Advances in QCD Theory

Christopher Lee
Los Alamos National Laboratory
Theoretical Division

August 3, 2017

Los Alamos NATIONAL LABORATORY

APS | DIVISION OF PARTICLES & FIELDS
DPF 2017

A handful of
Advances in QCD Theory

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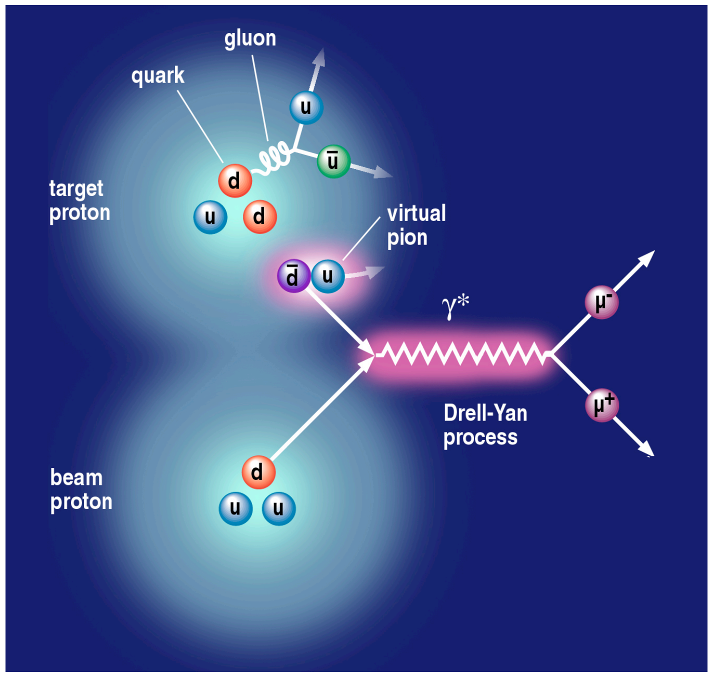
DPF 2017

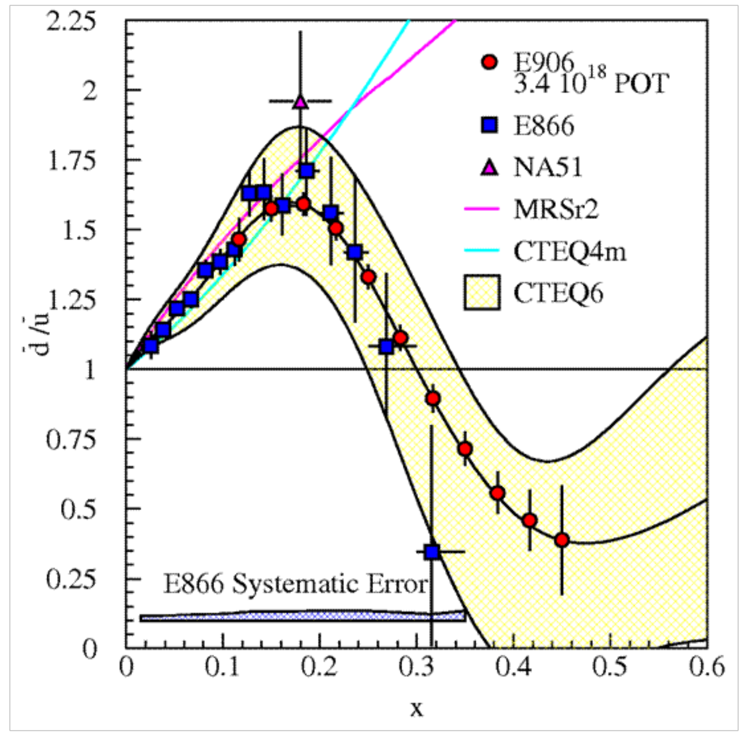
Fermi and Los Alamos

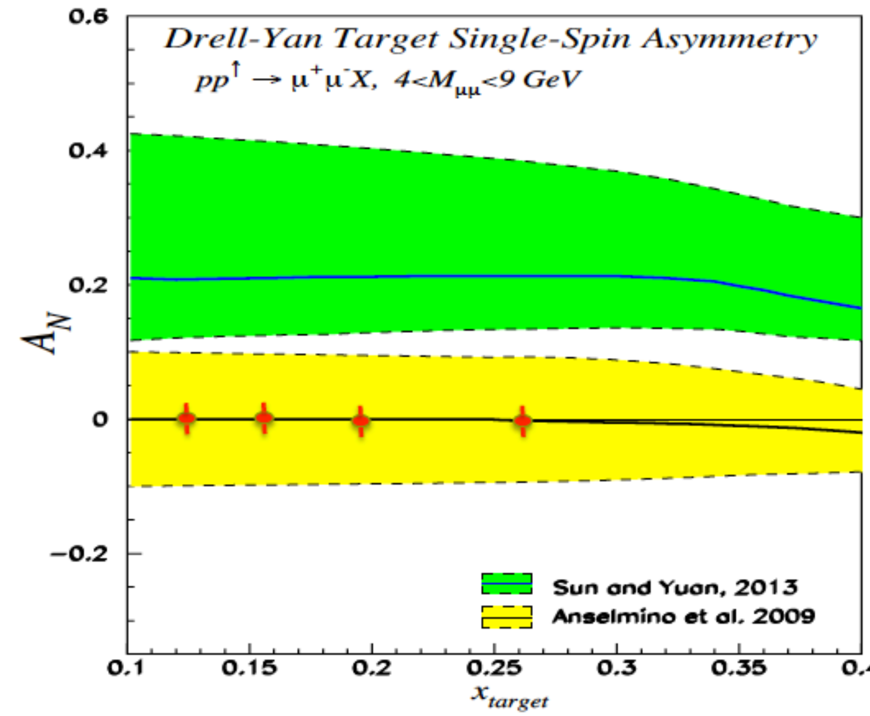
Valles Caldera, near Los Alamos, May 8, 1945

Fermilab and Los Alamos

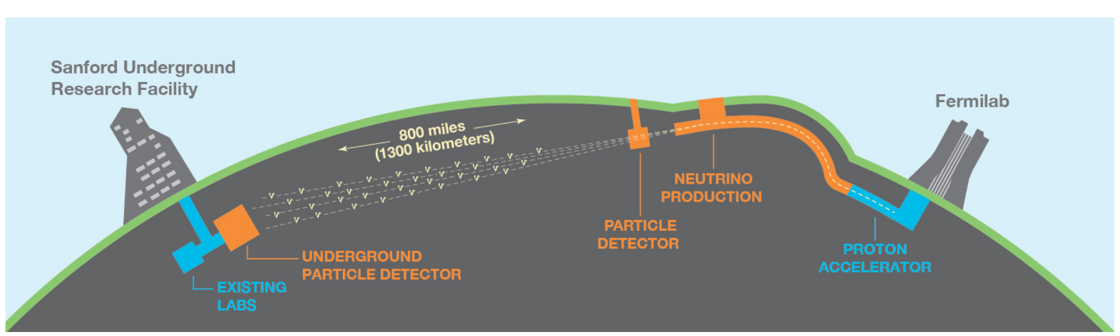
- Drell-Yan: E772/E789/E866 (NuSea)/E906 (SeaQuest), E1039

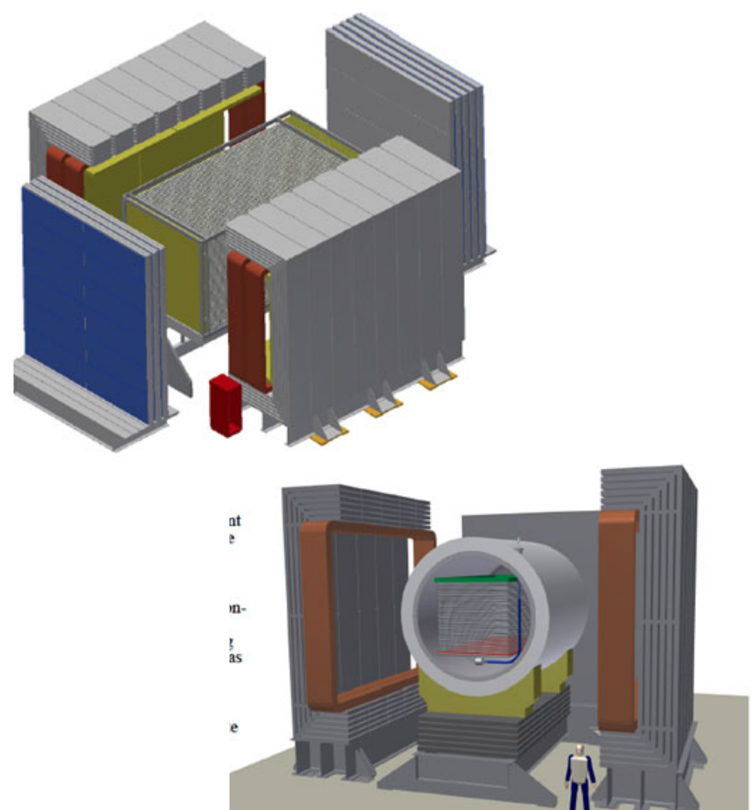






DEEP UNDERGROUND NEUTRINO EXPERIMENT



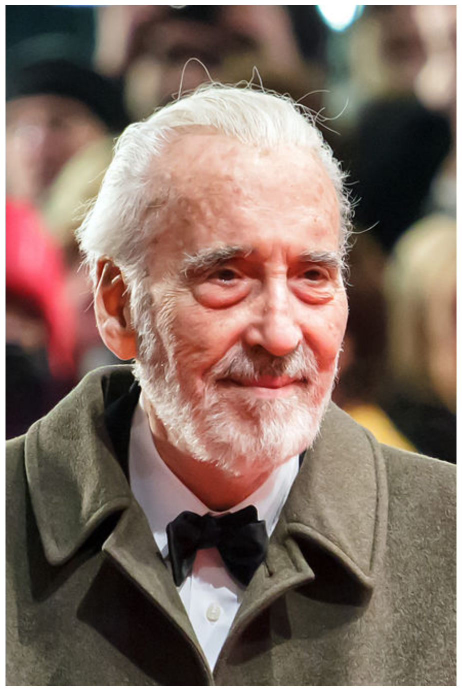


Geoff Mills, 1955-2017

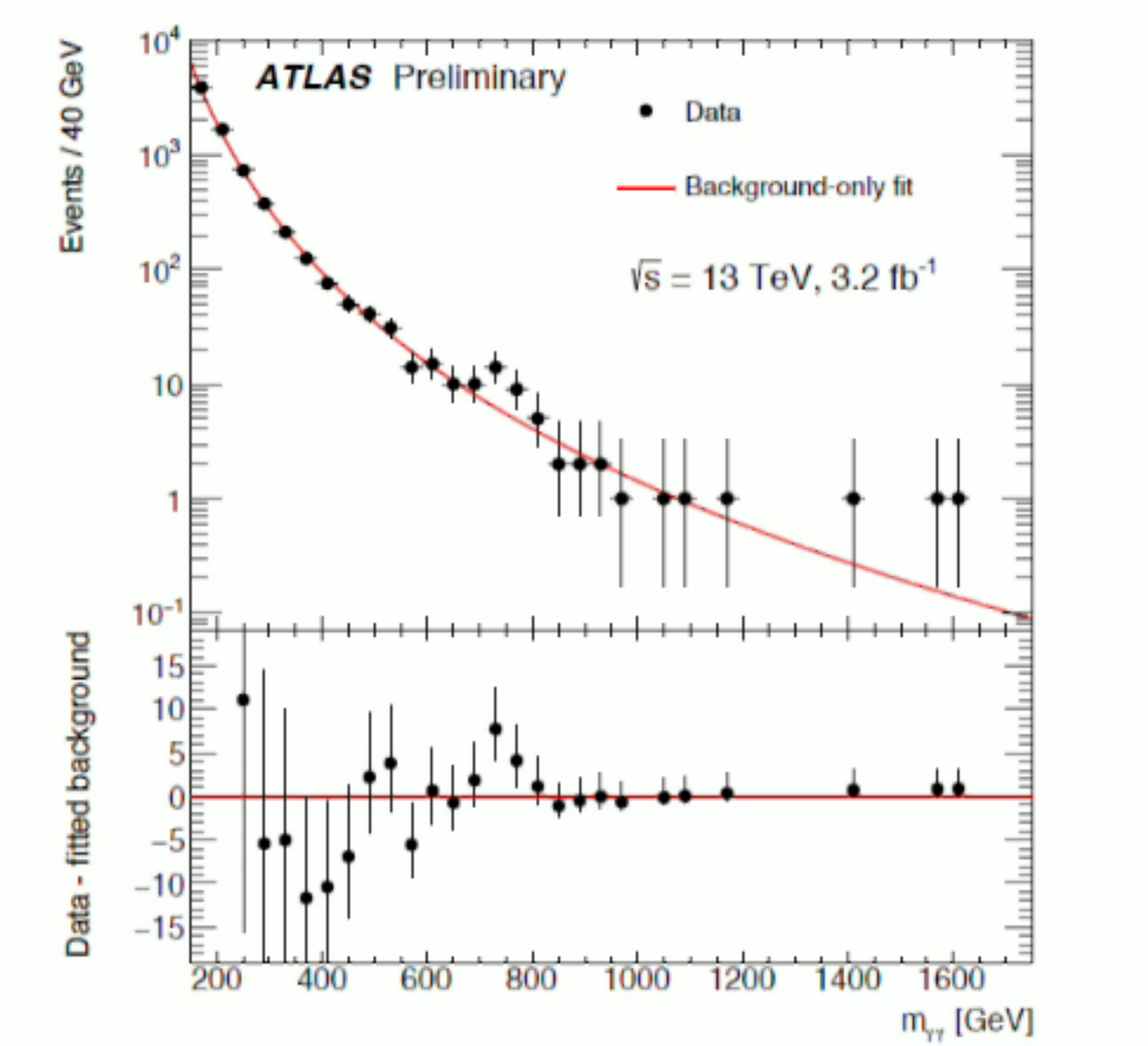
My charge

"QCD theory developments over the last ~2 years"

Two years ago



Christopher Lee,
May 27, 1922
— June 7, 2015



$$750 \text{ GeV}, \;\; 3.6\sigma$$

December 2015
— August 2016

CNN ORC INTERNATIONAL POLL

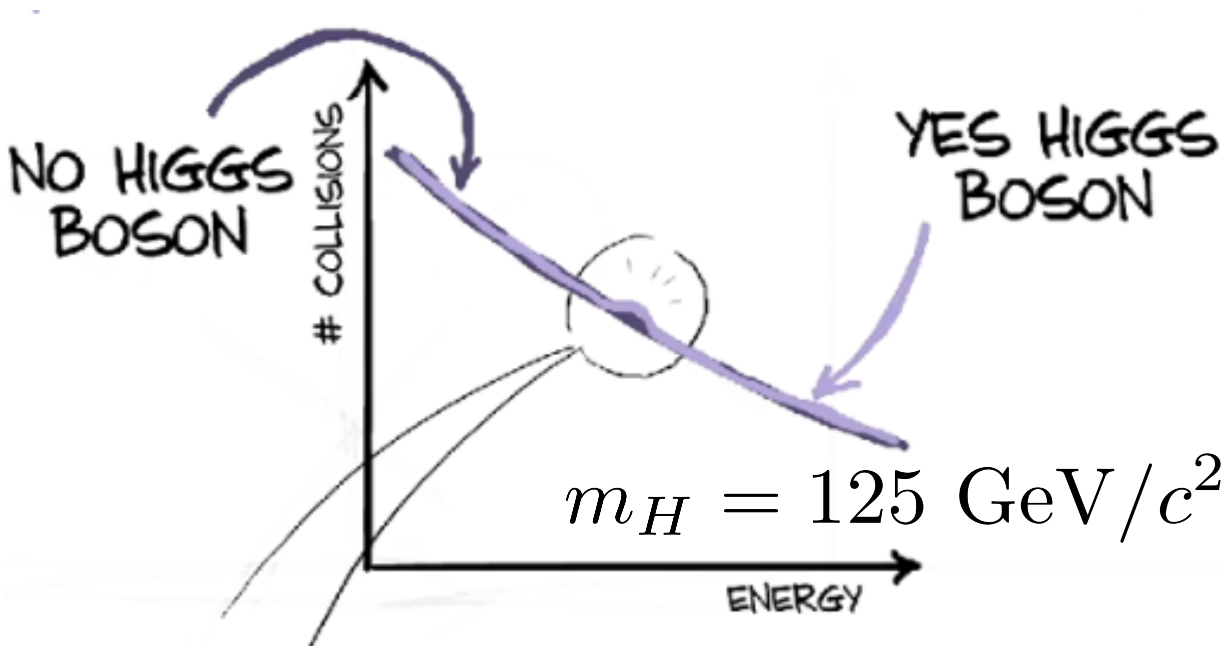
	June 26-28 2015
Bush	19%
Trump	12%
Huckabee	8%
Carson	7%
Paul	7%
Rubio	6%
Walker	6%
Perry	4%
Christie	3%
Cruz	3%
Santorum	3%
Jindal	2%
Kasich	2%
Fiorina	1%
Graham	1%

	Total
Clinton	59%
Trump	35%
Other	*
Neither	6%

Historic Day: July 4

Historic Day: July 4

2012

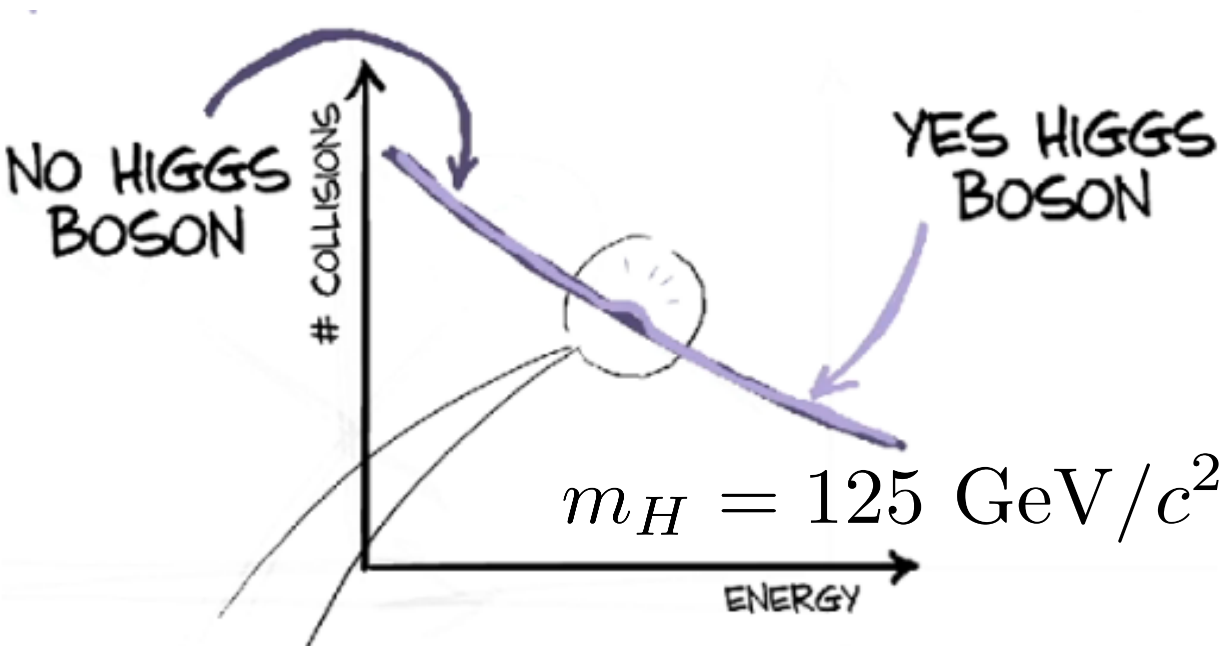


$$m_H = 125\ \mathrm{GeV}/c^2$$

San Diego
“Big Bay Boom”

Historic Day: July 4

2012



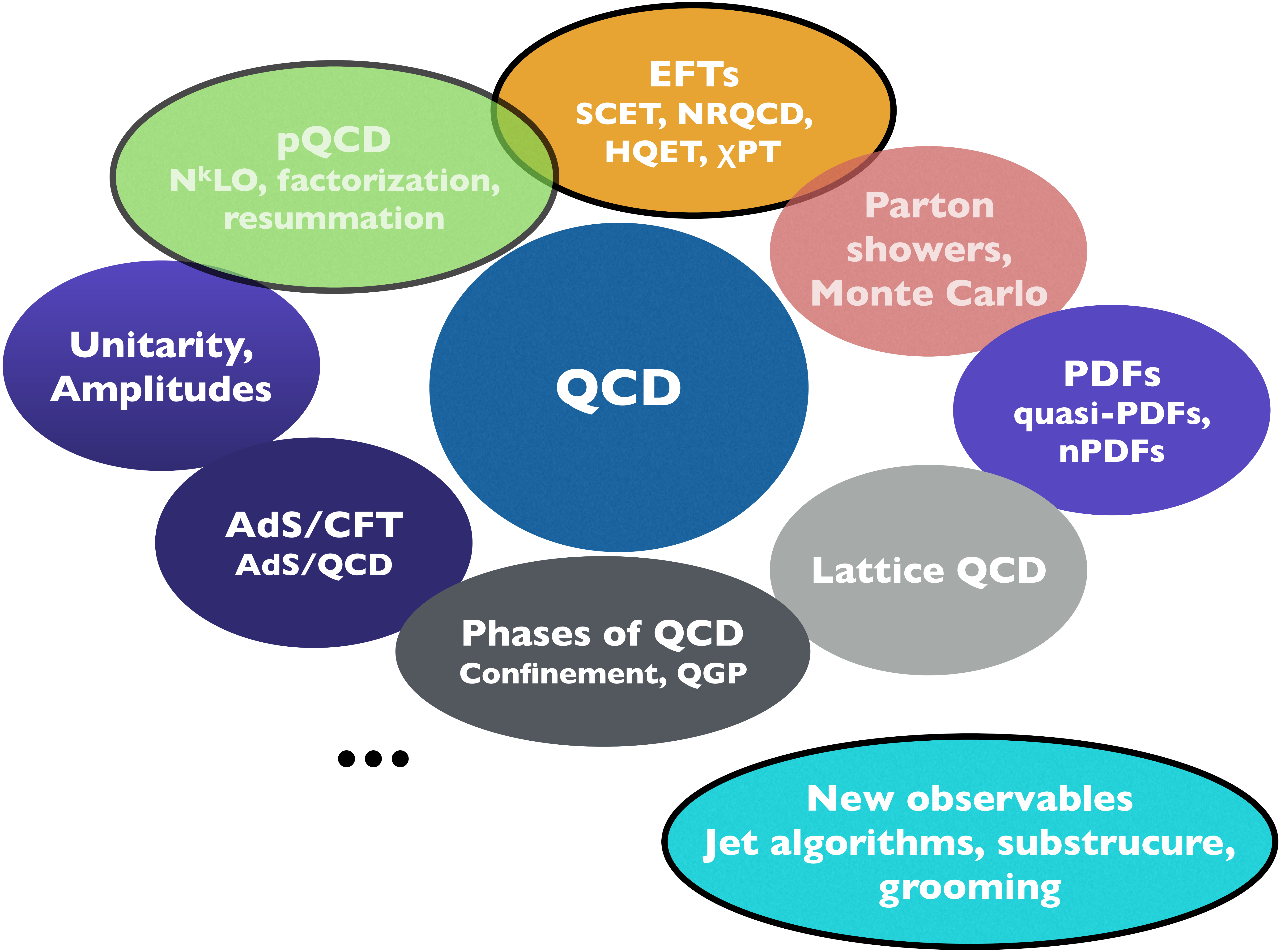
$$m_H = 125\ \mathrm{GeV}/c^2$$

San Diego
"Big Bay Boom"

2017

Andreas Maria Lee,
Los Alamos, NM

$$m_{\mathrm{AML}} = 1.813 \times 10^{27}\ \mathrm{GeV}/c^2$$


EFTs
SCET, NRQCD, HQET, χPT
pQCD
N^{k}LO, factorization, resummation
Parton showers, Monte Carlo
Unitarity, Amplitudes
QCD
PDFs
quasi-PDFs, nPDFs
AdS/CFT
AdS/QCD
Lattice QCD
Phases of QCD
Confinement, QGP
...
New observables
Jet algorithms, substrucure, grooming

Outline

- Soft Collinear Effective Theory
- *N*-Jettiness, $\mathrm{SCET}_{\mathrm{I}}$, $\mathrm{N}^3\mathrm{LL}$ resummation
- $\mathrm{SCET}_{\mathrm{II}}$ and $\mathrm{N}^3\mathrm{LL}$ resummed TMD distributions
- Subtractions and NNLO cross sections
- Non-Global Logarithms, fixed order and resummed
- Jet substructure and SCET_{+}
- Outlook

Formation of Jets in QCD

Perturbative soft and collinear splittings happen at intermediate time

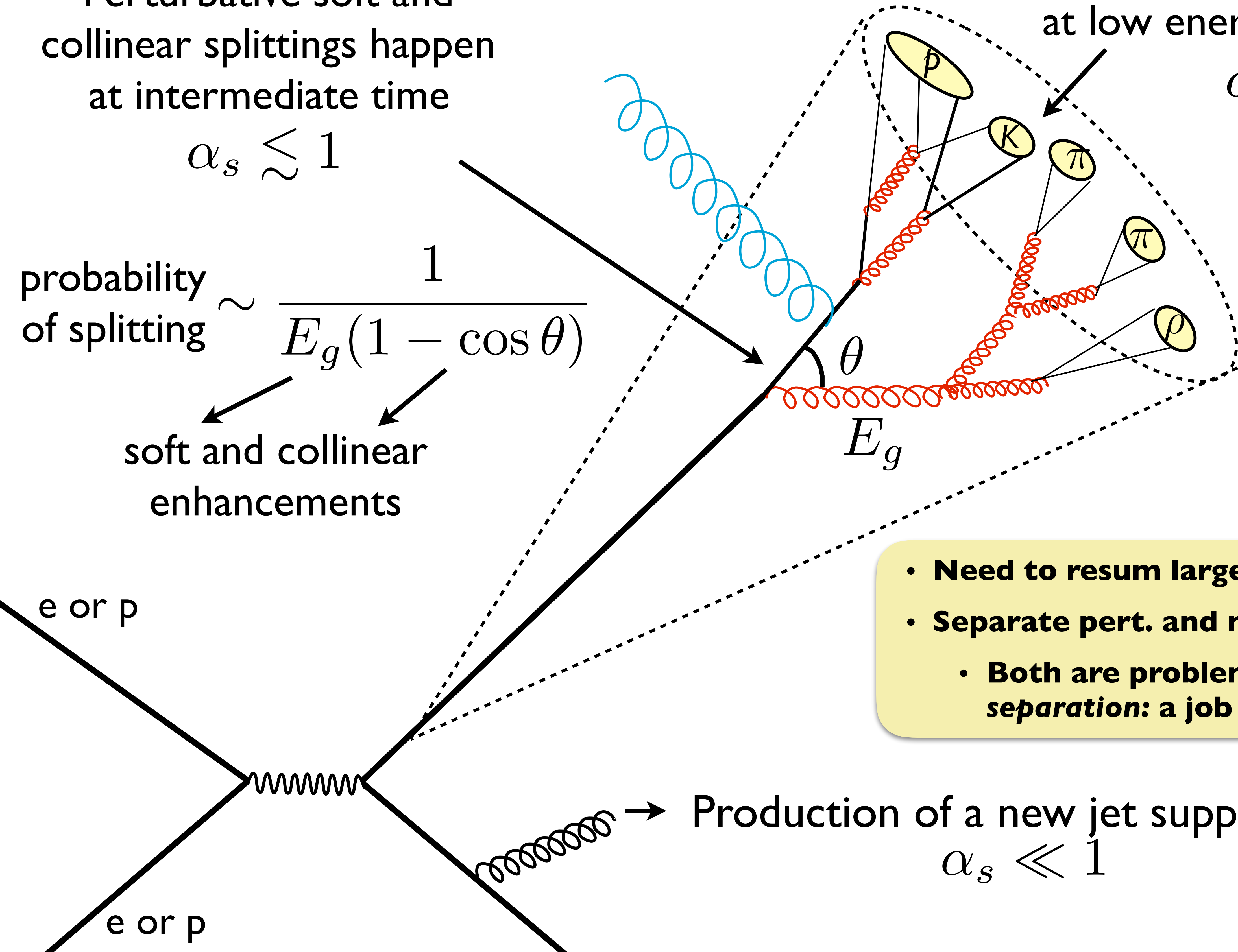
$\alpha_s \lesssim 1$

Hadronization at late time at low energy scale

$\alpha_s \gg 1$

probability of splitting $\sim \dfrac{1}{E_g(1-\cos\theta)}$

soft and collinear enhancements

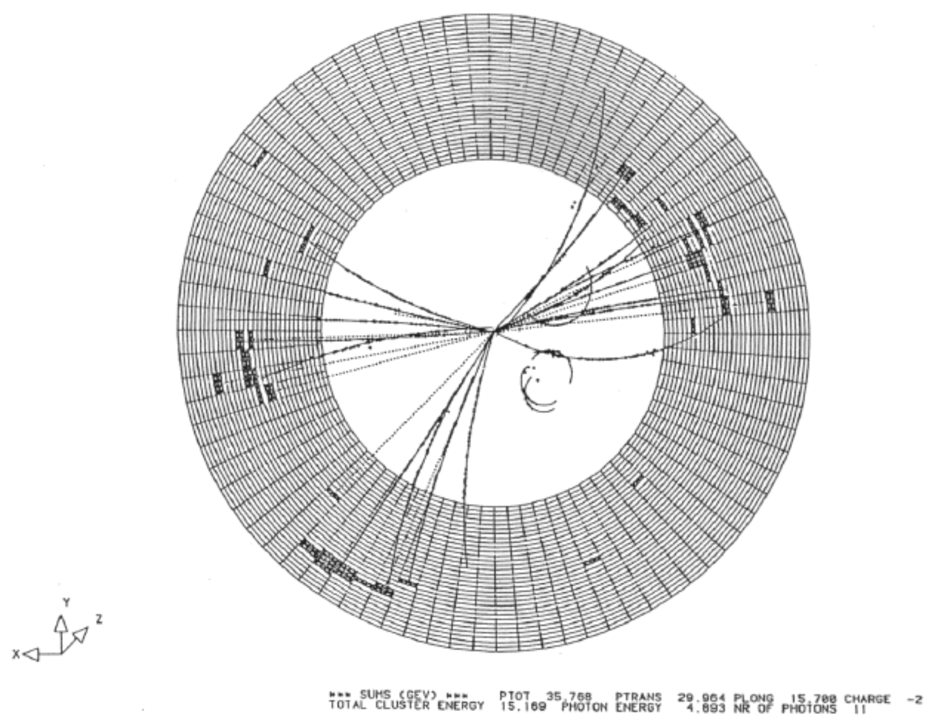


- **Need to resum large perturbative logs**
- **Separate pert. and non-pert. physics**
  - **Both are problems of *scale separation:* a job for EFT**

