Statistical Methods used in Reactor Neutrino Experiments

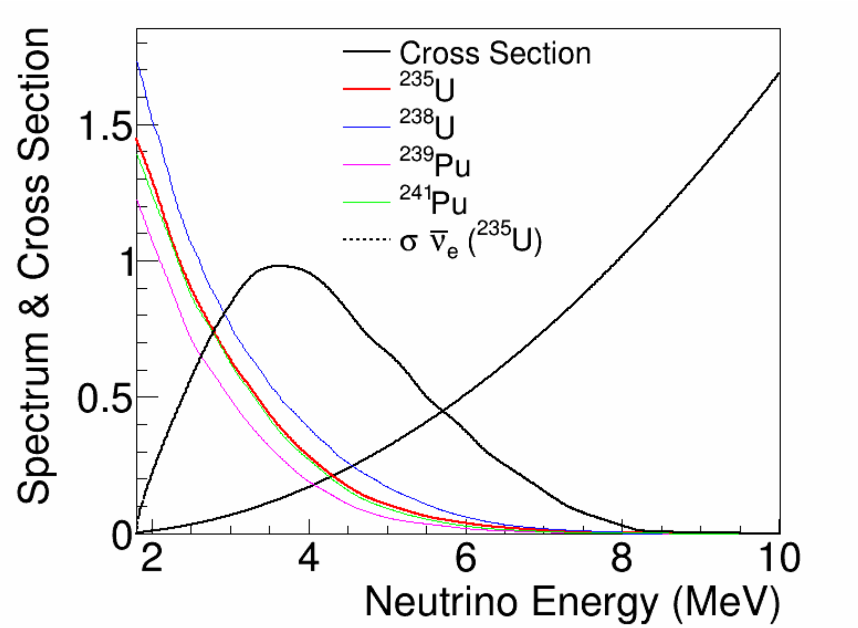
Xin Qian

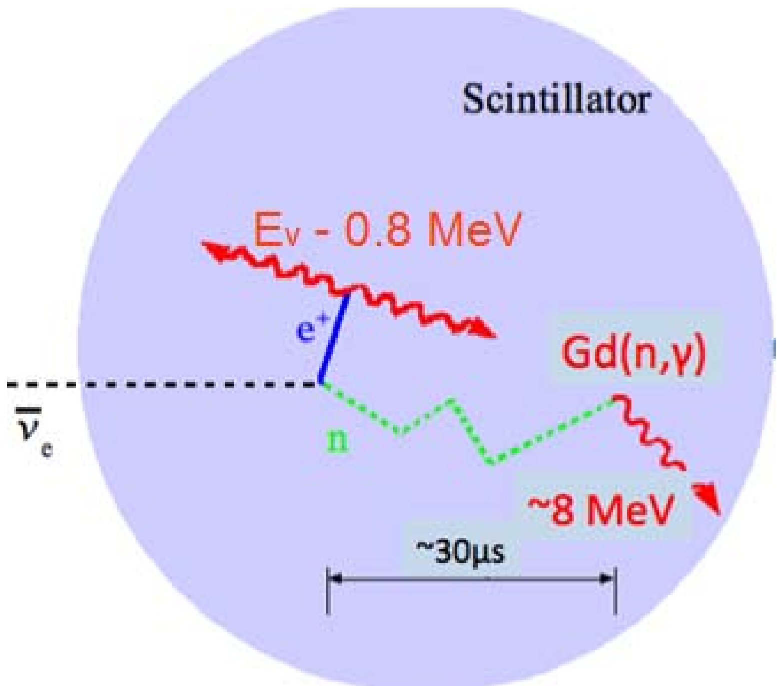
BNL

Reactor Neutrinos

- ~ 200 MeV per fission
- ~ 6 anti-ν_e per fission from daughters decay
- ~ 2 x 10^{20} anti-ν_e/GW_{th}/sec

$$\boxed{\overline{\nu_e} + p \rightarrow e^+ + n}$$





- Coincidence signal from Inverse Beta Decay:
  - Prompt: e^+ & annihilation →

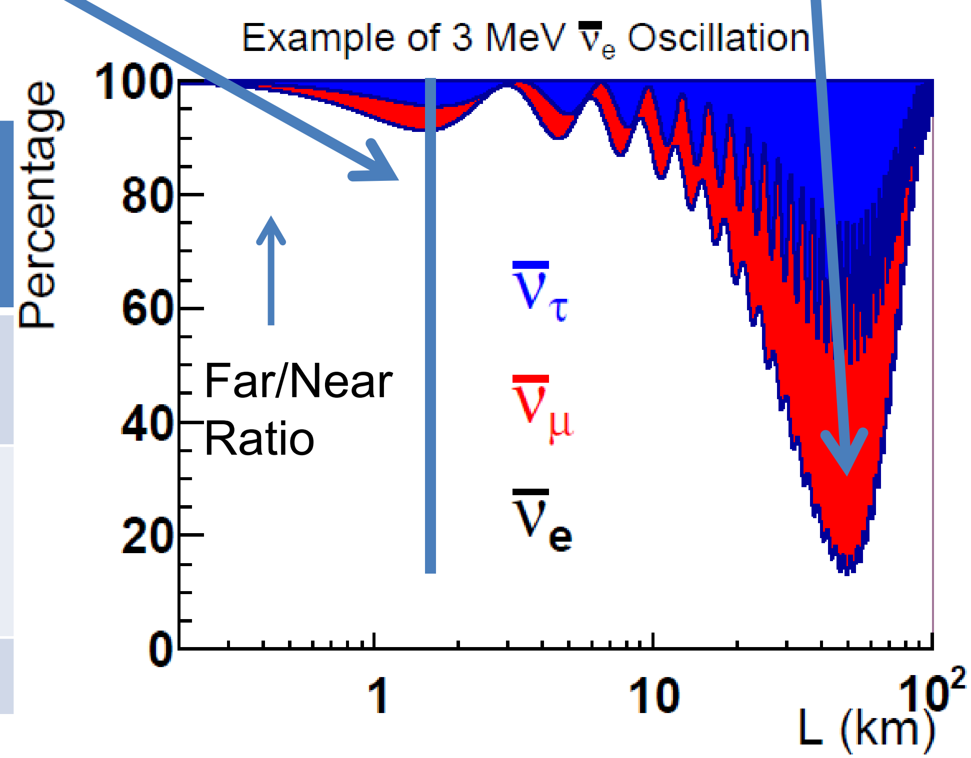
$$E_{prompt} \approx E_\nu - E_n - 0.78MeV$$

  - Delayed: n + Gd → 8 MeV with 30 us capture time

Anti-ν_e Disappearance

$$\boxed{P(\overline{\nu}_e \rightarrow \overline{\nu}_e) \approx 1 - \sin^2 2\theta_{13} \cdot \sin^2\left(\Delta m^2_{ee} \cdot \frac{L}{E}\right) - \cos^4\theta_{13} \cdot \sin^2 2\theta_{12} \cdot \sin^2\left(\Delta m^2_{21} \cdot \frac{L}{E}\right)}$$

	Daya Bay (China)	RENO (Korea)	Double Chooz (France)
Gd Target	80 ton	16.1 ton	10 ton
Reactor Thermal Power	17.4 GW	16.4 GW	8.7 GW
Baseline	~1.7 km	~1.4 km	~1.0 km



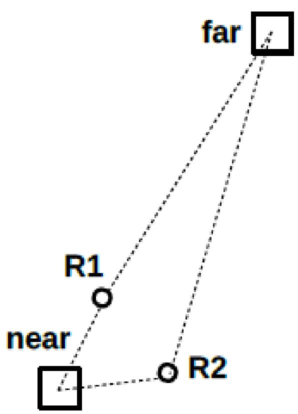
Near/far ratio to cancel uncertainty in reactor flux, firstly proposed by Mikaelyan&Sinev Phys. Atmo. Nucl. 63, (2000)

$$|\Delta m^2_{ee}| \sim |\Delta m^2_{3x}| \approx 2.4\times10^{-3}\,eV^2$$

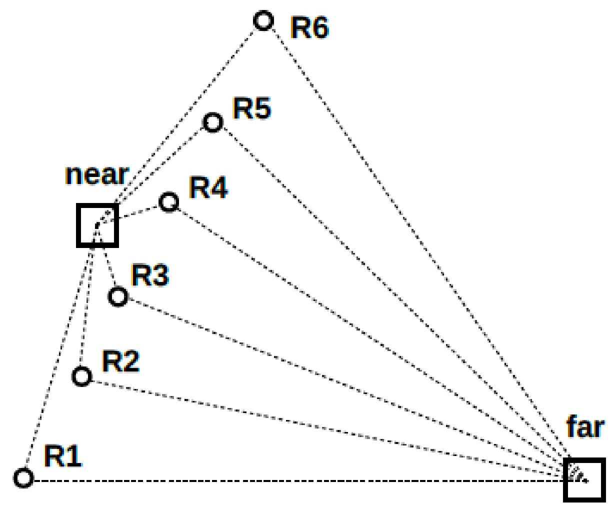
$$>> |\Delta m^2_{21}| \approx 7.6\times10^{-5}\,eV^2$$

Near/Far Ratio

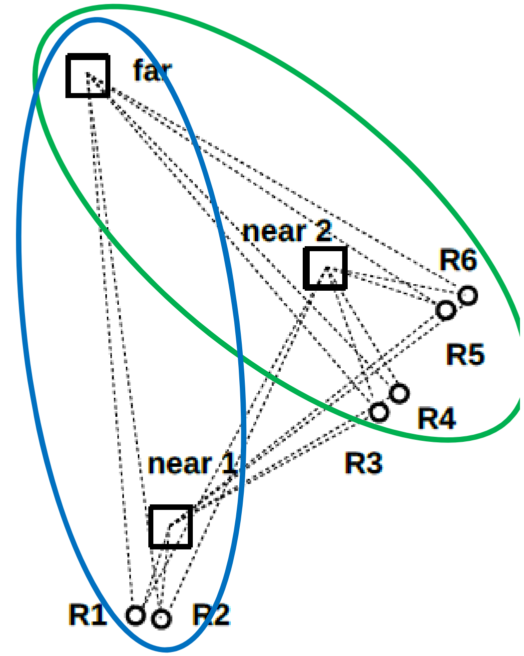
- 100% cancellation of flux uncertainty with one reactor, one near and one far detector



