QFT vs Gravity:

Why do we need string theory?

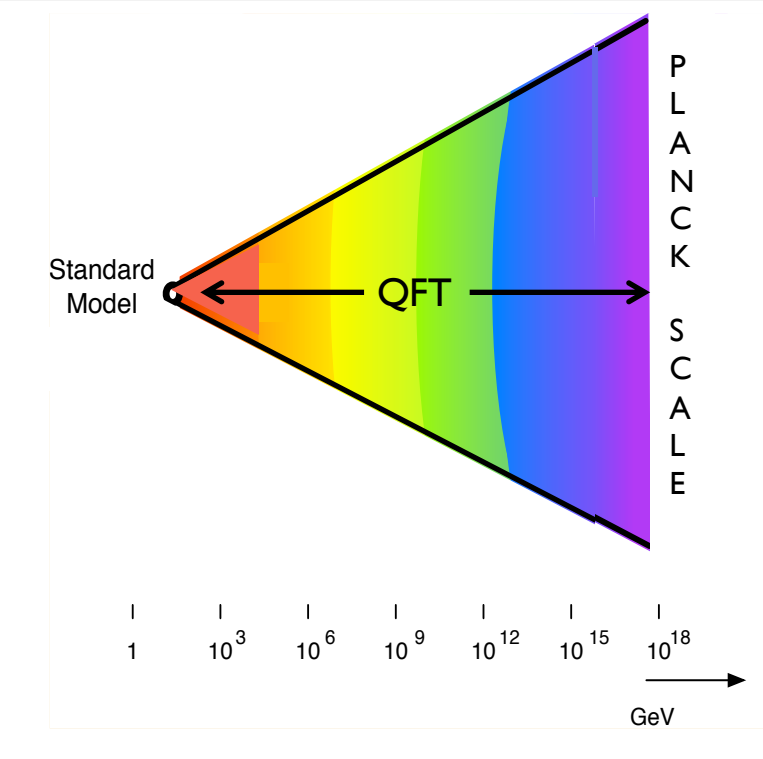
Herman Verlinde

Simplicity II

Fermilab 9/9/16

The Standard Model is an effective QFT, that, as far we know, successfully describes all physics from the MeV scale all the way to the Planck scale.

But it is not UV complete.



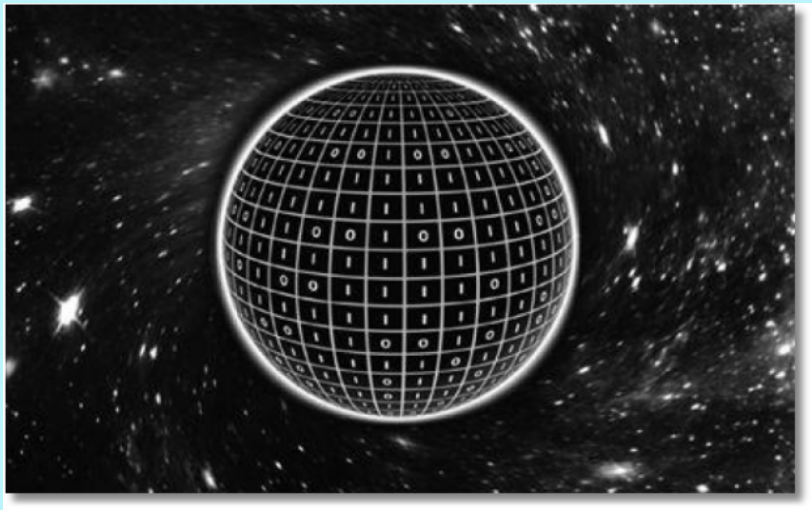
If we run the RG back up to M_{pl}, the effective QFT description will inevitably break down, due to non-renormalizable interactions.

