Jet Substructure at the LHC: New Observables and New Calculations

Ian Moult

Berkeley Center for Theoretical Physics
Lawrence Berkeley Laboratory

$D_2^{(2,2)}$ Spectrum
Vincia $p_T$ Ordered Hadron Level
1TeV, $e^+e^- \rightarrow$ dijets
$m_j \in [80, 100]$ GeV, $R=1$

Relative Probability

$D_2^{(2,2)}$

Z Boson Analytic
Z Boson Monte Carlo
Quark Jet Analytic
Quark Jet Monte Carlo

Boosted Z Selection
QCD

• QCD is an $SU(3)$ gauge theory:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \sum_f \bar{q}_f (i \slashed{D} - m_f) q_f$$

• Microscopic degrees of freedom are quarks and gluons.

• In scattering experiments we observe collimated sprays of hadrons, called jets:

  • Jets act as proxies for quarks and gluons.
  • Jets are our probe of the underlying microscopic dynamics.
Jets at PETRA

- Kinematics of jets used to infer gluon emission in hard scattering.

2-Jet Event

3-Jet Event
Jets at the LHC: Internal Structure

- Internal structure of jets resolved due to excellent detector resolution.
- Electroweak scale objects, $W/Z/H$ or $t$ can have sufficiently high $p_T$ to appear inside a jet.

Boosted Tops

Event Display

- Revolutionizes the types of questions we can/must ask about jets: $\rightarrow$ jets have substructure!
• Very complicated structure!

• Involves interactions at many hierarchical energy scales.

$Q \sim \text{TeV}$

$p_{TJ} \sim 500 \text{ GeV}$

$m_J \sim 100 \text{ GeV}$

$m_J^2/p_{TJ} \sim 20 \text{ GeV}$

$m_b \sim 4 \text{ GeV}$

$\Lambda_{\text{QCD}} \sim 100 \text{ MeV}$
A Theorists View

- Tractable due to factorization:

\[ \frac{d\sigma}{dM_1 \cdots} = \sum_{\{\kappa\}} \text{tr} H_{\kappa} I I J_{\kappa_j} \otimes \cdots \otimes J_{\kappa_j} S_{\kappa_s} \otimes f_{p/i} f_{p/j} \otimes f_{k \rightarrow H} \otimes \cdots \otimes f_{l \rightarrow H} \otimes F \]

- Written as a convolution of single scale objects.
- High energy dynamics integrated out, replaced by sources.
- Convenient formulation in effective field theory.

Energy Scale

- $Q \sim \text{TeV}$
- $p_{T,J} \sim 500 \text{ GeV}$
- $m_J \sim 100 \text{ GeV}$
- $m_J^2/p_{T,J} \sim 20 \text{ GeV}$
- $m_b \sim 4 \text{ GeV}$
- $\Lambda_{\text{QCD}} \sim 100 \text{ MeV}$
Recent Progress

- Significant recent progress in description of more complex final states.

- Hard Scattering: $H_{\alpha}$

\[ \text{Inclusive Jet Multiplicity} \]

\[ \text{NJet + Sherpa} \]

\[ \text{More Legs} \quad pp \rightarrow \text{jets at 7 TeV} \]

\[ \text{More Loops} \quad pp \rightarrow h + X \text{ gluon fusion} \]

[Badger, Biedermann, Uwer, Yundin]

[Anastasiou, Duhr, Dulat, Herzog, Mistlberger]

[Gehrmann et al.], [Baikov et al.]
Recent Progress

- Significant recent progress in description of more complex final states.
  - Dynamics of QCD radiation (SCET):
    $$\mathcal{I} \mathcal{I} \mathcal{I} \mathcal{J}_{\kappa_i} \times \cdots \times \mathcal{J}_{\kappa_j} \otimes S_{\kappa_s} \otimes F$$
  - Calculate properties of individual jets.

$$m^2_j = \sum_{i,j \in J} p_i \cdot p_j$$

Gluon Jets

Quark Jets

[Jouttenus, Stewart, Tackmann, Waalewijn]
Jet Substructure

- What if I want to look inside a jet?

