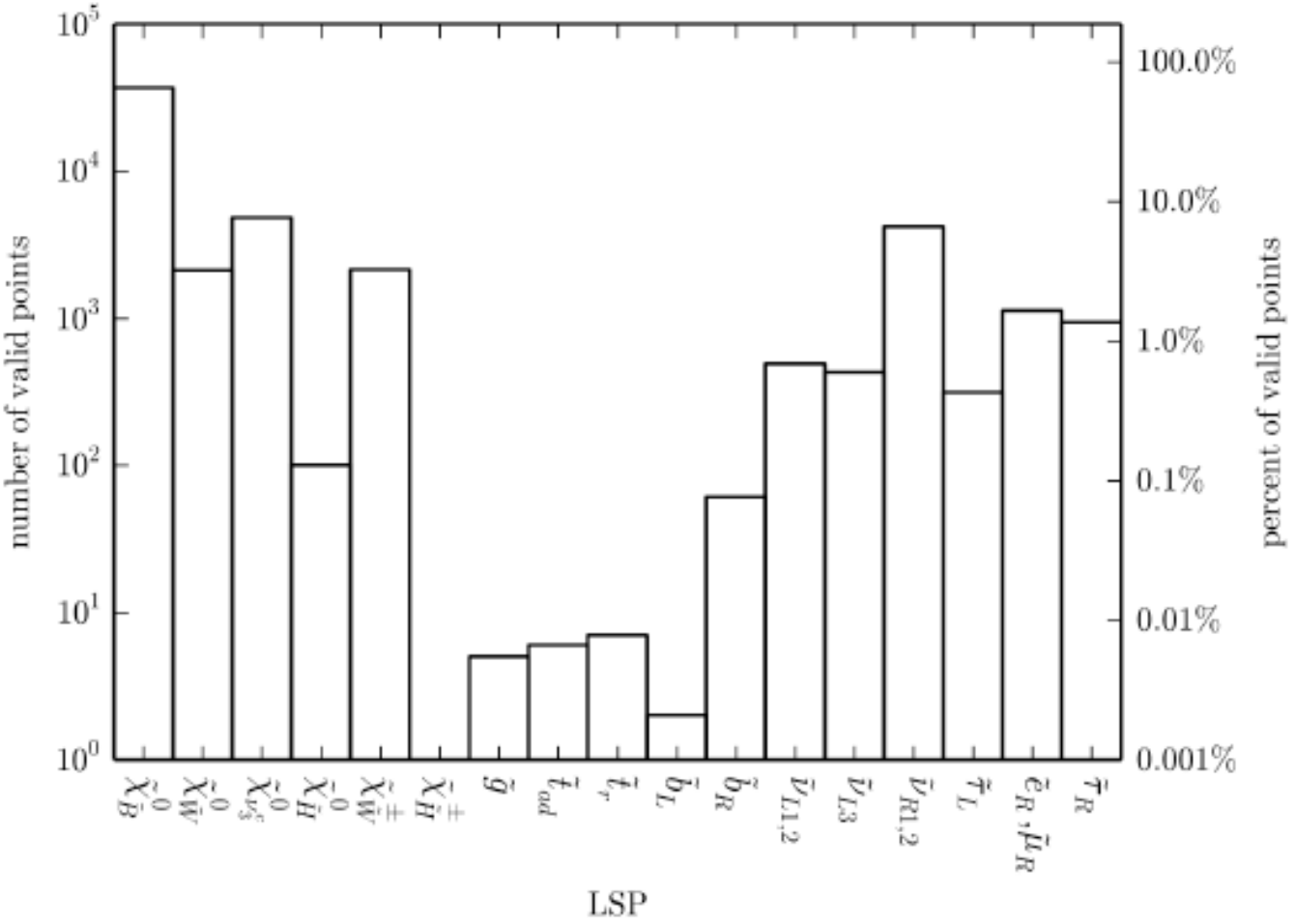


(b)

Two sample physical spectra . The $B - L$ scale is represented by a black dot-dash-dot line. The SUSY scale is represented by a black dashed line. The electroweak scale is represented by a solid black line. (a) and (b) have a neutralino and admixture stop LSP respectively.

The phenomenologically acceptable vacua can have different LSP's.

Statistically, over the 58,096 good points we find

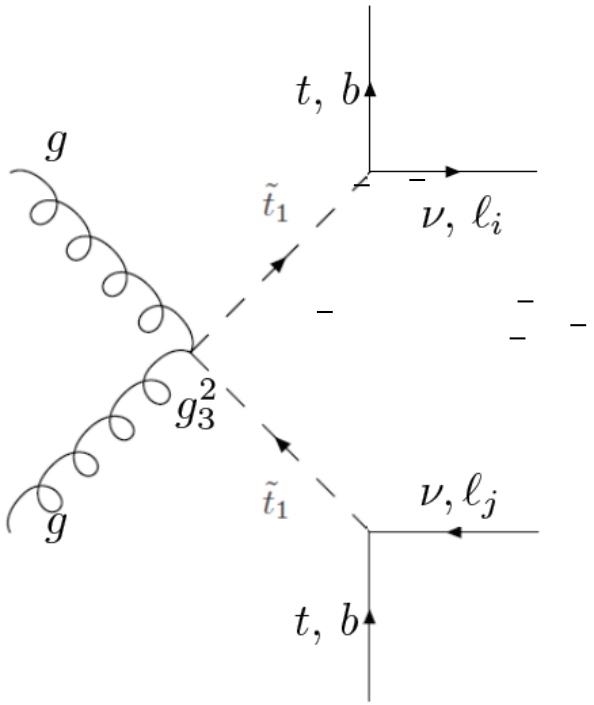


These include $\tilde{B},\ \tilde{\nu},\ \tilde{\tau},\ \tilde{t}, \ldots$. Note that they now can be charged and colored since they decay sufficiently quickly due to RPV interactions.

Some low energy "physics":

Pick black points with a stop LSP.

The left and right stops diagonalize to mass eigenstates $m_{\tilde{t}_1} < m_{\tilde{t}_2}$ with mixing angle $0 < \theta_t < 90^\circ$. Generically, $\tilde{t}_1$ decays via RPV interactions as a "leptoquark" $\Longrightarrow$ $\tilde{t}_1 \to t\,\nu_i, \;\text{or}\; \tilde{t}_1 \to b\,\ell_i^+$



For an "admixture" LSP $(\theta_t \lesssim 80^\circ)$, the dominant channel is

$$\tilde{t}_1 \to b\,\ell_i^+$$

After analyzing the partial widths for the LSP decay under the the assumption of "prompt" decays, and the associated neutrino mass matrix one determines the following.

Conclusion: The VEV of the right-handed third-family sneutrino $\Rightarrow$

a) The partial widths of the stop LSP decays via RPV interactions.

b) Majorana masses for the neutrinos via a "see-saw" mechanism.

$\Rightarrow$ Relationship between stop LSP decays and the neutrino mass hierarchy!