Quantum gravity takes over at

$$M_{planck} = \frac{c^3}{\hbar G}$$

Black Hole Entropy

Bound on the number of
microscopic degrees of freedom
inside the black hole

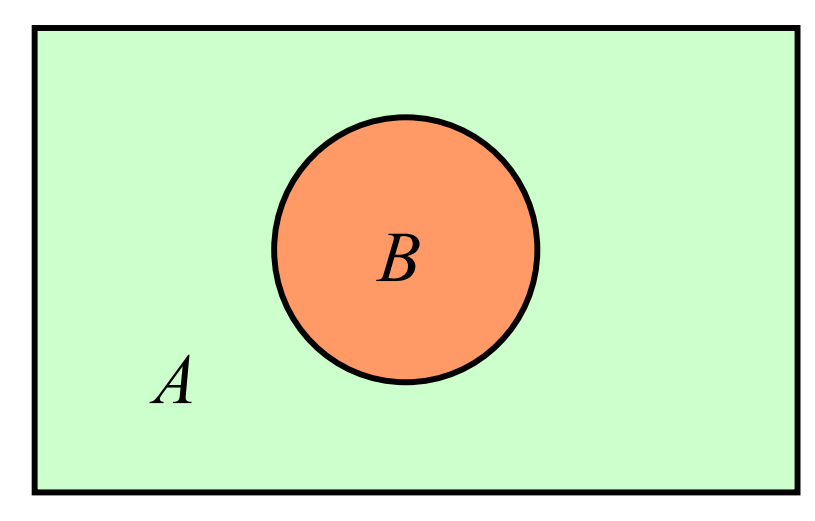


$$S_{BH} = k_B \frac{Ac^3}{4G\hbar}$$

The Planck Scale is the universal UV cut off

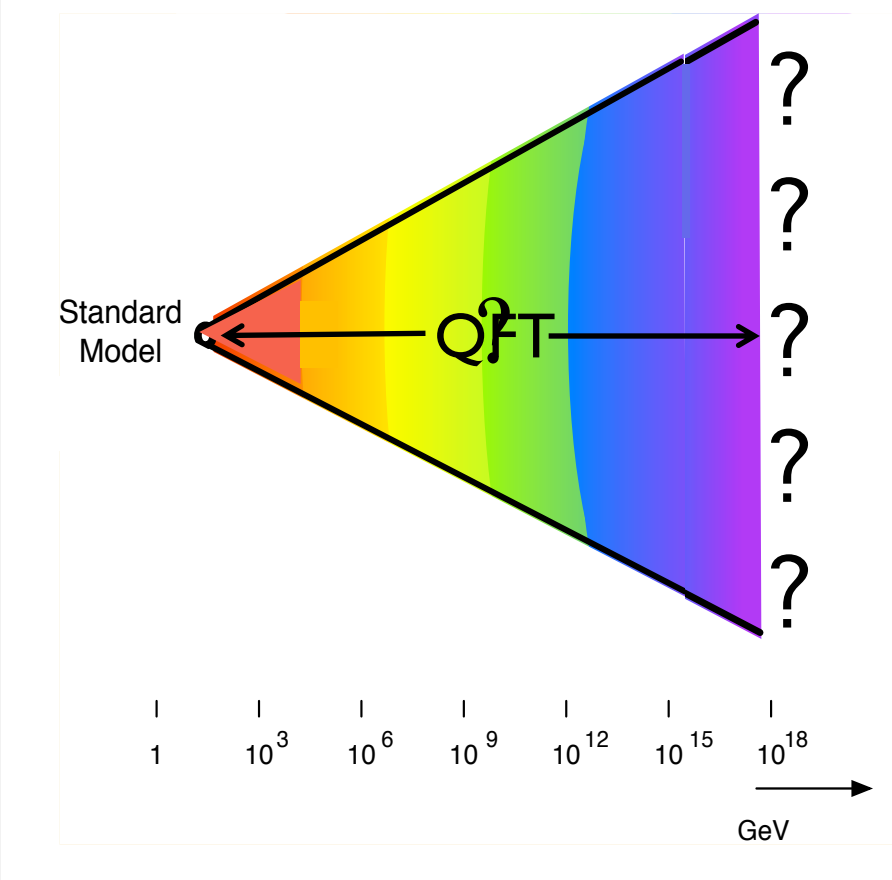
Entanglement Entropy

Quantifies the shared quantum information between A and B.



$$S_{BH} = k_B \frac{Ac^3}{4G\hbar}$$

$$S_A = -\mathrm{tr}(\rho_A \log \rho_A) \qquad \rho_A = \mathrm{tr}_B(|\psi\rangle\langle\psi|)$$

