On HQET and NRQCD Operators of Mass Dimension 8 and Above

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Based on: A.G and G. Paz JHEP 07(2017)137 [arxiv:1702.08904]

Outline

Introduction

Construction of HQET Matrix elements

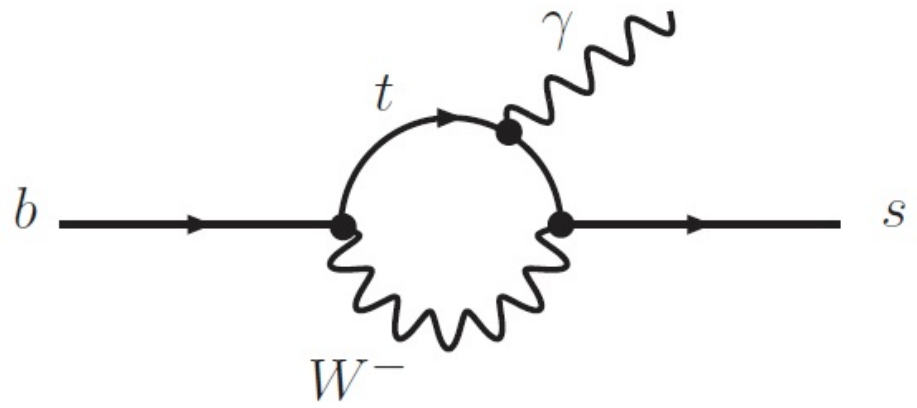
New Results

INTRODUCTION

Introduction

- Our Universe is mostly made out of matter instead of anti-matter.
- CP Violation: particle behaves differently than anti-particles
- For the radiative decays $\bar{B} \to X_s\gamma$ and $B \to X_s\gamma$ the CP asymmetry defined as

$$A_{X_s} = \frac{\Gamma(\bar{B} \to X_s\gamma) - \Gamma(B \to X_s\gamma)}{\Gamma(\bar{B} \to X_s\gamma) + \Gamma(B \to X_s\gamma)}$$



Introduction

- The Leading power correction to the decay rate is given by,

$$\Gamma = \sum_{n=0}^{\infty} \frac{1}{M_H} \sum_k c_{k,n} \langle O_{k,n} \rangle$$

- The $c_{k,n}$ are the Wilson Coefficients that can calculable from perturbative QCD.
- $O_{k,n}$ are non perturbative HQET matrix elements measured, e.g., in
  [P. Gambino, K. J Healey, S. Turczyk PLB 763, 60 (2016)]
- These Matrix elements have the form

$$O_{k,n} \backsim \bar{h} i D^{\mu_1} i D^{\mu_2} ... i D^{\mu_n} (s^{\lambda}) h T_{\mu_1 \mu_2 ... \mu_n(\lambda)}$$

$$D^{\mu} = \partial^{\mu} + igA^{\mu a} T^a$$

Introduction

- HQET matrix elements are important
  - To calculate decay rates accurately
    $\Rightarrow$ A probe of new physics
  - To analyze the data from Belle-II experiment

