Hilbert Series for Effective Field Theory

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based on 1503.07537, 1510.00372 AM, L.Lehman,
also Henning et al 1512.03433, 1507.07240

Simplicity II, Fermilab, Sept 8th, 2016

Wilsonian picture of field theory

$$\mathcal{L} = \int d^4x \sum_i c_i \, \mathcal{O}_i$$

take all degrees of freedom, form local operators of increasing dimension

all operators consistent with symmetries must be included

lowest mass dimension operators dominate IR physics

SM is a poster child EFT: SMEFT

degrees of freedom are: Q, u^c, d^c, L, e^c, H, gauge fields

symmetry is: Lorentz $\otimes$ $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$

low-dimension operators are easy, but quickly gets more complicated

dim ≤ 4: Standard Model

dim 5: 1 operator (neutrino mass) [Weinberg '79]

dim 6: 63 terms (neglecting flavor) [Büchmuller, Wyler '86, Grzadkowski et al '10]

dim 7: 20 terms [Lehman '14]

dim 8: no complete set known (as of Oct. 2015)

Can this be extended?

1.) to dimension-8?

2.) to all orders?

3.) to other EFTs?

higher dimension operators are complicated because there are more fields = number ways to contract indices grows rapidly!

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Yes, using algebraic technique known as
Hilbert Series

Outline

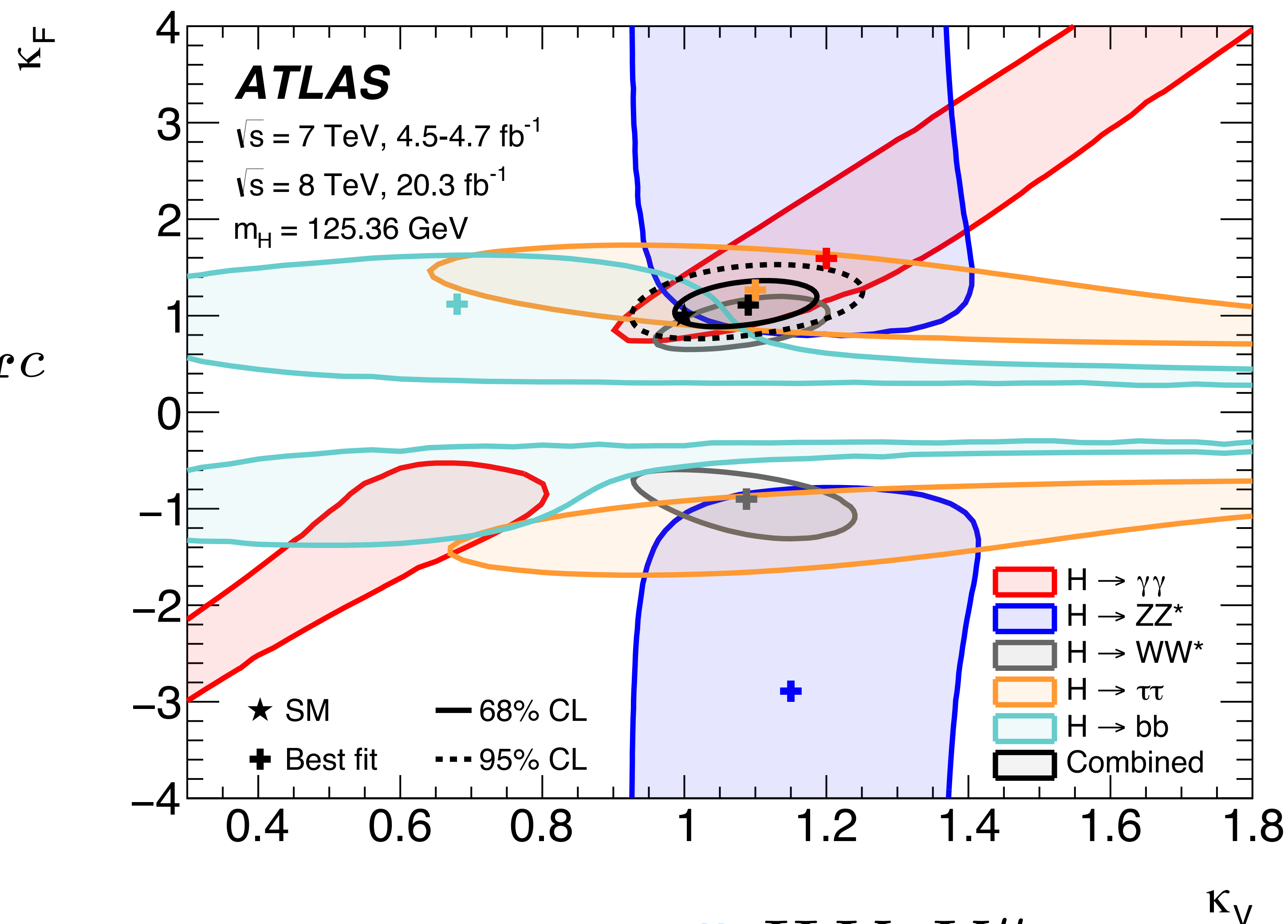
- motivation for d > 6 in the SMEFT
- introduction to Hilbert series, simple example
- towards full SMEFT, no derivatives
- adding derivatives: EOM and IBP troubles
- 'final' form: d = 8,9,10… in SMEFT