Double Chooz
~88% suppression of systematic uncertainties



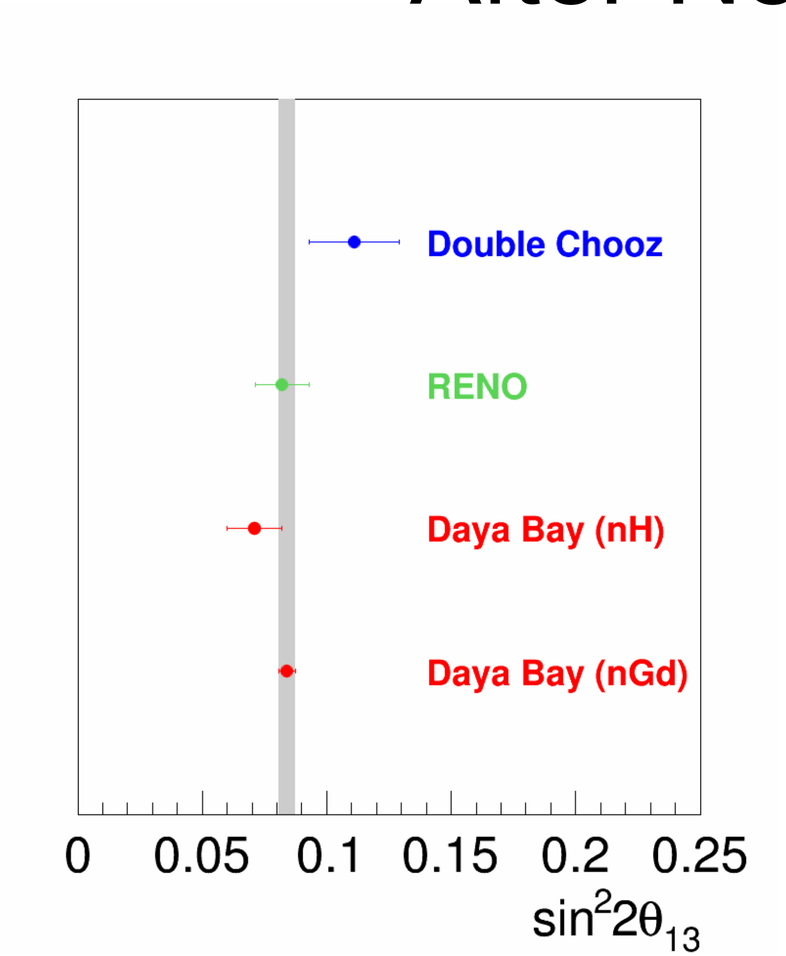
RENO
~77%

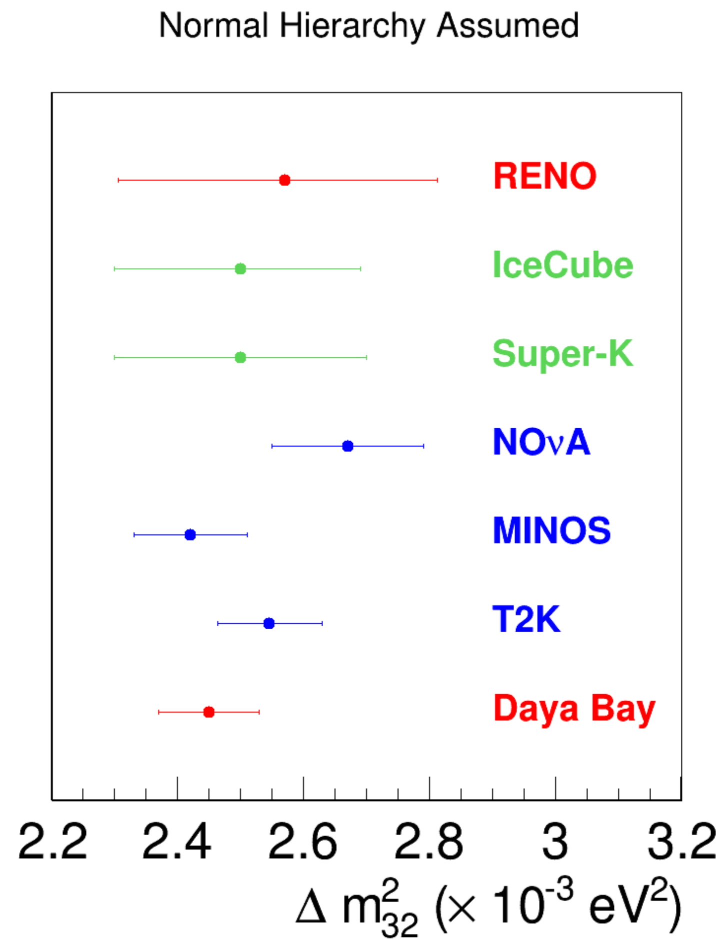


Daya Bay
~95%

Statement (~80% suppression) in arXiv:1501.00356 regarding DYB is incorrect

Current Status of $\sin^2 2\Theta_{13}$ and Δm^2_{32} After Neutrino 2016





In the following, I will focus on the statistical methods used in Daya Bay in fitting these parameters

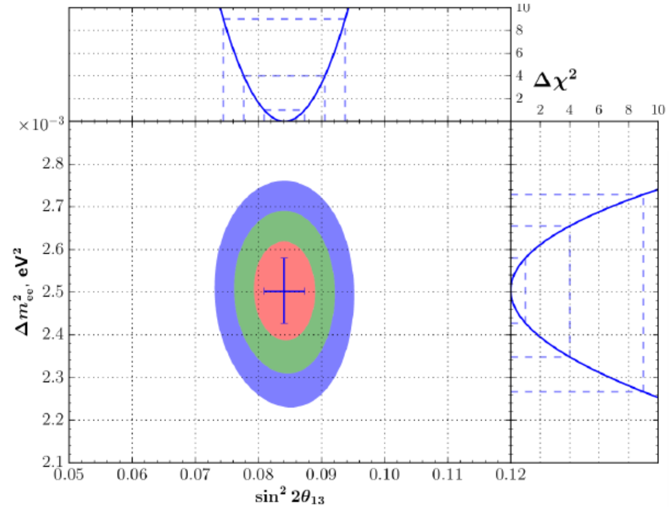
Log-likelihood profiling

- Also Pearson chi-square with pull terms in PRL, 108, 171803 (2012)

$$T = -2\mathrm{Log}\left(L_{stat}\right) - 2\mathrm{Log}\left(L_{sys}\right) + C$$

$$T_{stat} = 2\sum_{j}^{ADs,bin}\left(N_j^{pred} - N_j^{obs} + N_j^{obs}\cdot\mathrm{Log}\left(\frac{N_j^{obs}}{N_j^{pred}}\right)\right)$$

$$T_{sys} = T_{Detector} + T_{Background} + T_{\mathrm{Re}actor} + T_{Oscillation}$$



Example format $T_{sys}^{\eta} = \dfrac{(\eta - \bar{\eta})^2}{\delta\eta^2}$ with $\mathrm{N}_j^{pred}(\eta)$

AD: Antineutrino Detector

- According to Wilks' theorem, assuming $\Delta T = T - T_{min}$ following a chi-square distribution
- Advantages: simple to program and easy to examine
- Disadvantages: When number of nuisance parameters is large, can be slow to minimize

Covariance matrix in PRL, 115, 11802 (2015)

$$T=\sum_{i,j}\left(F_i^{obs}-F_i^{pred}\right)\cdot\left(V^{stat}+V^{sys}\right)_{ij}^{-1}\cdot\left(F_j^{obs}-F_j^{pred}\right)$$

"F" is a function of observed events

$$V_{ij}^{\eta}=\int_{-\infty}^{\infty}\frac{1}{\delta\eta\cdot\sqrt{2\pi}}\exp\left(-\frac{(\eta-\overline{\eta})^2}{2\delta\eta^2}\right)\cdot\left(F_i^{obs}(\eta)-F_i^{pred}(\overline{\eta})\right)\cdot\left(F_j^{obs}(\eta)-F_j^{pred}(\overline{\eta})\right)\mathrm{d}\eta$$

$$=\frac{1}{N}\sum_{k=1}^{N}\left(F_i^{psuedo,k}-F_i^{pred}(\overline{\eta})\right)\cdot\left(F_j^{psuedo,k}-F_j^{pred}(\overline{\eta})\right)$$

"i" is a energy bin label for a detector

- Approximating impacts of all systematics on the event counts as normal distributions
- Advantages: Since "V" can be pre-calculated, the minimization process to obtain T_{min} can be very fast
- Disadvantages: "V" may have dependences on the parameters of interest (i.e. θ_{13} and Δm^2), additional cares are needed
  - Also Gaussian-Hermite technique to calculate integration in-flight

$$E\left[h(y)\right]=\int_{-\infty}^{\infty}\frac{1}{\sigma_y\cdot\sqrt{2\pi}}\exp\left(-\frac{(\mathrm{y}-\overline{y})^2}{2{\sigma_y}^2}\right)\cdot h(\mathrm{y})\,\mathrm{dy}\approx\frac{1}{\sqrt{\pi}}\sum_{i=1}^{n}w_i\cdot h\left(\sqrt{2}\sigma_y x_i+\overline{y}\right)$$

Hybrid Approach in PRL,112, 061801 (2014)

- Sometimes, the number of nuisance parameters can be too many → numerical instability in finding the minimum
- For example, for reactor-related systematics (26 energy bins),

