Sensitivity to CP violation in neutrino oscillation experiments

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Based on:
Blennow, PC, Fernandez-Martinez, arXiv: 1407.3274 [hep-ph]
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The leptonic mixing matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric | Reactor | Solar

$$\times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}$$

$$\theta_{13} \sim 9^\circ$$

$$\theta_{23} \sim 45^\circ$$

$$\theta_{12} \sim 33^\circ$$

$$\Delta m^2_{21} \sim 7.5 \times 10^{-5}\,\mathrm{eV}^2$$

$$|\Delta m^2_{31}| \sim 2.5 \times 10^{-3}\,\mathrm{eV}^2$$

For up-to-date results and/or precise values, see e.g. www.nu-fit.org or Capozzi et al, 1601.07777

CP violation searches

Three-family ν_e appearance oscillation probability (vacuum):

Atmospheric

CP interference

$$
\begin{aligned}
P_{\mu e} \simeq\ & \boxed{(\sin\theta_{23}\sin 2\theta_{13})^2 \sin^2(\Delta_{31})} \\
+\ & \boxed{\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\Delta_{21}\sin(\Delta_{31})\cos(\pm\delta+\Delta_{31})} \\
+\ & \boxed{(\cos\theta_{23}\sin 2\theta_{12})^2\Delta_{21}^2}
\end{aligned}
$$

