Beauty-full Tetraquarks

Yang Bai

with Sida Lu and James Osborne, arxiv:1612.00012

University of Wisconsin-Madison

@DPF 2017, Fermilab, August 1, 2017
Outline

- Motivations for understanding QCD
- The static potential for four heavy quarks
- Numerical methods for many-body system
- Energy level for SI potential
- Spin-dependent correction
- Conclusions
### QED

- **Hydrogen system, solving the non-relativistic Schrodinger equation**

\[
\left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V(\vec{r}, t) \right] \Psi(\vec{r}, t) = E \Psi(\vec{r}, t)
\]

- The electric potential between two charge particles is just the Coulomb form

\[
V(\vec{r}) = -\frac{\alpha}{r}
\]

- **Quantized spectra**

![Image of quantized spectra](image-url)
Many-body QED

- To understand materials in ordinary life, it is extremely important to understand the many-electronic system.

- As a warm up, let’s consider a simple system with two electrons and two positrons:

  \[ V(\vec{r}_i) = -\alpha \left( \frac{1}{r_{13}} + \frac{1}{r_{24}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} \right) + \alpha \left( \frac{1}{r_{12}} + \frac{1}{r_{34}} \right) \]

- In 1946, Wheeler suggested a possible di-positronium molecular state, based on a simple variation calculation to estimate the additional binding energy.
Calculation for \( \text{Ps}_2 \)

- Many methods have been adopted to calculate the ground state energy: including the variation method and Quantum Monte Carlo method

<table>
<thead>
<tr>
<th>Author</th>
<th>Binding energy (( \omega_{\text{Ps}_2} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hylleraas and Ore (1947)</td>
<td>-0.0085 Ryd, -0.116 eV</td>
</tr>
<tr>
<td>Ore (1947)</td>
<td>-0.009 Ryd, -0.122 eV</td>
</tr>
<tr>
<td>Akimoto and Hamamura (1972b)</td>
<td>-0.0135 Ryd, -0.184 eV</td>
</tr>
<tr>
<td>Brinkman et al (1973)</td>
<td>-0.0145 Ryd, -0.197 eV</td>
</tr>
<tr>
<td>Lee et al (1983)</td>
<td>-0.03±0.002 Ryd, -0.408±0.027 eV</td>
</tr>
<tr>
<td>Ho (1986)</td>
<td>-0.0302 Ryd, -0.411 eV</td>
</tr>
<tr>
<td>Kinghorn and Poshusta (1993)</td>
<td>-0.03198 Ryd, -0.435 eV</td>
</tr>
<tr>
<td>Kozlowski and Adamowicz (1993)</td>
<td>-0.03198 Ryd, -0.435 eV</td>
</tr>
<tr>
<td>Present calculation at ( n_6 = 1 )</td>
<td>-0.0421 Ryd, -0.573 eV</td>
</tr>
</tbody>
</table>


- The additional binding energy is -0.435 eV
- The ratio is \( 0.435/13.6 = 3.2\% \); a small binding
Calculation for Ps$_2$

- The ratio is $0.435/13.6 = 3.2\%$; a small binding

- This small binding energy can be understood as the London-van der Waals force $\sim -\alpha/R^6$

- The wave-functions are modified compared to the hydrogen one

Suzuki, et. al, PRA 58 (1998) 1918
Experimental Confirmation for Ps$_2$

- The existence of di-positronium molecular is only demonstrated in 2007

$\text{f}_d$ is a measure of the amount of o-Ps created

\[
\frac{dn}{dt} = -\gamma n (1 + \beta n)
\]

parameter $\beta$ describes strength of SEQ and/or Ps$_2$ formation

QCD

- QCD behaves different from QED, especially at a long distance

\[ \alpha_s(M_Z) = 0.1181 \pm 0.0013 \]

- It is asymptotically free at a short distance and has confinement at a long distance
QCD Static Potential

- At short-distance, one could use the approximate one-gluon exchange Coulomb force.

- **Appelquist and Politzer** used this fact to predict the spin-one charmonium state based on Quantum Mechanics calculation, which is soon measured as J/Psi.