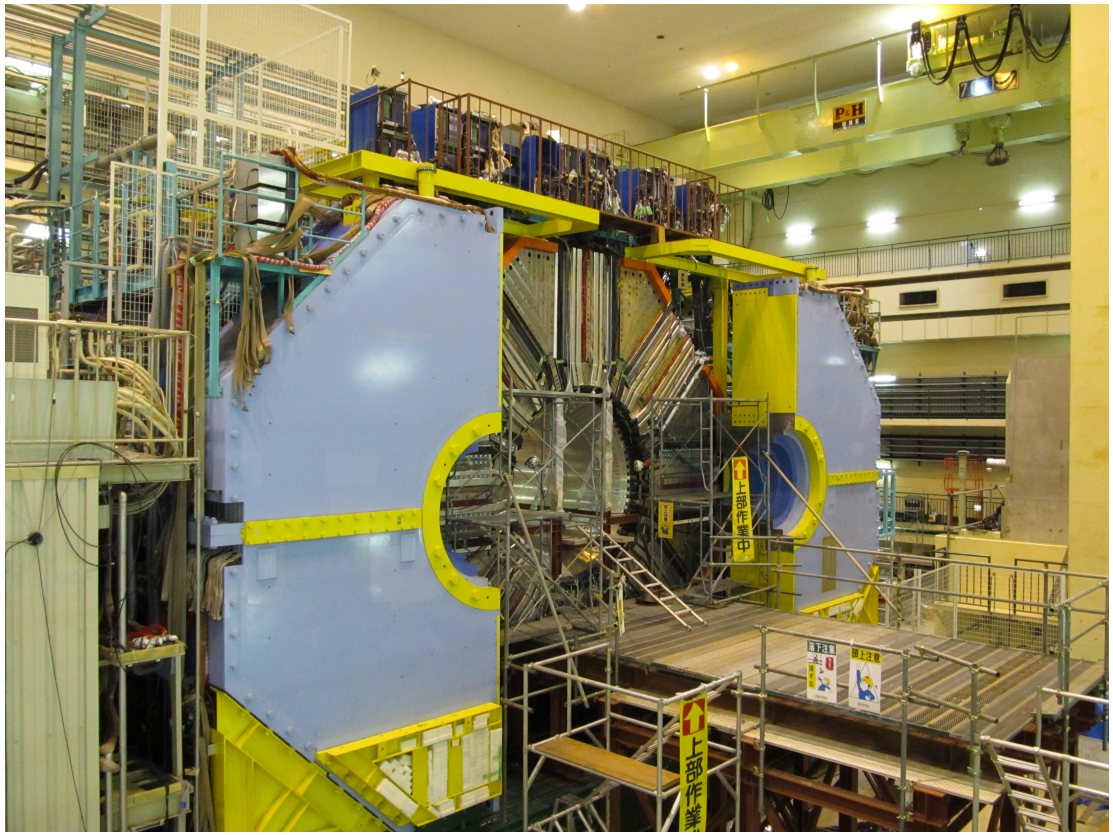


Figure: Belle-II experiment

Introduction

- List of these matrix elements up to mass dimension 8 was provided in
  [Mannel, Turczyk, Uraltsev JHEP 1011, 109(2010)]

- In our work we provide:
  [A. Gunawardana and G. Paz, JHEP 07(2017)137 [arXiv:1702.08904]]
  - A general method for a systematic construction of these matrix elements to any order of 1/M.
  - Construction of matrix elements up to mass dimension 9
  - Connection between the matrix elements and NRQCD/HQET

CONSTRUCTION OF MATRIX ELEMENTS

Definitions

- In the rest frame of Heavy quark $v^\mu = (1, 0, 0, 0) \therefore v.v = 1$.

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Definitions

- In the rest frame of Heavy quark $v^\mu = (1, 0, 0, 0) \therefore v.v = 1$.
- $(-i)\sigma_{\mu\nu} \to iv^\alpha \epsilon_{\alpha\mu\nu\lambda} s^\lambda$
- Matrix elements only depend on:
  - v^μ
  - $g^{\mu_i \nu_j} \Rightarrow \Pi^{\mu_i \mu_j} = g^{\mu_i \mu_j} - v^{\mu_i} v^{\mu_j}$: Parity even
  - $\epsilon^{\mu_i \mu_j \mu_k \mu_l}$: Parity odd

Constraints on matrix elements

- From PT symmetry:

$$\langle H|\bar{h}\, iD^{\mu_1} \ldots iD^{\mu_n} h|H\rangle \overset{PT}{=} \langle H|\bar{h}\, iD^{\mu_1} \ldots iD^{\mu_n} h|H\rangle^* ,$$

$$\langle H|\bar{h}\, iD^{\mu_1} \ldots iD^{\mu_n} s^{\lambda} h|H\rangle \overset{PT}{=} -\langle H|\bar{h}\, iD^{\mu_1} \ldots iD^{\mu_n} s^{\lambda} h|H\rangle^* .$$

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- Hermitian conjugation:

$$\langle H|\bar{h}\, iD^{\mu_1} \ldots iD^{\mu_n}(s^{\lambda}) h|H\rangle = \langle H|\bar{h}\, iD^{\mu_n} \ldots iD^{\mu_1}(s^{\lambda}) h|H\rangle^* .$$

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- PT+ Hermitian Conjugation$\Rightarrow$ Inversion Symmetry
  - Spin Independent : Symmetric under inversion of indices.
  - Spin Dependent : Anti-symmetric under inversion of indices