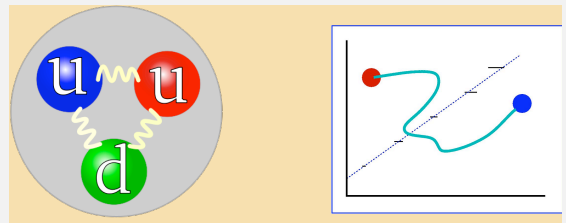


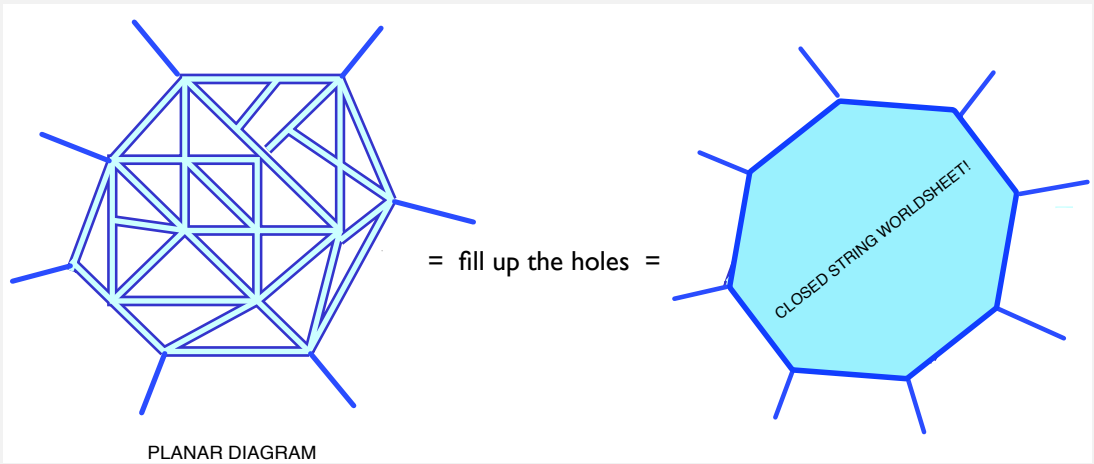
Why do we need String Theory?

‘ t Hooft’s argument

* 1/N expansion = string perturbation theory

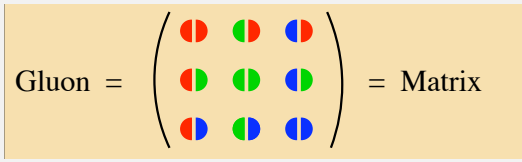
* planar diagram = triangulation of world sheet



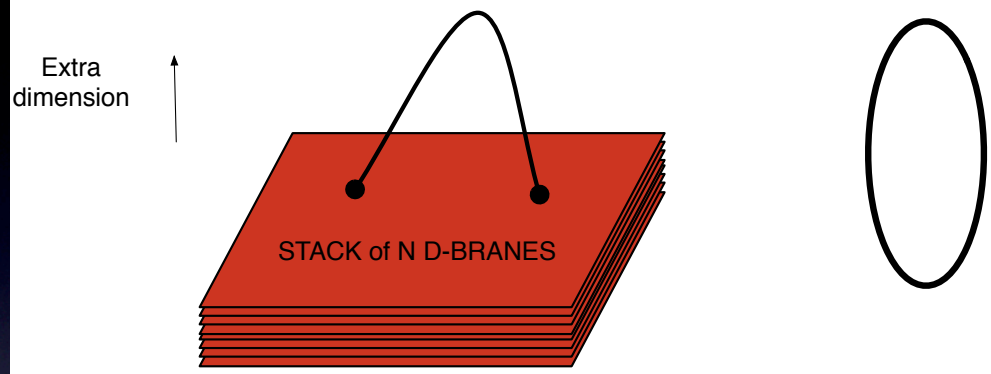


Large N QCD

<u>defines</u> a particular string theory, in a highly curved target space. In QCD, this string is strongly coupled.