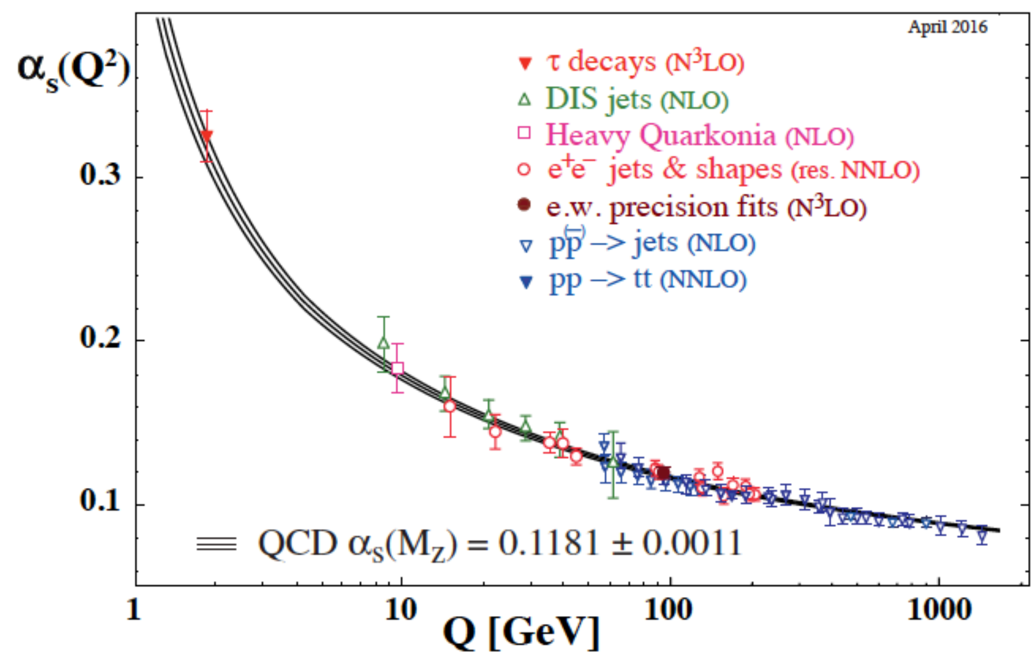
→ Production of a new jet suppressed by $\alpha_s \ll 1$

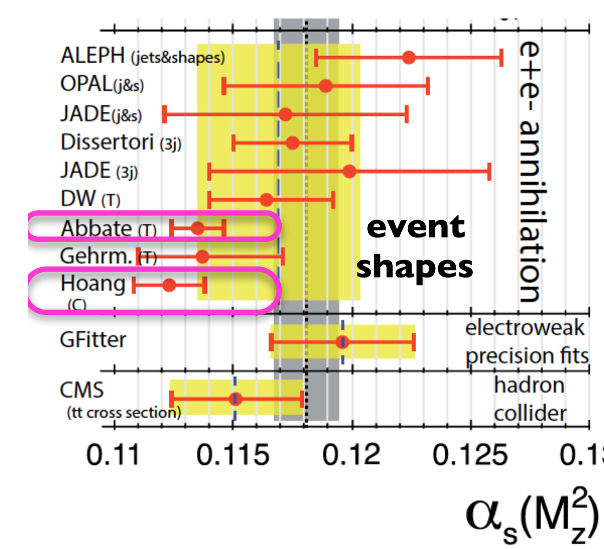
History of Jets in QCD

- **Existence of gluons:**



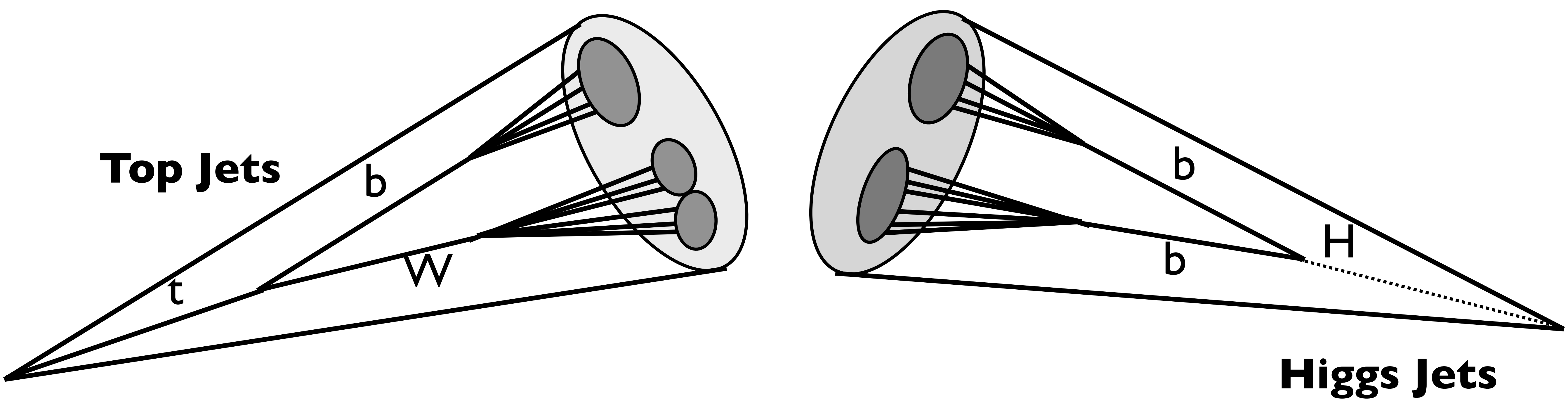
- **Measurements of strong coupling:**





PDG, RPP (2015-16)

- **Boosted heavy particles in SM and BSM**



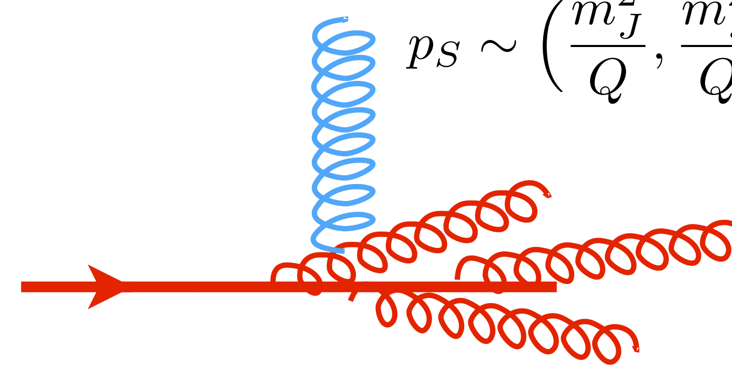
Separation of scales

- Large logs in QCD arise from large ratios of physical scales defining the measurement or degree of exclusivity of a jet cross section.
- For jet cross sections, these are precisely ratios of hard to soft scales and ratios of collinear momentum components.
  - e.g. measurement of jet mass

$$p_J^2 = (p_c + p_s)^2 = m_J^2$$

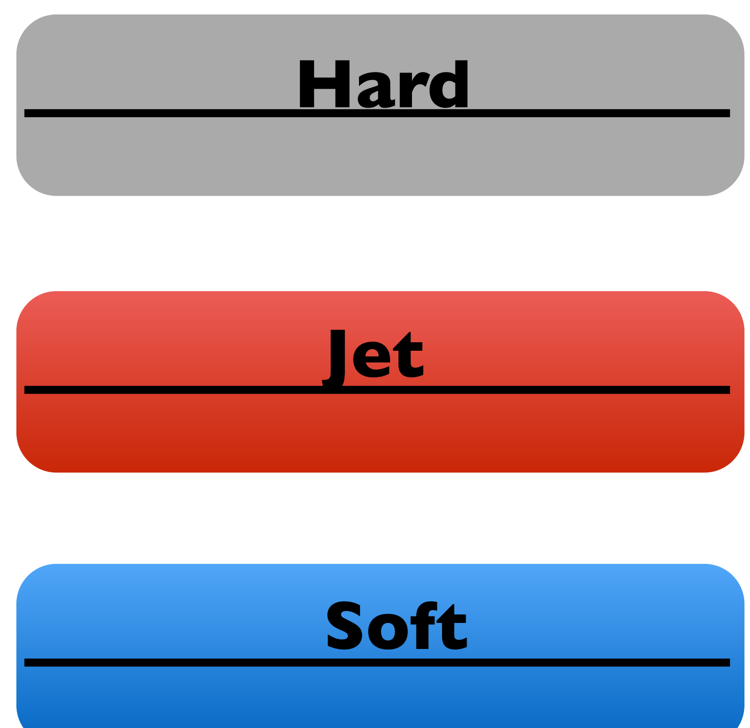
$$p = (\bar{n} \cdot p, n \cdot p, p_\perp)$$

Hierarchy of scales



$$p_S \sim \left(\frac{m_J^2}{Q}, \frac{m_J^2}{Q}, \frac{m_J^2}{Q}\right)$$

$$p_c \sim \left(Q, \frac{m_J^2}{Q}, m_J\right)$$



$$\mu_H = Q$$

$$\mu_J = m_J$$

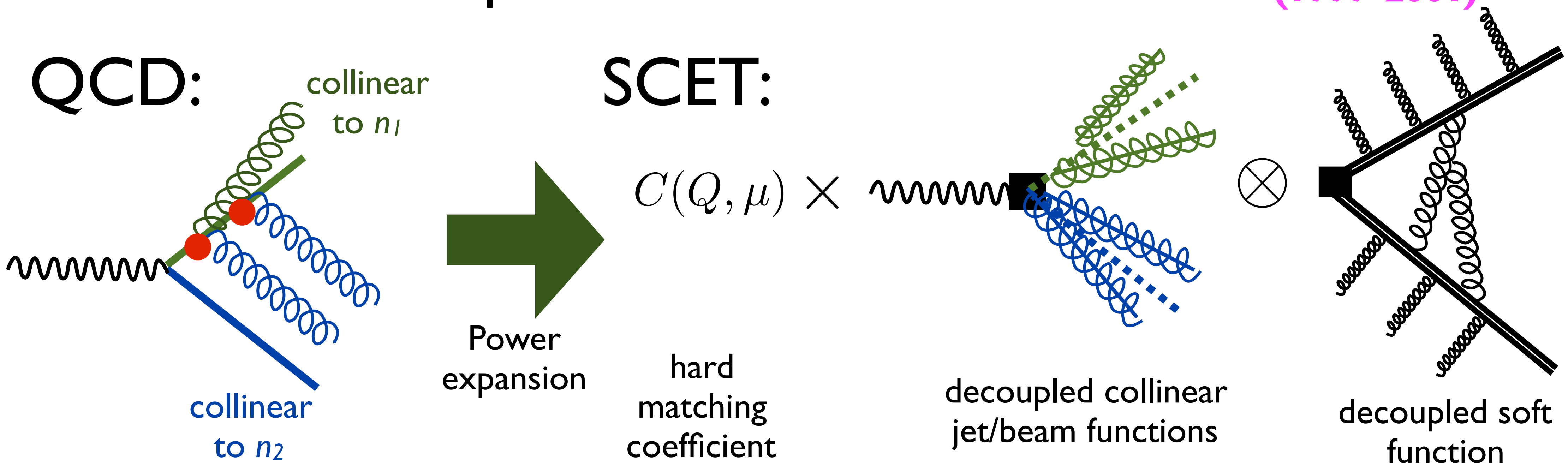
$$\mu_S = \frac{m_J^2}{Q}$$

Factorize cross section into pieces depending on only one of these scales at a time.

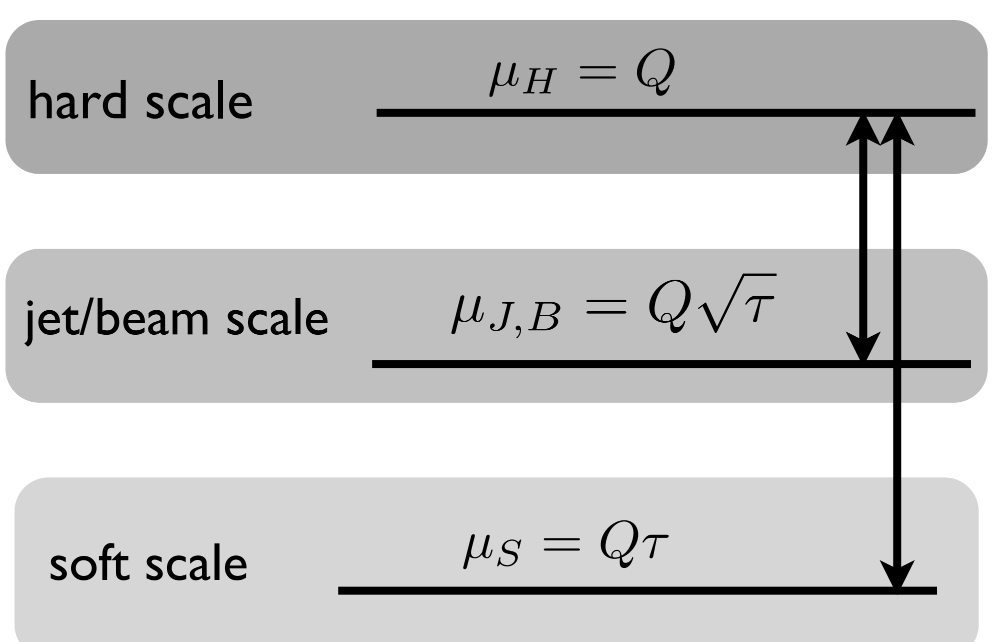
Soft Collinear Effective Theory

- Modern tools for high precision resummation, factorization of perturbative and nonperturbative effects

Bauer, Fleming, Luke, Pirjol, Stewart (1999-2001)



- RG Evolution



- Resummation of large logs

$$\begin{aligned}\ln\sigma(\tau) \sim\ & \alpha_s(\ln^2\tau + \ln\tau) \\ & + \alpha_s^2(\ln^3\tau + \ln^2\tau + \ln\tau) \\ & + \alpha_s^3(\ln^4\tau + \ln^3\tau + \ln^2\tau + \ln\tau) \\ & + \quad \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots\end{aligned}$$

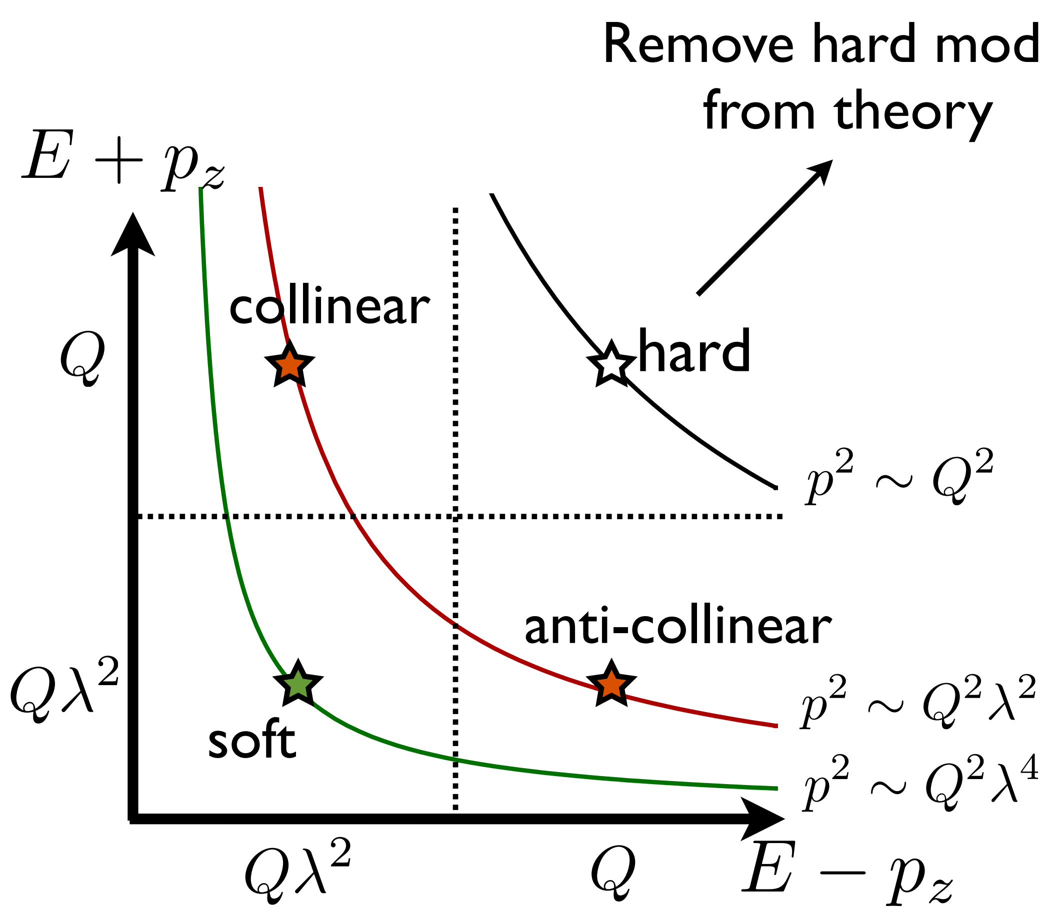
Leading Log (LL)	Next-to-Leading Log (NLL)	NNLL	N^{3}LL

Bauer, Fleming, Luke (2000)
Bauer, Fleming, Pirjol, Stewart (2001)
Bauer, Fleming, Pirjol, Rothstein, Stewart (2002)

Soft Collinear Effective Theory

- SCET_{I}

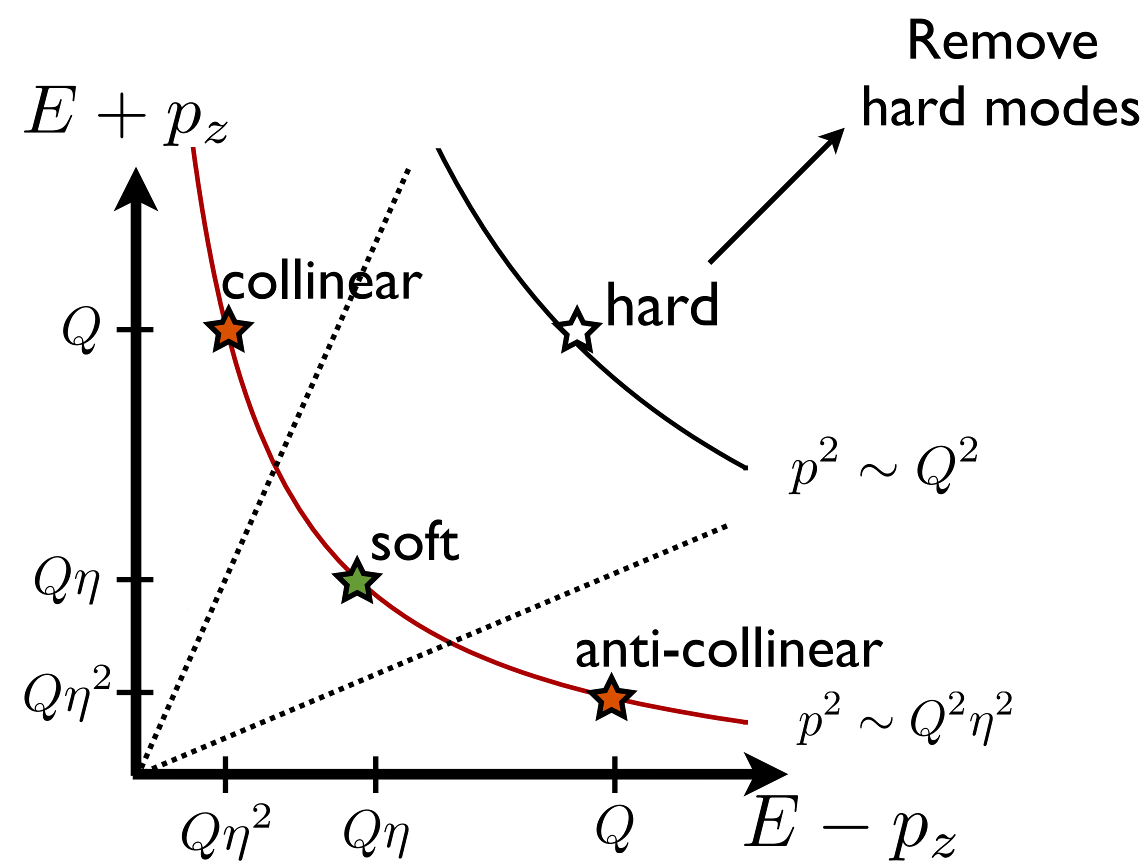
  Theory for jets constrained by mass



  - Hard, collinear, soft all separated by **virtuality**
  - Collinear/soft decoupling and factorization
  - Dim. Reg. regulates all divergences

- SCET_{II}

  Theory for jets constrained by transverse momentum or for exclusive collinear hadrons



  - Hard separated from coll. and soft by virtuality, collinear & soft separated by **rapidity**
  - Inherits SCET_{I} collinear-soft decoupling
  - Dim. Reg. regulates virtuality divergences but not rapidity divergences ➡ need additional regulator

Chiu, Jain, Neill, Rothstein (2011, 2012)

Challenges to Precision Jet Cross Sections

- Jet cross sections typically depend on
    - choice of jet algorithm
    - jet sizes
    - jet vetoes (for exclusive jet cross sections)
- These parameters generate a number of logarithms (non-global logs, logs of radii *R*, etc.) in perturbation theory which are challenging to resum
- ***N*-Jettiness**: a *global* observable picking out *N*-jet final states by measurement of a *single* parameter, logs of which *can* be resummed in perturbation theory by standard RGE

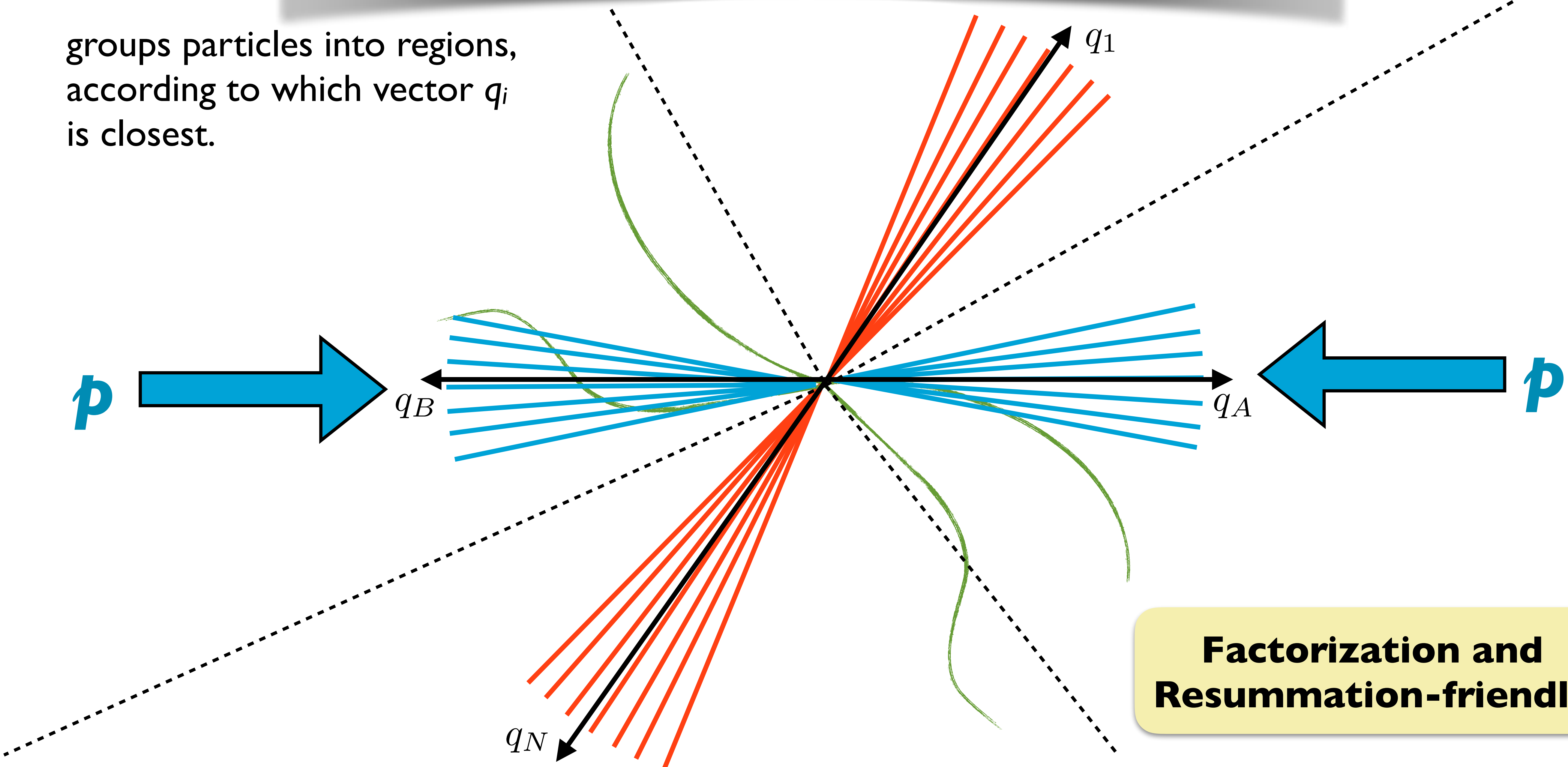
N-jettiness

Stewart, Tackmann, Waalewijn (2010)

- A global event shape measuring degree to which final state is *N*-jet-like. (small *N*-jettiness vetoes events with more than *N* jets.)

$$\tau_N = \frac{2}{Q^2} \sum_k \min\{\underbrace{q_A \cdot p_k, q_B \cdot p_k}_{\#\text{ beams}}, \underbrace{q_1 \cdot p_k, \ldots, q_N \cdot p_k}_{\#\text{ jets}}\}$$

groups particles into regions, according to which vector q_i is closest.