- Jet substructure: measure properties (charge, energy, etc) of radiation in a jet to extract information about its origin.
Outline

- New Observables for Jet Substructure
- Effective Field Theories for Jet Substructure
- Analytic Calculations for Jet Substructure
New Observables for Jet Substructure
Back to Basics

- What is an observable?

\[ F_N(P) = \sum E_{i_1} \cdots E_{i_N} f_N(\hat{p}_{i_1}, \cdots, \hat{p}_{i_N}) \]

- Linear in the energies by Infrared and Collinear (IRC) safety.
- \( f_N \) is symmetric, and \( f_N \to 0 \) if \( \hat{p}_i \parallel \hat{p}_j \)

- Known that from this one can reconstruct any IRC safe observable.

- Is this useful for jet substructure?
  \[ \implies \text{Need to choose a basis.} \]
Generalized Energy Correlation Functions

\[ R_{ij} = \sqrt{\Delta y_{ij}^2 + \Delta \phi_{ij}^2} \]

\[ i e_j^{(\beta)} = \frac{1}{p_{TJ}^{ij}} \sum_{1 \leq n_1 < \ldots < n_j \leq n} p_{Tn_1} p_{Tn_2} \cdots p_{Tn_j} \min \left( \prod_{s,t} R_{st}^{\beta} \right) \]

- Example: Three different ways to probe three particle correlations.

1. \[ e_3^{(\beta)} = \frac{1}{p_{TJ}^3} \sum_{1 \leq i < j < k \leq n_J} p_{Ti} p_{Tj} p_{Tk} \min \left[ R_{ij}^{\beta}, R_{ik}^{\beta}, R_{jk}^{\beta} \right] \]  
2. \[ e_3^{(\beta)} = \frac{1}{p_{TJ}^3} \sum_{1 \leq i < j < k \leq n_J} p_{Ti} p_{Tj} p_{Tk} \min \left[ R_{ij}^{\beta} R_{ik}^{\beta}, R_{ij}^{\beta} R_{jk}^{\beta}, R_{ik}^{\beta} R_{jk}^{\beta} \right] \]  
3. \[ e_3^{(\beta)} = \frac{1}{p_{TJ}^3} \sum_{1 \leq i < j < k \leq n_J} p_{Ti} p_{Tj} p_{Tk} R_{ij}^{\beta} R_{ik}^{\beta} R_{jk}^{\beta} = e_3^{(\beta)} \]

- Flexible basis for substructure observables.
Power Counting Observables

- How can we combine these observables to identify features of a jet?
- Example: Boosted $W/Z/H$ discrimination:

1. Write down Effective Field Theory (EFT) description of each configuration.
2. Identify region of validity of EFTs in terms of observables $\epsilon^{(\beta)}_i$.
3. Shared boundaries define how to separate.
Power Counting Observables: \( D_2 \)

- Consider using \( e_2^{(\beta)}, e_3^{(\beta)} \).
- Phase space separated by contours of \( D_2^{(\beta)}: e_3^{(\beta)} = D_2^{(\beta)} (e_2^{(\beta)})^3 \)

\[
\left( e_2^{(\beta)} \right)^3 < e_3^{(\beta)} < \left( e_2^{(\beta)} \right)^2
\]

\[
e_3^{(\beta)} < \left( e_2^{(\beta)} \right)^3
\]
Phase Space

- Define the discriminant

\[
D_2^{(\beta)} = \frac{e_3^{(\beta)}}{(e_2^{(\beta)})^3} = \frac{(\beta)}{3}
\]

([Larkoski, Moult, Neill])