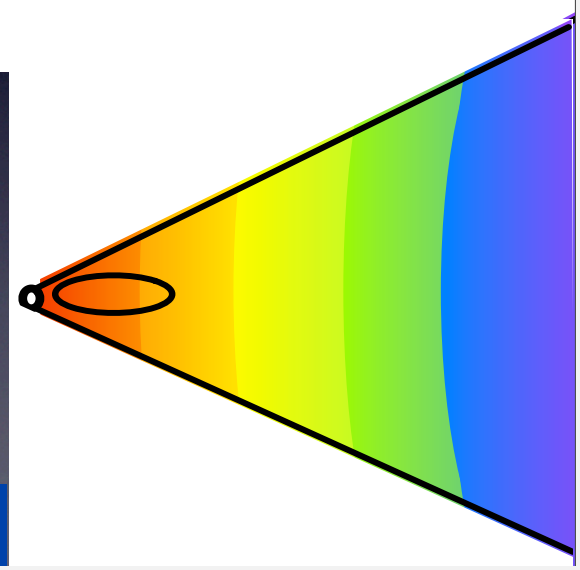
Maldacena's argument



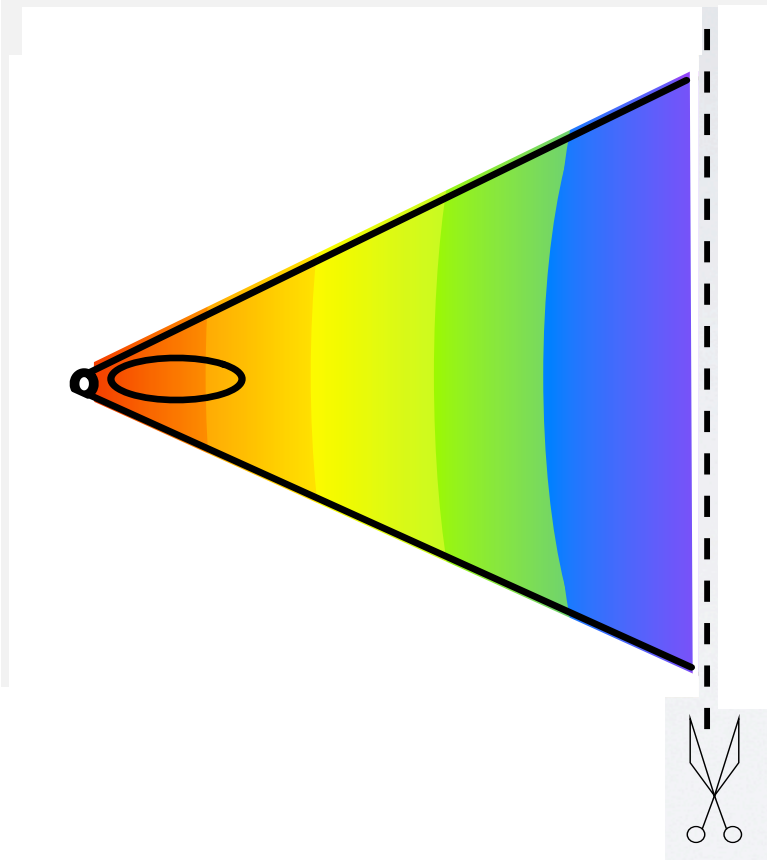
Gravitational back reaction creates a warped geometry