Let us analyze the case for an "admixture" stop LSP. The result is

Defining $\mathrm{Br}(\tilde{t}_1 \to b\ell_i^+) \equiv \dfrac{\Gamma(\tilde{t}_1 \to b\ell_i^+)}{\sum_{i=1}^{3} \Gamma(\tilde{t}_1 \to b\ell_i^+)}$ and using $\mathrm{Br}(\tilde{t}_1 \to b\,e^+) + \mathrm{Br}(\tilde{t}_1 \to b\,\mu^+) + \mathrm{Br}(\tilde{t}_1 \to b\,\tau^+) = 1$

$\Rightarrow$

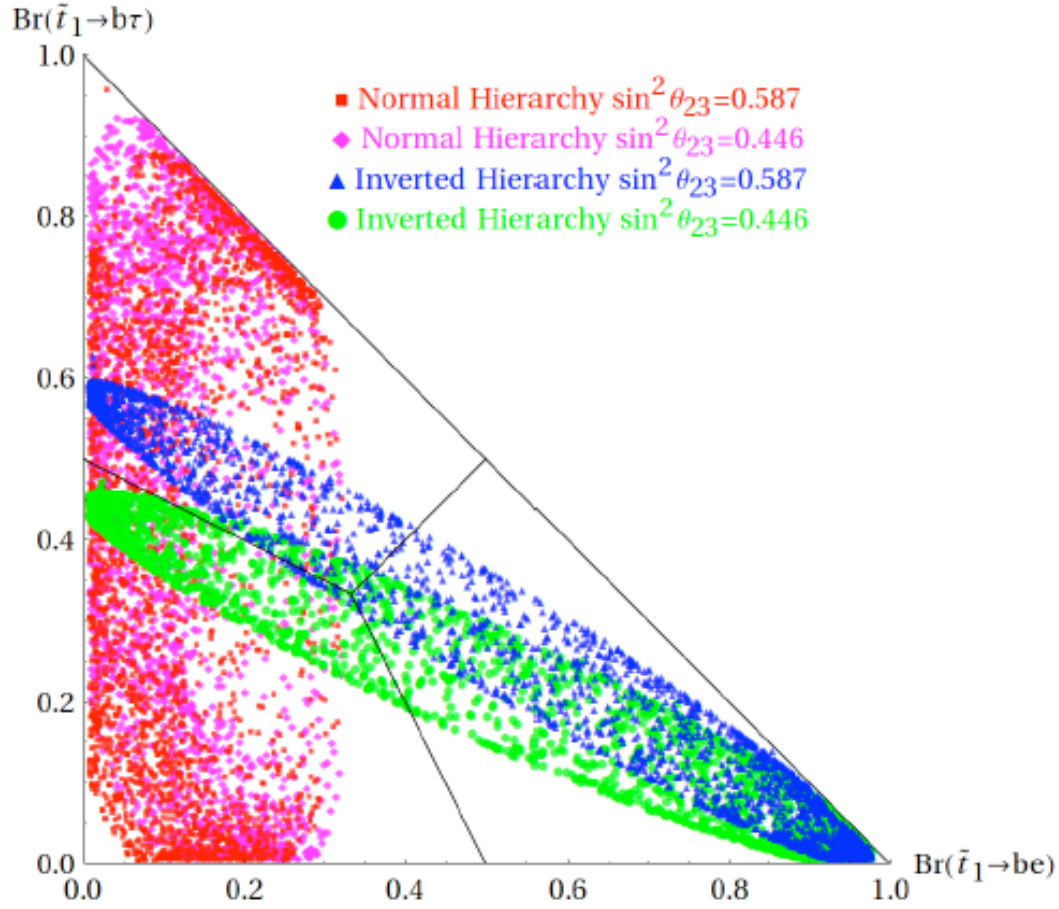


Figure 1: The results of the scan specified in Table 1 using the central values for the measured neutrino parameters in the $\mathrm{Br}(\tilde{t}_1 \to b\tau^+)$ - $\mathrm{Br}(\tilde{t}_1 \to be^+)$ plane. Due to the relationship between the branching ratios, the $(0,0)$ point on this plot corresponds to $\mathrm{Br}(\tilde{t}_1 \to b\mu^+) = 1$. The plot is divided into three quadrangles, each corresponding to an area where one of the branching ratios is larger than the other two. In the top left quadrangle, the bottom–tau branching ratio is the largest; in the bottom left quadrangle the bottom–muon branching ratio is the largest; and in the bottom right quadrangle the bottom–electron branching ratio is the largest. The two different possible values of θ_{23} are shown in blue and green in the IH (where the difference is most notable) and in red and magenta in the NH.

Using previous leptoquark searches at the LHC, one can put lower bounds on the LSP stop. We find that

