Alternative Parameterization of the form factors for semileptonic B decays

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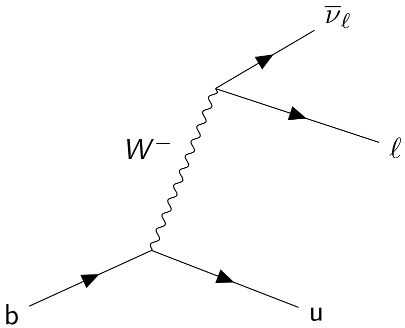
Background

- The Decay Process:
  $B \to \pi \ell \nu_\ell$



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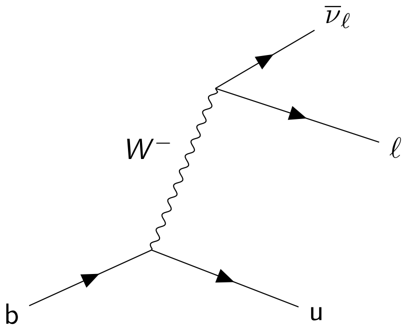
- Decay Rate Expression

Decay Rate

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda(q^2)^{3/2} |f_+(q^2)|^2$$



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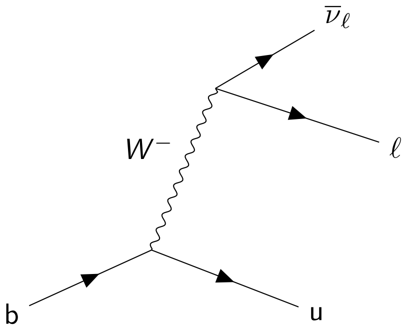

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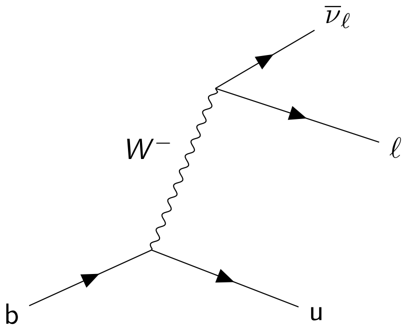
- Exclusive and inclusive decays have measurements of V_{ub} which differ by 2.4σ



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$$z(q^2;t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \tag{1}$$

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- Constraints:
  - Kinematics requires: $f_+(q^2 = 0) = f_0(q^2 = 0)$
  - QCD scaling: $f_+(q^2) \sim \frac{1}{q^2}$ as $q^2 \rightarrow \infty$
  - Conservation of angular momentum: $\frac{d}{dz} f_+(z)|_{z=-1} = 0$

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$$f_0(q^2) = \frac{\sqrt{2m_B}}{m_B^2 - m_\pi^2}\left[(m_B - E_\pi(q^2))f_{\|}(q^2) + (E_\pi(q^2)^2 - m_\pi^2)f_{\perp}(q^2)\right] \quad (4)$$

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- $f_{||}$ and f_+ are useful because they appear in χPT and are used to calculate the value of the form factors from the lattice

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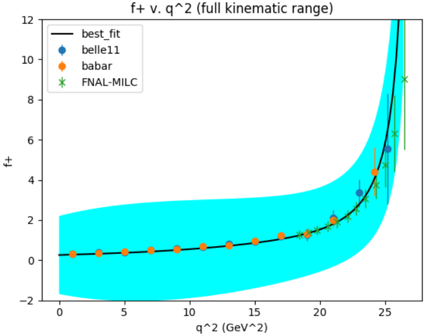

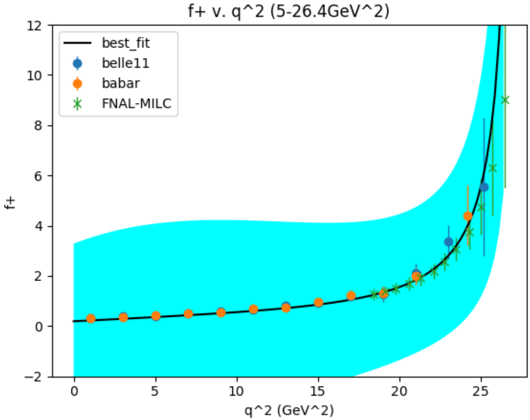
f+ v. q^2 (5-26.4GeV^2)
best_fit
belle11
babar
FNAL-MILC
f+
q^2 (GeV^2)
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10
8
6
4
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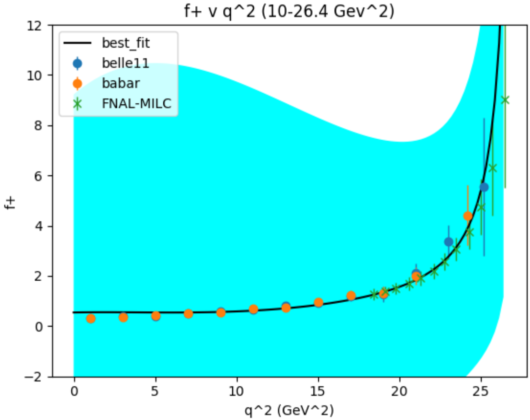
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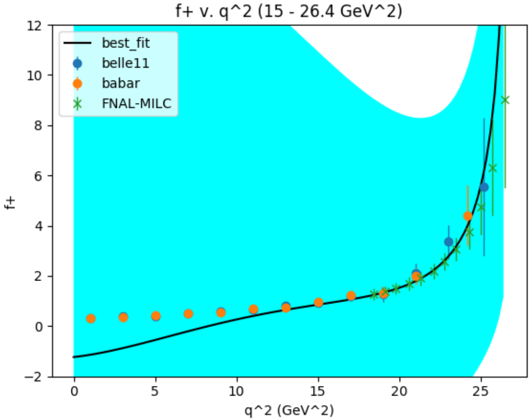
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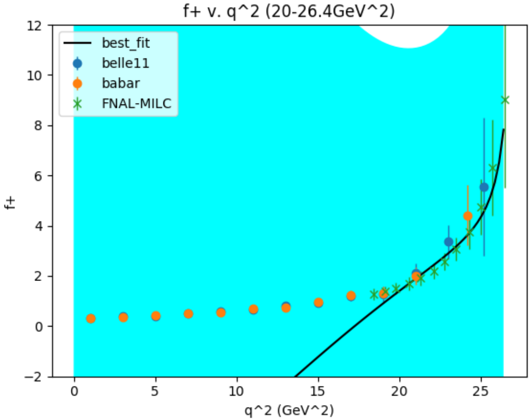
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Current Parameterizations

