Semiclassical Gravitational Collapse in $(N+1)$ Dimensions

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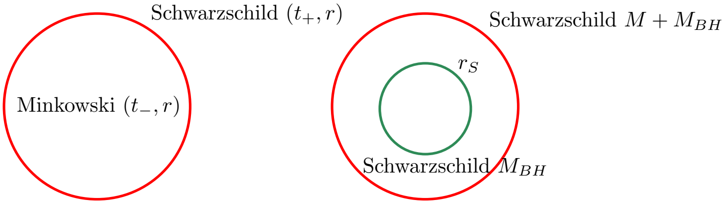
Outline

Review of *Semiclassical S-matrix for black holes*

$(N+1)$ dimensional extension
- Model
- Reflection probability and time delay for the shell

F. Bezrukov, D. Levkov, S. Sibiryakov, *Semiclassical S-matrix for black holes*. JHEP, 2015.

Spacetime picture



Why do we need such a method?

- Semiclassical treatment of Hawking radiation.
- Black holes are stable asymptotic states of classical gravity, evaporation of the black hole is not taken into account.

Modified semiclassical method

$$\langle\Psi_f|S|\Psi_i\rangle = \int \mathcal{D}\Phi\ e^{i\mathcal{S}_{tot}[\Phi]}$$

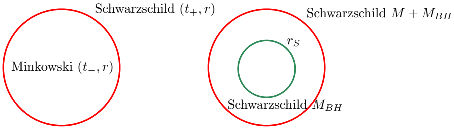
$$= \int \mathcal{D}\Phi\ e^{i\mathcal{S}_{tot}[\Phi]} \int dT_0\ \delta(T_{int}[\Phi] - T_0)$$

$$= \int \frac{dT_0 d\epsilon}{2\pi i} e^{\epsilon T_0} \int \mathcal{D}\Phi\ e^{i(\mathcal{S}_{tot}[\Phi] + i\epsilon T_{int}[\Phi])}$$

Extension to arbitrary spacetime dimensions

- Application to higher dimensional theories.
- Probing of the Bekenstein-Hawking entropy and Hawking temperature in higher dimensions.
- Simpler expressions due to disappearence of some classical divergences.

Physical situation



$$ds^2 = -f_\pm(r)dt_\pm^2 + \frac{dr^2}{f_\pm(r)} + r^2 d\Omega_{N-1}^2$$

$$f_-(r) = 1$$

$$f_+(r) = 1 - \frac{2\omega_{N-1}M}{r^{N-2}}$$

$$f_-(r) = 1 - \frac{2\omega_{N-1}M_{BH}}{r^{N-2}}$$

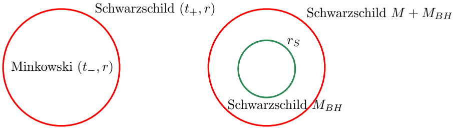
$$f_+(r) = 1 - \frac{2\omega_{N-1}(M+M_{BH})}{r^{N-2}}$$

where $\omega_{N-1} = \frac{8\pi G}{(N-1)\Omega_{N-1}}$ and $\Omega_{N-1} = \frac{2\pi^{N/2}}{\Gamma(\frac{N}{2})}$

Tangherlini, *Schwarzschild field in n dimensions and the dimensionality of space problem*, Nuovo Cim. 27, 1963

Model

Physical situation



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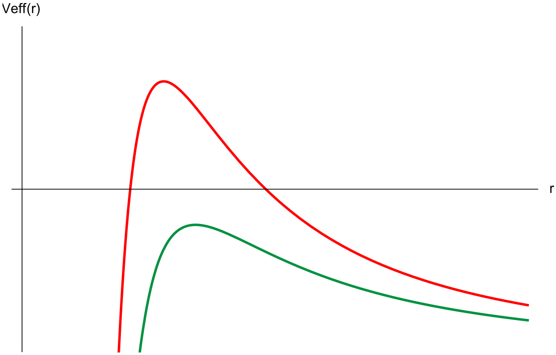
Equations of motion for the shell