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- From HQET equation of Motion:
  - $iv \cdot Dh = 0, \bar{h} iv \cdot D = 0$
  - $v.s = 0$

Possibility of multiple color structures

- The symmetric product of SU(3) color matrices

$$\{T^a, T^b\} = \frac{1}{3}\delta^{ab} + d^{abc}\,T^c$$

[A. Kobach and S. Pal [arXiv:1704.00008]]

- $\delta^{ab} \Rightarrow$ singlet part
- $d^{abc}\,T^c \Rightarrow$ octet part

- Example:
  - The symmetric product of color matrices in $\psi^\dagger E^i_a T^a E^i_b T^b \psi$ gives 2 operators
    1. $\psi^\dagger E^i_a E^i_b \{T^a, T^b\}\psi$ **:** generated by commutator and anti-commutators of covarient derivatives
    2. $\psi^\dagger E^i_a E^i_b \delta^{ab}\psi$ **:** generated by one-loop corrections to the previous operator

Spin Independent Matrix elements

- The spin independent matrix elements are invariant under parity.
- Dimension 3

$$\frac{1}{2M_H}\langle H|\bar{h}h|H\rangle = 1.$$

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- Dimension 5

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} h|H\rangle = a^{(5)}\Pi^{\mu_1\mu_2}$$

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- Dimension 6

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} h|H\rangle = a^{(6)}\Pi^{\mu_1\mu_3} v^{\mu_2}.$$

Spin Independent Matrix elements

- Dimension 7

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h|H\rangle = a_{12}^{(7)}\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4} + a_{13}^{(7)}\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4} + a_{14}^{(7)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3} + b^{(7)}\Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}.$$

- Checking the multiple color structures from $\bar{h}\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]h\} \Rightarrow$
  2 linearly independent combinations ($a_{13}^{(7)} - a_{14}^{(7)}$ and $b^{(7)}$)

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- Each combination gives singlet and octet structures:

Spin Independent Matrix elements

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- Each combination gives singlet and octet structures:
  - $g^2\psi^\dagger(\boldsymbol{B}_a^i\boldsymbol{B}_b^i - \boldsymbol{E}_a^i\boldsymbol{E}_b^i)T^a T^b\psi, g^2\psi^\dagger(\boldsymbol{B}_a^i\boldsymbol{B}_b^i - \boldsymbol{E}_a^i\boldsymbol{E}_b^i)\delta^{ab}\psi$
  - $g^2\psi^\dagger\boldsymbol{E}_a^i\boldsymbol{E}_b^i T^a T^b\psi, g^2\psi^\dagger\boldsymbol{E}_a^i\boldsymbol{E}_b^i\delta^{ab}\psi$

Spin Independent Matrix elements

- Dimension 8

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} h|H\rangle = a_{12}^{(8)}\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5} v^{\mu_4} + \Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_4} v^{\mu_2}\right) + a_{13}^{(8)}\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5} v^{\mu_4} + \Pi^{\mu_5\mu_3}\Pi^{\mu_4\mu_1} v^{\mu_2}\right) + a_{15}^{(8)}\left(\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_4} v^{\mu_2} + \Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_2} v^{\mu_4}\right) + b_{12}^{(8)}\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_5} v^{\mu_3} + b_{14}^{(8)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5} v^{\mu_3} + b_{15}^{(8)}\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4} v^{\mu_3} + c^{(8)}\Pi^{\mu_1\mu_5} v^{\mu_2} v^{\mu_3} v^{\mu_4}.$$

(Symmetric under inversion of indices)

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$$a_{13}^{(8)}\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5} v^{\mu_4} + \Pi^{\mu_5\mu_3}\Pi^{\mu_4\mu_1} v^{\mu_2}\right) + a_{15}^{(8)}\left(\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_4} v^{\mu_2} + \Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_2} v^{\mu_4}\right) +$$
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$$c^{(8)}\Pi^{\mu_1\mu_5} v^{\mu_2} v^{\mu_3} v^{\mu_4}.$$

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- Similarly checking the multiple color structure:
  $\bar{h}\{iD^{\mu_m}, \{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\}h \Rightarrow$ one combination

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- Similarly checking the multiple color structure: $\bar{h}\{iD^{\mu_m}, \{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\}h \Rightarrow$ one combination

