Analytic resummation for TMD observables

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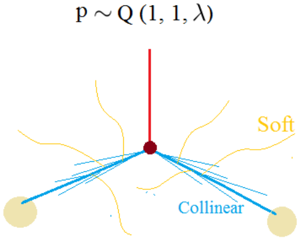
Factorization in SCET

- P+P $\to$ H+X, P+P $\to$ $l^+ + l^-$ +X.



$$p_c \sim Q\,(1, \lambda^2, \lambda)\,,$$
$$p_{\bar{c}} \sim Q\,(\lambda^2, 1, \lambda)\,,$$
$$p_s \sim Q\,(\lambda, \lambda, \lambda)\,,$$
$$\lambda = qT / Q$$

Motivation

- Resum large logs of qT/Q by setting renormalization scales in momentum space
- Obtain, for the first time, an analytic expression for resummed cross section.

Factorization

Transverse momentum cross section

$$\frac{d\sigma}{dq_T^2 dy} \propto H(\frac{\mu}{Q}) \times \int d^2\vec{q}_{Ts} d^2\vec{q}_{T1} d^2\vec{q}_{T2} S(\vec{q_{Ts}}, \mu, \nu) \times$$
$$f_1^{\perp}(x_1, \vec{q_{T1}}, \mu, \nu, Q) f_2^{\perp}(x_2, \vec{q_{T2}}, \mu, \nu, Q) \delta^2(\vec{q_T} - \vec{q_{Ts}} - \vec{q_{T1}} - \vec{q_{T2}})$$

- The function $f_i^{\perp}$ along with the soft function S forms the **TMDPDF**.
- RG equations in two scales, μ, ν.
- RG equations in momentum space are convolutions of distribution functions and hard to solve directly.

$$\nu \frac{d}{d\nu} G_i(\vec{q}_{Ti}, \nu) = \gamma_\nu^i(\vec{q}_{Ti}) \otimes G_i(\vec{q_{Ti}}, \nu)$$

Factorization

b space formulation

$$\frac{d\sigma}{dq_T^2 dy} \propto H(\frac{\mu}{Q}) \int bdbJ_0(bq_T)S(b,\mu,\nu)f_1^{\perp}(x_1,b,\mu,\nu,Q)f_2^{\perp}(x_2,b,\mu,\nu,Q)$$

RG equations in b space are simple

$$\mu\frac{d}{d\mu}F_i(\mu,\nu,b) = \gamma_\mu^i F_i(\mu,\nu,b), \qquad F_i \in (H,S,f_i^{\perp})$$

$$\nu\frac{d}{d\nu}G_i(\mu,\nu,b) = \gamma_\nu^i G_i(\mu,\nu,b), \qquad G_i \in (S,f_i^{\perp})$$

$$\sum_{F_i}\gamma_\mu^i = \sum_{G_i}\gamma_\nu^i = 0$$

Resummation schemes

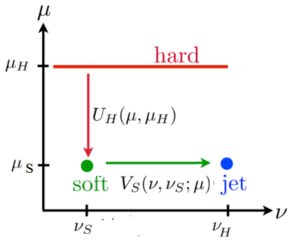


Figure: Choice of resummation path.

- b space resummation: Default choice of $\mu = \nu = 1/b$,1007.2351 De Florian et.al., 1503:00005 V. Vaidya et. al
- Momentum space resummation:Both μ, ν in momentum space, distributional scale setting, 1611.08610 Tackmann et.al., 1604.02191 P. Monni et. al.
- Hybrid: μ in momentum space(1007.4005, 1109.6027 Becher et.al.)

Scale choice in momentum space

- Can we choose scales in momentum space for μ and ν?
- Naively, we expect, $\mu_H, \nu_H \sim Q, \mu_L, \nu_L \sim qT$.
- Assume a power counting
  $\alpha_s \log(Q/\mu_L), \log(\mu_L b_0), \alpha_s \log(Q/\nu_L), \log(\nu_L b_0) \sim 1$

Attempt at NLL $\rightarrow$ running Soft function in ν [a]

[a] $b_0 = be^{-\gamma_E}/2$

$$\frac{d\sigma}{dq_t^2} \propto U_H^{NLL}(H, \mu_L) \int dbbJ_0(bq_t) U_S(\nu_H, \nu_L, \mu_L)$$

$$= U_H^{NLL}(H, \mu_L) \int dbbJ_0(bq_t) e^{\int_{\nu_L}^{\nu_H} d\ln\nu (\gamma_\nu^{(0)S})}$$

$$= U_H^{NLL}(H, \mu_L) \int dbbJ_0(bq_t) e^{-\Gamma_0 \frac{\alpha_s}{\pi} \log\left(\frac{\nu_H}{\nu_L}\right) \log(\mu_L b_0)}$$

- Cross section is singular due to divergence at small b.

