Monodromy in QCD: Insights From Supersymmetric Theories with Soft Breakings

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Two related issues in QCD: θ dependence and the $U(1)$ problem.

Instantons provide an understanding of both, but in ordinary QCD, plagued by infrared issues (Appelquist, Shankar). These are strong coupling questions.

Qualitative expectations from instantons

- Pure gauge theory: $V(\theta) = \Lambda^4 \sum_n c_n \cos(n\theta)$ with c_n's not calculable systematically (ir problems)
- With matter, η' potential: $V(\theta, \eta') \sim \Lambda^4 \cos(\theta + \eta'/f'_\eta)$

Singling out the η' is heuristic, at best; η' is not particularly light.

Witten long ago suggested thats instantons are not a reliable guide, and proposed an alternative: large N.

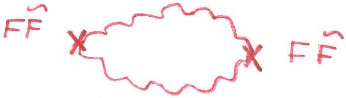
- Large N approximation: $N \to \infty$, $g^2 N$ fixed: consistent with many qualitative features of QCD (existence of resonances, Zweig's rule...)
- At large N, instantons effects should behave as $e^{-\frac{8\pi^2}{g^2}} \sim e^{-c\,N}$. $U(1)$ problem reappears.
- Instead, θ, η' potentials from resumming of perturbation theory. N dependence as from Feynman diagrams.

Implications of this viewpoint:

- Correlation functions at zero momentum of $F\tilde{F}$ behave as:

$$\frac{d^n}{d\theta^n}E(\theta) \propto \langle \left(\int d^4x\, F\tilde{F} \right)^n \rangle \propto N^{2-n}$$

  This is not compatible with a simple $\cos(\theta)$, or more generally $\sum_n c_n \cos(n\theta)$ behavior for E.
- If $N_f \ll N$, quarks as a perturbation – anomaly as a perturbation. η' a pseudogoldstone boson, with mass of order $1/N$.



Branched structure of QCD: multiple (metastable) ground states at fixed θ

How then to account for the 2π periodicity expected of θ? Witten suggested that QCD should be branched. For pure QCD:

$$E(\theta) = \min_k (\theta + 2\pi k)^2.$$

Then $\theta \to \theta + 2\pi$ compensated by $k \to k - 1$.

Including quarks, $\theta \to \theta + \eta'/f_\pi$: Now one can write an expression in terms of the Goldstone boson matrix, $U = e^{i\eta'/N_f + \pi^A \lambda^A/2/f_\pi}$:

$$E(\theta, \eta') = f_\pi^2 \left(\mathrm{Tr}\mu_i^2 U + (\theta + 2\pi k + \frac{\eta'}{f_\pi})^2 \right) .$$

Focus on $m_q \ll \Lambda/N$:

1. Mass of the η': $m_\eta'^2 \propto 1/N$ ($F_\pi^2 \propto \sqrt{N}$).
2. Interactions of the η' suppressed as: $\frac{1}{N^{n-2}}(\frac{\eta'}{f_\pi})^n$.

Supersymmetric Gauge Theories: A Testing Grounds

In QCD these problems are hard. In Supersymmetric gauge theories with small soft breakings, we can address all of these questions. Today we will see:

- In pure gauge case, with small m_λ, branches, N-dependence, N-dependence exactly as anticipated.
- Branches arise from spontaneous breaking of approximate, discrete symmetries.
- Small numbers of matter fields can be treated systematically as a perturbation
- η' lagrangian as anticipated.

But:

- While N dependence as anticipated, interpretation in terms of Feynman graphs obscure
- When instanton computations are possible, they are unsuppressed at large N – indeed, same N counting as anticipated from perturbation theory.
- Critical role of spontaneously broken, approximate discrete symmetries; these symmetries badly broken as $m_\lambda \to \Lambda_{QCD}$. Do these branches survive?

We will see that even in the regime of weak coupling, phase transitions as function of m_λ/m_q between N and N_f ground states.

Whether the N branches survive, or disappear, as the soft breakings increase and we approach real QCD is a question for lattice experiments.

Supersymmetric Gauge Theory Without Matter

We have understood much about strongly interacting supersymmetric gauge theories, exploiting, particularly, holomorphy of quantities like the superpotential as functions of couplings.