- Gives two operators:
  - $g^2\psi^\dagger\{i\boldsymbol{D}^i, \epsilon^{ijk}\boldsymbol{E}_a^i\boldsymbol{B}_b^k\{T^a, T^b\}\}\psi, g^2\psi^\dagger\{i\boldsymbol{D}^i, \epsilon^{ijk}\boldsymbol{E}_a^i\boldsymbol{B}_b^k\delta^{ab}\}\psi$

Spin Dependent Matrix elements

- Spin Dependent matrix elements are parity odd
- Dimension 3

$$\frac{1}{2M_H}\langle H|\bar{h}s^{\lambda}h|H\rangle = 0 \ (\because v.s = 0)$$

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$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}s^{\lambda}h|H\rangle = 0 \ (\because \Pi^{\mu_1\lambda} : \ Parity\ even)$$

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- Dimension 5

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} s^\lambda h|H\rangle = i\tilde{a}^{(5)}\epsilon^{\rho\mu_1\mu_2\lambda} v_\rho.$$

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- Dimension 6

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} s^\lambda h|H\rangle = i\tilde{a}^{(6)} v^{\mu_2}\epsilon^{\rho\mu_1\mu_3\lambda} v_\rho.$$

Spin Dependent Matrix elements

- Dimension 7

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^{\lambda} h|H\rangle =$$

$$i\tilde{a}_{12}^{(7)}\left(\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - \Pi^{\mu_4\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{a}_{13}^{(7)}\left(\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho - \Pi^{\mu_4\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right)$$

$$+\, i\tilde{a}_{14}^{(7)}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_3\lambda}v_\rho + i\tilde{a}_{23}^{(7)}\Pi^{\mu_2\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho + i\tilde{b}^{(7)}v^{\mu_2}v^{\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho,$$

(Anti Symmetric under inversion of indices)

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$$+\, i\tilde{a}_{14}^{(7)}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_3\lambda}v_\rho + i\tilde{a}_{23}^{(7)}\Pi^{\mu_2\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho + i\tilde{b}^{(7)}v^{\mu_2}v^{\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho,$$

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- Checking the multiple color structure:
  $\bar{h}\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}h \Rightarrow$ All the combinations vanish.

Spin Dependent Matrix elements

- Dimension 8

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} s^{\lambda} h|H\rangle =$$

$$i\tilde{a}_{12}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{a}_{14}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$

$$+ i\tilde{a}_{15}^{(8)} v^{\mu_3}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho + i\tilde{a}_{24}^{(8)} v^{\mu_3}\Pi^{\mu_2\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho +$$

$$+ i\tilde{b}_{13}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{b}_{14}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_3\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$

$$+ i\tilde{b}_{15}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_2\lambda}v_\rho\right) + i\tilde{b}_{34}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_2}\epsilon^{\rho\mu_5\mu_1\lambda}v_\rho\right) +$$

$$+ i\tilde{b}_{35}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\rho\mu_5\mu_2\lambda}v_\rho\right) + i\tilde{b}_{45}^{(8)}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_1\mu_3\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\rho\mu_5\mu_3\lambda}v_\rho\right) +$$

$$+ i\tilde{c}^{(8)} v^{\mu_2} v^{\mu_3} v^{\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho.$$

Spin Dependent Matrix elements

- Dimension 8

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} s^\lambda h|H\rangle =$$
$$i\tilde{a}_{12}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{a}_{14}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$
$$+ i\tilde{a}_{15}^{(8)} v^{\mu_3}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho + i\tilde{a}_{24}^{(8)} v^{\mu_3}\Pi^{\mu_2\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho +$$
$$+ i\tilde{b}_{13}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{b}_{14}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_3\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$
$$+ i\tilde{b}_{15}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_2\lambda}v_\rho\right) + i\tilde{b}_{34}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_2}\epsilon^{\rho\mu_5\mu_1\lambda}v_\rho\right) +$$
$$+ i\tilde{b}_{35}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\rho\mu_5\mu_2\lambda}v_\rho\right) + i\tilde{b}_{45}^{(8)}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_1\mu_3\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\rho\mu_5\mu_3\lambda}v_\rho\right) +$$
$$+ i\tilde{c}^{(8)} v^{\mu_2} v^{\mu_3} v^{\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho.$$