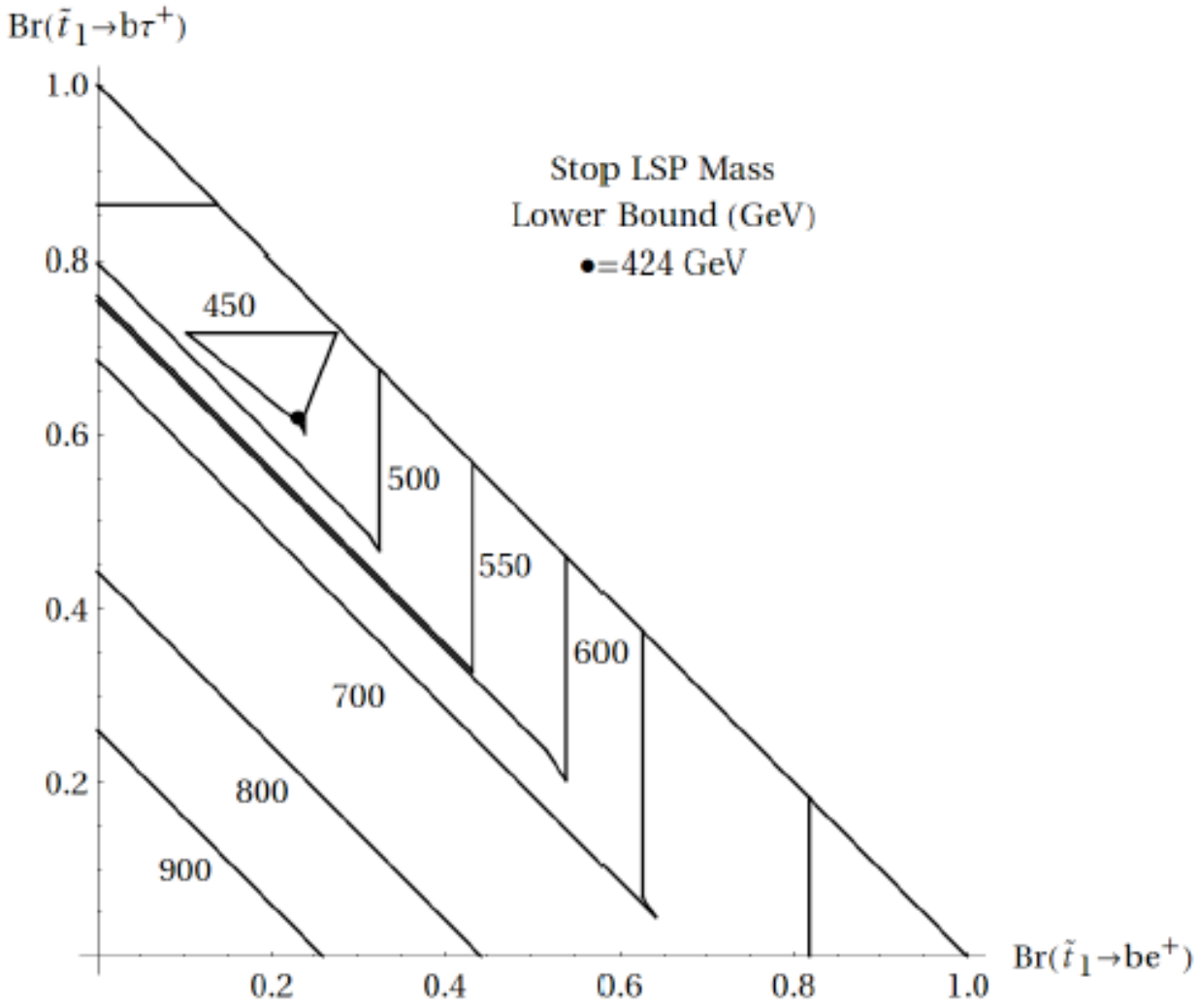
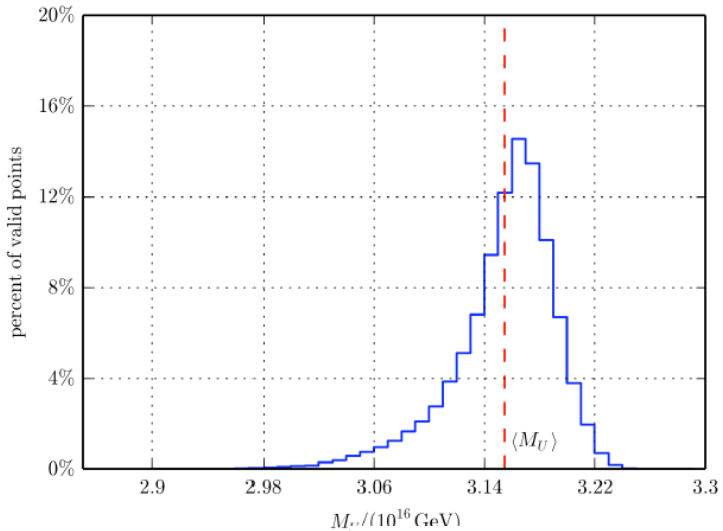
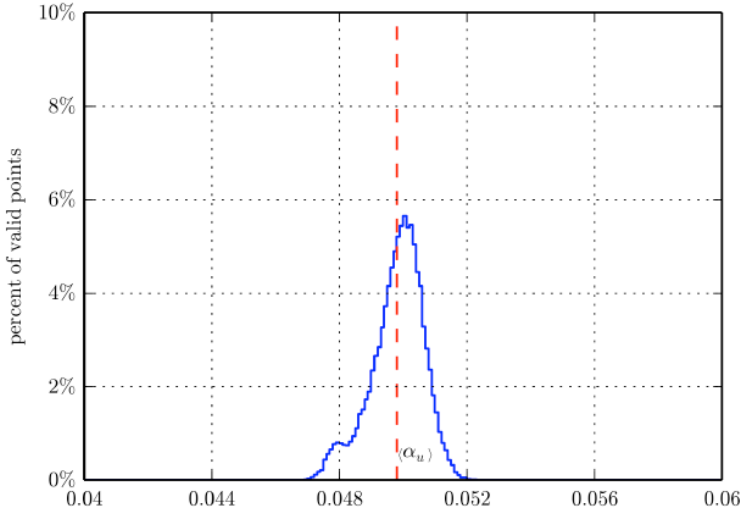


Figure 2: Lines of constant stop lower bound in GeV in the Br($\tilde{t}_1 \to b\tau^+$) - Br($\tilde{t}_1 \to be^+$) plane. The strongest bounds arise when the bottom–muon branching ratio is largest, while the weakest arise when the bottom–tau branching ratio is largest. The dot marks the absolute weakest lower bound at 424 GeV.

Summary: under the assumption of a left-right $M_{\chi_{B-L}} > M_{\chi_{T_{3R}}}$ interval, we found that out of 10 million random points in initial parameter space, 58,096 satisfied all low energy phenomenological constraints.

Various possible LHC signatures were analyzed in detail. Note that





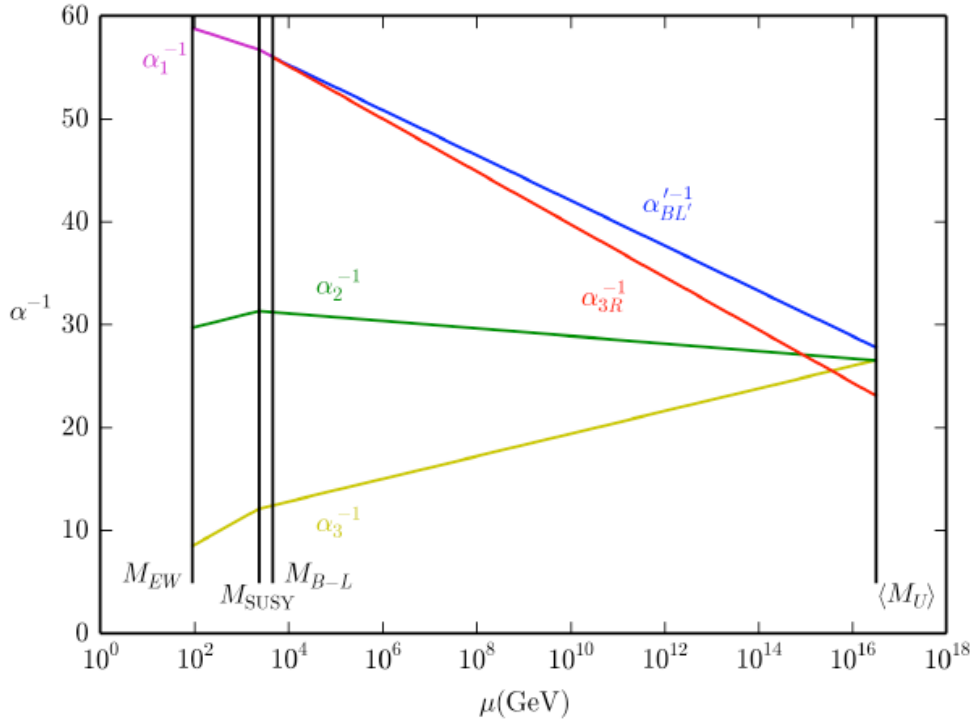
$\Rightarrow \quad \langle M_U \rangle = 3.15 \times 10^{16}\ \text{GeV} \ , \quad \langle \alpha_u \rangle = 0.0498$

However!

