Interpretation of the semi-classical stress-energy tensor for a Schwarzschild black hole

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The context

The focus of this talk is on large Schwarzschild black holes with mass $M \gg m_{\mathrm{p}}$, m_{p} the Planck mass, formed by spherically symmetric gravitational collapse in an asymptotically flat spacetime, such that in the distant past the spacetime is approximately Minkowskian. The collapse leads to the formation of trapped surfaces and an *apparent* horizon at $r \cong 2M$ that persists for many dynamical times. There may or may not be an *event* horizon.

Quantum fields are considered as small perturbations on a classical background spacetime, whose metric is a solution of the classical Einstein equations. There is every reason to think that this semi-classical approximation is extraordinarily well justified for the black holes known to exist astrophysically, for which the quantum corrections are of order $m_{\mathrm{p}}^2 / M^2 < 10^{-76}$ for $M > 1 M_{\odot}$. The semi-classical approximation does break down in the deep interior of the black hole, and will eventually break down around the horizon as the black hole evaporates.

My units are $G = c = 1$, $\hbar = m_{\mathrm{p}}^2$.

Outline

1. Review of calculations of semi-classical stress-energy tensor (SCSET) for Schwarzschild black holes from the 1980's and 1990's.
   - A. assumptions and methods
   - B. fits to numerical results
   - C. physical interpretation -where is Hawking radiation created?
   - D. backreaction

2. Implications for the black hole information paradox.
   - A. black hole horizons versus Rindler horizons
   - B. storage of quantum information in a "stretched horizon"?
   - C. entanglement entropy versus Bekenstein-Hawking entropy
   - D. can firewalls exist?
   - E. Resolutions?

The semi-classical energy-momentum tensor

Analytic estimates and numerical results are known for conformally coupled spin 0 and spin 1 fields on a Schwarzschild background *outside* the horizon at $r = 2M$.

The four independent physical components in a static frame are:

energy density $E = -T_t^t$, energy flux $F = -(1-2M/r)^{-1} T_t^r$,

radial stress $P_r = T_r^r$, transverse stress $P_t = T_\theta^\theta = T_\varphi^\varphi$.

Conservation equations $\nabla_\nu T_\mu^\nu = 0$, assuming no t-dependence:

energy conservation $\partial_r \left[r^2 (1-2M/r) F \right] = 0,$

momentum conservation $(E+P_r)\dfrac{M/r}{1-2M/r} + \dfrac{1}{r}\partial_r\left(r^2 P_r\right) - 2P_t = 0.$

The trace anomaly arising from the renormalization of classically conformally invariant quantum fields is (with $x \equiv 2M/r$)

$$T_\mu^\mu = \sum_s q_s \frac{m_{\mathrm{p}}^2}{2880\pi^2}\left(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = 48\frac{M^2}{r^6}\right) = 96 q_s \left(\frac{2}{3}\sigma T_{\mathrm{H}}^4\right) x^6,$$

$q_0 = 1$, $q_1 = -13$, $q_2 = 212$, for Ricci-flat spacetimes.

Applying the conservation laws

Hawking energy flux: $F = \frac{L_{\mathrm{H}}}{4\pi r^2}(1-x)^{-1} = \frac{3}{8}k_s\left(\frac{2}{3}\sigma T_{\mathrm{H}}^4 \equiv P_0\right)\frac{x^2}{1-x}$,

$T_{\mathrm{H}} = \frac{m_{\mathrm{p}}^2}{8\pi M}$, $k_0 = 14.36$, $k_1 = 6.4924$, $k_2 = 0.7404$. The singularity on the horizon is because the static frame moves outward at light speed relative to any inertial frame there, infinitely blueshifting ingoing radiation (and infinitely redshifting outgoing radiation).

Momentum conservation:

A particular solution for radially streaming radiation is $E = P_r \propto x^2/(1-x)$, $P_t = 0$. For the SCSET to be nonsingular in a falling frame at the horizon, the singular components must be purely ingoing, $E^{\mathrm{sing}} = P_r^{\mathrm{sing}} = -F$.

The "regular" parts $E^{\mathrm{reg}} = E + F$ and $P_r^{\mathrm{reg}} = P_r + F$ satisfy

$$\left(P_r^{\mathrm{reg}} + P_t - \frac{1}{2}T^{\mu}_{\mu}\right)\frac{x}{1-x} - x\frac{dP_r^{\mathrm{reg}}}{dx} + 2P_r^{\mathrm{reg}} - 2P_t = 0.$$

Solve for P_r^{reg} given $P_t(x)$ from numerically calculated data.