Why?

precision: LHC, HL-LHC, etc. will soon test SM to unprecedented precision = sensitivity to effects from even higher dimension

1507.04548v1

$\# \, H \, f_L \, f^c$



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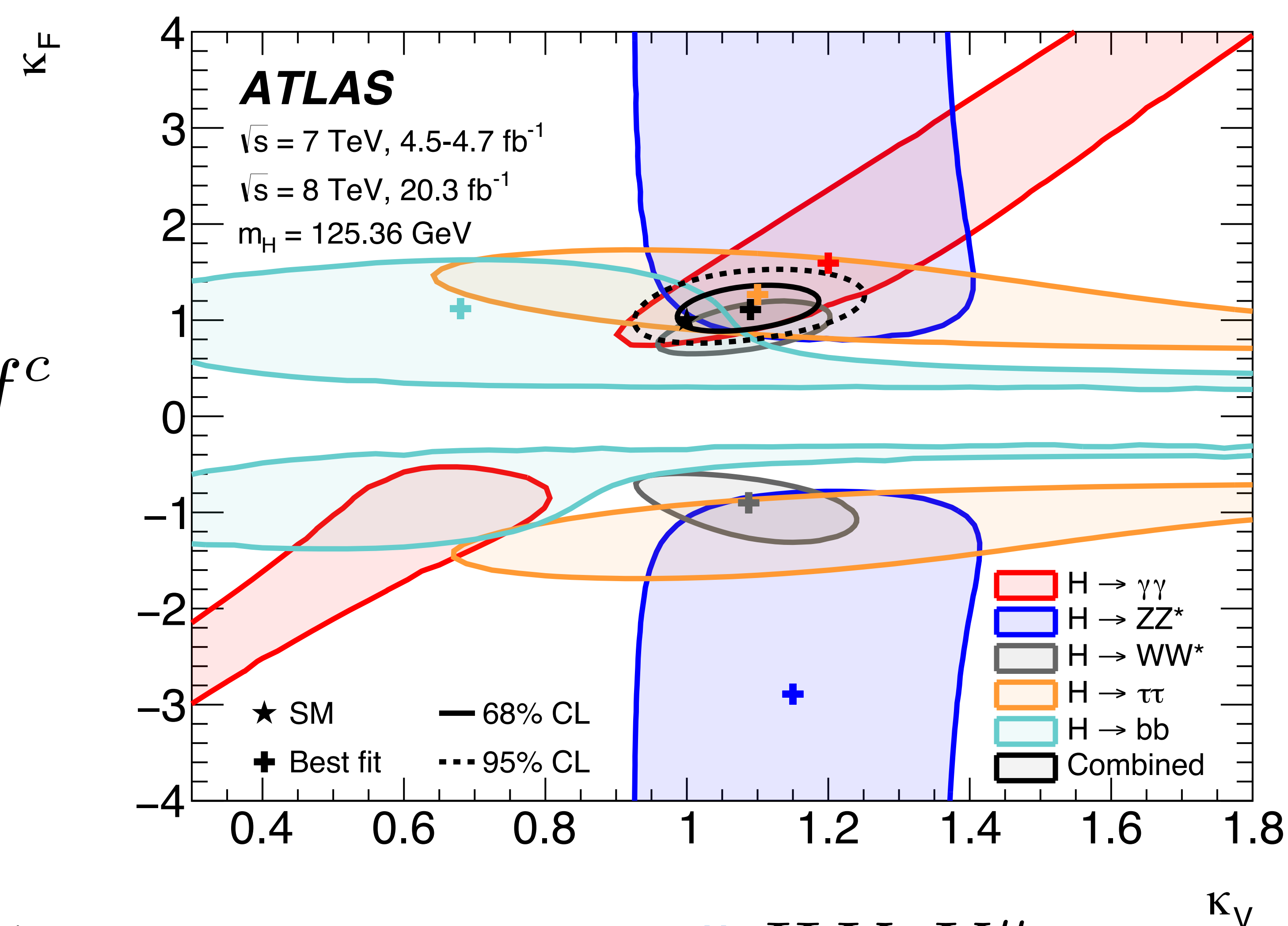
$$\# \; \frac{(H^\dagger H)}{\Lambda^2}\, H f_L f^c$$



$$\# \; \frac{(H^\dagger H)}{\Lambda^2}\, D_\mu H^\dagger D^\mu H$$

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cool

How?

