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Beauty-full Tetraquarks

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Outline

- ❖ **Motivations for understanding QCD**
- ❖ **The static potential for four heavy quarks**
- ❖ **Numerical methods for many-body system**
- ❖ **Energy level for SI potential**
- ❖ **Spin-dependent correction**
- ❖ **Conclusions**

QED

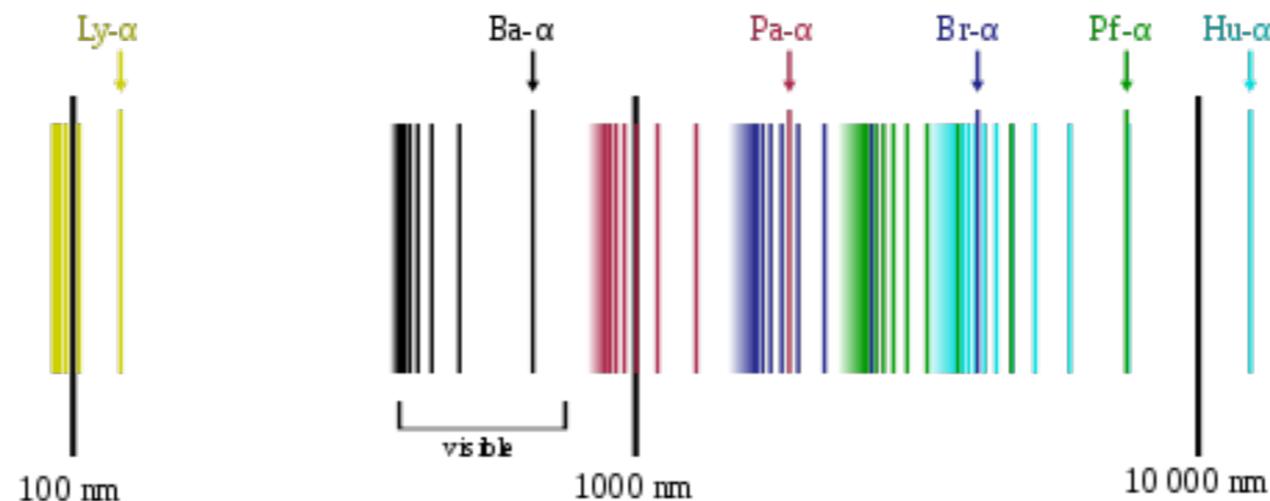
- ❖ **Hydrogen system, solving the non-relativistic Schrodinger equation**

$$\left[\frac{-\hbar^2}{2\mu} \nabla^2 + V(\vec{r}, t) \right] \Psi(\vec{r}, t) = E \Psi(\vec{r}, t)$$

- ❖ **The electric potential between two charge particles is just the Coulomb form**

$$V(\vec{r}) = -\frac{\alpha}{r}$$

- ❖ **Quantized spectra**

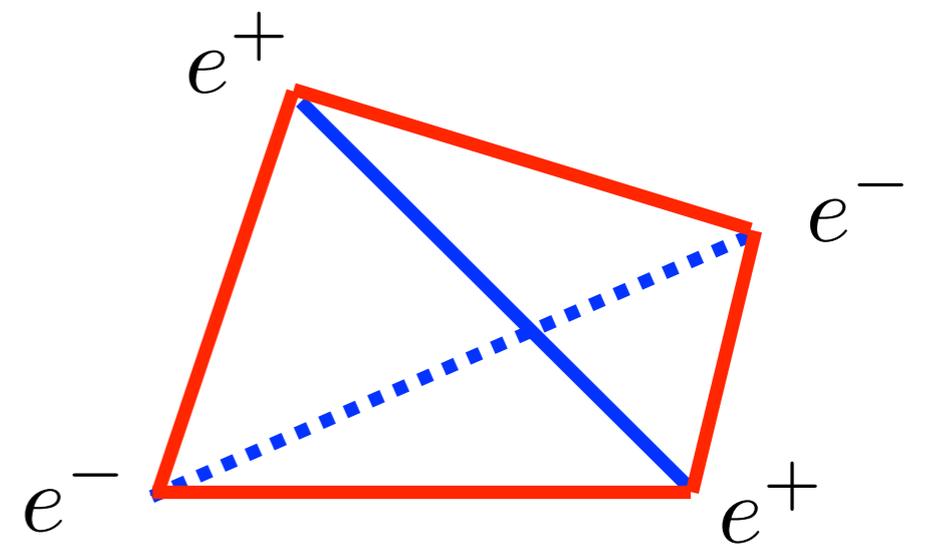


Many-body QED

- ❖ To understand materials in ordinary life, it is extremely important to understand the many-electronic system
- ❖ As a warm up, let's consider a simple system with two electrons and two positrons

$$V(\vec{r}_i) = -\alpha \left(\frac{1}{r_{13}} + \frac{1}{r_{24}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} \right) + \alpha \left(\frac{1}{r_{12}} + \frac{1}{r_{34}} \right)$$

- ❖ In 1946, Wheeler suggested a possible di-positronium molecular state, based on a simple variation calculation to estimate the additional binding energy



Calculation for Ps_2

- ❖ Many methods have been adopted to calculate the ground state energy: including the variation method and Quantum Monte Carlo method

Table 10. Calculations of binding energies of the positronium molecule ω_{Ps_2}

Author	Binding energy (ω_{Ps_2})	
	Ryd	eV
Hylleraas and Ore (1947)	-0.0085	-0.116
Ore (1947)	-0.009	-0.122
Akimoto and Hamamura (1972b)	-0.0135	-0.184
Brinkman <i>et al</i> (1973)	-0.0145	-0.197
Lee <i>et al</i> (1983)	-0.03 ± 0.002	-0.408 ± 0.027
Ho (1986)	-0.0302	-0.411
Kinghorn and Poshusta (1993)	-0.031 98	-0.435
Kozłowski and Adamowicz (1993)	-0.031 98	-0.435
Present calculation at $m_6 = 1$	-0.0421	-0.573

El-Gogary, et. al., J. Phys. B, 28 (1995) 4927

- ❖ The additional binding energy is -0.435 eV
- ❖ The ratio is $0.435/13.6 = 3.2\%$; a small binding

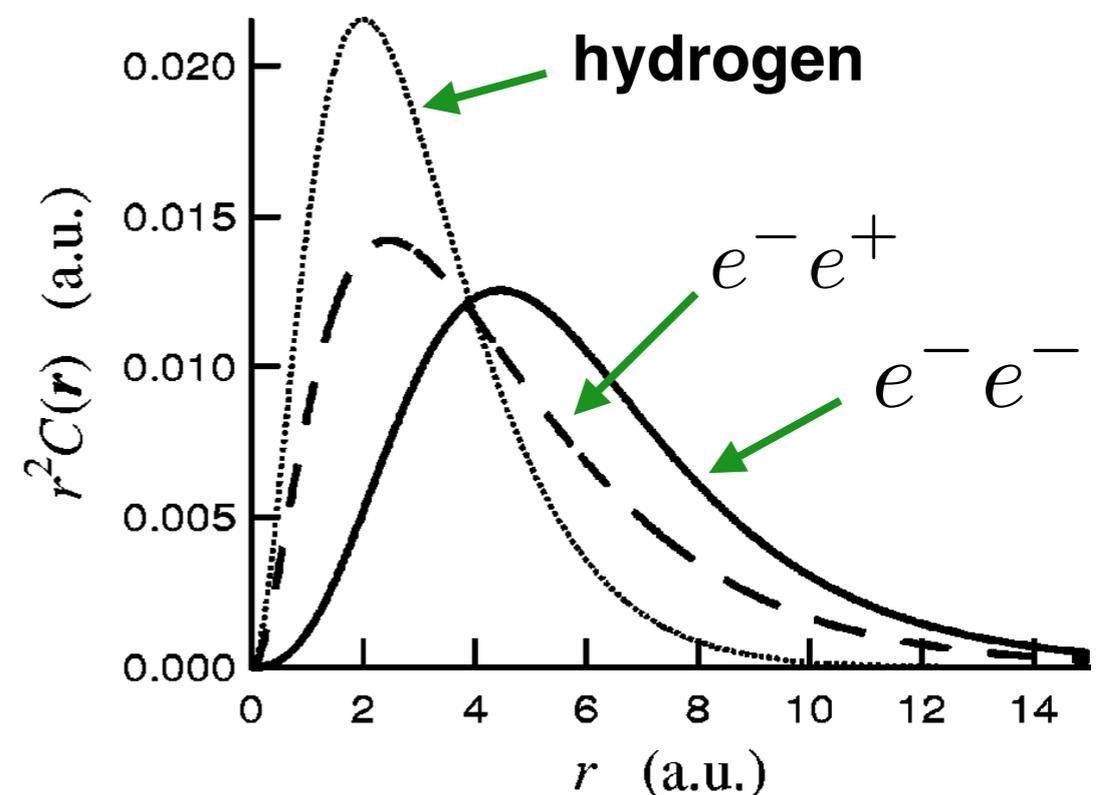
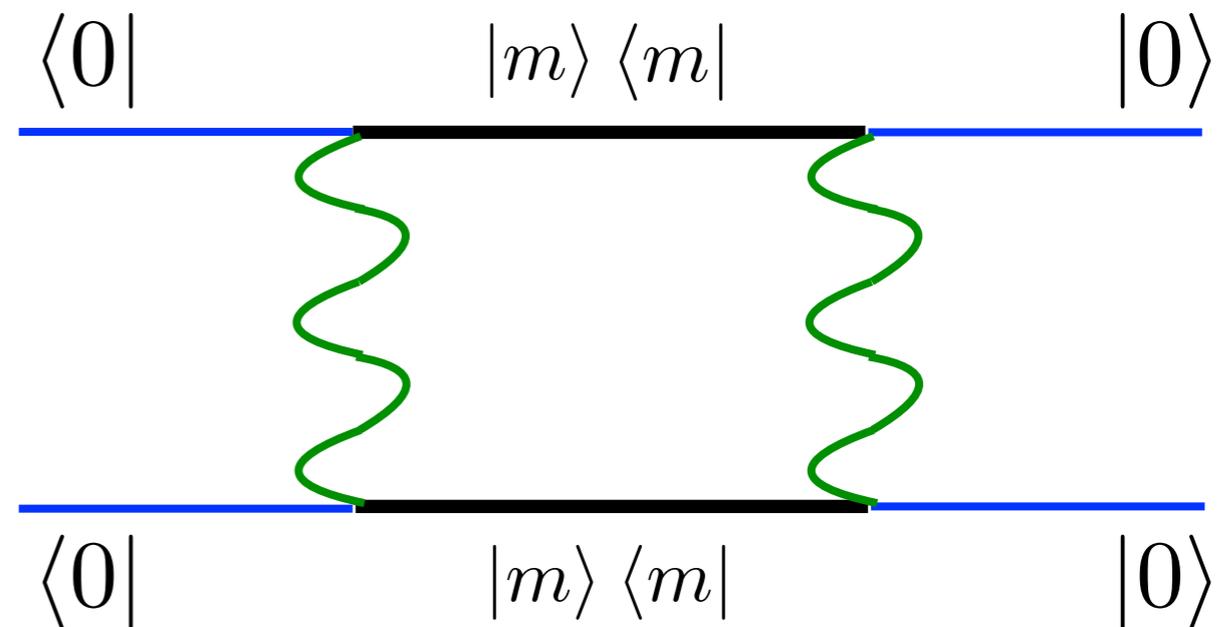
Calculation for Ps_2

❖ The ratio is $0.435/13.6 = 3.2\%$; a small binding

❖ This small binding energy can be understood as the London-van der Waals force $\sim -\alpha/R^6$