A simple ansatz is that P_t is a polynomial of the form $P_t = h_s P_0 \sum_{n=0}^{N} t_n x^n$,

where h_s is the number of helicities for spin s, 1 for spin 0 and 2 for spin 1. Terms of the form $x^n \log(x)/(1-x)$ would also have been consistent with regularity at $x=1$.

For a polynomial P_t, and using the trace anomaly, the solution of the momentum conservation equation for P_r^{reg} is:

$$P_r^{\text{reg}} = h_s P_0 \left[\begin{array}{l} t_0 + 2(t_1 - t_0)x + (3t_1 - 2t_2)x^2 \dfrac{\log(x)}{1-x} + \dfrac{48}{5}\dfrac{q_s}{h_s}\sum_{n=2}^{6} x^n \\ + \sum_{n=3}^{N} \dfrac{2}{n-2} x^2 \left(1 + x + \ldots + x^{n-3}\right) t_n - \sum_{n=2}^{N} \dfrac{3}{n-1} x^2 \left(1 + x + \ldots + x^{n-2}\right) t_n \end{array} \right].$$

If $t_2 = 3t_1/2$, then $P_r^{\text{reg}} = h_s P_0 \sum_{n=0}^{N} r_n x^n$, and

$$r_2 = \frac{48}{5}\frac{q_s}{h_s} - 3t_2 - \sum_{n=3}^{N} \frac{n-4}{(n-1)(n-2)} t_n .$$

A known value of r_2 is a constraint on P_t, and vice versa. For $N=6$,

$$t_6 = -10 r_2 + 96(q_s/h_s) - 30t_2 + 5t_3 - 5t_5/6 .$$

Hartle-Hawking (HH) state

The HH state is a static thermal state for an eternal black hole, with zero energy flux in the static frame everywhere. At large r the SCSET approaches that of thermal radiation in equilibrium in the gravitational field, with a temperature equal to the Hawking termperature. The SCSET is non-singular on the horizon, and as $r \to \infty$,

$$P_{\mathrm{t}} \simeq P_{\mathrm{r}} \simeq E/3 \simeq h_s P_0 (1-x)^{-2} = h_s P_0 \left(1 + 2x + 3x^2 + \ldots\right).$$

Deviation from the thermal expansion and departures from pressure isotropy may occur at order x^3, due to geodesic deviation associated with the background curvature, but it should hold at lower orders, with $t_0 = 1$, $t_1 = 2$, $t_2 = 3$, and the same for the r_n. The value $r_2 = 3$ constrains the higher-order t_n.

While not appropriate for isolated black holes formed by gravitational collapse, the HH state simplifies the renormalization calculations of the SCSET tangential stress, and is an important intermediate step in most calculations of the SCSET for the Unruh state.

The Unruh state

The Unruh state is the state that evolves from the Minkowski vacuum in the distant past, before the collapse that formed the black hole. After the transient effects of the collapse have dissipated, the SCSET becomes approximately stationary. There is no incoming radiation at past null infinity and the SCSET is regular in a *falling* frame at the (future) horizon. At future null infinity there is outgoing radiation with the Hawking luminosity for each field of spin s,

$$L_{\mathrm{H}} = 4\pi k_s M^2 \sigma T_{\mathrm{H}}^4, \quad \sigma = \frac{\pi^2}{60 m_{\mathrm{p}}^6}, \quad T_{\mathrm{H}} = \frac{m_{\mathrm{p}}^2}{8\pi M}.$$

The Hawking energy flux

$$F = \frac{1}{4} k_s \sigma T_{\mathrm{H}}^4 \frac{x^2}{1-x} = \frac{3}{8} k_s P_0 \frac{x^2}{1-x}$$

is an outward flow of positive energy at large r, so

$$E, P_r \to +F \Rightarrow E^{\mathrm{reg}}, P_r^{\mathrm{reg}} \to 2F \Rightarrow r_2 = \frac{3}{4}\frac{k_s}{h_s}.$$

The tangential stress has $t_0 = t_1 = t_2 = 0$, and $r_2 = 10.77$ for spin 0, 2.435 for spin 1, constraining the nonzero t_n.

Results

Analytic approximations:

For spin 0 HH, Page (1982):

$$P_t = P_0\left(1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 - 9x^6\right).$$

For spin 1 HH, Jensen and Ottewill (1989):

$$P_t = 2P_0\left(1 + 2x + 3x^2 + 44x^3 - 305x^4 + 66x^5 - 579x^6\right).$$

For spin 0 and spin 1 Unruh, Matyjasek(1998):

$$P_t = P_0\left(8.75x^4 + 6x^5 - 9x^6\right),\ \ 2P_0\left(-119.5x^4 - 83.3x^5 - 579x^6\right).$$

Numerical results based on point-splitting renormalization:

For spin 0 HH, Howard (1984) and Anderson, et al (1993).
For spin 1 HH, Jensen and Ottewill (1989).
For spin 0 and spin 1 Unruh, Jensen, et al (1991).
Table of HH and Unruh spin 0 data preserved in Visser (1997).