Consider a simple setup: ϕ, ϕ^* with charge +1, -1

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Hilbert series is defined as: $$h = \sum_n \kappa_n \, t^n$$

number of invariants of degree n

invariants

for us:

degree = mass dimension, t = symmetry-invariant operators

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treat ϕ, ϕ* as complex #, modulus 1 rather than quantum fields (call it a `spurion')… then we can formally sum series

$$h_\phi = \frac{1}{1 - (\phi\phi^*)}$$

rewrite

$$h_\phi = \frac{1}{2\pi} \int_0^{2\pi} \frac{d\theta}{(1 - \phi\, e^{i\theta})(1 - \phi^* e^{-i\theta})}$$

change to $z = e^{i\theta}$

$$h_\phi = \frac{1}{2\pi i} \oint_{|z|=1} \frac{dz}{z} \frac{1}{(1 - \phi z)(1 - \frac{\phi^*}{z})}$$

overly complicated for simple example, but will be generalizable to more fields, symmetries

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$$\frac{1}{(1-\phi z)(1-\frac{\phi^*}{z})} = 1 + (\phi\phi^*) + (\phi\phi^*)^2 + (\phi\phi^*)^3 + \cdots$$

$$+z(\phi + \phi(\phi\phi^*) + \phi(\phi\phi^*)^2 + \phi(\phi\phi^*)^3 + \cdots)$$

$$+\frac{1}{z}(\phi^* + \phi^*(\phi\phi^*) + \phi^*(\phi\phi^*)^2 + \phi^*(\phi\phi^*)^3 + \cdots)$$

$$+\cdots$$

generates **all** possible combinations of ϕ, ϕ*. Combinations can be grouped according to their charge

only the combinations at O(1) (charge zero) are picked out by the contour integral dz/z

manipulate further

$$\frac{1}{(1-\phi z)(1-\frac{\phi^*}{z})} = \exp\Big(- \log(1-\phi z) - \log(1-\frac{\phi^*}{z})\Big)$$

$$= \exp\Big(\sum_{r=1}^{\infty}\Big\{\frac{(\phi z)^r}{r} + \frac{1}{r}\Big(\frac{\phi^*}{z}\Big)^r\Big\}\Big)$$

this will be the most useful (= generalizable) form

generating function written as “Plethystic exponential” = PE

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objects `charge'

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Plethystic exponential

Hilbert series

$$h = \int d(\text{measure}) \Bigg(\quad\quad \Bigg)$$

more complicated example:

$$\phi_1,\ \phi_1^*,\ \phi_2,\ \phi_2^*$$

charge: $+1,\ -1,\ +2,\ -2$

now there are four invariants

$$(\phi_1\phi_1^*),\ (\phi_2\phi_2^*),\ (\phi_1^2\phi_2^*),\ (\phi_1^{*2}\phi_2)$$

based on last example, may guess that

$$h_{\phi_1\phi_2} = \frac{1}{(1-(\phi_1\phi_1^*))(1-(\phi_2\phi_2^*))(1-(\phi_1^2\phi_2^*))(1-(\phi_1^{*2}\phi_2))}$$

generates all invariants

not correct! misses relations among invariants:

$$(\phi_1^2\phi_2^*)(\phi_1^{*2}\phi_2) = (\phi_1\phi_1^*)^2\,(\phi_2\phi_2^*)$$

correct series is

$$h_{\phi_1\phi_2} = \frac{1-\phi_1^2\phi_1^{*2}\phi_2\phi_2^*}{(1-(\phi_1\phi_1^*))(1-(\phi_2\phi_2^*))(1-(\phi_1^2\phi_2^*))(1-(\phi_1^{*2}\phi_2))}$$

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“syzygy”