The assumption that $M_{\chi_{B-L}} > M_{\chi_{T_{3R}}}$ is only valid in a restricted region of moduli space. A far more general analysis would require

$$M_{\chi_{T_{3R}}} \simeq M_{\chi_{B-L}} \left(\simeq \ M_U \right)$$

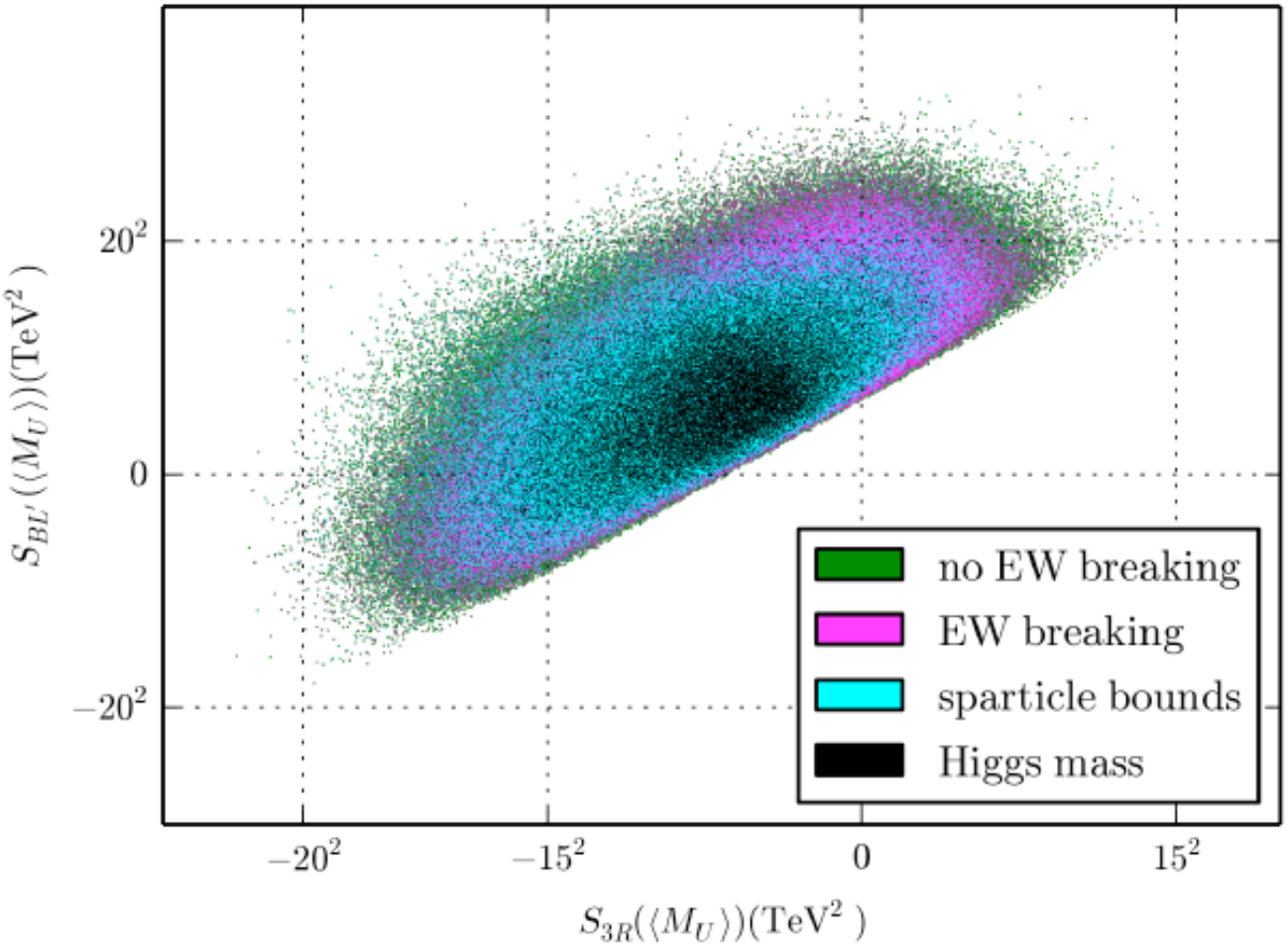
We now carry out this analysis. Note that the gauge couplings no longer unify. For example, for a "valid" point below



$\Longrightarrow$ Henceforth, we statistically scatter the 24 soft supersymmetry parameters in the same range $(\frac{M}{f}, Mf)$ where $M = 2.7\ TeV$, $f = 3.3$ at the average "unification" scale $\langle M_U \rangle = 3.15 \times 10^{16}\ \mathbf{GeV}$.

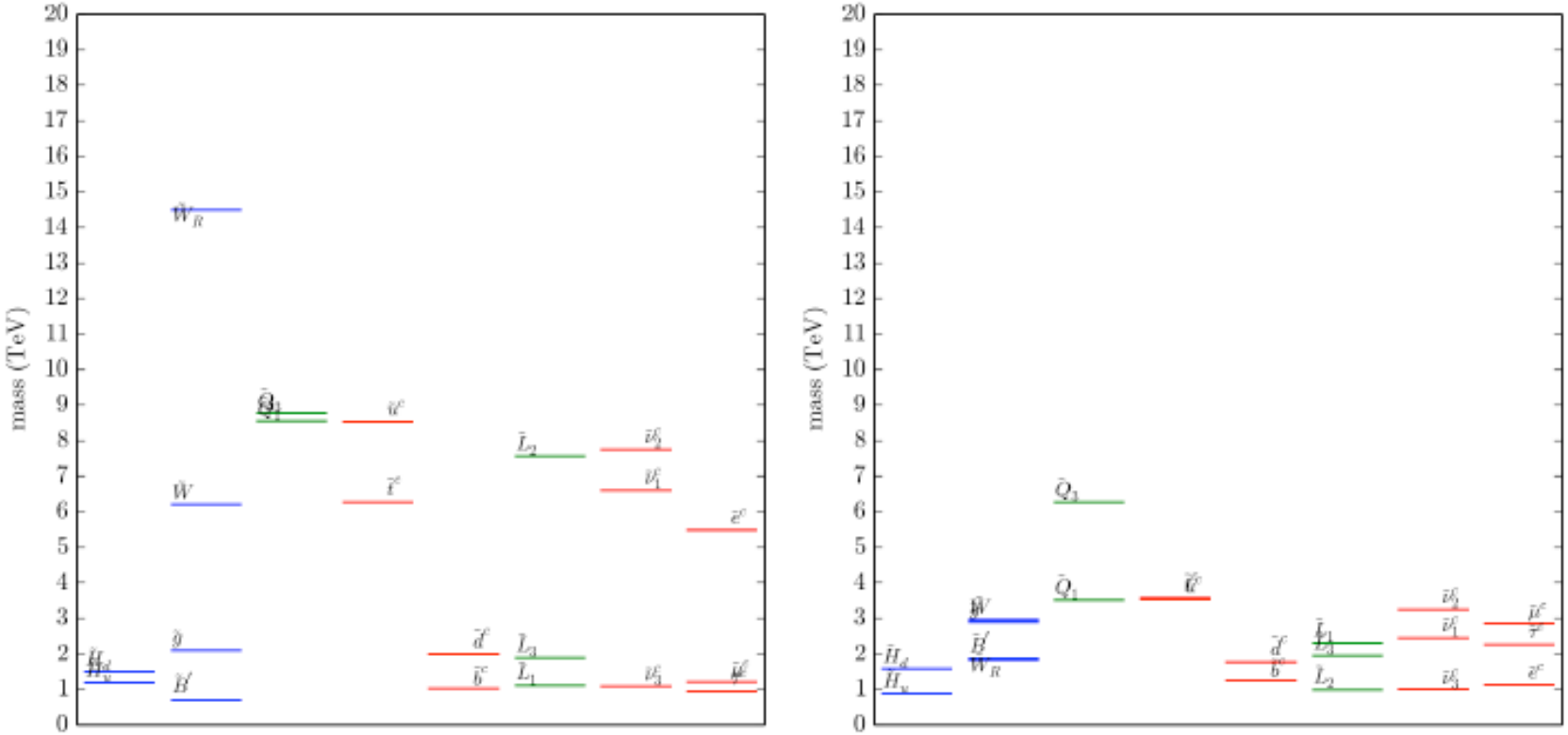
The results are subjected to all the same phenomenological constraints.