In the supersymmetric limit, the gauge theory without matter possesses a Z_N symmetry, spontaneously broken by a gaugino condensate:

$$\langle\lambda\lambda\rangle = 32\pi^2\Lambda_{hol}^3 e^{\frac{2\pi ik}{N}}$$

First question: is this consistent with expectations from large N. For this we need to understand:

1. What is the dependence on N we expect from Feynman diagrams?
2. How is Λ_{hol} related to Λ_{QCD}, N?

N-Dependence of the Gaugino Condensate

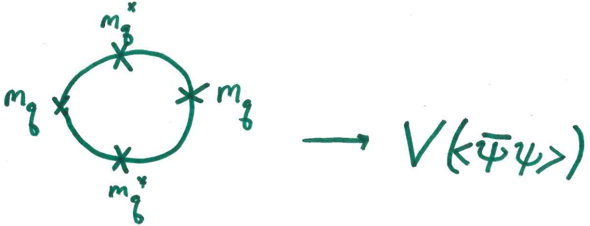
Reminder: Coleman-Witten argument for quark condensate at large N.

$$\mathcal{M} = \langle \bar{\psi}\psi \rangle$$

the effective potential for $\mathcal{M}$ takes the form

$$V(\mathcal{M}) = N\, F\left(\frac{\mathcal{M}^\dagger \mathcal{M}}{N^2 \Lambda_{QCD}^6}\right)$$

so $\mathcal{M} \propto \tilde{N}\Lambda$



Establishing existence of condensate invokes anomaly matching; not our concern here.

Note, in large N, $F_\pi^2 \propto N$, so from

$$m_\pi^2 F_\pi^2 = m_q \mathcal{M}$$

we have $m_\pi^2 \propto N^0 \; m_q \Lambda_{QCD}$

Scaling of $\langle\lambda\lambda\rangle$ with N

For supersymmetric gauge theories, similar counting of diagrams

$$V(\langle\lambda\lambda\rangle) = N^2\ F(\frac{\langle\lambda\lambda\rangle\langle\lambda\lambda\rangle^*}{N^2\Lambda^6}).$$

So if diagrammatic analysis is meaningful, $\langle\lambda\lambda\rangle = N\Lambda^3_{QCD}$. Note here we are normalizing the fields so $1/g^2$ sits in front of the whole lagrangian, and $g^2 \sim 1/N$.

Λ_{hol} vs. Λ_{QCD}

The holomorphic Λ parameter differs from the more conventional Λ parameter, as defined by the *Particle Data Group* by an N-dependent factor:

$$\Lambda_{hol}^3 = N\Lambda_{QCD}^3$$

So $\langle\lambda\lambda\rangle \propto N\Lambda_{QCD}^3$, as anticipated from perturbation theory, though the connection is obscure.

Monodromy in Supersymmetric $SU(N)$ Gauge Theory Without Matter

In the presence of a small, holomorphic gluino mass, $m_\lambda = |m_\lambda| e^{\frac{i\theta}{N}}$,

$$V(\theta, k) \approx \frac{|m_\lambda|}{2} \Lambda_{hol}^3 \cos \frac{\theta + 2\pi k}{N} \tag{1}$$

In terms of physical quantities,

$$m_\lambda \Lambda_{hol}^3 = N^2 m_{phys} \Lambda_{QCD}^3.$$

Note for fixed θ, k, large N:

$$V(\theta, k) = N^2 m_{phys} \Lambda^3 (\frac{\theta + 2\pi k}{N})^2. \tag{2}$$

Stability of the Different Branches

For small m_λ, can ask about the tunneling between states of different k (Shifman).

Thin wall, $\Delta k = 1$:

1. Domain wall tension: $T = \Delta W = N\Lambda_{QCD}^3$
2. Energy splitting: $\Delta E = 32\pi^2 m_\lambda \Lambda_{QCD}^3$.

So

$$\Gamma^3 e^{-B}; \quad B \propto \frac{T^4}{\Delta E^3} \propto N^4 \left(\frac{\Lambda_{QCD}}{m_\lambda}\right)^3.$$

Suggests highly metastable even as $m_\lambda > \Lambda_{QCD}$ for large N.

Supersymmetric QCD with $N_f < N$ flavors, $m_q = 0$ possesses an $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$ symmetry. With small m_q, $N \gg N_f$:

$$W = N \frac{\Lambda_{hol}^3}{(\det \frac{\bar{Q}Q}{\Lambda^2})^{\frac{1}{N}}} + \bar{Q} m_q Q.$$

Adding soft breakings:

First:

$$\delta V = \tilde{m}^2 \sum_f \left(|Q_f|^2 + |\bar{Q}_f|^2 \right).$$

With $\tilde{m}^2_{f\bar{f}} = \tilde{m}^2 \delta_{f\bar{f}}$, symmetry is still $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$.