Thermal power for reactor i
6

Fission fraction for isotope k at reactor i
4 x 6 = **24**

$$\phi_i^{\nu} = \frac{W_i}{\sum_k f_{ik} E_k} \sum_k f_{ik} S_k$$

Mean energy released per fission for isotope k
4

Anti-neutrino spectrum for isotope k
26 x 4 = **104**

Off-equilibrium correction **6**
Spent fuel correction **6**

Spectrum correlation **1**

4 isotopes
6 reactors
26 bins

Total: 151 uncertainties

Given three sites, the number of event bins is about 26x3=78

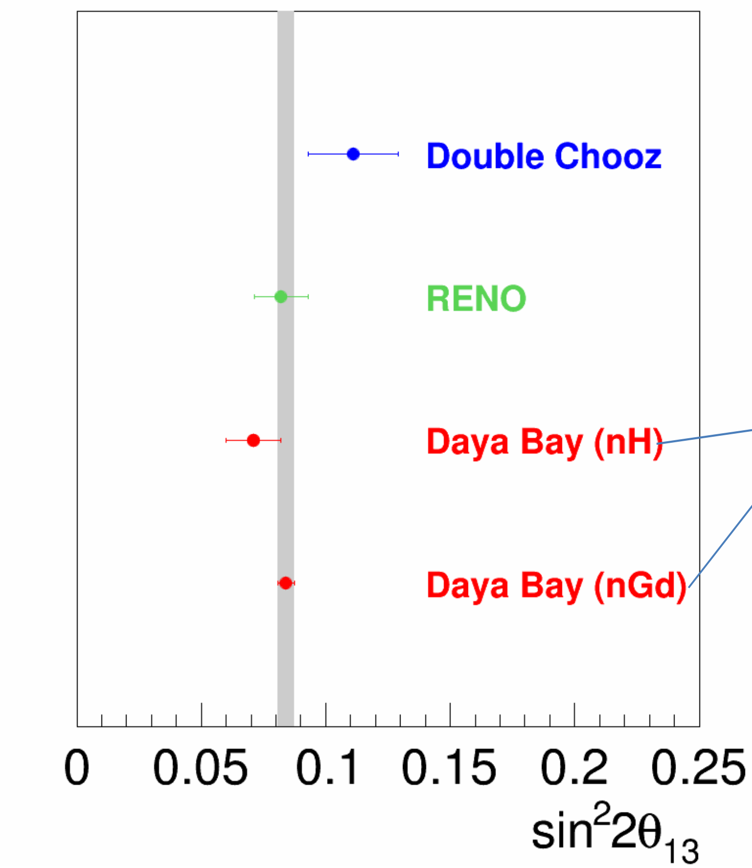
Given the nature of these systematics, expect many degeneracies → potential difficulties in finding the minimum

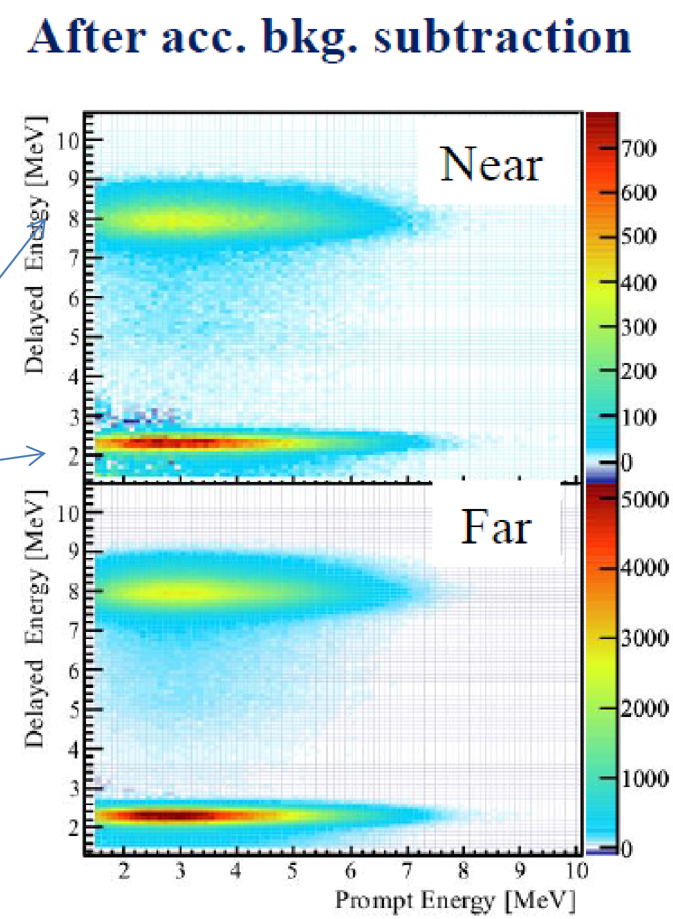
Use Covariance Matrix (rank 78) to reduce 151 uncertainties → 78 nuisance parameters (one on each event bin

$$T = -2\mathrm{Log}\left(L_{stat}\left(\eta\right)\right) - 2\mathrm{Log}\left(L_{other\ sys}\right) + \sum_{i,j}\left(\eta_i\right)\cdot\left(V_{reactor}\right)_{ij}^{-1}\cdot\left(\eta_j\right) + C$$

Also NDF difference can be used to check the covariance matrix

Combining nH + nGd (I)





$$\overline{\nu_e} + p \rightarrow e^+ + n$$

- n + Gd (nGd) → ~ 8 MeV gammas
- n + p (nH) → 2.2 MeV gamma

Combing nH + nGd (II)

- Approximately, one can estimate the combination through the Best Linear Unbiased Estimate (BLUE)
  A. C. Aitken, Proc. Ry. Soc. Edinburgh 55, 42 (1935)
  Lyons&Gibaut&Clifford, NIMA 270, 110 (1988)

$$\sigma^2 = \frac{\sigma_{Gd}^2 \sigma_H^2 (1-\rho^2)}{\sigma_{Gd}^2 + \sigma_H^2 - 2\rho\sigma_{Gd}\sigma_H}$$

Combining Daya Bay, RENO, and Double Chooz? Expect <10% improvement

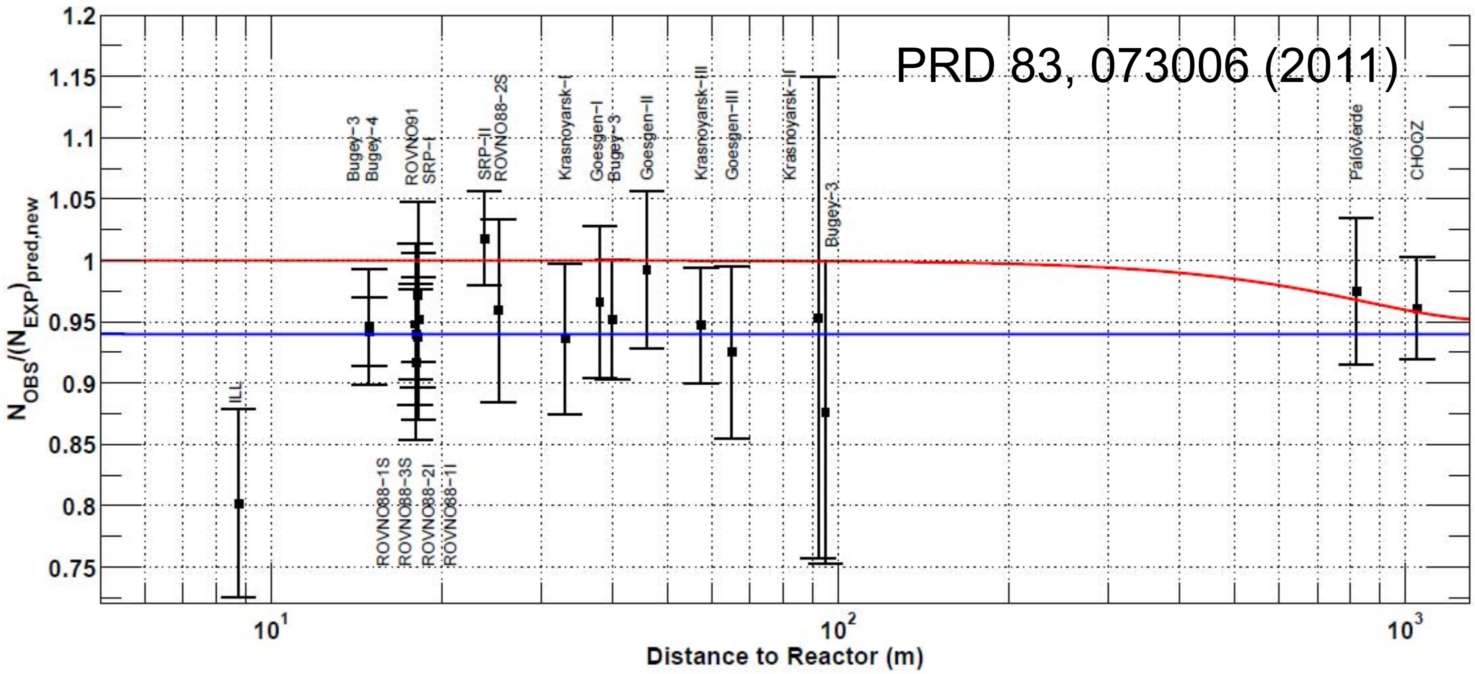
- Alternatively, a single fitter can be written to take into account all correlations in systematics
- Both methods reach similar results
- Combined result reported in PRD 90, 071101(R) (2014) PRD 93, 072011 (2016).