Again, we can plot our results in a two-dimensional space. We find that out of 10 million random initial points in SUSY breaking parameter space, all points that break B-L symmetry with $M_{B-L} > 2.5\ TeV$ are



Of these, there are 44,884 "valid" black points that satisfy all phenomenological requirements.

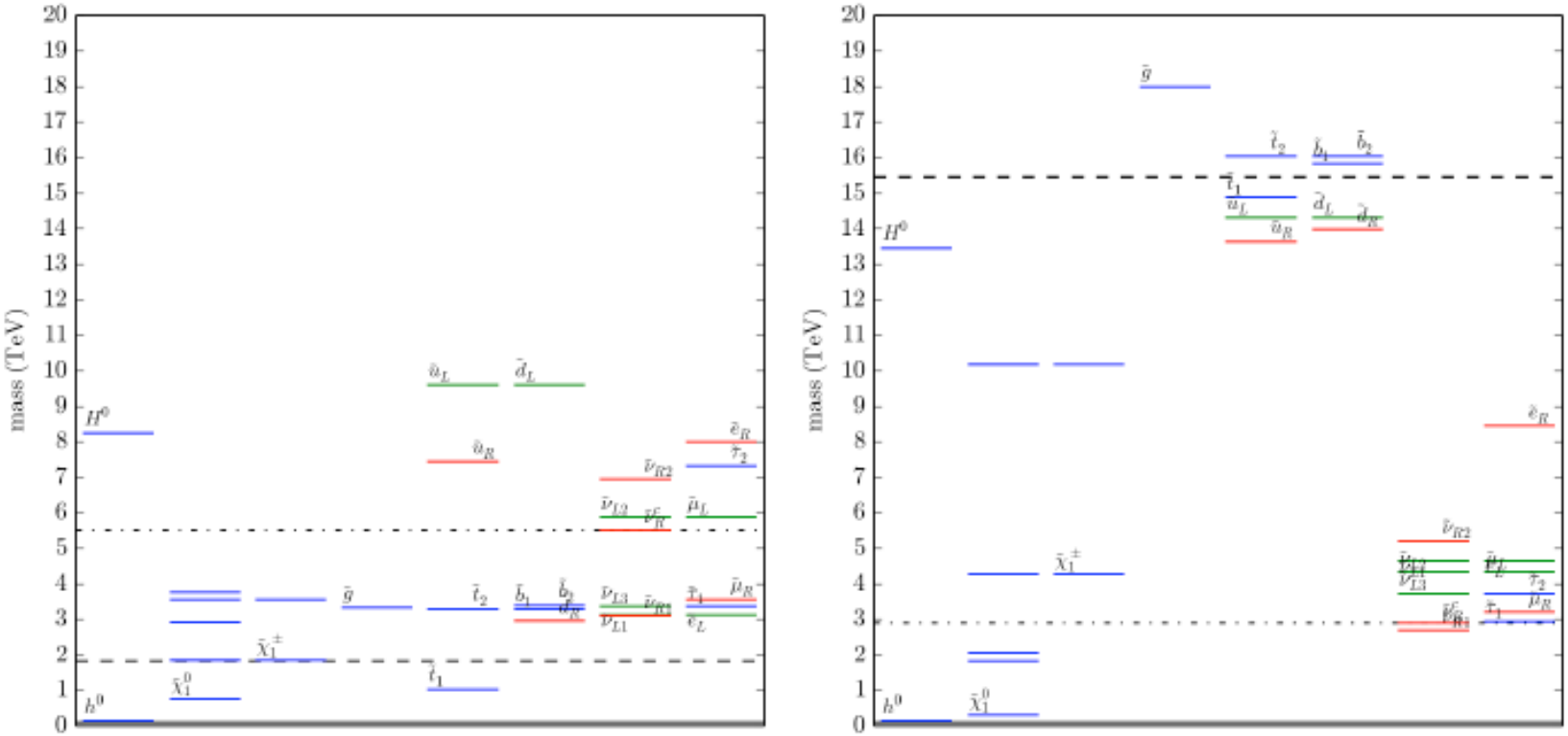
Phenomenologically "valid" black points arise from a wide range of initial conditions. For example



Example high-scale boundary conditions at $\langle M_U \rangle$ for the two valid points with the largest and smallest amount of splitting.

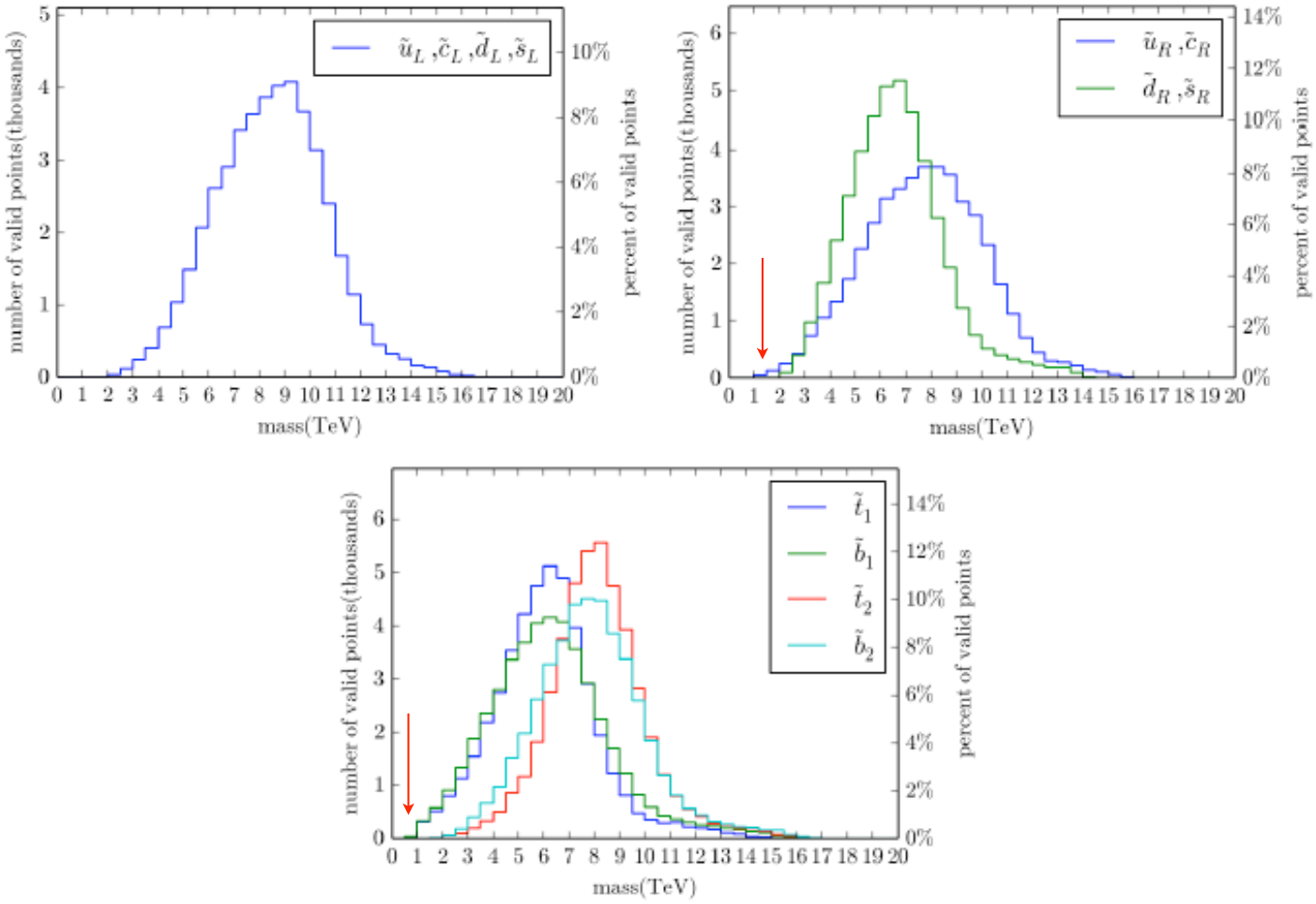
$\Rightarrow$ No special initial conditions--such as parameter "universality" or tuning for the Higgs mass-- are required.