Ignoring, at first, the gaugino mass, the potential

$$V = \sum_f \left(|\frac{\partial W}{\partial Q_f}|^2 + |\frac{\partial W}{\partial \bar{Q}_f}|^2 \right) + \delta V \tag{3}$$

yields a minimum at

$$Q_f \bar{Q}_g = v^2 U_{fg} \quad U = e^{\frac{i(\eta' + \pi^a \lambda^a)}{2F_\pi}}.$$

v is given by:

$$v = \Lambda_{hol} \left(\frac{\Lambda_{hol}^2}{\tilde{m}^2} \right)^{1/4}.$$

If we take $\tilde{m}^2 \sim \Lambda^2$, and note that $\Lambda_{hol}^3 \sim N\Lambda^3$, then $v = f_{\eta'} \propto \sqrt{N}$, as expected by standard large N arguments.

Small m_λ breaks the Z_N symmetry. Can take $m_\lambda = |m_\lambda| e^{i\theta/N}$. Now a potential for U,

$$V(\theta, \eta') = |m_\lambda| \Lambda^3_{hol} \cos(\frac{\theta + 2\pi k + \frac{\eta'}{v}}{N})$$

where $\arg \det U = \frac{\eta'}{v}$.

Expanding for very large N, gives

$$V(\theta, \eta') = |m_\lambda| \Lambda^3_{hol} \left(\frac{\theta + 2\pi k + \frac{\eta'}{v}}{N} \right)^2$$

Witten's potential for the η'; scaling with N is exactly as predicted, for $m_\lambda^{phys} \sim \Lambda$.

Two possible behaviors as soft breakings become large and we approach real QCD: N vacua persist, metastable. Or vacua disappear. Were it not for the argument that instantons are suppressed, the latter would be very plausible. Supersymmetric theories in some cases permit reliable instanton computations. Suppressed for large N?

Cut Off Instantons

A picture of cutoff instantons would suggest

$$V(\theta) = \Lambda_{QCD}^4 \sum c_n \cos(n\theta)$$

E.g. QCD without flavors. One instanton contribution to $V(\theta)$ has form:

$$V(\theta) = \int d\rho \rho^{-5+\frac{11N}{3}} M^{\frac{11N}{3}} N e^{-\frac{8\pi^2}{g(M)^2}} \cos(\theta)$$

Since $g^2(M) \sim 1/N$, this is formally exponentially suppressed. The expression is, however, infrared divergent, complicating the argument.

Suppose that this expression is cut off in the infrared at $\rho \approx \Lambda_{QCD}^{-1}$.

$$V(\theta) = \mathrm{C}\Lambda_{QCD}^{4} \cos(\theta).$$

$\Lambda_{QCD} \sim \mathcal{O}(1)$ (in the sense of large N counting).

This argument is handwaving at best (Witten). If the cutoff is $c\,\Lambda$, with c an order one constant, then the result can be exponentially suppressed or enhanced by c^N.

Scaling of (Reliable) Instanton Computations with N

For $N_f = N - 1$, instanton computations are reliable. Infrared divergences are cut off by the vev's of the fields $\bar{Q}Q \equiv v^2$. The ρ integrals take the form

$$W \sim \int d\rho \Lambda \rho^{2N+1} v^{*\ 2N-2} \rho^{4N-5} e^{-c^2 \rho^2 |v|^2}$$

$$\sim \frac{\Lambda^{2N+1}}{v^{2N-2}}.$$

A careful analysis yields:

$$W = \frac{\Lambda_{hol}^{2N+1}}{\det \bar{Q}Q}$$

$\Lambda_{hol} = Me^{-\frac{8\pi^2}{g_{hol}^2(M)N}}$ **so the numerator is order e^{-N}.**

However, v^2 also depends on Λ. For simplicity, taking all of the quarks to have equal mass,

$$v^N = \Lambda_{hol}^N \left(\frac{\Lambda_{hol}}{m_q} \right)$$

At the stationary point,

$$\langle W \rangle = a \Lambda_{hol}^2 m_q \left[\frac{\Lambda_{hol}}{m_q} \right]^{1/N} .$$

$$\Lambda_{hol}^{2+\frac{1}{N}} = M^{2+\frac{1}{N}} e^{-\frac{8\pi^2}{g^2 N}}.$$

No e^{-N} suppressions, no factors like π^N or 2^N which might have obstructed a suitable large N limit.