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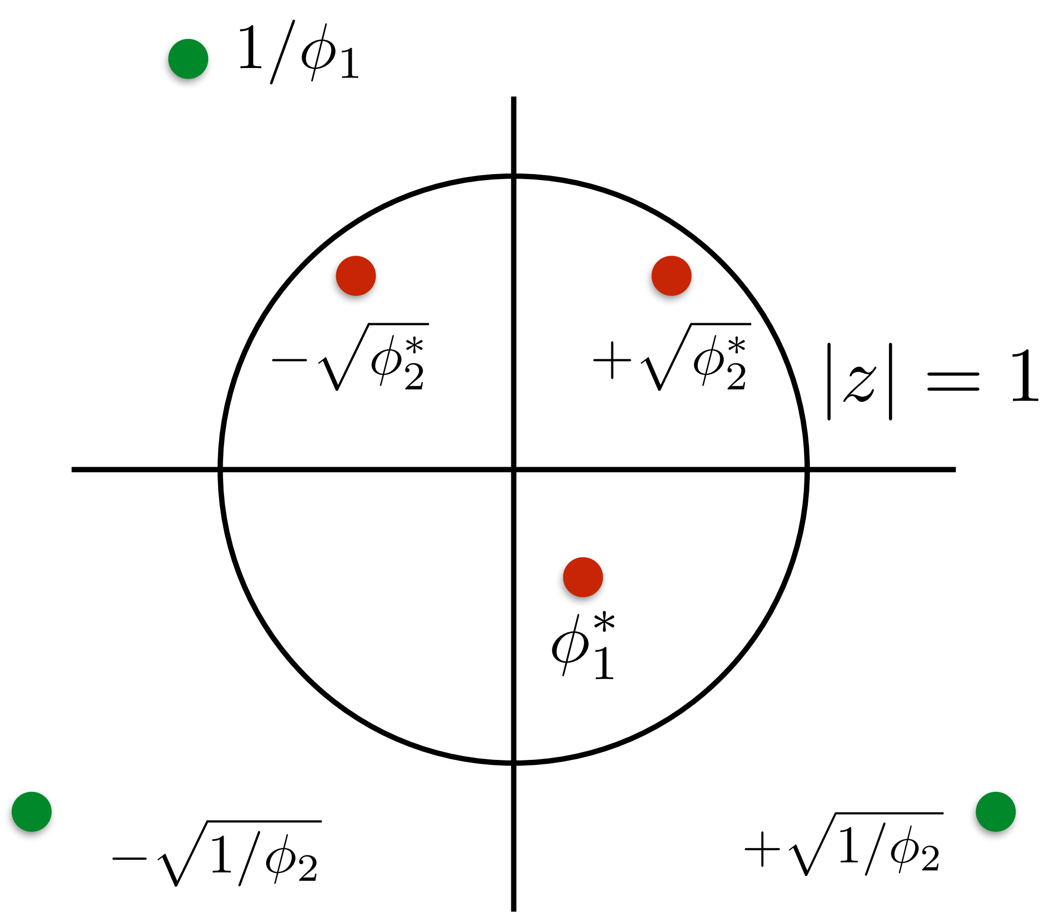
however, if we work with the PE, we get this **automatically.**

extend

$$\exp\Big(\sum_{r=1}^{\infty}\Big\{\frac{(\phi_1 z)^r}{r}+\frac{1}{r}\Big(\frac{\phi_1^*}{z}\Big)^r+\frac{(\phi_2 z^2)^r}{r}+\frac{1}{r}\Big(\frac{\phi_2^*}{z^2}\Big)^r\Big\}\Big)$$

$$\frac{1}{2\pi i}\oint_{|z|=1}\frac{dz}{z}\frac{1}{(1-\phi_1 z)(1-\frac{\phi_1^*}{z})(1-\phi_2 z^2)(1-\frac{\phi_2^*}{z^2})}$$

multiple poles, but not all reside in $|z| < 1$ (ϕ_1, ϕ_2 are also mod <1)



Molien form = PE

developed to capture invariants correctly

[Melia]

all invariants, keeping track of redundancies captured by the PE approach. We want to use this to generate all EFT operators; $\phi \to Q, u^c, d^c, H, F_{\mu\nu}$, etc.

Need to:

1.) expand to other larger groups

2.) deal with anticommuting objects

3.) incorporate derivatives ; brings difficulty of equations of motion (EOM) and integration by parts (IBP)

Other groups:

$$\exp\Big(\sum_{r=1}^{\infty}\Big\{\frac{(\phi_1 z)^r}{r}+\frac{1}{r}\Big(\frac{\phi_1^*}{z}\Big)^r+\frac{(\phi_2 z^2)^r}{r}+\frac{1}{r}\Big(\frac{\phi_2^*}{z^2}\Big)^r\Big\}\Big)$$

for a \`field' in a representation R of a group G,

z → $\chi_R(z_i)$, the **character** of the representation R

character?

if, under G $\phi_i \to D_{R,ij}\phi_j$ then $\chi_R = tr(D_R)$

χ_R are functions of **j** complex numbers, **j = rank of G**

(1 for SU(2), 2 for SU(3), etc..)

ex:

U(1), charge Q: $\chi_Q = z^Q$

$$\exp\left(\sum_{r=1}^{\infty}\left\{\frac{(\phi_1 z)^r}{r} + \frac{1}{r}\left(\frac{\phi_1^*}{z}\right)^r + \frac{(\phi_2 z^2)^r}{r} + \frac{1}{r}\left(\frac{\phi_2^*}{z^2}\right)^r\right\}\right)$$

SU(2), doublet: $\chi = (z + \frac{1}{z})$

triplet: $\chi = (1 + z^2 + \frac{1}{z^2})$

SU(3), triplet: $\chi = (z_1 + \frac{z_2}{z_1} + \frac{1}{z_2})$

charged under multiple groups: total character is product of each group characters

Other groups:

$$\frac{1}{2\pi i}\oint \frac{dz}{z} \to \int d\mu \text{ , Haar measure}$$

Haar measure: volume of compact group expressed as an integral over the j complex variables = Cartan subalgebra variables