$$ds^2 = a^2(r)\,\eta_{\mu\nu}dx^\mu dx^\nu + dr^2$$

Extra dimension = RG scale



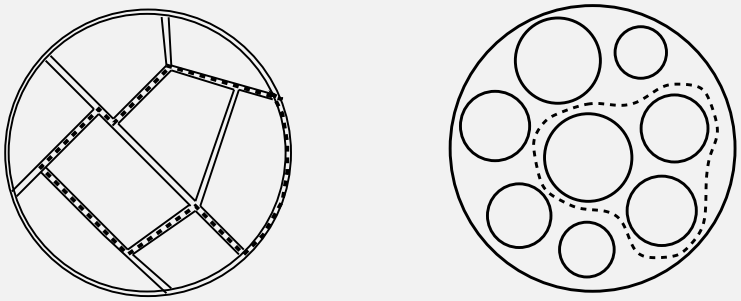
Holographic Renormalization Group

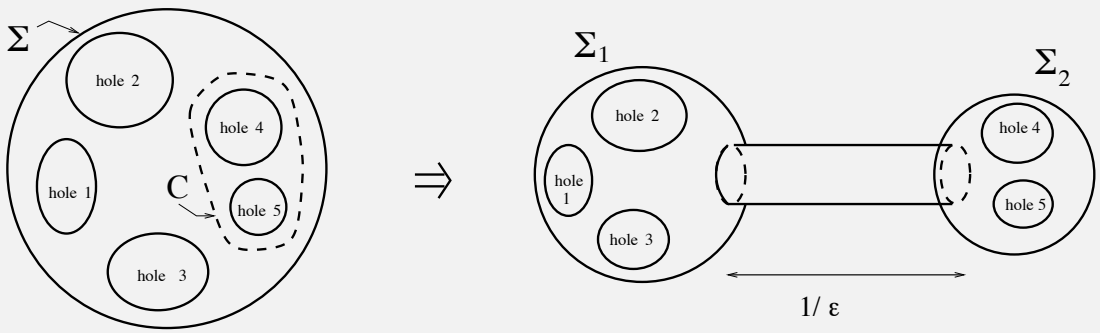


- Introduce a cut-off scale Λ_c = radial location r_c
- Integrate out UV = everything at scales > $\Lambda_c = r_c$
- Wilsonian action $S_{eff}(\Lambda_c)$ = gravity action $S_{class}(r_c)$
- RG evolution eqn = radial Schrodinger eqn

Why is this true?

Renormalization Group = Open/Closed String Duality





$$\hbar\partial_\Lambda e^{-\frac{1}{\hbar}S_{\rm int}(A,\Lambda)} \;=\; \hat{H}_{\rm gauge}\; e^{-\frac{1}{\hbar}S_{\rm int}(A,\Lambda)}$$

$$\hat{H}_{\rm gauge} = \hbar^2\,\alpha^{ij}\,{\rm tr}\left(\frac{\partial^2}{\partial A^i \partial A^j}\right)$$

$$\delta_\Lambda e^{-\frac{1}{\hbar}S_{\rm int}(A,\Lambda)} \;=\; \int \mathcal{D}a\; e^{-\frac{1}{\hbar}\left(\frac{\alpha_{ij}}{2\delta\Lambda}{\rm tr}(a^i a^j)\;+S_{\rm int}(A+a;\Lambda)\right)}$$

$$e^{-\frac{1}{\hbar}S_{\rm int}(A,\Lambda)} = \int \mathcal{D}\phi\; e^{-\frac{1}{\hbar}\left(S_0(A;\phi)+S_{\rm grav}(\phi;\Lambda)\right)}$$

$$S_0(A;\phi) = \sum_{(i_1\ldots i_n)} \phi_{(i_1\ldots i_n)}\,{\rm tr}\Big(A^{i_1}\ldots A^{i_n}\Big)$$

$$\boxed{\hbar\partial_\Lambda e^{-\frac{1}{\hbar}S_{grav}(\phi,\Lambda)} = \hat{H}_{\rm grav}\, e^{-\frac{1}{\hbar}S_{grav}(\phi,\Lambda)}}$$

$$\boxed{\check{H}_{\rm grav} = \beta_I\frac{\partial}{\partial\phi_I} + g_{IJ}\frac{\partial^2}{\partial\phi_I\partial\phi_J}}$$

Holographic RG eqn
=
Schrodinger evolution
in the scale direction

$$\mathcal{H} = \frac{1}{\sqrt{g}} g_{ij} g_{kl} \left(\pi^{ik} \pi^{jl} - \pi^{ij} \pi^{kl} \right) - \sqrt{g} \left(R - 2\Lambda \right) .$$

2+1 Gravity from 1+1 CFT with a T T deformation

$$S = S_{\mathrm{CFT}} - \mu \int dx d\tau \; T\bar{T}$$

Integrable QFT with a irrelevant interaction => UV cut off scale

$$\boxed{\langle \Theta \rangle = \frac{c}{24\pi} R + 2\mu \left\langle T\bar{T} \right\rangle}$$

Trace anomaly equation with TT deformation

$$\boxed{\left\langle T\bar{T} \right\rangle = \langle T \rangle \left\langle \bar{T} \right\rangle - \langle \Theta \rangle \langle \Theta \rangle}$$

Remarkable result due to A.B. Zamolodchikov (2004)

$$\boxed{\langle \Theta \rangle = \frac{c}{24\pi} R + 2\mu \left(\langle T \rangle \left\langle \bar{T} \right\rangle - \langle \Theta \rangle \langle \Theta \rangle \right).}$$

This is the Wheeler-de Witt equation in $2+1$ Gravity!

$$\langle n|T\bar{T}|n\rangle = \langle n|T|n\rangle\langle n|\bar{T}|n\rangle - \langle n|\Theta|n\rangle\langle n|\Theta|n\rangle$$