	Uncertainty Fraction (%)	Correlation
Statistical	51.8	0
Detector	39.2	0.07
Reactor	4.2	1
^{9}Li/^{8}He	4.4	0
Accidental	0.4	0
Fast neutron	0.3	0
Am-C	0.1	0.7
Combined	100.4	0.02

One Note About Global Average

- We reported 0.943 +- 0.008 (exp.)
- Many literatures reported 0.928 (~ 1.5% lower)

PRL, 116, 061801 (2016)
and arXiv:1607.05378



- A tricky statistical mistake, they used the measured values to build the theoretical covariance matrix

- See G. D'Agostini NIMA 346, 306 (1994), V. Blobel, SLAC-R-0703, p101, B. Roe arXiv:1506.09077

$$\chi^2(\mathrm{R}_g^{past}) = (\mathrm{R}_g^{past} - R_i)\cdot \mathrm{V}_{ij}^{-1}\left(\mathrm{R}_g^{past} - R_j\right)$$

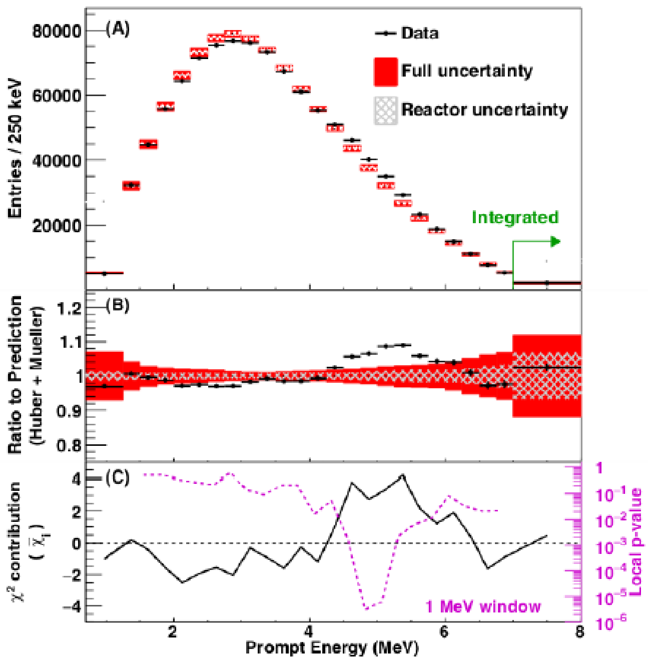
$$V = V^{\exp} + V^{theory}$$

$$V^{theory} = R_i^{obs} R_j^{obs} (\sigma^{theory})^2$$

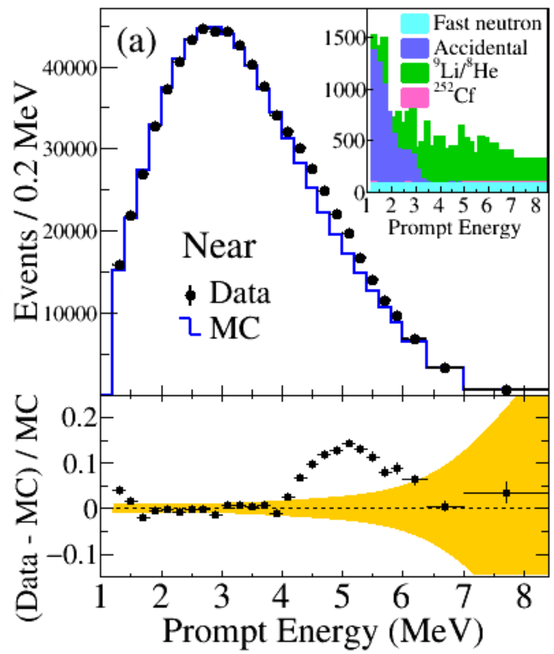
should be $V^{theory} = R_i^{theory} R_j^{theory} (\sigma^{theory})^2$

The 5-MeV “Bump”

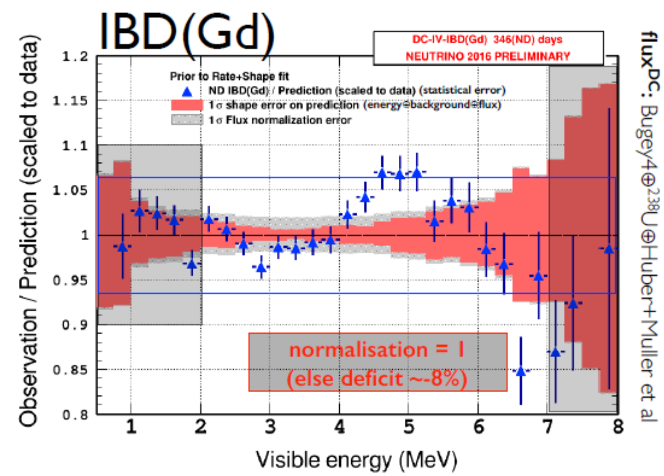
Daya Bay



RENO



Double Chooz



- Unambiguous observations of discrepancies between data and spectrum calculation at ~ 5 MeV from all three experiments
- Uncertainties in flux calculation is underestimated (> 5% from Hayes et al. PRL 112, 202501, 2014)
- Also saw in NEOS. Which isotopes? arXiv:1609.03910 (Huber)

Absolute Neutrino Spectrum

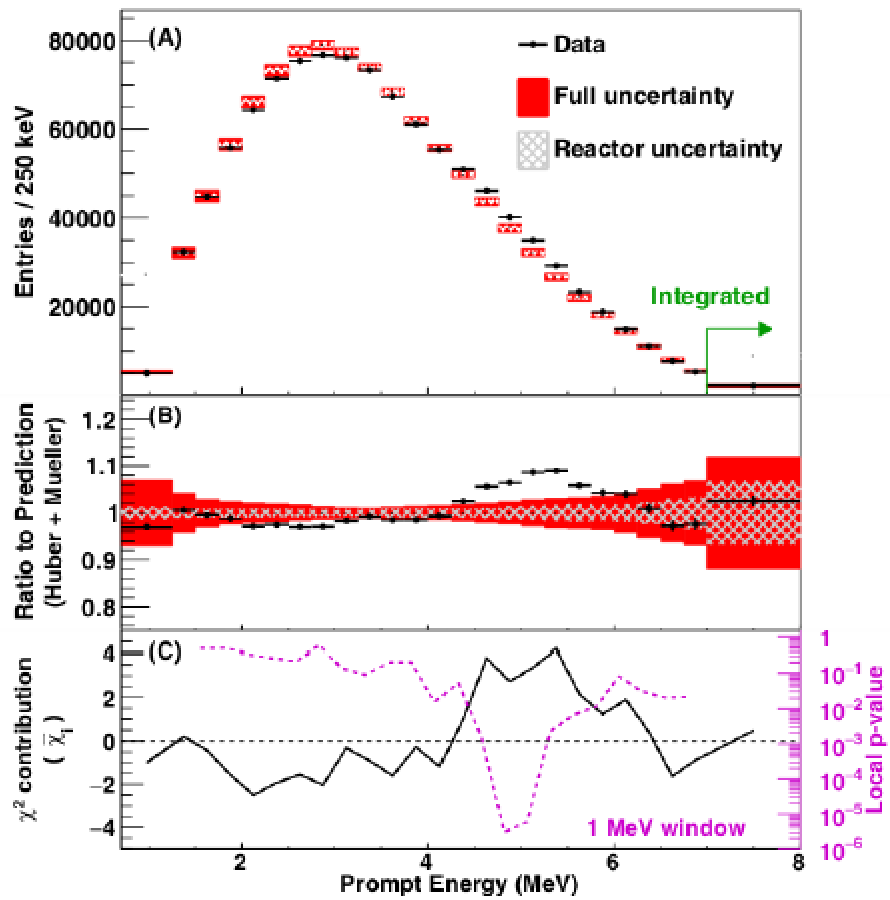
- Compare to Huber+Mueller model
- 3σ discrepancy at the full energy range

$$\frac{\chi^2}{NDF} = \frac{48.1}{24}$$

- 4.4σ local significance at 4~6 MeV

$$\frac{\Delta\chi^2}{\Delta NDF} = \frac{37.4}{8}$$

Nested-hypothesis test: eight nuisance parameters controlling the shape in 2 MeV window are allowed to freely move

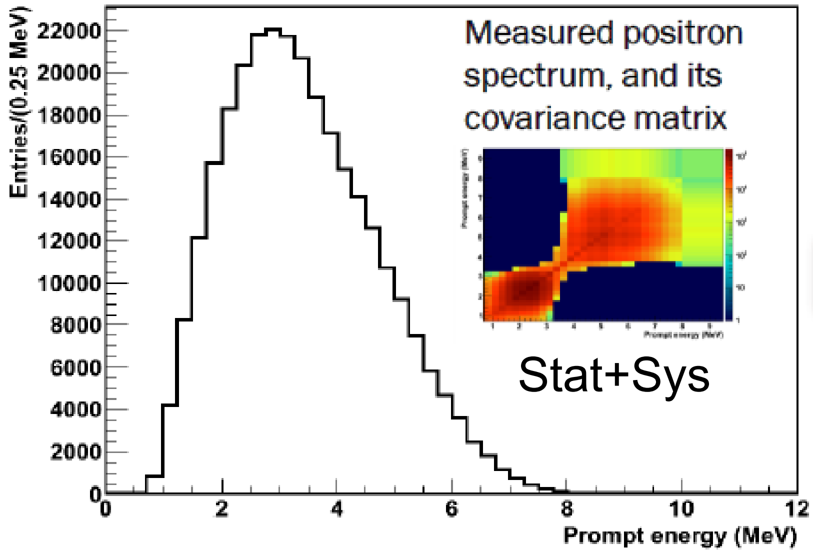


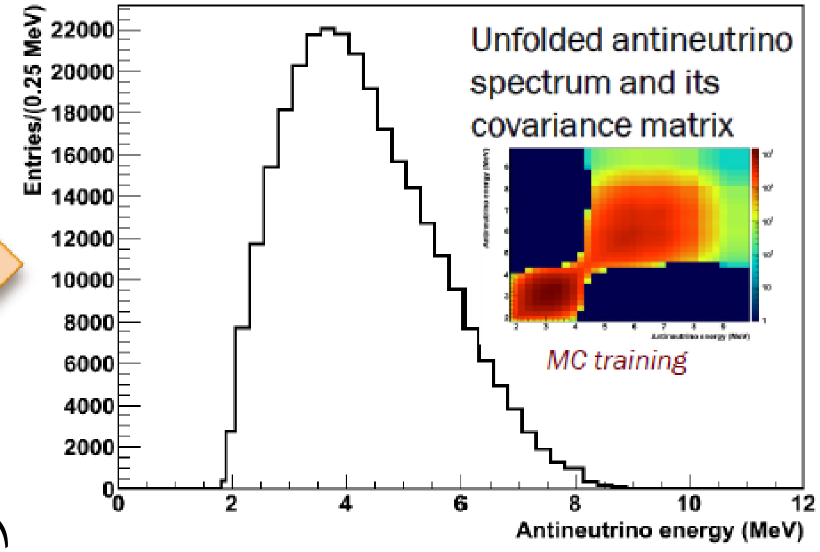
arXiv:1607.05378

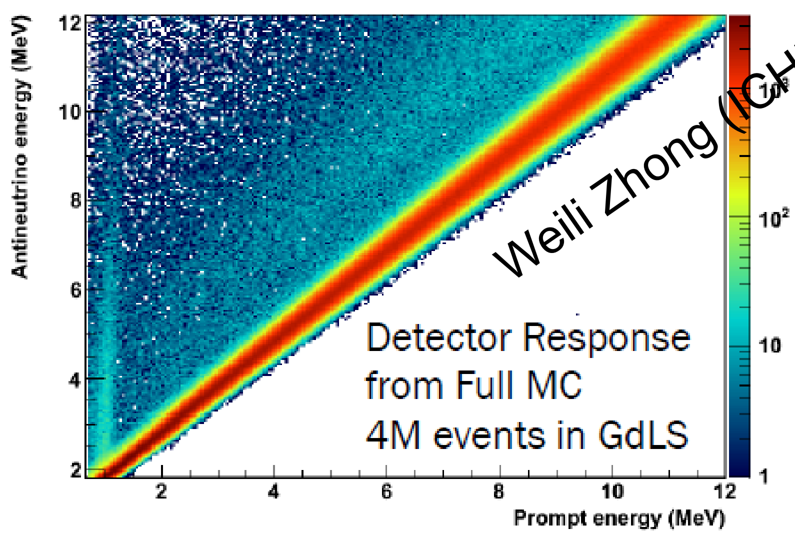
2.6σ and 4.0σ in PRL 116, 061801

Neutrino Spectrum Extraction (Unfolding)