SU(2): $$\int d\mu_{SU(2)} = \frac{1}{2\pi i}\oint dz\,\frac{(z^2-1)}{z}$$

SU(3): $$\int d\mu_{SU(3)} = \frac{1}{(2\pi i)^2}\oint dz_1\,dz_2\,\frac{(1-z_1 z_2)}{z_1 z_2}\Big(1-\frac{z_1^2}{z_2}\Big)\Big(1-\frac{z_2^2}{z_1}\Big)$$

Peter-Weyl theorem: characters of compact Lie groups form an orthonormal basis set for functions of the j complex variables

$$\int_G d\mu\, \chi_M(z_i)\, \chi_N^*(z_i) = \delta_{MN}$$

and we can expand any function of z_i as a linear combination of $\chi_M(z_i)$

$$F(z_i) = \sum_M A_M\, \chi_M(z_i)$$

coefficient, indep. of z_i

can project out any A_M using orthonormality exactly like Fourier series:

$$f(\theta) = \sum_{n=-\infty}^{\infty} A_n \, e^{i\,n\,\theta}$$

$$= A_0 + \sum_n \tilde{A}_n \, \cos(n\theta) + \sum_n \tilde{B}_n \sin(n\theta)$$

project out individual coefficient

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} d\theta \, f(\theta) = A_0$$

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in fact: set $z = e^{i\theta}$

Fourier series = character orthonormality for U(1)

Generalizes to multiple symmetry groups

1.) form the PE: PE[$\phi_1(\chi_1(z_1), \chi_2(z_2)\ldots) + \phi_2(\chi_1'(z_1), \chi'_2(z_2)) + \ldots$]

2.) PE is a function of the complex variables parameterizing the groups, z_i can be expanded in terms of characters

$$PE = \prod_G \Big(\sum_M A_M^G\, \chi_M^G(z_i) \Big)$$ (combo of all reps of all groups)

3.) Integrate over Haar measure

$$\int \prod_G d\mu_G \prod_G \Big(\sum_M A_M^G\, \chi_M^G(z_i) \Big) = \prod_G A_0^G$$

only piece that survives is A_0, coefficient of **overall** singlet/ invariant irrep

Ex: doublet scalar with Higgs charges under $SU(2)_W \otimes U(1)_Y$

$$PE[H(0, \frac{1}{2}, -\frac{1}{2}) + H^\dagger(0, \frac{1}{2}, \frac{1}{2})]$$

$$PE[H\left(z + \frac{1}{z}\right)u^{-1/2} + H^\dagger\left(z + \frac{1}{z}\right)u^{1/2}]$$

$$\frac{1}{(2\pi i)^2}\oint_u \frac{du}{u}\oint_z dz\frac{(z^2-1)}{z}PE[H, H^\dagger]$$

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$$1+(H^\dagger H)+(H^\dagger H)^2+(H^\dagger H)^3+\cdots$$

Fermions:

asymmetric, plus they transform under Lorentz group

Asymmetry:

[Hanany '14]

Plethystic Exponential (PE)

→ Fermionic Plethystic Exponential (PEF)

$$PEF[\psi] = \exp\left\{\sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{r} \left(\psi\, \chi(z_i)\right)^r\right\}$$

character

Lorentz group:

LH, RH fermions are in 2D reps of the Lorentz group

just two more groups: $SO(3,1) \rightarrow SO(4) \cong SU(2)_R \otimes SU(2)_L$

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example: QQQL operators, $N_f = 3$

$$PEF[3\,Q(0,1/2;3,2,1/6) + 3\,L(0,1/2;1,2,-1/2)]$$

x, y for $SU(2)_R \times SU(2)_L$; (w1, w2) for SU(3), z for $SU(2)_W$, u for $U(1)_Y$

$$PEF[3\,Q\left(y+\frac{1}{y}\right)\left(z+\frac{1}{z}\right)\left(w_1+\frac{w_2}{w_1}+\frac{1}{w_2}\right)u^{1/6}$$
$$+3\,L\left(y+\frac{1}{y}\right)\left(z+\frac{1}{z}\right)u^{-1/2}]$$

$$\int d\mu_{\mathrm{Lorentz}}(x,y)\,d\mu_{SU(3)}(w_1,w_2)d\mu_{SU(2)}(z)d\mu_{U(1)}(u)\,PEF[3Q,3L]$$

$$1+57\,LQ^3+4818\,L^2\,Q^6+162774\,L^3\,Q^9+\cdots$$

derivatives:

general EFT expansion can have derivatives on fields
as well as fields

$$\mathcal{L} \supset \phi^n,\ (\partial_\mu \phi)^n \phi^m, \text{etc}$$

since PE generates all combinations, we need to add $\partial_\mu \phi$ to PE

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.. and also $\partial^2{}_\mu \phi$, $\partial^3{}_\mu \phi$ ..