❖ The wave-functions are modified compared to the hydrogen one

Suzuki, et. al, PRA 58 (1998) 1918



Experimental Confirmation for Ps₂

- ❖ The existence of di-positronium molecular is only demonstrated in 2007

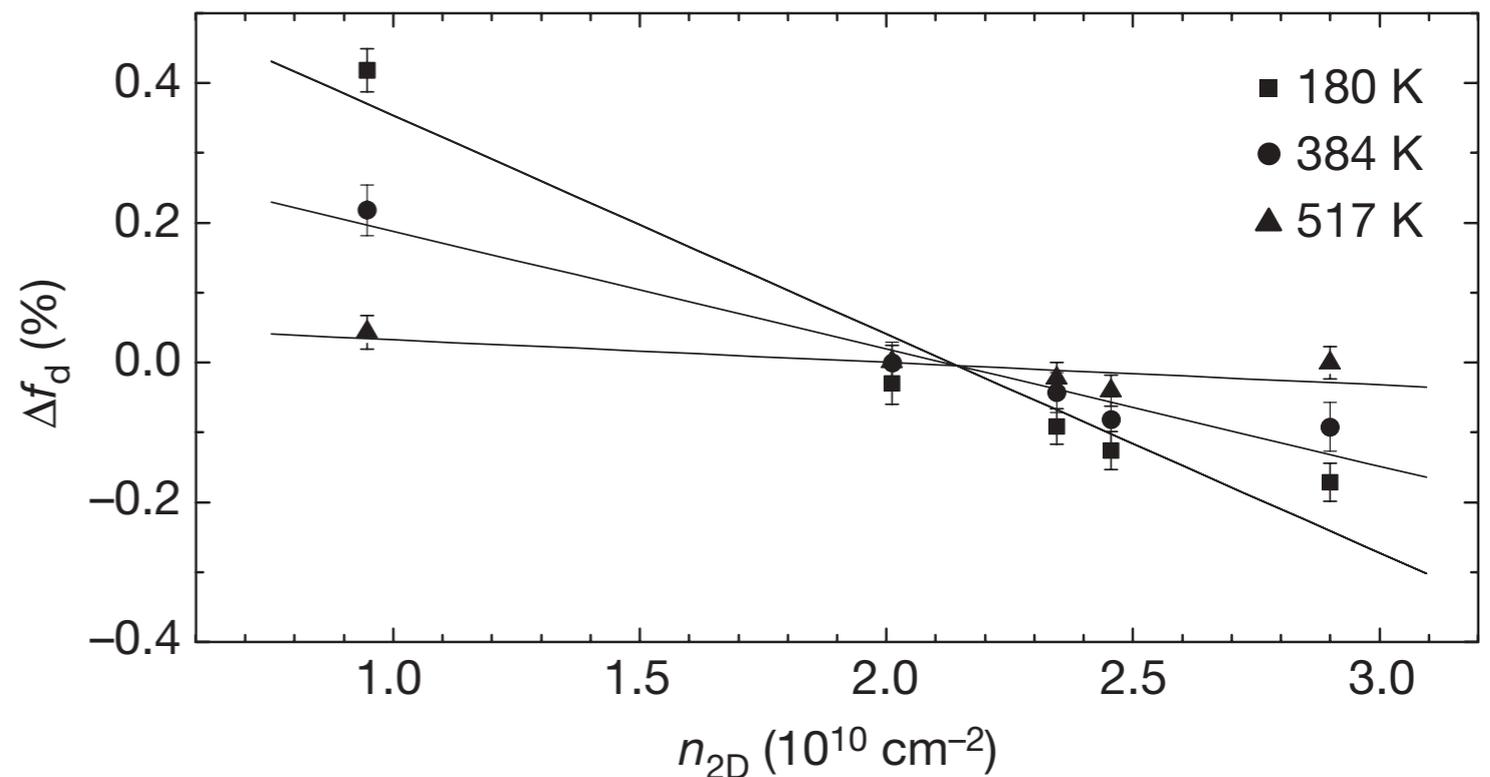
f_d is a measure of the amount of o-Ps created

Ordinary (pick-off) decay
decay due to Ps-Ps interactions

$$\frac{dn}{dt} = -\gamma n(1 + \beta n)$$

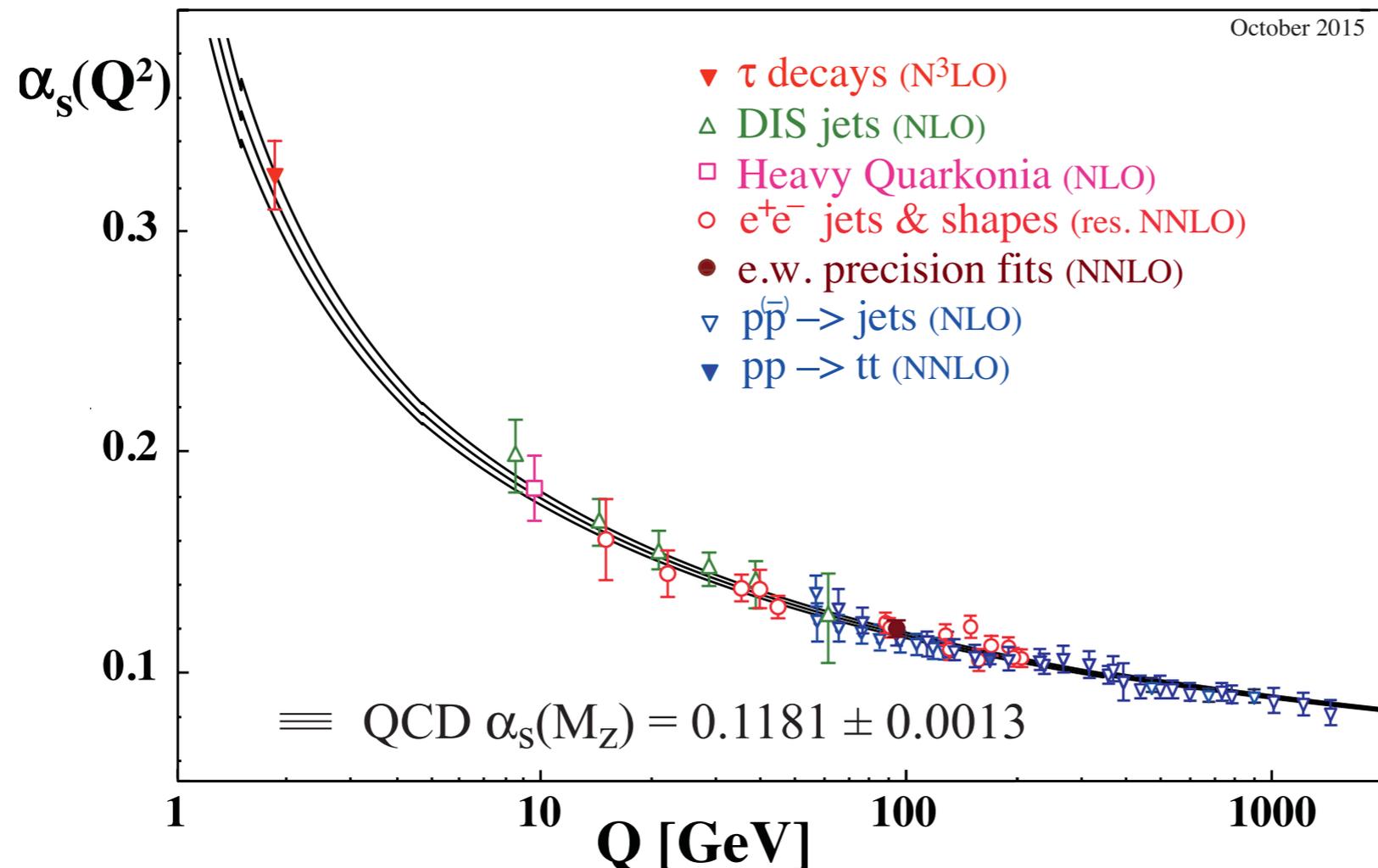
parameter β describes strength of SEQ and/or Ps₂ formation

Cassidy and Mills, Nature 449, (2007) 195



QCD

- ❖ QCD behaves different from QED, especially at a long distance

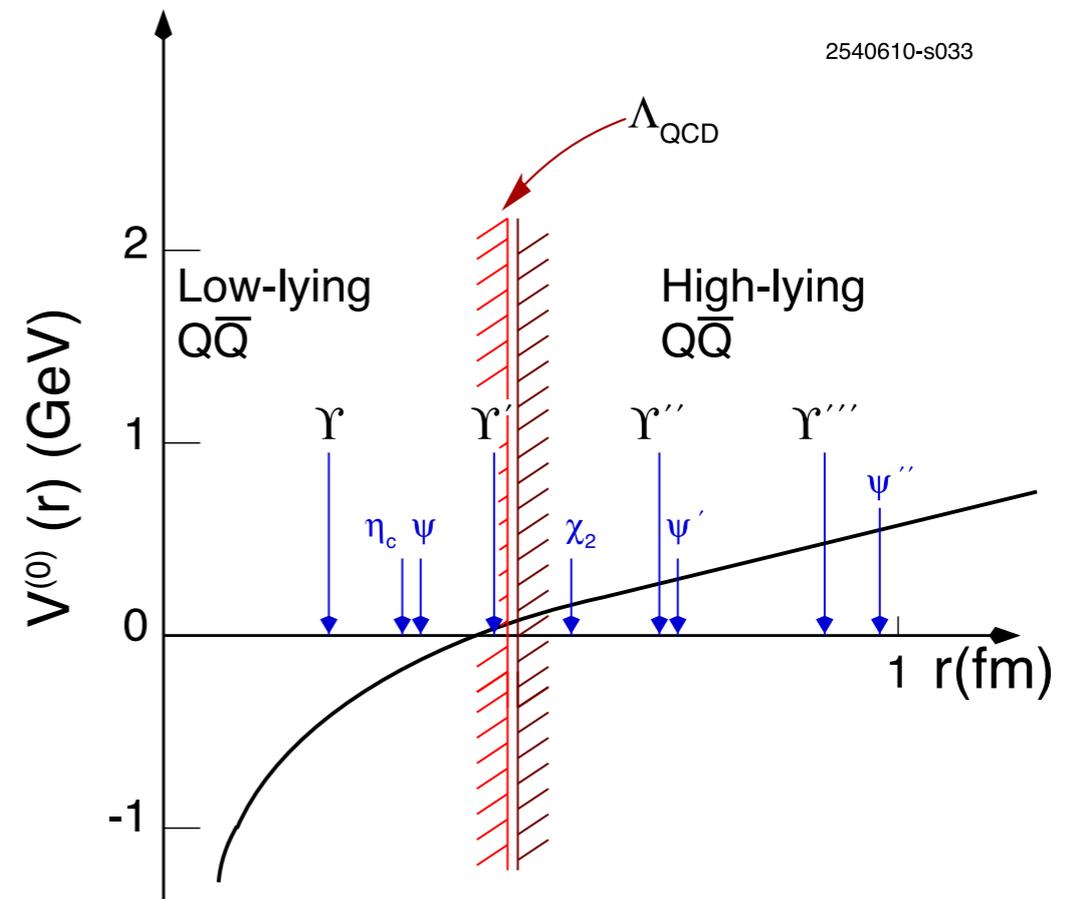


- ❖ It is asymptotically free at a short distance and has confinement at a long distance

QCD Static Potential

- ❖ At short-distance, one could use the approximate one-gluon exchange Coulomb force
- ❖ **Appelquist and Politzer** used this fact to predict the spin-one charmonium state based on Quantum Mechanics calculation, which is soon measured as J/Psi
- ❖ To have a more precise QM calculation for the quarkonium spectroscopy, phenomenological potential is widely used

$$V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + \frac{r}{a^2}$$



Brambilla, et. al., arXiv:1010.5827

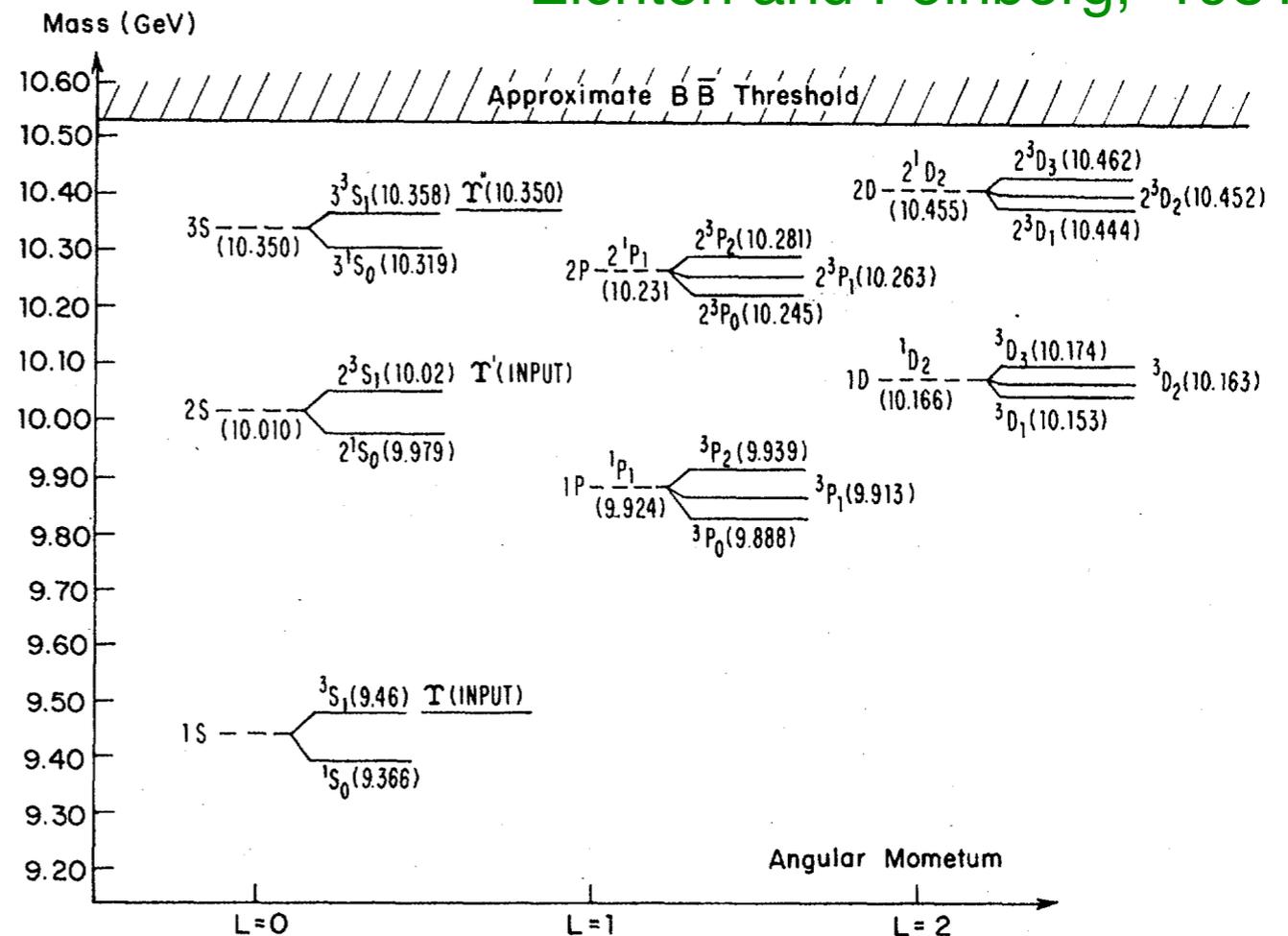
QCD Spin-dependent Potential

- ❖ The situation is similar to the QED for the short-distance SD interactions
- ❖ The delta function part of the potential is given by

$$V(\vec{r}) = -\frac{4}{3}\alpha_s \frac{2}{3m_1m_2} \vec{S}_1 \cdot \vec{S}_2 4\pi\delta(\vec{r})$$