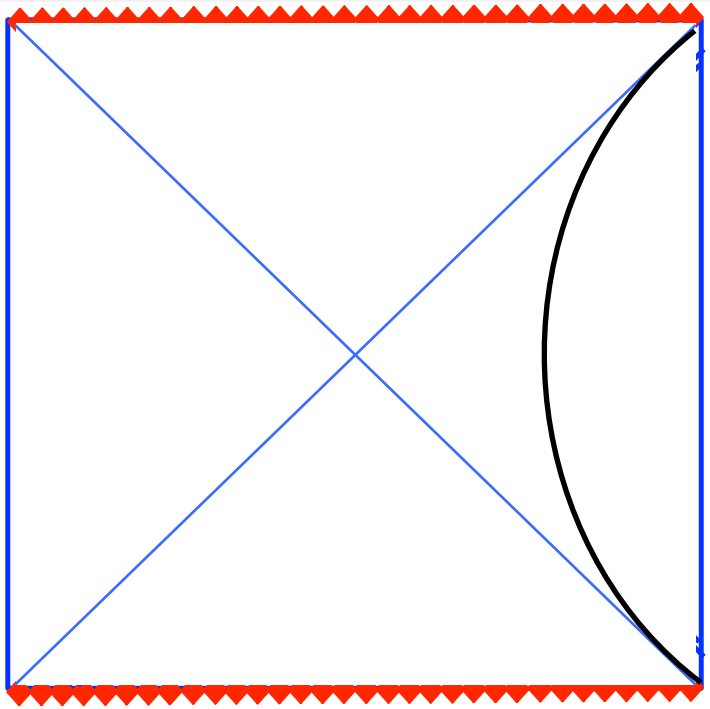
$$\frac{\partial E_n(R,\mu)}{\partial \mu} + E_n(R,\mu)\,\frac{\partial E_n(R,\mu)}{\partial R} + \frac{J_n^2}{R} = 0$$

Burgers equation!

$$E(R) = \frac{R}{4\mu}\left[1 - \sqrt{1 - \frac{\mu M}{R^2} + \frac{\mu^2 J^2}{4R^4}}\right]$$

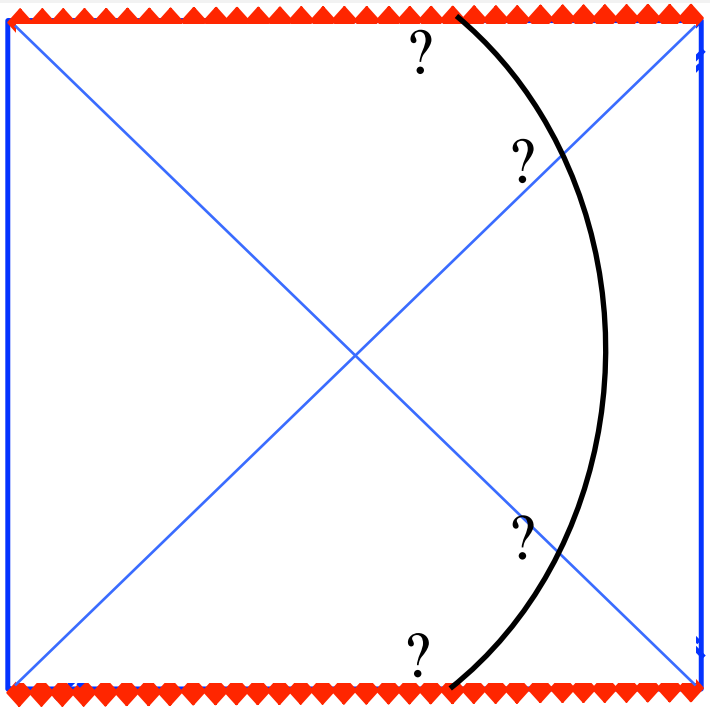
Total energy of a rotating black hole with radial cut-off!