- To have a more precise QM calculation for the quarkonium spectroscopy, phenomenological potential is widely used.

\[ V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + \frac{r}{a^2} \]

Brambilla, et. al., arXiv:1010.5827
QCD Spin-dependent Potential

- The situation is similar to the QED for the short-distance SD interactions
- The delta function part of the potential is given by

\[ V(\vec{r}) = -\frac{4}{3} \alpha_s \frac{2}{3m_1 m_2} \vec{S}_1 \cdot \vec{S}_2 4\pi \delta(\vec{r}) \]

- This can generate the spin splitting energy from perturbation calculations

Eichten and Feinberg, ‘1981
Outline

- Motivations for understanding QCD
- The static potential for four heavy quarks
- Numerical methods for many-body system
- Energy level for SI potential
- Spin-dependent correction
- Conclusions
Static Potential for Three Quarks

- One can use the static potential to study baryon spectrum in the quark model.
- Based on the flux-tube model from the strong-coupling Hamiltonian lattice formation of QCD, one has two configurations to consider

\[ V^{3Q} = -\frac{1}{2} \frac{4 \alpha_s}{3} \left( \frac{1}{r_{12}} + \frac{1}{r_{13}} + \frac{1}{r_{23}} \right) + \frac{1}{a^2} L_{\text{min}} \]

Isgur and Paton, ‘1985

**FIG. 2.** The flux-tube configuration in a three-quark state at strong coupling, when none of the interior angles of the triangle \( \vec{r}_1 \vec{r}_2 \vec{r}_3 \) are greater than 120°.

**FIG. 3.** The flux-tube configuration in a three-quark state when the angle at \( \vec{r}_2 \) is greater than 120°.
Static Potential for Three Quarks

- The lattice simulation has supported the flux-tube model for static potential

\[ V_{3Q} = - \lim_{T \to \infty} \frac{1}{T} \ln \langle W_{3Q} \rangle \]

Takahashi, et. al., hep-lat/0204011
Static Potential for Four Quarks

- For the four-quark state, more configurations exist

\[ V_{\text{di-meson}}^{(13,24)} = -\frac{4\alpha_s}{3} \left( \frac{1}{r_{13}} + \frac{1}{r_{24}} \right) + \frac{1}{a^2} (r_{13} + r_{24}) \]

\[ V^{\text{flip-flop}} \equiv \min \left[ V_{\text{di-meson}}^{(13,24)}, V_{\text{di-meson}}^{(14,23)} \right] \]
Static Potential for Four Quarks

- diquark-diquark configuration

- the “good” diquark is used here; triplet color contraction

- for the “bad” sextet diquark contraction, replace the coefficients \((-1/3, -2/3)\) by \((-5/6, 1/3)\), which provides a larger value

\[
V_{\text{butterfly}} = -\frac{\alpha_s}{3} \left( \frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} + \frac{1}{r_{24}} \right) \\
- \frac{2\alpha_s}{3} \left( \frac{1}{r_{12}} + \frac{1}{r_{34}} \right) + \frac{1}{a^2} L_{\text{min}}
\]
Static Potential for Four Quarks

- The four-quark static potential is to find the minimum value of all three configurations

\[ V^{4Q} \equiv \min (V^{\text{flip-flop}}, V^{\text{butterfly}}) \]

- This potential is again supported by Lattice QCD simulation

**Figure 6:** A planar configuration of the tetraquark system.

**Figure 7:** A twisted configuration of the tetraquark system.

**Diagram:**
- **Symmetric planar**
- **Symmetric twisted**
Static Potential for Four Quarks

- Confirmation from Lattice QCD simulations

\[ V_{4Q} \]

\[
\begin{align*}
V_{4Q} & \quad \text{symmetric planar} \\
V_{4Q} & \quad \text{symmetric twisted}
\end{align*}
\]

Takahashi, et al., hep-lat/0412012
Outline

- Motivations for understanding QCD
- The static potential for four heavy quarks
- Numerical methods for many-body system
- Energy level for SI potential
- Spin-dependent correction
- Conclusions
Lmin for Butterfly

- Two numerical technical issues

- The first one is how to determine the minimum flux-tube length for the butterfly configuration

\[ V^{\text{butterfly}} = -\frac{\alpha_s}{3} \left( \frac{1}{r_{13}} + \frac{1}{r_{14}} + \frac{1}{r_{23}} + \frac{1}{r_{24}} \right) - \frac{2}{3} \frac{\alpha_s}{\alpha^2} \left( \frac{1}{r_{12}} + \frac{1}{r_{34}} \right) + \frac{1}{\alpha^2} L_{\text{min}} \]
Similar to the Fermat-Torricelli point

- Which has three vertexes in two-dimensional space
Steiner tree problem

- The more general problem is called Steiner tree problem

- This problem is NP-hard.