Eichten and Feinberg, '1981

- ❖ This can generate the spin splitting energy from perturbation calculations



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Static Potential for Three Quarks

- ❖ One can use the static potential to study baryon spectrum in the quark model.
- ❖ Based on the flux-tube model from the strong-coupling Hamiltonian lattice formation of QCD, one has two configurations to consider

Isgur and Paton, '1985

$$V^{3Q} = -\frac{1}{2} \frac{4\alpha_s}{3} \left(\frac{1}{r_{12}} + \frac{1}{r_{13}} + \frac{1}{r_{23}} \right) + \frac{1}{a^2} L_{\min}$$

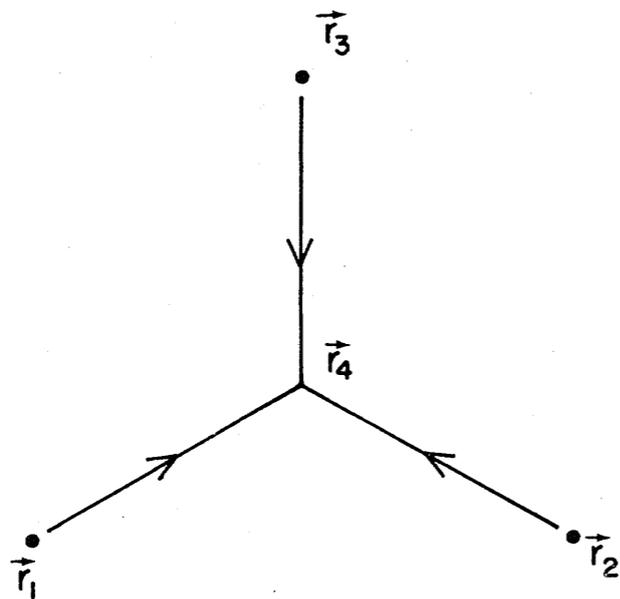


FIG. 2. The flux-tube configuration in a three-quark state at strong coupling, when none of the interior angles of the triangle $\vec{r}_1\vec{r}_2\vec{r}_3$ are greater than 120° .

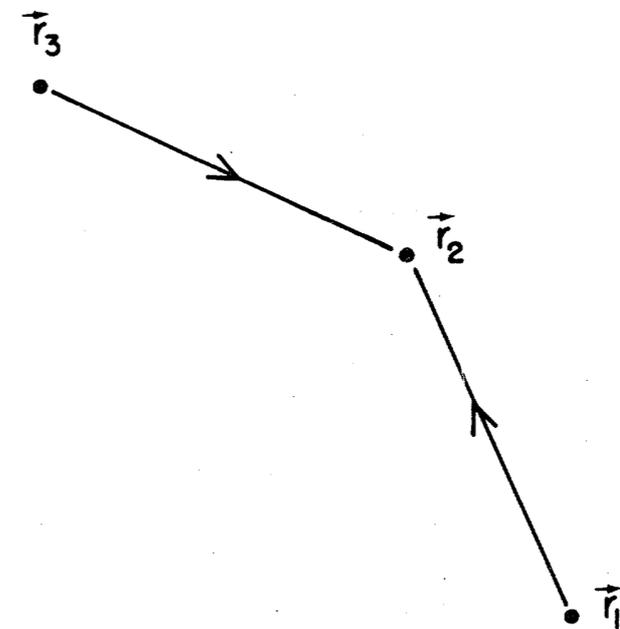
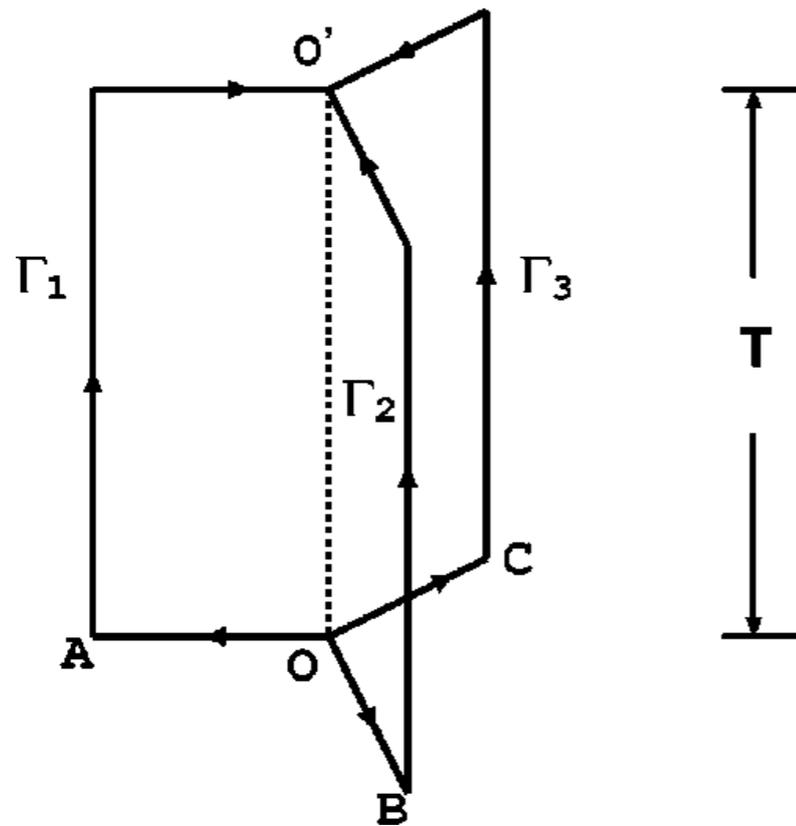


FIG. 3. The flux-tube configuration in a three-quark state when the angle at \vec{r}_2 is greater than 120° .

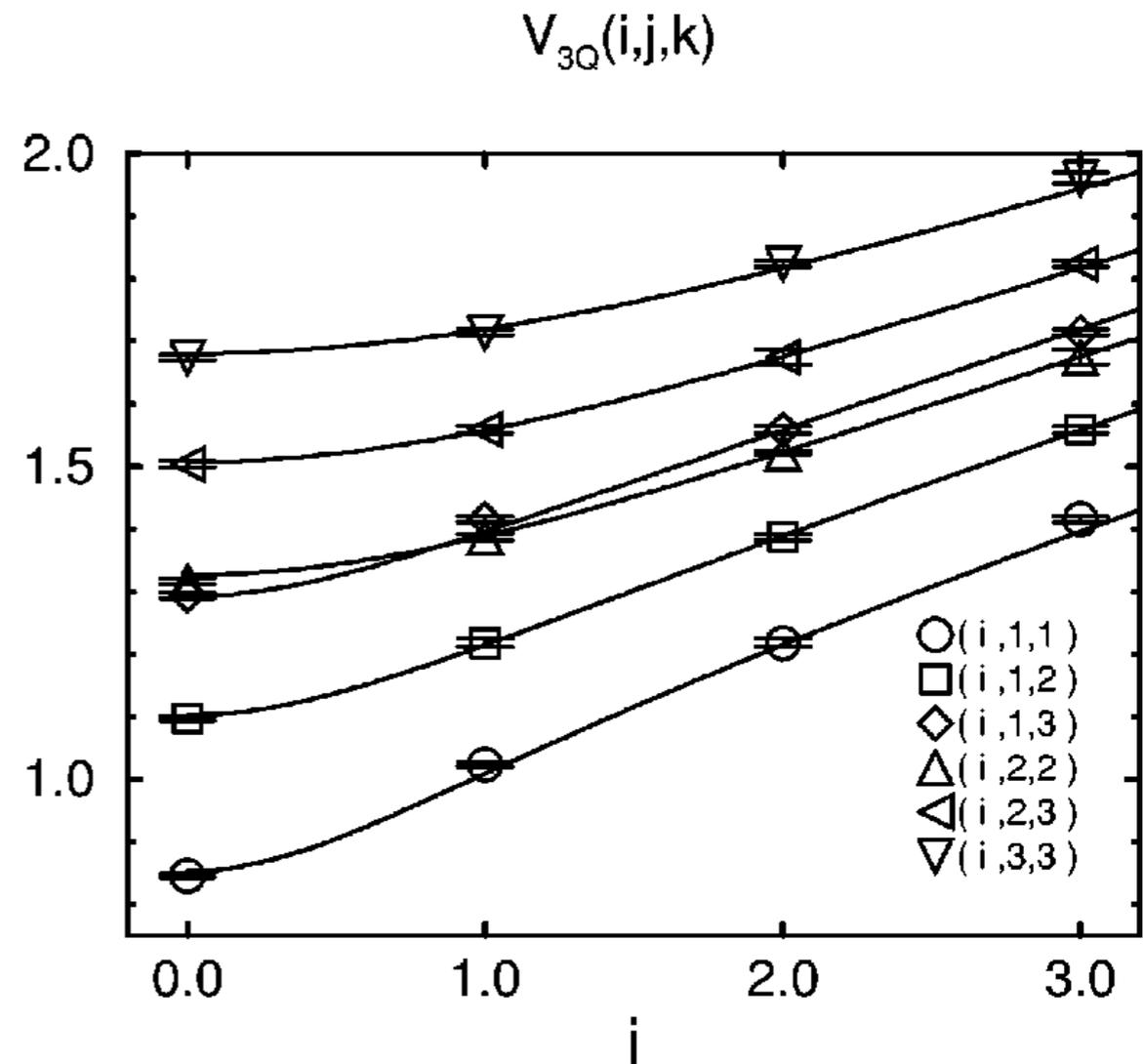
Static Potential for Three Quarks

- ❖ The lattice simulation has supported the flux-tube model for static potential