Adiabatic expansion or contraction



$$E(R) = \frac{R}{4\mu}\left[1 - \sqrt{1 - \frac{\mu M}{R^2} + \frac{\mu^2 J^2}{4R^4}}\right]$$

Isobaric expansion and contraction



$$S = S_{\mathrm{CFT}} + \int dx d\tau \left[\frac{\xi\bar{\xi}}{\mu} + \bar{\xi}\, T + \xi\, \bar{T} \right]$$

$$ds^2 = dx_+ dx_- + \frac{\xi}{2} dx_+^2 + \frac{\bar{\xi}}{2} dx_-^2$$

$$\langle \xi \rangle = -\mu \langle T \rangle \qquad \langle \bar{\xi} \rangle = -\mu \langle \bar{T} \rangle$$

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2} dt^2 + \frac{r^2}{(r^2 - r_+^2)(r^2 - r_-^2)} dr^2 + r^2 \left(d\phi - \frac{r_+ r_-}{r^2} dt \right)^2$$

Metric of a rotating black hole in 2+1 dimensions

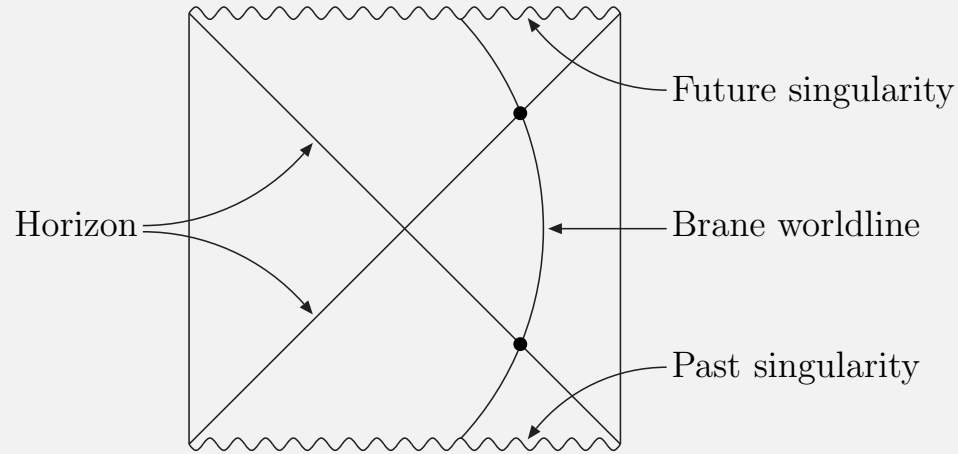
$$M = r_+^2 + r_-^2 \qquad J = 2 r_+ r_- \qquad \beta = \frac{2\pi r_+}{r_+^2 - r_-^2}$$

RS Brane world cosmology

$$ds^2_{n+2} = \frac{1}{h(a)}da^2 - h(a)dt^2 + a^2 d\Omega_n^2$$

$$h(a) = \frac{a^2}{L^2} + 1 - \frac{\omega_{n+1}M}{a^{n-1}},$$

$$\frac{1}{h(a)}\left(\frac{da}{d\tau}\right)^2 - h(a)\left(\frac{dt}{d\tau}\right)^2 = -1$$