- Fortunately, we only have four b’s
Steiner tree problem

- It requires iterative method to find the Steiner points

\[ V_4 \leq \left( k_{x_1} + k_{y_1} \right)^{3/2} + k_{z_1} \]

Ay, Richard, Rubinstein, 0901.3022

Smith, algorithmica, 7 (1992) 137

- Usually, after 20 steps, the results become stable
DMC for many-body Schrödinger

- One could use the standard variation methods. It turns out to be very timing consuming.
- Instead, we adopt the Diffusion Monte Carlo method.
- The time-dependent wave-function evolves as
  \[
  \sum_{n} e^{-iE_{n}t} \Psi_{n}(\vec{x})
  \]
- Replace the time by an imaginary time and change all energy levels by a guess groundstate energy
  \[
  \sum_{n} e^{-(E_{n} - E_{g})\tau} \Psi_{n}(\vec{x})
  \]
DMC for many-body Schrödinger

\[ \sum_n e^{-(E_n - E_g)\tau} \Psi_n(x) \]

- After a sufficiently long time, only when \( E_g = E_0 \), the wave-function is stabilized.

- Practically, the wave function in the DMC method is represented by random walks of many particles in the phase space.

- To observe the behavior of wave function with respect to \( E_g \), a “born-death” mechanism is designed such that when \( E_g \) is too large, the particles will replicate themselves and increase the total number of particles, and vice versa.
DMC for di-positronium molecular

- We can reproduce the binding energy for the $\text{Ps}_2$; the true answer is $-0.435 \text{ eV}$
Outline

- Motivations for understanding QCD
- The static potential for four heavy quarks
- Numerical methods for many-body system
  - Energy level for SI potential
- Spin-dependent correction
- Conclusions
DMC for 4-b Tetraquark

- Two benchmark Cornell potential will be used

BM-I: $m_b = 4.79$ GeV, $\alpha_s = 0.38$, $a = 2.43$ GeV$^{-1}$, Quigg and Rosner, ‘1979

The $k$ parameter is introduced to check numerical stability.
Additional Binding Energies

- Additional binding energies from dissociated di-meson structure
  
  **BM-I**:  
  \( m_b = 4.79 \text{ GeV}, \alpha_s = 0.38, \ a \ = \ 2.43 \text{ GeV}^{-1} \),  
  
  **BM-II**:  
  \( m_b = 5.17 \text{ GeV}, \alpha_s = 0.36, \ a \ = \ 2.34 \text{ GeV}^{-1} \).

<table>
<thead>
<tr>
<th></th>
<th>Benchmark-I</th>
<th>Benchmark-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_0/2 ) (di-meson)</td>
<td>9.455 GeV</td>
<td>9.460 GeV</td>
</tr>
<tr>
<td>( \Delta E ) (flip-flop)</td>
<td>−52 MeV</td>
<td>−51 MeV</td>
</tr>
<tr>
<td>( \Delta E ) (flip-flop+butterfly)</td>
<td>−80 MeV</td>
<td>−78 MeV</td>
</tr>
</tbody>
</table>

- Additional \( \sim 80 \text{ MeV} \) binding energy for the tetra-quark compared to the dissociated di-meson state
Wave-function

Making approximation that the wave-function is flat in $r_{12}$
Wave-function around Origin

- The wave-function-squared at origin is around 1/2 of the dissociated di-meson configuration.
Outline

- Motivations for understanding QCD
- The static potential for four heavy quarks
- Numerical methods for many-body system
- Energy level for SI potential
- Spin-dependent correction
- Conclusions
Hyperfine splitting for two-body

- Similar to QED, the one-gluon exchange has the short-range SD interactions
  \[ H_{SD} \supset \Delta C_2 \alpha_s \frac{2}{3 m_1 m_2} \mathbf{s}_1 \cdot \mathbf{s}_2 4\pi \delta(r_{12}) \]
  - Rujula, Georgi, Glashow, ‘1975
  - Eichten and Feinberg, ‘1981