Scale choice in momentum space

- Resum all logarithms of the form $\alpha_s \log^2(\mu_L b_0)$

A choice for ν in b space $\rightarrow$ include sub-leading terms

$$\nu_L = \frac{\mu_L^n}{b_0^{1-n}}, \qquad n = \frac{1}{2}\left(1 - \alpha(\mu_L)\frac{\beta_0}{2\pi}\log(\frac{\nu_H}{\mu_L})\right)$$

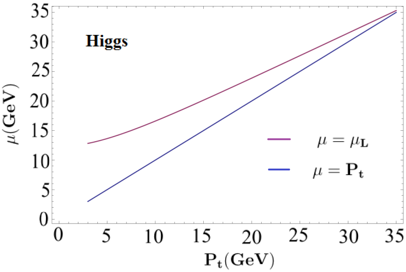
Soft exponent at NLL $\rightarrow$ Quadratic in $\log(\mu_L b_0)$

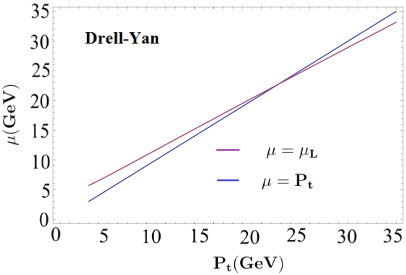
$$\log(U_S^{NLL}(\nu_H, \nu_L, \mu_L)) = -2\Gamma_0 \frac{\alpha(\mu_L)}{2\pi} \times$$
$$\left(\log(\frac{\nu_H}{\mu_L})\log(\mu_L b_0) + \frac{1}{2}\log^2(\mu_L b_0) + \alpha(\mu_L)\frac{\beta_0}{4\pi}\log^2(\mu_L b_0)\log(\frac{\nu_H}{\mu_L})\right)$$

Scale choice in momentum space

A choice for μ_L in momentum space

- A choice that justifies the power counting $log(\mu_L b_0) \sim 1$
- Scale shifted away from q_T due to the scale Q in b space exponent.





Analytical expression for cross section

Mellin-Barnes representation of Bessel function

- Polynomial integral representation for Bessel function is needed

$$J_0(z) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dt \frac{\Gamma[-t]}{\Gamma[1+t]}\left(\frac{1}{2}z\right)^{2t}$$

b space integral

$$U_S = C_1 Exp[-A\log^2(Ub)]$$

$$I_b = \int_0^\infty dbbJ_0(bq_T)U_S \qquad \text{No Landau pole}$$

$$= \quad C_1\int_{-i\infty}^{i\infty} dt \frac{\Gamma[-t]}{\Gamma[1+t]}\int_0^\infty dbb(\frac{bq_T}{2})^{2t} Exp[-A\log^2(Ub)]$$

Analytical expression for cross section

$$
\begin{aligned}
I &= \frac{2C_1}{iq_T^2}\frac{1}{\sqrt{4\pi A}}\int_{c-i\infty}^{c+i\infty} dt\frac{\Gamma[-t]}{\Gamma[1+t]}Exp[\frac{1}{A}(t-t_0)^2] \\
t_0 &= -1 + A\log(2U/q_T) \rightarrow \text{saddle point}
\end{aligned}
$$

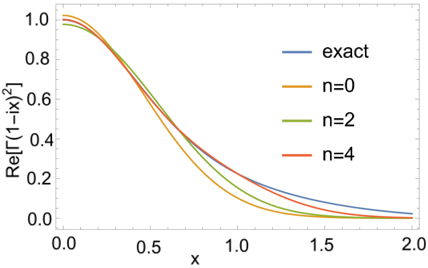
- Path of steepest descent is parallel to the imaginary axis
- Suppression controlled by 1/A $\sim \frac{4\pi}{\alpha_s} 1/\Gamma_{cusp}^{(0)}$
- $t = c + ix, \quad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin(\pi z)}.,$

$$
I = \frac{2C_1}{q_T^2}\frac{1}{\sqrt{4\pi A}}\int_{-\infty}^{\infty} dx\Gamma[-c-ix]^2\sin[\pi(c+ix)]Exp[-\frac{1}{A}(x-i(c-t_0))^2]
$$