Polynomial fits to numerical results:

For spin 0 HH, $N = 8$ Matyjasek (1996) and $N = 6$ Bardeen.
For spin 0 Unruh, $N = 6$ Visser (1997) and $N = 6$ Bardeen..
For spin 1 HH, $N = 6$, and Unruh, $N = 7$, Bardeen.

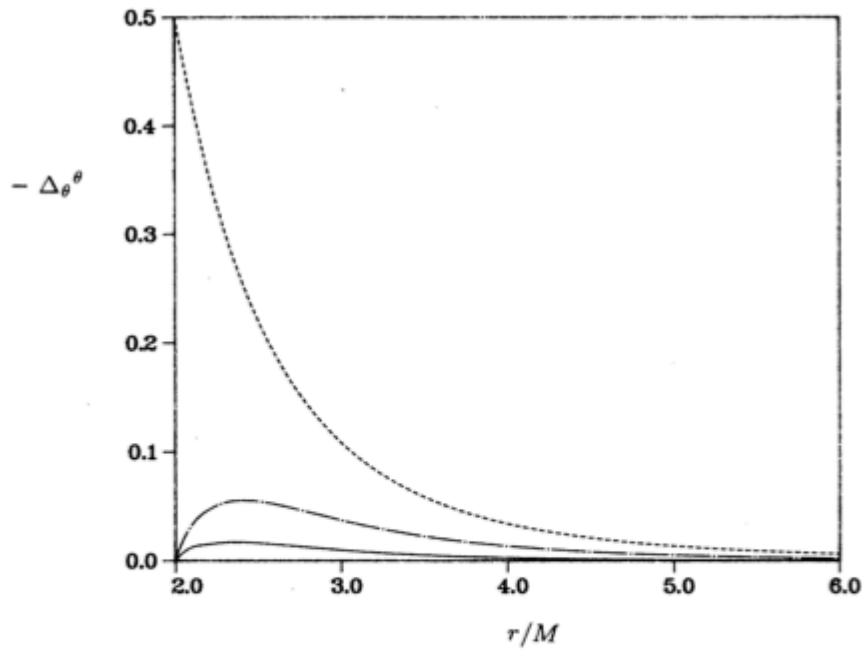


FIG. 4. $\Delta_\theta{}^\theta = (\pi^2/\kappa^4)(\langle T_\theta{}^\theta\rangle - \langle T_\theta{}^\theta\rangle_{\text{approx}})$ for $\langle T_\theta{}^\theta\rangle_{\text{approx}} = \langle T_\theta{}^\theta\rangle_{\text{analytic}}$ (solid curve), $\langle T_\theta{}^\theta\rangle_{\text{BOP}}$ (dashed curve), and $\langle T_\theta{}^\theta\rangle_{\text{FZ}}$ (dashed-dotted curve).

Fits

Spin 0 HH (Bardeen), max error ~ 0.07%,

$$P_{\mathrm{t}} = P_0\left(1 + 2x + 3x^2 + 3.650x^3 + 14.398x^4 - 48.170x^5 + 34.392x^6\right).$$

Spin 0 Unruh (Visser), max error ~ 0.7%,

$$P_{\mathrm{t}} = P_0\left(26.5652x^4 - 59.0214x^5 + 38.2068x^6\right),$$

Spin 0 Unruh (Bardeen), max error ~0.2%,

$$P_t = P_0\left(0.264x^3 + 25.438x^4 - 57.460x^5 + 37.503x^6\right).$$

Spin 1 HA - H (Bardeen), max error ~ 5%,

$$-\Delta_\theta^\theta = 2P_0\left[6.89x^3 - 106.20x^4 + 389.17x^5 - 289.86x^6\right],$$