- Unfolding “original” neutrino spectrum with reduced information from the measured prompt energy spectrum is desired for simpler usage







Weili Zhong (ICHEP 2014)

Input of unfolding:

1. Measured positron spectrum, covariance matrix
2. Detector response matrix

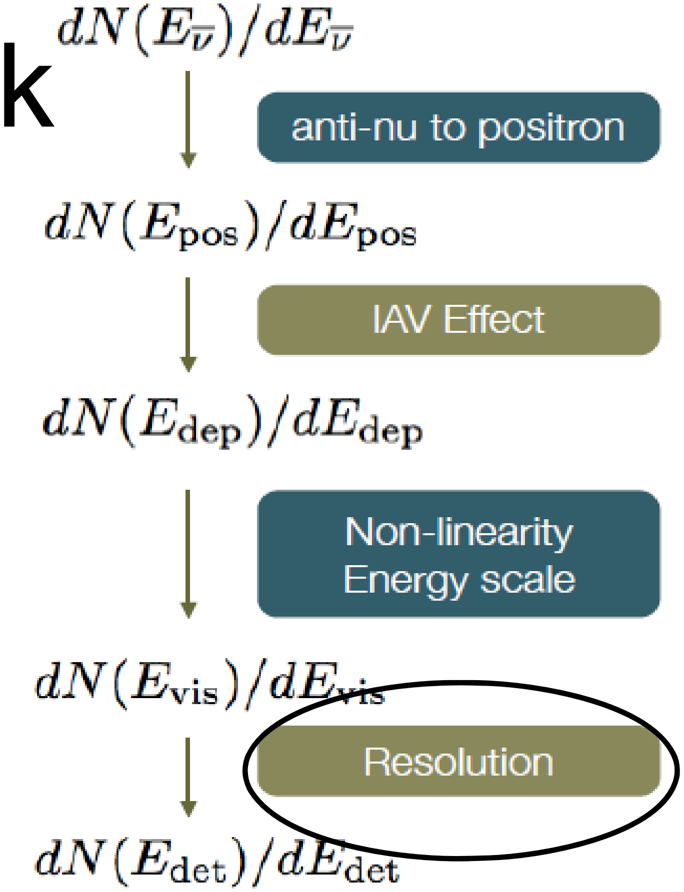
Unfolding methods:

Bayesian iterative and SVD

Output of unfolding:

Antineutrino spectrum, covariance matrix

An independent Check



- One challenge of the unfolding is the smearing due to finite energy resolution and statistical fluctuations
- Therefore, regularization is needed

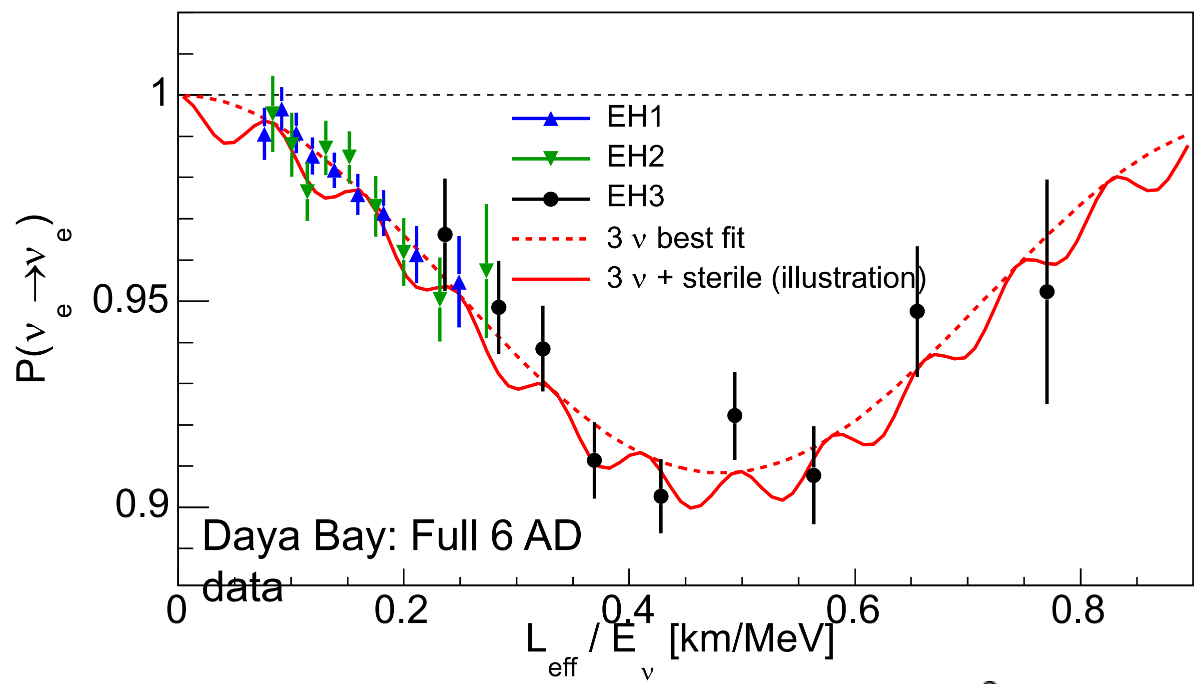
$$\chi^2 = \sum_i \left(M_i - \sum_j R_{ij} \cdot S_j \right)^2 + \chi^2_{regularization}$$

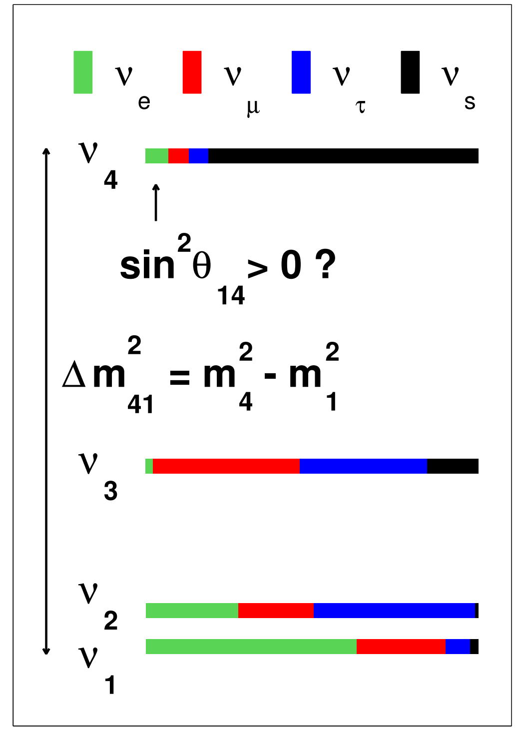
$$\chi^2_{regularization} = c^2 \sum_i (S_i")^2 = \sum_i \left(\sum_j F_{ij} \cdot S_j \right)^2$$

$$\frac{\partial \chi^2}{\partial S_k} = 0 \rightarrow \mathrm{S} = (1 + \frac{F^2}{R^2})^{-1} R^{-1} M$$

- Basically, smearing due to detector response “R” (typically irregular) is replaced by a regular response $(1+F^2/R^2)^{-1}$
  - With existence of uncertainties, smearing represents an information loss, and cannot be fully recovered
  - The optimal regularization depends on the existing smearing and statistics

Possible light sterile neutrino oscillation





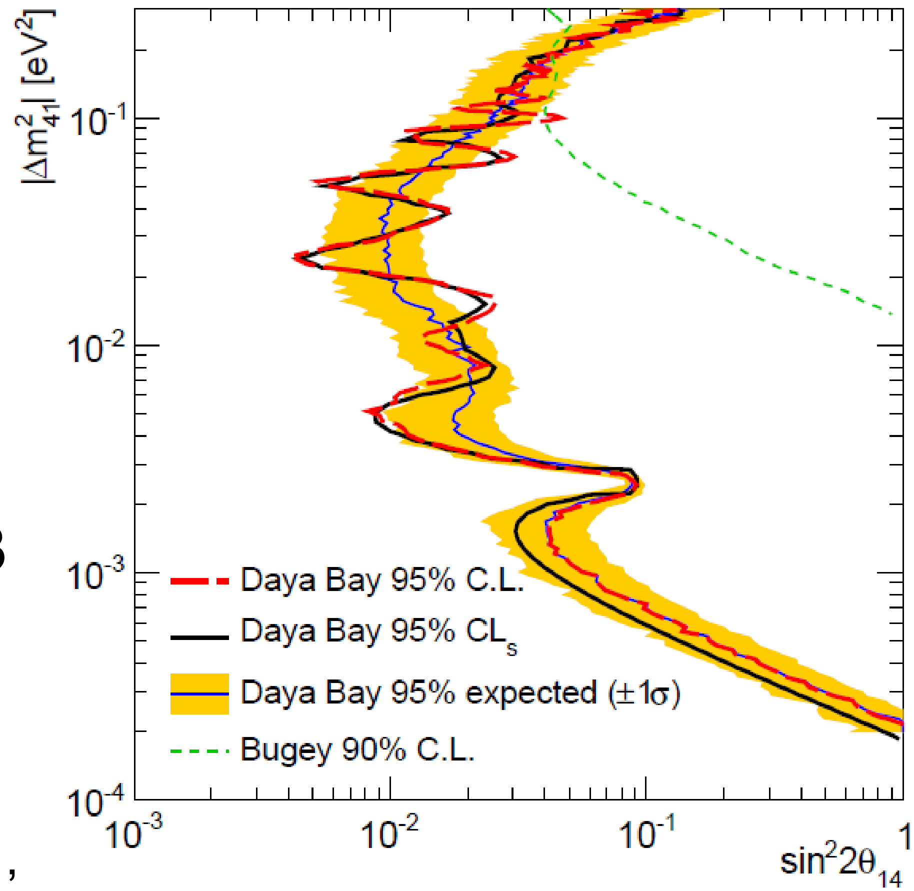
$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \cong 1 - \boxed{\cos^4\theta_{14} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{ee} L}{4E_\nu}\right)} - \boxed{\sin^2 2\theta_{14} \sin^2\left(\frac{\Delta m^2_{41} E}{4E_\nu}\right)}$$

