Small Instantons and the Strong CP Problem

Kiel Howe. Fermi National Accelerator Laboratory, khowe@fnal.gov

DPF

K.H., Prateek Agrawal

Strong CP Problem

$$\mathcal{L} = \theta G\tilde{G} + y_u HQu^c + y_d HQd^c$$

Two SM CP violating Phases:

Strong CP (non-perturbative) $\bar{\theta} = \theta - \mathrm{Argdet} y_u - \mathrm{Argdet} y_d$

CKM $V_{CKM} \supset e^{i\delta_{\mathrm{CKM}}}$

Puzzle: CKM Phase O(1) but $\bar{\theta} \lesssim 10^{-10}$

(neutron EDM)

Sequestering?

SM is special $\Delta\bar{\theta} \ll \delta_{\mathrm{CKM}}$

Massless Up Quark Solution

Two-Flavor (single generation) QCD:

$$\mathcal{L} = \theta \tilde{G}_{\mu\nu} G^{\mu\nu} + m_u u u^c + m_d d d^c + \text{h.c.}$$

Chiral rotation: $u u^c \to e^{i\alpha} u' u^{c\prime}$, $d d^c \to e^{i\beta} d' d^{c\prime}$

Axial anomaly: $\theta \to \theta + \alpha + \beta$

Invariant CP Phase

$$\bar{\theta} = \theta - \mathrm{Arg} m_u - \mathrm{Arg} m_d$$

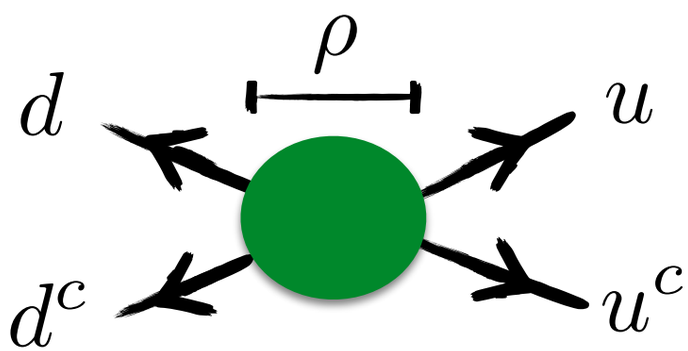
$m_u = 0$ → $U(1)_{PQ}$ → No Strong CP violation

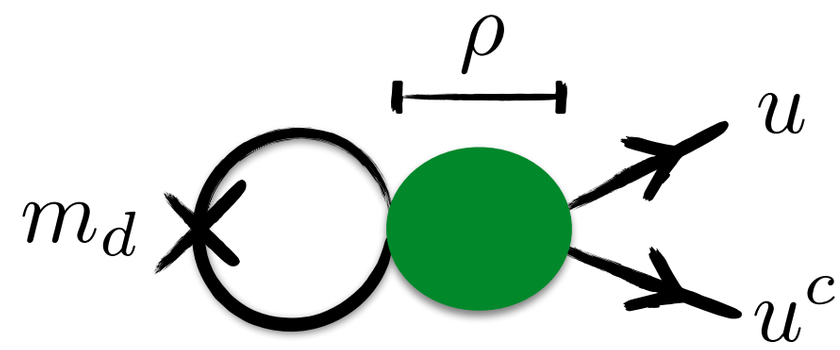
BUT Anomalous → Non-perturbative QCD generates up mass $m_u \neq 0$?

Dine et. al. 1410.8505,
Choi et. al. 1988, Georgi et. al. 1981

Massless Up Quark Solution

Dilute Instanton Gas Approximation



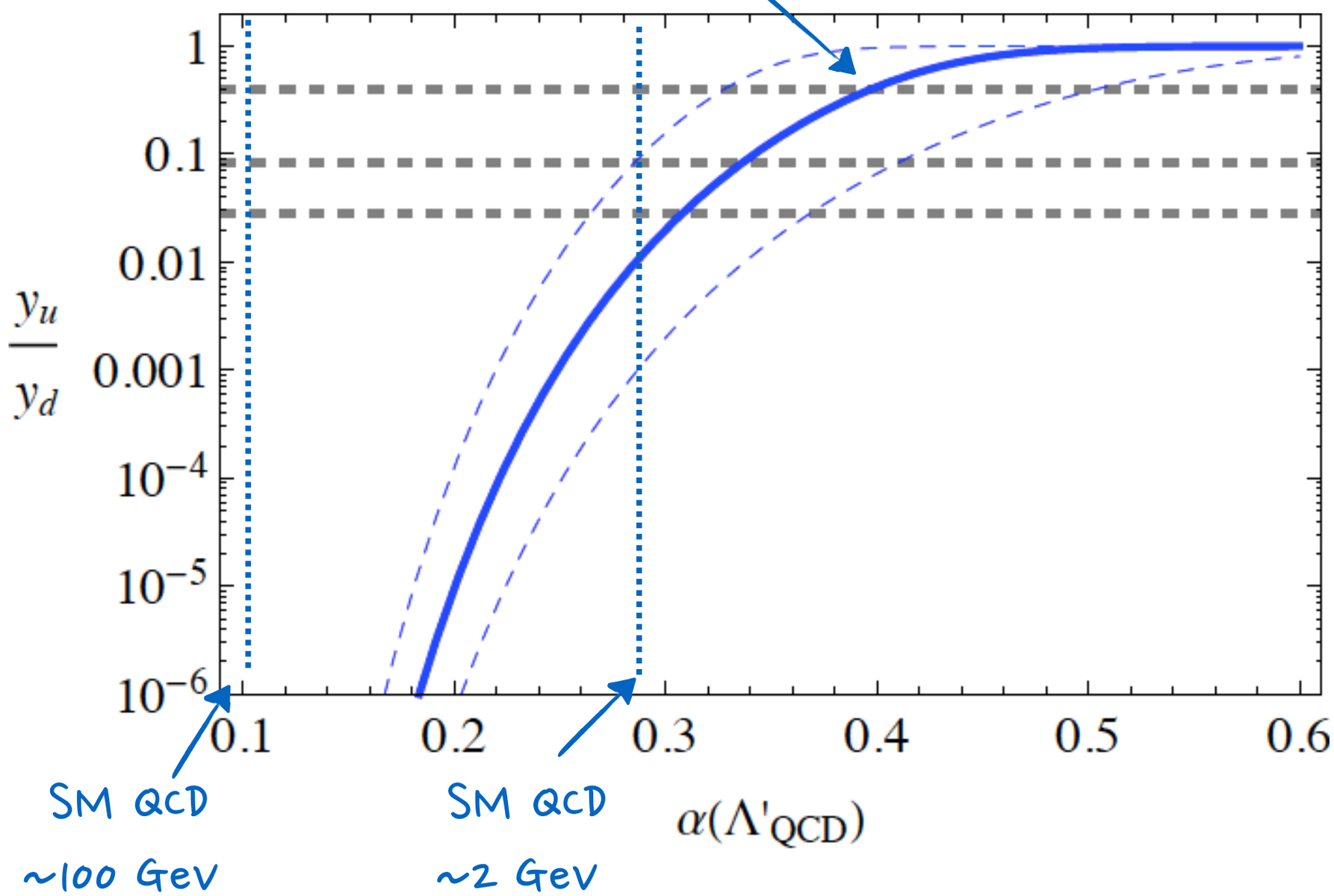


$$\Delta m_u = \underbrace{\int_{\rho_0}^{\rho_1} \frac{d\rho}{\rho}}_{\text{instanton sizes}} \times \overbrace{C_0 \times \left(\frac{8\pi^2}{g(\rho)^2}\right)^6 e^{-\frac{8\pi^2}{g(\rho)^2}}}^{\text{instanton "density" } D(\rho)} \times \underbrace{m_d^* e^{i\theta}}_{\text{invariant phase}}$$