Contours of \(D_2^{(\beta)}\): 1-Prong, 2-Prong

\(D_2^{(2,2)}\) Spectrum
Vincia \(p_T\) Ordered Hadron Level
1 TeV, \(e^+e^-\) \(\rightarrow\) dijets
\(m_J \in [80, 100]\) GeV, \(R=1\)

Relative Probability

- Z Boson Analytic
- Z Boson Monte Carlo
- Quark Jet Analytic
- Quark Jet Monte Carlo
Summarizing the Observables

- Observables can be designed for a variety of purposes

Quark vs. Gluon

W/Z/H Tagging

Top Tagging

Focus on $W/Z/H$:

Searches at ATLAS

![Probability Density](U_3^{(0.2)})

$U_3^{(0.2)}$

R=0.6, $p_T>500$ GeV (Pythia 8.219)

Probability Density

Quark

Gluon

Searches at CMS

$D_2^{(\beta)} = \frac{e_3^{(\beta)}}{(e_2^{(\beta)})^3} = \frac{1}{(e_2^{(\beta)})^3}$

$N_2^{(\beta)} = \frac{2e_3^{(\beta)}}{(e_2^{(\beta)})^2 (e_2^{(\beta)})^3}$
The Shape of Jets at the LHC: $D_2$

- $D_2$ is default tagger for ATLAS.

\[ D_2^{(\beta)} = \frac{e_3^{(\beta)}}{(e_2^{(\beta)})^3} = \left(\frac{\Delta R(large\ jet, b\text{-jet}) > 1.0}{\Delta R(large\ jet, b\text{-jet}) > 1.0}ight)^3 \]

### $D_2$ on QCD Jets

<table>
<thead>
<tr>
<th>ATLAS Preliminary</th>
<th>Data 2015+2016</th>
<th>$\sqrt{s} = 13$ TeV, 36.5 fb$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trimmed anti-$k$, $R=1.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dijet Selection</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_T &gt; 450$ GeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_{comb} &gt; 50$ GeV</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Events / 0.1**

- $10^5$
- $5 	imes 10^4$
- $2.5 	imes 10^4$
- $1.5 	imes 10^4$
- $1 	imes 10^4$
- $0.5 	imes 10^4$
- $1 	imes 10^3$
- $5 	imes 10^2$
- $1 	imes 10^2$
- $5 	imes 10^1$
- $1 	imes 10^1$
- $5 	imes 10^0$
- $1 	imes 10^0$

**Data/Pred.**

- $1.5$
- $1.25$
- $1$
- $0.75$
- $0.5$
- $0$

Leading Large-$R$ Jet $D_2^{\beta=1}$

### $D_2$ on $W$ Jets

<table>
<thead>
<tr>
<th>ATLAS Preliminary</th>
<th>Data 2015+2016</th>
<th>$\sqrt{s} = 13$ TeV, 36.5 fb$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trimmed anti-$k$, $R=1.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jet $p_T &gt; 200$ GeV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_{comb} &gt; 50$ GeV</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Events / 0.12**

- $16000$
- $14000$
- $12000$
- $10000$
- $8000$
- $6000$
- $4000$
- $2000$
- $1000$
- $500$
- $0$

Large-$R$ Jet $D_2^{\beta=1}$
Applications: High Mass Resonances

- Particular interest in possibility of high mass resonances decaying to dibosons: $WW$, $WZ$, $HZ$, $\gamma\gamma$, $\gamma Z$, ...
Applications: Dark Matter Searches

- Dark Matter Searches:

\[ E_{miss}^{T} \]

\[ \text{Observed 95\% CL Limits} \]
\[ \text{Expected 95\% CL Limits} \]
\[ \sigma_1 \pm \text{Expected} \]
\[ \sigma_2 \pm \text{Expected} \]

\[ \text{Run 1 Observed} \]

\[ m_{\chi} [GeV] \]

\[ m_{Z} [GeV] \]