- Checking the multiple color structure:
  $\bar{h}\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, [iD^{\mu_l}, iD^{\mu_m}]\}h \Rightarrow$ 6 linearly independent combinations

Spin Dependent Matrix elements

- Dimension 8

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} s^\lambda h|H\rangle =$$
$$i\tilde{a}_{12}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{a}_{14}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$
$$+ i\tilde{a}_{15}^{(8)} v^{\mu_3}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho + i\tilde{a}_{24}^{(8)} v^{\mu_3}\Pi^{\mu_2\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho +$$
$$+ i\tilde{b}_{13}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{b}_{14}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_3\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$
$$+ i\tilde{b}_{15}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_2\lambda}v_\rho\right) + i\tilde{b}_{34}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_2}\epsilon^{\rho\mu_5\mu_1\lambda}v_\rho\right) +$$
$$+ i\tilde{b}_{35}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\rho\mu_5\mu_2\lambda}v_\rho\right) + i\tilde{b}_{45}^{(8)}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_1\mu_3\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\rho\mu_5\mu_3\lambda}v_\rho\right) +$$
$$+ i\tilde{c}^{(8)} v^{\mu_2} v^{\mu_3} v^{\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho.$$

- Checking the multiple color structure: $\bar{h}\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, [iD^{\mu_l}, iD^{\mu_m}]\}h \Rightarrow$ 6 linearly independent combinations

- Results agree with
  - [Mannel, Turczyk, Uraltsev JHEP 1011, 109(2010)]
  - [A. Kobach and S. Pal [arXiv:1704.00008]]

New Results

New Result: Dimension 9 HQET matrix element

$$
\begin{aligned}
&\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} iD^{\mu_6} h|H\rangle = a^{(9)}_{12,34}\, \Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}\Pi^{\mu_5\mu_6} + \\
&+a^{(9)}_{12,35}\, (\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}\Pi^{\mu_4\mu_6} + \Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}\Pi^{\mu_5\mu_6}) + a^{(9)}_{12,36}\, (\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}\Pi^{\mu_4\mu_5} + \Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}\Pi^{\mu_5\mu_6}) + \\
&+a^{(9)}_{13,25}\, \Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}\Pi^{\mu_4\mu_6} + a^{(9)}_{13,26}\, (\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}\Pi^{\mu_4\mu_5} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_6}) + \\
&+a^{(9)}_{14,25}\, \Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_6} + a^{(9)}_{14,26}\, (\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_5} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_6}) + \\
&+a^{(9)}_{15,26}\, \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_4} + a^{(9)}_{16,23}\, \Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_5} + a^{(9)}_{16,24}\, \Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_5} + \\
&+a^{(9)}_{16,25}\, \Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_4} + b^{(9)}_{12,36}\, (\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1\mu_4}\Pi^{\mu_5\mu_6} v^{\mu_2} v^{\mu_3}) + \\
&+b^{(9)}_{12,46}\, (\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_6} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_6} v^{\mu_2} v^{\mu_4}) + b^{(9)}_{12,56}\, \Pi^{\mu_1\mu_2}\Pi^{\mu_5\mu_6} v^{\mu_3} v^{\mu_4} + \\
&+b^{(9)}_{13,26}\, (\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1\mu_5}\Pi^{\mu_4\mu_6} v^{\mu_2} v^{\mu_3}) + \\
&+b^{(9)}_{13,46}\, \Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_6} v^{\mu_2} v^{\mu_5} + b^{(9)}_{14,26}\, (\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_6} v^{\mu_2} v^{\mu_4}) + \\
&b^{(9)}_{14,36}\, \Pi^{\mu_1\mu_4}\Pi^{\mu_3\mu_6} v^{\mu_2} v^{\mu_5} + b^{(9)}_{15,26}\, \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6} v^{\mu_3} v^{\mu_4} + \\
&b^{(9)}_{16,23}\, (\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3} v^{\mu_4} v^{\mu_5} + \Pi^{\mu_1\mu_6}\Pi^{\mu_4\mu_5} v^{\mu_2} v^{\mu_3}) + \\
&+b^{(9)}_{16,24}\, (\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4} v^{\mu_3} v^{\mu_5} + \Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_5} v^{\mu_2} v^{\mu_4}) + b^{(9)}_{16,25}\, \Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5} v^{\mu_3} v^{\mu_4} + \\
&+b^{(9)}_{16,34}\, \Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_4} v^{\mu_2} v^{\mu_5} + c^{(9)}\, \Pi^{\mu_1\mu_6} v^{\mu_2} v^{\mu_3} v^{\mu_4} v^{\mu_5}.
\end{aligned}
$$