Analytical expression for cross section

An expansion for $\Gamma[1-ix]^2$ in weighted Hermite polynomials

$$\Gamma(1-ix)^2 = \left[\sum_{n=0}^{\infty} c_{2n} H_{2n}(\alpha x) e^{-a_0 x^2} + \frac{i\gamma_E}{\beta} \sum_{n=0}^{\infty} d_{2n+1} H_{2n+1}(\beta x) e^{-b_0 x^2}\right]$$



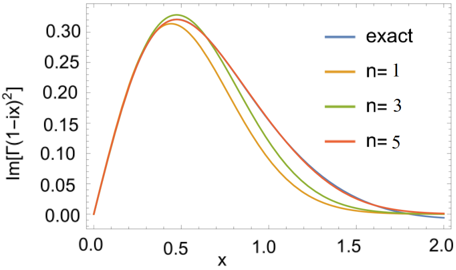


Figure: Expansion in Hermite polynomials

Analytical expression for cross section

Expression for resummed Soft function

$$I_b = \frac{2C}{\pi q_T^2} \sum_{n=0}^{\infty} \mathrm{Im}\left\{ c_{2n}\mathcal{H}_{2n}(\alpha, a_0) + \frac{i\gamma_E}{\beta} d_{2n+1}\mathcal{H}_{2n+1}(\beta, b_0) \right\}$$

$\mathcal{H}_n$ to all orders

$$\mathcal{H}_n(\alpha, a_0) = \mathcal{H}_0(\alpha, a_0)\frac{(-1)^n n!}{(1+a_0A)^n}$$

$$\times \sum_{m=0}^{Floor[n/2]} \frac{1}{m!}\frac{1}{(n-2m)!}\left\{[A(\alpha^2 - a_0) - 1](1 + a_0A)\right\}^m (2\alpha z_0)^{n-2m}$$

with $\mathcal{H}_0(\alpha, a_0) = e^{\frac{-A(L-i\pi/2)^2}{1+a_0A}} \frac{1}{\sqrt{1+a_0A}}, \quad z_0 = iA(L - i\pi/2)$

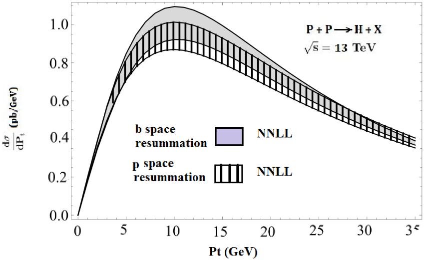
Analytical expression for cross section

Fixed order terms

- I_b acts as a generating function for residual fixed order logs

$$
\begin{aligned}
I_{even} &= C_1 \int bdbJ_0(bq_T)\log^{2n}(Ub)Exp[-A\log^2(Ub)] \\
&= (-1)^n \frac{d^n}{dA^n} I_b(A,L) \\
I_{odd} &= C_1 \int bdbJ_0(bq_T)\log^{2n+1}(Ub)Exp[-A\log^2(Ub)] \\
&= (-1)^n \frac{d^n}{dA^n}\frac{(-1)}{2A}\frac{d}{dL} I_b(A,L)
\end{aligned}
$$

Comparison with b space resummation



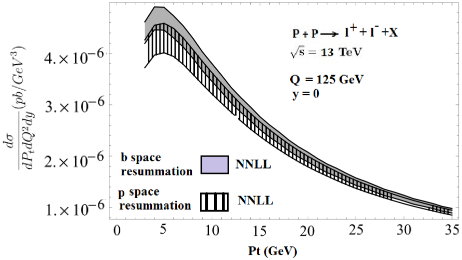


Figure: comparison of nnll cross section in two schemes

- Difference of the order of sub-leading terms.
- More reliable perturbative error estimation in the absence of Landau pole.

Matching to fixed order

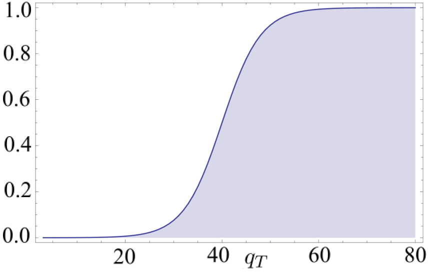
- Implement profiles in μ and ν to turn off resummation

$$S = S_L^{(1-z(q_T))} Q^{z(q_T)} \qquad \mathrm{S} \in \mu, \nu$$

- Soft exponent scales as $(1 - z(q_T))$

$$U_S = Exp[(1-z)\gamma_S^{\nu} log \left(\frac{Q}{\nu_L}\right)]$$

- This is equivalent to A $\rightarrow$ A(1-z) in I_b(A,L)



$$z(q_T) = \frac{1}{2}\left(1 + \tanh\left[r\left(\frac{q_T}{t} - 1\right)\right]\right)$$

Summary

- Implementation momentum space resummation for transverse spectra of gauge bosons
- Rapidity choice in impact parameter space
- Virtuality choice in momentum space.
- Analytical expression for cross section across the entire range of q_T obtained for the first time.
- Numerical accuracy controlled by the accuracy of the expansion for process independent function $\frac{\Gamma[-t]}{\Gamma[1+t]}$
- Outlook
  - Promising approach for other observables with similar factorization structure.
  - Non-perturbative effects need to be included for low Q as well as the low q_T regime.