Takahashi, et. al., hep-lat/0204011

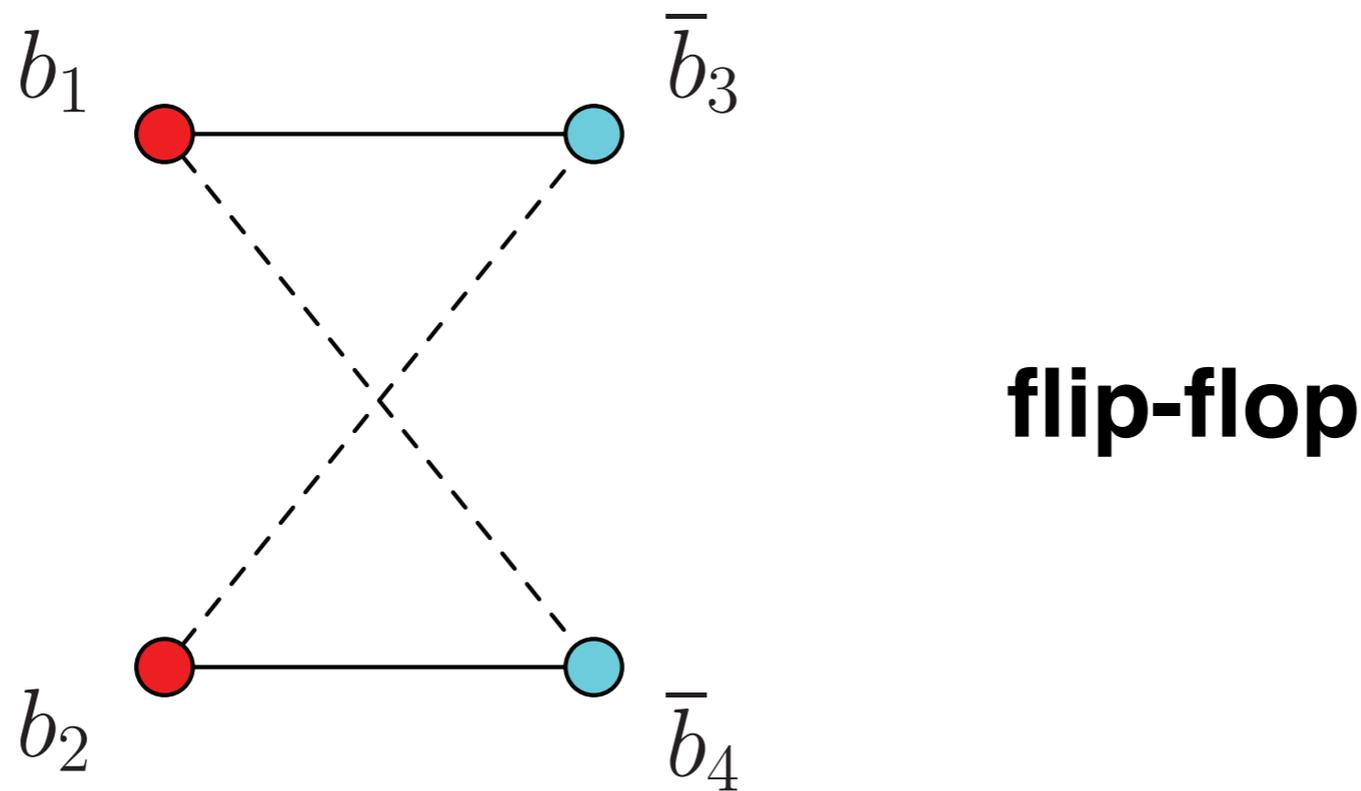


$$V_{3Q} = - \lim_{T \rightarrow \infty} \frac{1}{T} \ln \langle W_{3Q} \rangle$$



Static Potential for Four Quarks

- ❖ For the four-quark state, more configurations exist

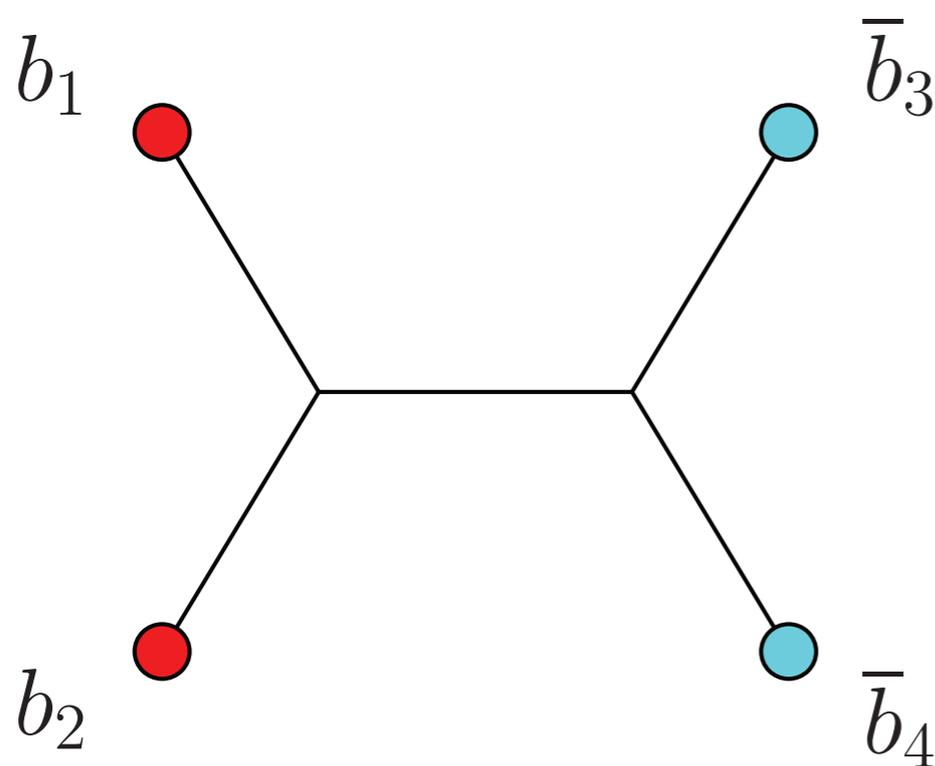


$$V_{(13,24)}^{\text{di-meson}} = -\frac{4\alpha_s}{3} \left(\frac{1}{r_{13}} + \frac{1}{r_{24}} \right) + \frac{1}{a^2} (r_{13} + r_{24})$$

$$V^{\text{flip-flop}} \equiv \min \left[V_{(13,24)}^{\text{di-meson}}, V_{(14,23)}^{\text{di-meson}} \right]$$

Static Potential for Four Quarks

❖ diquark-diquark configuration



butterfly

$$V^{\text{butterfly}} = -\frac{\alpha_s}{3} \left(\frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} + \frac{1}{r_{24}} \right) - \frac{2\alpha_s}{3} \left(\frac{1}{r_{12}} + \frac{1}{r_{34}} \right) + \frac{1}{a^2} L_{\min}$$

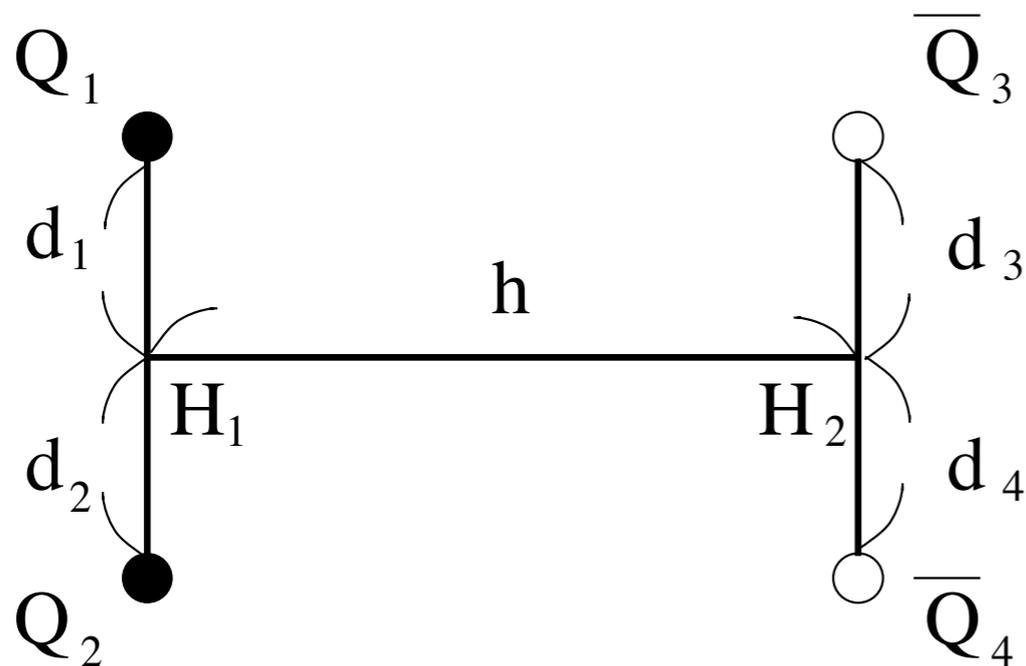
- ❖ the “good” diquark is used here; triplet color contraction
- ❖ for the “bad” sextet diquark contraction, replace the coefficients $(-1/3, -2/3)$ by $(-5/6, 1/3)$, which provides a larger value

Static Potential for Four Quarks

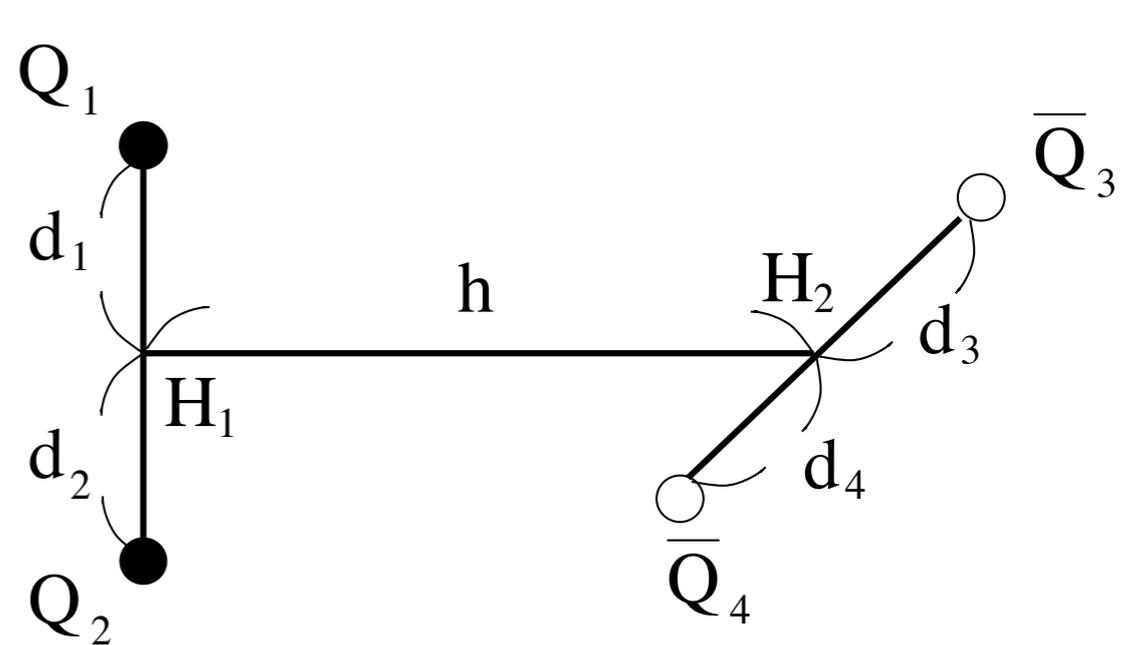
- ❖ The four-quark static potential is to find the minimum value of all three configurations