$$\arg(\Delta m_u) + \arg(m_d) - \theta = 0$$

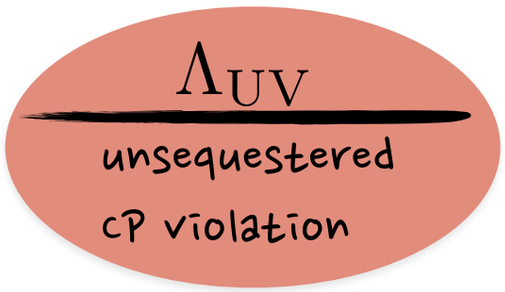
Massless Up Quark Solution

Leading chiral Lagrangian: $m_u \approx 0.5\ m_d$

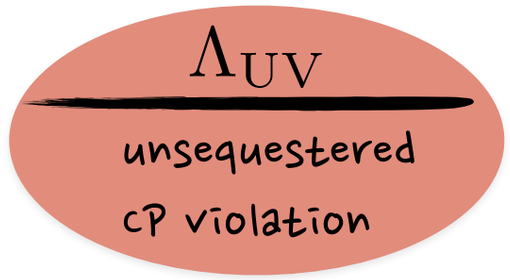


3-flavor lattice —> massless up quark solution fails

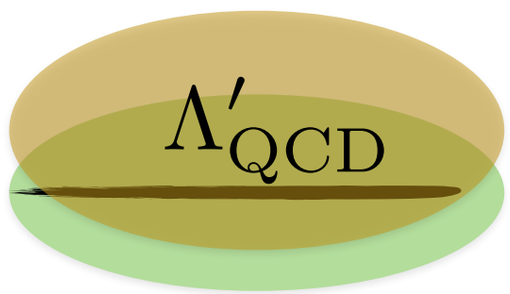
UV Instantons and PQ?



$m_u \approx 0$



$m_u \approx 0$

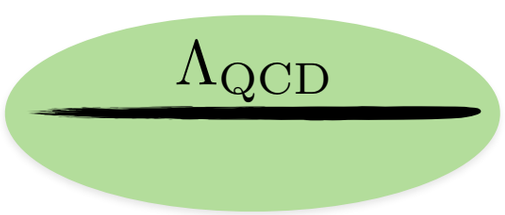


cP-violation remains sequestered

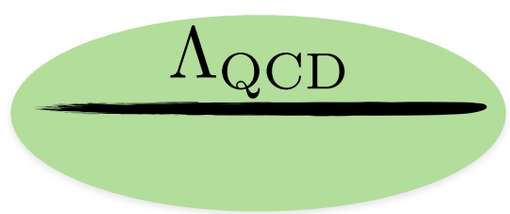
Non-perturbative effects important again

$\Delta m_u \sim m_d^* e^{i\delta}$

SM, m_u nonvanishing



Normal
(m_u = 0 not viable)



UV Instanton solutions

One 'wrong' way

$\Lambda_{\rm UV}$

SU(3 + N)

$\Lambda'_{\rm QCD}$

Higgsing

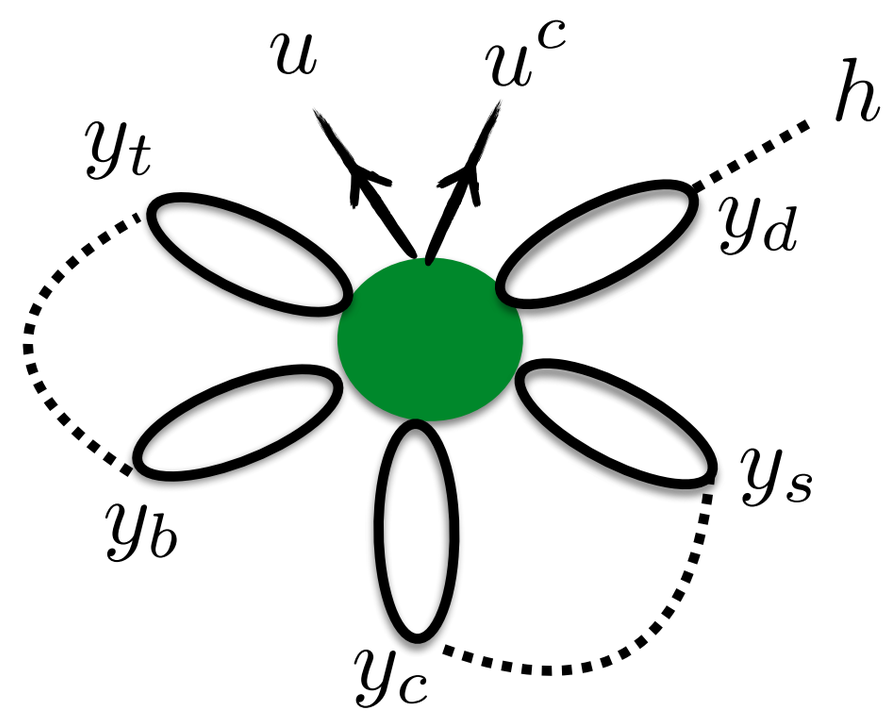
(+ any extra chiral symmetries strongly broken)

SM + Extra Matter

New colored states

$\Lambda_{\rm QCD}$

6-flavor instanton:



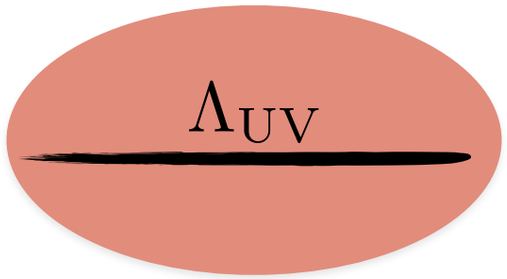
$$\frac{\Delta y_u}{y_d} \sim D(\Lambda'_{\rm QCD})\frac{y_t y_b y_s y_c}{(16\pi^2)^2}$$

too small

(extra breaking of SM chiral symmetries typically introduces phases, extra matter leads to further suppressions)

Axions: Holdom '81, Holdom & Peskin '85. Dine & Seiberg '86, Randall & Flynn '87

Flavorful Instantons



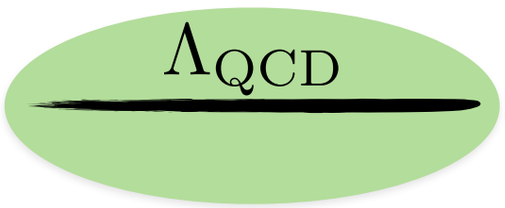
SU(3) × SU(3) × SU(3) (×⋯)

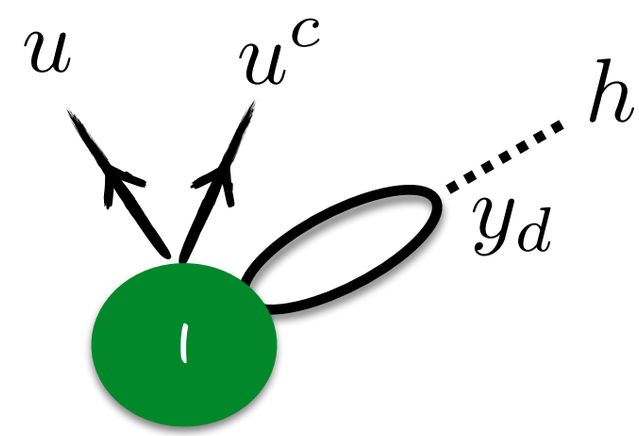
coincidence of scales?