Solar

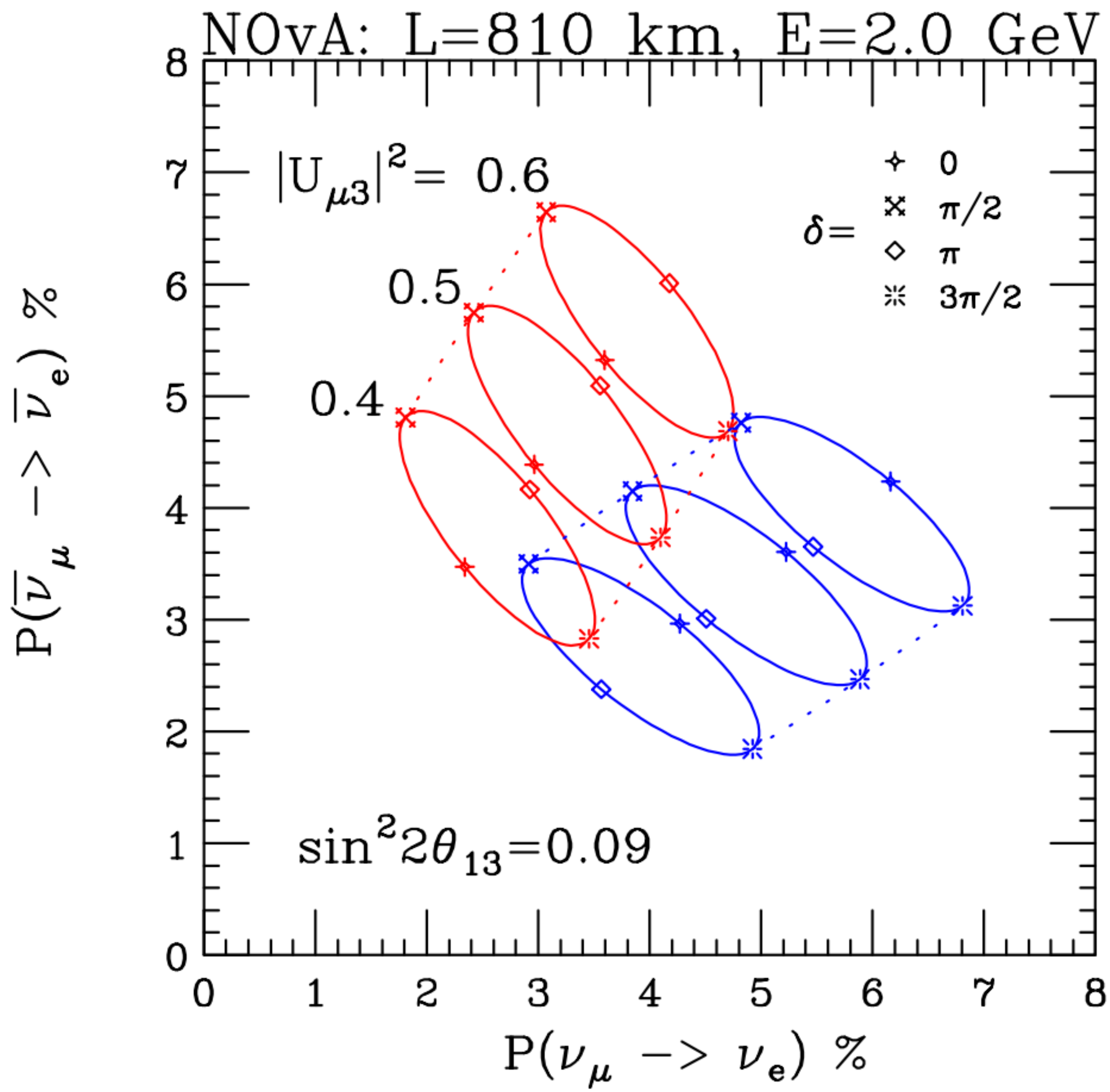
Neutrino/Antineutrino

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

$$\Delta_{21} \sim 0.03\Delta_{31}$$

Cervera et al, hep-ph/0002108
(see also Akhmedov et al, hep-ph/0402175, and
Asano and Minakata, 1103.4387)

CP violation searches



(Figure by Stephen Parke)

A test statistic for CP conservation

$$\chi^2 = -2 \log \mathcal{L}$$

Likelihood

$$\boxed{S = \min_{\delta=0,\pi} \chi^2 - \min_{\text{global}} \chi^2}$$

In the literature, the sensitivity to CP violation is usually computed under several assumptions:

$\rightarrow$ It is assumed that S is distributed as a chi-square with 1 d.o.f.

$\rightarrow$ The Asimov data set (data without statistical fluctuations) is assumed to be representative for the median outcome of the experiment

A test statistic for CP conservation

$$\boxed{S = \min_{\delta=0,\pi} \chi^2 - \min_{\text{global}} \chi^2}$$

- This problem is similar to the case of neutrino mass hierarchy determination, with some differences:
  - The test statistic in this case is always defined to be positive
  - The parameter in this case is continuous, and we are testing just two points in parameter space
  - The parameter we are testing is cyclic!
  - We can have multiple minima (degeneracies $\rightarrow$ more on this later)

Testing the CP conservation hypothesis

$$\boxed{S = \min_{\delta=0,\pi} \chi^2 - \min_{\text{global}} \chi^2}$$

Standard procedure to test the CP conservation hypothesis:

1) Simulate a large number of experimental realizations using the null hypothesis (CP conservation)
2) The value of S is computed for each realization. This gives the distribution of S
3) CP conservation is rejected at CL x if the experimental outcome gives an S among the 1-x most extreme values

This outline is valid if we have an experiment that has already taken data

To determine future sensitivities, we need to repeat this procedure for all possible true values of the oscillation parameters (frequentist approach)

External constraints

$$\chi^2 = \chi_0^2(\xi) + \frac{(\xi - \bar{\xi})^2}{\sigma_\xi^2}$$

When determining the distribution of S (ie, for $\delta = 0,\pi$), statistical fluctuations are also considered for the experiments determining the priors*:

- Calibration is done assuming a true value of ξ (ξ_{true})
- The values of $\bar{\xi}$ are randomized according to a normal distribution around ξ_{true}
- The final chi-square is obtained after minimization over the nuisance parameters (ξ)

*Prior constraints = our prior knowledge on other oscillation parameters and/or systematic uncertainties relevant to the experiment (xsec, flux, etc)

External constraints

$$\chi^2 = \chi_0^2(\xi) + \frac{(\xi - \bar{\xi})^2}{\sigma_\xi^2}$$

When computing the median experimental outcome expected for a CP-violating true value of delta, however, the value of $\bar{\xi}$ is kept fixed.