- A minimum extension of the 3-ν model: 3(active) + 1(sterile)-ν model
- Search for a higher frequency oscillation pattern besides $|\Delta m^2_{ee}|$

Search for a Light Sterile Neutrino

- Confidence Intervals are obtained from Covariance matrix method (fast) with the Feldman-Cousins (FC)
  - PRD 57, 3873 (1998)
- Due to FC's computing demands, CLs method (A.L. Read, J. Phys. G28, 2693 T. Junk, NIMA 434,435) is chosen for "likelihood + pull"
  - Gaussian CLs method is used
    - G. Cowan et al. Eur. Phys. J. C71, 1544 (2011)
    - XQ, A. Tan et al. NIMA 827, 63 (2016)

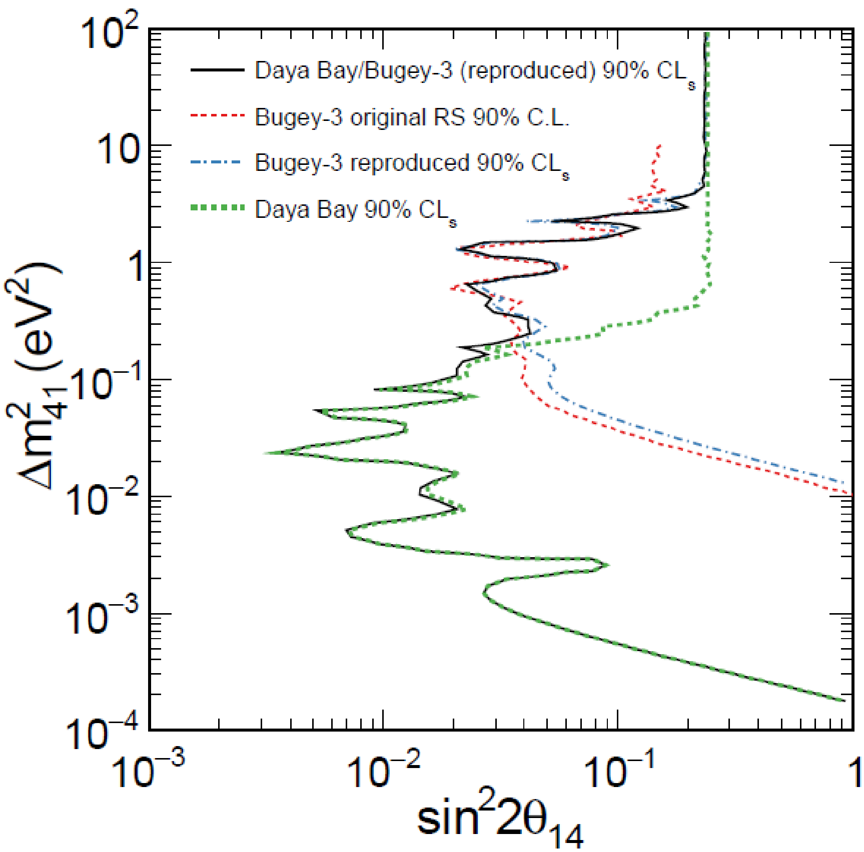
See A. Tan's talk

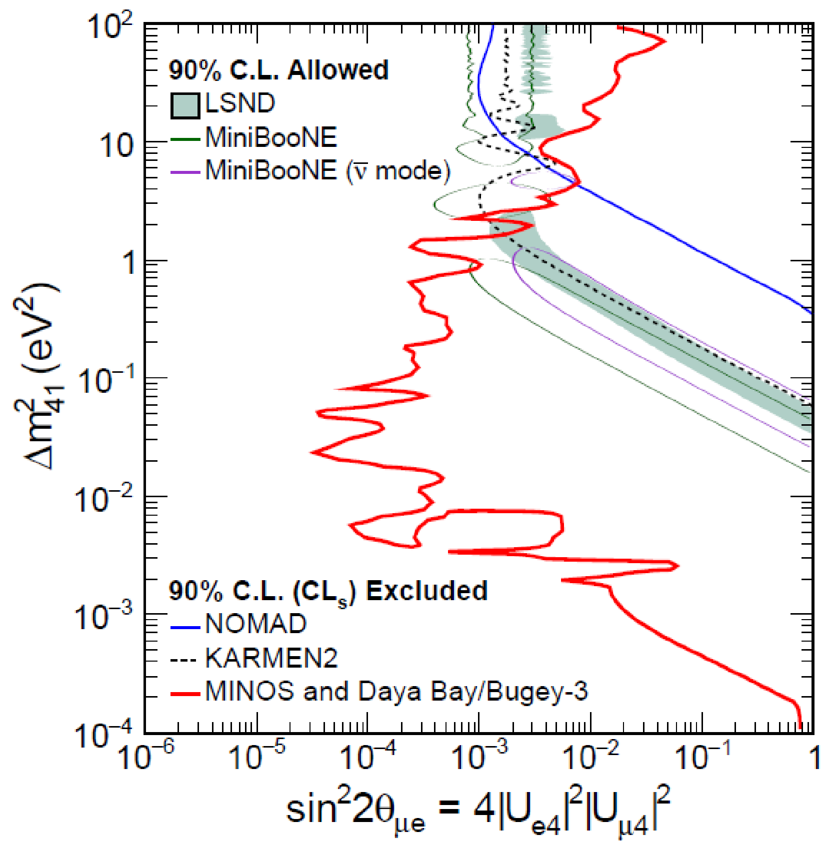


arXiv:1607.01174 (to be published in PRL), factor of 2 improvement to the previous result (PRL 113, 141802, 2014)

Combined Sterile Search

- CLs method is easy to combine results





arXiv:1607.01177 (DYB+MINOS) to be published in PRL

MINOS → θ_{24} with ν_μ disappearance
Daya Bay/Bugey-3 → θ_{14} with (anti)ν_e disappearance

- See past Wine&Cheese seminars

Further Prospect of Current Reactor Neutrino Experiments

- Daya Bay:
  - Expect to reach < 3% uncertainty for both $\sin^2 2\theta_{13}$ and Δm^2_{ee} by 2020
  - Another factor of two improvement in the limit of sterile neutrino search at low Δm^2_{41}
    - Complimentary to the expected results from short-baseline reactor experiments (i.e, PROSPECT) at high Δm^2_{41}

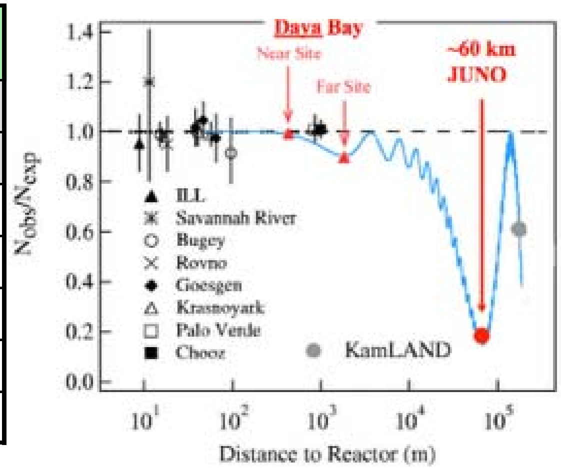
- Combination among Daya Bay, RENO, and Double Chooz is under discussion
  - Below 3% precision of $\sin^2 2\theta_{13}$ by 2017

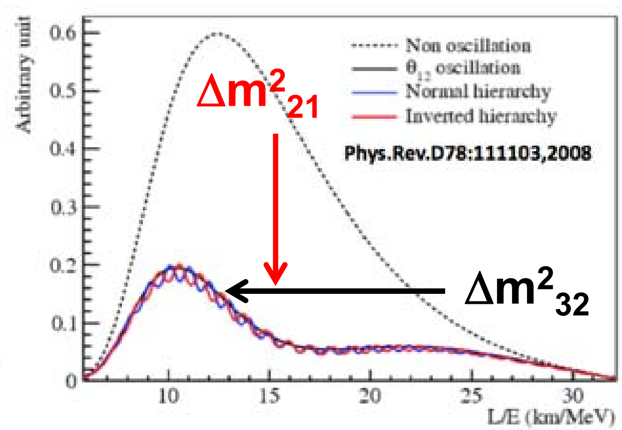
JUNO

- Reactor Power: 36 GW
- Baseline: 53 km
- Detector: 20 kton LS
- σ_E: 3% (2% at 2.5 MeV)
- ν rate: ~60/day
- Background:
  - Accidentals (10%)
  - ^{9}Li (<1%)
  - Fast neutrons (<1%)