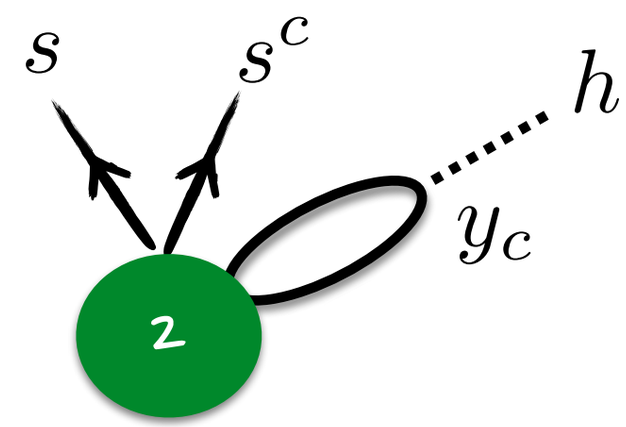
Higgsing

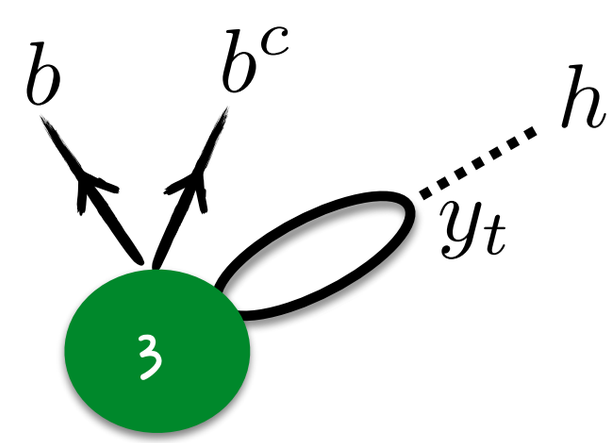
(+ any extra chiral symmetries strongly broken)

SM

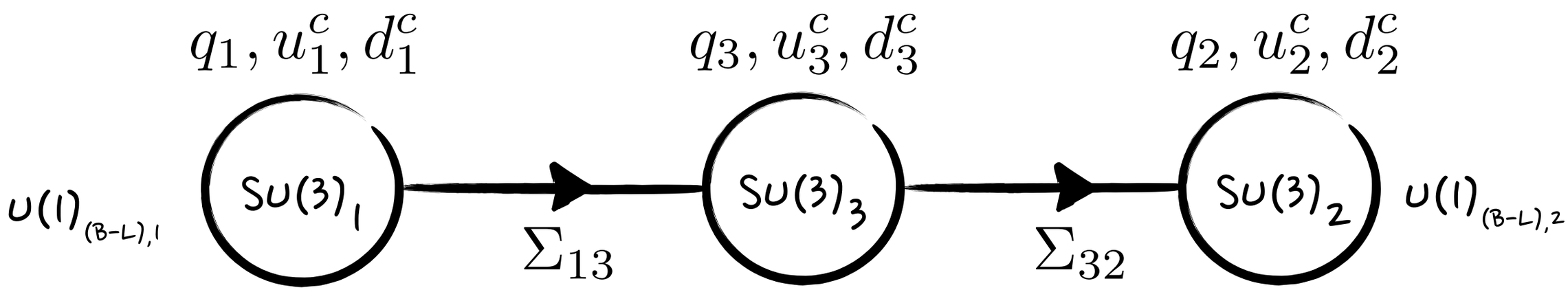




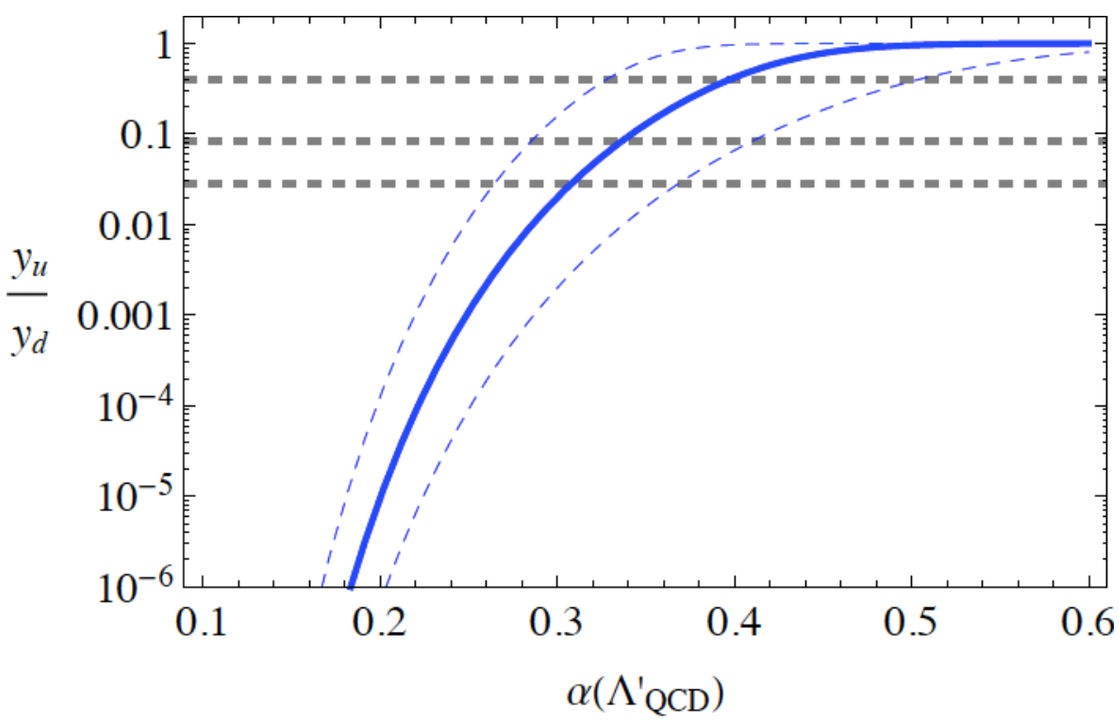




3x(2-flavor) models



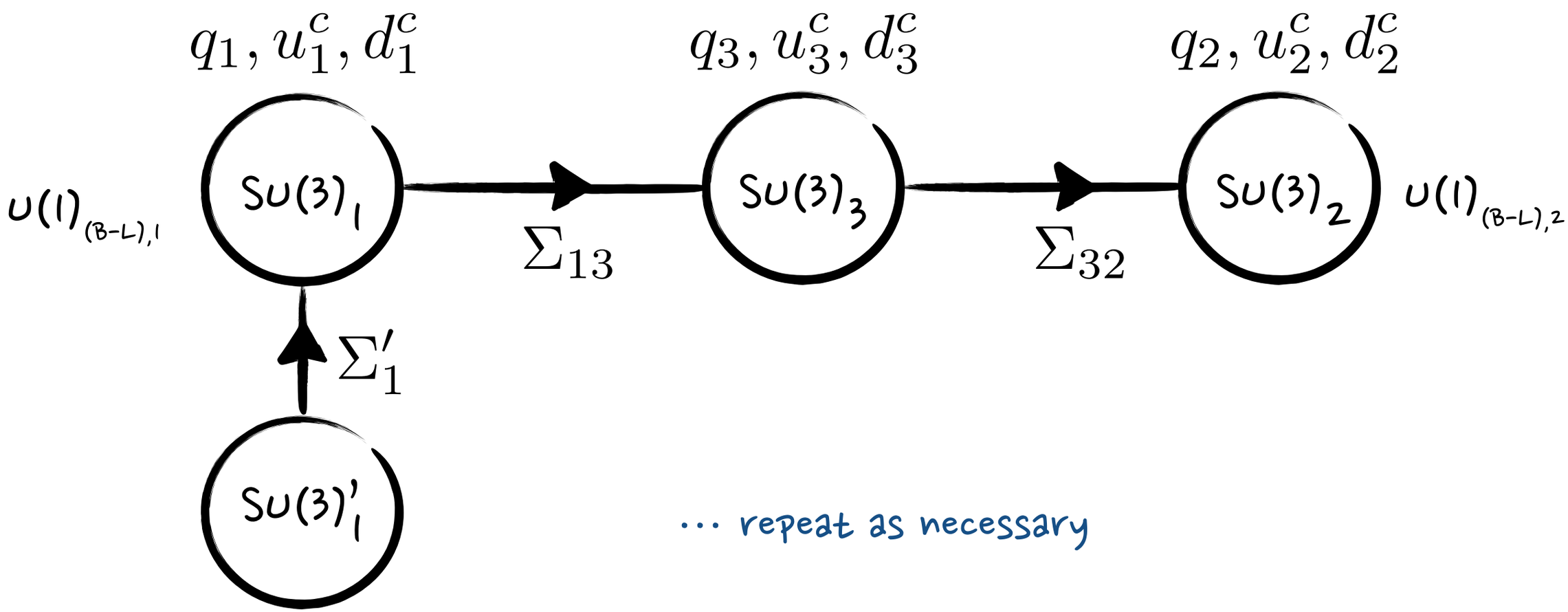
$\Delta y_u/y_d \approx 1/2$ $\Delta y_b/y_t \approx 1/35$ $\Delta y_s/y_c \approx 1/10$



$$\frac{1}{\alpha_s} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3}$$

$\longrightarrow$ $\alpha_s \sim 0.1 - 0.2$

weak coupling



... repeat as necessary

e.g. vector-like matter w/PQ

$$\psi_1', \overline{\psi}_1' \qquad m_\psi = 0$$

$$\Delta m_\psi = D(gv_1') \times (gv_1) \sim 10^{-6} \Lambda_{\mathrm{QCD}'}$$

CKM

Above instanton scale:

(background fields)

Higher dim $\sim \frac{\langle\Sigma_{13}\rangle\langle\Sigma_{32}\rangle}{\Lambda^2}$

only 1 phase
(depends on 4 elements)

Anomalous U(1)'s
remove theta

$$(Q_i y_u^{ij} u_j^c)H \qquad\qquad (Q_i y_d^{ij} d_j^c)H^\dagger$$

$$\begin{array}{cc|ccc} & & U(1)_{u_1} & -1 & -1 \\ \hline U(1)_{q12} & +1 & 0 & y_c s_{\theta_{12}} & 0 \\ & +1 & 0 & y_c c_{\theta_{12}} & 0 \\ U(1)_{q3} & +1 & 0 & 0 & y_t \end{array} \qquad \begin{array}{ccc} -1 & U(1)_{d_2} & -1 \\ \hline y_d & 0 & V_{31} y_b e^{i\delta_0} \\ \tilde{y}_d \sim 0 & 0 & V_{32} y_b \\ 0 & 0 & 0 \end{array}$$

Spurion/radiative:

$$\tilde{y}_d \sim y_d \times \frac{1}{16\pi^2}(\sim y_b^2 V_{31} V_{32} + \sim y_c^2 s_{\theta_{12}} c_{\theta_{12}})$$

Why this structure?

Need at least 3 orthogonal anomalous U(1) to remove phases

CKM

Below instanton scale:

$$(Q_i y_u^{ij} u_j^c) H \qquad\qquad (Q_i y_d^{ij} d_j^c) H^\dagger$$

$$\begin{pmatrix} \frac{y_u}{c_{\theta_{12}}} & y_c s_{\theta_{12}} & 0 \\ 0 & y_c c_{\theta_{12}} & 0 \\ 0 & 0 & y_t \end{pmatrix} \qquad \begin{pmatrix} y_d & 0 & \overset{\sim 1/100}{V_{31}} y_b \overset{\sim \pi/8}{e^{i\delta_0}} \\ \tilde{y}_d \sim 0 & y_s & V_{32} y_b \\ 0 & 0 & y_b \end{pmatrix}$$

$\sim 1/25$

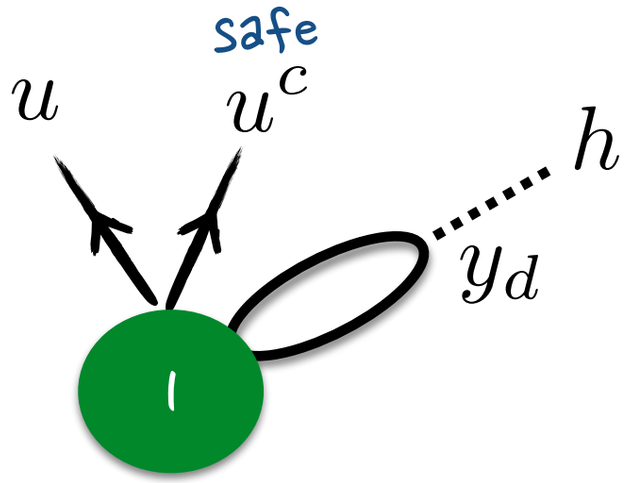
Why this structure?

Need at least 3 orthogonal anomalous U(1) to remove phases

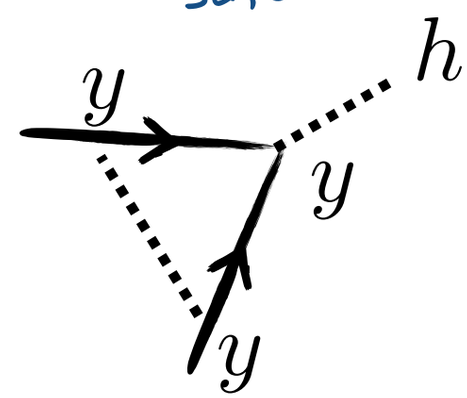
Radiative Sequestering of CPV?

Looks a lot like the SM…
BUT SU(3)xSU(3)xSU(3) breaks many of the flavor symmetries.

safe



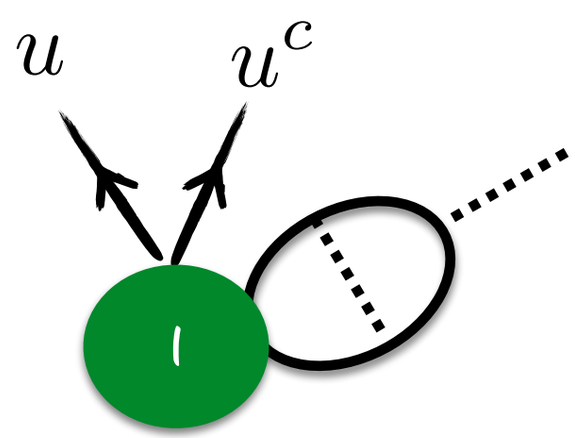
safe



running CP phase of mass?

Threshold effect: $$\Delta m_u = \int_{\rho_0}^{\rho_1} \frac{d\rho}{\rho} \times D(\rho) \times m_d(\rho)^* e^{-i\theta}$$

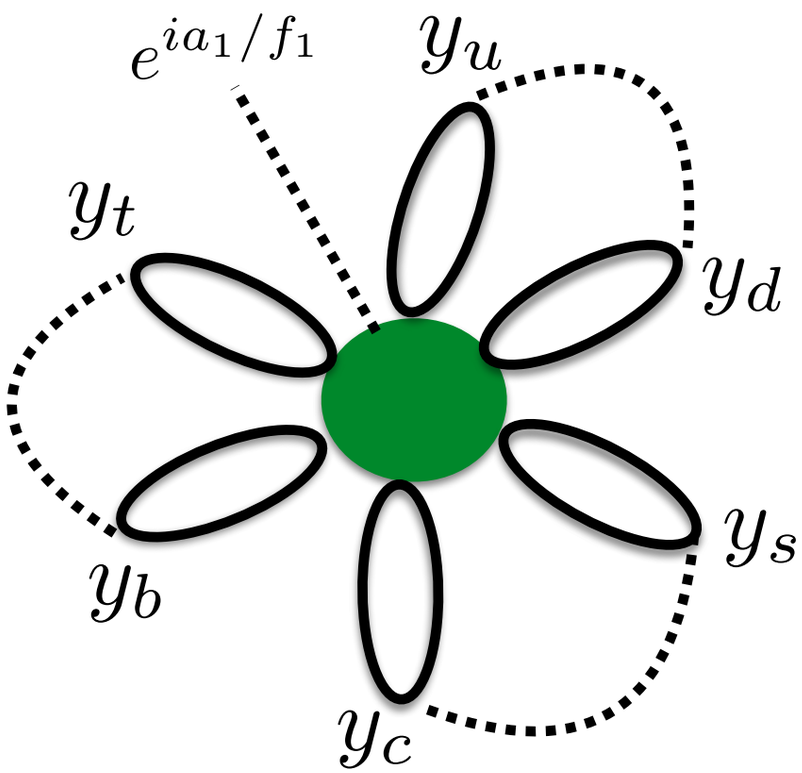
dangerous



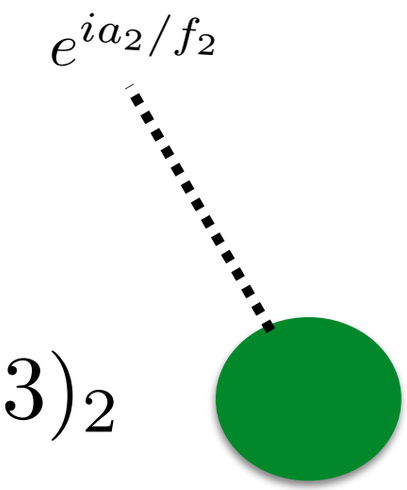
$$\Delta\bar{\theta} \approx \frac{6}{64\pi^2} \sin\delta_0 \tan\theta_{12} V_{31} V_{32} y_b^2$$