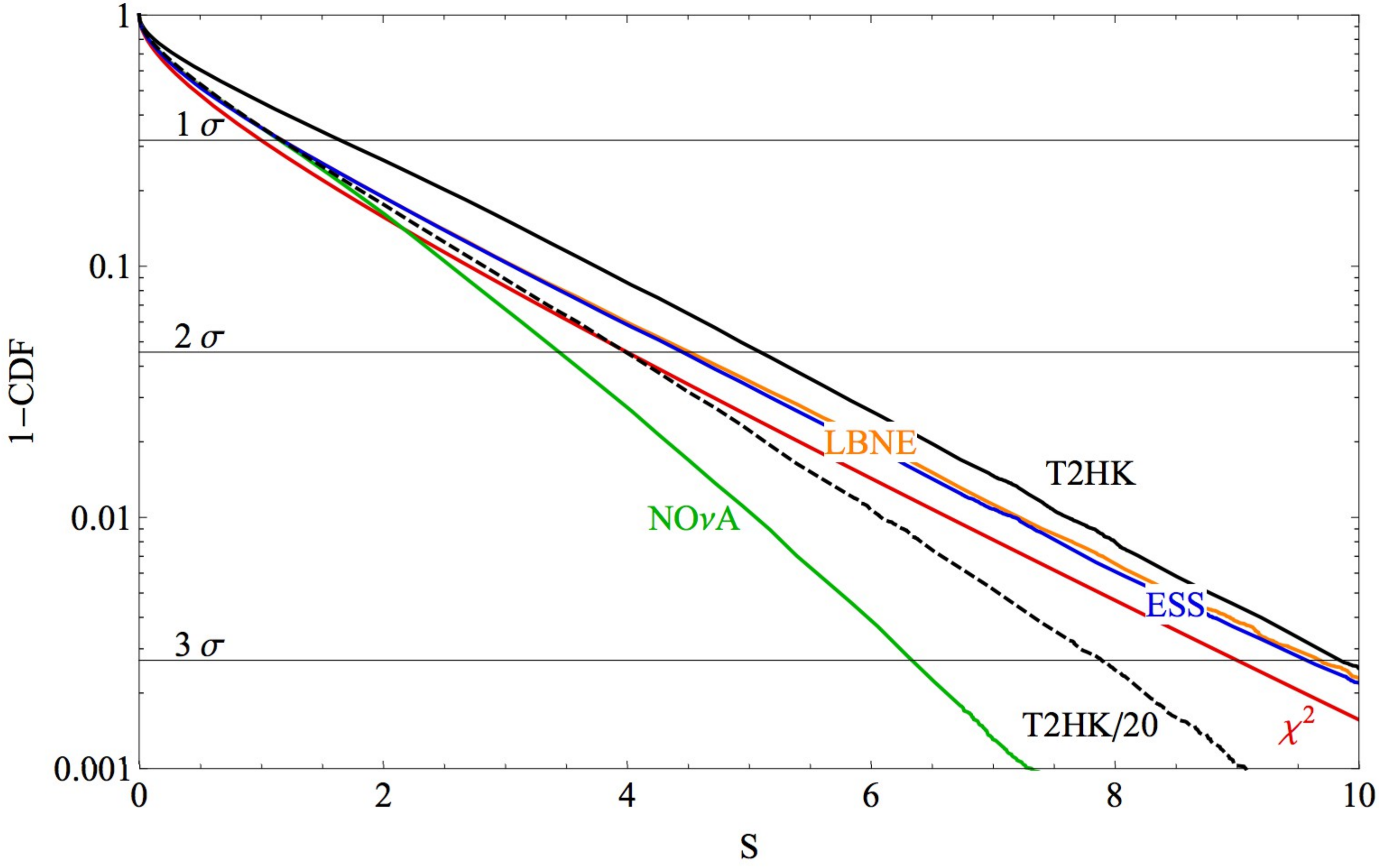
$\rightarrow$ This gives the probability of reaching a given confidence level <u>assuming</u> a certain set of true values for the external constraints

*Prior constraints = our prior knowledge on other oscillation parameters and/or systematic uncertainties relevant to the experiment (xsec, flux, etc)

Experiments considered

- T2HK (560 kton fiducial, 750 kW, 7+3 yrs)
  - Events in nu/nubar: 4800/2000
- LBNE (1.2 MW, 34kt, 5+5 yrs)
  - Events in nu/nubar: 1500/400
- ESSnuSB (L = 540 km, 2.5 GeV protons, 8+2 yrs)
  - Events in nu/nubar: 200/160
- NOvA (6e20 PoT/yr, 3+3 yrs, 13 kton)
  - Events in nu/nubar: 60/18