$\partial_\mu \sim (1/2, 1/2)$ of Lorentz, so doesn't look too terrible

but even at ∂^2 there are two possibilities:

$$\partial_{\{\mu,\nu\}}\phi, \quad \Box\phi$$

$$(1,1), \quad (0,0)$$

but any polynomial containing any $\Box\phi$ formed by the PE

i.e. $\phi^m \Box\phi$

always reduces via the EOM

$$\Box\phi = m^2\phi^2 + \lambda\phi^3 \qquad \text{(for } \phi^4 \text{ theory)}$$

form of RHS of EOM is not important. We only care that $\Box\phi$ can always be replaced by terms with fewer derivatives

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so: $$PE[\phi, \Box\phi]_{EOM} = PE[\phi]$$

(all polynomials in ϕ and $\partial^2\phi$ = all polynomials in ϕ)

by same logic, at higher derivative order, only keep the fully symmetric term

$$PE[\phi] \to PE[\phi(0,0) + D\,\phi(1/2,1/2) + D^2\,\phi(1,1) + \cdots]$$

similar story for fermions and field strengths by same logic, at higher derivative order, only keep the fully symmetric term

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similar story for fermions and field strengths

$$\underset{\partial_\mu}{(\frac{1}{2},\frac{1}{2})} \otimes \underset{Q}{(0,\frac{1}{2})} = \boxed{\underset{\text{EOM:}}{(\frac{1}{2},0)}} \oplus (\frac{1}{2},1)$$

therefore:

$$PEF[\psi] = PEF[\psi(0,\frac{1}{2}) + D\psi(\frac{1}{2},1) + D^2\psi(1,\frac{3}{2}) + \cdots]$$

Integration by parts (IBP)

derivative-extended PE still contains redundancy from IBP:

ex.)

$$D_\mu H D^\mu H H^{\dagger 2}, \quad D_\mu H^\dagger D^\mu H^\dagger H^2, \quad D_\mu H D^\mu H^\dagger (H^\dagger H)$$

are not all independent

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options:

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options:

1.) brute force.. may suffice for dim 8

2.) better idea?

Lehman, AM 1510.00372

all $\mathcal{O}(D^m \phi^n)$ must come from $D \times \mathcal{O}(D^{m-1}\phi^n)$

if we can count the number of $\mathcal{O}(D^{m-1}\phi^n)$, thats a set of constraints on the $\mathcal{O}(D^m \phi^n)$

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Lehman, AM 1510.00372

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net result:

$$\int(\prod_i d\mu_{G_i})\Big(1 - D(\frac{1}{2},\frac{1}{2})\Big)\,PE[\phi + D\phi(\frac{1}{2},\frac{1}{2}) + \cdots]$$

cross-checks:

easily extended to multiple scalars, complex scalars

fermion-scalar theory

works with gauge theory, D → covariant derivative

gets SM dim 6 correct, $N_F = 1, 3$

gets SM dim 7 correct, predicts dim-8^+

some dim-8, according to this algorithm:

$$
\begin{array}{cccc}
(d_c^\dagger d_c)(e_c^\dagger e_c)F^L & (u_c^\dagger u_c)(e_c^\dagger e_c)F^L & 2\,(d_c^\dagger d_c)(u_c^\dagger u_c)F^L & (d_c^\dagger d_c)(L^\dagger L)F^L \\
(u_c^\dagger u_c)(L^\dagger L)F^L & (e_c^\dagger e_c)(L^\dagger L)F^L & (e_c^\dagger e_c)(Q^\dagger Q)F^L & (d_c Q)(e_c^\dagger L^\dagger)F^L \\
(d_c Q)(e_c^\dagger L^\dagger)F^R & 2\,(L^\dagger L)(Q^\dagger Q)F^L & 2\,(d_c^\dagger d_c)(Q^\dagger Q)F^L & 2\,(u_c^\dagger u_c)(Q^\dagger Q)F^L \\
3\,(e_c L)(u_c Q)F^L & 3\,(u_c d_c)\,Q^2\,F^L & (d_c^\dagger d_c)(L^\dagger L)W^L & (e_c^\dagger e_c)(L^\dagger L)W^L \\
(e_c^\dagger e_c)(Q^\dagger Q)W^L & (u_c^\dagger u_c)(L^\dagger L)W^L & (L^\dagger L)^2 W^L & (e_c^\dagger L^\dagger)(d_c Q)W^L \\
(e_c L)(d_c^\dagger Q^\dagger)W^L & 2\,(d_c^\dagger d_c)(Q^\dagger Q)W^L & 2\,(u_c^\dagger u_c)(Q^\dagger Q)W^L & 3\,(L^\dagger L)(Q^\dagger Q)W^L \\
2\,(Q^\dagger Q)^2 W^L & 3\,(e_c L)(u_c Q)W^L & 3\,(u_c d_c)\,Q^2\,W^L & (d_c^\dagger)^2\,d_c^2\,G^L \\
(u_c^\dagger)^2\,u_c^2\,G^L & (d_c^\dagger d_c)(e_c^\dagger e_c)\,G^L & (u_c^\dagger u_c)(e_c^\dagger e_c)\,G^L & 4\,(d_c^\dagger d_c)(u_c^\dagger u_c)\,G^L \\
(Q^\dagger Q)(e_c^\dagger e_c)\,G^L & (d_c^\dagger d_c)(L^\dagger L)\,G^L & (u_c^\dagger u_c)(L^\dagger L)\,G^L & 2\,(Q^\dagger Q)(L^\dagger L)\,G^L \\
4\,(d_c^\dagger d_c)(Q^\dagger Q)\,G^L & 4\,(u_c^\dagger u_c)(Q^\dagger Q)\,G^L & 2\,(Q^\dagger)^2\,Q^2 G^L & (d_c Q)(e_c^\dagger L^\dagger)\,G^L \\
(d_c Q)(e_c^\dagger L^\dagger)\,G^R & 3\,(e_c L)(u_c Q)\,G^L & 6\,(d_c u_c)\,Q^2\,G^L &
\end{array}
$$