- The correction to spin-one is \(-1/3\) compared to the correction to spin-zero

- Wave-function at origin enters into the calculation

- For the four-b state, we need to have spin-configuration to satisfy the Pauli principle
Ground-state Wave Function

- Focusing on the flip-flop configure part of the wave-function

\[
\Psi = \begin{cases} 
\psi(r_{13}) \psi(r_{24}) \otimes |(b_1 \bar{b}_3)_1 (b_2 \bar{b}_4)_1 \rangle \otimes \chi^{(0,0)}_{13,24} & \text{for } R_1 , \\
\psi(r_{14}) \psi(r_{23}) \otimes |(b_1 \bar{b}_4)_1 (b_2 \bar{b}_3)_1 \rangle \otimes -\chi^{(0,0)}_{14,23} & \text{for } R_2 .
\end{cases}
\]

- The total spin-zero wave function is

\[
\chi^{(0,0)}_{13,24} = \frac{1}{2} (b_1^{\uparrow} b_3^{\downarrow} - b_1^{\downarrow} b_3^{\uparrow}) (b_2^{\uparrow} b_4^{\downarrow} - b_2^{\downarrow} b_4^{\uparrow})
\]

\[
\chi^{(0,0)}_{14,23} = \frac{1}{2} (b_1^{\uparrow} b_4^{\downarrow} - b_1^{\downarrow} b_4^{\uparrow}) (b_2^{\uparrow} b_3^{\downarrow} - b_2^{\downarrow} b_3^{\uparrow})
\]

- The spin-dependent correction is

\[
\Delta E_{SD} = - \frac{4}{3} \frac{\alpha_s(\mu)}{m_b^2} \left[ \psi^2(r_{13} = 0) + \psi^2(r_{24} = 0) \right] \approx -145 \pm 30 \text{ MeV}
\]
Ground-state Tetraquark Energy

- Altogether, we have ground-state energy for the 4-b tetraquark as

\[ M(0^{++}) = 18.69 \pm 0.03 \text{ GeV} \]

- Which is below the possible decay thresholds of

\[ 2M(\eta_b) = 18.798 \text{ GeV} \]
\[ 2M[Y(1S)] = 18.920 \text{ GeV} \]

- It is possible for this 18.69 GeV state decaying into Y(1S) and Y(1S)*, which decay to two leptons and observed at colliders
Decay of this 18.59 GeV State

- Schematically, the leading decay channel is like

- It can be searched in the four-lepton channel like Higgs boson
Conclusions

- A four-b tetraquark state with a mass around 18.7 GeV is predicted to exist.

- The calculation is based on the flux-tube model for the static potential of four quarks, which is supported by Lattice QCD.

- We used the diffusion Monte Carlo method to numerically solve this many-body system.

- This state, if with sufficient production cross sections, could be observed as a Y(1S)Y(1S)* resonance via the four-lepton final state at the LHC.
Thanks!
Other Approaches

- **Operator-product expansion plus sum rules**

  \[ \Pi^j(q^2) = \left(\frac{q^2}{\pi}\right)^n \int \frac{\text{Im} \Pi^j(s)}{s^n(s - q^2)} \, ds + \sum_{k=0}^{n-1} a_k(q^2)^k \]

- **It works reasonable good for charmonium, but not good for the bottomonium**

  Reinders, Rubinstein, Yazaki, Physics Report, 1985

- **One could also perform a more systematically EFT to integrate out scales step by step via the NRQCD**

  Bodwin, Braaten, Lepage
  hep-ph/9407339
Ground-state 4-c Tetraquark

- Using the benchmark-I with

\[ m_c = 1.37 \text{ GeV} \quad \alpha_s(2m_c) \approx 0.25 \]

- The ground-state energy for the 4-c tetraquark has

\[ M(0^{++}) = 5.97 \pm 0.04 \text{ GeV} \]

- Which is also below the decay thresholds of two J/Psi

\[ 2M(\eta_c) = 5.961 \text{ GeV} \]
\[ 2M(J/\psi) = 6.194 \text{ GeV} \]

- Within the calculation uncertainty, it may have a two-body decay into two \( \eta_c \) and a smaller branching ratio into \( \Psi(1S)\Psi(1S)^* \)