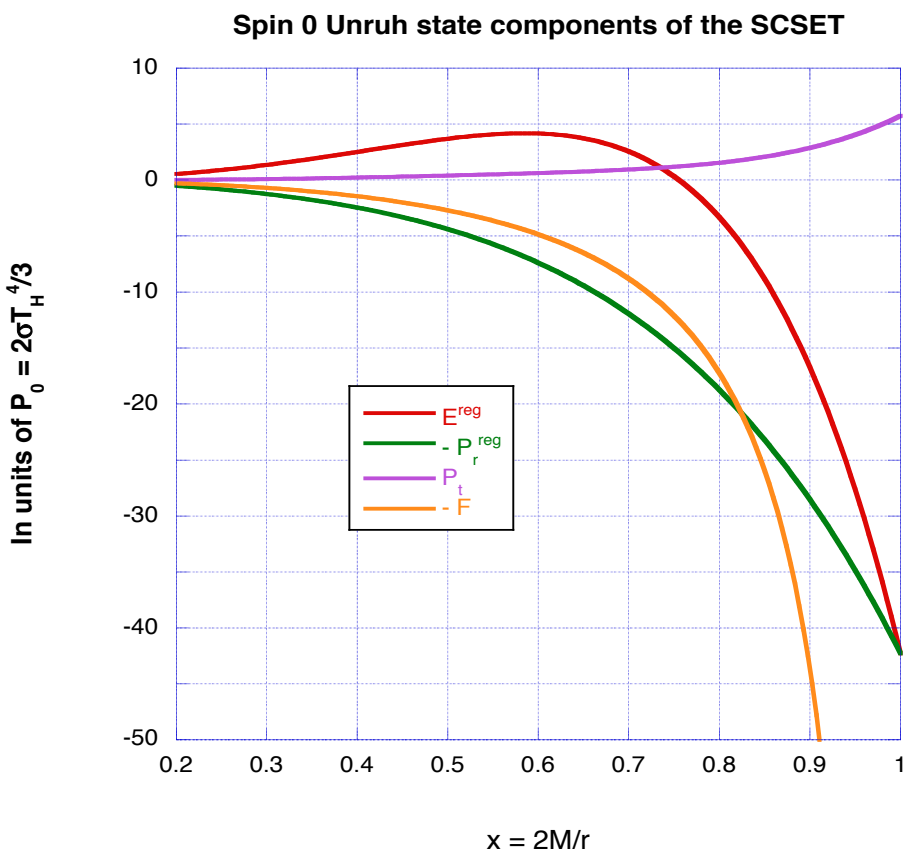
Spin 1 HH (Bardeen), max error ~0.5%,

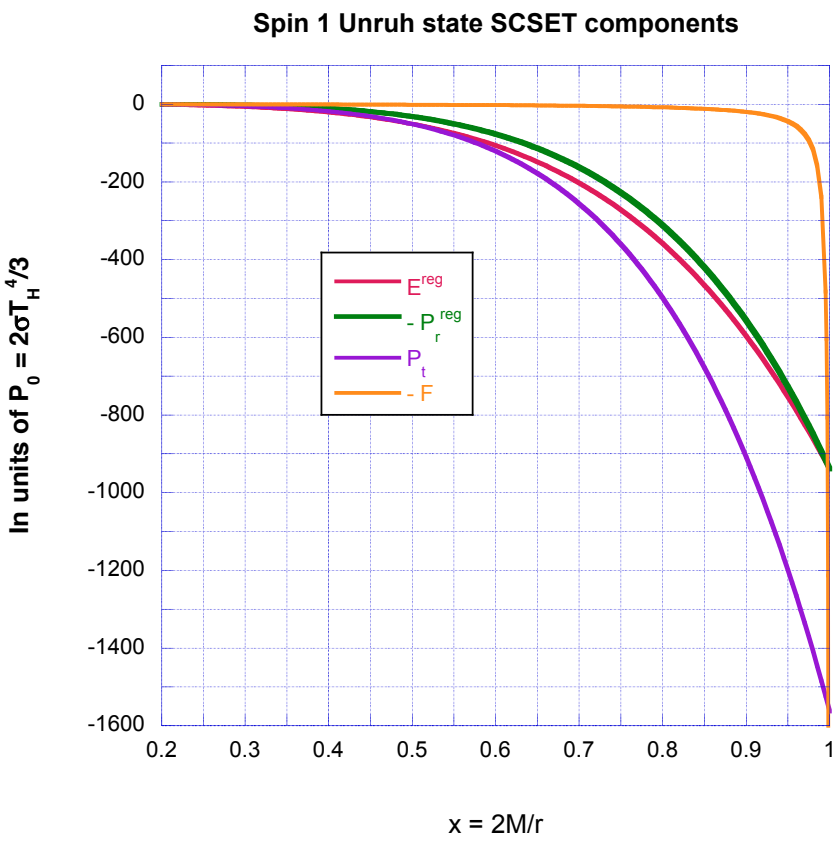
$$P_t = 2P_0\left(1 + 2x + 3x^2 + 37.11x^3 - 198.80x^4 - 323.17x^5 - 289.14x^6\right).$$