The most popular parameterization of the form factor is the BLC expansion

BLC Parameterization

$$f_+(q^2) = \frac{1}{1 - \frac{q^2}{m_{B*}^2}} \sum_{n=0}^{N-1} b_{+,n}(z^n - \frac{n}{N}(-1)^{N-n} z^N) \quad (5)$$

$$f_0(q^2) = \sum_{n=0}^{N} b_{0,n} z^n \quad (6)$$

Benefits of this parameterization:

- naturally satisfies the QCD scaling and conservation of angular momentum

New Parameterization

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$$f_+(q^2) = C + \frac{1}{1 - \frac{q^2}{m_{B*}^2}}\left(A + B\left(\frac{1+z}{1-z}\right)^2\right) \quad (7)$$

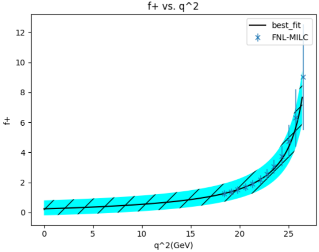
Alternative Parameterization

$$f_+(q^2) = -\frac{Bm_{B*}^2}{t_+ - t_0} + \frac{1}{1 - \frac{q^2}{m_{B*}^2}}\left(A + B\left(\frac{1+z}{1-z}\right)^2\right) \quad (8)$$

Advantages of this parameterization

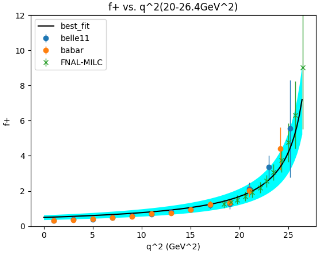
- Naturally satisfies the contraints imposed by analyticity, conservation of angular momentum and the $\frac{1}{q^2}$ fall off.
- relatively simple expression

Results of Lattice Fit



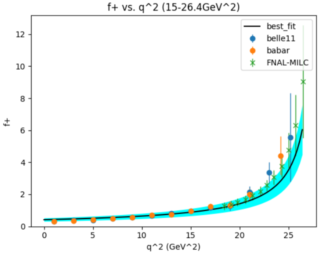
- $A = 0.34(6)$
- $B = 1.90(5)$
- $Q = 0.99$
- $\chi^2/dof = 0.27$
- $dof = 10$

Results of Experimental Fit



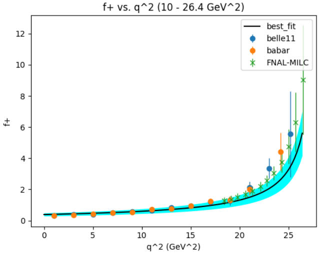
- $A = 0.229(60)$
- $B = 1.88(60)$
- $Q = 0.85$
- $\chi^2/dof = 0.4$
- $dof = 5$

Results of Experimental Fit



- $A = 0.211(60)$
- $B = 1.88(66)$
- $Q = 0.061$
- $\chi^2/dof = 1.7$
- $dof = 11$

Results of Experimental Fit



- $A = 0.182(67)$
- $B = 1.88(60)$
- $Q = 0.0017$
- $\chi^2/dof = 2.4$
- $dof = 15$

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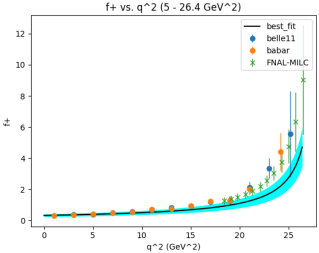
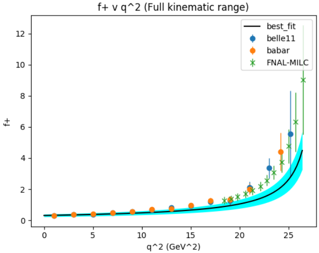


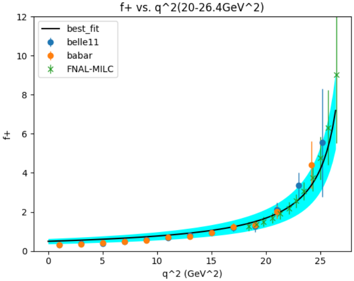
f

- $A = 0.121(48)$
- $B = 1.88(66)$
- $\chi^2/dof = 7.2$
- $dof = 21$

Results of Experimental Fit



- $A = 0.106(40)$
- $B = 1.87(66)$
- $\chi^2/dof = 8.1$
- $dof = 21$



Possible Avenue: Use our model and a power series in the variable z

$$f_+(z(q^2;t0)) = C + \frac{A + B\left(\frac{1+z}{1-z}\right)^2}{1 - q^2/m_{B*}^2}\left(\sum_{n=0} c_n z^n\right) \qquad (9)$$