Backup

Analytical expression for cross section

- What choice do we make for c? Obvious choice $c = t_0$? c depends on A and hence on the details of the process.
- For percent level accuracy, we need info about $F(x) = \frac{\Gamma[-c-ix]}{\Gamma[1+c+ix]}$ out to $x_l \sim \sqrt{2A\log(10)}$
- Worst case scenario A $\sim$0.5 $\implies x_l \sim 1.5$
- A Taylor series expansion around the saddle point is not enough.
- Choose c = -1, the saddle point in the limit $A \to 0$ for all observables and use a more suitable basis for expanding F(x)

Analytical expression for cross section

Guidelines for choosing a basis for expansion

- Fixed order cross section

$$I_{exact}^{O(\alpha_s)} = -2\Gamma_{cusp}^{(0)}\frac{\alpha(\mu_L)}{4\pi}\frac{2}{q_T^2}\left(F'[0]\log\left(\frac{\mu_L e^{-\gamma_E}}{q_T}\right)+\frac{F''[0]}{4}\right)$$

- To correctly reproduce the fixed order cross section upto α_s^n, we need $2n^{th}$ derivative of the expansion to match the exact function F(x)
- We need the expansion in a basis to be accurate upto $x \sim 1.5$
- The basis functions for the expansion should be chosen so as to yield a rapidly converging and analytical result.

(A more accurate) Analytical expression for cross section

An expansion for F(x)=$\Gamma[-1 - ix]/\Gamma[ix]$

A general basis $x^n e^{\alpha x^2+\beta x}$ for expansion

$$\hat{F}_R(x) = g_1(Exp[-g_2 x^2] - \cos[g_3 x]) + g_4 x^2 Exp[-g_5 x^2]$$

$$\hat{F}_I(x) = f_1 \sin[f_2 x^2] + f_3 \sinh(f_4 x)$$

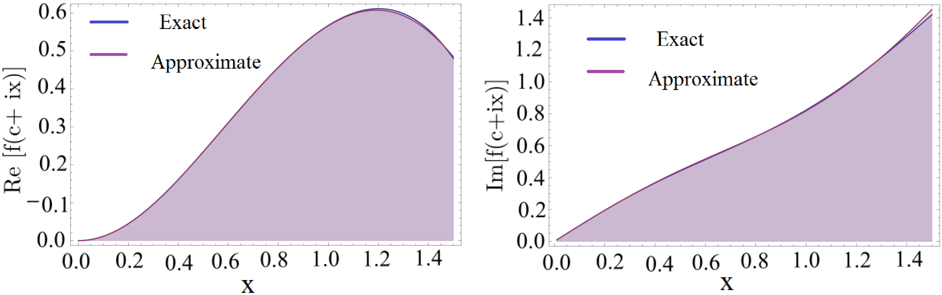


Figure: (Expansion for real and imaginary parts of f(t), c is chosen to be -1

Numerical results

- Easily extended to NNLL, b space exponent kept quadratic in $\log(\mu b)$

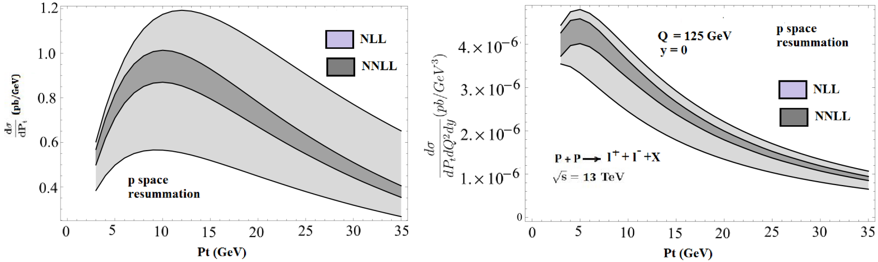


Figure: Resummation in momentum space.

- Excellent convergence for both the Higgs and Drell-Yan spectrum
- No arbitrary b space cut-off while estimating perturbative errors.