$$\sim 10^{-10}$$

Heavy Axion



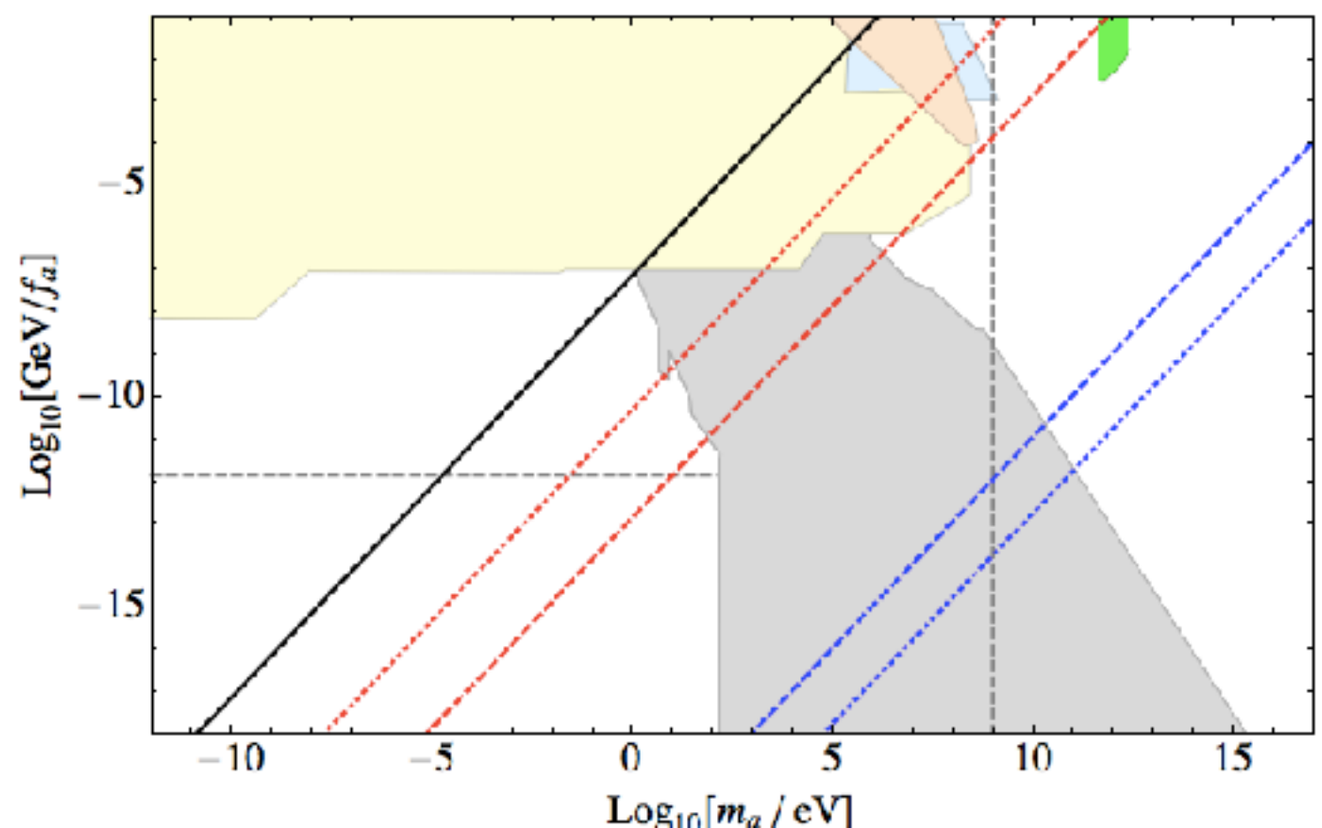
$$SU(3)_1 \times SU(3)_2$$



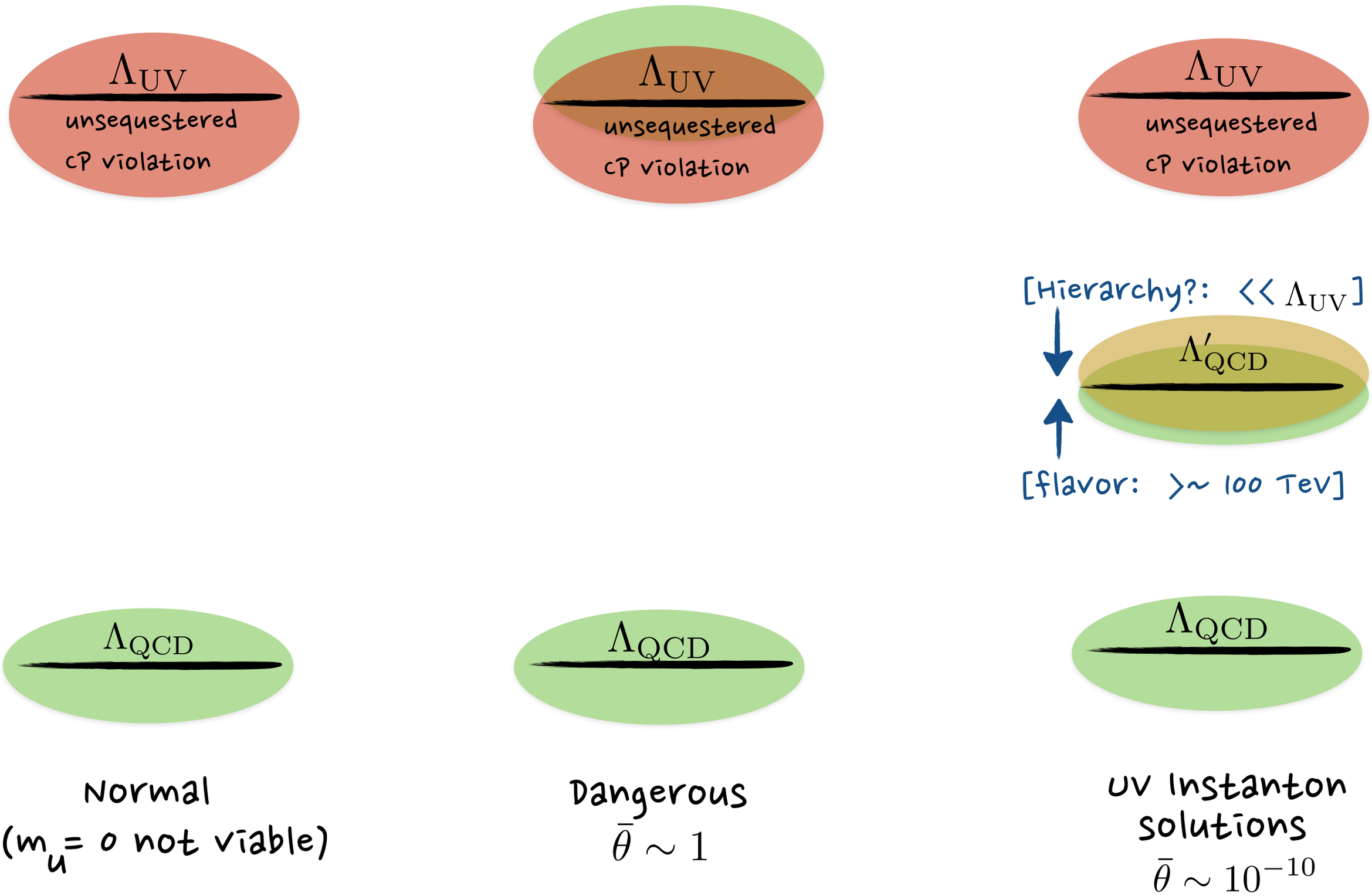
$$\Lambda_2^4 \approx \frac{2}{7} D[\alpha_3^{(2)}(\Lambda'_{QCD}{}^{-1})]\Lambda'_{QCD}{}^4$$