$$V^{4Q} \equiv \min (V^{\text{flip-flop}}, V^{\text{butterfly}})$$

- ❖ This potential is again supported by Lattice QCD simulation



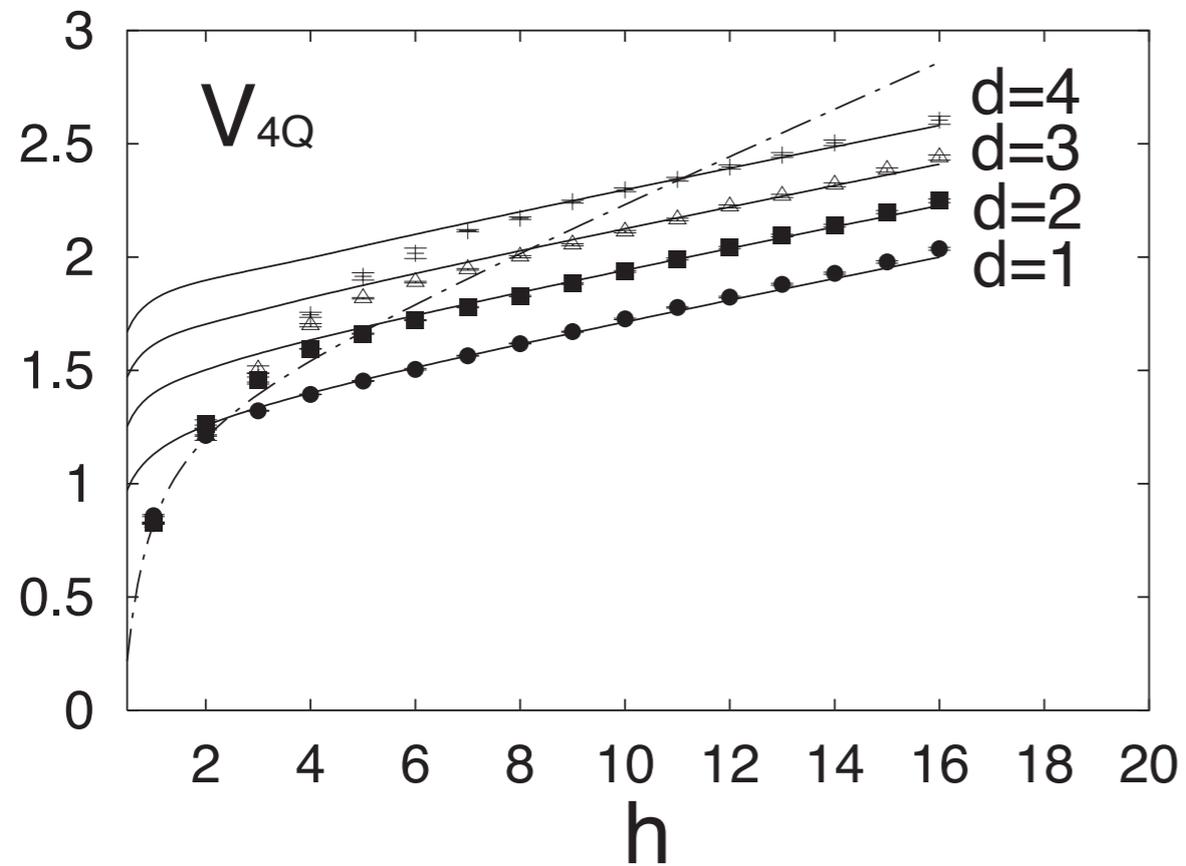
symmetric planar



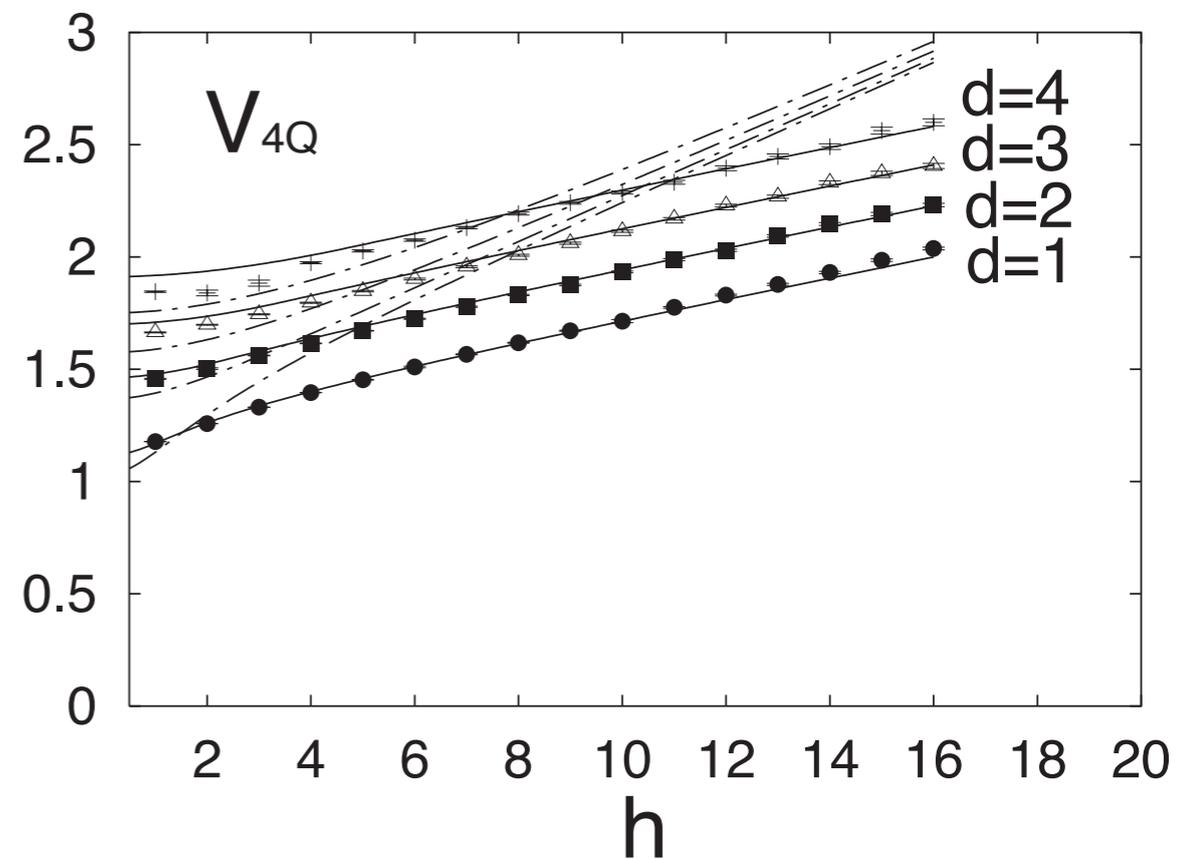
symmetric twisted

Static Potential for Four Quarks

❖ Confirmation from Lattice QCD simulations



symmetric planar



symmetric twisted

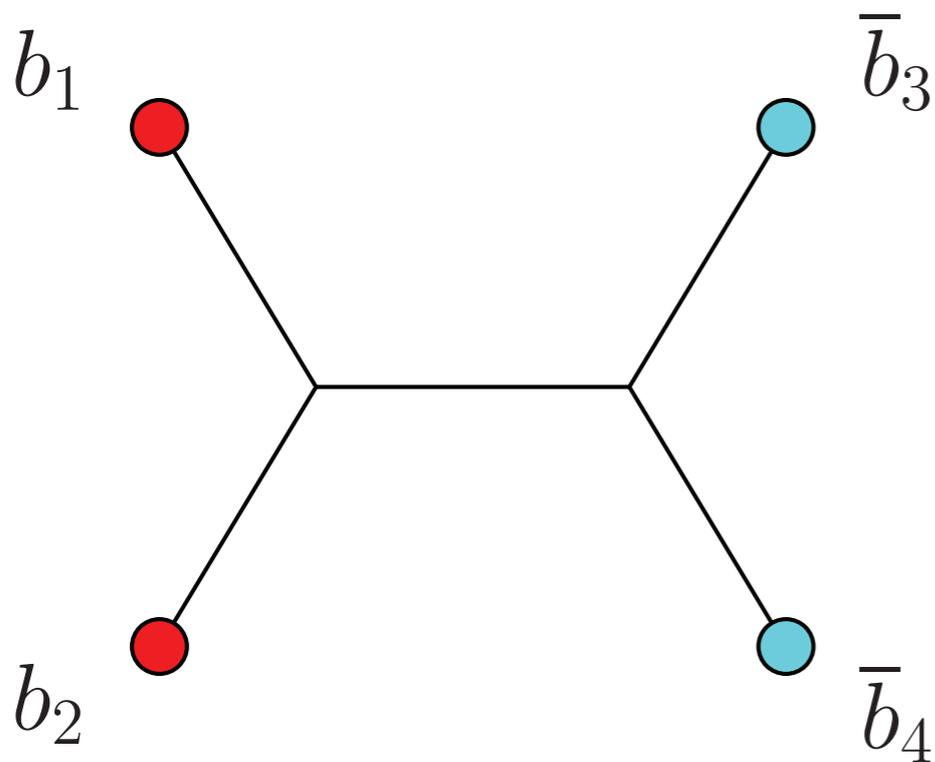
Takahashi, et. al., hep-lat/0412012

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Lmin for Butterfly

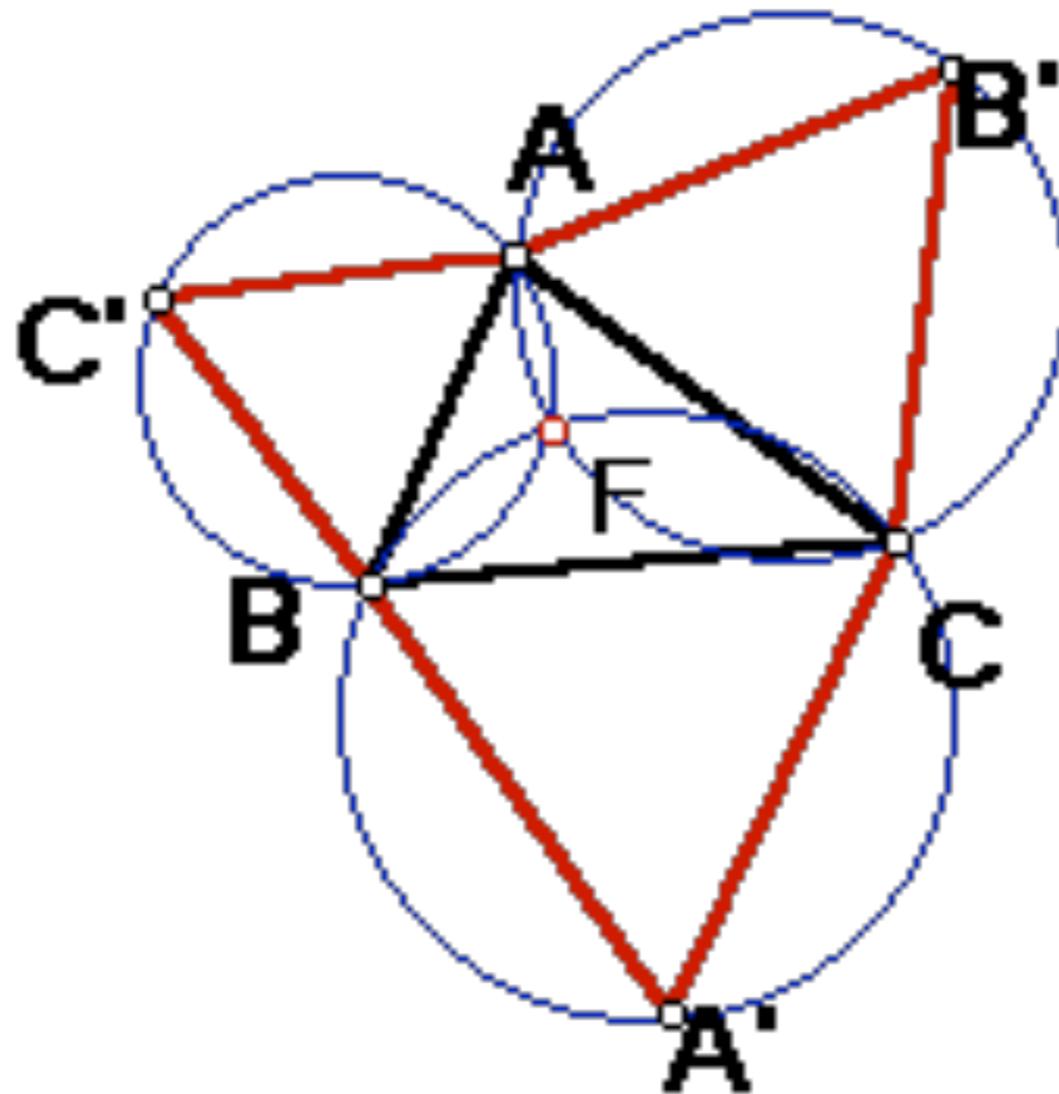
- ❖ Two numerical technical issues
- ❖ The first one is how to determine the minimum flux-tube length for the butterfly configuration



$$V^{\text{butterfly}} = -\frac{\alpha_s}{3} \left(\frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} + \frac{1}{r_{24}} \right) - \frac{2\alpha_s}{3} \left(\frac{1}{r_{12}} + \frac{1}{r_{34}} \right) + \frac{1}{a^2} L_{\min}$$