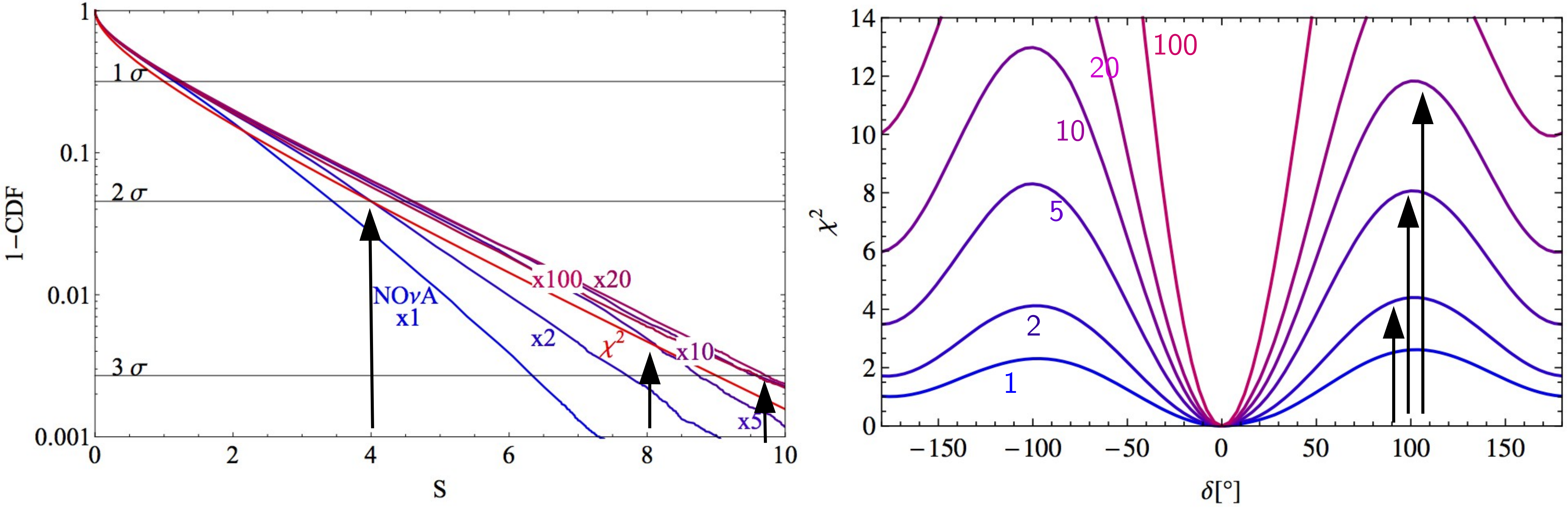
Distribution of S



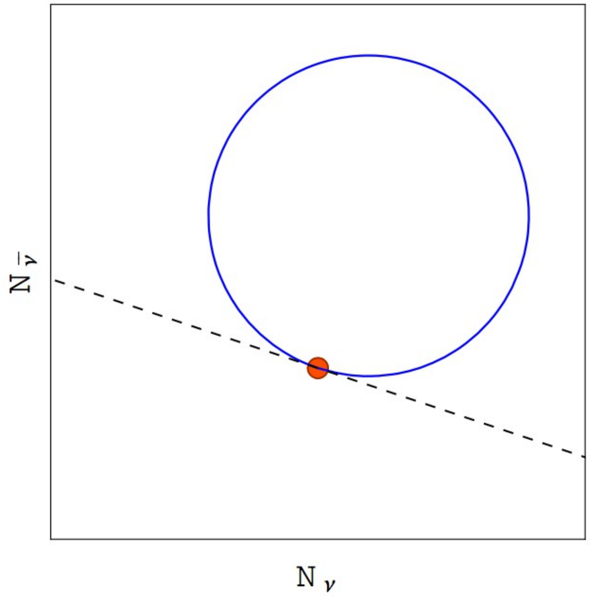
*CDF = Cumulative Distribution Function

Distribution of S

The deviations from the chi-squared behavior depend with the sensitivity of the experiment. The distribution is also modified if sign degeneracies are present:



Toy-model

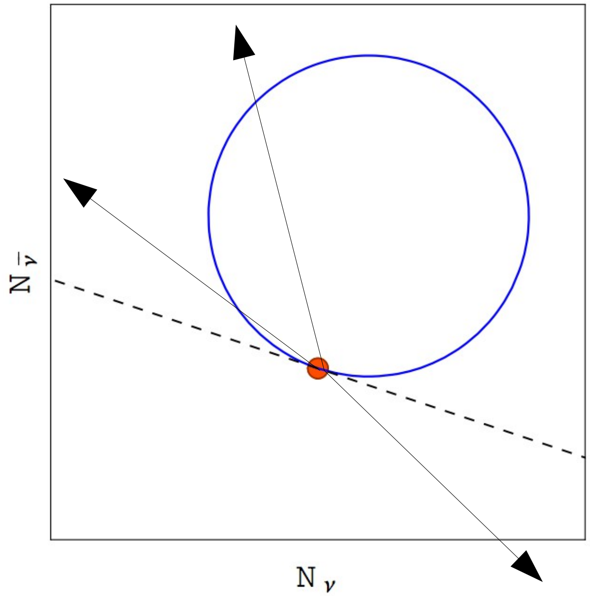


The outcome space is determined by the number of neutrino and antineutrino events