$$\Lambda_1^4 = K \int_{\rho=0}^{\rho=\Lambda'^{-1}_{QCD}} 2\frac{d\rho}{\rho^5} D[\alpha_3^{(1)}(\rho)] \approx K\frac{2}{3} D[\alpha_3^{(1)}(\Lambda'_{QCD}{}^{-1})]\Lambda'_{QCD}{}^4,$$

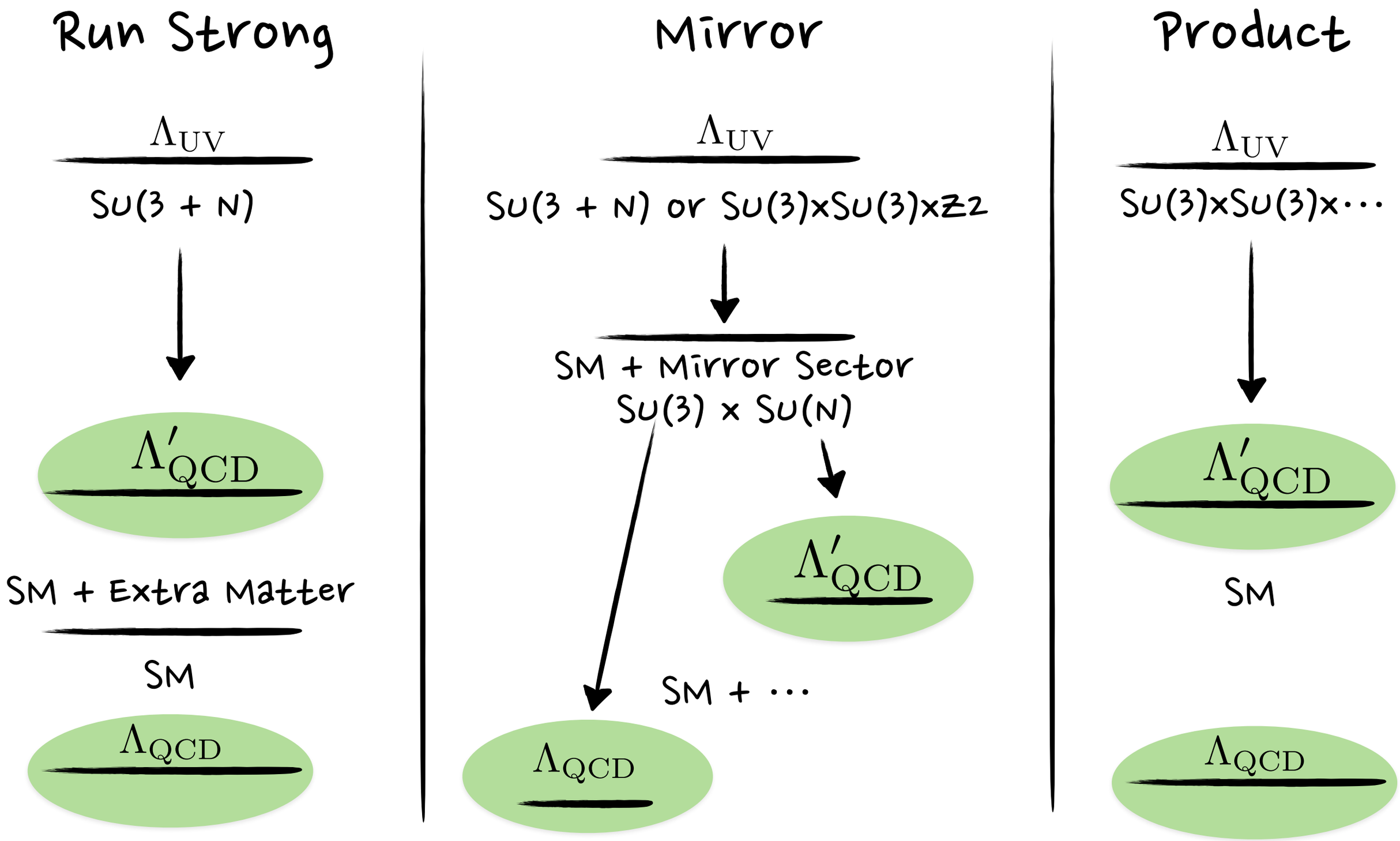
$$K = \left(\frac{y_u}{4\pi}\right)\left(\frac{y_d}{4\pi}\right)\left(\frac{y_c}{4\pi}\right)\left(\frac{y_s}{4\pi}\right)\left(\frac{y_t}{4\pi}\right)\left(\frac{y_b}{4\pi}\right) \approx 10^{-24}$$



Summary


$\Lambda_{\rm UV}$
unsequestered
CP violation
$\Lambda_{\rm UV}$
unsequestered
CP violation
$\Lambda_{\rm UV}$
unsequestered
CP violation
[Hierarchy?: << $\Lambda_{\rm UV}$]
$\Lambda'_{\rm QCD}$
[flavor: >~ 100 TeV]
$\Lambda_{\rm QCD}$
$\Lambda_{\rm QCD}$
$\Lambda_{\rm QCD}$
Normal
(m_u = 0 not viable)
Dangerous
$\bar{\theta} \sim 1$
UV Instanton
solutions
$\bar{\theta} \sim 10^{-10}$

Conclusions - Paths to Strong Coupling



Holdom '81,
Holdom & Peskin '85.
Dine & Seiberg '86,
Randall & Flynn '87

Rubakov et al. hep-ph/9703409,
Gianotti et. al. hep-ph/0009290,
Hook et al. 1411.3325, …
Yanagida et. al. 1504.06084,…

Thanks!

Protection from UV?

running CP phase of mass?

$$\Delta m_u = \int_{\rho_0}^{\rho_1} \frac{d\rho}{\rho} \times D(\rho) \times m_d(\rho)^* e^{-i\theta}$$

$\Lambda_{\rm UV}$

unsequestered CP violation

$$m'_d = e^{i\delta} m_d$$

$$\Delta m'_u \sim D(\Lambda_{\rm UV}^{-1}) m_d^* e^{-i\delta}$$

$$m_u = \Delta m_u + \Delta m'_u$$

—> Phase mismatch

$$\Delta\bar{\theta} \sim D(\Lambda_{\rm UV}^{-1}) \frac{|m_d|}{|m_u|} \times \delta$$