The exact sparticle spectrum can be derived for each valid point. For example, for the two black points just presented, their low energy spectra respectively are



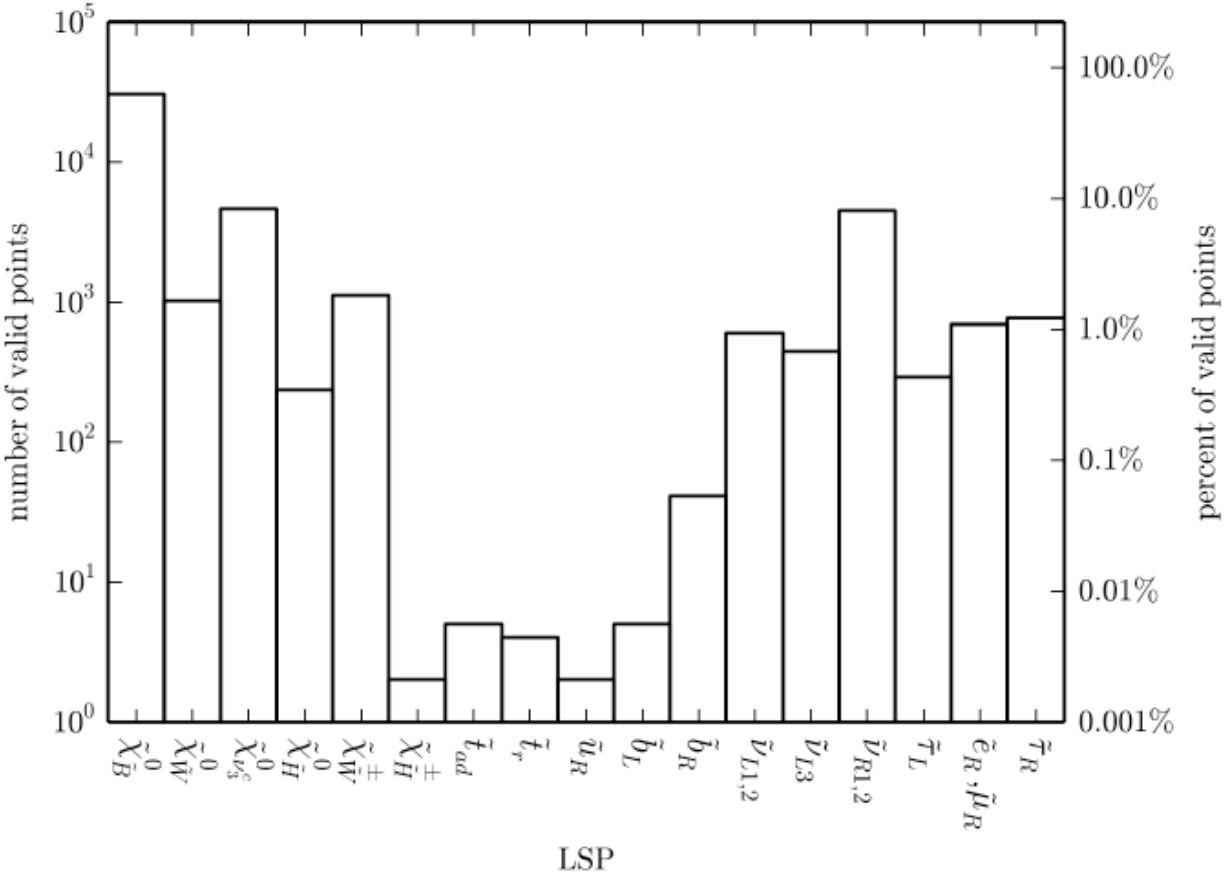
Two sample physical spectra with a right-side-up hierarchy and upside-down hierarchy. The $B-L$ scale is represented by a black dot-dash-dot line. The SUSY scale is represented by a black dashed line. The electroweak scale is represented by a solid black line.

For each sparticle type, one can present a histogram of its mass with respect to the 44,884 valid points. For example



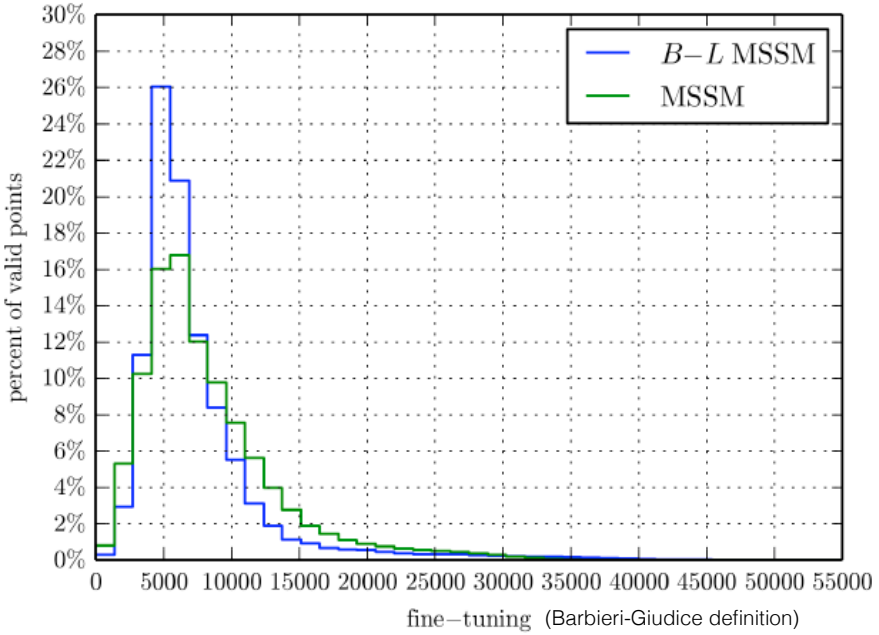
Histograms of the squark masses from the valid points in the main scan. The first- and second-family left-handed squarks are shown in the top-left panel. The first- and second-family right-handed squarks are shown in the top-right panel. The third family squarks are shown in the bottom panel.