- The red point is CP-conserving
- All possible experimental outcomes for different CP phases form an ellipse

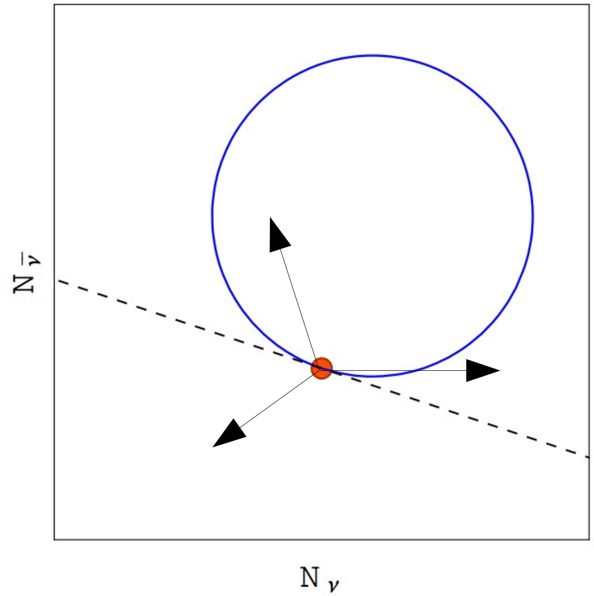
$\rightarrow$ The dashed line represents the expectation for a linear space (Wilks' theorem)

Toy-model



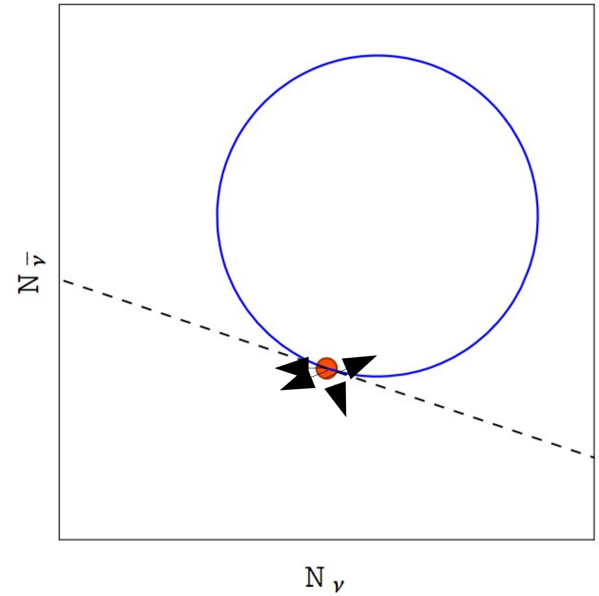
Case 1: experiment with limited sensitivity to the CP phase