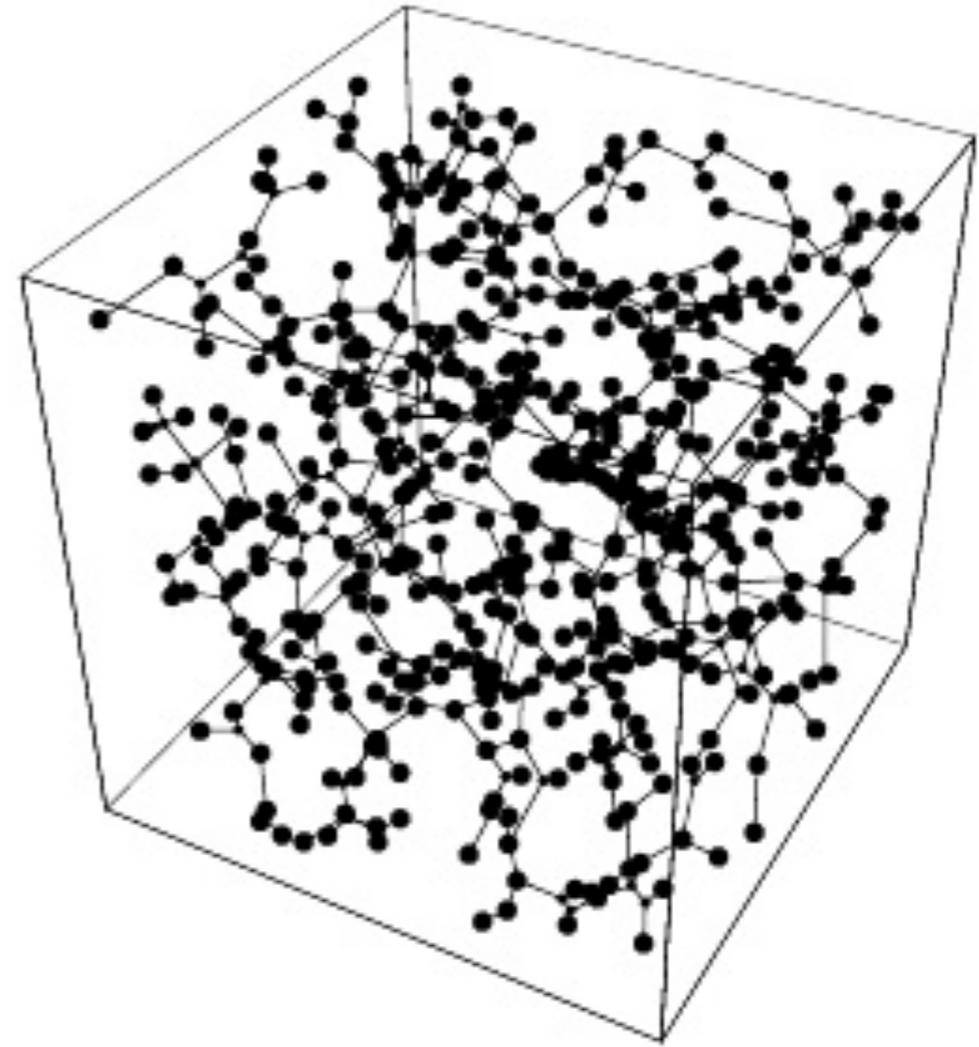
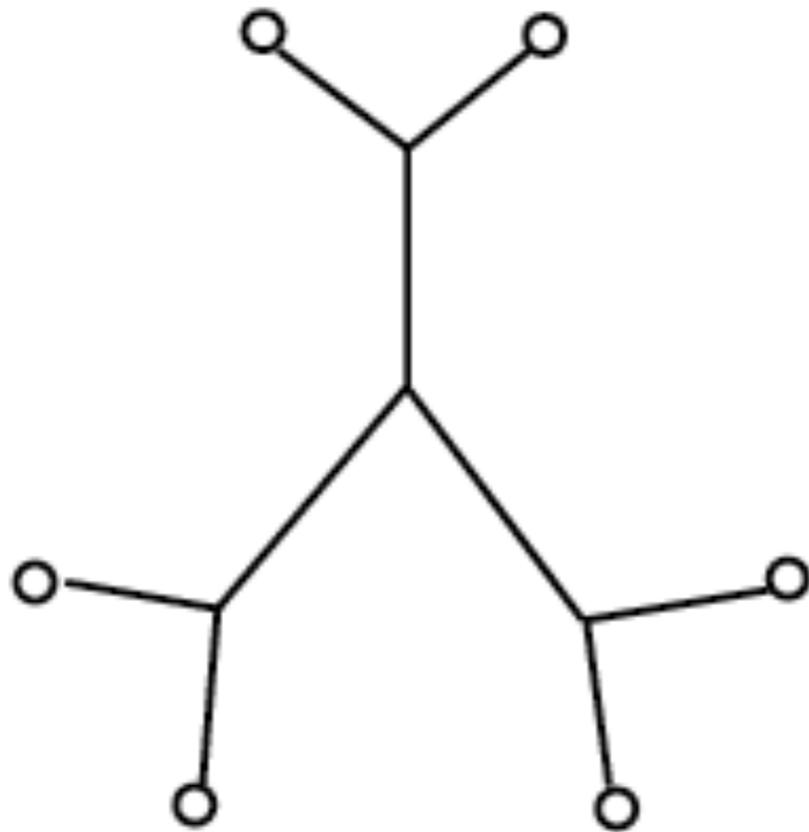
Similar to the Fermat-Torricelli point

- ❖ Which has three vertexes in two-dimensional space



Steiner tree problem

- ❖ The more general problem is called Steiner tree problem



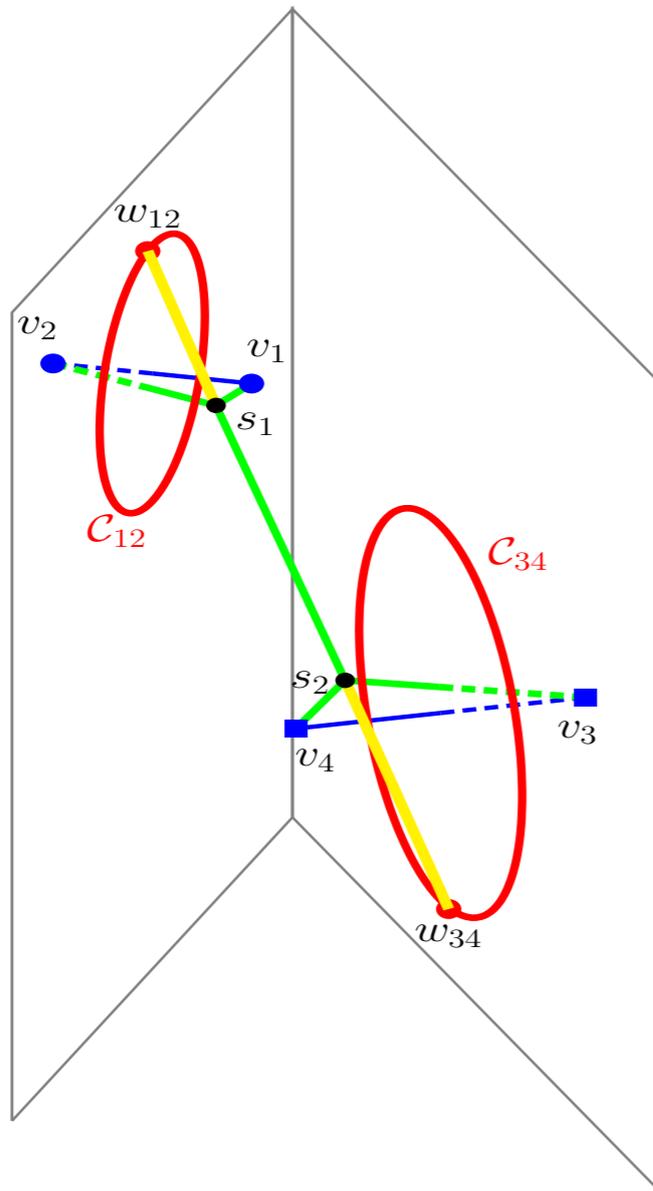
- ❖ This problem is NP-hard.
- ❖ Fortunately, we only have four b's

Steiner tree problem

- ❖ It requires iterative method to find the Steiner points

Ay, Richard, Rubinstein, 0901.3022

Smith, algorithmica, 7 (1992) 137



$$\sum_{kj \text{ links}} \frac{(\vec{x}_k^{(i+1)} - \vec{x}_j^{(i+1)})}{(|\vec{x}_k^{(i)} - \vec{x}_j^{(i)}|)} = 0$$

- ❖ Usually, after 20 steps, the results become stable

DMC for many-body Schrodinger

- ❖ One could use the standard variation methods. It turns out be very timing consuming
- ❖ Instead, we adopt the Diffusion Monte Carlo method
- ❖ The time-dependent wave-function evolves as

$$\sum_n e^{-iE_n t} \Psi_n(\vec{x})$$

- ❖ Replace the time by an imaginary time and change all energy levels by a guess groundstate energy

$$\sum_n e^{-(E_n - E_g)\tau} \Psi_n(\vec{x})$$

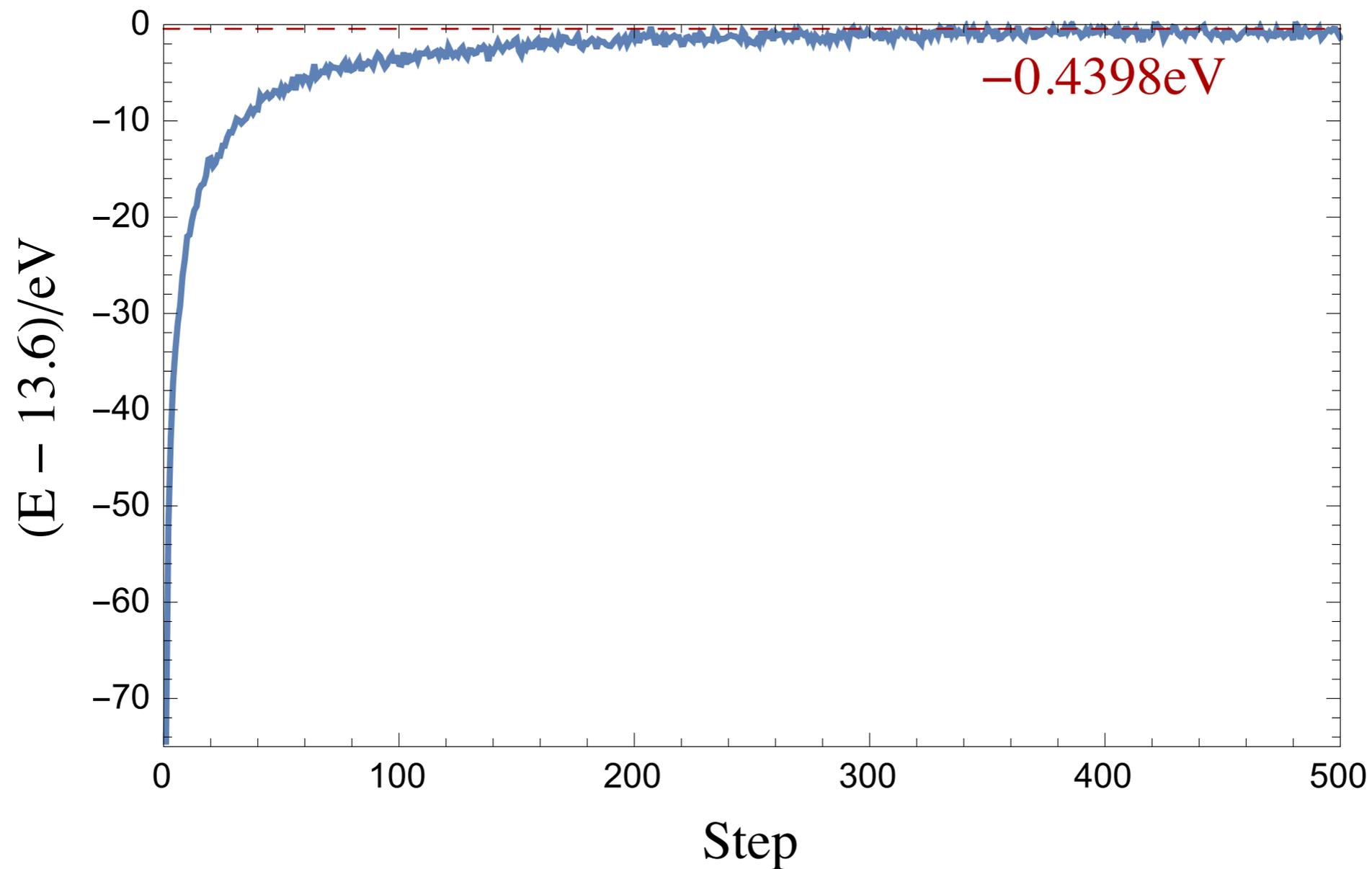
DMC for many-body Schrodinger

$$\sum_n e^{-(E_n - E_g)\tau} \Psi_n(\vec{x})$$

- ❖ After a sufficiently long time, only when $E_g = E_0$, the wave-function is stabilized
- ❖ Practically, the wave function in the DMC method is represented by random walks of many particles in the phase space
- ❖ To observe the behavior of wave function with respect to E_g , a “born-death” mechanism is designed such that when E_g is too large, the particles will replicate themselves and increase the total number of particles, and vice versa

DMC for di-positronium molecular

- ❖ We can reproduce the binding energy for the Ps_2 ; the true answer is -0.435 eV