ATLAS
\[ \sqrt{s} = 13 \text{ TeV}, 3.2 \text{ fb}^{-1} \]
\[ Z\text{-2HDM Simplified Model} \]
\[ \tan(\beta) = 1, m_{\chi} = 100 \text{ GeV} \]

Boosted regime essential to explore heavy mediator masses.

\[ h \]

\[ \tilde{\chi} \]

\[ \chi \]

\[ m_{\tilde{\chi}} = 1 \text{ GeV}, m_{Z'} = 2 \text{ TeV} \]

\[ \beta \tan(\beta) \]

\[ \text{Resolved} \]

\[ \text{Merged} \]

\[ \text{Substructure} \]
$N_2$ at CMS

- $N_2$ measured by CMS.

\[ N_2^{(\beta)} = \frac{2e_3^{(\beta)}}{(e_2^{(\beta)})^2} \left( \frac{3}{\beta} \right)^3 \]
Low Mass Searches

- Low mass $Z'$ search in dijet channel.

Jet Mass Distribution

- Find $Z' = Z/W$!
- Probe new low mass, low x-sec region of parameter space!
Applications: $H \rightarrow b\bar{b}$

- Towards boosted $H \rightarrow b\bar{b}$.

**Without $b$-tag**

**With $b$-tag**
Effective Field Theory Frameworks for Jet Substructure

Soft Subjet
\[ e_3^{(\alpha)} \ll (e_2^{(\alpha)})^3 \]
\[ e_2^{(\alpha)} \sim e_2^{(\beta)} \]
A Standard QCD Calculation

- QCD has soft and collinear singularities.
- If one measures an observable $\tau$ on a jet,

$$\frac{d\sigma}{d\tau} = -\frac{2\alpha_s C_F}{\pi} \frac{\log \tau}{\tau} \left( 1 - \frac{\alpha_s C_F \log^2 \tau}{\pi} + \frac{1}{2} \left( \frac{\alpha_s C_F \log^2 \tau}{\pi} \right)^2 - \frac{1}{6} \left( \frac{\alpha_s C_F \log^2 \tau}{\pi} \right)^3 + \cdots \right)$$

$$= -\frac{2\alpha_s C_F}{\pi} \frac{\log \tau}{\tau} \left( e^{-\frac{\alpha_s C_F \log^2 \tau}{\pi}} \right)$$

- All orders resummation necessary.
$D_2$

- Recall $D_2^{(\beta)} = \left(\frac{e_3^{(\beta)}}{e_2^{(\beta)}}\right)^3$
- First non-zero at two-emissions
- Cross section for $D_2$ computed by marginalization:

$$\frac{d\sigma}{dD_2} = \int d\alpha \frac{d\sigma}{de_2^{(\alpha)} de_3^{(\alpha)}} \delta \left(D_2 - \frac{e_3^{(\alpha)}}{\left(e_2^{(\alpha)}\right)^3}\right) \frac{d\sigma}{de_2^{(\alpha)} de_3^{(\alpha)}}$$
• Cross section for \( D_2 \) computed by marginalization:

\[
\frac{d\sigma}{dD_2} = \int d(e_2^{(\alpha)}) d(e_3^{(\alpha)}) \delta \left( D_2 - \frac{e_3^{(\alpha)}}{(e_2^{(\alpha)})^3} \right) \frac{d\sigma}{d(e_2^{(\alpha)}) d(e_3^{(\alpha)})}
\]

• For each value of \( D_2 \) contour of integration passes through singular region of phase space \( \Rightarrow \) not computable in fixed order perturbation theory!
Cross section for $D_2$ computed by marginalization:

$$\frac{d\sigma}{dD_2} = \int d\epsilon_2^{(\alpha)} d\epsilon_3^{(\alpha)} \delta \left( D_2 - \frac{\epsilon_3^{(\alpha)}}{(\epsilon_2^{(\alpha)})^3} \right) \frac{d\sigma}{d\epsilon_2^{(\alpha)} d\epsilon_3^{(\alpha)}}$$