$$S < \chi^2$$



Case 2: some sensitivity to the CP phase, but still sees it is a cyclic variable

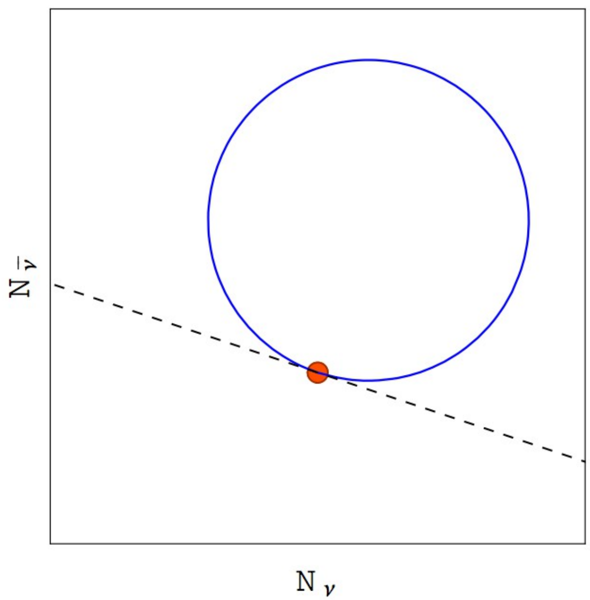
$$S > \chi^2$$

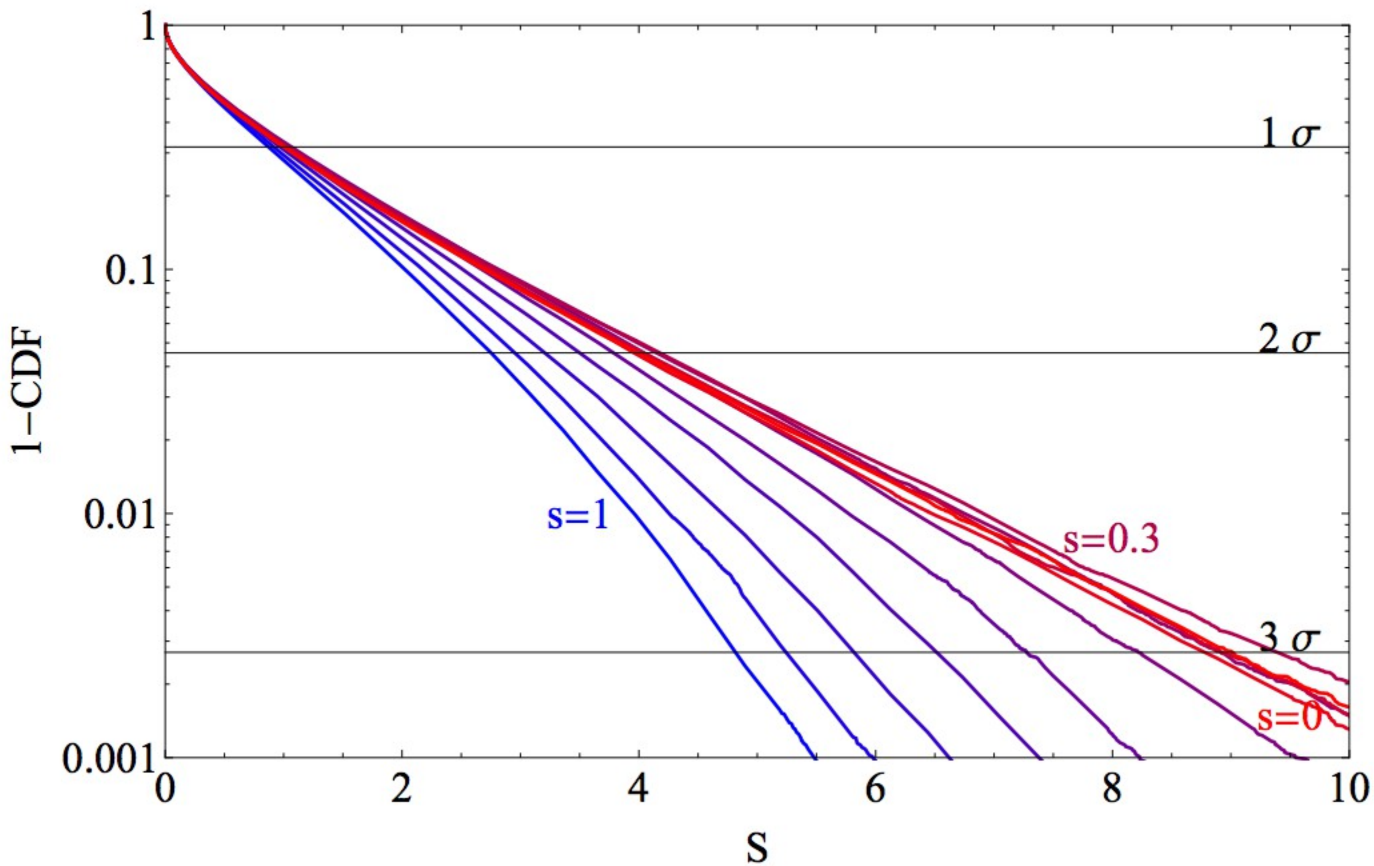


Case 3: so sensitive to CP phase, it does not see it's cyclic anymore

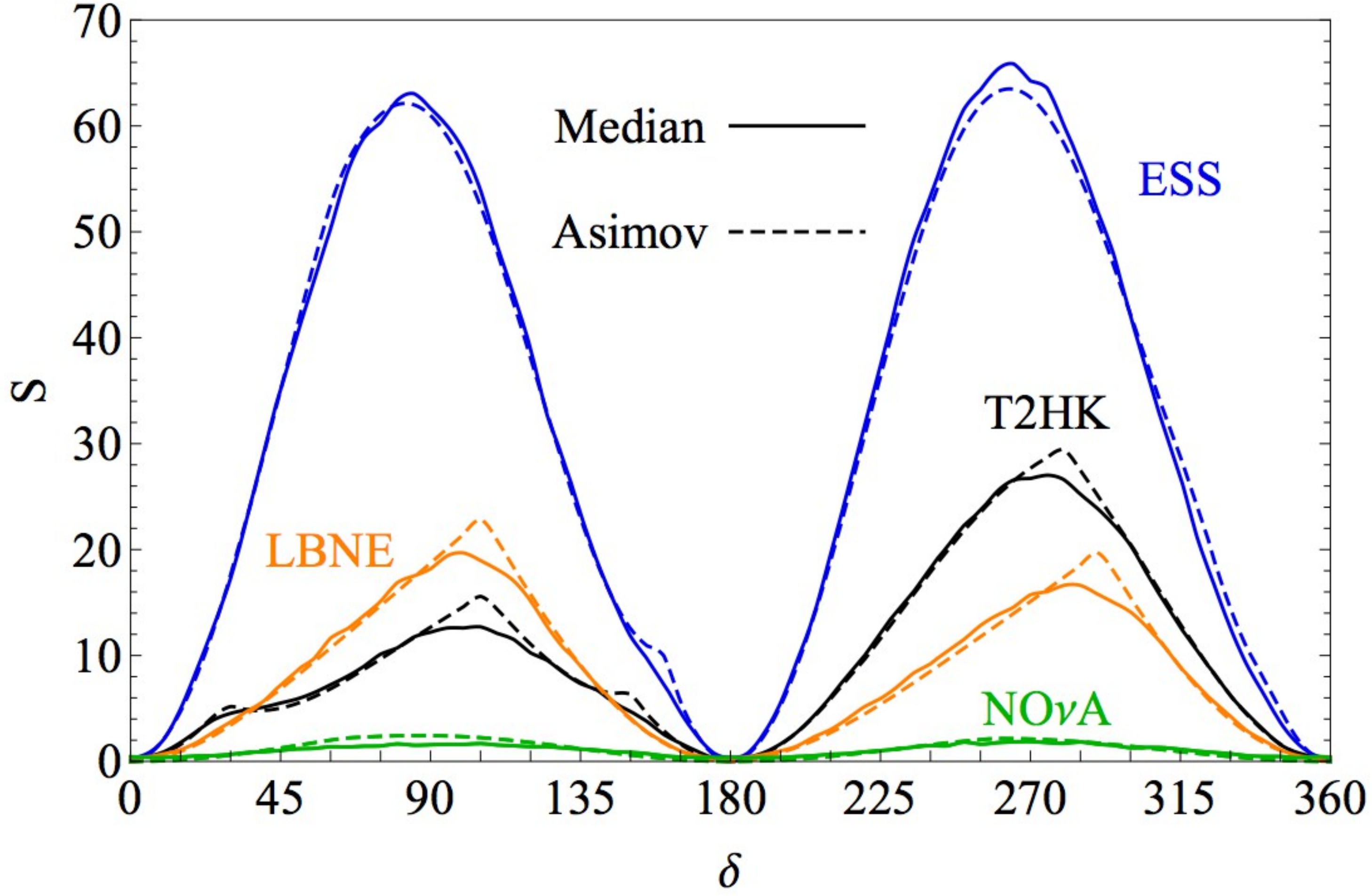
$$S \to \chi^2$$

Toy-model



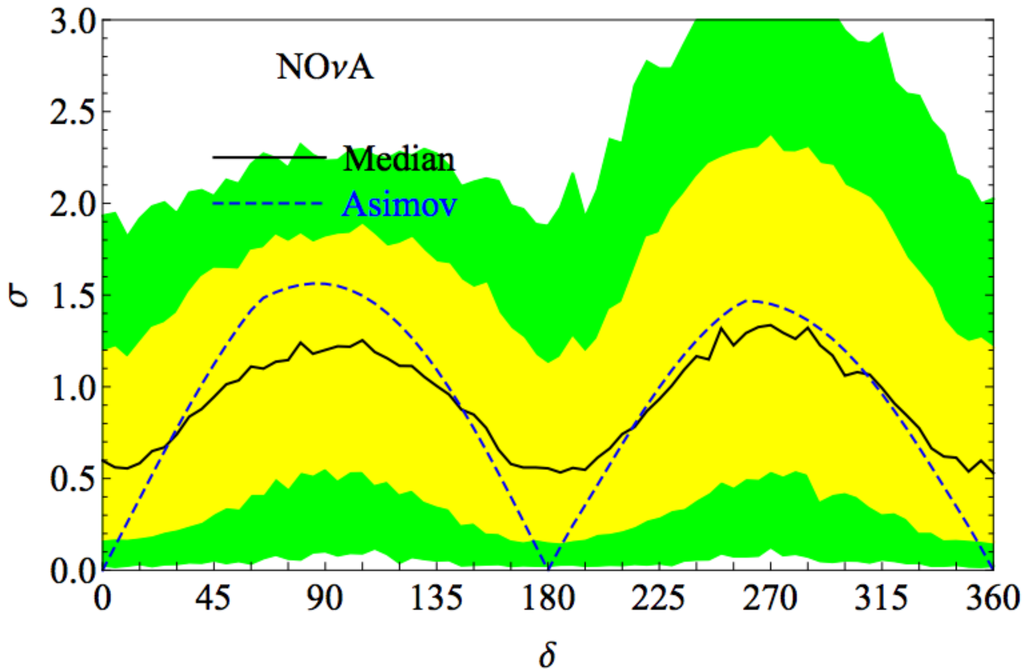


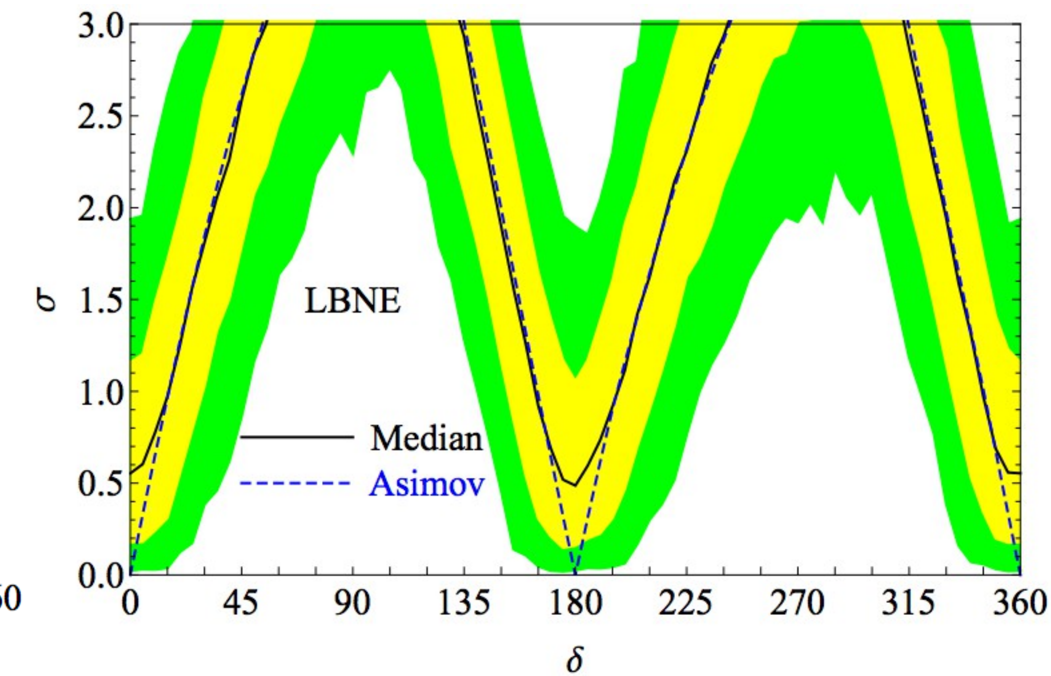
Median outcome?



Final sensitivities

Now we can compute the final sensitivities to CP violation, ie, the CL at which CP conservation can be rejected





68% of experiments

95% of experiments

Summary and conclusions

- The sensitivity to CP violation in neutrino oscillation experiments is always computed assuming a chi-squared distribution
  - But the assumptions needed for Wilks' theorem are violated
- We have simulated statistical fluctuations for several experimental setups and studied the distribution of S
  - We find large deviations from a chi-squared distribution
  - Size of deviations depend on the sensitivity of a given experiment and on the presence of degeneracies
  - Final sensitivities not so far off, though. Largest deviations observed for NOvA
- We have explained the deviations using a very simple toy model based on geometric arguments

Thank you!!