New Result:Dimension 8 HQET/NRQCD Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm NRQCD}^{\rm dim=8} = \psi^\dagger \Bigg\{ & c_{X1} g \frac{[\boldsymbol{D}^2, \{\boldsymbol{D}^i, \boldsymbol{E}^i\}]}{M^4} + c_{X2} g \frac{\{\boldsymbol{D}^2, [\boldsymbol{D}^i, \boldsymbol{E}^i]\}}{M^4} + c_{X3} g \frac{[\boldsymbol{D}^i, [\boldsymbol{D}^i, [\boldsymbol{D}^j, \boldsymbol{E}^j]]]}{M^4} \\
& + i c_{X4a}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}^j_a \boldsymbol{B}^k_b \{T^a, T^b\}\}}{2M^4} + i c_{X4b}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}^j_a \boldsymbol{B}^k_b\, \delta^{ab}\}}{M^4} \\
& + i c_{X5} g \frac{\boldsymbol{D}^i \boldsymbol{\sigma} \cdot (\boldsymbol{D} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{D}) \boldsymbol{D}^i}{M^4} + i c_{X6} g \frac{\epsilon^{ijk} \sigma^i \boldsymbol{D}^j [\boldsymbol{D}^l, \boldsymbol{E}^l] \boldsymbol{D}^k}{M^4} \\
& + c_{X7a}\, g^2 \frac{\{\boldsymbol{\sigma} \cdot \boldsymbol{B}_a T^a, [\boldsymbol{D}^i, \boldsymbol{E}^i]_b T^b\}}{2M^4} + c_{X7b}\, g^2 \frac{\boldsymbol{\sigma} \cdot \boldsymbol{B}_a [\boldsymbol{D}^i, \boldsymbol{E}^i]_a}{M^4} \\
& + c_{X8a}\, g^2 \frac{\{\boldsymbol{E}^i_a T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_b T^b\}}{2M^4} + c_{X8b} g^2 \frac{\boldsymbol{E}^i_a [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_a}{M^4} \\
& + c_{X9a}\, g^2 \frac{\{\boldsymbol{B}^i_a T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}]_b T^b\}}{2M^4} + c_{X9b}\, g^2 \frac{\boldsymbol{B}^i_a [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}]_a}{M^4} \\
& + c_{X10a}\, g^2 \frac{\{\boldsymbol{E}^i_a T^a, [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i]_b T^b\}}{2M^4} + c_{X10b}\, g^2 \frac{\boldsymbol{E}^i_a [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i]_a}{M^4} \\
& + c_{X11a}\, g^2 \frac{\{\boldsymbol{B}^i_a T^a, [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{E}^i]_b T^b\}}{2M^4} + c_{X11b}\, g^2 \frac{\boldsymbol{B}^i_a [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{E}^i]_a}{M^4} \\
& + \tilde{c}_{X12a}\, g^2 \frac{\epsilon^{ijk} \boldsymbol{\sigma}^i \boldsymbol{E}^j_a [D_t, \boldsymbol{E}^k]_b \{T^a, T^b\}}{2M^4} + \tilde{c}_{X12b}\, g^2 \frac{\epsilon^{ijk} \boldsymbol{\sigma}^i \boldsymbol{E}^j_a [D_t, \boldsymbol{E}^k]_a}{M^4} \\
& + i c_{X13} g^2 \frac{[\boldsymbol{E}^i, [D_t, \boldsymbol{E}^i]]}{M^4} + i c_{X14} g^2 \frac{[\boldsymbol{B}^i, (\boldsymbol{D} \times \boldsymbol{E} + \boldsymbol{E} \times \boldsymbol{D})^i]}{M^4} + i c_{X15} g^2 \frac{[\boldsymbol{E}^i, (\boldsymbol{D} \times \boldsymbol{B} + \boldsymbol{B} \times \boldsymbol{D})^i]}{M^4} \\
& + c_{X16} g^2 \frac{[\boldsymbol{\sigma} \cdot \boldsymbol{B}, \{\boldsymbol{D}^i, \boldsymbol{E}^i\}]}{M^4} + c_{X17} g^2 \frac{[\boldsymbol{B}^i, \{\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}\}]}{M^4} + c_{X18} g^2 \frac{[\boldsymbol{E}^i, \{\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i\}]}{M^4} \Bigg\} \psi.
\end{aligned}
$$