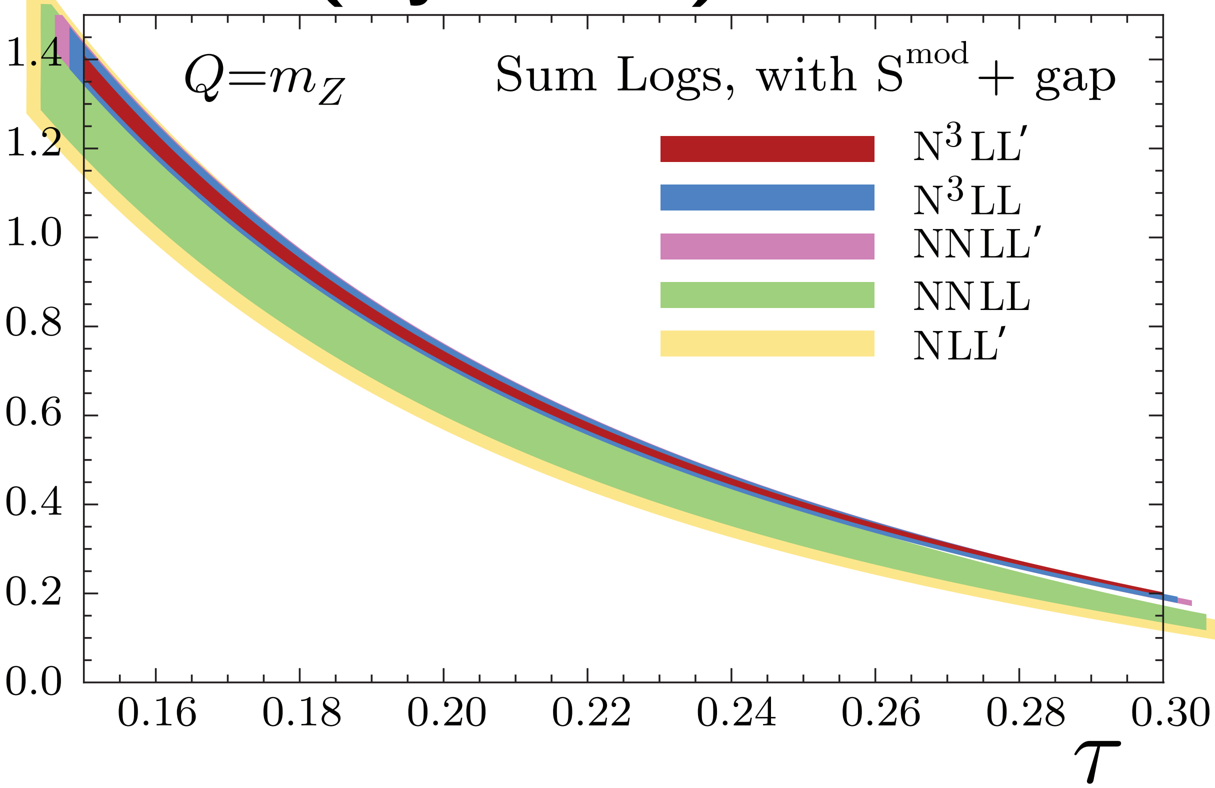


Factorization and Resummation-friendly

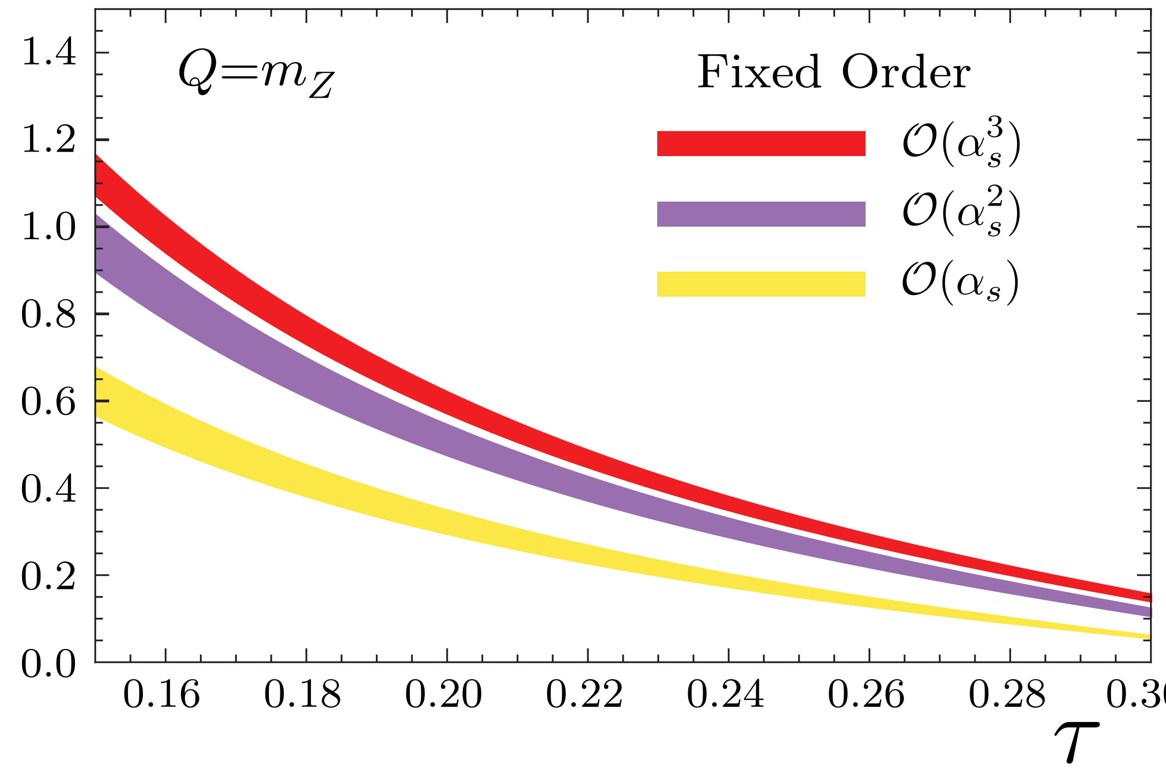
N^{3}LL resummation with SCET

e$^+$e$^-$ Thrust (2-Jettiness)

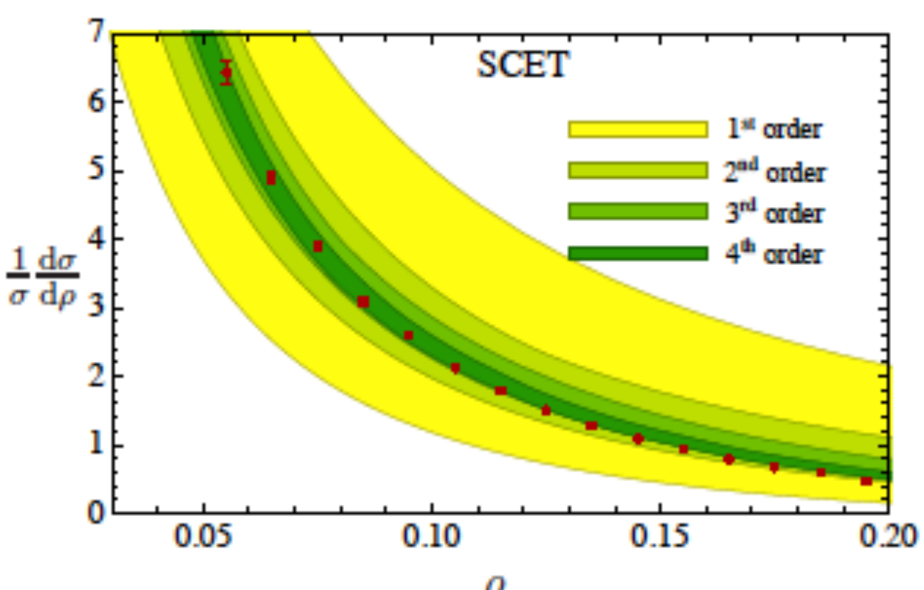
Abbate, Fickinger, Hoang, Mateu, Stewart (2010)



Compare fixed order:

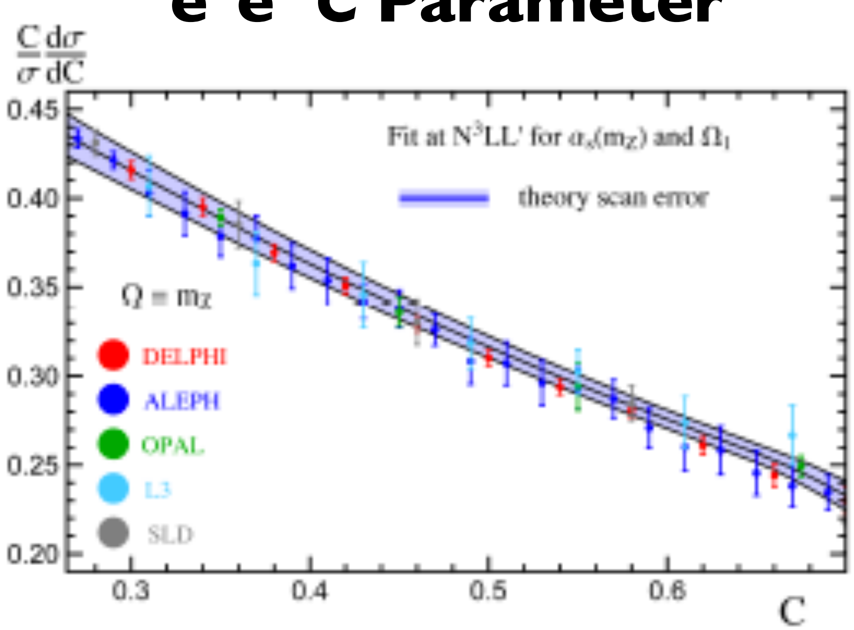


e$^+$e$^-$ Hemisphere Jet Mass



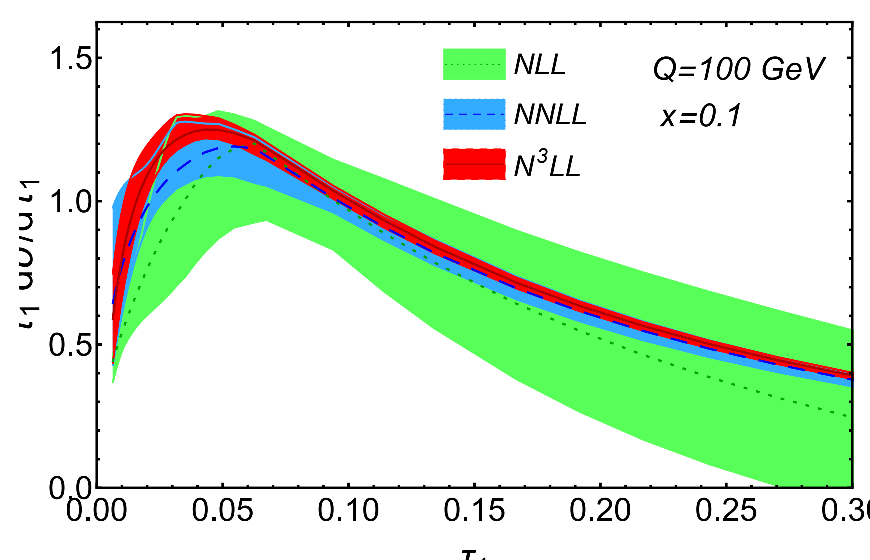
Chien, Schwartz (2010)

e$^+$e$^-$ C Parameter



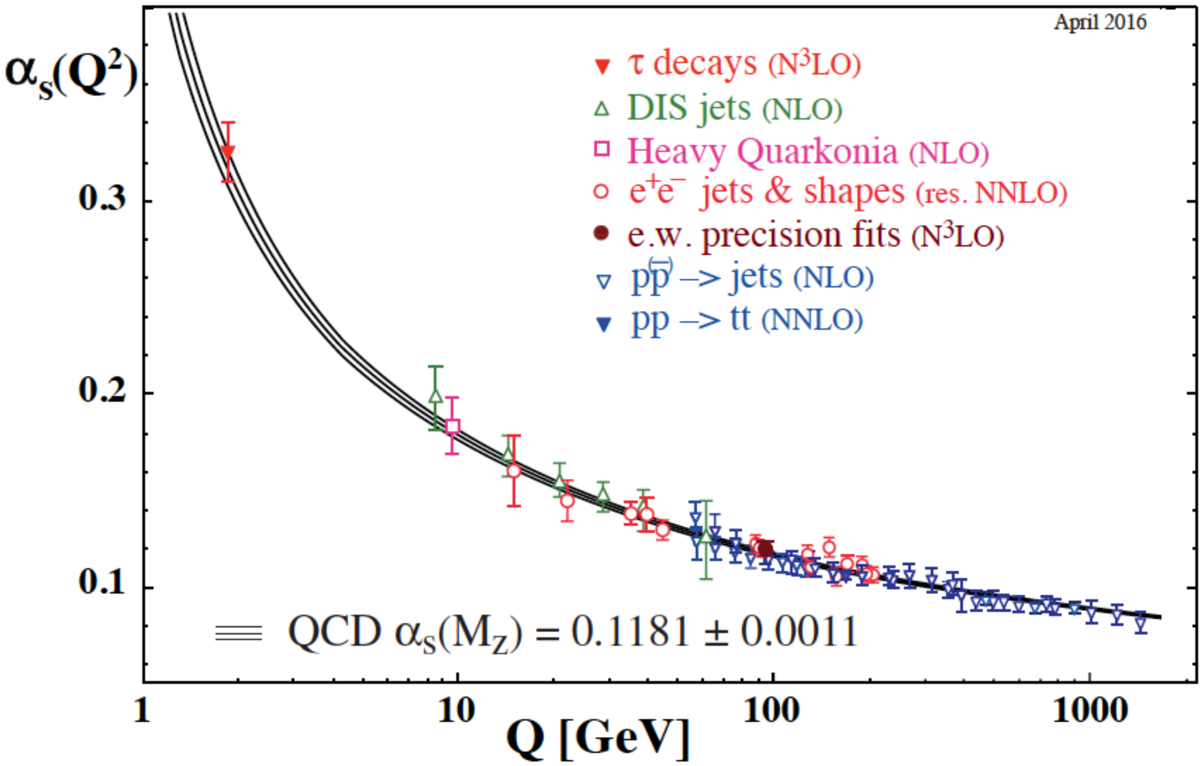
Hoang, Kolodrubetz, Mateu, Stewart (2014)

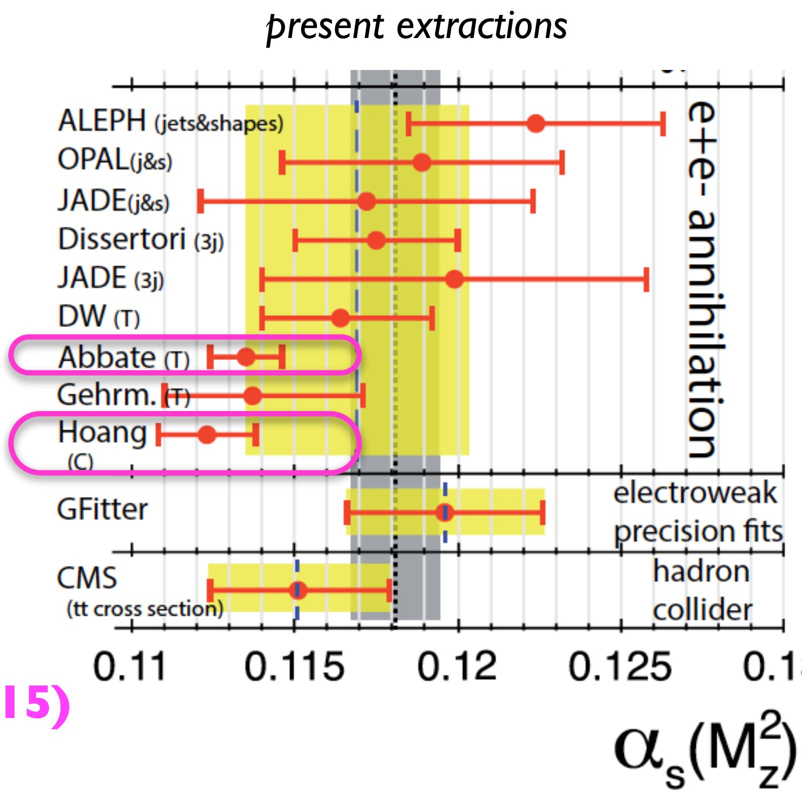
DIS *ep* 1-Jettiness



Kang, CL, Stewart (preliminary, 2017)

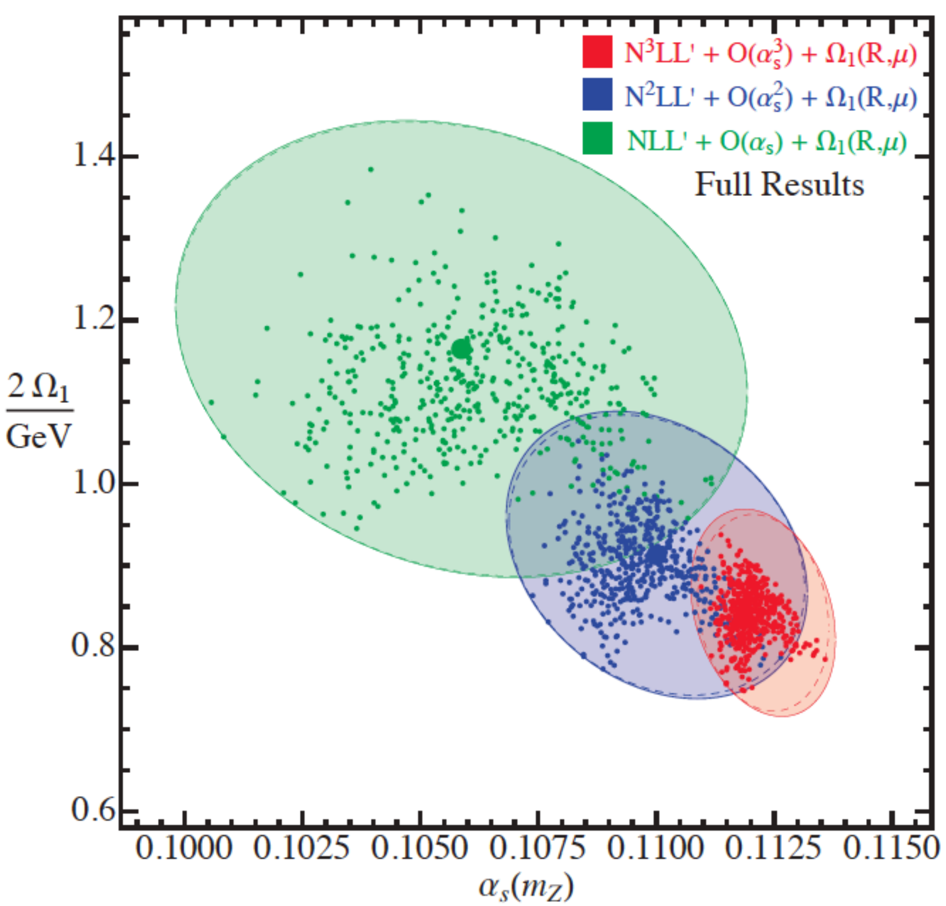
High precision strong coupling

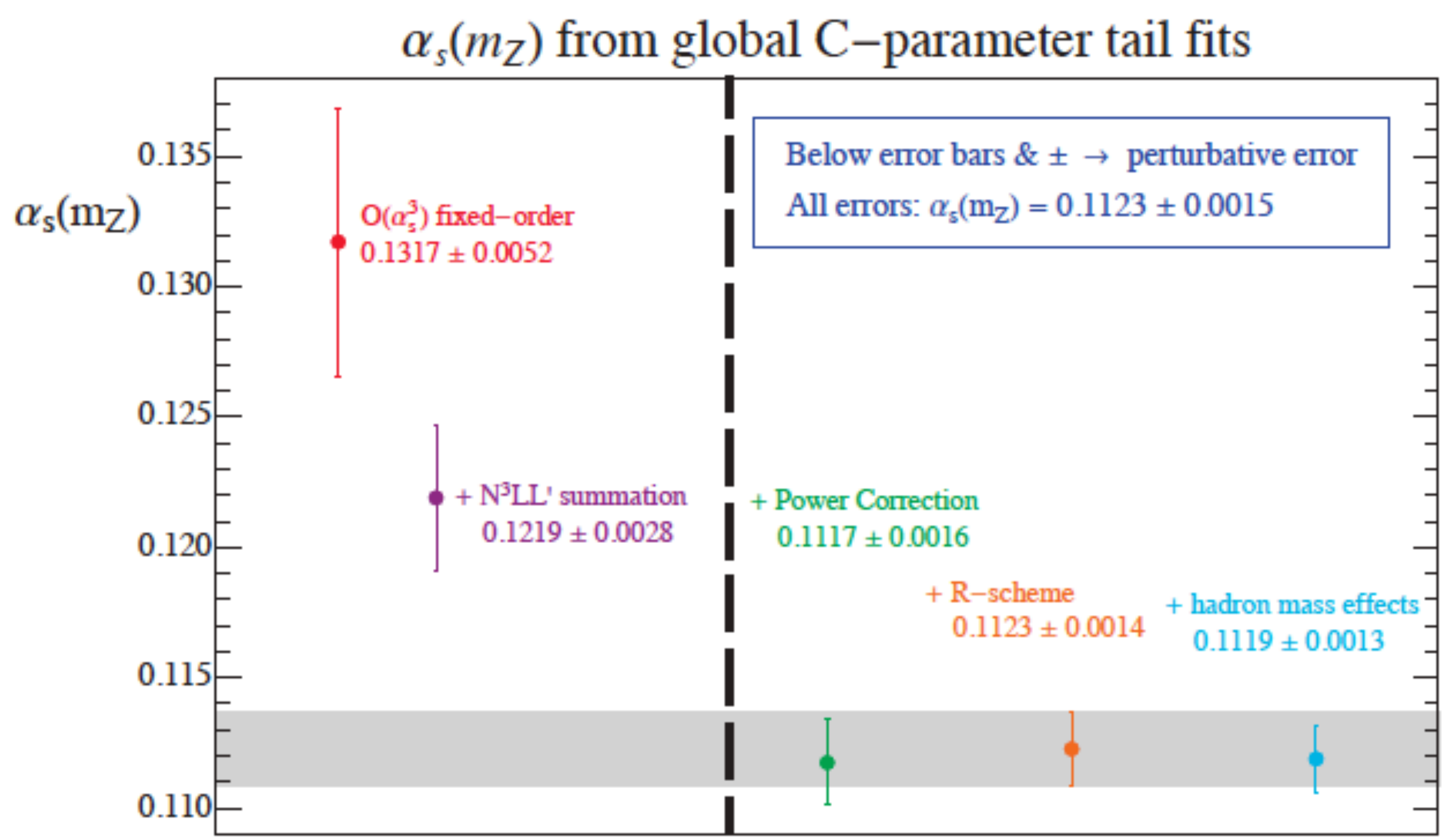




from SCET predictions for e$^+$e$^-$ event shapes

PDG, RPP (2015)

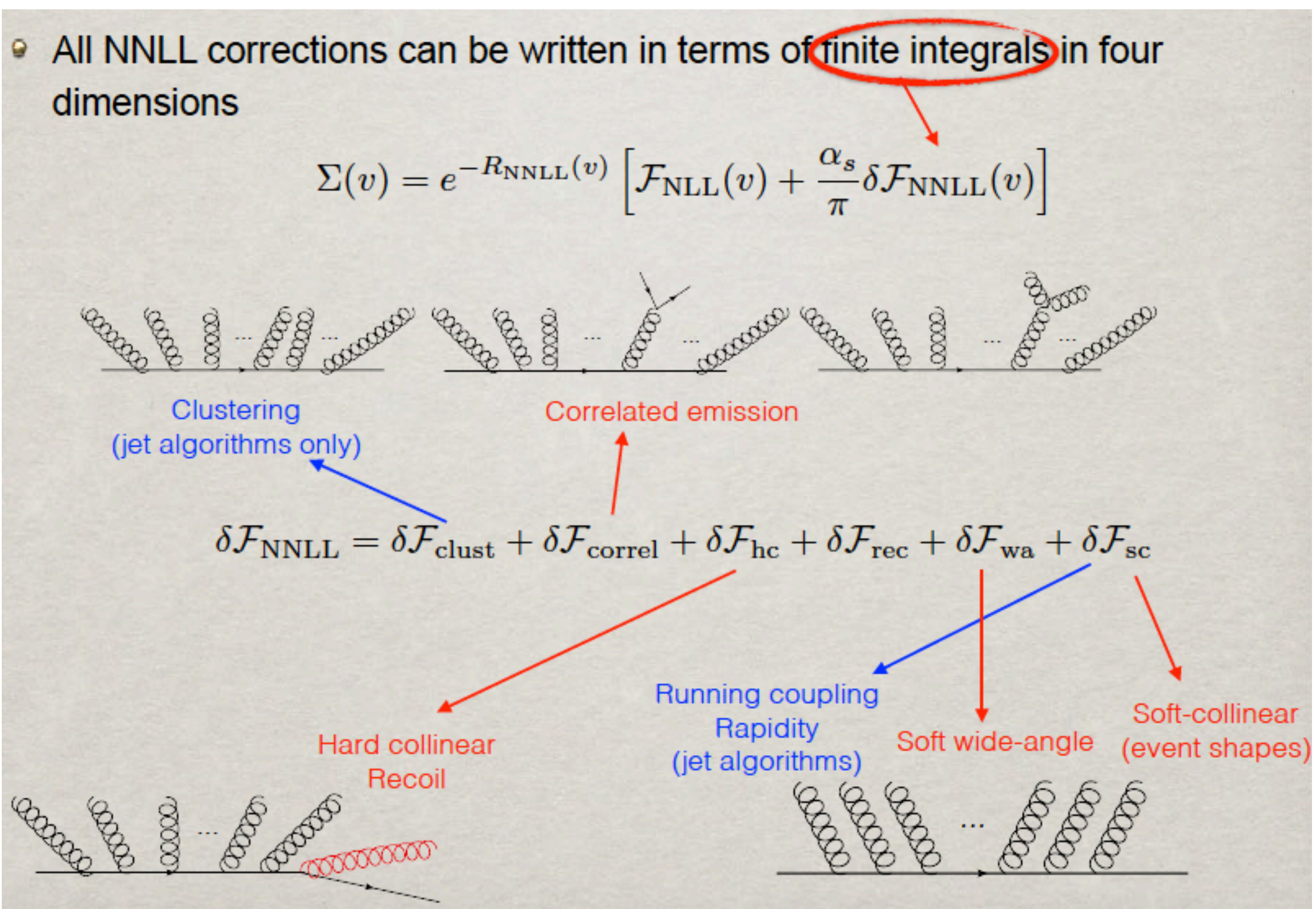




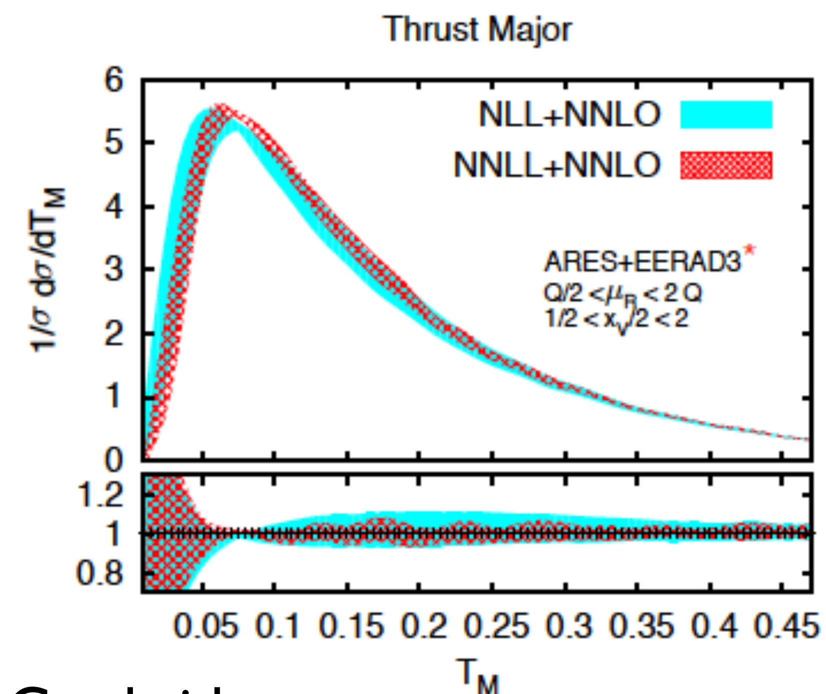
Hoang, Kolodrubetz, Mateu, Stewart (2015)

NNLL resummation for generic observables

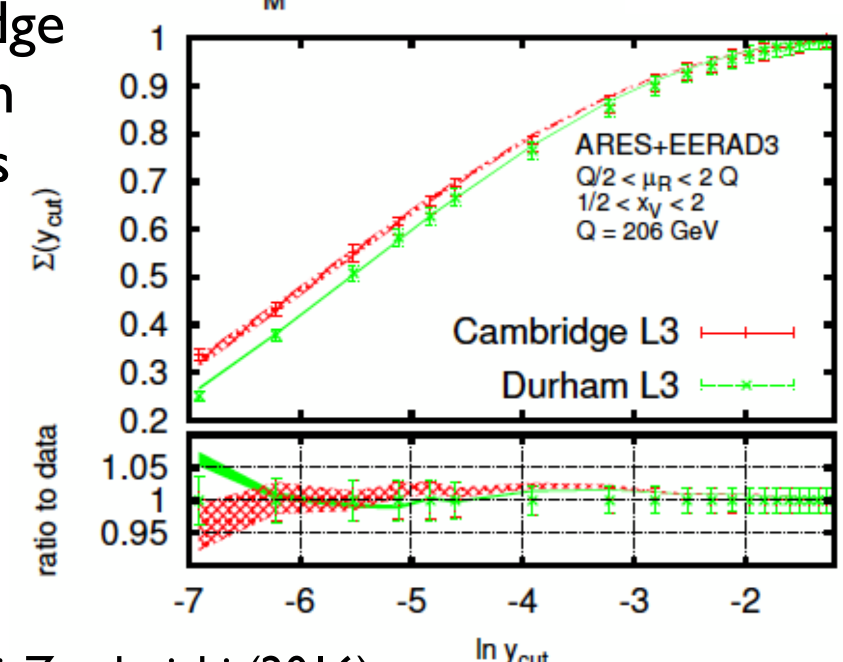
- We do not always have a factorization theorem available to make SCET and its RG evolution to achieve resummation
- Monte Carlo implementation ARES (successor to NLL CAESAR) of emission amplitudes needed for NNLL



Banfi, LoopFest 2017

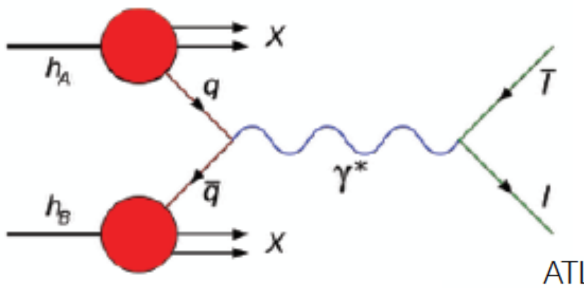


Cambridge /Durham Jet Rates



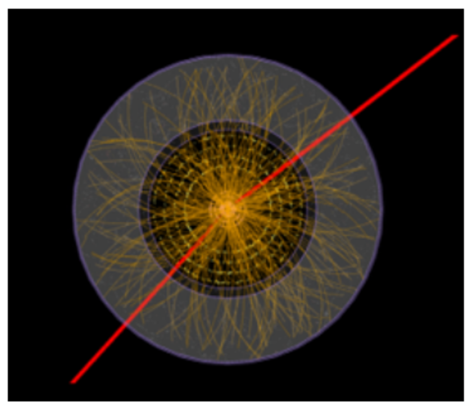
Banfi, McAslan, Monni, Zanderighi (2016)