Spin 1 Unruh (Bardeen), max error ~ 1%,

$$P_t = 2P_0\left(57.31x^3 - 593.12x^4 + 696.98x^5 - 1268.71x^6 + 326.10x^7\right),$$

$$P_r^{\mathrm{reg}} = 2P_0\begin{pmatrix} 2.435x^2 - 112.185x^3 + 566.90x^4 \\ \qquad - 490.87x^5 + 666.21x^6 - 326.10x^7 \end{pmatrix}.$$


Spin 0 Unruh state components of the SCSET
10
0
-10
-20
-30
-40
-50
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x = 2M/r
In units of $P_0 = 2\sigma T_H^4/3$
E^{reg}
$-P_r^{reg}$
P_t
$-F$


Spin 1 Unruh state SCSET components
0
-200
-400
-600
-800
-1000
-1200
-1400
-1600
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
In units of $P_0 = 2\sigma T_H^4/3$
x = 2M/r
E^{reg}
$-P_r^{reg}$
P_t
$-F$

Interpretation

An often-expressed mythology about Hawking radiation is that it is created very close to the horizon by pair creation or tunneling. A Hawking partner with negative Killing energy is left inside the horizon, decreasing the mass of the black hole. If so, positive energy should propagate outward from close to the horizon. This is inconsistent with the Unruh state SCSET.

Define the "outward" part of the energy flux by

$$F^{\text{out}} = \frac{1}{4}\left(E + P_r + 2F\right) = \frac{1}{4}\left(E^{\text{reg}} + P_r^{\text{reg}}\right) \equiv \frac{1}{2}(1-x)Z_s(x).$$

For spin 0,

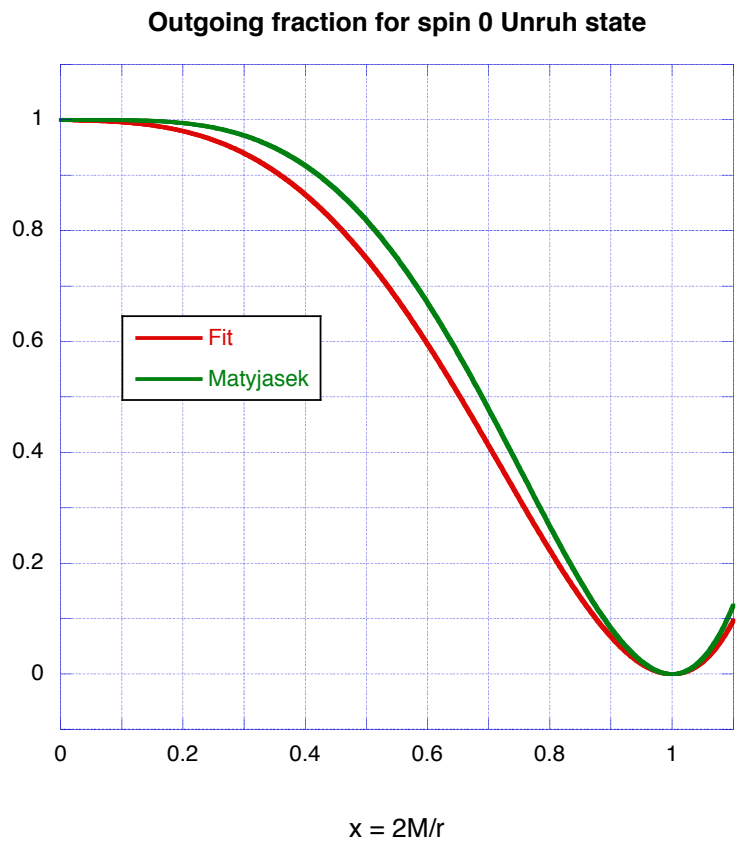
$$Z_0 = P_0\left(10.77x^2 + 21.276x^3 + 31.914x^4 + 23.399x^5\right).$$