This is because the infrared cutoff is itself determined by the instanton computation (W_{np}).

In the presence of a gaugino mass, $m_\lambda = |m_\lambda| e^{i\frac{\theta}{N}}$, and writing $v^2 = |v^2| e^{\frac{2\pi i k}{N}}$,

$$E(\theta) = m_\lambda \langle W \rangle = m_\lambda \Lambda_{hol}^3 \cos(\frac{2\pi k + \theta}{N})$$

Complete agreement with expectations based on N counting of perturbative Feynman diagrams. In particular, correlators of n $F\tilde{F}$ operators at zero momentum behave as N^{2-n}, precisely as expected. Yet at the same time, the result arises from instantons! In terms our earlier cutoff argument, the ir cutoff is Λ_{hol}, yielding $\cos(\theta/N)$.

Does the Branched Structure Survive?

So, on the one hand, we see evidence for a branched structure, a structure originally suggested by a presumed suppression of instanton effects. On the other hand, we see that instantons are not suppressed. Moreover, we see again that the branches are associated with an approximate discrete symmetry. So we might imagine that the branched structure survives as the supersymmetry breaking grows, or that it disappears.

Phases of the Theory with General N_f, N with Small Soft Breakings

Already in the limit of soft breakings, for general N_f, N, there is an intricate phase structure as one varies the soft breaking parameters. We would anticipate this because of the symmetry pattern:

1. $m_\lambda \neq 0, m_q = 0$: Z_{N_f}
2. $m_\lambda = 0, m_q \neq 0$: Z_N

As one varies the parameter $x = \frac{m_\lambda}{m_q}$ the number of local minima of the potential changes from N at small x to N_f at large x.

Just before states disappear, they become highly unstable.

To see this, take $|m_q|^2, |m_\lambda|^2 \ll \tilde{m}^2$, and $\tilde{m}^2$, m_q proportional to the unit matrix in flavor space. Then we can take $\bar{Q}Q = v_0^2 e^{i\eta'}$. The potential for η' is (large N):

$$V(\eta') = m_q \, \Lambda_{hol}^3 \cos(\frac{\eta'}{f_\pi}) + N m_\lambda \, \Lambda_{hol}^3 \cos(\eta' \frac{N_f}{N}).$$

This potential has N vacua in the limit of small x, and N_f in the limit of large x. The transition occurs for x of order one. As the vacua are about to disappear, they become less and less metastable.

Apart from illustrating the possibility of a rich phase structure, we can use this result in considering actual QCD. We can consider approaching QCD in two ways. Take $\tilde{m}^2, m_q, m_\lambda \to \infty$, with $\tilde{m}^2 \gg m_q \gg m_\lambda$ or $\tilde{m}^2 \gg m_\lambda \gg m_q$. In the first case, until we lose control of the computation we have N branches; in the second N_f. If there are to be N branches in actual QCD, then in the latter case, we must cross a line from N_f to N vacua. This is plausible, but these observations also make plausible the possibility that the branches of SBQCD simply disappear.

Conclusion: Two possible behaviors

So two possible behaviors for real QCD:

1. N branches
2. $O(1)$ branches.

Implications, e.g., for the behavior of the η'. Also for QCD-like theories as models for monodromy inflation.

To settle: Lattice computations

1. Study correlation functions, $(F\tilde{F})^n$. Some results from Bonati et al. Support the branched picture.
2. Look for the unstable states. E.g. current lattices not too large. Might expect for different members of the ensemble, different values of k. Measure $\int d^4x\, F\tilde{F} \propto \Lambda_{QCD}^4 k\, VT$.

Supplementary Material

Quarks as a Perturbation

One of the attractive features of the large N limit is that the anomaly, in large N, can be treated as a perturbation, and thus the η' is a pseudogoldstone boson. This follows from the fact that the matter fields are themselves a perturbation. Can actually implement the idea of quarks ($N_f \ll N$) as a perturbation in supersymmetric QCD with small quark mass.