High precision p_T resummation at LHC

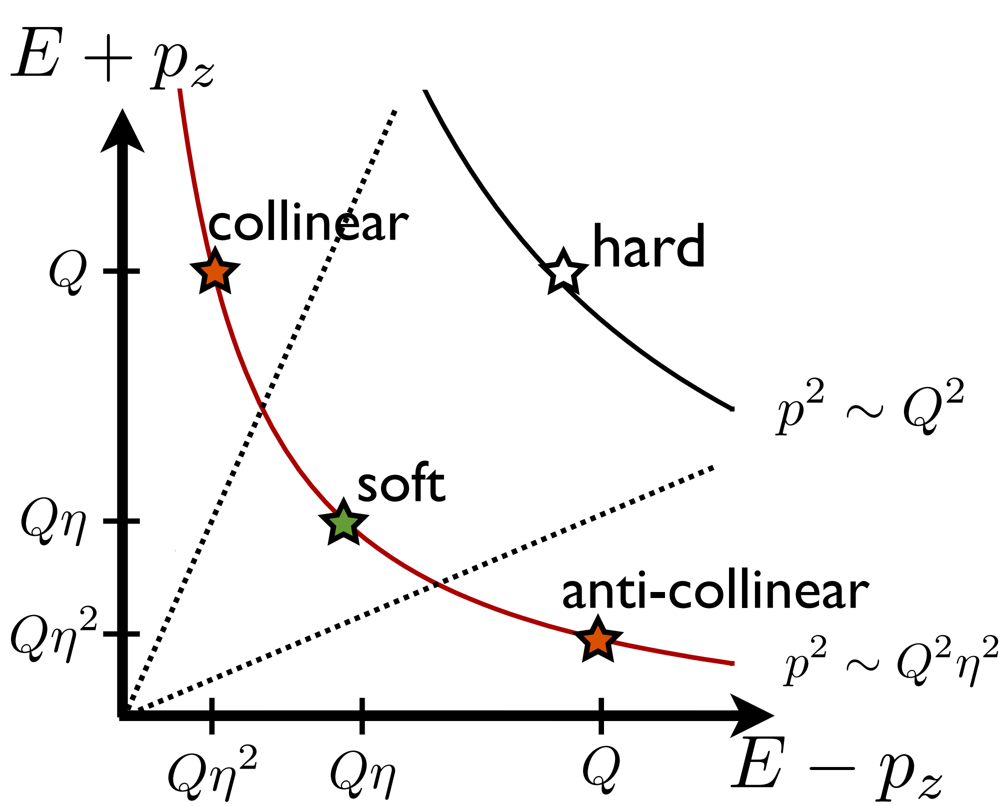


$$\frac{d\sigma}{dq_T^2 dy} = \sigma_0 H(Q^2,\mu) \int d^2\vec{q}_{Ts} d^2\vec{q}_{T1} d^2\vec{q}_{T2} \delta(\vec{q_T}^2 - |\vec{q_{T}}_s + \vec{q_T}_1 + \vec{q_T}_2|^2)$$
$$\times S(\vec{q}_{Ts},\mu,\nu) f_1^{\perp}(x_1 = \frac{Q}{\sqrt{s}}e^{y}, Q, \vec{q}_{T1},\mu,\nu) f_2^{\perp}(x_2 = \frac{Q}{\sqrt{s}}e^{-y}, Q, \vec{q}_{T2},\mu,\nu)$$

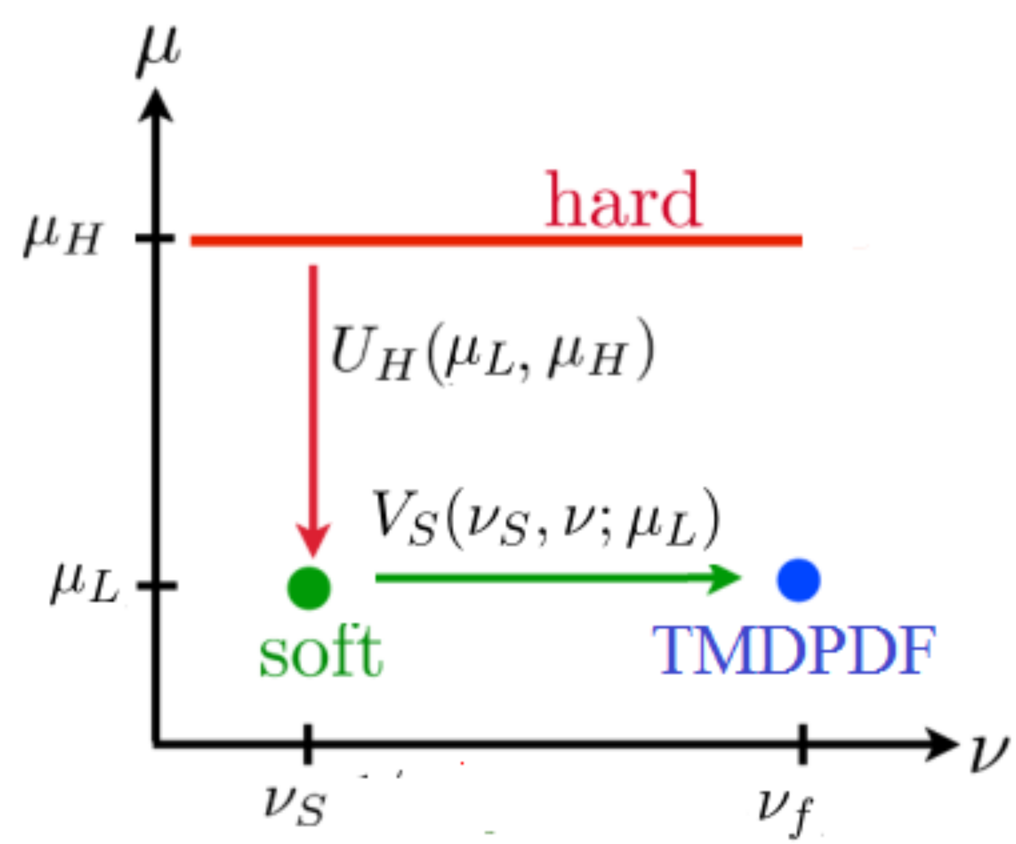
ATLAS event: 242090708



- $\mathrm{SCET_{II}}$



- Rapidity Renormalization Group



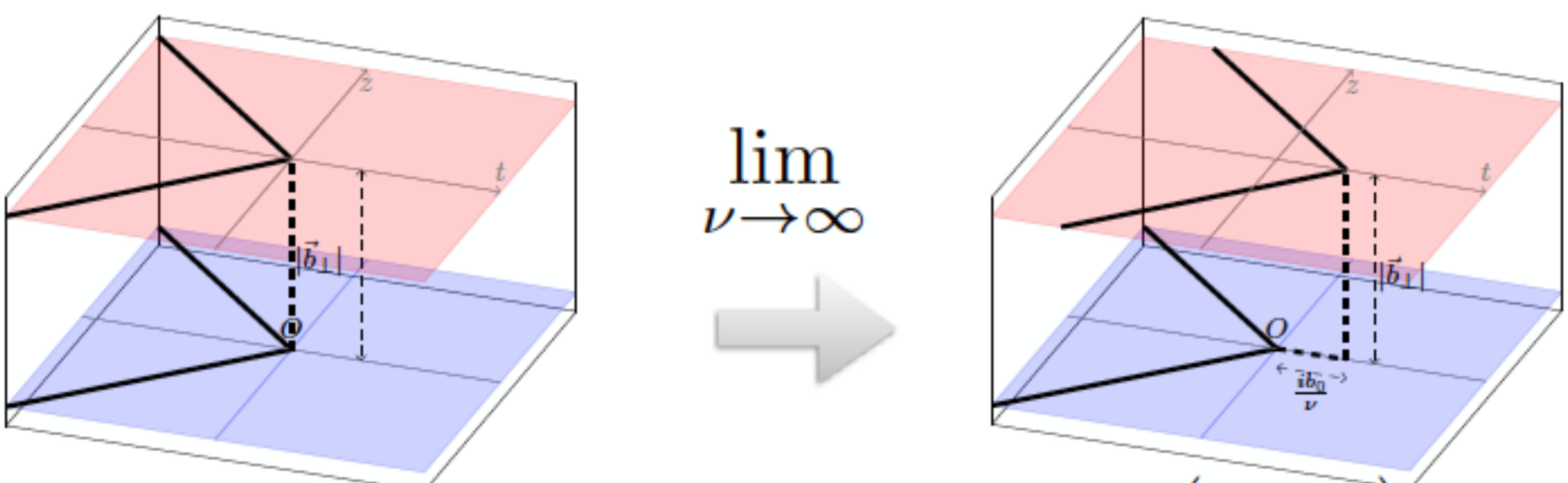
Chiu, Jain, Neill, Rothstein (2011, 2012)

New rapidity regulator and 3-loop anomalous dimension

$$\frac{d\sigma}{dq_T^2 dy} = \sigma_0 \pi (2\pi)^2 H(Q^2,\mu) \int db\, b J_0(bq_T) \widetilde{S}(b,\mu,\nu) \widetilde{f}_1^{\perp}(b,Q,x_1,\mu,\nu) \widetilde{f}_2^{\perp}(b,Q,x_2,\mu,\nu)$$

$$\widetilde{f}(\vec{b}) \equiv \int \frac{d^2 q_T}{(2\pi)^2} e^{i\vec{b}\cdot\vec{q_T}} f(\vec{q_T}) \qquad \widetilde{f}(\vec{b}) \equiv \frac{1}{2\pi} \widetilde{f}(b)\,, \quad b \equiv |\vec{b}|$$

- Computation of beam or soft functions requires regulation of rapidity divergences: $\displaystyle\int_0^\infty \frac{dk^+}{k^+}$
- Regulator: shift separation of soft Wilson lines defining soft function in Euclidean time



Li, Neill, Zhu (2016)

✦ In momentum space: $\displaystyle\prod_i \int \frac{d^{d-1}k_i}{2k_i^0(2\pi)^3} \Rightarrow \left(\prod_i \int \frac{d^{d-1}k_i}{2k_i^0(2\pi)^3}\right) \exp\left(-\sum_j \frac{b^0 k_j^0}{\nu}\right)$

N^{3}LL resummed p_T spectrum

- 3-loop soft function diagrams:

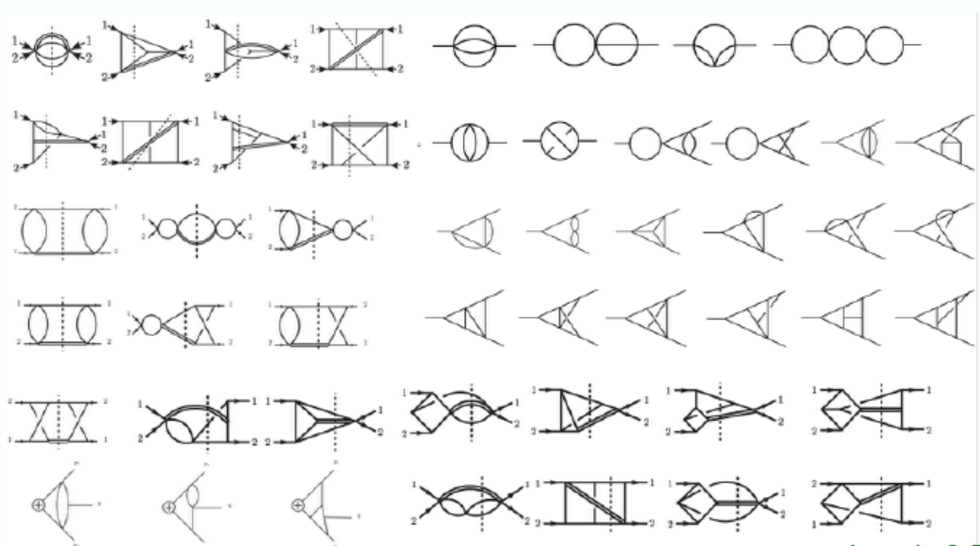


Image by C. Duhr

✦ All three-loop integrals for threshold soft function known

Anastasiou et al, 2015; Y. Li et al, 2014

- 3-loop rapidity anomalous dimension:

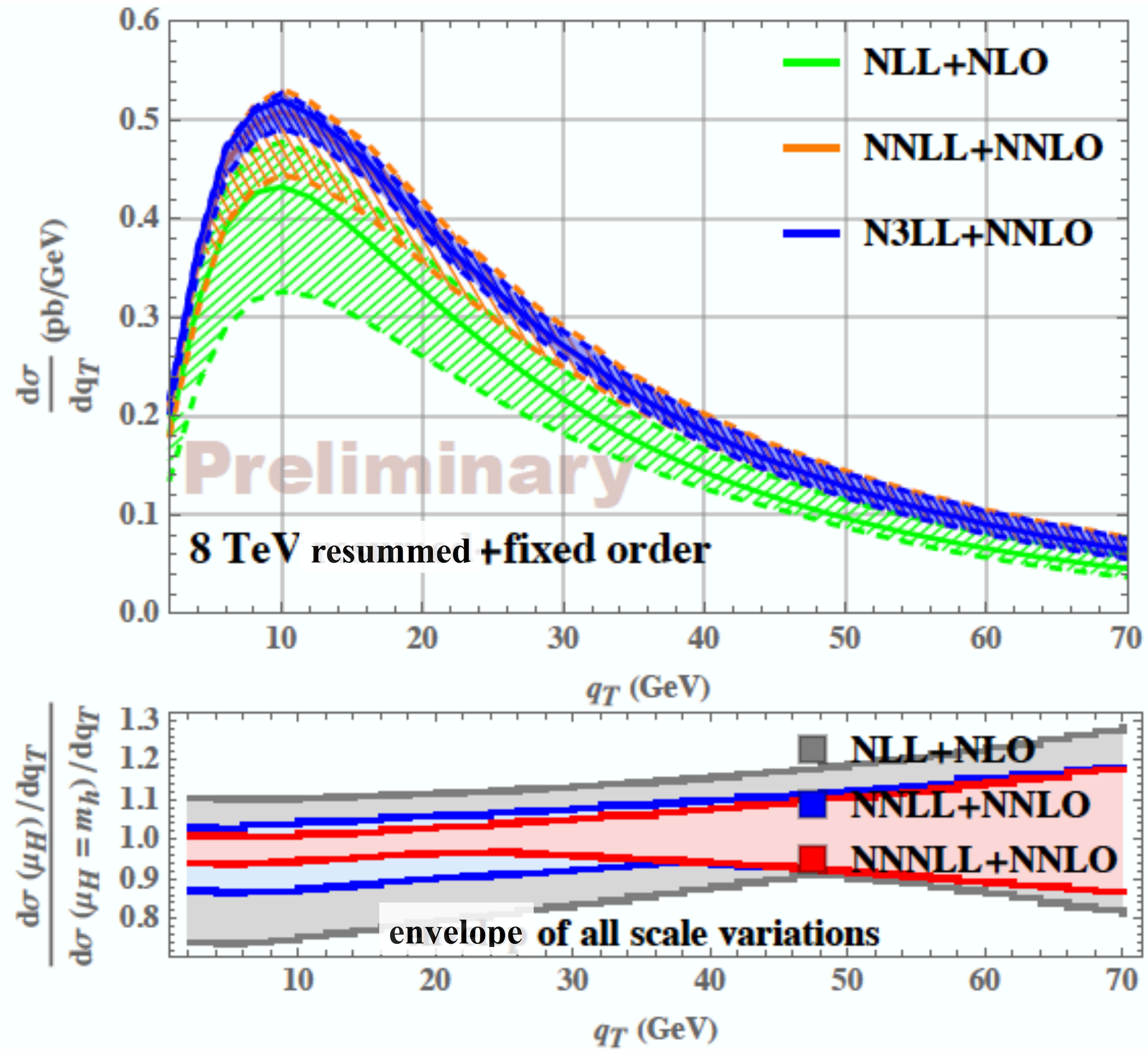
$$\gamma_0^R = 0$$

$$\gamma_1^R = C_a C_A \left(28\zeta_3 - \frac{808}{27}\right) + \frac{112 C_a n_f}{27}$$

$$\gamma_2^R = C_a C_A^2 \left(-\frac{176}{3}\zeta_3\zeta_2 + \frac{6392\zeta_2}{81} + \frac{12328\zeta_3}{27} + 44\zeta_4 - 192\zeta_5 - \frac{297029}{729}\right) + C_a C_A n_f \left(-\frac{824\zeta_2}{81} - \frac{904\zeta_3}{27} + 8\zeta_4 + \frac{62626}{729}\right) + c\beta_0 + C_a n_f^2 \left(-\frac{32\zeta_3}{9} - \frac{1856}{729}\right) + C_a C_F N_f \left(-\frac{304\zeta_3}{9} - 16\zeta_4 + \frac{1711}{27}\right)$$

New three loop results!

- N^{3}LL resummed results:

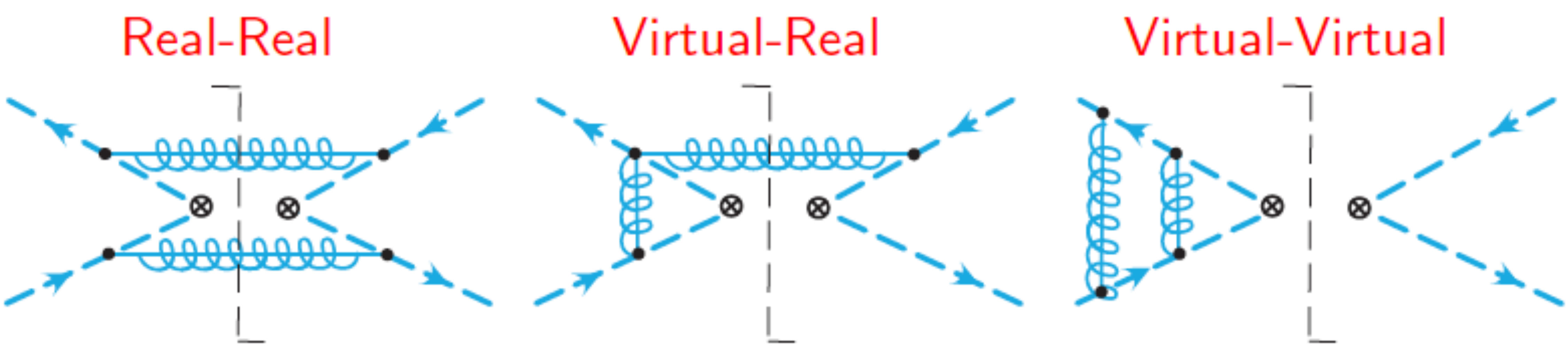


Li, Neill, Schulze, Stewart, Zhu
(SCET2016, Argonne *Advances in QCD* 2016)

NNLO Subtractions

- NNLO:

Moult (LoopFest 2017)



- Local Subtractions: Colorful NNLO, Sector Decomposition, Antenna Subtraction, ... [Del Duca, Duhr, Kardos, Somogyi, Trocsanyi, Tulipant] [Anastasiou, Melnikov, Petriello] [Gehrmann-De Ridder, Gehrmann, Glover et al.]
- Global Subtractions: q_T, N-jettiness, ... [Catani, Grazzini] [Boughezal, Focke, Petriello, Liu] [Gaunt, Stahlhofen,Tackmann, Walsh]

$$\sigma(X) = \int_0 d\mathcal{T} \frac{d\sigma(X)}{d\mathcal{T}} = \int_0^{\mathcal{T}^{\text{cut}}} d\mathcal{T} \frac{d\sigma(X)}{d\mathcal{T}} + \int_{\mathcal{T}^{\text{cut}}} d\mathcal{T} \frac{d\sigma(X)}{d\mathcal{T}}$$

N-Jettiness Subtractions

- Exploit factorization and 2-loop computations of ingredients for small τ_N

$$\sigma(\mathcal{T}^{\text{cut}}) = \int_0^{\mathcal{T}^{\text{cut}}} d\mathcal{T} \frac{d\sigma(X)}{d\mathcal{T}}$$

Compute using factorization in soft/collinear limits:

$$\frac{d\sigma}{d\mathcal{T}} = HB_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1}$$

$$\int_{\mathcal{T}^{\text{cut}}} d\mathcal{T} \frac{d\sigma(X)}{d\mathcal{T}}$$

Additional jet resolved. Use NLO subtractions.

Boughezal, Liu, Focke, Petriello (2015)
Gaunt, Stahlhofen, Tackmann, Walsh (2015)

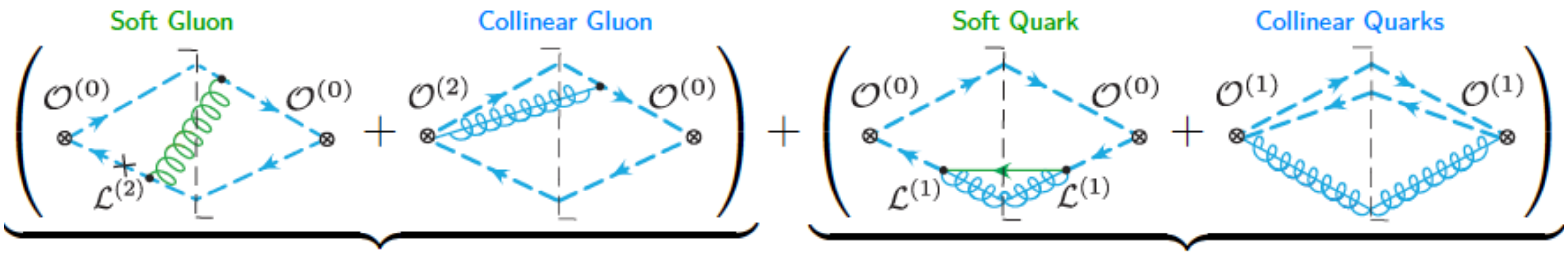
- High precision, numerical stability requires power corrections:

$$\sigma(\tau_{\text{cut}}) = \int_0^{\tau_{\text{cut}}} d\tau \frac{d\sigma}{d\tau} = \underbrace{\sum_{n=0}^{n=\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} \tilde{c}_{nm}^{(0)} \log^m(\tau_{\text{cut}})}_{\text{Leading Power}} + \underbrace{\tau_{\text{cut}} \sum_{n=0}^{n=\infty} \left(\frac{\alpha_s}{\pi}\right)^n \sum_{n=0}^{2n-1} \tilde{c}_{nm}^{(2)} \log^m(\tau_{\text{cut}}) + \cdots}_{\text{Power Corrections}}$$

Subleading Power Corrections

- SCET well formulated to compute power corrections:

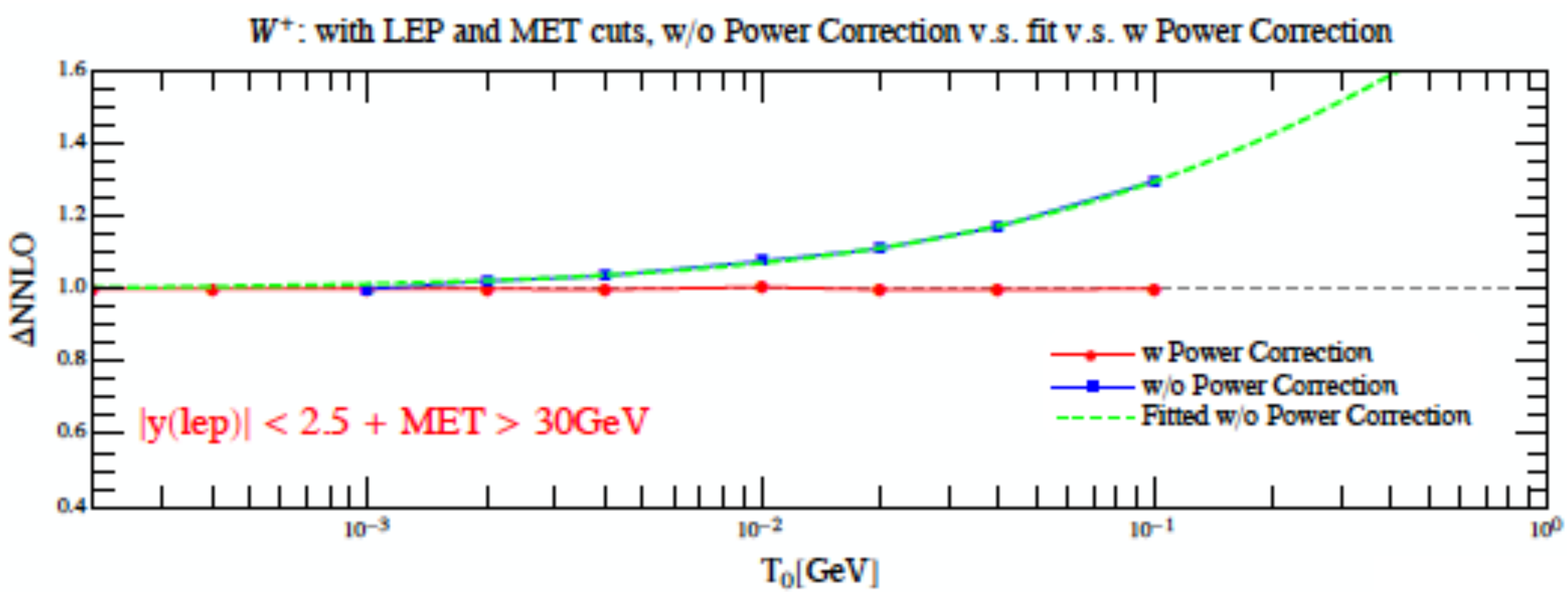
Moult, Rothen, Stewart, Tackmann, Zhu (2016)



$$-8C_F\left[\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{Q^2\tau^2}\right)\right]+8C_F\left[\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{Q^2\tau}\right)\right]$$
$$=8C_F\log(\tau)$$

$$4C_F\left[\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{Q^2\tau^2}\right)\right]-4C_F\left[\frac{1}{\epsilon}+\log\left(\frac{\mu^2}{Q^2\tau}\right)\right]$$
$$=-4C_F\log(\tau)$$

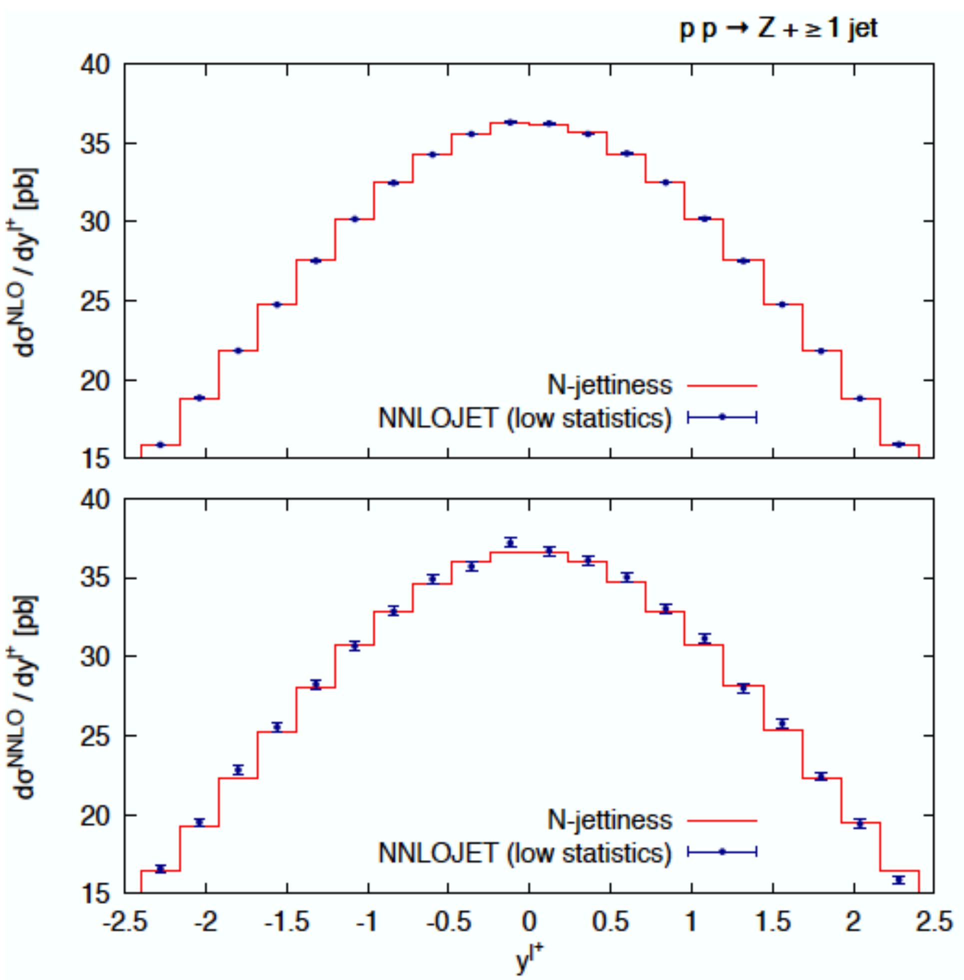
- Also computable in fixed-order QCD, dramatic improvement in τ_{cut} independence:



Boughezal, Liu, Petriello (2016)

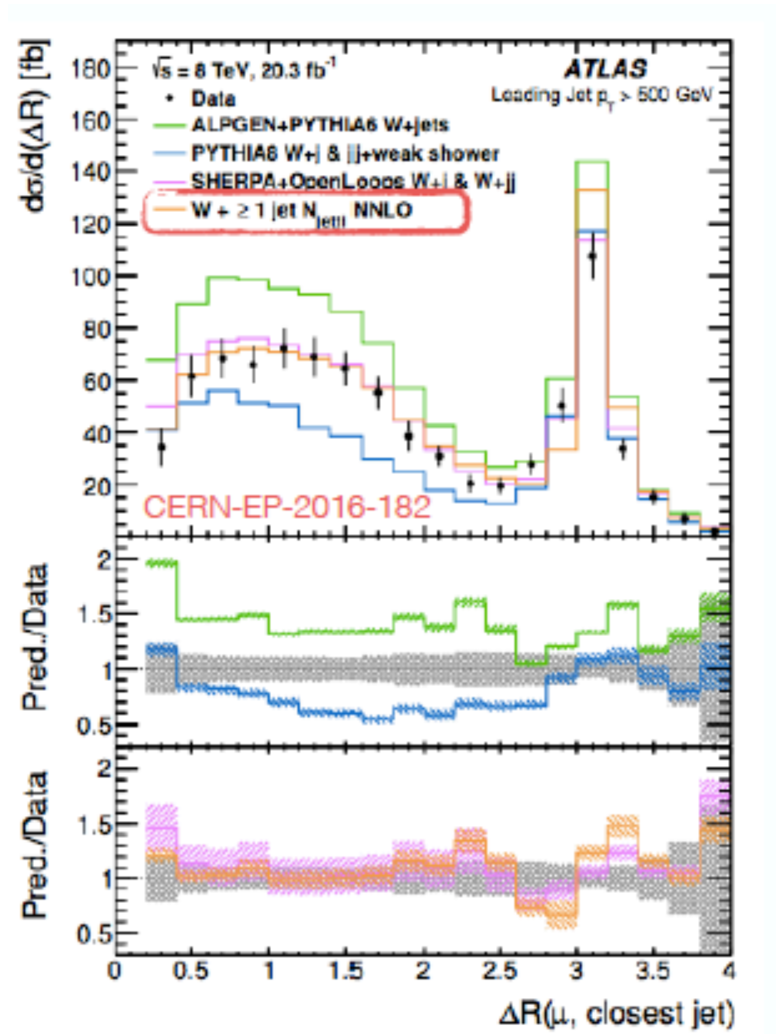
NNLO Results *V*+jet

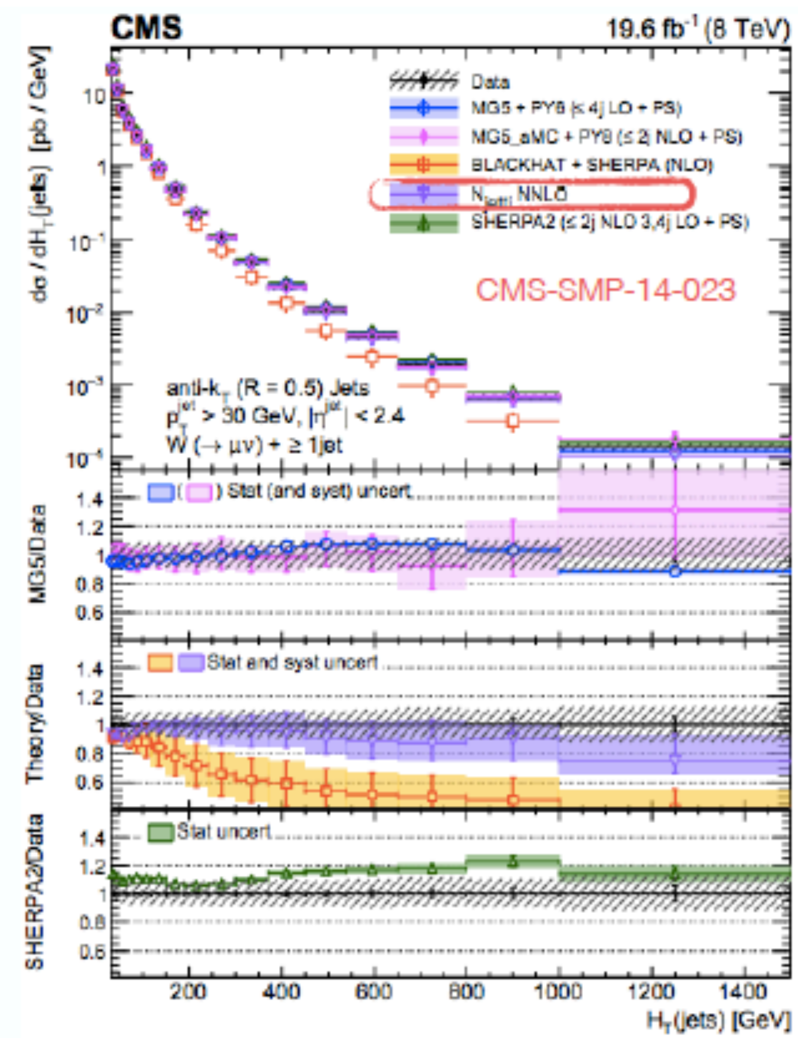
- *N*-jettiness subtraction method *vs.* antenna subtraction:



Boughezal, Campbell, Ellis, Focke, Giele, XL, Petriello
Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan

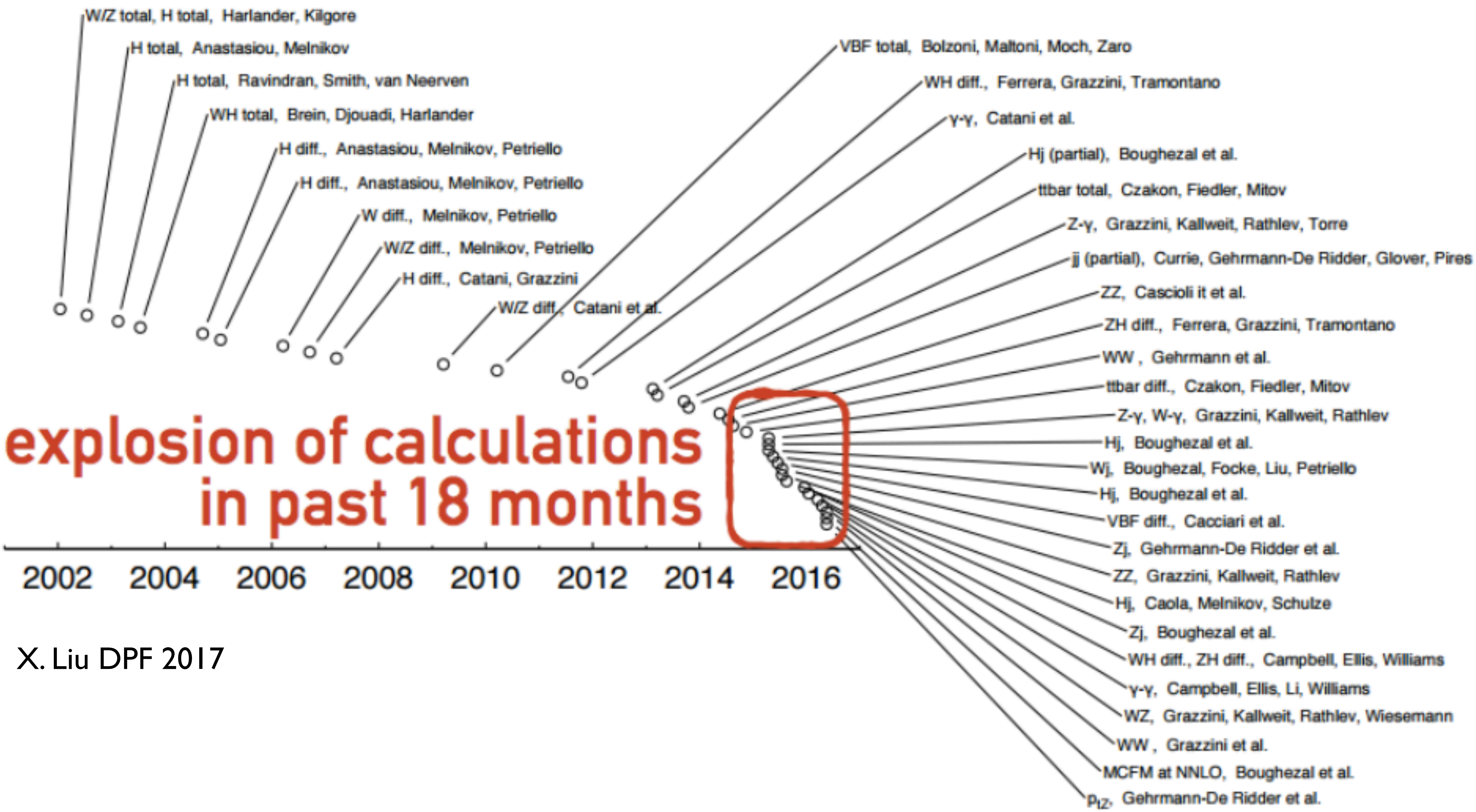
- *vs.* data:





Boughezal, Liu, Petriello (2016)

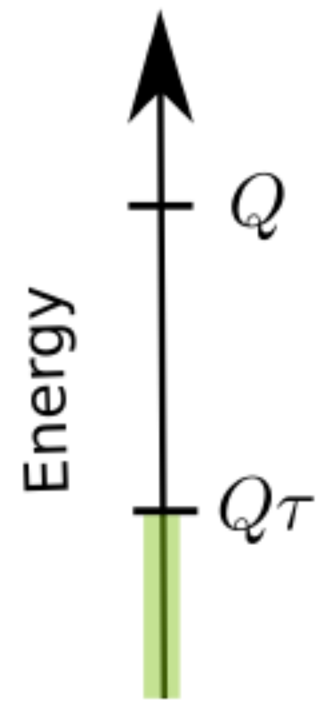
NNLO Revolution

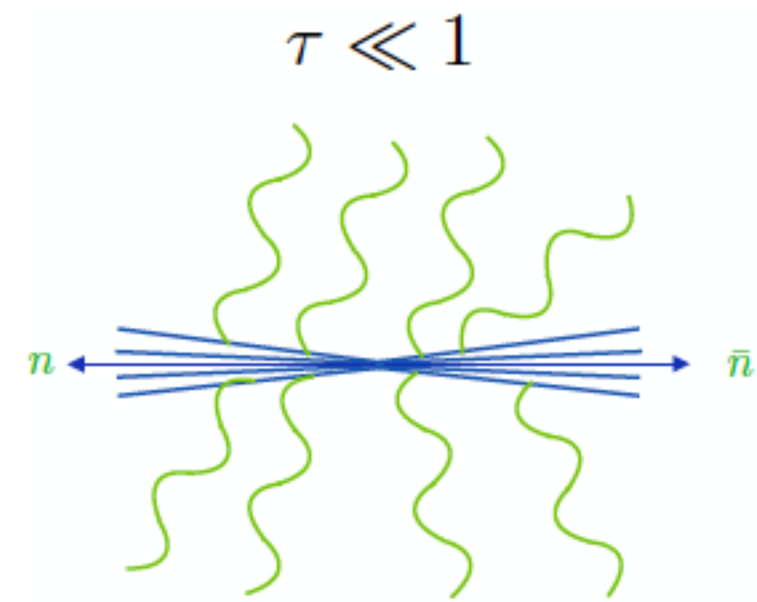


X. Liu DPF 2017

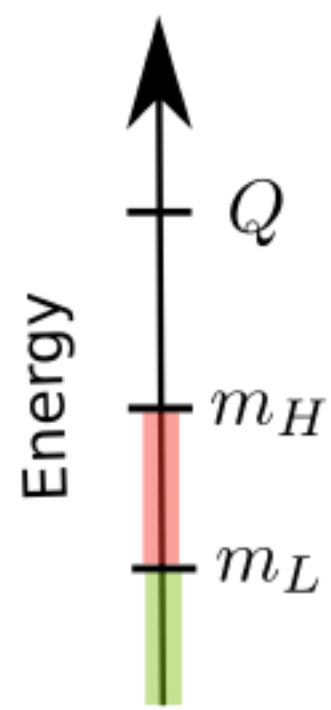
Non-global logs

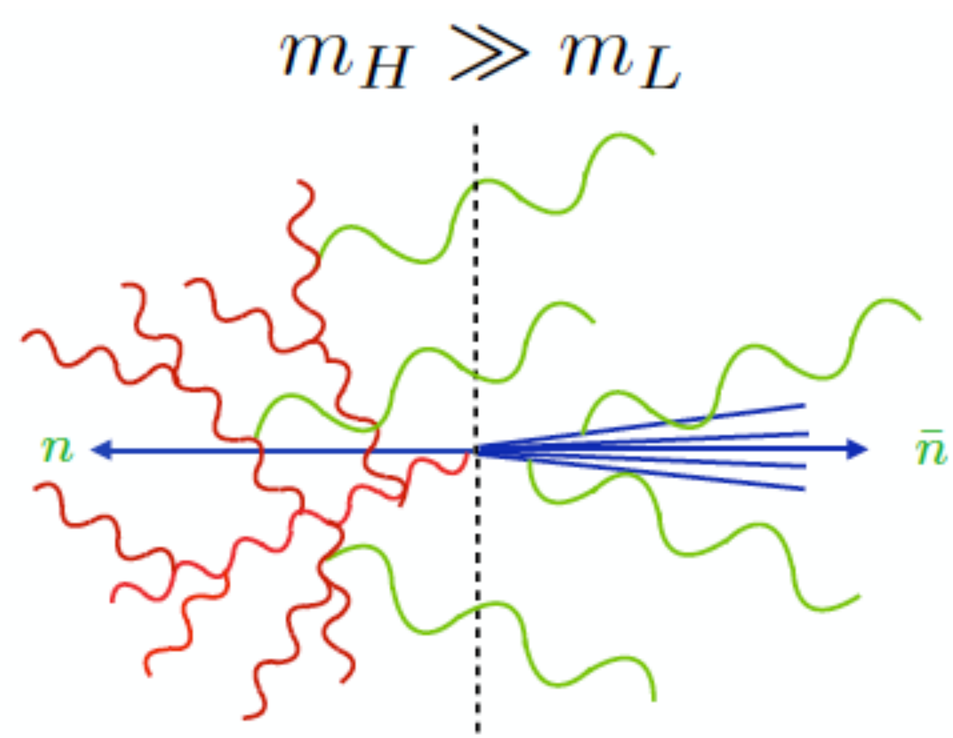
- **Global observable:**
  (thrust, N-jettiness)





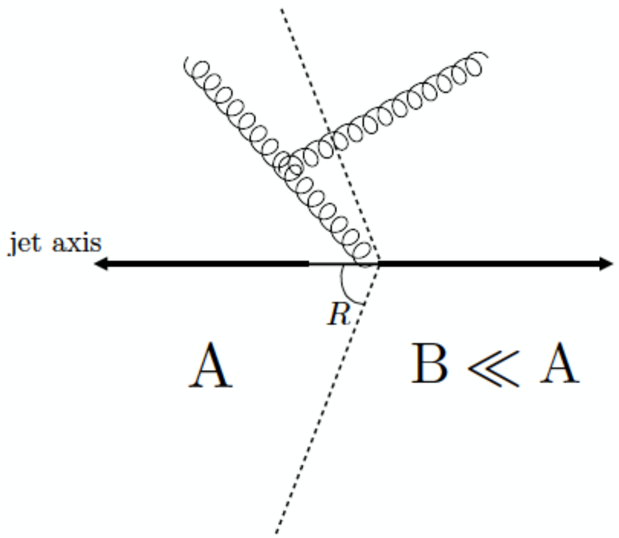
- **Non-global observable:**
  (double hemisphere mass,
  jet vetoes)





D. Neill SCET 2017

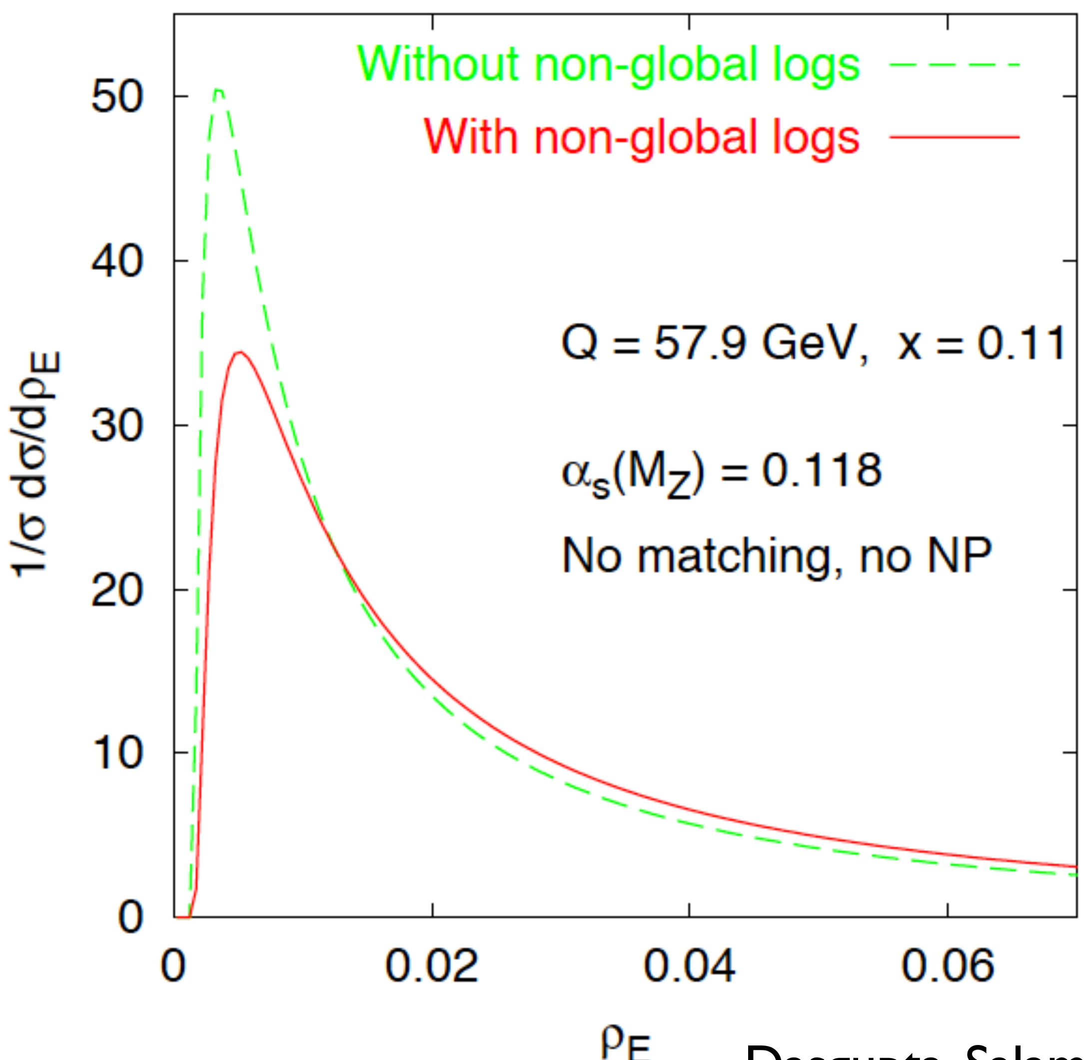
Non-global logs



Dasgupta, Salam (2001)

- Start to spoil "global" resummation at 2 loops:

$$\sigma(m_H/m_L) = \sigma_{\rm gl}(m_H/m_L)\left[1 + \frac{\alpha_s^2}{(2\pi)^2} C_F C_A \frac{\pi^2}{3} \ln^2 \frac{m_H}{m_L} + \cdots\right]$$



Dasgupta, Salam (2002)

Conjecture / fit to Monte Carlo resummation (large N_c):

$$\mathcal{S}_{\rm ng} = \exp\left[-C_F C_A \frac{\pi^2}{3}\left(\frac{1+(at)^2}{1+(bt)^c}\right) t^2\right]$$

$$t = \frac{1}{4\pi\beta_0} \ln \frac{1}{1 - 2\beta_0 \alpha_s L} \qquad L = \ln \frac{m_H}{m_L}$$

$$a = 0.85 C_A\,, b = 0.86 C_A\,, c = 1.33$$

Fixed-order computations

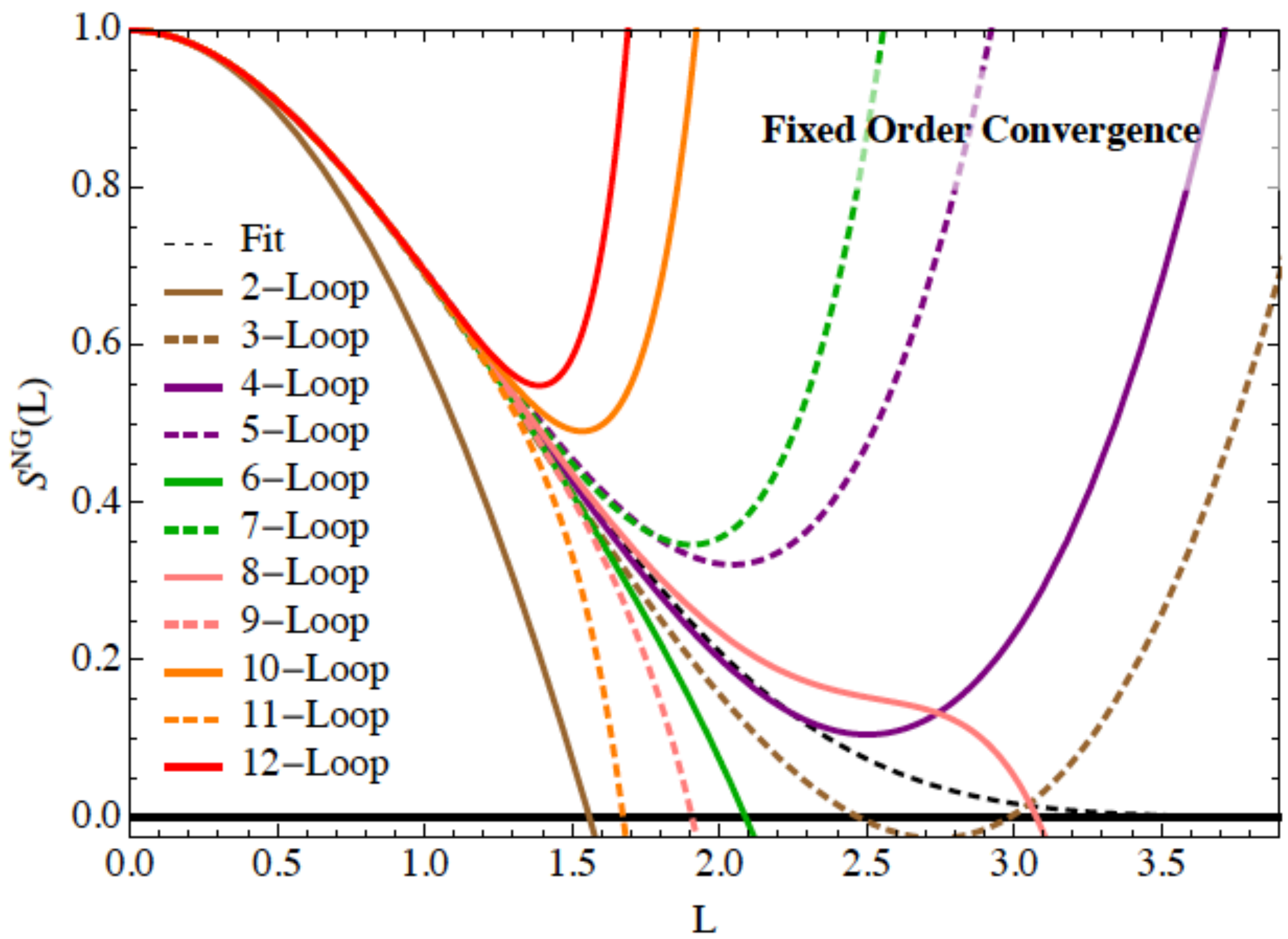
- Soft functions for non-global observables in SCET, two-loop computations, and subleading (single) NGLs

Kelley, Schabinger, Schwartz, Zhu (2011); Hornig, CL, Stewart, Walsh, Zuberi (2011)

- 5 loops: $$\boxed{g_{n\bar{n}}(L) = 1 - \frac{\pi^2}{24}L^2 + \frac{\zeta(3)}{12}L^3 + \frac{\pi^4}{34560}L^4 + \left(-\frac{\pi^2\zeta(3)}{360} + \frac{17\zeta(5)}{480}\right)L^5 + \ldots}$$

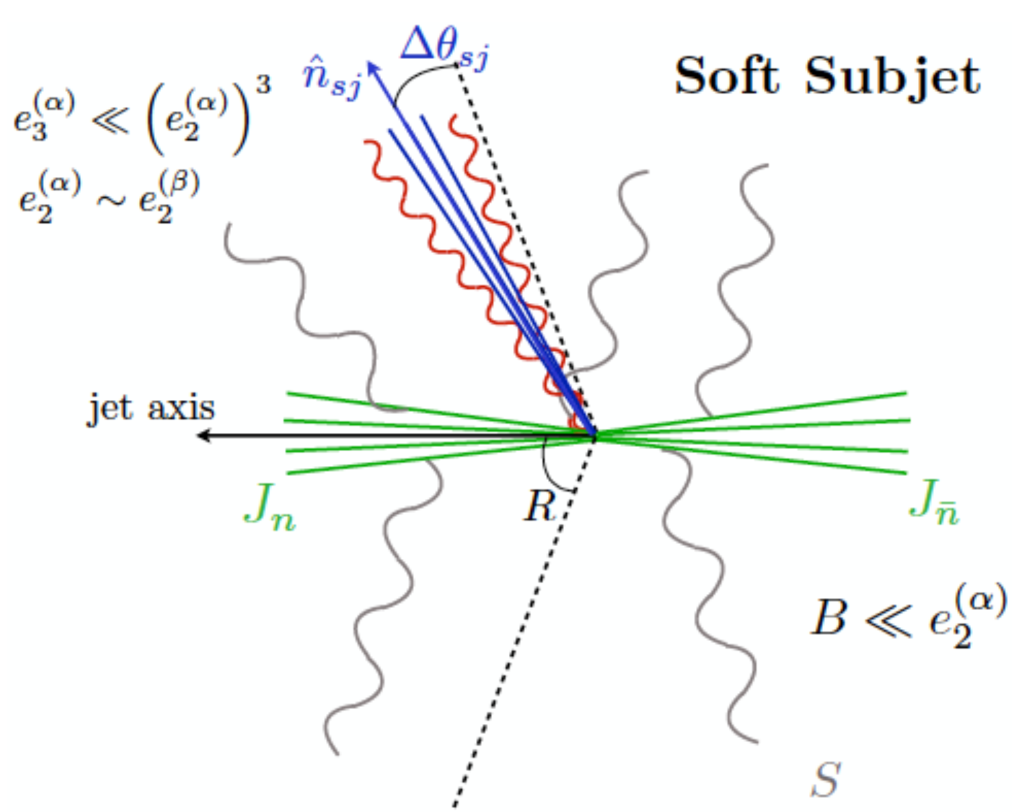
Schwartz, Zhu (2014)

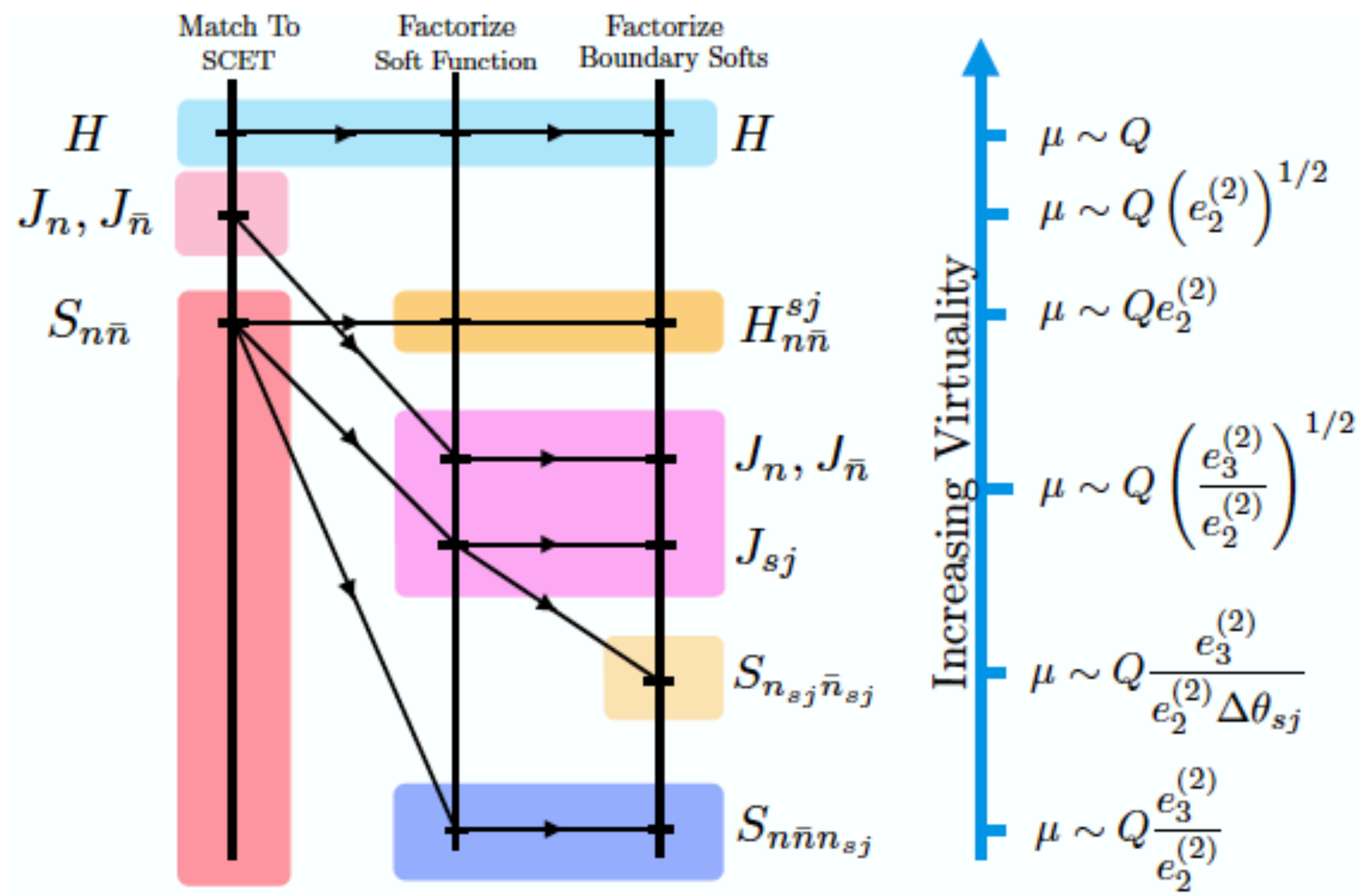
- 12 loops!