$\Lambda_{\rm QCD}$

$$m_d$$

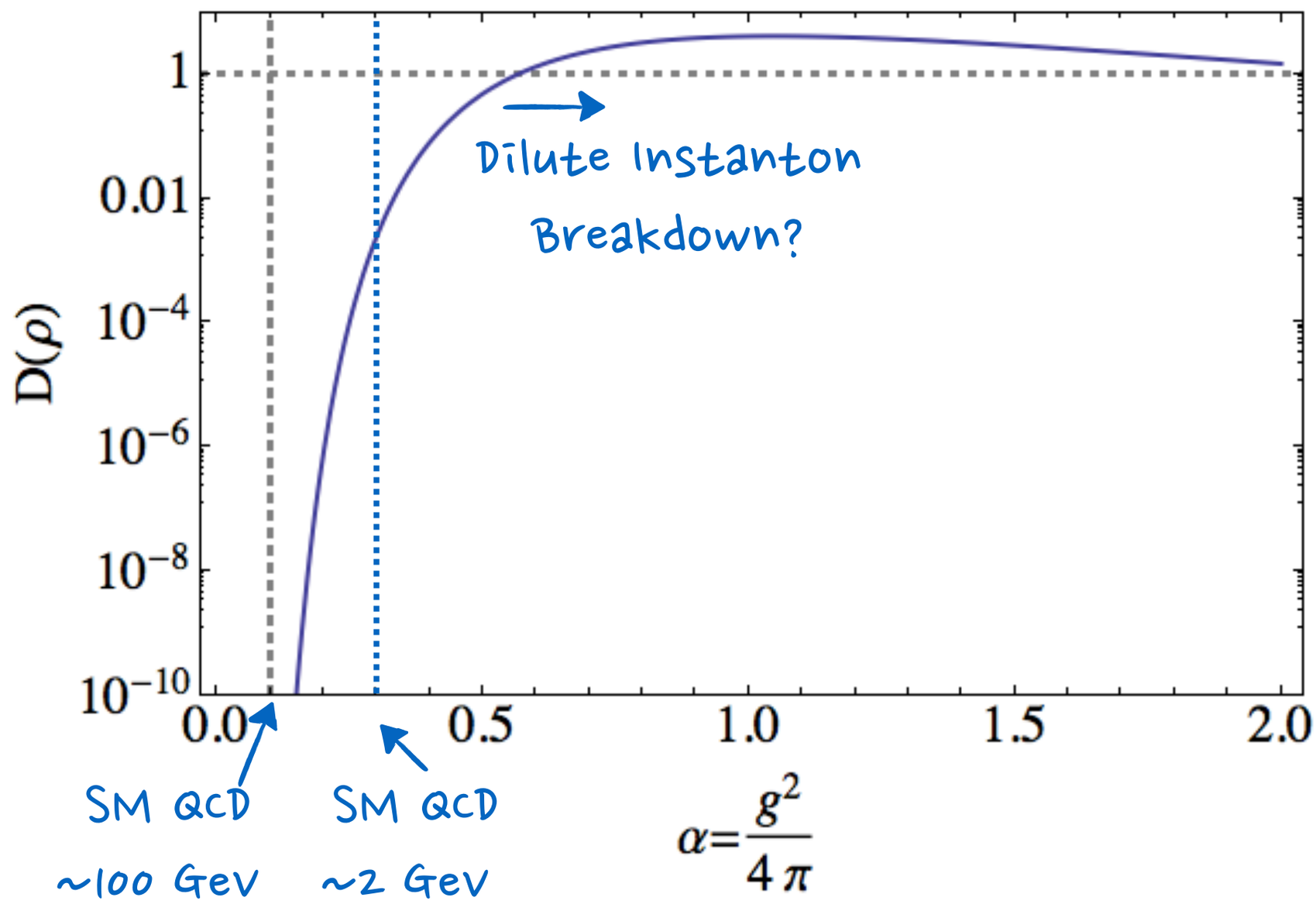
$$\Delta m_u \sim D(\Lambda_{\rm QCD}^{-1}) m_d^*$$

Suppression of extra PQ Breaking?

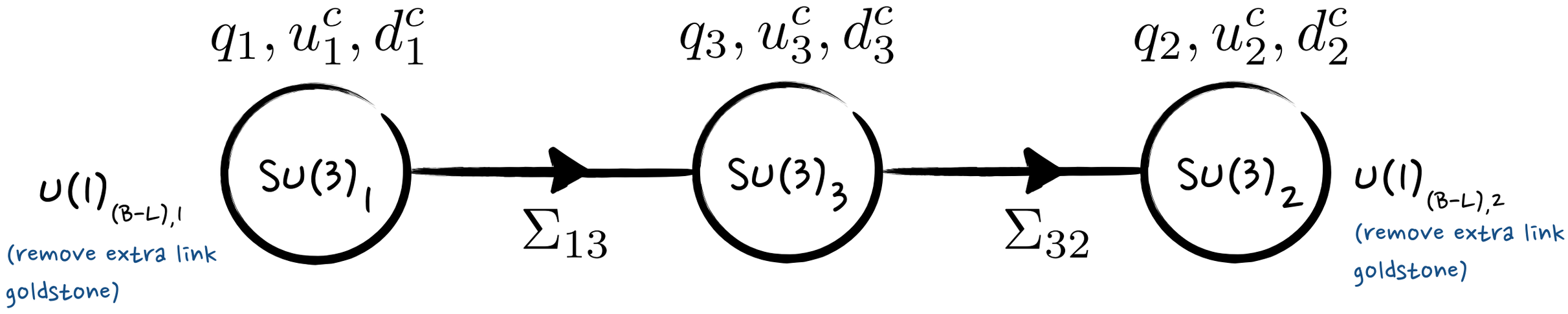
$$m_u = \Delta m_u + \Delta m'_u + m_u^0 \qquad \Delta\bar{\theta} \sim |m_u^0|/|m_u|$$

Massless Up Quark Solution

instanton "density" $$D(\rho) = C_0 \times \left(\frac{8\pi^2}{g(\rho)^2}\right)^6 e^{-\frac{8\pi^2}{g(\rho)^2}}$$



3x(2-flavor) models



PQ x 3

$$y_u = 0 \qquad y_b = 0 \qquad y_s = 0$$

Higgsing

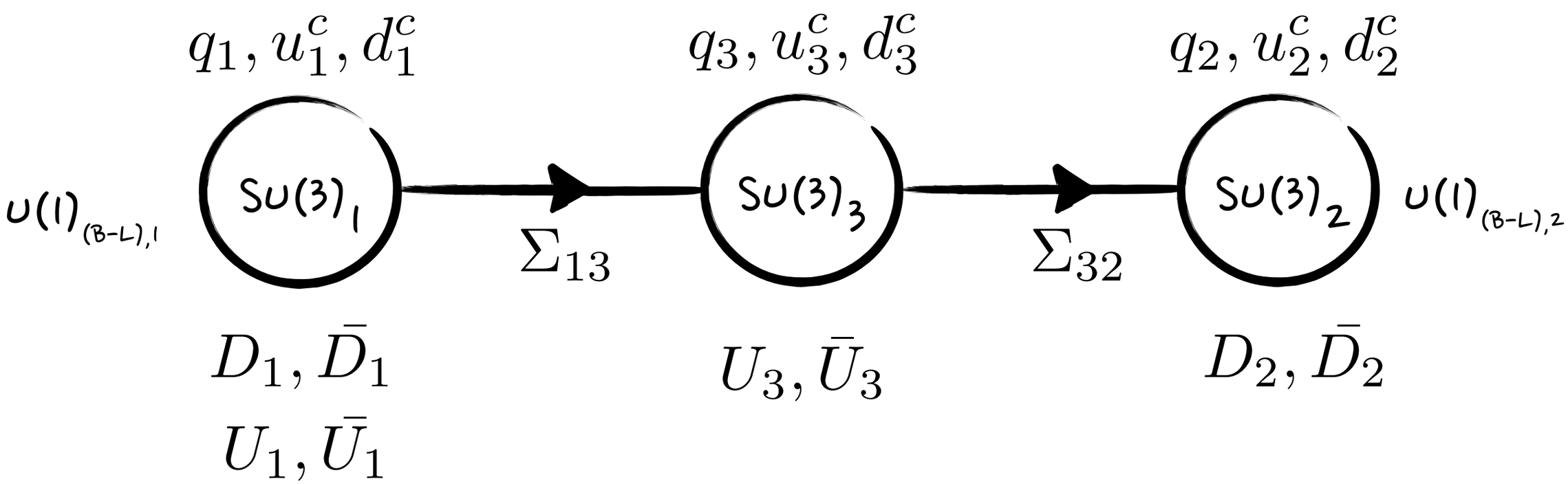
$$V(\Sigma_{13}) = (|\Sigma_{13}|^2 - v_{13}^2)^2 \qquad V(\Sigma_{32}) = (|\Sigma_{32}|^2 - v_{32}^2)^2$$