$$\dot{r}^2 + V_{eff}(r) = 0$$

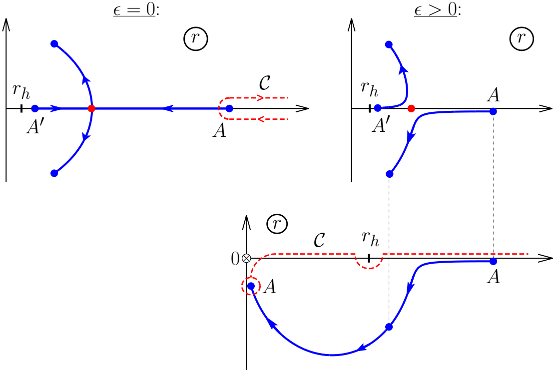
$$V_{eff}(r) = \frac{f_+ + f_-}{2} - \frac{r^{2N-4}}{\omega_{N-1}^2 m_{eff}^2}\left(\frac{f_+ - f_-}{2}\right)^2 - \frac{\omega_{N-1}^2 m_{eff}^2}{4r^{2N-4}}$$

$$m_{eff}^2 = m^2 + \frac{L^2}{r^2}$$

Effective potential for the shell



Motion of the turning points



F. Bezrukov, D. Levkov, S. Sibiryakov, *Semiclassical S-matrix for black holes*. JHEP, 2015.

Total action

$$\mathcal{S}_{tot} = \underbrace{\mathcal{S}_{shell} + \mathcal{S}_{EH} + \mathcal{S}_{GH}}_{\mathcal{S}(t_i, t_f)} + \mathcal{S}_0(0^-, t_i) + \mathcal{S}_0(t_f, 0^+) - i \log \Psi_i - i \log \Psi_f^*$$

Model

Case of a collapsing shell:

$$\mathcal{S}_{tot}|_{m=L=0} = \frac{N-2}{N-1}\int_{\mathcal{C}} dr \frac{M}{f_+(r)} - M\left[\frac{N-2}{N-1}(r_i + r_f) - 2r_0\right]$$

Case of a shell collapsing on a black hole:

$$\mathcal{S}_{tot}|_{m=L=0} = \frac{N-2}{N-1}\int_{\mathcal{C}} dr \left[\frac{M_{BH}+M}{f_+(r)} - \frac{M_{BH}}{f_-(r)}\right] - M\left[\frac{N-2}{N-1}(r_i + r_f) - 2r_0\right]$$

Shell reflection probability

$$P_{fi} = \exp\left(-2\mathrm{Im}\mathcal{S}_{tot}\right)$$
$$P_{fi}\big|_{m=L=0} = \exp\left[-2M\frac{N-2}{N-1}\mathrm{Res}\left(\frac{1}{f_+(r)}, r_S\right)\right]$$

Reflection probability and time delay for the shell

Case of a collapsing shell:

$$P_{fi} = \exp\left(-\frac{4\pi M}{N-1} r_S\right) = \exp(-S_{BH})$$

Case of a shell collapsing on a black hole:

$$P_{fi} = \exp\left(-\frac{4\pi(M+M_{BH})}{N-1} r_S^+ + \frac{4\pi M_{BH}}{N-1} r_S^-\right)$$
$$\to \exp\left(-\frac{M}{T_H}\right) \text{ when } M_{BH} \gg M$$

Time delay

$$\Delta t = \mathrm{Re}\frac{\mathrm{d}\mathcal{S}_{tot}}{\mathrm{d}M} - 2(r_0 - r_S)$$

Reflection probability and time delay for the shell

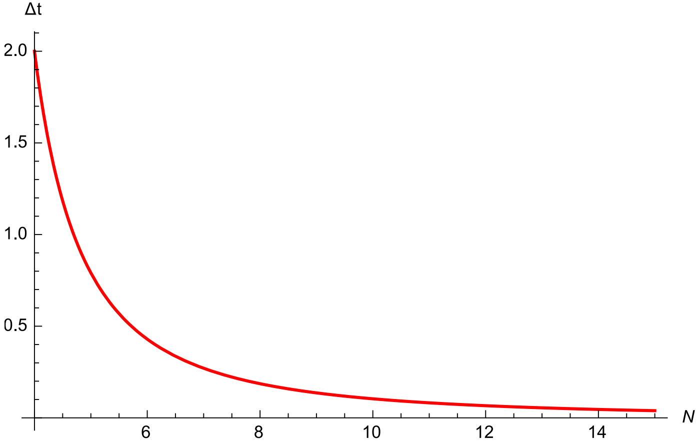
Case of a collapsing shell:

$$\Delta t = 2r_S \left[1 - \frac{\pi}{N-2}\frac{1}{\tan\left(\frac{\pi}{N-2}\right)}\right]$$