112 at $O(D^0)$

$$
\begin{array}{cccc}
3\,D\,(d_c^\dagger d_c)(L\,H\,e_c) & D\,(e_c^\dagger e_c)(L\,H\,e_c) & 3\,D\,(L^\dagger L)(L\,H\,e_c) & 3\,D\,(d_c^\dagger d_c)(Q\,H\,d_c) \\
3\,D\,(e_c^\dagger e_c)(Q\,H\,d_c) & 6\,D\,(L^\dagger L)(Q\,H\,d_c) & 6\,D\,(Q^\dagger Q)(L\,H\,e_c) & 6\,D\,(Q^\dagger Q)(Q\,H\,d_c) \\
3\,D\,(d_c^\dagger u_c)(L\,H^\dagger e_c) & 6\,D\,(d_c^\dagger d_c)(Q\,H^\dagger u_c) & 3\,D\,(e_c^\dagger e_c)(Q\,H^\dagger u_c) & 6\,D\,(L^\dagger L)(Q\,H^\dagger u_c) \\
6\,D\,(Q^\dagger Q)(Q\,H^\dagger u_c) & 3\,D\,(u_c^\dagger u_c)(L\,H\,e_c) & 6\,D\,(u_c^\dagger u_c)(Q\,H\,d_c) & 3\,D\,(u_c^\dagger u_c)(Q\,H^\dagger u_c)
\end{array}
$$

181 at $O(D)$

535 total (931 counting +h.c. separately)

cross-checks:

easily extended to multiple scalars, complex scalars

fermion-scalar theory

works with gauge theory, D → covariant derivative

gets SM dim 6 correct, $N_F = 1, 3$

gets SM dim 7 correct, predicts dim-8$^+$

But, fails if constraints not independent..

(happens more often for higher D)

also, seems ad hoc…

Henning, Lu, Melia, Murayama 1512.03433

$$\int (\prod_i d\mu_{G_i}) \Big(1 - D(\frac{1}{2}, \frac{1}{2}) + D^2((0,1) + (1,0)) - D^3(\frac{1}{2}, \frac{1}{2}) + D^4\Big) PE[\phi + D\phi(\frac{1}{2}, \frac{1}{2}) + \cdots]$$

Henning, Lu, Melia, Murayama 1512.03433

$$\int (\prod_i d\mu_{G_i}) \Big(1 - D(\frac{1}{2},\frac{1}{2}) + D^2((0,1)+(1,0)) - D^3(\frac{1}{2},\frac{1}{2}) + D^4\Big)\, PE[\phi + D\phi(\frac{1}{2},\frac{1}{2}) + \cdots]$$

works, free of issues

- extend d=8 SMEFT set to 992 (+62 from Lehman, AM)
- count d=9,10,11,12 SMEFT operators (560, 15456, 11962..)
- possible compact ‘all orders’ form

Why $\Big(1 - D(\frac{1}{2},\frac{1}{2}) + D^2((0,1)+(1,0)) - D^3(\frac{1}{2},\frac{1}{2}) + D^4\Big)$?

Start with irreps of conformal symmetry SO(4,2)

operators in conformal theory: primary $\mathcal{O}$, descendents $\partial_\mu \mathcal{O}$

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integration over **SO(4,2)/SO(3,1)** (dilations, conformal trans) + highest weight projection conspire to give 1 - D…prefactor

What now?

- knowing all dim-8 SMEFT, we can study which operators have an impact at LHC. Specifically, dim-8 important to understand uncertainty on dim-6

$$|\mathcal{A}_{SM} + A_6 + A_8|^2 \supset |A_{SM}|^2 + 2\,Re(A_{SM}\,A_6) + |A_6|^2 + 2\,Re(A_{SM}A_8)\cdots$$

[pp → hV, Lehman, AM in progress]

- analytic properties?

- application to EFT with nonlinear fields?

