Little hierarchy in the minimally specified MSSM

Radovan Dermisek
Indiana University, Bloomington

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Fine tuning problem in EWSB

in a bottom up approach is seen as the cancellation of two numbers:

the top loop contribution to the Higgs mass squared

\[ \Delta m_H^2 = -\frac{\lambda_t^2}{8\pi^2} \Lambda^2 + \ldots \]

and the boundary condition at the scale of new physics.
Quantifying fine tuning

In a model with two parameters $A, B \sim 1$ contributing to $X$,

$$X = A - B$$

in order to get e.g. $X = 0.001$, parameters $A$ and $B$ have to be specified with 3 digits and carefully chosen/tuned

- e.g.: $A = 0.963$
  - $B = 0.962$

Tuning often quantified by:

$$\max_{p=\{A,B\}} \left| \frac{\partial \ln X}{\partial \ln p} \right| \simeq \frac{A}{X} \simeq 1000$$

$0.1\%$ tuning
Quantifying fine tuning

In a model with two parameters $A$, $B \sim 1$ contributing to $X$, $X = A - B$

in order to get e.g. $X = 0.001$, parameters $A$ and $B$ have to be specified with 3 digits and carefully chosen/tuned

Tuning often quantified by:

$$\max_{p=\{A,B\}} \left| \frac{\partial \ln X}{\partial \ln p} \right| \simeq \frac{A}{X} \simeq 1000$$

Note that ~10% tuning corresponds to specifying $A$ and $B$ with one digit: no matter what the remaining digits are, we need to adjust just the first digit of model parameters to get $X \sim 0.1$
In the MSSM there are several parameters contributing to the electroweak scale, e.g:

\[ M_Z^2 \simeq -1.9 \mu^2 + 5.9 M_3^2 - 1.2 m_{H_u}^2 + 1.5 m_t^2 - 0.8 A_t M_3 + 0.2 A_t^2 + \ldots \]

boundary conditions at the GUT scale \( \tan \beta = 10 \)

The usual naturalness measure,

\[ \max_p \left| \frac{\partial \ln M_Z^2}{\partial \ln p} \right| \]

only cares about the largest individual contribution. It doesn't tell us how the model parameters need to be tuned/specified.
Little hierarchy from complexity

e.g. in a model with more parameters contributing to $X$:

\[ X = A - B - c - d \]

in order to get $X = 0.001$ for randomly chosen $A \sim 1$, no parameter needs to be carefully chosen, e.g.:

\[
\begin{align*}
A &= 0.963 \\
B &= 0.9 \\
c &= 0.06 \\
d &= 0.002
\end{align*}
\]

only the first digit of all parameters need to be adjusted no matter what the following digits are

what is a tuned outcome in a model with 2 parameters may be a completely ordinary outcome in a more complex model

RD, arXiv:1611.03188
Naturalness criteria based on the largest contribution are too strong and do not necessarily indicate how model parameters have to be tuned.

Natural outcome for an observable in a given model is any outcome that does not require specifying more than one digit of model parameters, regardless of what the remaining digits are.

Let’s see what it means for the CMSSM.
Minimally specified CMSSM

Assume model parameters of the same order

\[ M_{1/2}, m_0, \mu, -A_0 \simeq M_{\text{SUSY}} \]

and vary them in ± 50% range keeping only one digit specifying the departure from \( M_{\text{SUSY}} \), e.g.:

\[
\begin{align*}
M_{1/2} &= 0.6 M_{\text{SUSY}} \\
m_0 &= 1.1 M_{\text{SUSY}} \\
\mu &= 1.4 M_{\text{SUSY}} \\
-A_0 &= 0.9 M_{\text{SUSY}}
\end{align*}
\]

\[
\begin{align*}
M_{1/2} &= 0.9 M_{\text{SUSY}} \\
m_0 &= 1.2 M_{\text{SUSY}} \\
\mu &= 0.6 M_{\text{SUSY}} \\
-A_0 &= 0.8 M_{\text{SUSY}}
\end{align*}
\]

\[
\begin{align*}
M_{1/2} &= 0.5 M_{\text{SUSY}} \\
m_0 &= 1.1 M_{\text{SUSY}} \\
\mu &= 1.5 M_{\text{SUSY}} \\
-A_0 &= 0.8 M_{\text{SUSY}}
\end{align*}
\]
Minimally specified CMSSM

Maximal hierarchy from minimally specified inputs:

RD and N. McGinnis, arXiv:1705.01910

The smallest outcome that does not depend on specifying parameters with more than 1 digit is indicated by the largest gap found in the distribution. Outcomes smaller than the largest gap are accidental.

The largest gaps ~0.001 (fairly uniform away from edges of the distribution)
Minimally specified CMSSM

Maximal hierarchy from minimally specified inputs:

The largest gaps $\sim 0.001$ (fairly uniform away from edges of the distribution)

The smallest outcome that does not depend on specifying parameters with more than 1 digit is indicated by the largest gap found in the distribution. Outcomes smaller than the largest gap are accidental.

$m_\tilde{\tau} \sim 30 M_Z$

is an ordinary outcome from minimally specified parameters

RD and N. McGinnis, arXiv:1705.01910
Minimally specified CMSSM

For different central values of parameters, the peak of the distribution changes

\[ \mu \simeq 2M_{\text{SUSY}} \]

\[ \mu \simeq \sqrt{2}M_{\text{SUSY}} \]

\[ \mu \simeq M_{\text{SUSY}} \]

but the maximal gap size remains almost the same.

Prediction for maximal hierarchy is very robust.

\[ M_{1/2} \simeq m_0 \simeq -A_0 \simeq M_{\text{SUSY}} \]

\[ M_{\text{SUSY}} = 3 \text{ TeV} \]

\[ \tan \beta = 10 \]
Minimally specified CMSSM

Higgs boson mass from minimally specified inputs:

\[ m_\tilde{t} \lesssim 30 M_Z \]

Only scenarios with negative A-terms can have sufficiently heavy stops to explain the Higgs boson mass

\[ -A_0 \approx M_{\text{SUSY}} \]

\[ M_{\text{SUSY}} = 3 \, \text{TeV} \]

\[ \tan \beta = 10 \]

\[ A_0 = 0 \]

\[ A_0 \approx M_{\text{SUSY}} \]

FeynHiggs 2.13.0-beta
Conclusions

The usual naturalness criteria do not necessarily indicate how model parameters have to be tuned.

I advocated considering any outcome that does not require specifying more than one digit of model parameters as natural.

In the CMSSM, up to ~3 TeV superpartners are natural in this sense. Only one digit of model parameters needs to be adjusted to get ~100 GeV electroweak scale, no matter what the remaining digits are. Parameter choices with negative A terms can give the correct Higgs mass.