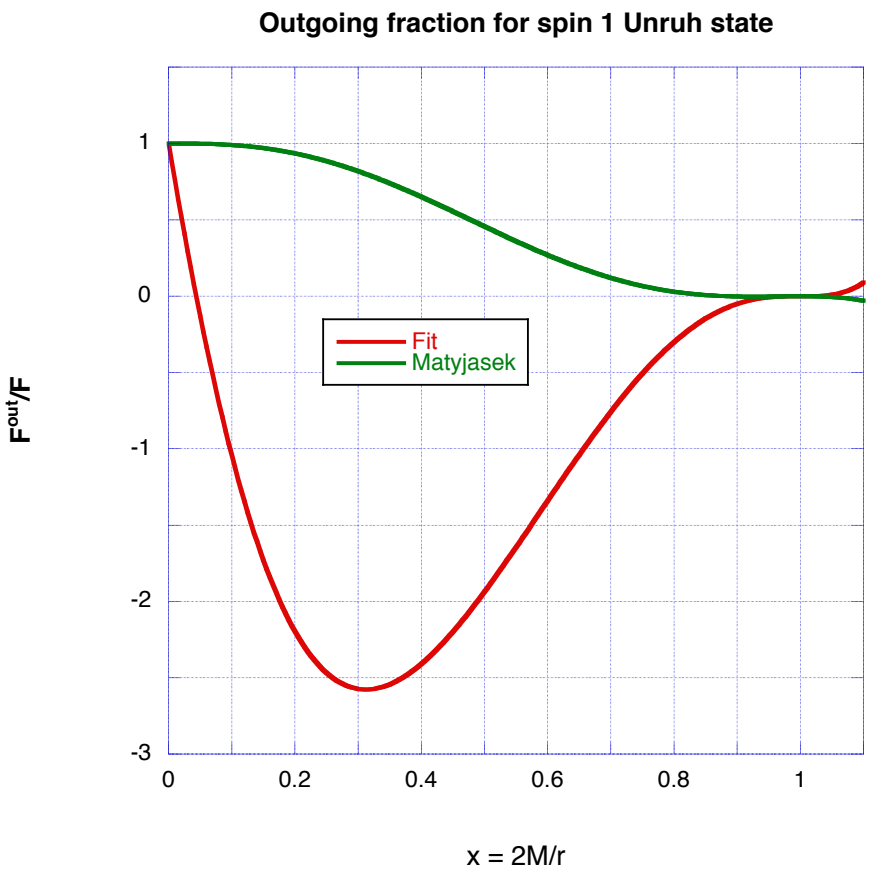
The fraction of the net flux that is outgoing is

$$F^{\text{out}} / F = (1-x)^2 Z_0 / \left(r_2 x^2\right),$$

which decreases monotonically from 1 at $x = 0$ to 0 at $x = 1$.

The alternative, as argued originally by Unruh (1977) and Fulling (1977), is that the Hawking radiation originates from vacuum fluctuations well outside the horizon, in the general vicinity of $r = 3M$.


Outgoing fraction for spin 0 Unruh state
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
Fit
Matyjasek
x = 2M/r


Outgoing fraction for spin 1 Unruh state
1
0
-1
-2
-3
0
0.2
0.4
0.6
0.8
1
Fit
Matyjasek
F^out/F
x = 2M/r

The SCSET in the free-fall frame

The velocity of a FF frame falling from rest at large r, relative to the static frame, is $v=-\sqrt{x}$. The Lorentz transformation to the FF frame gives

$$E^{\mathrm{FF}}=\frac{E+P_r v^2-2vF}{1-v^2}=2Z_s-P_r^{\mathrm{reg}}-\frac{(1-x)F}{\left(1+\sqrt{x}\right)^2},$$

$$P_r^{\mathrm{FF}}=\frac{Ev^2+P_r-2vF}{1-v^2}=2xZ_s+P_r^{\mathrm{reg}}-\frac{(1-x)F}{\left(1+\sqrt{x}\right)^2},$$

$$F^{\mathrm{FF}}=\frac{-v\left(E+P_r\right)+\left(1+v^2\right)F}{1-v^2}=2\sqrt{x}Z_s+\frac{(1-x)F}{\left(1+\sqrt{x}\right)^2}.$$

What if I had defined $E^{\mathrm{reg}}=E-F$ and $P_r^{\mathrm{reg}}=P_r-F$? This has no effect on the form of the momentum conservation equation relating P_r^{reg} to P_t. The difference is that now the "singular" part of the SCSET is an outgoing flow of positive energy and by itself contains the asymptotic Hawking radiation, so $r_2=0$. The only changes in the Lorentz transformation to the FF frame are the signs of some terms and replacing $1+\sqrt{x}$ by $1-\sqrt{x}$. Why isn't this a good model for Hawking radiation being generated very close to the horizon?

Hawking radiation is not created close to the horizon

The formal singularity of this alternate SCSET in a falling frame at $x = 1$ is not the primary issue. Close to is not "at", and the divergence would be cut off somewhere around a Planck distance from the apparent horizon.

As noted earlier, the value of r_2 is a constraint on the t_n. The spin 0 value $r_2 = 10.77$ was consistent with an excellent fit to the numerical results for P_t. Letting r_2 be an additional free parameter in the fit gives $r_2 = 10.8 \pm 0.3$, which is not at all consistent with the zero value required if the outgoing flow of positive energy starts close to the horizon.