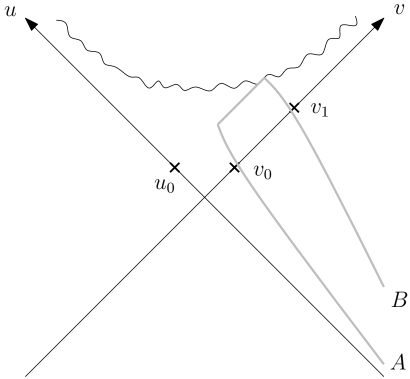
Case of a shell collapsing on a black hole:

$$\Delta t = 2r_S^+ \left[1 - \frac{\pi}{N-2}\frac{1}{\tan\left(\frac{\pi}{N-2}\right)}\right]$$

Reflection probability and time delay for the shell



Gedanken experiment used to compute the scrambling time



Susskind, Thorlacius, *Gedanken experiments involving black holes*, Phys. Rev. D, 1993.

Comparison of the time delay with the scrambling time

When $N \to \infty$,

$$\Delta t = \frac{2r_S}{3}\left(\frac{\pi}{N-2}\right)^2 + O\left(\frac{1}{(N-2)^3}\right)$$

$$t_{scr} = \frac{2r_S}{N-2}\log\left(\frac{r_S}{l_P}\right)$$

To sum up:

- Modify semiclassical approach by constraining path integral.
- Disappearence of some classical logarithmic divergences in the total action for $(N+1) > 4$ spacetime dimensions.
- P_{fi} is exponentially suppressed with an exponent equal to the Bekenstein-Hawking entropy.
- For a shell collapsing on a pre-existing very massive black hole this exponent is proportional to $\frac{1}{T_H}$.
- Δt and t_{scr} are both decreasing functions of the number of dimensions with different power laws.

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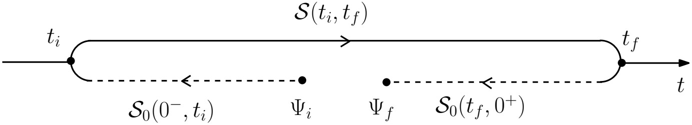
Thank you!

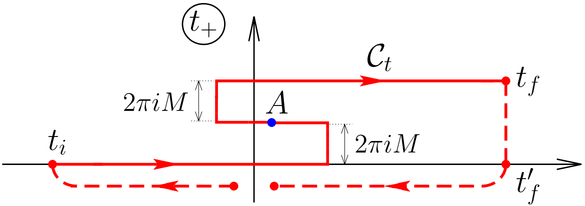
Arnowitt Deser Misner (ADM) mass

$$M_{ADM} = -\frac{1}{8\pi G} \lim_{S_t \to \infty} \oint_{S_t} (k - k_0)\sqrt{\sigma} d^{N-1}x$$

Poisson, *A Relativist's Toolkit, The Mathematics of Black Hole Mechanics, Cambridge University Press, 2004.*

Integration contours





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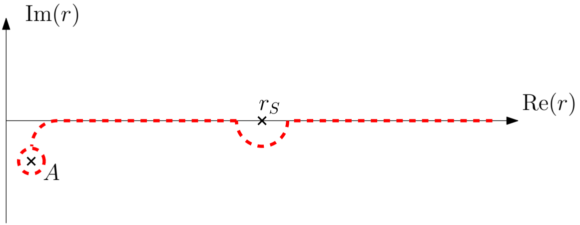

Total action

$$\mathcal{S}_{tot}|_{N=3} = \int_{\mathcal{C}} \frac{dr}{\sqrt{-V_{eff}}}\left[\frac{m^2 - 2m_{eff}^2}{2m_{eff}} + \frac{M}{2}\frac{\sqrt{f_+ - V_{eff}}}{f_+}\right] - \sqrt{M^2 - m^2}\left[\frac{(r_i + r_f)}{2} - 2r_0\right] + \frac{M(2M^2 - m^2)}{2\sqrt{M^2 - m^2}}\left[1 - \log(r_i r_f / r_0^2)\right]$$

$$\mathcal{S}_{tot}|_{N} = \int_{\mathcal{C}} \frac{dr}{\sqrt{-V_{eff}}}\left[\frac{m^2 - (N-1)m_{eff}^2}{(N-1)m_{eff}} + M\frac{N-2}{N-1}\frac{\sqrt{f_+ - V_{eff}}}{f_+}\right] - \sqrt{M^2 - m^2}\left[\frac{N-2}{N-1}(r_i + r_f) - 2r_0\right]$$

$N = 3$: F. Bezrukov, D. Levkov, S. Sibiryakov, *Semiclassical S-matrix for black holes*. JHEP, 2015.

Hawking temperature

$$T_H = \frac{N-2}{4\pi r_S^-}$$