Caron-Huot

Factorization and Resummation of NGLs





Larkoski, Moult, Neill (2015)

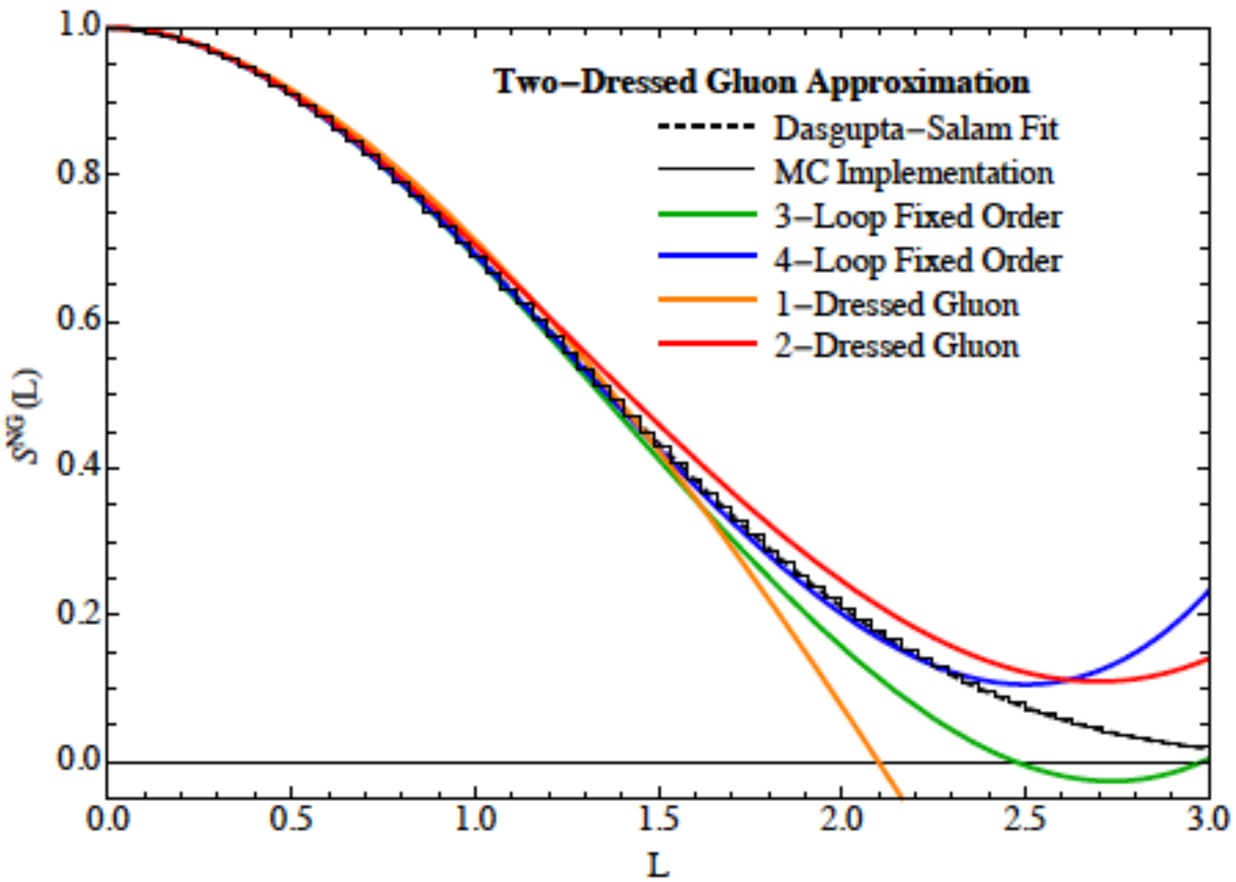
$e^+e^- \to 2_j + 1_{sj}$:

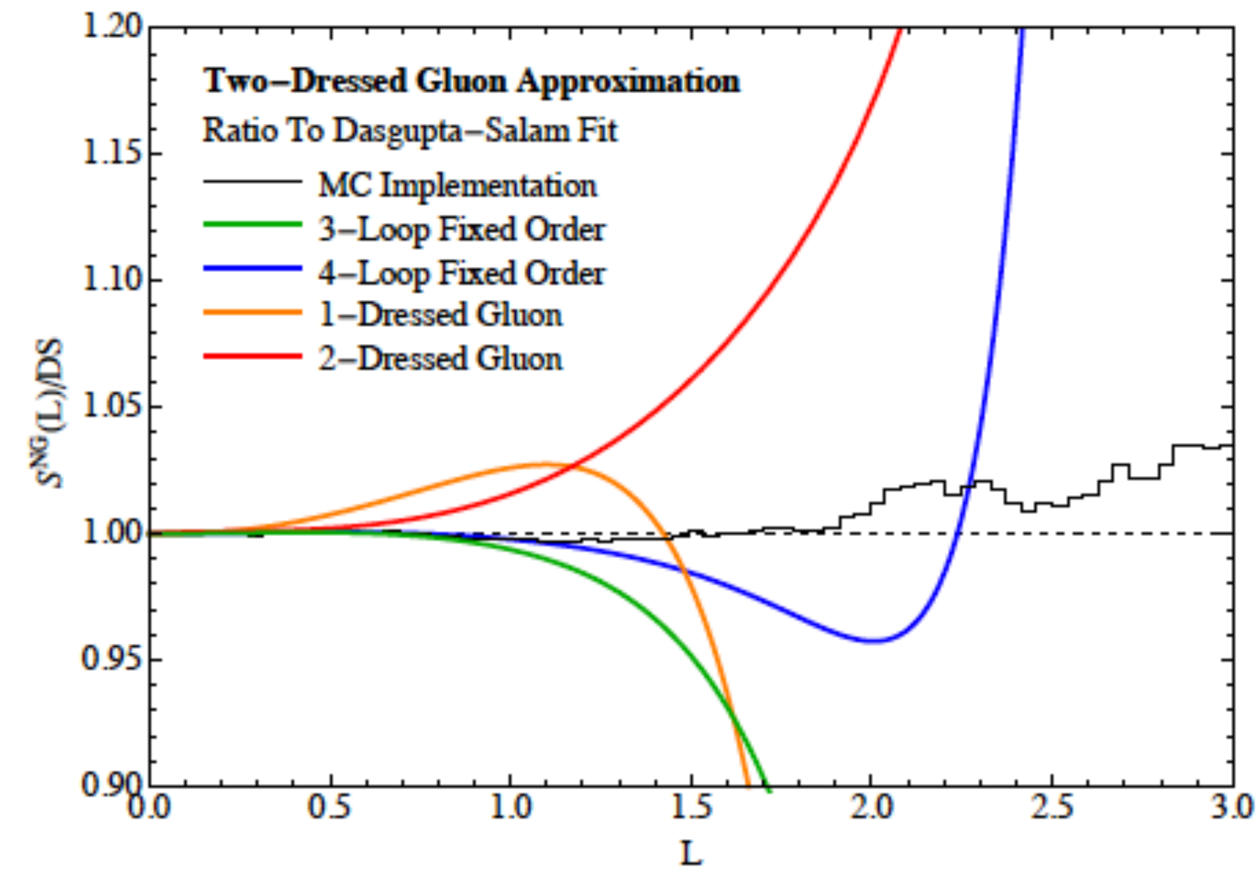
$$\frac{d\sigma}{de_2^{(\alpha)}de_2^{(\beta)}de_3^{(\beta)}dB} = H_{n\bar{n}}H_{n\bar{n}}^{sj}(e_2^{(\alpha)}, e_2^{(\beta)})J_{n_{sj}}(e_3^{(\beta)}) \otimes S_{n_{sj}\bar{n}_{sj}}(e_3^{(\beta)})$$
$$\otimes S_{n\bar{n}n_{sj}}(e_3^{(\beta)}; B) \otimes J_n(e_3^{(\beta)}) \otimes J_{\bar{n}}(B)$$

RG evolve, integrate over to obtain original non-global distribution, now resummed!

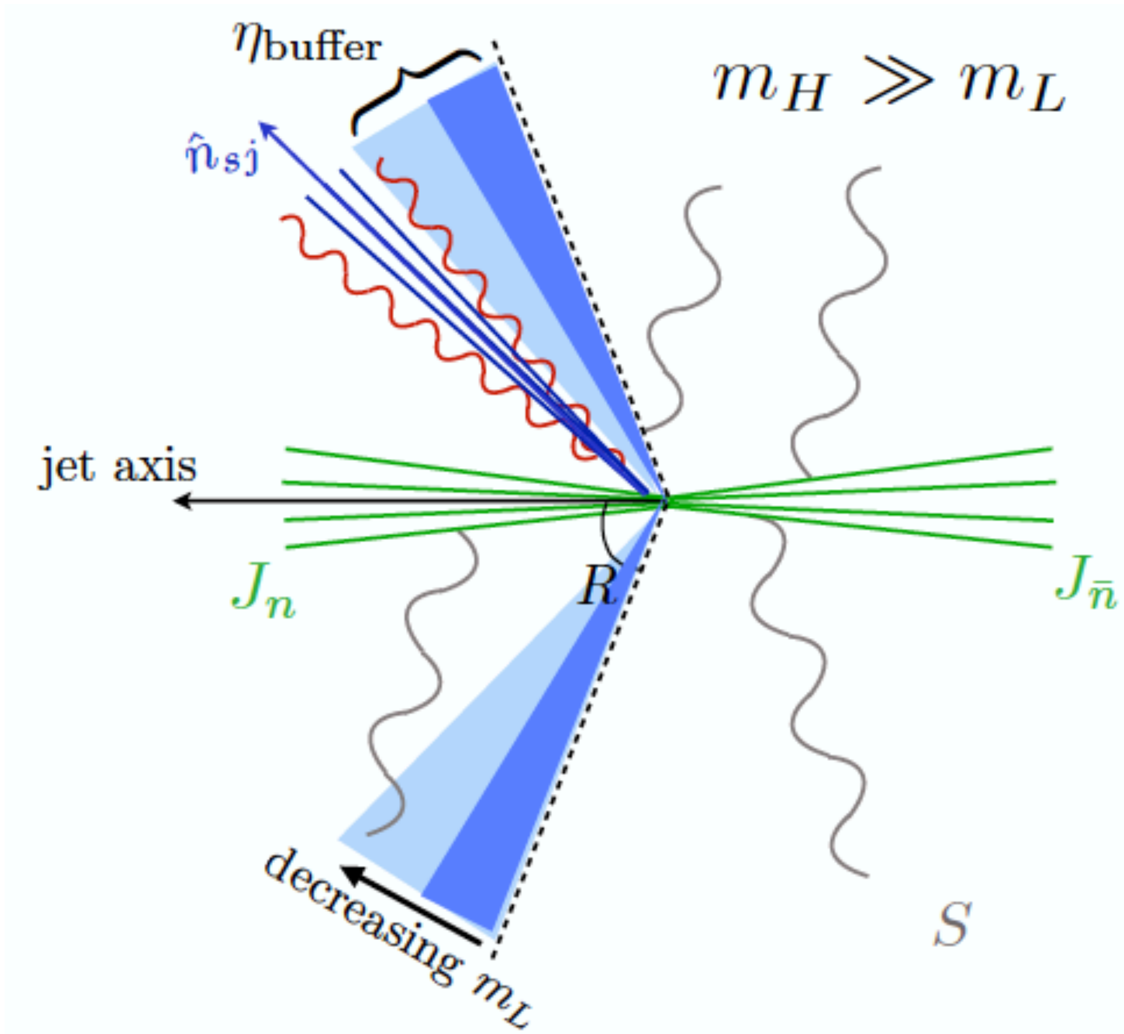
Resummed computations

Larkoski, Moult, Neill (2015)





Singularities and Buffers



- Boundary Soft RG implies:
  $$U_{abj} \propto \left(1 - \frac{\tan^2 \frac{\theta_{sj}}{2}}{\tan^2 \frac{R}{2}}\right)^L$$
- Cross-section for production of a jet at the boundary vanishes!

- Buffer region and singularities in *L* reproduced by resummed calculation with jets, but not fixed-order calculation with partons

- Take fixed order series and apply conformal mapping obeying proper singularities in *L* and buffer region:

- $g_{ab}(L) \to g_{ab}(u)$.
- $g_{ab}(u) = 1 + b_1 u + b_2 u^2 + \ldots.$

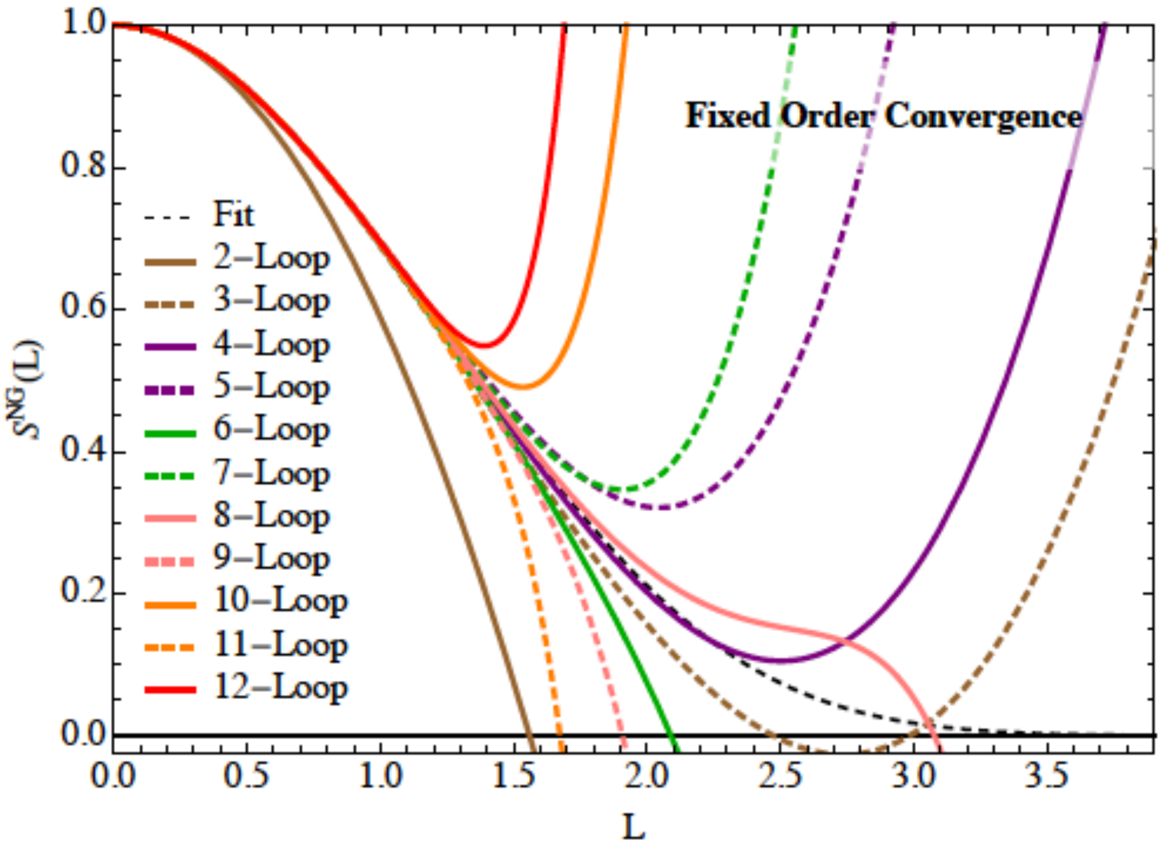
Example u's:

$$u(L) = \begin{cases} \ln(1+L), & \text{log mapping} \\ \frac{\sqrt{1+L}-1}{\sqrt{1+L}+1}, & \text{disc mapping} \end{cases}$$

Determine b's by matching taylor series at $L = 0$.

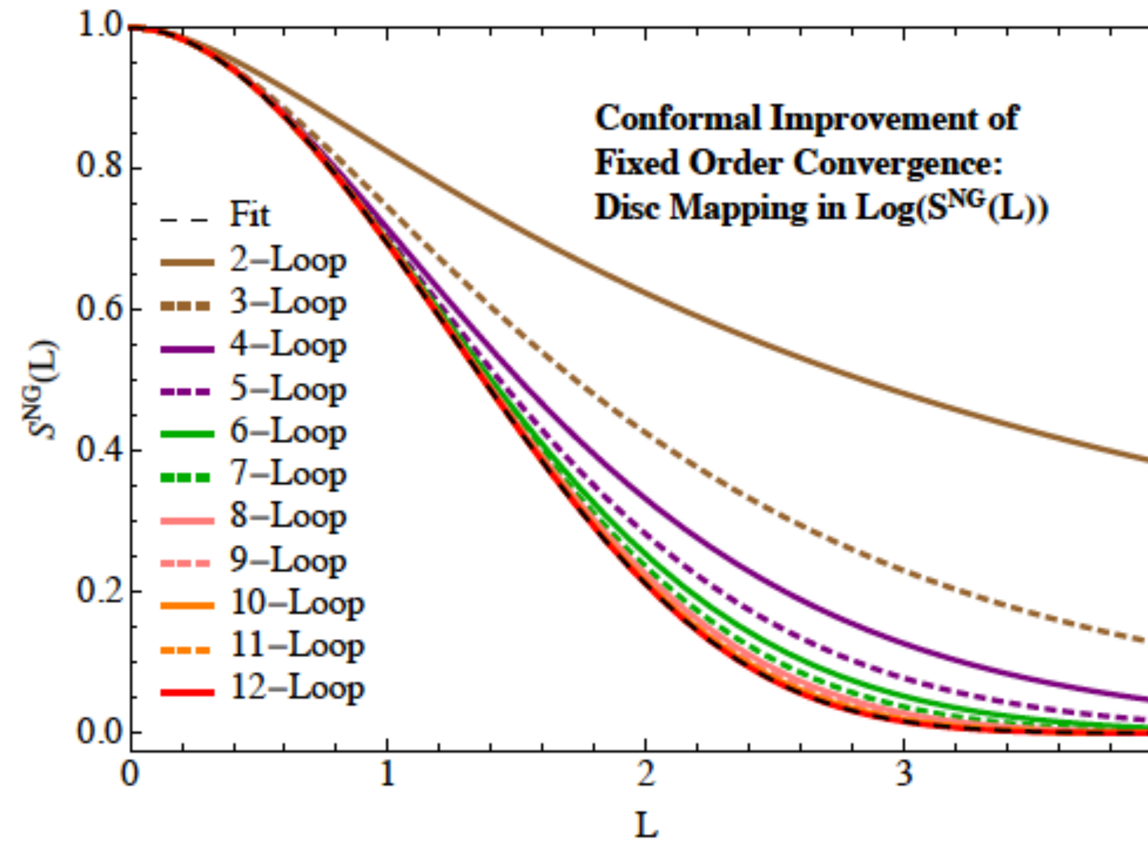
Larkoski, Moult, Neill (2016)

Conformal improvement of fixed-order NGLs



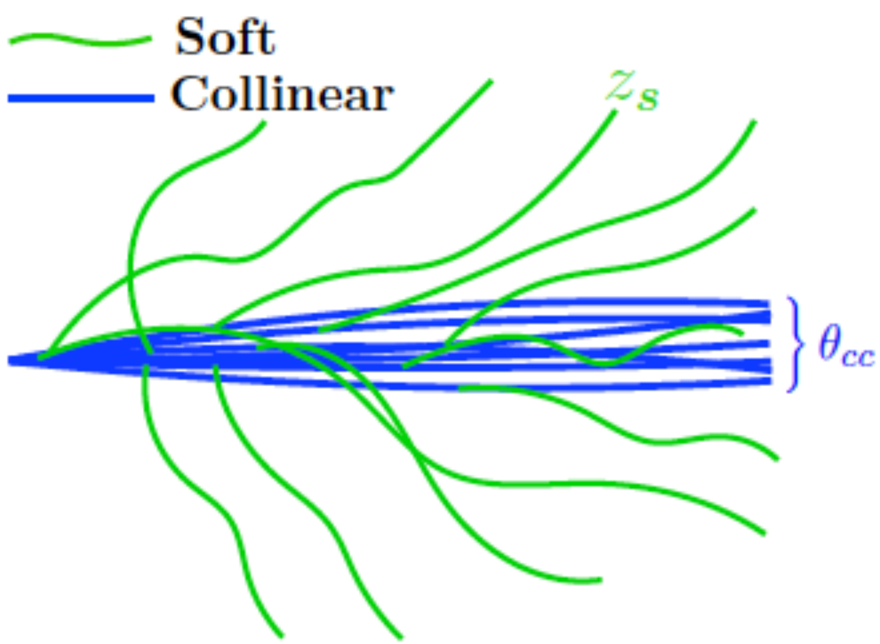
Caron-Huot

→

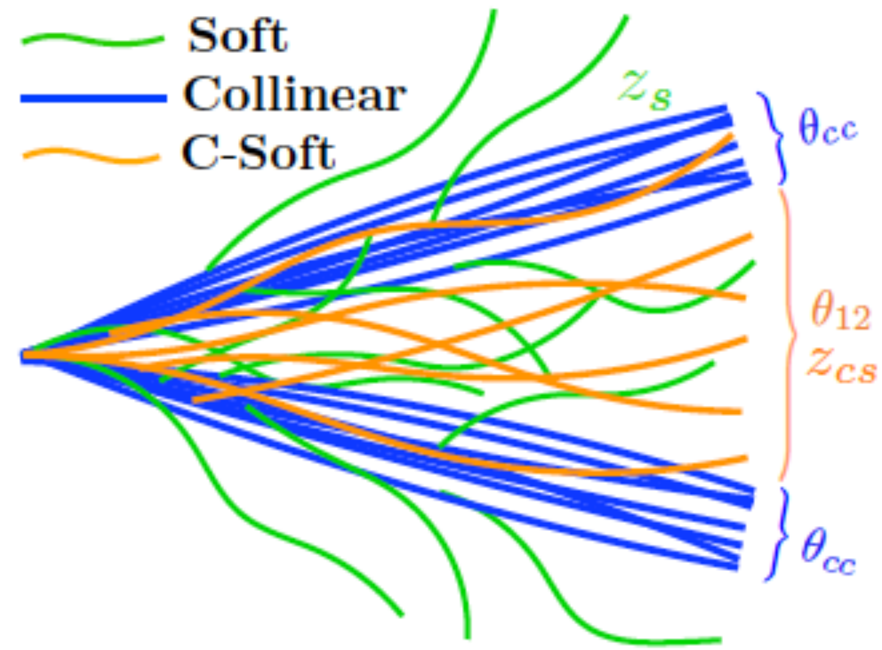


Larkoski, Moult, Neill (2016)

Jet Substructure

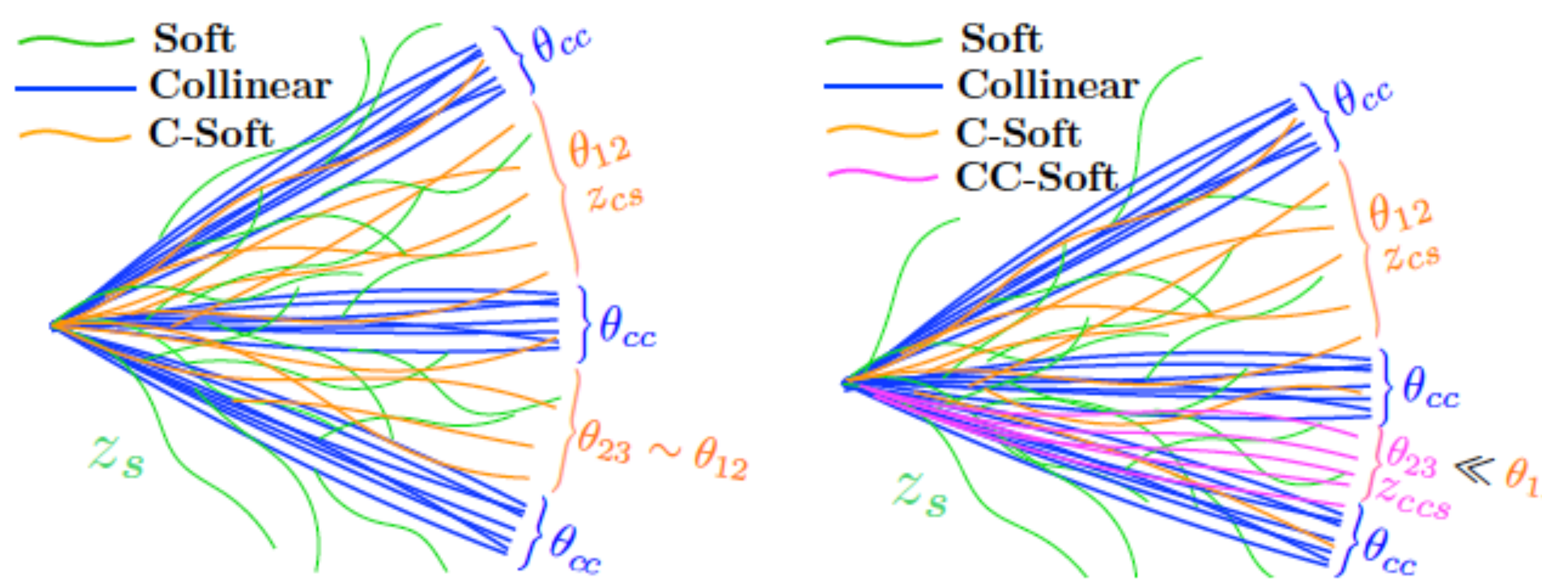


1-prong



2-prong

3-prong:



(a) (b)

Energy Correlators

cf. Basham, Brown, Ellis, Love (1978); Larkoski, Salam, Thaler (2013)

$$_v e_n^{(\beta)} = \sum_{1 \le i_1 < i_2 < \cdots < i_n \le n_J} z_{i_1} z_{i_2} \dots z_{i_n} \prod_{m=1}^{v} \overset{(m)}{\min_{s<t\in\{i_1,i_2,\dots,i_n\}}} \left\{ \theta_{st}^{\beta} \right\}$$

Moult, Necib, Thaler (2016)

good discriminants:

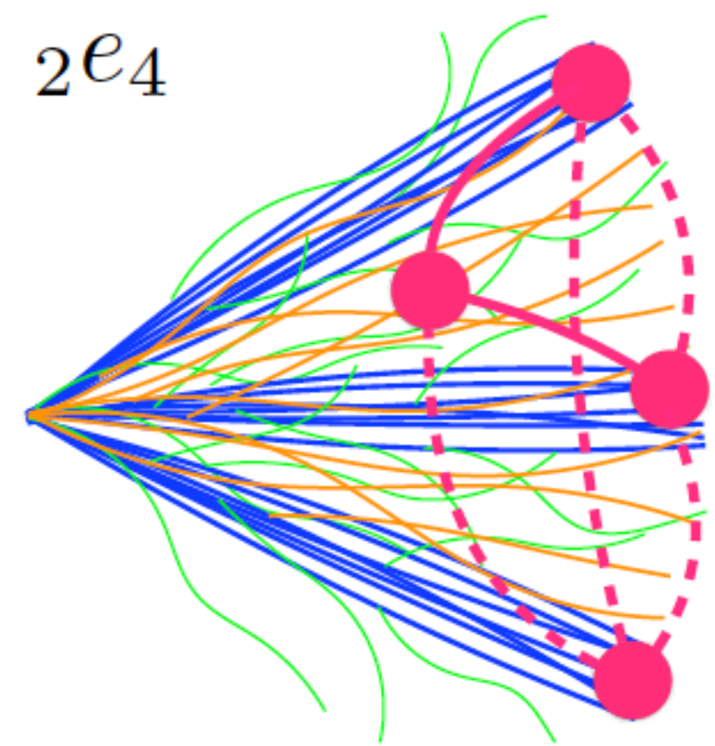
definite power counting, amenable to factorization and precision calculation

3-prong (top): $N_3^{(\beta)} = \dfrac{_2e_4^{(\beta)}}{(_1e_3^{(\beta)})^2}$

2-prong: $M_2^{(\beta)} = \dfrac{_1e_3^{(\beta)}}{_1e_2^{(\beta)}}$

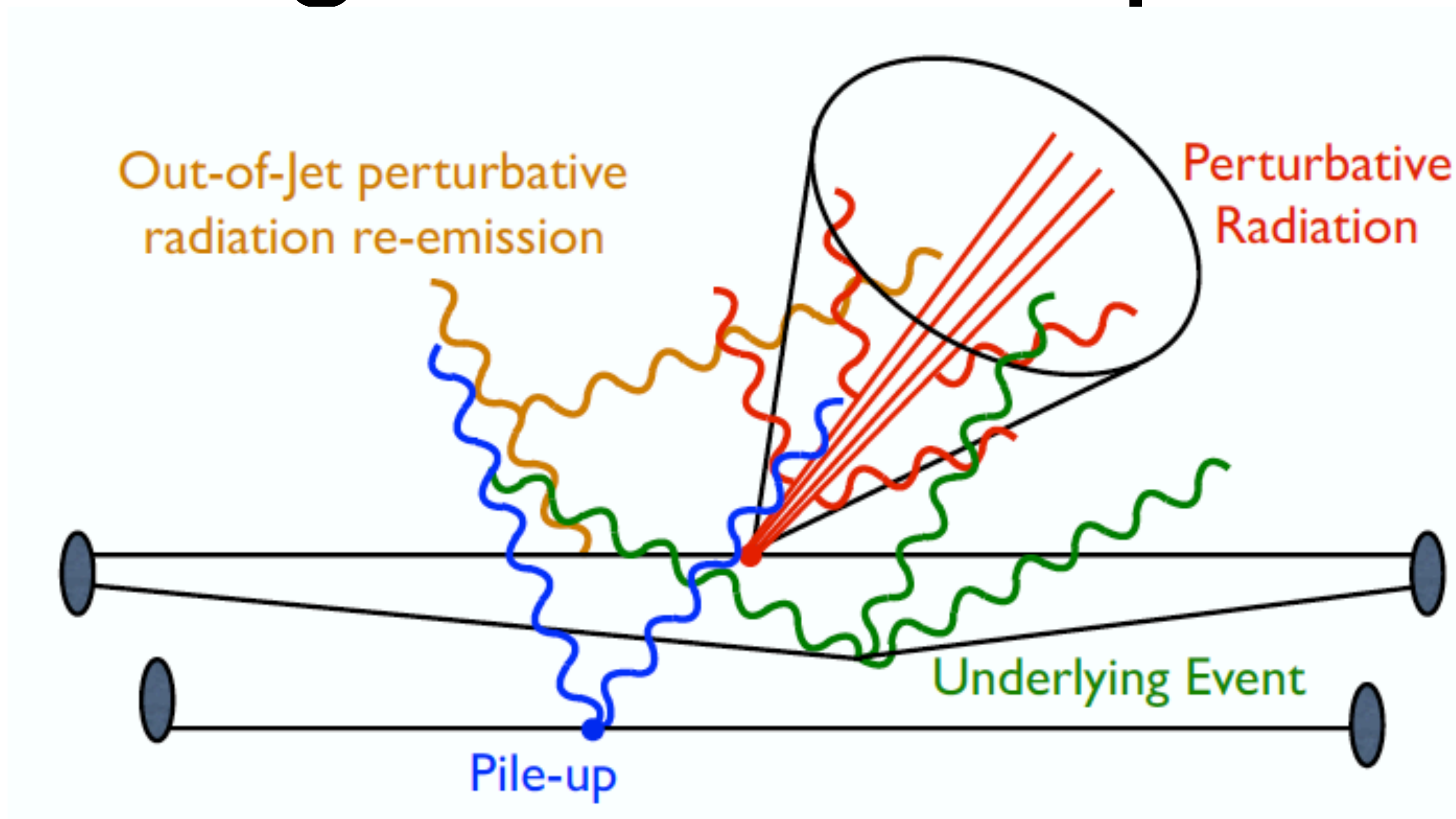
1-prong (q vs g): $U_i^{(\beta)} = {_1e_{i+1}^{(\beta)}}$