New Results: Moments of shape function

- The leading order shape function:

$$S(\omega,\mu) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} \frac{e^{-i\omega t}}{2M_B} \langle \bar{B}(v)|\bar{h}(tn)S_n(tn)S_n^\dagger(0)h(0|\bar{B}(v)\rangle$$

- The moments of leading order shape function

$$\int d\omega\, S(\omega) = 1, \qquad \int d\omega\, \omega\, S(\omega) = 0, \qquad \int d\omega\, \omega^2\, S(\omega) = -a^{(5)} = -\lambda_1/3,$$

$$\int d\omega\, \omega^3\, S(\omega) = -a^{(6)} = -\rho_1/3, \qquad \int d\omega\, \omega^4\, S(\omega) = a_{12}^{(7)} + a_{13}^{(7)} + a_{14}^{(7)} - b^{(7)} = m_1/5 - m_2/3,$$

$$\int d\omega\, \omega^5\, S(\omega) = 2a_{12}^{(8)} + 2a_{13}^{(8)} + 2a_{15}^{(8)} + b_{12}^{(8)} + b_{14}^{(8)} + b_{15}^{(8)} - c^{(8)} =$$

$$= \left(-8r_1 + 2r_2 + 2r_3 + 2r_4 + r_5 + r_6 + r_7\right)/15,$$

$$\int d\omega\, \omega^6\, S(\omega) = -a_{12,34}^{(9)} - 2a_{12,35}^{(9)} - 2a_{12,36}^{(9)} - a_{13,25}^{(9)} - 2a_{13,26}^{(9)} - a_{14,25}^{(9)} - 2a_{14,26}^{(9)} - a_{15,26}^{(9)} - a_{16,23}^{(9)}$$

$$-a_{16,24}^{(9)} - a_{16,25}^{(9)} + 2b_{12,36}^{(9)} + 2b_{12,46}^{(9)} + b_{12,56}^{(9)} + 2b_{13,26}^{(9)} + b_{13,46}^{(9)} + 2b_{14,26}^{(9)} + b_{14,36}^{(9)}$$

$$+b_{15,26}^{(9)} + 2b_{16,23}^{(9)} + 2b_{16,24}^{(9)} + b_{16,25}^{(9)} + b_{16,34}^{(9)} - c^{(9)}.$$

Summary

- Higher dimensional HQET matrix elements are important for inclusive B decays.
- Presented a new method for systematic construction of these matrix elements.
- We present the tensor decomposition of spin independent matrix element at mass dimension 9
- Possibility of extra color structures was pointed out in
  [A. Kobach and S. Pal [arXiv:1704.00008]]
  Incorporated into our method in the updated version of
  [A. Gunawardana and G. Paz, arXiv:1702.08904[hep-ph]]

Summary

- Applications:
  - ✓ Calculation of the moments of Leading order shape function.
  - ✓ HQET/NRQCD Lagrangian at mass dimension 8
- Future Work:
  - Estimation of CP asymmetry for radiative B decays.

THANK YOU

BACKUP SLIDES

- CP asymmetry(experimental)$= 0.015 \pm 0.02$
- $\mathcal{B}(B \to X_s\gamma)\ (E_\gamma > 1.6\,\text{GeV}) = (3.36 \pm 0.16) \times 10^{-4}$

  [Heavy Flavor Averaging Group (2016)]
- $\mathcal{B}(B \to X_s\gamma)\ (E_\gamma > 1.6\,\text{GeV}) = 3.15 \pm 0.23 \times 10^{-4}$

  [Misiak et al]
- $\mathcal{A}^{SM}_{X_s\gamma} = (0.44^{+0.15}_{-0.10} \pm 0.03^{+0.19}_{-0.09})\%$
- $\mathcal{A}_{X_s\gamma} = -(1.2 \pm 2.8)\%$ [Y. Amhis et al.[HFAG], arXiv:1207.1158[hep-ex]]