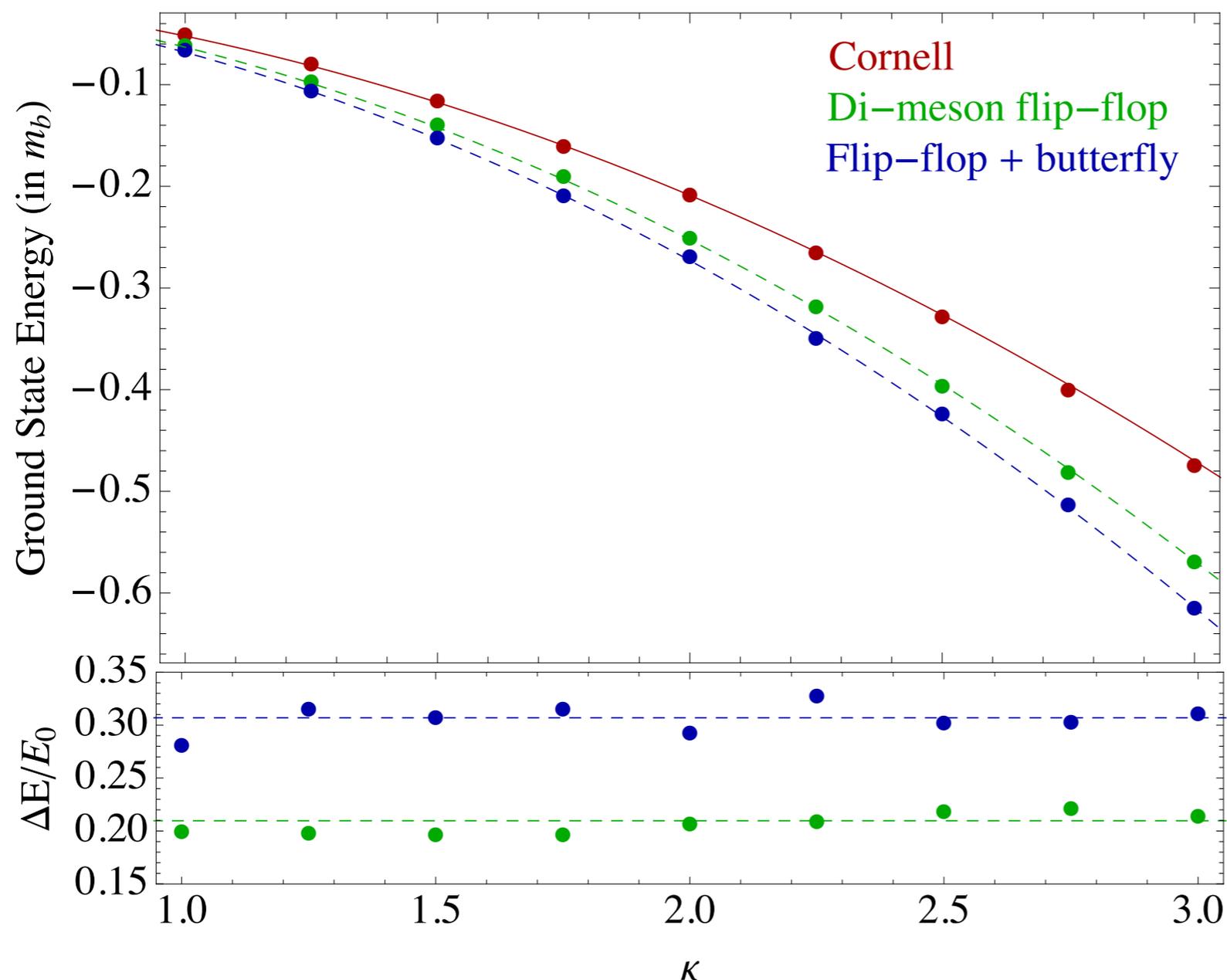
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DMC for 4-b Tetraquark

- ❖ **Two benchmark Cornell potential will be used**

BM-I : $m_b = 4.79 \text{ GeV}$, $\alpha_s = 0.38$, $a = 2.43 \text{ GeV}^{-1}$, Quigg and Rosner, '1979



the k parameter is introduced to check numerical stability

Additional Binding Energies

❖ Additional binding energies from dissociated di-meson structure

$$\text{BM-I : } m_b = 4.79 \text{ GeV}, \alpha_s = 0.38, a = 2.43 \text{ GeV}^{-1},$$

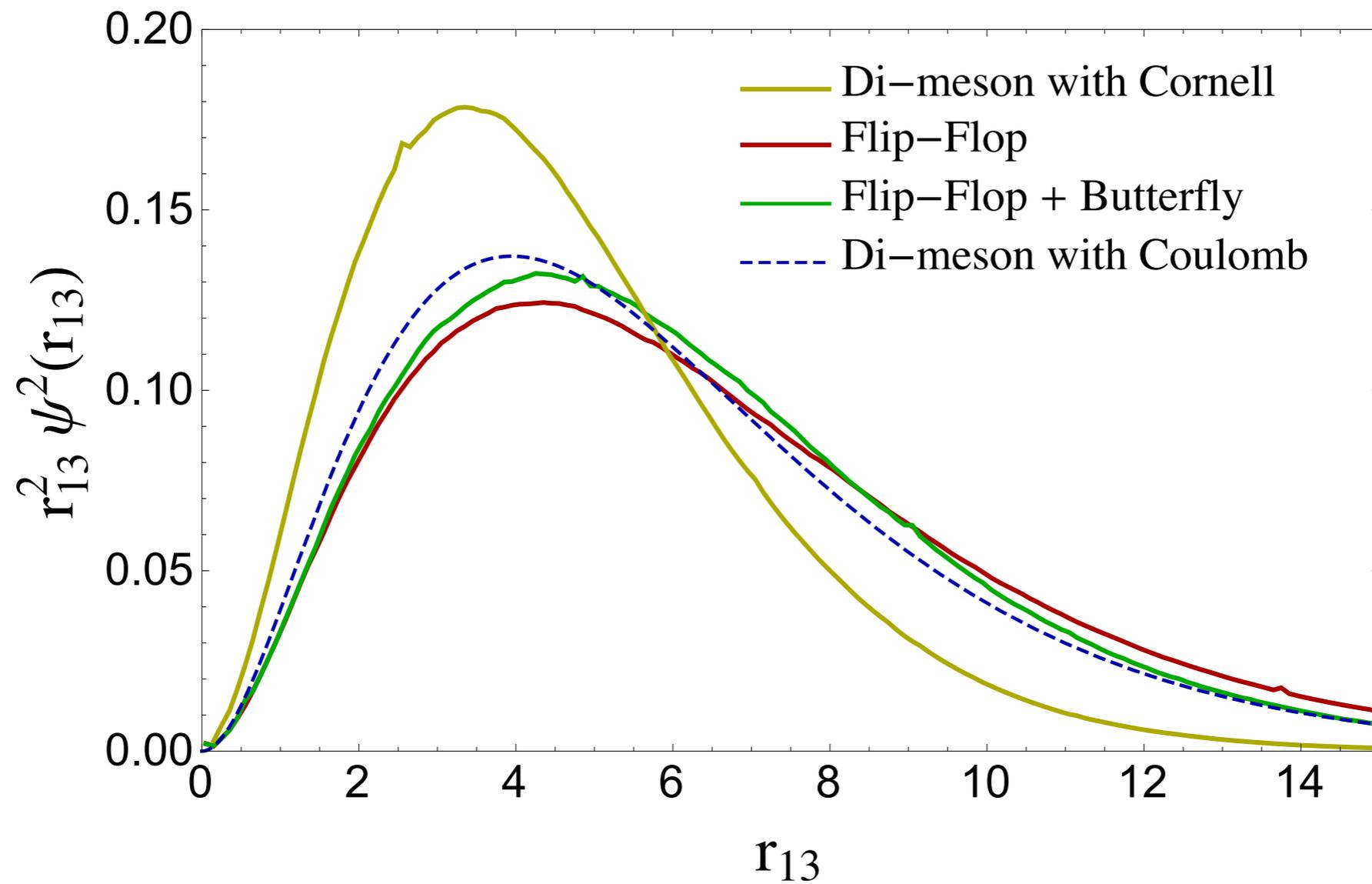
$$\text{BM-II : } m_b = 5.17 \text{ GeV}, \alpha_s = 0.36, a = 2.34 \text{ GeV}^{-1}.$$

	Benchmark-I	Benchmark-II
$E_0/2$ (di-meson)	9.455 GeV	9.460 GeV
ΔE (flip-flop)	-52 MeV	-51 MeV
ΔE (flip-flop+butterfly)	-80 MeV	-78 MeV

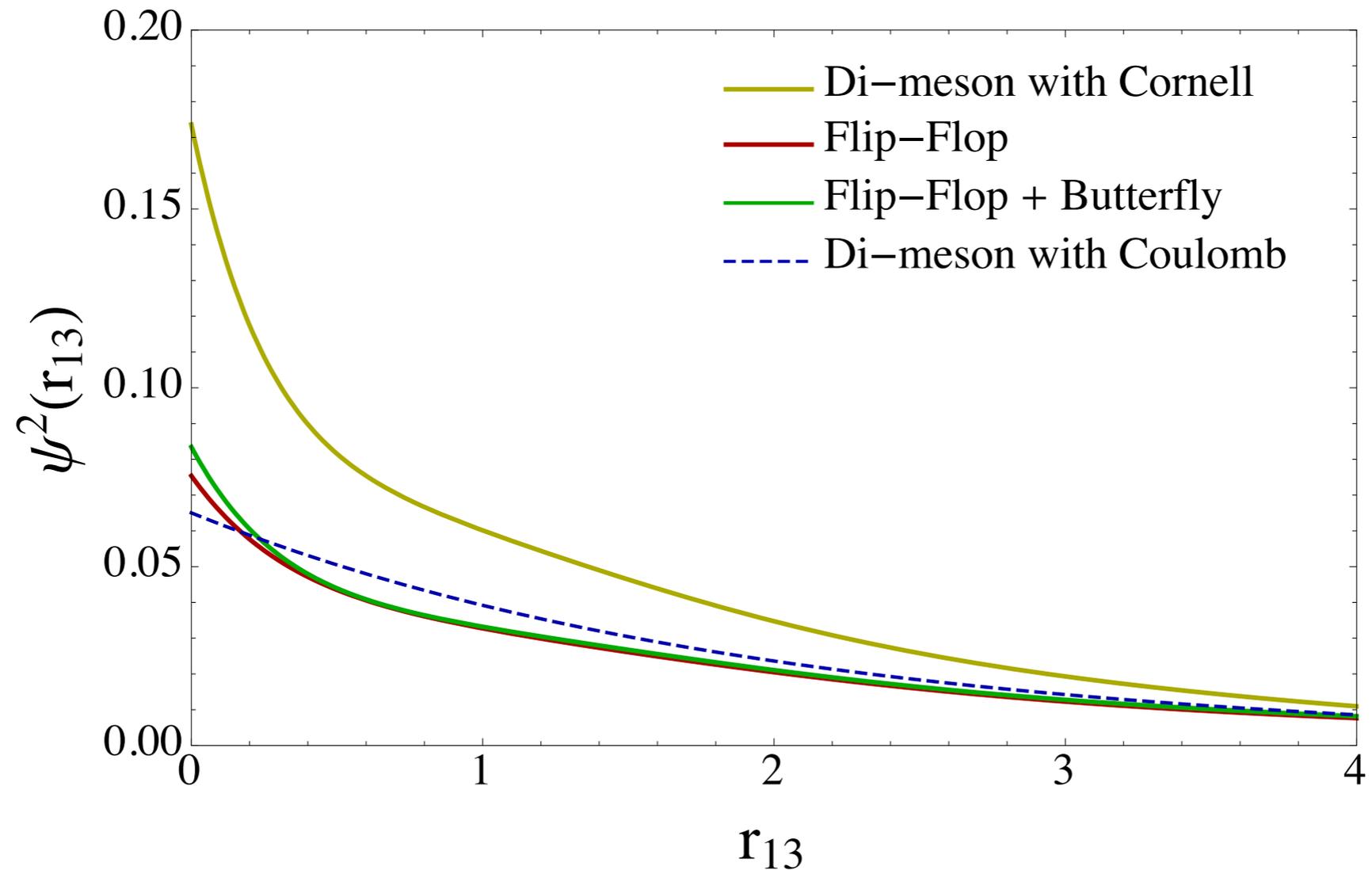
❖ Additional ~ 80 MeV binding energy for the tetra-quark compared to the dissociated di-meson state

Wave-function

- ❖ Making approximation that the wave-function is flat in r_{12}



Wave-function around Origin



- ❖ **The wave-function-squared at origin is around 1/2 of the dissociated di-meson configuration**

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Hyperfine splitting for two-body

- ❖ **Similar to QED, the one-gluon exchange has the short-range SD interactions**

Rujula, Georgi, Glashow, '1975
Eichten and Feinberg, '1981

$$H_{\text{SD}} \supset \Delta C_2 \alpha_s \frac{2}{3 m_1 m_2} \vec{s}_1 \cdot \vec{s}_2 4\pi \delta(r_{12})$$

- ❖ **The correction to spin-one is -1/3 compared to the correction to spin-zero**
- ❖ **Wave-function at origin enters into the calculation**
- ❖ **For the four-b state, we need to have spin-configuration to satisfy the Pauli principle**

Ground-state Wave Function

- ❖ **Focusing on the flip-flop configure part of the wave-function**

$$\Psi = \begin{cases} \psi(r_{13}) \psi(r_{24}) \otimes |(b_1 \bar{b}_3)_1 (b_2 \bar{b}_4)_1\rangle \otimes \chi_{13,24}^{(0,0)} & \text{for } R_1 , \\ \psi(r_{14}) \psi(r_{23}) \otimes |(b_1 \bar{b}_4)_1 (b_2 \bar{b}_3)_1\rangle \otimes -\chi_{14,23}^{(0,0)} & \text{for } R_2 . \end{cases}$$

- ❖ **The total spin-zero wave function is**

$$\chi_{13,24}^{(0,0)} = \frac{1}{2} (b_1^\uparrow b_3^\downarrow - b_1^\downarrow b_3^\uparrow) (b_2^\uparrow b_4^\downarrow - b_2^\downarrow b_4^\uparrow)$$

$$\chi_{14,23}^{(0,0)} = \frac{1}{2} (b_1^\uparrow b_4^\downarrow - b_1^\downarrow b_4^\uparrow) (b_2^\uparrow b_3^\downarrow - b_2^\downarrow b_3^\uparrow)$$

- ❖ **The spin-dependent correction is**

$$\Delta E_{\text{SD}} = -\frac{4\alpha_s(\mu)}{3} \frac{1}{m_b^2} [\psi^2(r_{13} = 0) + \psi^2(r_{24} = 0)] \approx -145 \pm 30 \text{ MeV}$$