$_2e_4$



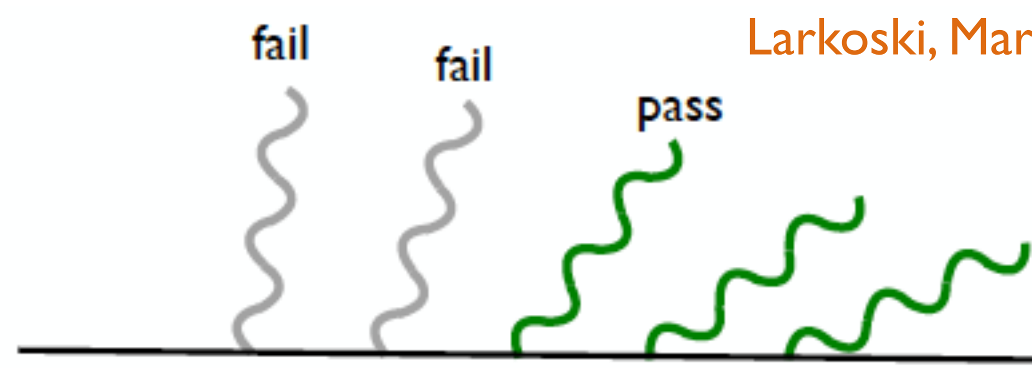
Grooming and Soft Drop

contamination:



Larkoski, Marzani, Soyez, Thaler (2014)

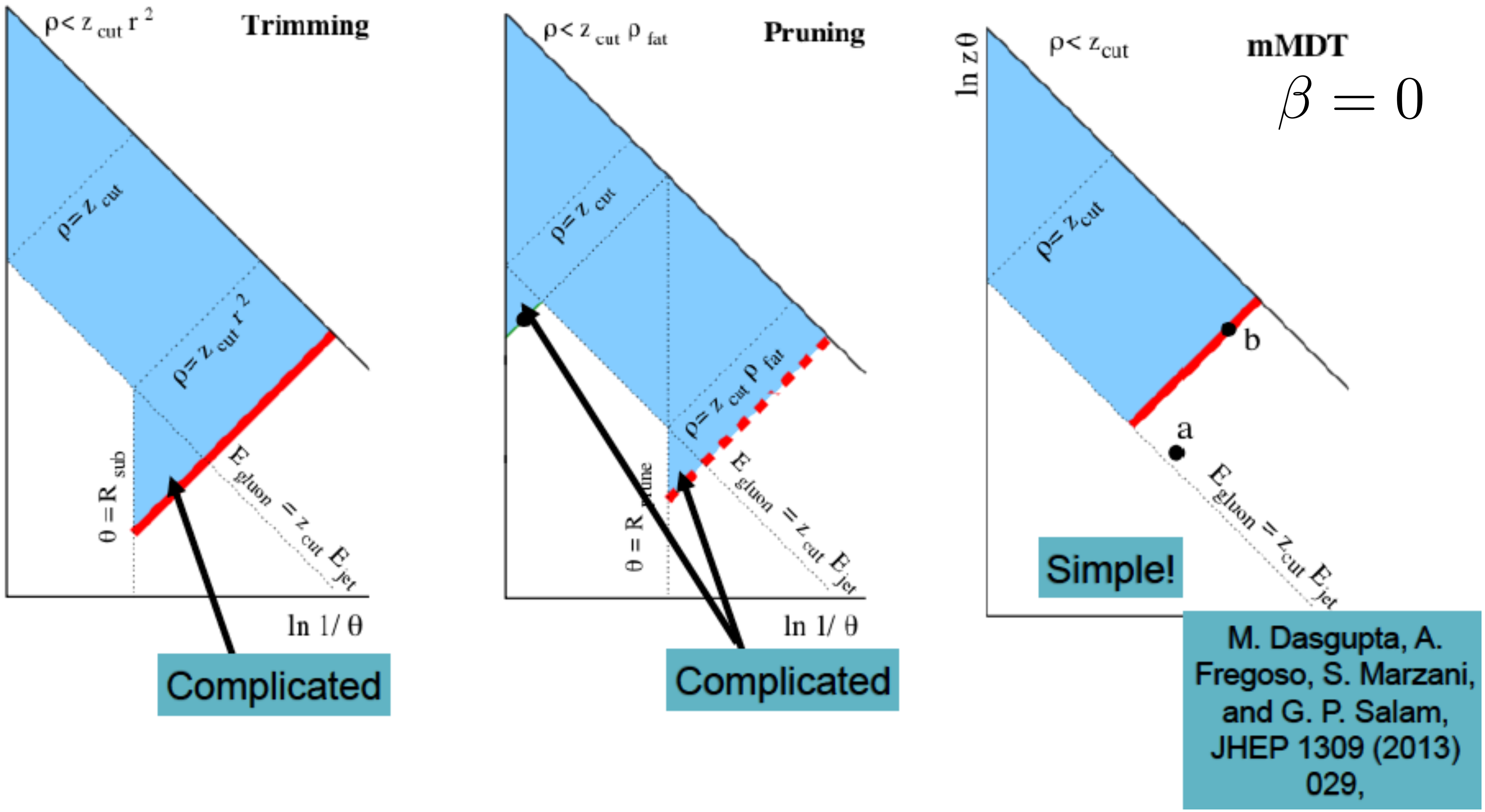
grooming:



Soft Drop: $$\frac{\min[p_{Ti}, p_{Tj}]}{p_{Ti} + p_{Tj}} > z_{\text{cut}} \left(\frac{R_{ij}}{R}\right)^{\beta}$$

Soft Drop

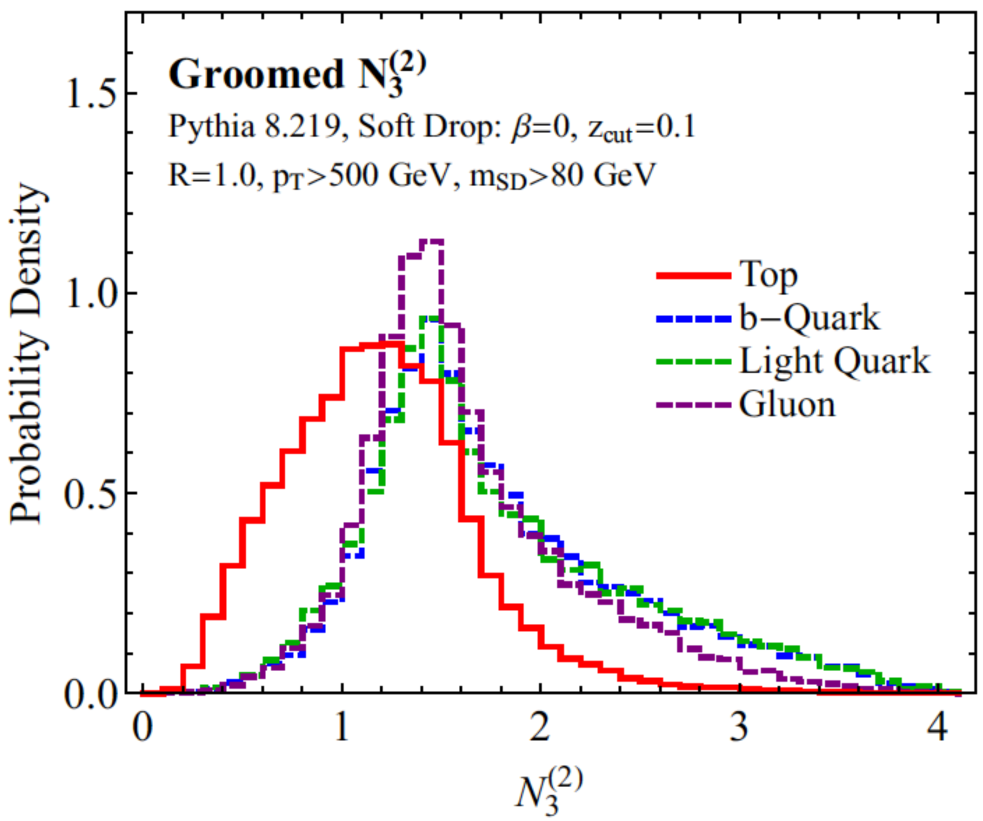
- Simplifies theoretical calculations:

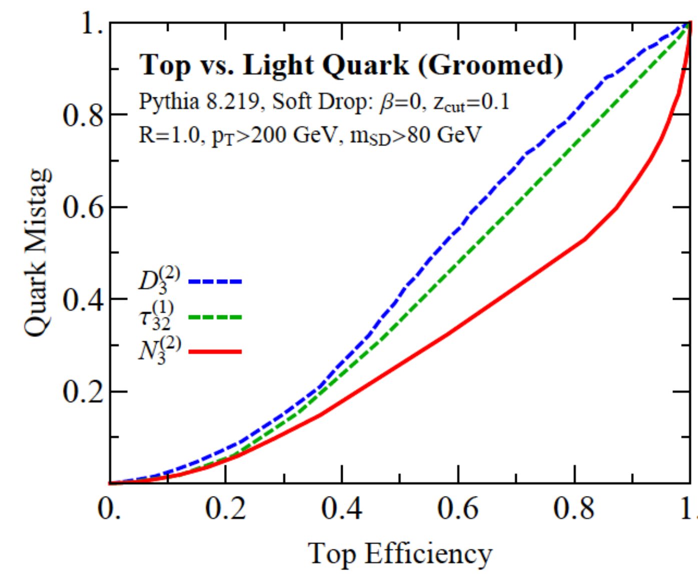


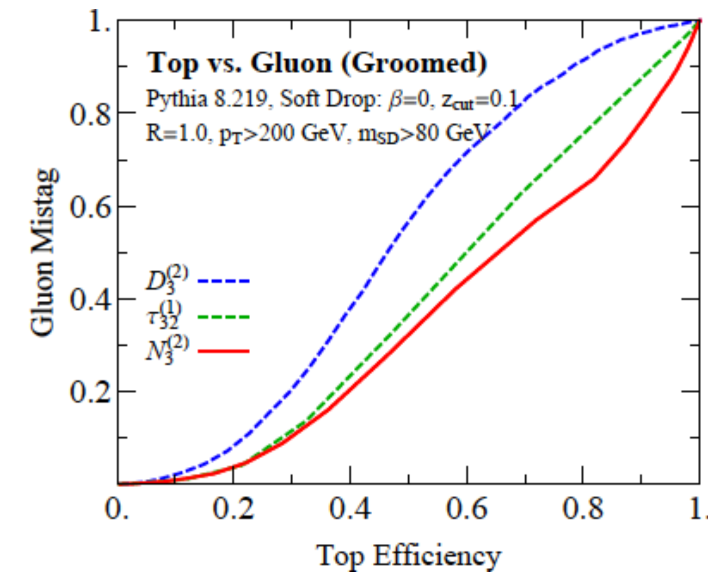
Groomed substructure

Moult, Necib, Thaler (2016)

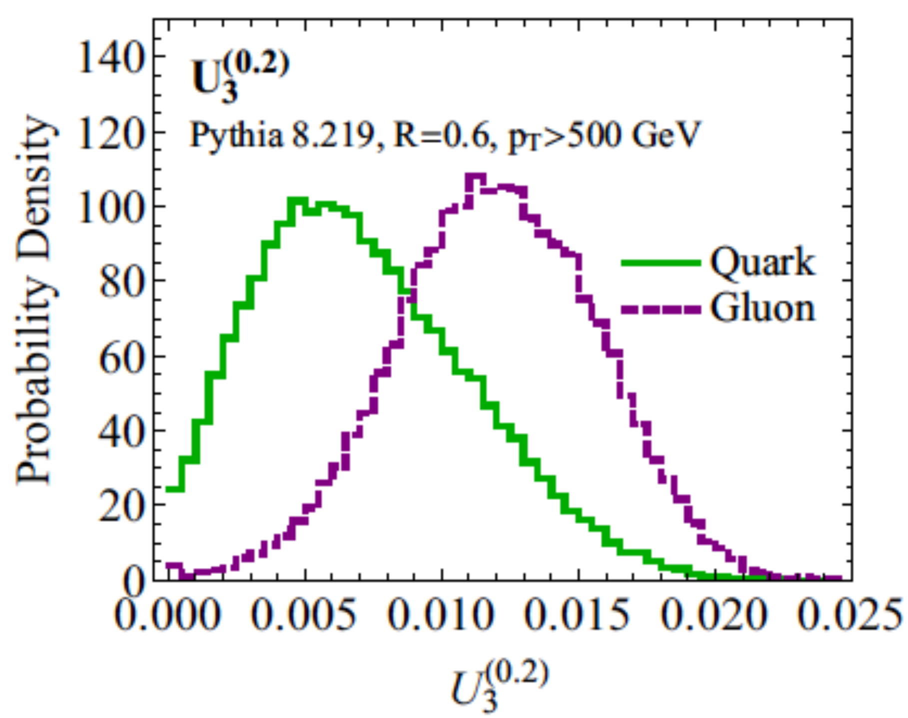
- Top tagging:

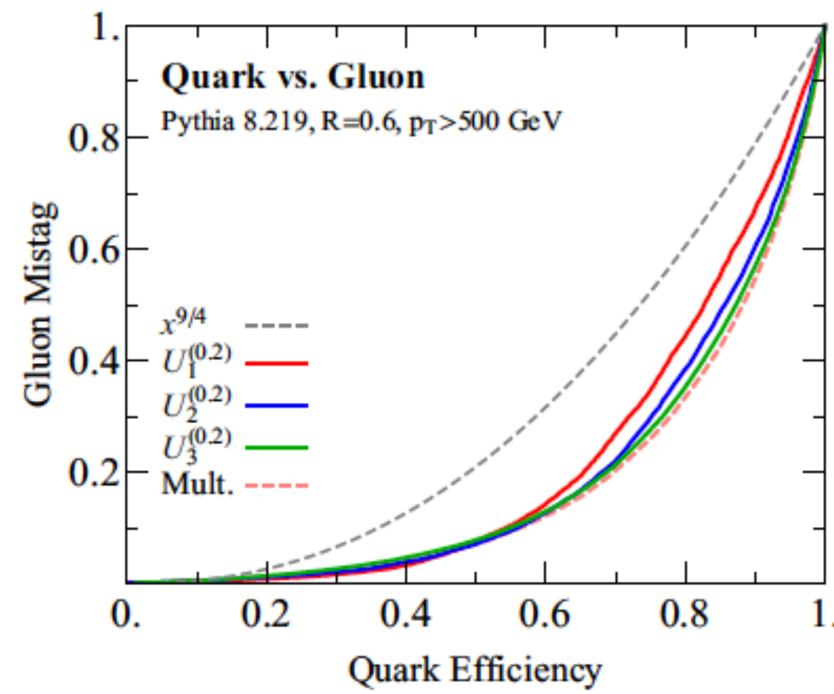






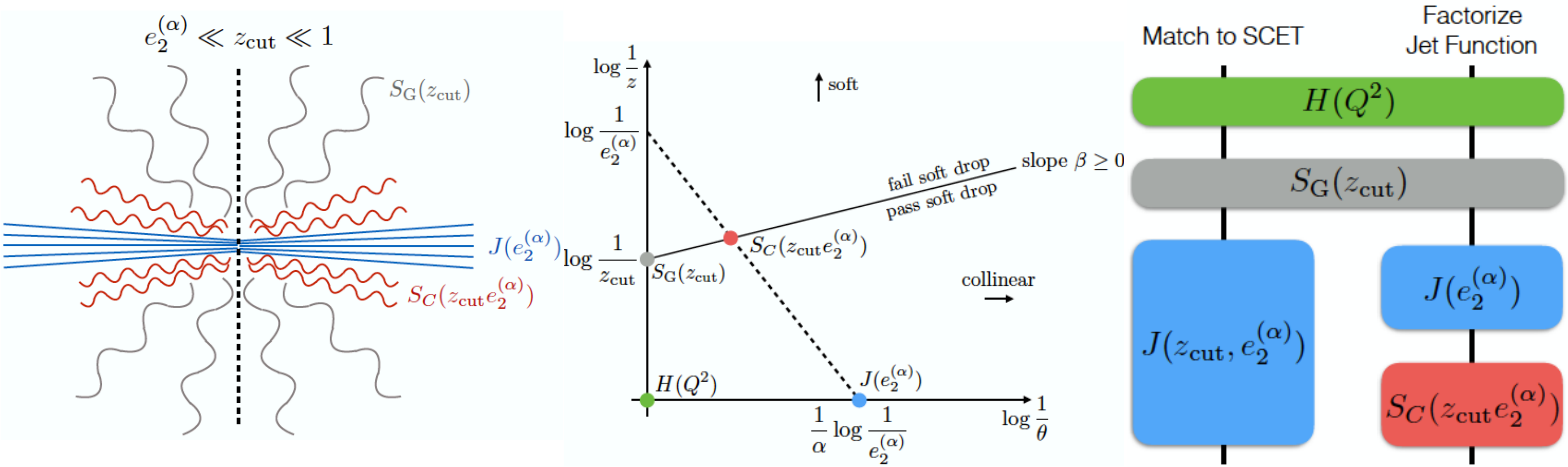
- q vs g:





Groomed substructure and SCET+

- Soft drop groomed energy correlators:



SCET+: Bauer, Tackmann, Walsh, Zuberi (2011)

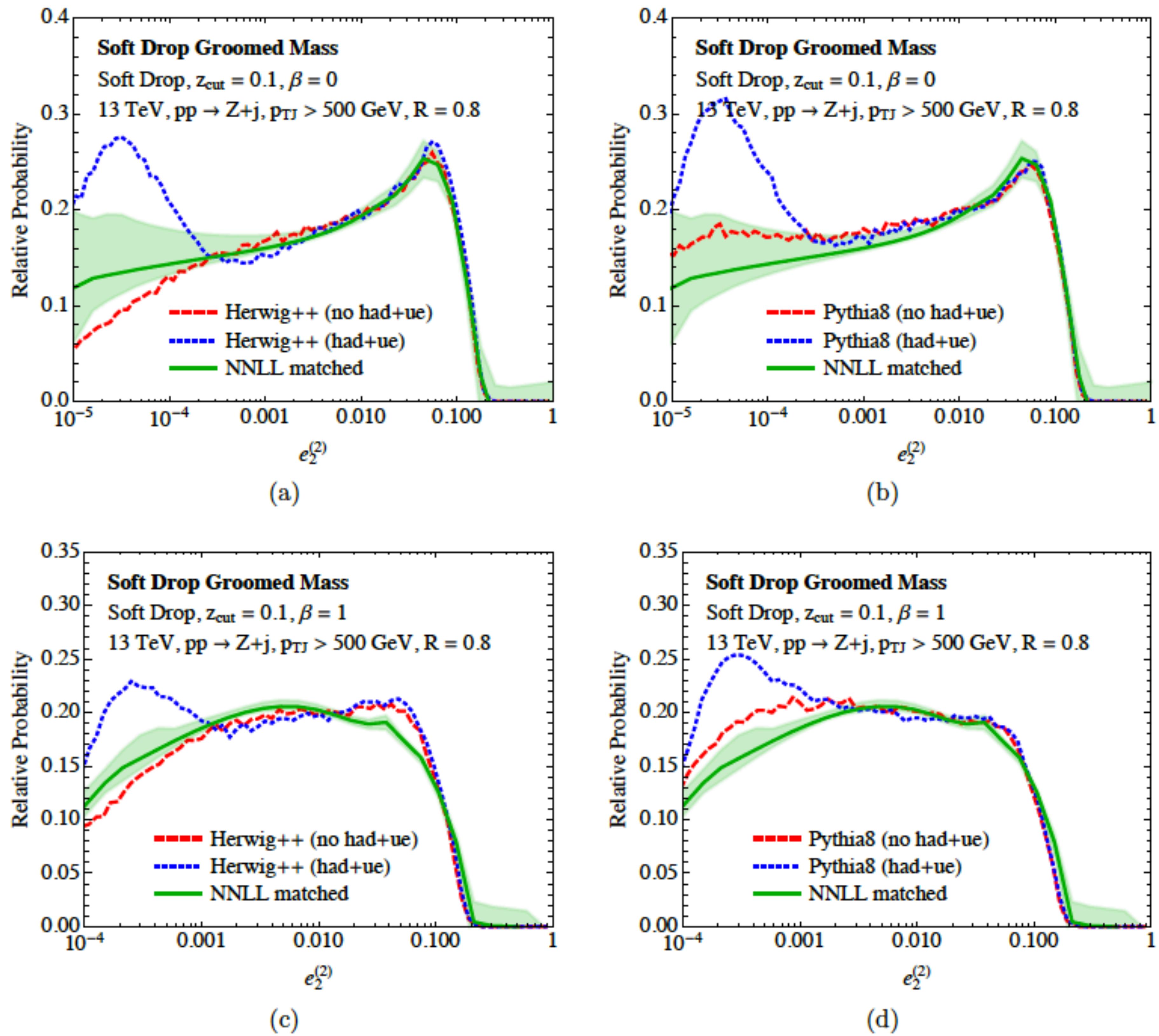
Frye, Larkoski, Schwartz, Yan (2016)

$$\frac{d^2\sigma}{de_{2,L}^{(\alpha)}\, de_{2,R}^{(\alpha)}} = H(Q^2) S_G(z_{\rm cut}) \left[S_C\left(z_{\rm cut} e_{2,L}^{(\alpha)}\right) \otimes J(e_{2,L}^{(\alpha)}) \right] \left[S_C\left(z_{\rm cut} e_{2,R}^{(\alpha)}\right) \otimes J(e_{2,R}^{(\alpha)}) \right]$$

free of NGLs; correlated hierarchical emissions groomed away

NNLL substructure calculations

Frye, Larkoski, Schwartz, Yan (2016)



Many other EFT directions

- Connection of NGLs and small-x evolution (BFKL)
- SCET with Glauber modes for factorization violating effects, small-x resummation, forward scattering

I. Rothstein and I. Stewart (2016)

- SCET_G for jets in heavy-ion collisions

A. Idilbi and A. Majumder (2008)
G. Ovanesyan and I. Vitev (2011)

- SCET + NRQCD for improved description of quarkonia in jets, discriminate production mechanisms

Baumgart, Leibovich, Mehen, Rothstein (2014)
Bain, Dai, Leibovich, Makris, Mehen (2016-17)

- SCET_{EW} for resummation of electroweak logs in colliders, dark matter production and annihilation

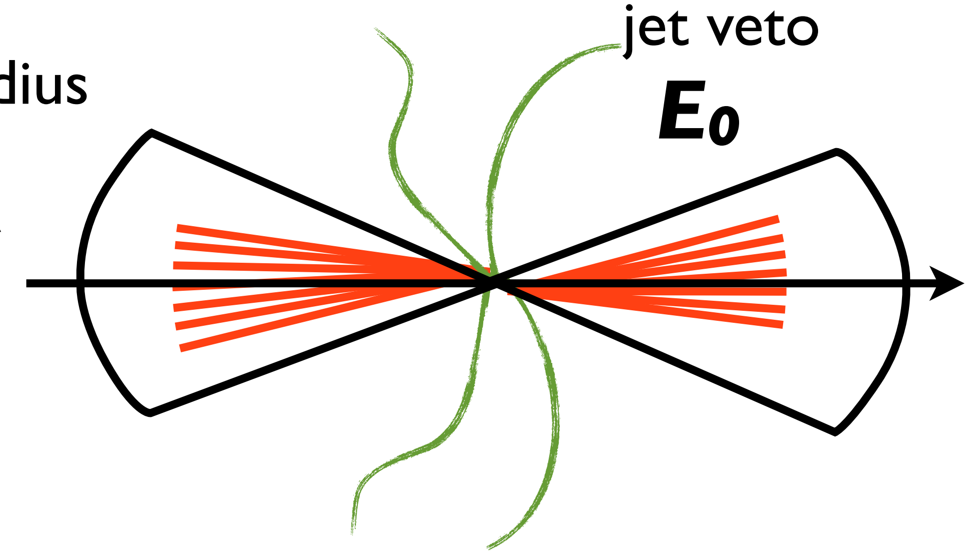
Chiu, Golf, Kelley, Manohar (2007)
Ovanesyan, Slatyer, Stewart (2014)
Baumgart, Rothstein, Vaidya (2014)
etc.

In the last several years, we have gained a collection of EFT and other powerful tools for high precision calculations of observables in multiscale jet-like processes, making possible fixed-order and resummed calculations to orders previously unachievable and the solution of problems previously intractable in QCD.

The future holds great promise

Extra slides

Jet Algorithms and Radii



- Example: e^+e^- to **two** jet cross section:
- One-loop cross section in QCD:
  - in a cone algorithm:

$$\frac{\sigma_{\text{2-jet}}}{\sigma_0} = 1 + \frac{\alpha_s C_F}{\pi}\left(-4\ln\frac{2E_0}{Q}\ln R - 3\ln R - \frac{1}{2} + 3\ln 2\right)$$

  - in a kT-type recombination (or Sterman-Weinberg) algorithm:

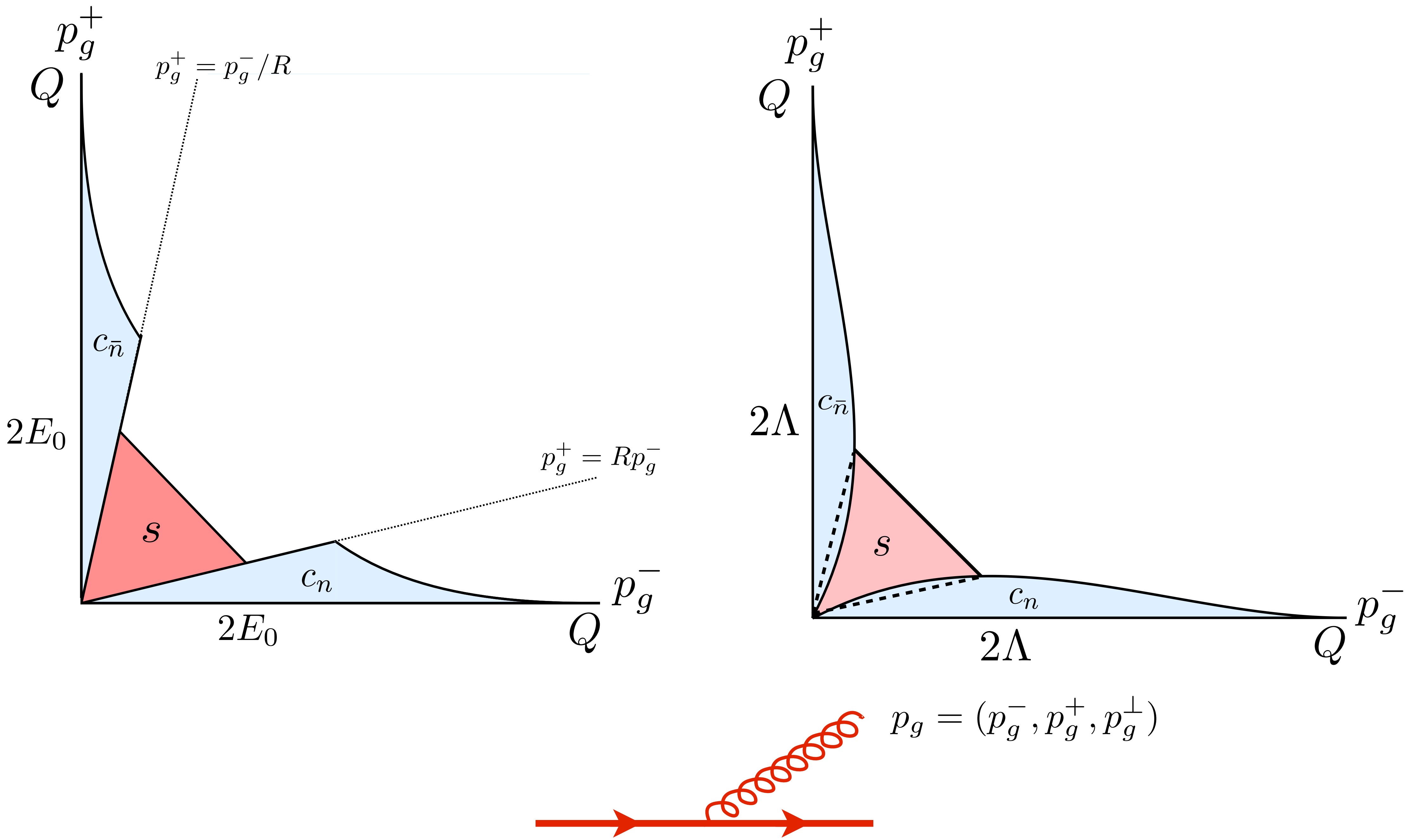
$$\frac{\sigma_{\text{2-jet}}}{\sigma_0} = 1 + \frac{\alpha_s C_F}{\pi}\left(-4\ln\frac{2E_0}{Q}\ln R - 3\ln R - \frac{\pi^2}{3} + \frac{5}{2}\right)$$

- Natural to use **SCET** to factorize and resum, but structure of logs is surprisingly subtle.