The most compelling reason is that there is no energy source for the very large energy the Hawking particle has to have in a FF frame close to the horizon to compensate for the redshift factor of order $(1-x)^2$ in propagating out to large r. There is no gravitational potential energy in a FF frame, so the fact that the Killing energy may be small is irrelevant. The Hawking partner must have a correspondingly large *positive* energy in the FF frame, even though it is inside the apparent horizon. Furthermore, to have negative Killing energy the partner, like Hawking particle, must be propagating *outward* in the local FF frame, which violates conservation of momentum as well as conservation of energy.

What of the many calculations that associate outgoing modes just outside the apparent horizon with outgoing modes just inside the apparent horizon to derive the existence and spectrum of the Hawking radiation? Visser (Int. J. Mod. Phys., 2003) has given a clear discussion of this kind of derivation using Painleve-Gullstrand coordinates. In a WKB approximation, the phase of an outgoing mode of frequency ω is $\mp i\left(\omega t - \int^{r} k(r')dr'\right)$, where near the horizon $k \approx \omega / \left[\kappa\left(r - r_H\right) + i\varepsilon\right]$, κ the surface gravity of the horizon. Any physical excitation is a wave packet, integrated over a range of frequencies. The rapid oscillation of phase near the horizon means any such wave packet will have very small amplitude close to the horizon due to destructive interference between neighboring frequencies.

The proper interpretation of a quantum field in the Unruh state is that close to the apparent horizon the fluctuations associated with outgoing modes are just vacuum fluctuations, carrying no energy. It is only well outside the horizon that they take on the character of physical particles. The "outgoing" modes inside the horizon also are vacuum fluctuations near the horizon and may (or may not) become Hawking "partners" as they propagate deeper into the interior. It is ingoing vacuum fluctuations that carry negative energy into the black hole (Unruh (1977), Fulling (1977)).

Backreaction

The general spherically symmetric metric in advanced Eddington-Finkelstein coordinates is

$$ds^2 = -e^{2\psi}(1-2m/r)dv^2 + 2e^{\psi}dvdr + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

The inverse metric has

$$g^{vv} = 0, \quad g^{vr} = e^{-\psi}, \quad g^{rr} = 1-2m/r.$$

The stress-energy tensor in terms of the static frame components is

$$T_v^v = -E - F = -E^{\text{reg}}, \quad T_r^r = P_r + F = P_r^{\text{reg}},$$

$$T_v^r = -(1-x)F = -\frac{3}{8}k_s P_0 x^2, \quad T_\theta^\theta = T_\varphi^\varphi = P_t,$$

$$T_r^v = (1-x)^{-1}(E + P_r + 2F) = (1-x)^{-1}\left(E^{\text{reg}} + P_r^{\text{reg}}\right) = 2Z_s.$$

The Einstein equations give, with the initial value equations

$(\partial m/\partial r)_v = -4\pi r^2 T_v^v = 4\pi r^2 E^{\text{reg}}$ and

$\left(\partial e^{-\psi}/\partial r\right)_v = -4\pi r T_r^v = -8\pi r Z_s$ plus the evolution equation

$$(\partial m/\partial v)_r = -(3/2)\pi k_s (2M)^2 P_0.$$

The fact that $(\partial m/\partial v)_r$ is independent of r means that as long as $|\psi| \ll 1$ for all $r \geq 2m$ the geometry remains Schwarzschild to an excellent approximation. The boundary condition for ψ is that $\psi \to 0$ as $r \to \infty$, so roughly $\psi \approx -\log(r_c/r)(2M)^2 P_0$, where r_c is the radius of the future light cone from the formation event.

A black hole information paradox?

The properties of black hole event horizons were derived from classical general relativity. They rely on energy conditions that are violated by the SCSET. While the existence of trapped surfaces and apparent horizons is robust, there are no compelling reasons to think that quantum black holes have an event horizon or interior singularities.

An observer uniformly accelerating forever in Minkowski spacetime has an event horizon, called a Rindler horizon. A Rindler observer will lose track of quantum information crossing his event horizon, but this does not mean the information is destroyed or that there is any violation of unitarity. Locally, the horizon of a quiescent black hole is indistinguishable from a Rindler horizon.

Information is really lost in the case of black holes only if there is a singularity in the interior that prevents indefinite evolution of quantum fields entangled with degrees of freedom outside the black hole. Whether quantum gravity allows such a singularity to exist is an open question.