For each value of $D_2$ contour of integration passes through singular region of phase space $\Rightarrow$ not computable in fixed order perturbation theory!
\( D_2 \)

- \( D_2 \) can be computed in resummed perturbation theory.
- Resummation regulates singular region of phase space prior to integration over the contour: Sudakov Safety. [Larkoski, Thaler]
- Result starts at \( \alpha_s \) and with a single color factor:

\[
\frac{d\sigma}{dD_2} \propto \frac{\alpha_s C_F}{\pi} e^{-\frac{\alpha_s}{\pi} \frac{C_A}{2} \log^2 D_2} \]

\[
\propto \frac{\alpha_s C_F}{\pi} \frac{1}{D_2} + \ldots
\]

- Other examples exist with \( \sqrt{\alpha_s} \) behavior.
- Requires understanding of all order structure of perturbation theory.
EFTs for Jet Substructure

- Can understand all orders structure using Effective Field Theories.
- Dynamics of subjets iteratively integrated out and replaced by sources (Wilson lines)

**Collinear Subjets**
\[ e_3^{(\alpha)} \ll (e_2^{(\alpha)})^3 \]
\[ e_2^{(\beta)} \sim (e_2^{(\alpha)})^{3/\alpha} \]

**Soft Subjet**
\[ e_3^{(\alpha)} \ll (e_2^{(\alpha)})^3 \]
\[ e_2^{(\alpha)} \sim e_2^{(\beta)} \]

- Powerful approach to describe complicated situations.
Analytic Calculations for Jet Substructure

$D_2^{(2,2)}$ Spectrum

Vincia $p_T$ Ordered Hadron Level

1 TeV, $e^+e^- \rightarrow$ dijets

$m_J \in [80, 100]$ GeV, $R=1$

Boosted Z Selection
Calculation of $D_2$

- Set up EFT description in each region of phase space.
- Factorized description of phase space regions can be combined to make prediction for $D_2$. 

![Graph of $D_2$ Phase Space]

**Contours of $D_2^{(β)}$**

- **Soft Haze**
- **Collinear Subjets**
- **Soft Subjet**

**Axes:**
- $e_2^{(β)}$ vs $e_3^{(β)}$
A Warm Up: $e^+e^-$

- Warm up with $e^+e^-$

$D_2$ Spectrum

$D_2^{(2,2)}$ Spectrum
Vincia $p_T$ Ordered Hadron Level
1TeV, $e^+e^- \rightarrow$ dijets
$m_t \in [80, 100]$ GeV, $R=1$

Tagging Efficiency

$D_2^{(2,2)}$: Z Boson vs. QCD
1TeV, $e^+e^- \rightarrow$ dijets
$m_t \in [80, 100]$ GeV, $R=1$
Analytic Boosted Boson Discrimination at the LHC

- Difficulties in extending to $pp$:

  - Global color correlations
  - Hadronization corrections
  - Pile-Up
  - Underlying event

- All complications associated with soft radiation.
- Groomers remove soft radiation
  \[ \Rightarrow \text{ Makes calculations simpler and more universal.} \]
Analytic Boosted Boson Discrimination at the LHC

- Grooming removes all color correlations.

\[ f = f_g + f_q \]

- Jet can be considered in isolation!
- Enables calculations in complicated LHC environment.
Analytic Boosted Boson Discrimination at the LHC

- Calculation of groomed $D_2$ at the LHC.

- Analytic understand of modern jet substructure tools at LHC!
Conclusions

- Jet substructure provides novel ways to test the SM and to search for new physics at the LHC.

- Analytic calculations are a catalyst for further improvements in jet substructure.

- More Sophisticated Calculations

- More Sophisticated Techniques

- DPF 2017 August 4, 2017 36 / 37
Thanks!