Note that various sparticles can be quite light, depending on the initial valid point. We can plot a histogram of the "LSP" scanned over the black points. We find that



Note that, although most LSP's are neutralinos, due to R-parity violation some of the LSP's can carry electric and color charge while still being consistent with dark matter.

As always, the "little hierarchy problem" requires fine-tuning of the μ-parameter. This fine-tuning varies with the choice of initial valid point and can be expressed statistically as a histogram. We have also carried out a similar analysis in the standard MSSM. The results are



The blue line in the histogram shows the amount of fine-tuning required for valid points in the main scan of the simultaneous Wilson line $B - L$ MSSM. Similarly, the green line specifies the amount of fine-tuning necessary for the valid points of the R-parity conserving MSSM–computed using the same statistical procedure as for the $B - L$ MSSM with $M = 2700$ GeV and $f = 3.3$. The $B - L$ MSSM shows slightly less fine-tuning, on average, than the MSSM.

String Threshold Corrections:

It is expected that--at string tree level--all four gauge couplings unify with the dimensionless gravitational coupling

$$\sqrt{8\pi \frac{G_N}{\alpha'}}$$

to a single parameter $g_{\rm string}$ at a "string unification" scale

$$M_{\rm string} = g_{\rm string} \times 5.27 \times 10^{17}\ {\rm GeV}$$

$g_{\rm string}$ is set by the value of the dilaton and is typically of $\mathcal{O}(1)$.

Here, for specificity, we use a common value of $g_{\rm string} = 0.7 \Longrightarrow$

$$\alpha_{\rm string} = \frac{g_{\rm string}}{4\pi} = 0.0389\ , \quad M_{\rm string} = 3.69 \times 10^{17}\ {\rm GeV}$$

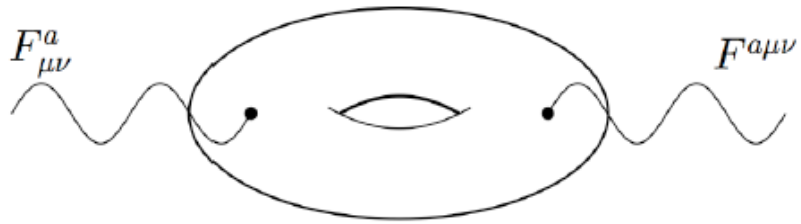
This introduces a fourth scaling regime

- $M_{\rm string} - M_U$: The RGEs are now

$$4\pi {\alpha_a}^{-1}(p) = 4\pi \alpha_{\rm string}^{-1} - b_a \ln\left(\frac{p^2}{M_{\rm string}^2}\right) + \tilde{\Delta}_a$$

where $a = 3, 2, 3R, BL'$.

Note that the one-loop running couplings no longer exactly unify at M_{string}. Rather, they are "split" by dimensionless threshold effects. These arise predominantly from massive genus-one string modes



contributing to the correleation function $\langle F^a_{\mu\nu} F^{a\mu\nu} \rangle$ and, hence, to α_a.

For each valid black point we can evaluate

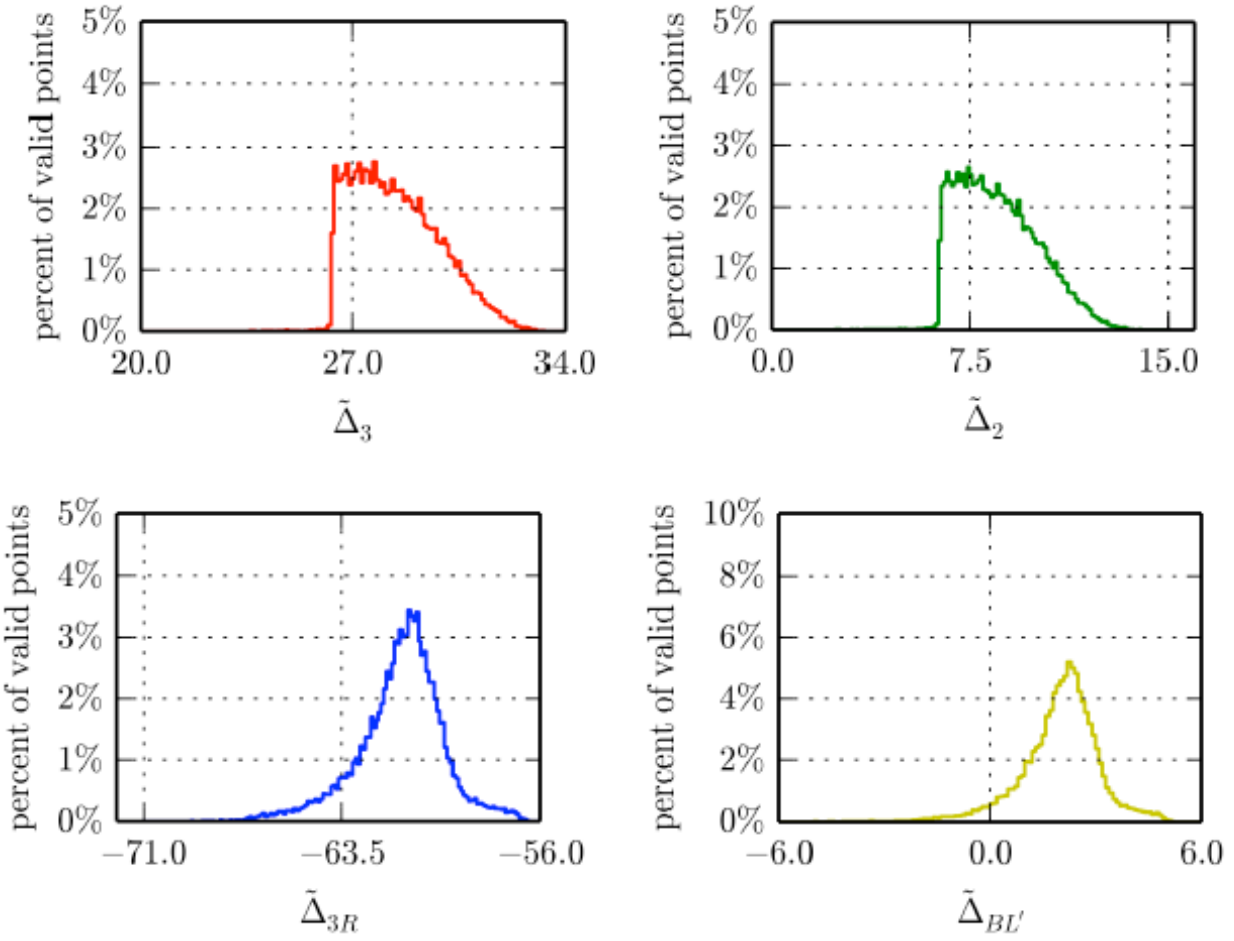
$$\alpha_3(\langle M_U \rangle),\ \alpha_2(\langle M_U \rangle),\ \alpha_{3R}(\langle M_U \rangle) \text{ and } \alpha'_{BL}(\langle M_U \rangle)$$