Compute $\langle \bar{Q} Q \rangle$ in the presence of the gaugino condensate.

$$\langle \bar{Q} Q \rangle = \langle \bar{Q}(x) Q(x) \int d^4 z_1 \; d^4 z_2 \; \sqrt{2} Q^*(z_1) \lambda(z_1) \psi_Q(z_1)$$

$$\sqrt{2} \bar{Q}^*(z_2) \lambda(z_2) \psi_{\bar{Q}}(z_2) \rangle.$$

Q, Q^*, ψ_Q, $\psi_{\bar{Q}}$ are contracted as in Wick's theorem. For the gauginos, make the replacement:

$$\lambda_i^j(z_1) \lambda_k^\ell(z_2) = \frac{1}{N^2} \delta_k^j \delta_i^\ell \langle \lambda \lambda \rangle.$$

$$\langle \bar{Q}Q \rangle = \frac{1}{32\pi^2} \frac{m_q}{|m_q|^2} \langle \lambda\lambda \rangle.$$

We know (Intriligator-Seiberg)

$$\langle \lambda\lambda \rangle = 32\pi^2 \Lambda_{hol}^3 \quad \langle \bar{Q}Q \rangle = \frac{m_q}{|m_q|^2} \Lambda_{hol}^3. \tag{4}$$

So while the calculation is arguably heuristic, the agreement is complete!

Shifman noted that the constant c is very small for $g \sim 1$.

$$c = \frac{3^3 g^6}{2^{18} \pi^8}. \tag{5}$$

For $m_\lambda \approx \Lambda_{QCD}$, this formula should roughly hold. As a result, these states would be highly unstable, unless $N > 300$ or so. This is a troubling result. If the estimate for small m_λ is correct once $m_\lambda \sim \Lambda_{QCD}$ at the order one level, then this would suggest that large N isn't even qualitatively valid for $N \approx 3$.

But objections to this estimate. If one is counting π's at strong coupling (a questionable practice in any case) one should consider the g's as well. One might expect that these would give additional factors of π, strong coupling taking over when g^2 is possibly as large as $16\pi^2$. Using $g^2 = 8\pi^2$ in the expression for c gives

$$c = \frac{27}{\pi^2 2^9} \approx 5 \times 10^{-3}. \tag{6}$$

This is of order one for $N = 3.6$. Needless to say, this estimate is subject to wild uncertainties.

Quantities in supersymmetric gauge theories are readily derived in terms of an object referred to as the *holomorphic scale*, Λ_{hol}. In the case of $SU(N)$ SUSY QCD without chiral fields, we can make this notion precise in a very simple way, embedding the theory in an $N = 4$ gauge theory with masses for the adjoint fields providing a cutoff for the SQCD theory. (Arkani-Hamed, Murayama).

In a presentation in which the $SU(4)$ symmetry is (almost) manifest, one writes the action as

$$\mathcal{L} = -\frac{1}{32\pi^2}\int d^2\theta\tau W_\alpha^2 + \frac{1}{g^2}\int d^4\theta\Phi_i^\dagger\phi^i + \int d^2\theta\frac{1}{g^2}f_{abc}\epsilon^{ijk}\Phi_i^a\Phi_j^b\Phi_k^c.$$

Here τ is

$$\tau = \frac{8\pi^2}{g^2} + i\theta.$$

In order that the superpotential be a holomorphic function of τ, we rescale the Φ^a fields. In this presentation, one can add also holomorphic mass terms for the Φ_i fields:

$$\mathcal{L} = -\frac{1}{32\pi^2} \int d^2\theta \tau W_\alpha^2 + \frac{1}{g^{2/3}} \int d^4\theta \Phi_i^\dagger \Phi^i + \int d^2\theta (f_{abc}\epsilon^{ijk} \Phi_i^a \Phi_j^b \Phi_k^c + M\Phi$$

Holomorphy of the gauge coupling function gives, for the renormalized coupling,

$$\frac{8\pi^2}{g^2(m)} = \frac{8\pi^2}{g^2(M)} + b_0 \log(m/M).$$

Here m and M are holomorphic parameters.