...

conclusions:

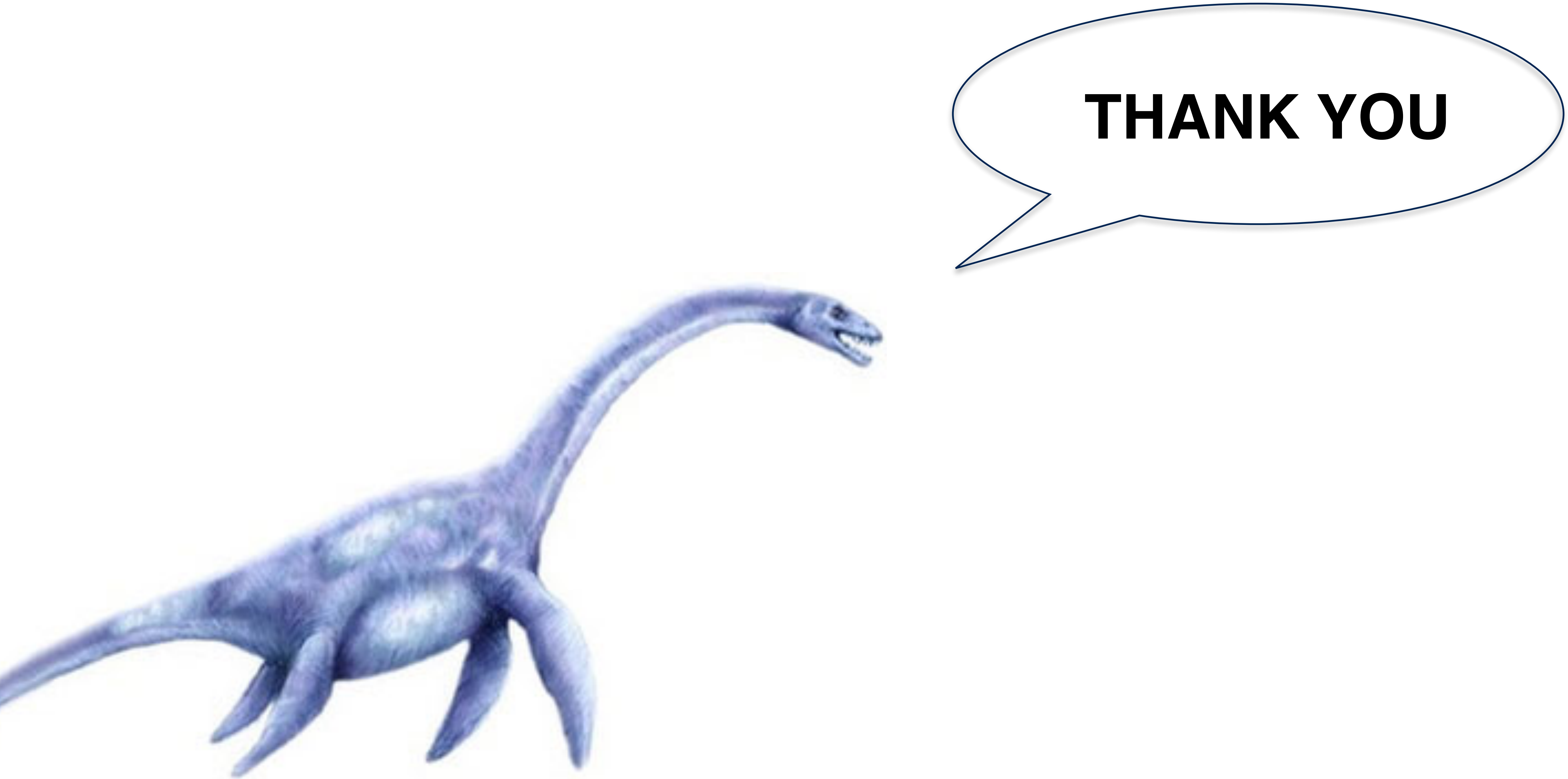
given symmetry group **G**, fields **ϕ_i, ψ_i, $X_i^{L,R}$**

$\xrightarrow{\text{Hilbert series}}$

and form of **all** invariant (Lorentz & gauge) operators, accounts for IBP, EOM

- generates all possible combinations of operators, uses character orthonormality to pick out invariants
- derivatives tricky, but issues recently overcome

lots of interesting directions to explore!


THANK YOU

$$\int \Big(d\mu_{SO(4,2)(q,x,y)} \times d\mu_{gauge} \Big) \Big(\sum_{n=1}^{\infty} D^n \chi^*_{[n,0,0]}(q,x,y) \Big) \times$$
$$\prod_i PE[\phi_i \chi_{[1,0,0]}(q,x,y) \chi_{\phi_i,gauge}] \prod_j PEF[\psi_j \, \chi_{[3/2,0,1/2]}(q,x,y) \chi_{\psi_j,gauge}]$$