But what if the black hole evaporates completely, leaving no remnant?

Can quantum information be stored in a "stretched horizon"?

The stretched horizon is a concept from *classical* general relativity to describe how black hole perturbations decay and how a black hole interacts with fields due to external sources. There is no reason to interpret this decay as "scrambling" or "thermalization" that somehow preserves quantum information.

The location of the black hole event horizon, it it exists, is only knowable precisely if one knows the entire future history of the black hole. The apparent horizon is a quasi-local quantity, but is a spacelike hypersurface for an accreting black hole . Quantum information at the apparent horizon ends up well inside later apparent horizons, in general. For a quiescent black hole, what propagates along the horizon are just vacuum fluctuations.

Can these vacuum fluctuations carry at least some quantum information? Hawking, Perry, and Strominger have argued "yes". Infalling disturbances do perturb affine parameters along the null generators of the event horizon. However, the same is true for neighboring null hypersurfaces that go off to infinity or into the black hole. They can't all be carrying the same quantum information, and there is no way to know which is the actual event horizon without knowing the black hole's future history.

Entropies

What is the entropy of a black hole? A black hole is not a conventional quantum system, and a fairly young black hole is really just a rather empty region of spacetime. It has an entanglement (von Neumann) entropy S_{vN} , renormalized so as not to include the short-range correlations of the vacuum across the horizon. The entanglement entropy is the number of entangled degrees of freedom involved in the initial collapse and any subsequent accretion, plus the trapped vacuum fluctuations or Hawking "partners" entangled with qbits of the emitted Hawking radiation.

The Bekenstein-Hawking entropy is $S_{\mathrm{BH}} = A/(4\hbar) = 4\pi\left(M^2/m_{\mathrm{p}}^2\right)$, where A is the area of the apparent horizon. As the black hole evaporates, S_{vN} increases and S_{BH} decreases. They become equal at the Page time, when the black hole has lost about 1/2 of its original mass. While technically the semi-classical approximation is no longer valid at that point, since the accumulated backreaction is no longer small, there is no obvious reason for the evaporation to stop. Continuing evaporation violates the entropy bound unless subsequent Hawking particles are entangled with earlier Hawking particles. This seems impossible unless quantum information can be transferred faster than light, or quantum information can be cloned. Either is a serious violation of conventional quantum mechanics.

Possible resolutions?

1.) Evaporation continues, with quantum information remaining trapped, until the black hole approaches the Planck scale, when it dissolves into vacuum fluctuations containing all the trapped information. Unruh and Wald have argued that there are no compelling reasons to reject this scenario, though theirs is a minority opinion.

2) The entropy bound is enforced by quantum gravitational backreaction that never lets the amount of entanglement across a 2-surface exceed one qubit per Planck area. This is suggested by the Bousso covariant entropy bound as applied in the interior of the black hole, assuming there is no $r = 0$ singularity. The quantum information trapped by the black hole ends up within an inner apparent horizon, whose radius grows with the increase in entanglement entropy. By the Page time the inner apparent horizon merges with the outer apparent horizon, resulting either in a degenerate horizon with zero Hawking temperature or in the disappearence of all horizons. I suggested a toy model of this type in arXiv:1406.4098.

3) A phase transition to a "fuzzball" (Mathur) or to a "bose condensate of gravitons" (Dvali), or some other breakdown of a spacetime described by GR, occurring perhaps around the Page time. It does not seem plausible that such a breakdown would occur in a "young", i.e., any astrophysical, black hole.

Loose ends

Adam Levi and Amos Ori have recently embarked on a program to greatly improve the accuracy and range of calculations of SCSETs for black holes. They extend the point-splitting renormalization technique from the separations in t of earlier calculations to separations in the θ and φ directions, in order to accommodate axisymmetric and evolving geometries. Preliminary results, for a minimally coupled scalar field in Schwarzschild, were presented in arXiv:1608.03806. They confirm the need for an asymptotic x^3 falloff of the tangential stress by calculating out to $r = 40M$, and calculate directly all components of the SCSET to confirm the ingoing flow of negative energy at the horizon.

Find a reasonable anomaly effective action, which can be used to find an anomaly stress-energy tensor. Earlier proposals by Brown and Ottewill (1985), by Mottola and Vaulin (2006), and by Mottola (2016) have severe deficiencies.

Can the ER=EPR proposal of Susskind, be applied to entanglement of individual qbits between the interior of the black holt and the exterior? If so, it may provide a mechanism for large backreaction on the macroscopic geometry as the density of Planck scale Einstein-Rosen bridges crossing a two-surface approaches one per Planck area.

Develop a complete theory of quantum gravity.