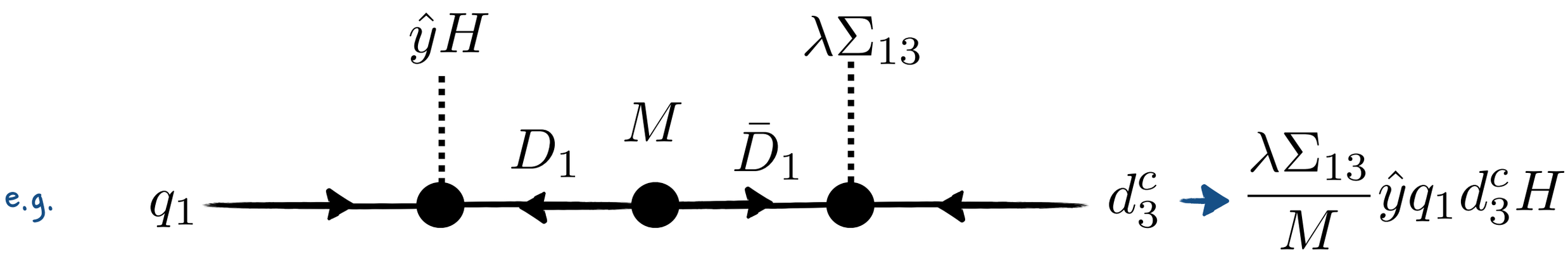
$$SU(3)_1 \times SU(3)_2 \times SU(3)_3 \to SU(3)_{\rm QCD}$$

Strongly coupled $\longrightarrow$ weakly coupled

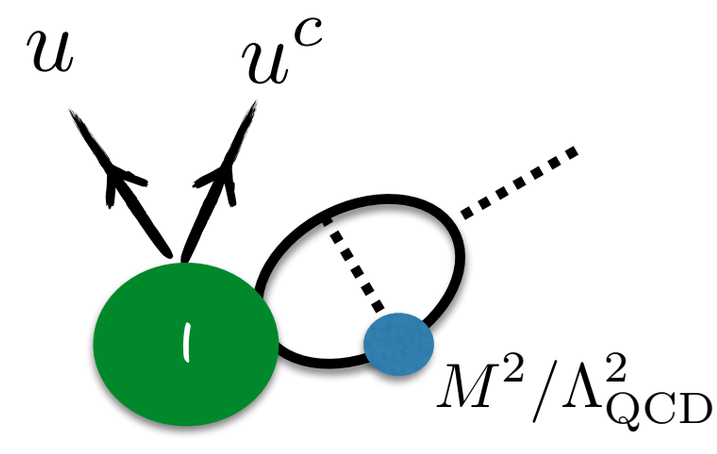
$$\Lambda'_{\rm QCD} \sim g v_{13}, g v_{32}$$

Flavor Completion

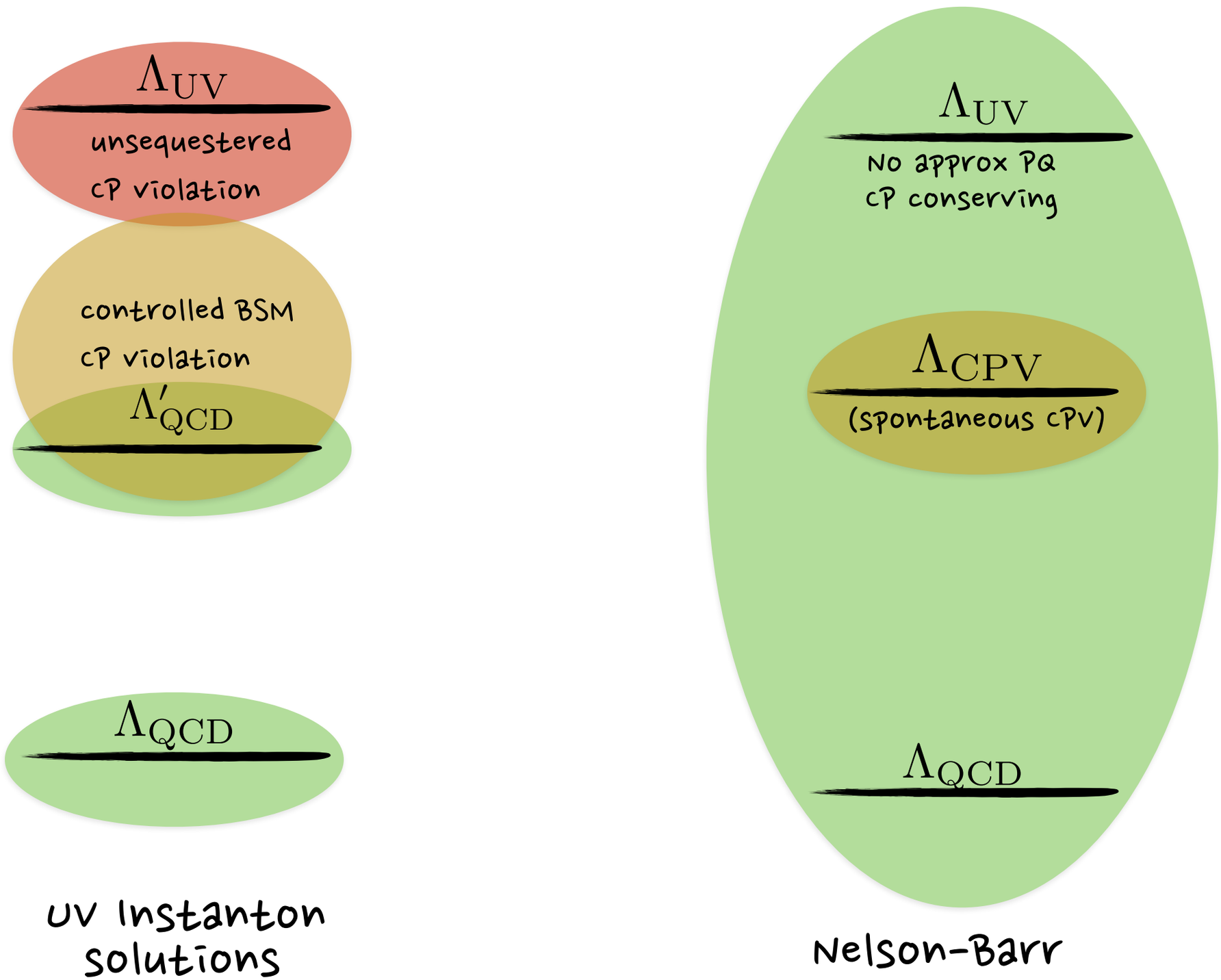




soft flavor? $M \lesssim \Lambda_{\mathrm{QCD}'}$ → soft CP violation



Radiative Phases & Nelson Barr



Alternative hierarchy

$$\begin{pmatrix} 0 & y_c s_{\theta_{12}} & 0 \\ 0 & y_c c_{\theta_{12}} & 0 \\ 0 & 0 & y_t \end{pmatrix} \qquad \begin{pmatrix} y_d & 0 & V_{31} y_b e^{i\delta_0} \\ \tilde{y}_d \sim 0 & 0 & V_{32} y_b \\ 0 & 0 & 0 \end{pmatrix}$$

$$y_d \to y_s, s_{\theta_{12}} \to c_{\theta_{12}}, V_{31} \to V_{32}$$

$$\Delta y_u \sim y_s, \Delta y_d \sim y_c$$ light quark masses from instantons!

$$\Delta\bar{\theta} \sim 3 \times 10^{-9}$$ —> soft flavor or tuning