Ground-state Tetraquark Energy

- ❖ **Altogether, we have ground-state energy for the 4-b tetra quark as**

$$M(0^{++}) = 18.69 \pm 0.03 \text{ GeV}$$

- ❖ **Which is below the possible decay thresholds of**

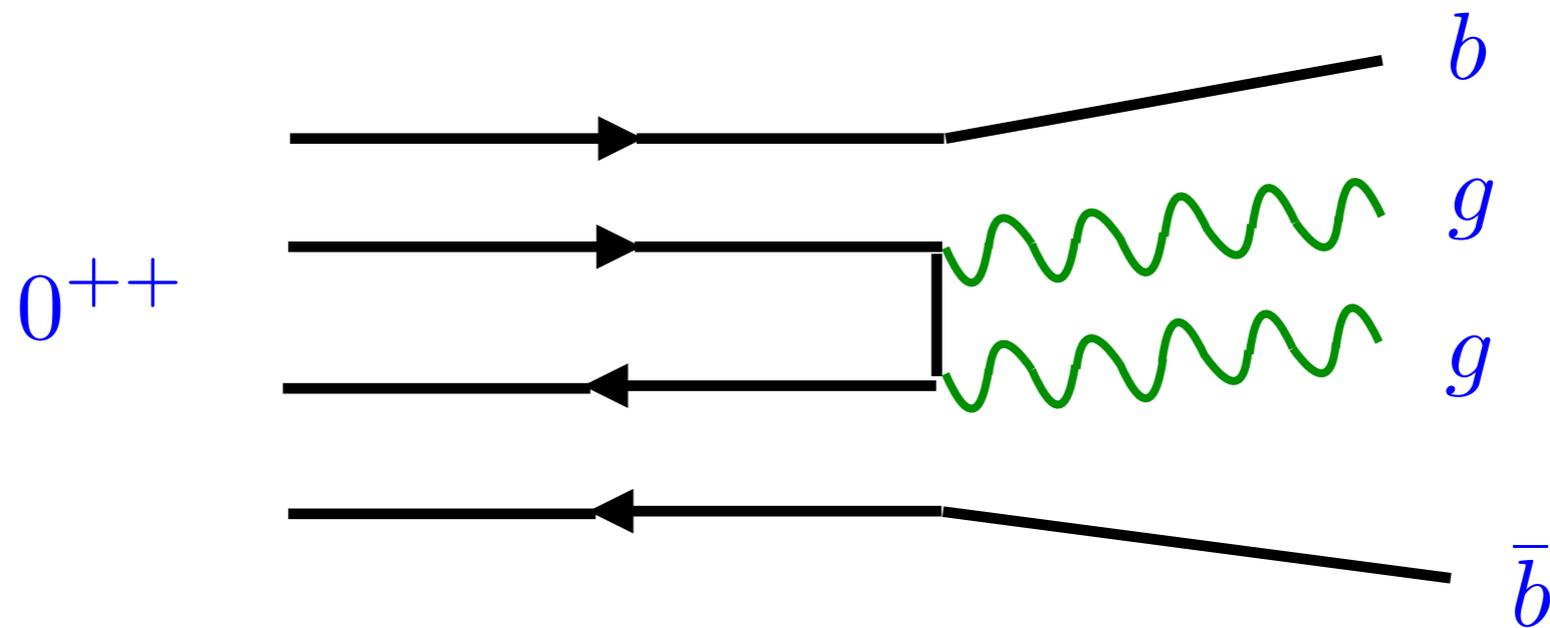
$$2M(\eta_b) = 18.798 \text{ GeV}$$

$$2M[Y(1S)] = 18.920 \text{ GeV}$$

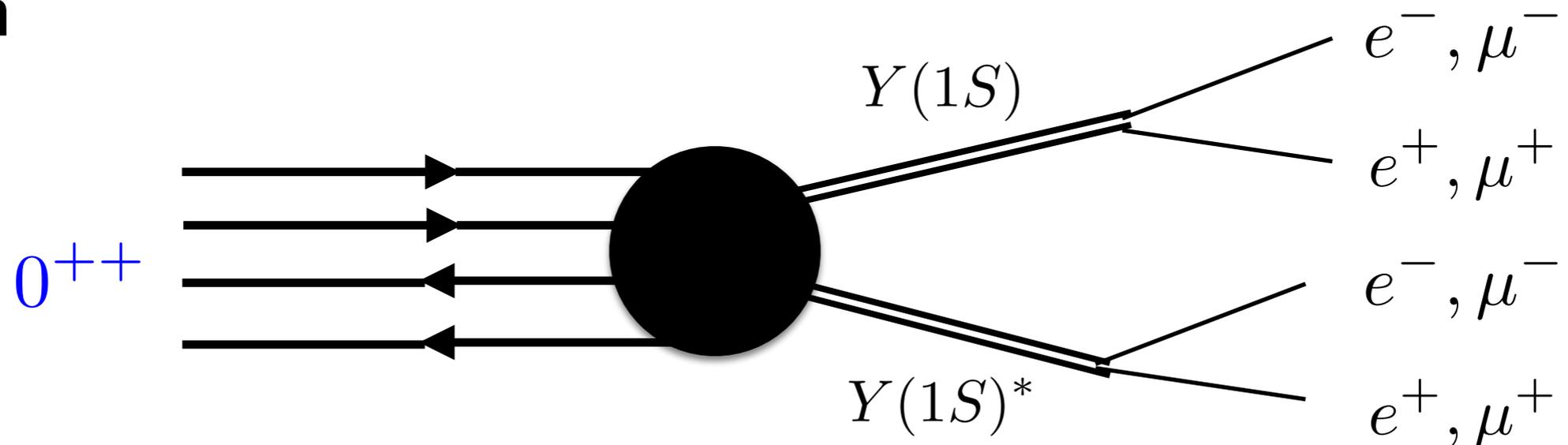
- ❖ **It is possible for this 18.69 GeV state decaying into $Y(1S)$ and $Y(1S)^*$, which decay to two leptons and observed at colliders**

Decay of this 18.59 GeV State

- ❖ Schematically, the leading decay channel is like

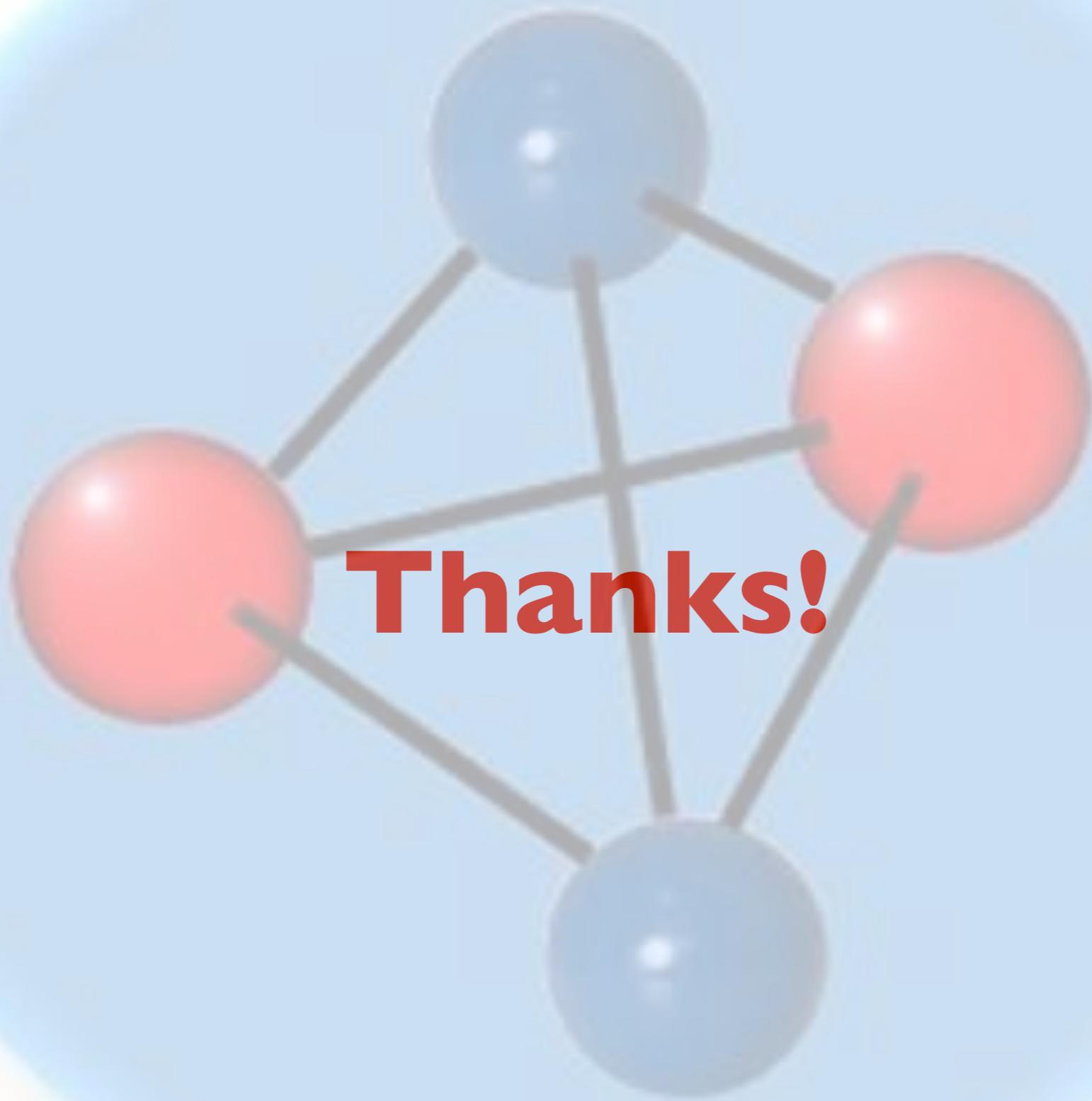


- ❖ It can be searched in the four-lepton channel like Higgs boson



Conclusions

- ❖ **A four-b tetraquark state with a mass around 18.7 GeV is predicted to exist**
- ❖ **The calculation is based on the flux-tube model for the static potential of four quarks, which is supported by Lattice QCD**
- ❖ **We used the diffusion Monte Carlo method to numerically solve this many-body system**
- ❖ **This state, if with sufficient production cross sections, could be observed as a $Y(1S)Y(1S)^*$ resonance via the four-lepton final state at the LHC**



Other Approaches

- ❖ **Operator-product expansion plus sum rules**

$$\Pi^j(q^2) = \frac{(q^2)^n}{\pi} \int \frac{\text{Im } \Pi^j(s)}{s^n (s - q^2)} ds + \sum_{k=0}^{n-1} a_k (q^2)^k$$

- ❖ **It works reasonable good for charmonium, but not good for the bottomonium**

Reinders, Rubinstein, Yazaki,
Physics Report, 1985

- ❖ **One could also perform a more systematically EFT to integrate out scales step by step via the NRQCD**

Bodwin, Braaten, Lepage
hep-ph/9407339

$$\begin{aligned} \delta \mathcal{L}_{\text{bilinear}} = & \frac{c_1}{8M^3} (\psi^\dagger (\mathbf{D}^2)^2 \psi - \chi^\dagger (\mathbf{D}^2)^2 \chi) \\ & + \frac{c_2}{8M^2} (\psi^\dagger (\mathbf{D} \cdot g\mathbf{E} - g\mathbf{E} \cdot \mathbf{D}) \psi + \chi^\dagger (\mathbf{D} \cdot g\mathbf{E} - g\mathbf{E} \cdot \mathbf{D}) \chi) \\ & + \frac{c_3}{8M^2} (\psi^\dagger (i\mathbf{D} \times g\mathbf{E} - g\mathbf{E} \times i\mathbf{D}) \cdot \boldsymbol{\sigma} \psi + \chi^\dagger (i\mathbf{D} \times g\mathbf{E} - g\mathbf{E} \times i\mathbf{D}) \cdot \boldsymbol{\sigma} \chi) \\ & + \frac{c_4}{2M} (\psi^\dagger (g\mathbf{B} \cdot \boldsymbol{\sigma}) \psi - \chi^\dagger (g\mathbf{B} \cdot \boldsymbol{\sigma}) \chi), \end{aligned}$$

Ground-state 4-c Tetraquark

- ❖ Using the benchmark-I with

$$m_c = 1.37 \text{ GeV} \quad \alpha_s(2m_c) \approx 0.25$$

- ❖ The ground-state energy for the 4-c tetraquark has

$$M(0^{++}) = 5.97 \pm 0.04 \text{ GeV}$$

- ❖ Which is also below the decay thresholds of two J/Psi

$$2M(\eta_c) = 5.961 \text{ GeV}$$

$$2M(J/\psi) = 6.194 \text{ GeV}$$

- ❖ Within the calculation uncertainty, it may have a two-body decay into two η_c and a smaller branching ratio into

$$\Psi(1S)\Psi(1S)^*$$