The physical masses are related to these by a factor of $g^{2/3}(m), g^{2/3}(M)$; substituting yields the standard β function through two loops (related to the NSVZ β-function). Λ_{hol} is then defined through:

$$\Lambda_{hol} = Me^{-\tau/b_0} = g^{-2/3} M_{phys} e^{-\tau/b_0}.$$

The Particle Data Group presents the strong coupling as

$$\alpha_s(\mu) = \frac{4\pi}{b_0 t}\left(1 - \frac{b_1}{b_0^2}\frac{\log t}{t}\right). \quad t = \log(\frac{\mu^2}{\Lambda^2}).$$

Comparing with the solution of the RGE:

$$\frac{8\pi^2}{g^2(\mu)} = \frac{8\pi^2}{g^2(M)} + b_0 \log(\mu/M) - \frac{b_1}{b_0}\log(g(\mu)/g(M)).$$

we see that taking

$$\Lambda = fMe^{-\frac{8\pi^2}{g^2(\mu)}}\left(\sqrt{\frac{b_0}{16\pi^2}}g\right)^{-b_1/b_0^2},$$

and writing

$$\log t \approx \log(\frac{4\pi}{b_0 g^2})$$

gives result identical to

$$\Lambda_{QCD} = e^{-\int^g \frac{dg'}{\beta(g')}}$$

Further circumstantial evidence for the role of instantons

Also instructive are instanton computations in the pure supersymmetric gauge theory. While one can't make rigorous statements, these are suggestive In pure $SU(N)$ supersymmetric QCD, Shifman et al calculated the correlation function

$$G^{2N} = \langle \lambda\lambda(x_1) \dots \lambda\lambda(x_N) \rangle$$

This receives a contribution from a single instanton, which turns out to be infrared finite. $G^{2N} \sim \Lambda^{3N}$, which is formally of order e^{-N}. But the authors argued that the N'th root of this expression was

$$G = \langle \lambda\lambda \rangle$$

In making this claim, that noted that G^{2N} is the correlator of the lowest component of a set of chiral fields. As a result, it must be independent of position. They argued, in addition, that there were not corrections to the result. More precisely they argued for a perturbative non-renormalization of the instanton effect, and, much in the spirit of large N, that one could ignore instanton-anti-instanton corrections. This, along with with cluster decomposition, would appear to support the claim.

But it is known that the single instanton computation makes an order one error in the computation of these quantities. The corrections can be understood as dilute gas instanton computations (in the sense that they can be shown to arise from the sector with topological number one (Hollowood et al, Dine, Festuccia). If the naive reasoning were correct, these effects would be suppressed by further powers of e^{-N}, but this is not the case.

More General Behavior: A Toy Field Theory

Illustration: theory of a complex scalar.

$$V(\phi) = -\mu^2|\phi|^2 + \frac{\lambda}{2}|\phi^4| - \Gamma\phi^N$$

respects a Z_N symmetry. If Γ is small, then we can write:

$$\phi = fe^{ia/f} \quad f = \sqrt{\frac{\mu^2}{\lambda}}$$

The field a has a potential:

$$V(a) = -\Gamma f^N \cos(N\frac{a}{f}).$$

The system has N degenerate minima, at $\frac{a}{f} = \frac{2\pi k}{N}$, reflecting the spontaneous breaking of the discrete symmetry.

Now add a coupling

$$\delta V = m_\lambda \phi + \text{c.c.}$$

This breaks the Z_N symmetry, the parameter m_λ behaving as a spurion just like m_λ in SUSY QCD. For small $m_\lambda = |m_\lambda| e^{i\alpha}$, ϕ does not shift significantly; the classical potential has the form:

$$E(\alpha, k) = |m^2| f^2 \cos(\alpha + \frac{2\pi k}{N}).$$

The potential respects the (spontaneously broken) spurious discrete symmetry. Quantum mechanically, E has a small imaginary part except for k such that $|\alpha + \frac{2\pi k}{N}| < \pi$.

Elsewhere in the parameter space, however, the branches disappear. For example, for μ^2 negative, the potential has a unique minimum; this is not altered by the addition of the m_λ term. Instead,

$$\langle \phi \rangle = \frac{m_\lambda^*}{|\mu^2|}$$

Now

$$E(\alpha, k) = \frac{|m_\lambda|^2}{|\mu^2|}.$$

Thinking of this as a model of supersymmetric QCD, the parameters $\mu^2 \to \mu^2(m_\lambda)$, $\Gamma \to \Gamma(m_\lambda)$. If, for example, $\mu^2(m_\lambda)$ becomes negative and Γ does not grow too rapidly for large m_λ, the branched structure disappears. Alternatively, if for large m_λ, $\mu^2 > 0$ and if Γ grows rapidly with m_λ, then the branched structure survives. Note that, at least in this model, the N vacua reflect an approximate Z_N symmetry which survives in the limit.