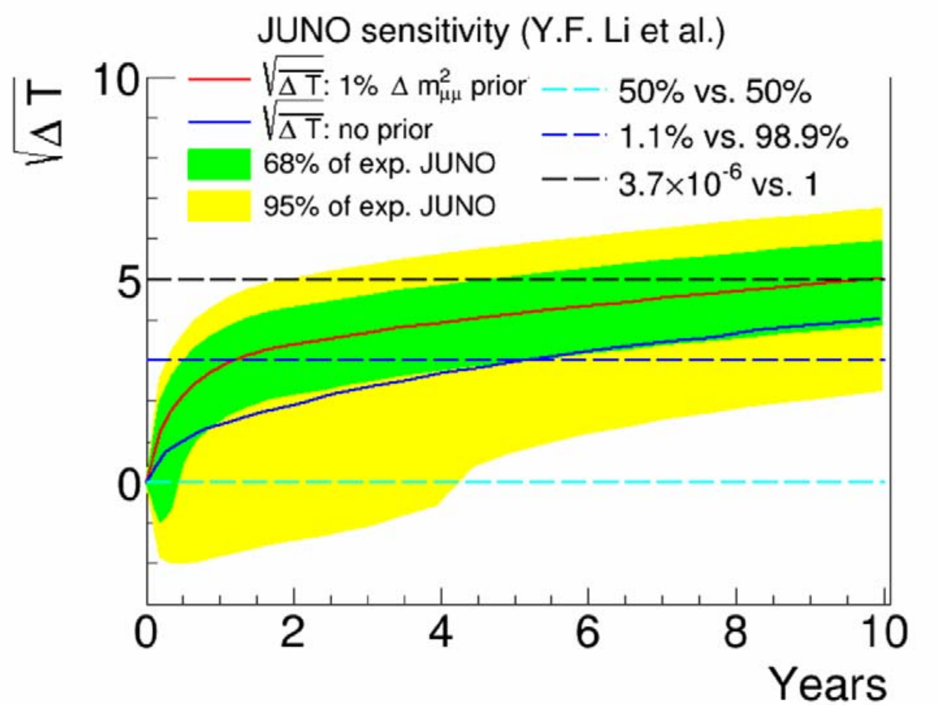
J. Phys. G: Nucl. Part. Phys. 43, 030401 (2016)

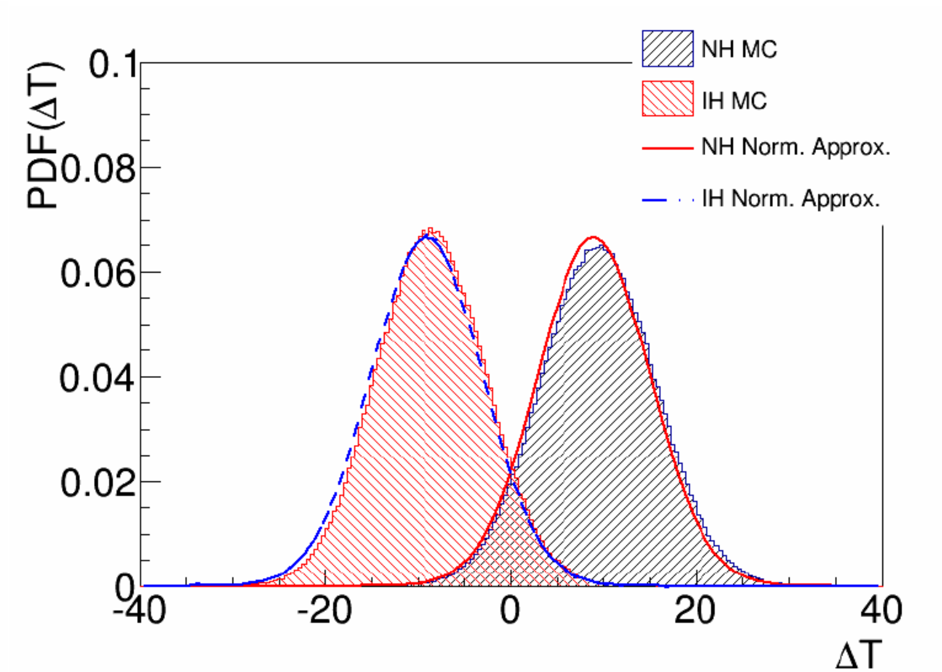
	JUNO	DUNE
$\sin^2 2\theta_{12}$	0.7%	
Δm^2_{21}	0.6%	
$\lvert\Delta m^2_{32}\rvert$	0.5%	0.3%
MH	3–4σ	>5σ
$\sin^2 2\theta_{13}$	14%	3%
$\sin^2 2\theta_{23}$		3%
δ_{CP}		10°





MH Sensitivity (Non-nested Hypothesis Test)





- What's the meaning of MH sensitivity?
  - XQ, A. Tan et al. PRD86, 113011 (2012)
  - M. Blennow et al. JHEP 03, 028 (2014) among others

Summary

- Reactor neutrinos have been and will continue to play an important role in understanding the neutrino properties
  - Previous: KamLAND
  - Current: Daya Bay, RENO, Double Chooz
  - Future: JUNO, PROSPECT …
- Data analysis of reactor neutrinos involves a wide range of statistical techniques
  - Parameter fit, (nested/non-nested) hypothesis tests, unfolding …

Rate-only vs. Shape-only

$$T_{stat} = 2\sum_{j,i}^{ADs,bin}\left(N_{ji}^{pred} - N_{ji}^{obs} + N_{ji}^{obs}\cdot \mathrm{Log}\left(\frac{N_{ji}^{obs}}{N_{ji}^{pred}}\right)\right) = \begin{cases} 2\sum_{j}^{ADs}\left(N_{j}^{pred} - N_{j}^{obs} + N_{j}^{obs}\cdot \mathrm{Log}\left(\frac{N_{j}^{obs}}{N_{j}^{pred}}\right)\right) \\ +2\sum_{j,i}^{ADs,bin} N_{ji}^{obs}\cdot \mathrm{Log}\left(\frac{N_{ji}^{obs}}{N_{ji}^{pred}}\right) - 2\sum_{j}^{ADs}\left(N_{j}^{obs}\cdot \mathrm{Log}\left(\frac{N_{j}^{obs}}{N_{j}^{pred}}\right)\right) \end{cases}$$

AD: Antineutrino Detector

- Rate-only:

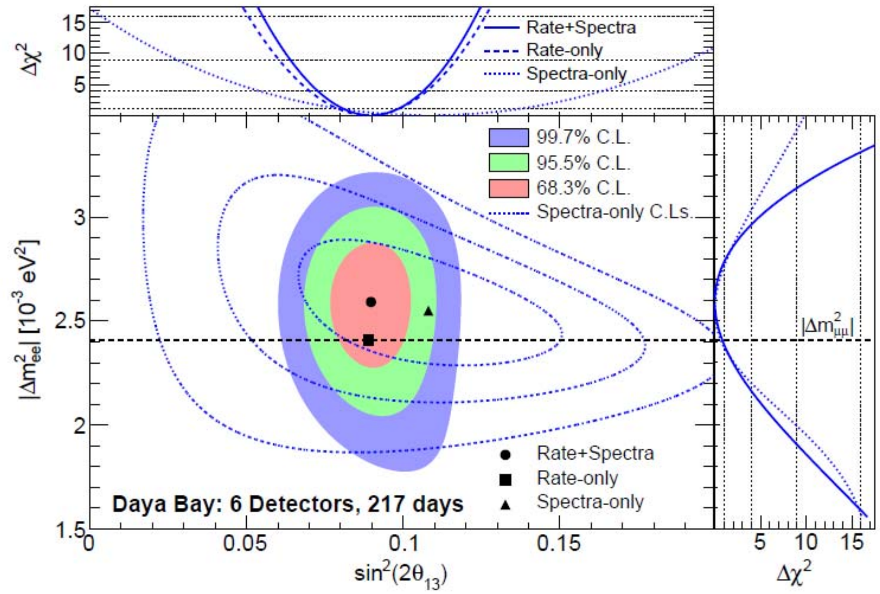
$$2\sum_{j}^{ADs}\left(N_{j}^{pred} - N_{j}^{obs} + N_{j}^{obs}\cdot \mathrm{Log}\left(\frac{N_{j}^{obs}}{N_{j}^{pred}}\right)\right)$$

- Shape-only:

$$2\sum_{j,i}^{ADs,bin} N_{ji}^{obs}\cdot \mathrm{Log}\left(\frac{N_{ji}^{obs}}{N_{ji}^{pred}}\right)$$

$$with \sum_{i} N_{ji}^{pred} = \sum_{i} N_{ji}^{obs}$$

PRL,112, 061801 (2014)



Multinomial distribution first discussed in Baker&Cousins, NIMA, 221, 437 (1984)

Absolute Reactor Anti-Neutrino Flux

- 621 days data
- Effective fission fraction

^{235}U	^{238}U	^{239}Pu	^{241}Pu
56.1%	7.6%	30.7%	5.6%

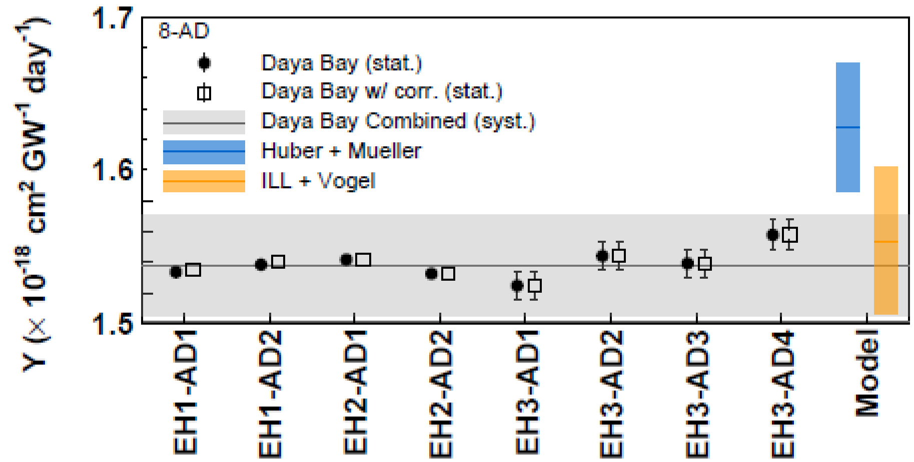
- Daya Bay result:

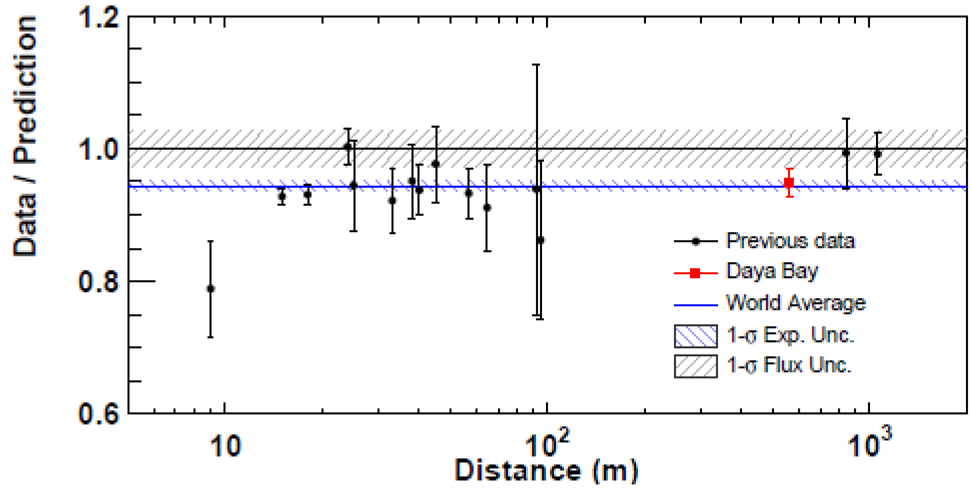
$R_{dyb} = 0.946 \pm 0.020$

- The World Average:

$R_{globe} = 0.943 \pm 0.008$

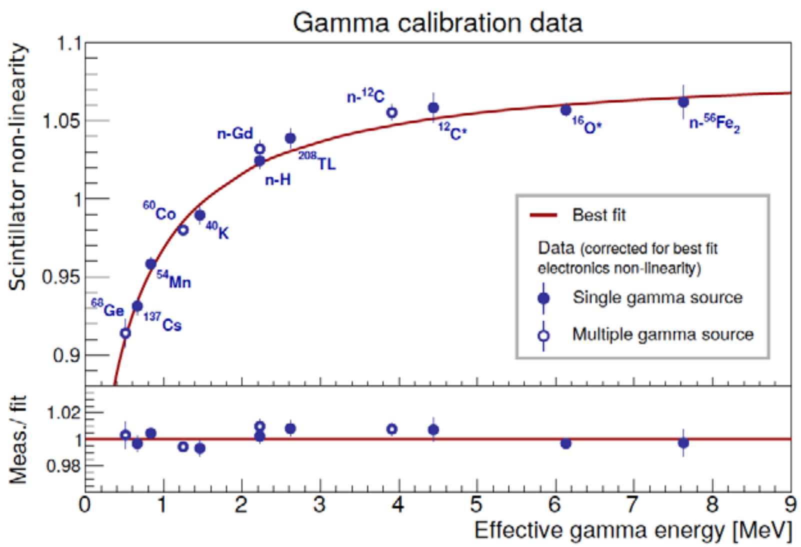
Daya Bay's absolute reactor flux measurement is consistent with previous short baseline experiments

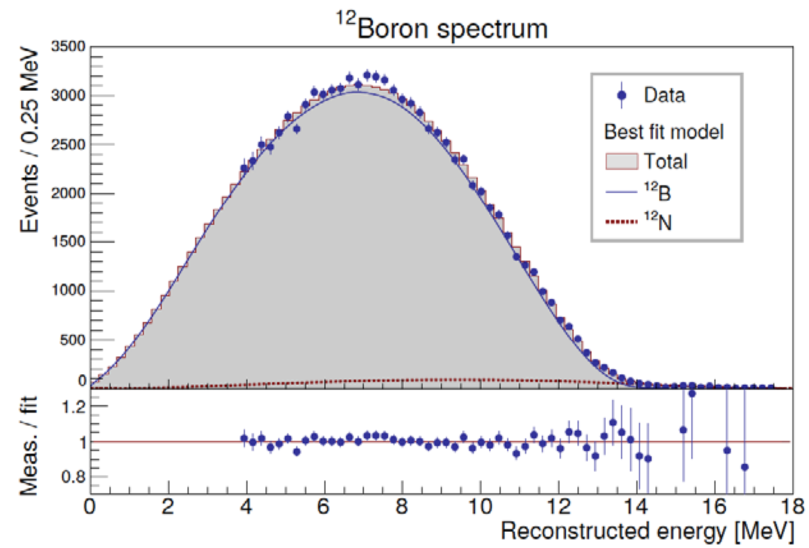




PRL, 116, 061801 (2016) and arXiv:1607.05378

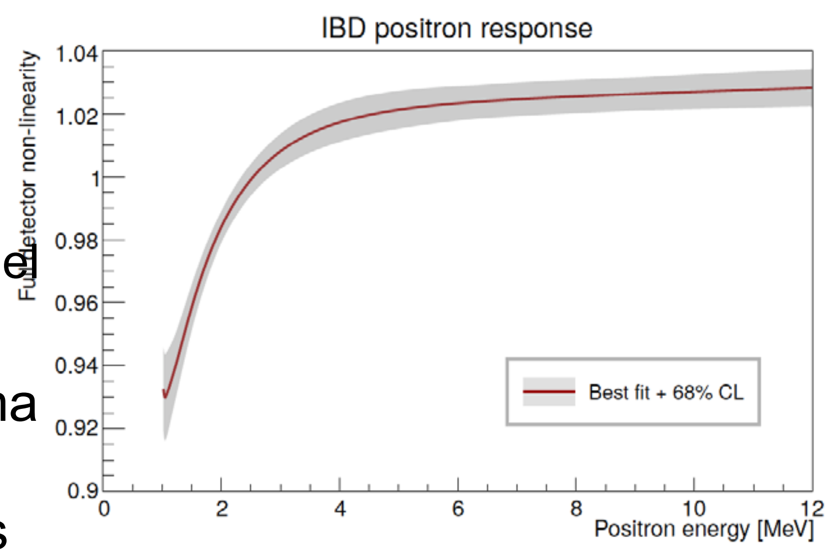
Energy Nonlinearity Calibration





Sources of detector energy nonlinearity

- Scintillator quenching (Birks Law)
- Cherenkov light
- PMT readout electronics
  - Modeled with MC and single channel FADC measurement



Energy model is constrained with gamma (Improved fitting upon Crystal Ball in arXiv:1603.04433) and electron sources

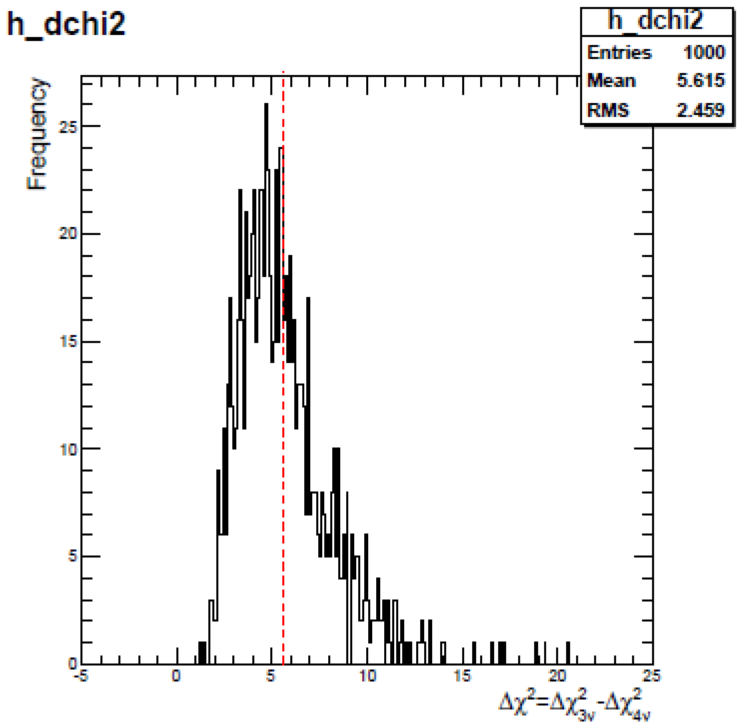
~1% uncertainty (correlated among detectors)

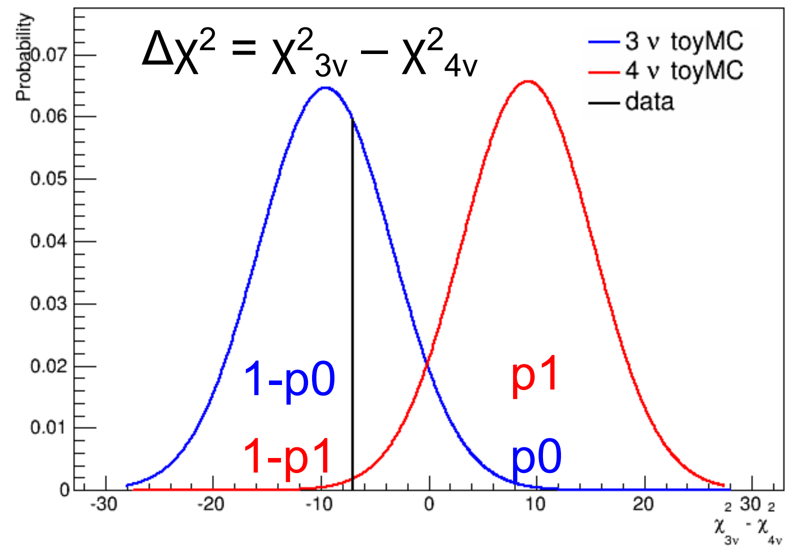
An Independent Check

- When treating the (still smeared) unfolded spectrum as the real (unsmeared) spectrum, additional uncertainties (bias) by are needed, which represents an additional information loss
  - The price that we have to pay for the simpler usage
  - Otherwise, same amount of information generally

- Bias be estimated through pseudo experiments
  - In Daya Bay, we use various predictions of neutrino spectrum → pseudo measurements → unfolded spectrum to be compared with MC truth → determine the size of bias and additional uncertainties needed

Statistical tests: 3-ν or 4-ν ?





$$CL_s = \frac{1-p_1}{1-p_0}$$

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$$CLs = \frac{\left(1 + Erf\left(\frac{\overline{\Delta\chi^2_{4\nu}} - \Delta\chi^2_{data}}{\sqrt{8\overline{\Delta\chi^2_{4\nu}}}}\right)\right)}{\left(1 + Erf\left(\frac{\overline{\Delta\chi^2_{3\nu}} - \Delta\chi^2_{data}}{\sqrt{8\overline{\Delta\chi^2_{3\nu}}}}\right)\right)}$$

- **Data is consistent with 3-ν hypothesis with FC test**
  No evidence for sterile neutrino

  - $\Delta\chi^2_{data} = 5.6$; p-value is 0.41

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