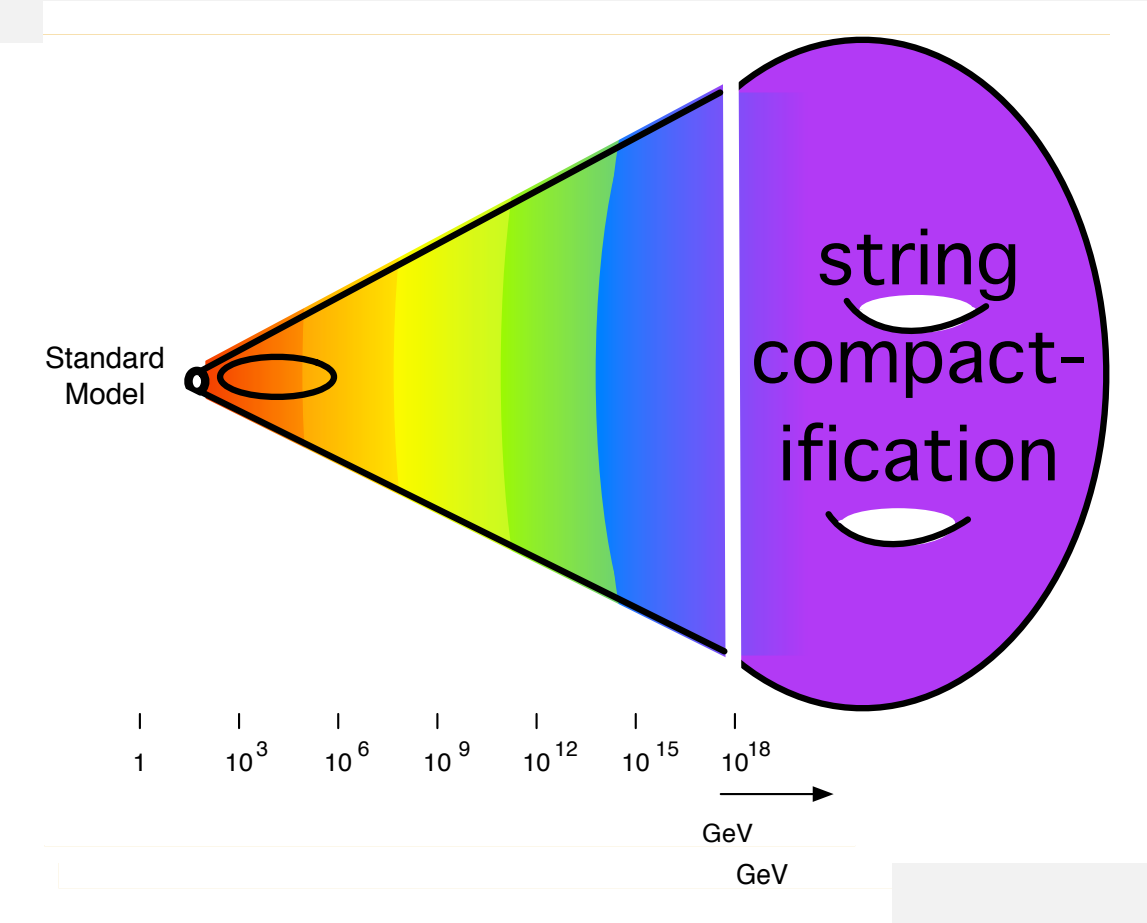


$$\boxed{ds^2_{n+1} = -d\tau^2 + a^2(\tau)d\Omega_n^2}$$

$$\boxed{H^2 = -\frac{1}{a^2} + \frac{\omega_{n+1}M}{a^{n+1}}}$$

FRW from CFT

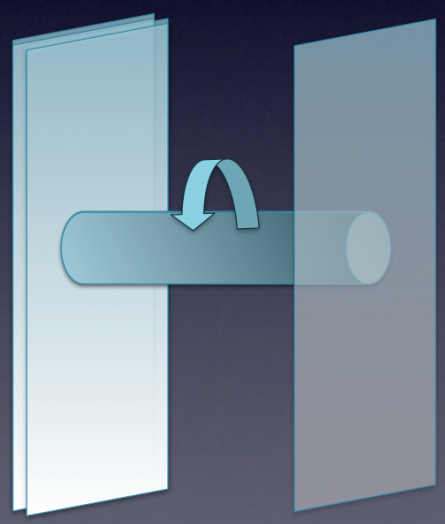
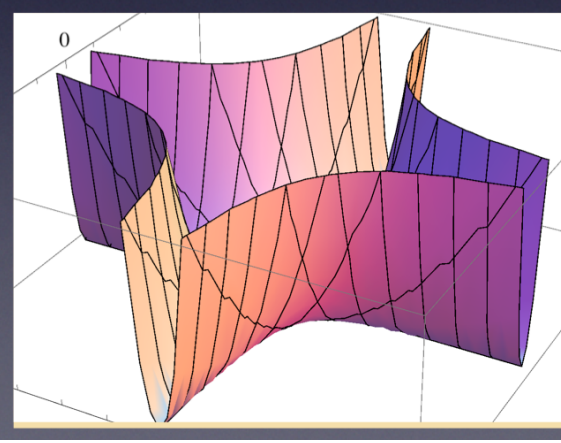
EA TERBY©


Standard
Model
string
compact-
ification
1
10 3
10 6
10 9
10 12
10 15
10 18
GeV
GeV

Matrix Theory

Early example of gravity arising from gauge theory:

* Large N matrix quantum mechanics
* Eigenvalues are positions of particles
* Off-diagonal modes are open strings
*Quantum fluctuations induce gravity:



$$H = \mathrm{tr}\left(P_I^2 + [X_I, X_J]^2 + \Psi^* X_I \Gamma^I \Psi\right)$$