Choosing $p = \langle M_U \rangle$, using these results and the values of α_{string} and M_{string} given above $\Longrightarrow$ for each phenomenologically valid point the string thresholds are given by

$$\tilde{\Delta}_a = 4\pi {\alpha_a}^{-1}(\langle M_U \rangle) - 4\pi \alpha^{-1}_{\text{string}} + b_a \ln\left(\frac{\langle M_U \rangle^2}{M^2_{\text{string}}}\right)$$

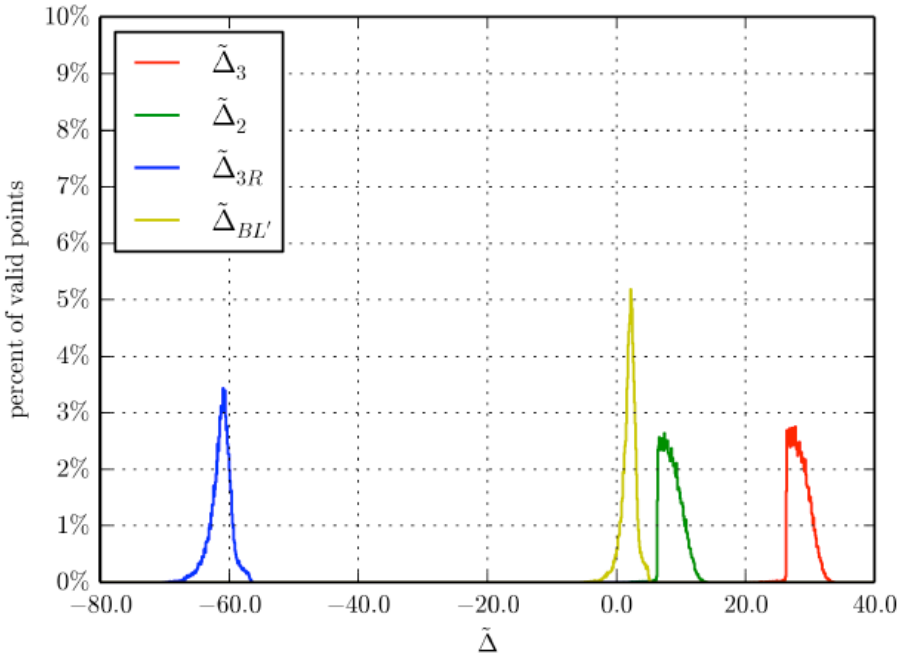
for $a = 3, 2, 3R, BL'$.

Evaluating these statistically over the set of valid initial points, we find



Histograms of each of the heavy string thresholds $\tilde{\Delta}_a$, $a = 3, 2, 3R, BL'$ arising from the $44,884$ phenomenologically valid points of our statistical survey. Each threshold value is plotted against the percentage of valid points giving rise to it. The bin width is 0.1.

Plotting these in a single histogram gives



It is well-known that each threshold breaks into

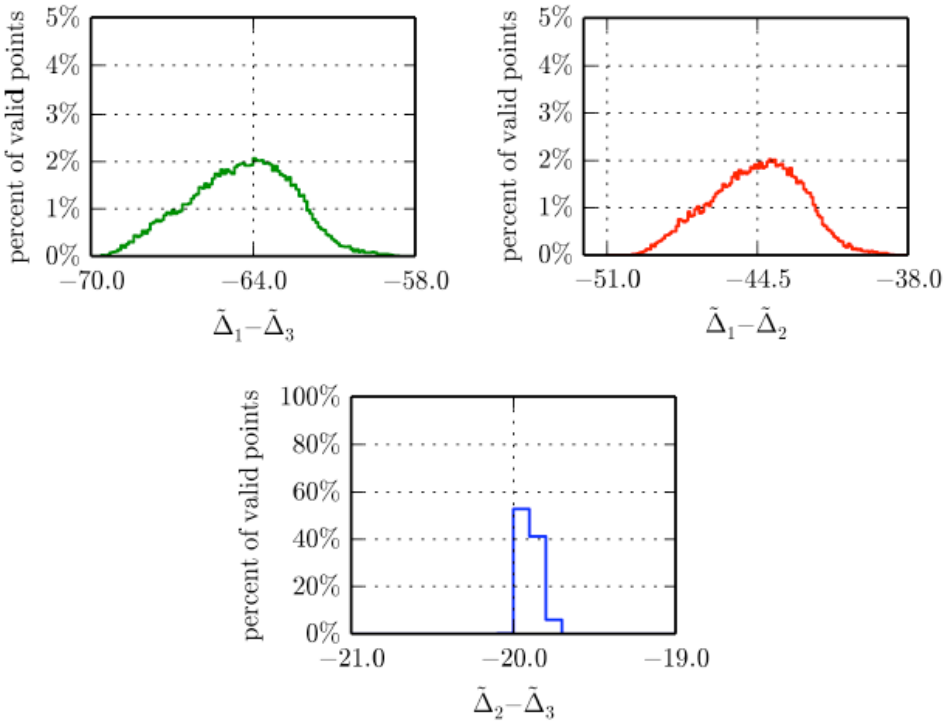
$$\tilde{\Delta}_a = \mathbb{Y} + \Delta_a$$

where $\mathbb{Y}$ is a "universal" piece independent of the gauge group and Δ_a are the moduli dependent "true" threshold terms for each gauge group. The universal piece is difficult to calculate due to infrared divergences, but Δ_a can be calculated using formulas by Kaplunovsky and Louis. This has not yet been carried out.

To now, the most we have done is to compute the Abelian hypercharge coupling using

$$\alpha_1^{-1} = \frac{3}{5}\alpha_{3R}^{-1} + \frac{2}{5}\alpha_{BL'}^{-1}$$

and the differences



Histograms of our statistical predictions for the values of $\tilde{\Delta}_1 - \tilde{\Delta}_2$, $\tilde{\Delta}_1 - \tilde{\Delta}_3$, and $\tilde{\Delta}_2 - \tilde{\Delta}_3$. The third of these plots looks different because the quantity $\tilde{\Delta}_2 - \tilde{\Delta}_3$ falls in a very narrow range. The bin width in all three plots is 0.1.

It would be interesting to compute these results from string theory.

Final Property of Simultaneous Wilson lines:

In both the sequential and the simultaneous Wilson lines cases, we chose the "average" SUSY breaking mass scale to be of $\mathcal{O}(2.7TeV)$; that is, "low scale" supersymmetry breaking. Can we raise the scale of SUSY breaking to $\mathcal{O}(10^{13}GeV)$ to allow for "split" or "high scale" supersymmetry breaking?

Answer:

A) For sequential Wilson lines, raising the SUSY scale to even $\mathcal{O}(10^{4}GeV)$ *widens the "left-right interval" to over three orders of magnitude. For larger SUSY scale, the calculation breaks down!*

B) For simultaneous Wilson lines, however, the SUSY breaking scale can be raised to an arbitrarily high value, including $\mathcal{O}(10^{13}GeV)$.