Chien, Hornig, CL (2015)

Soft and Soft-Collinear phase space

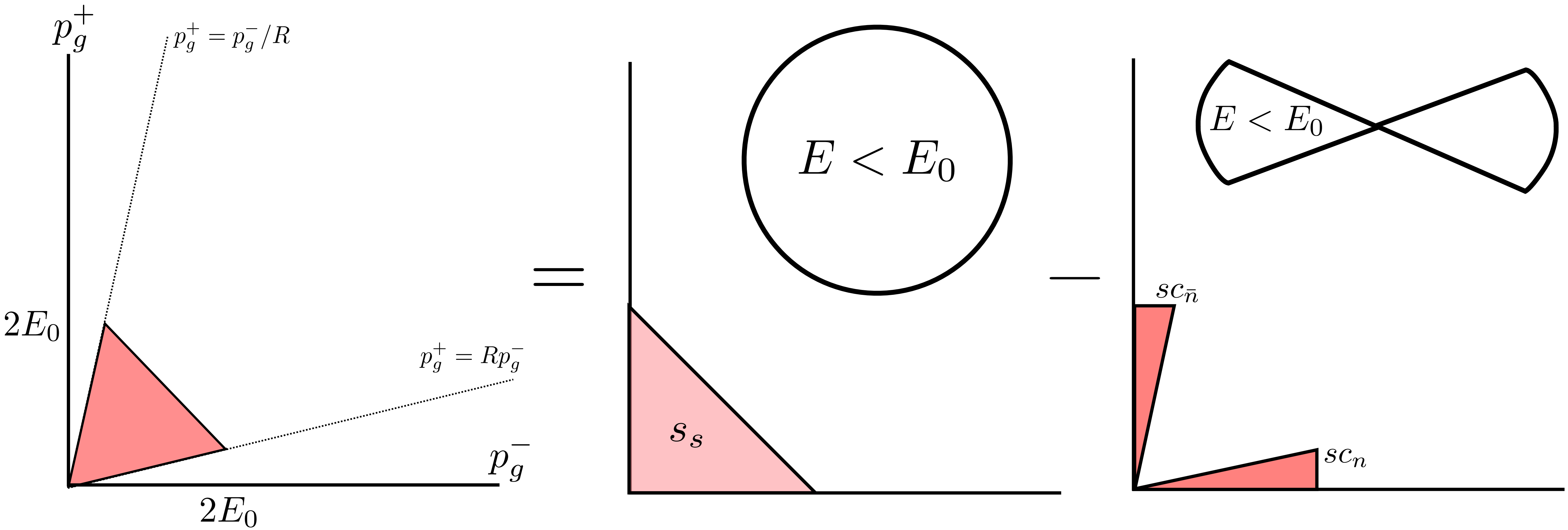
- collinear and soft phase space for cone and kT algorithms:



Ellis, Hornig, CL, Vermilion, Walsh (2010)
Chien, Hornig, CL (2015)

Soft and Soft-Collinear phase space

- Soft phase space splits into two, single-scale-sensitive regions:

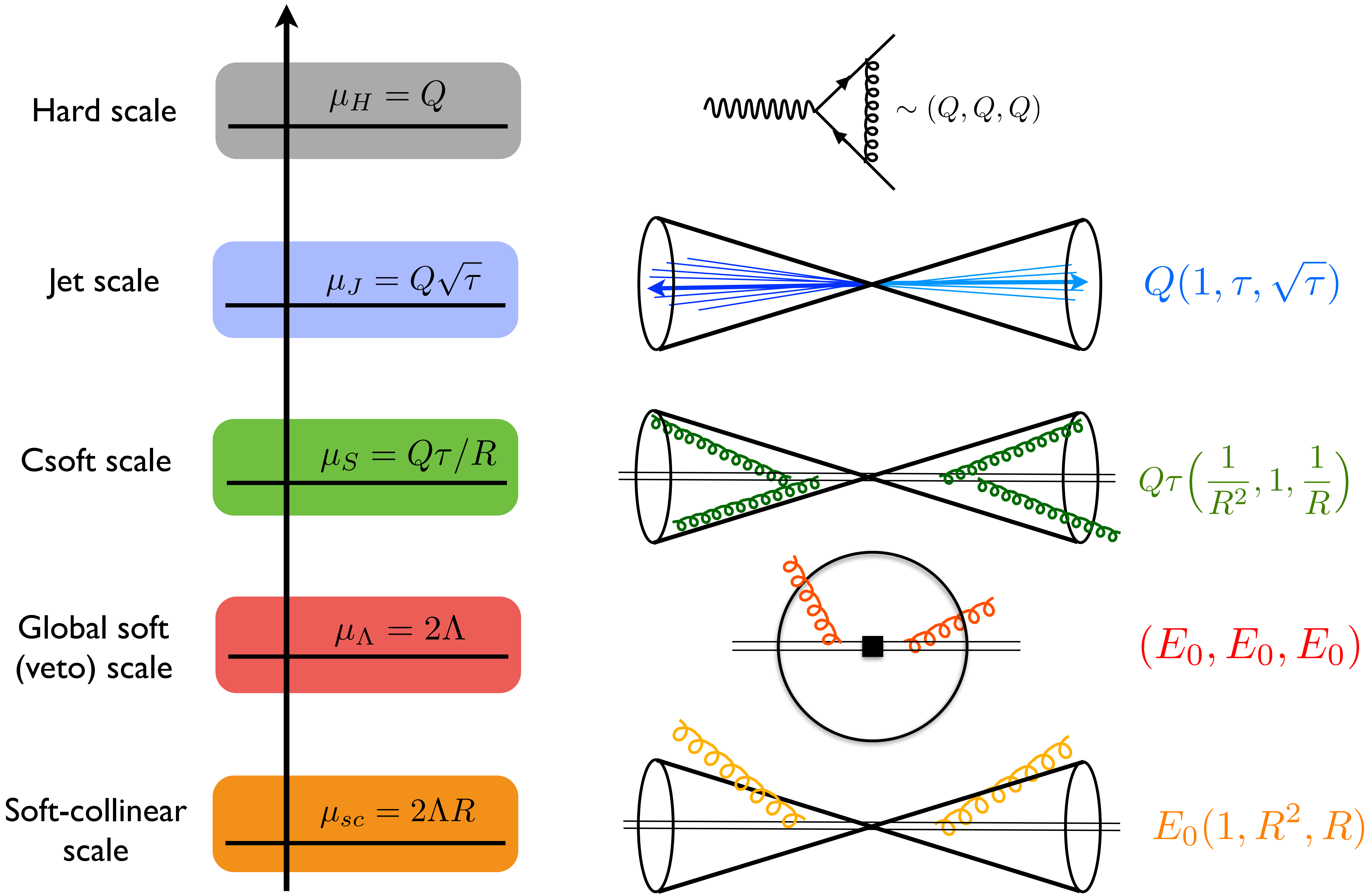


$S_{\text{veto}}(E_0, R, \mu)$ $\qquad$ $S_s(E_0, \mu)$ $\qquad$ $-2S_{sc}(E_0 R, \mu)$

$$\frac{\alpha_s C_F}{4\pi}\left(8\ln R \ln\frac{\mu^2}{4E_0^2 R} - \frac{2\pi^2}{3}\right) \qquad \frac{\alpha_s C_F}{4\pi}\left(8\ln^2\frac{\mu}{2E_0} - \pi^2\right) \qquad \frac{\alpha_s C_F}{4\pi}\left(-8\ln^2\frac{\mu}{2E_0 R} + \frac{\pi^2}{3}\right)$$

SCET++

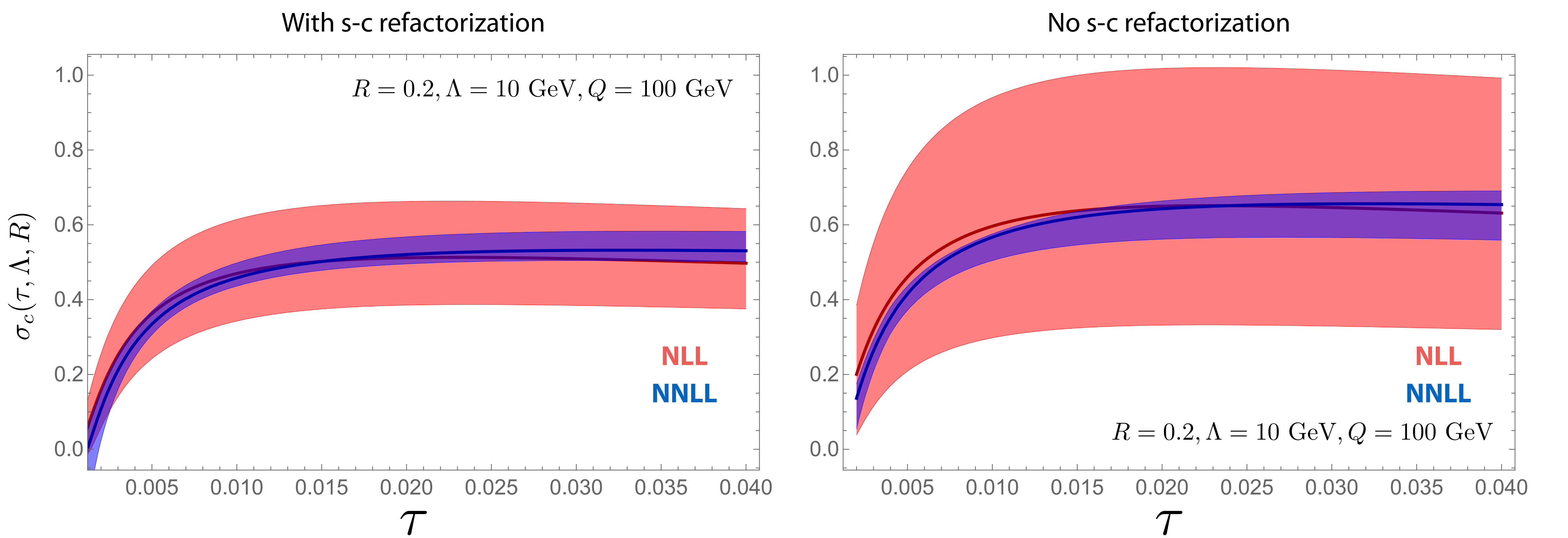
Chien, Hornig, CL (2015)



Chien, Hornig, CL (2015)

Resummed jet thrust cross section

- Integrated jet thrust in e^+e^-:

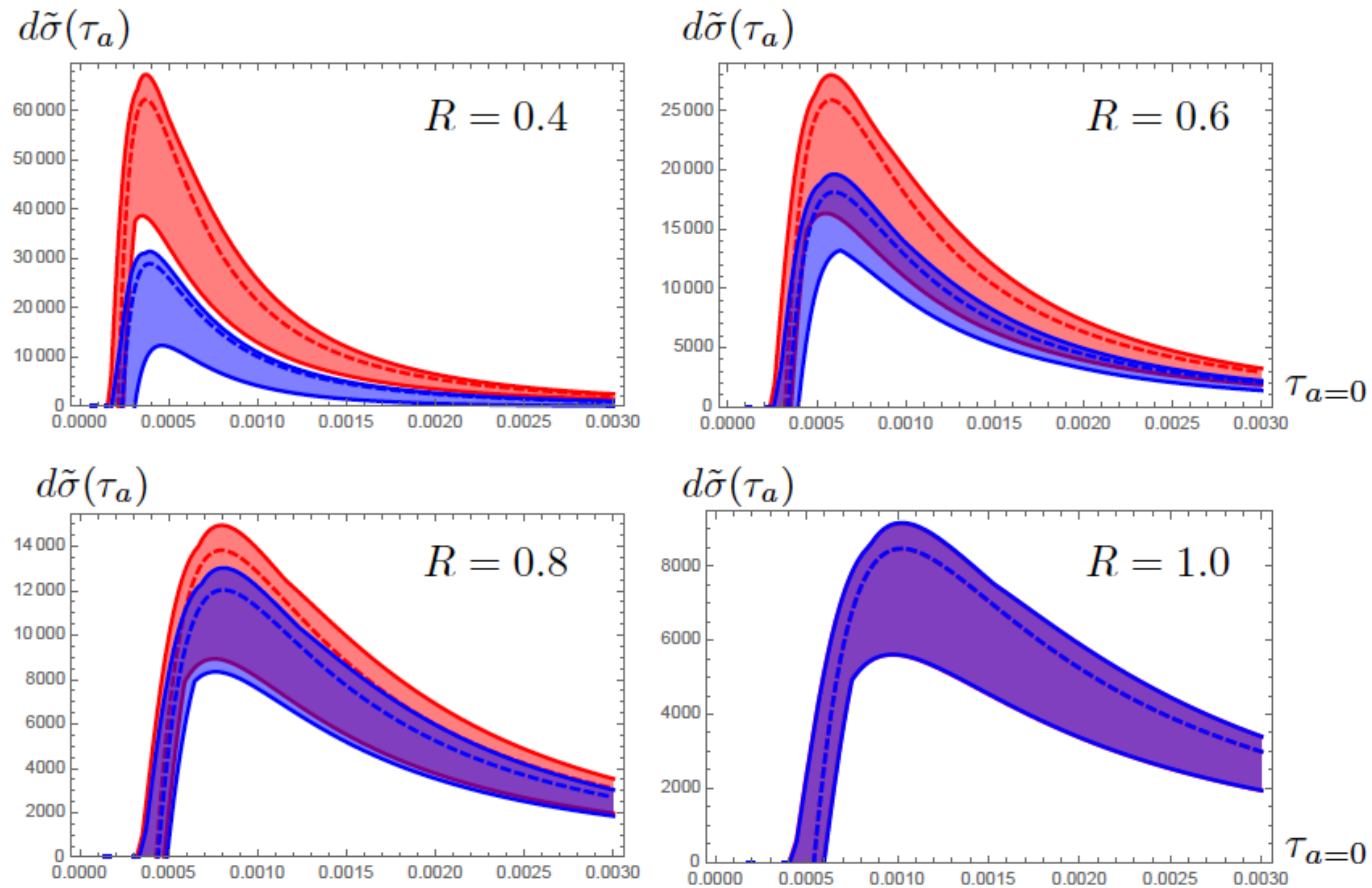


- Improved perturbative convergence thanks to additional logs resummed after soft-collinear refactorization

Resummed jet thrust cross section

- *pp* jet angularity differential distribution:

A. Hornig, Y. Makris, T. Mehen (2016)



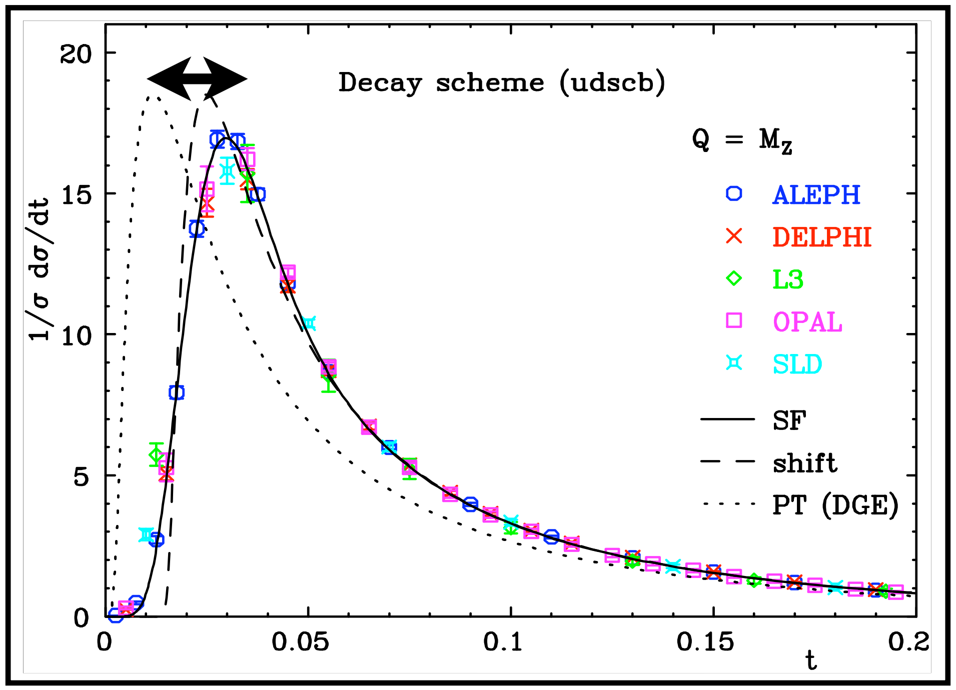
without soft-collinear refactorization

with soft-collinear refactorization

- Larger impact on differential shape

NP Corrections

- Reminder: Dokshitzer-Webber model



$$\langle e \rangle = \langle e \rangle_{\rm PT} + c_e \frac{\Omega_1}{Q}$$

conjecture from single soft gluon emission: Dokshitzer, Webber (1995, 1997)

c_e observable dependent, calculable coefficient

Ω_1 universal nonperturbative parameter

(one for each of ee, *ep*, *pp*)

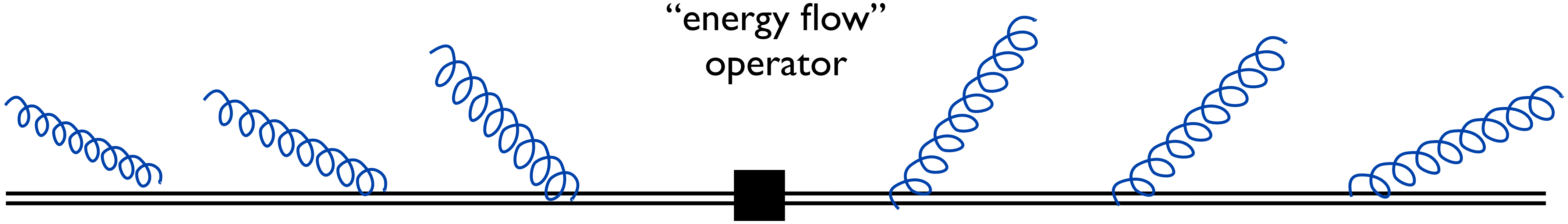
proof to all orders in soft gluon emission:

CL, Sterman (2006, 2007)

- SCET: First rigorous proof (and **field theory** definition of Ω_1) from factorization theorem and boost invariance of soft radiation:

$$\Omega_1 = \frac{1}{N_C} \mathrm{Tr} \langle 0 | \overline{Y}_{\bar n}^\dagger Y_n^\dagger \mathcal{E}_T(\eta) Y_n \overline{Y}_{\bar n} | 0 \rangle$$

"energy flow" operator



soft radiation sees only direction, not energy, of original collinear partons, invariant to boosts along z

Belitsky, Korchemsky, Sterman (2001)
Bauer, Fleming, **CL**, Sterman (2008)

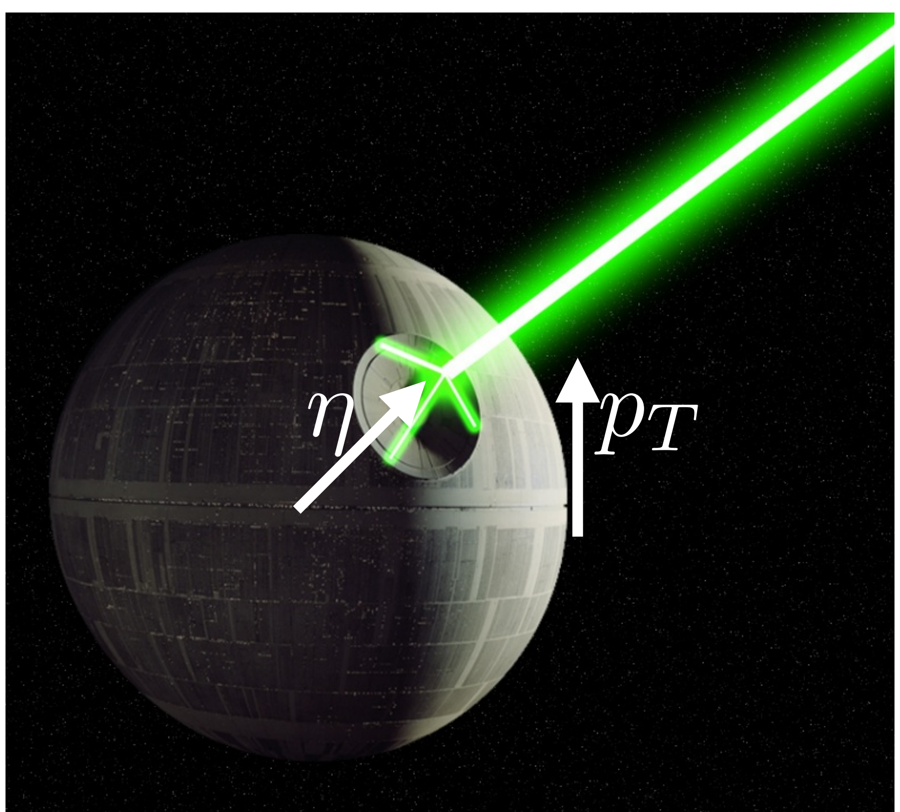
Momentum Flow Operators

generic form of event shapes: $e(X) = \frac{1}{Q}\sum_{i\in X} f_e(\eta_i)|\mathbf{p}_T^i|$ e.g. angularities $f_{\tau_a}(\eta) = e^{-|\eta|(1-a)}$

operator action in terms of transverse momentum flow operator:
$$\hat{e}\,|X\rangle \equiv e(X)\,|X\rangle = \frac{1}{Q}\int_{-\infty}^{\infty} d\eta\, f_e(\eta)\mathcal{E}_T(\eta;\hat{t})\,|X\rangle$$

$$\mathcal{E}_T(\eta)|X\rangle = \sum_{i\in X}|\mathbf{p}_T^i|\delta(\eta-\eta_i)|X\rangle$$

construct out of energy-momentum tensor of QCD:



$$\mathcal{E}_T(\eta) = \frac{1}{\cosh^3\eta}\int_0^{2\pi} d\phi \lim_{R\to\infty} R^2 \int_0^\infty dt\,\hat{n}_i T_{0i}(t, R\hat{n})$$

measures total transverse momentum $|\mathbf{p}_T|$ flowing through slice of sphere at rapidity η from collision time *t=0* to detector at $t\to\infty$
$R\to\infty$

since Lagrangian of SCET factors into collinear and soft sectors, so does the energy-momentum tensor:

$$T_{\mu\nu} \to T_{\mu\nu}^n + T_{\mu\nu}^{\bar{n}} + T_{\mu\nu}^s$$

CL, Sterman (2006, 2007)

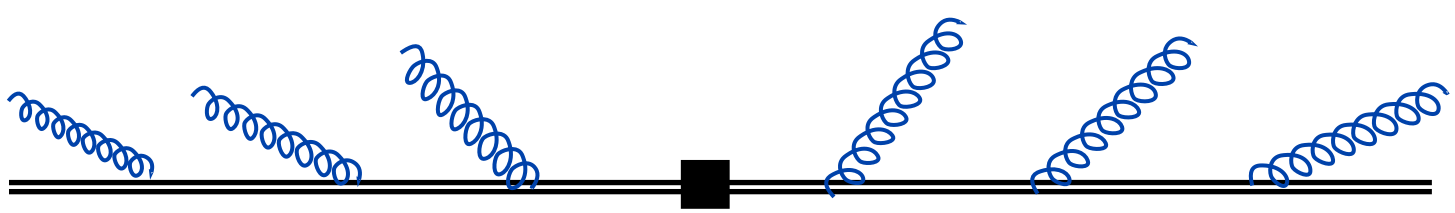
Proof of universality

- In general NP part of soft function must be modeled and is observable-dependent:

$$S(e,\mu,\Lambda) = \int_0^\infty de'\, S_{\rm PT}(e-e',\mu) F_{\rm NP}(e',\Lambda)$$

- The universality of the first moment, however, can be proven exactly:

$$\Delta\langle e\rangle_s = \frac{1}{Q}\int_{-\infty}^{\infty} d\eta\, f_e(\eta) \frac{1}{N_C}\,\mathrm{Tr}\langle 0|\overline{T}[Y_n^\dagger Y_{\bar n}]\mathcal{E}_T(\eta) T[Y_{\bar n}^\dagger Y_n]|0\rangle$$



$\Lambda_\alpha^{-1}\Lambda_\alpha$ Lorentz boosts by rapidity α along z:

$$Y_n = P\exp\left[ig\int_0^\infty ds\, n\cdot A_s(ns)\right] \rightarrow Y_n$$

$$|0\rangle \rightarrow |0\rangle$$

$$\mathcal{E}_T(\eta) \rightarrow \mathcal{E}_T(\eta+\alpha)$$

$$\Rightarrow \boxed{\Delta\langle e\rangle_s = \frac{1}{Q}\underbrace{\left\{\int_{-\infty}^{\infty} d\eta\, f_e(\eta)\right\}}_{c_e}\underbrace{\left\{\frac{1}{N_C}\,\mathrm{Tr}\langle 0|\overline{T}[Y_n^\dagger Y_{\bar n}]\mathcal{E}_T(0) T[Y_{\bar n}^\dagger Y_n]|0\rangle\right\}}_{\Omega_1}}$$

e.g. $c_\tau = 2$ $\quad c_C = 3\pi$ $\quad c_{\tau_a} = \dfrac{2}{1-a}$ $\quad$ for e$^+$e$^-$ scaling is obeyed well by LEP data

e^+e^- Thrust: Precision extraction of α_s

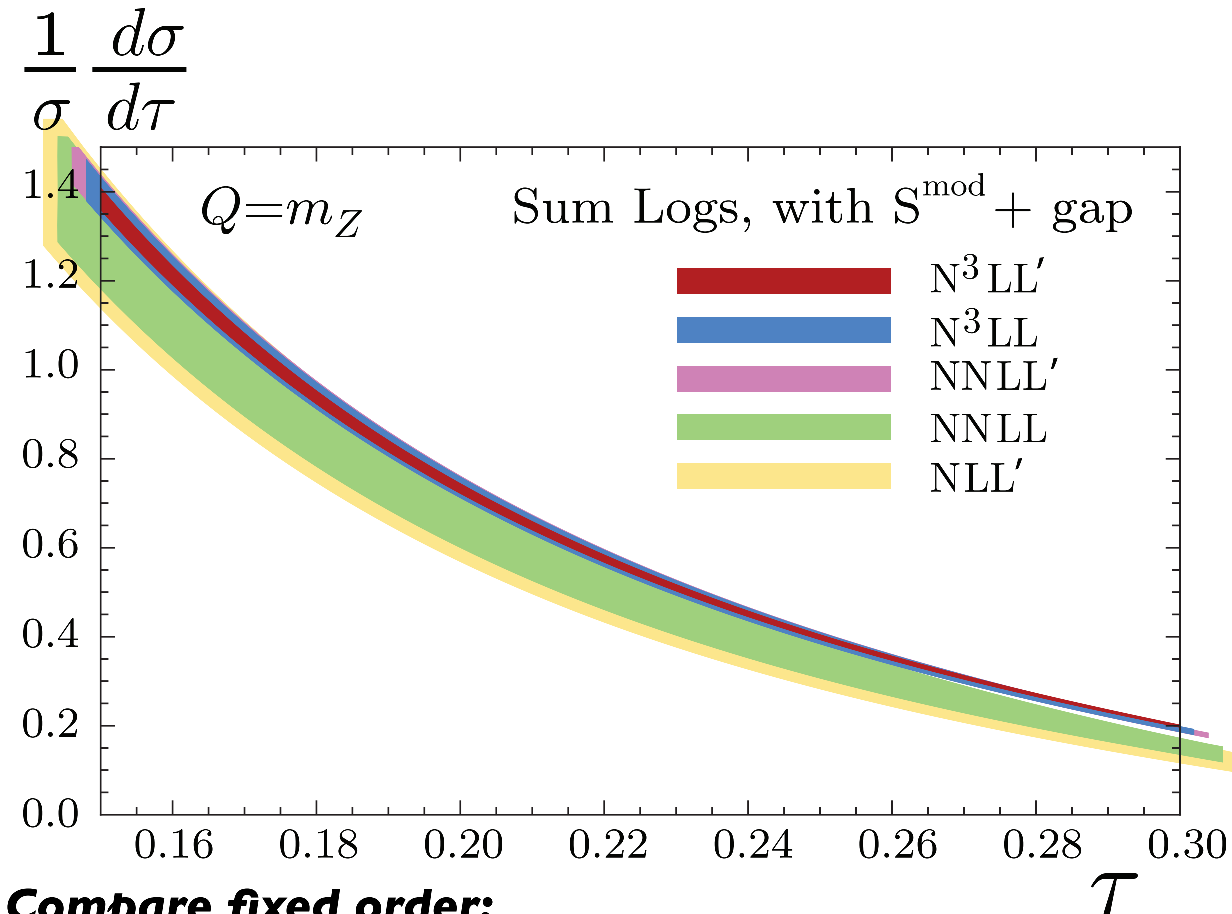
(2-jettiness)

NNNLL perturbative prediction + nonperturbative soft power correction led to most precise extraction of strong coupling from event shapes

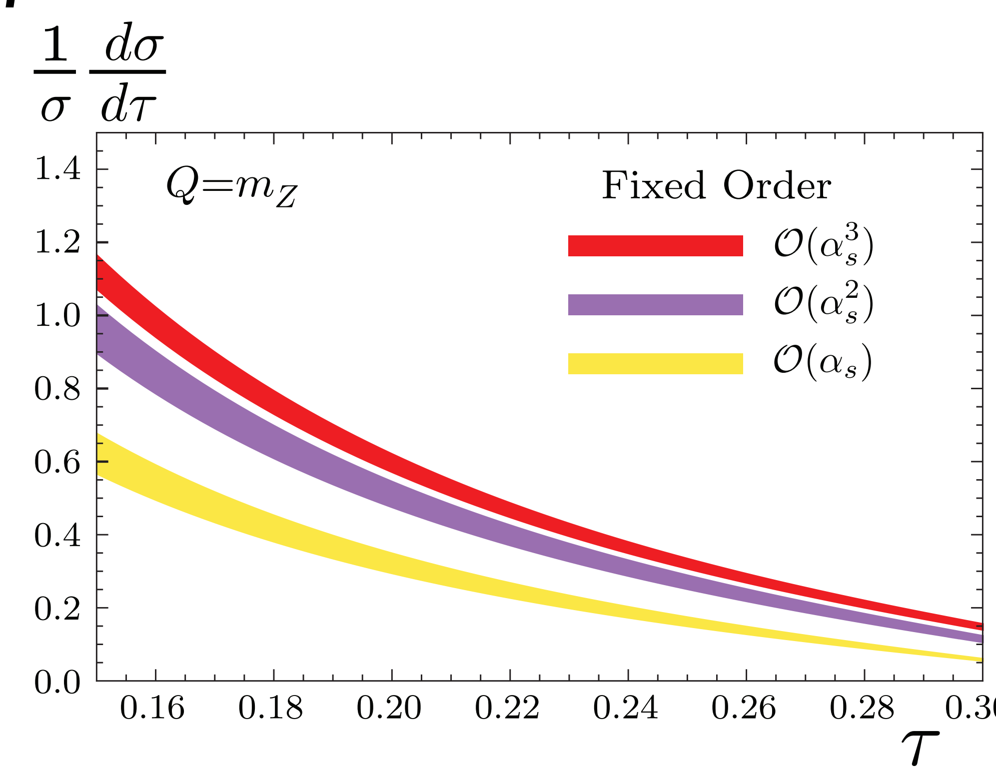
Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

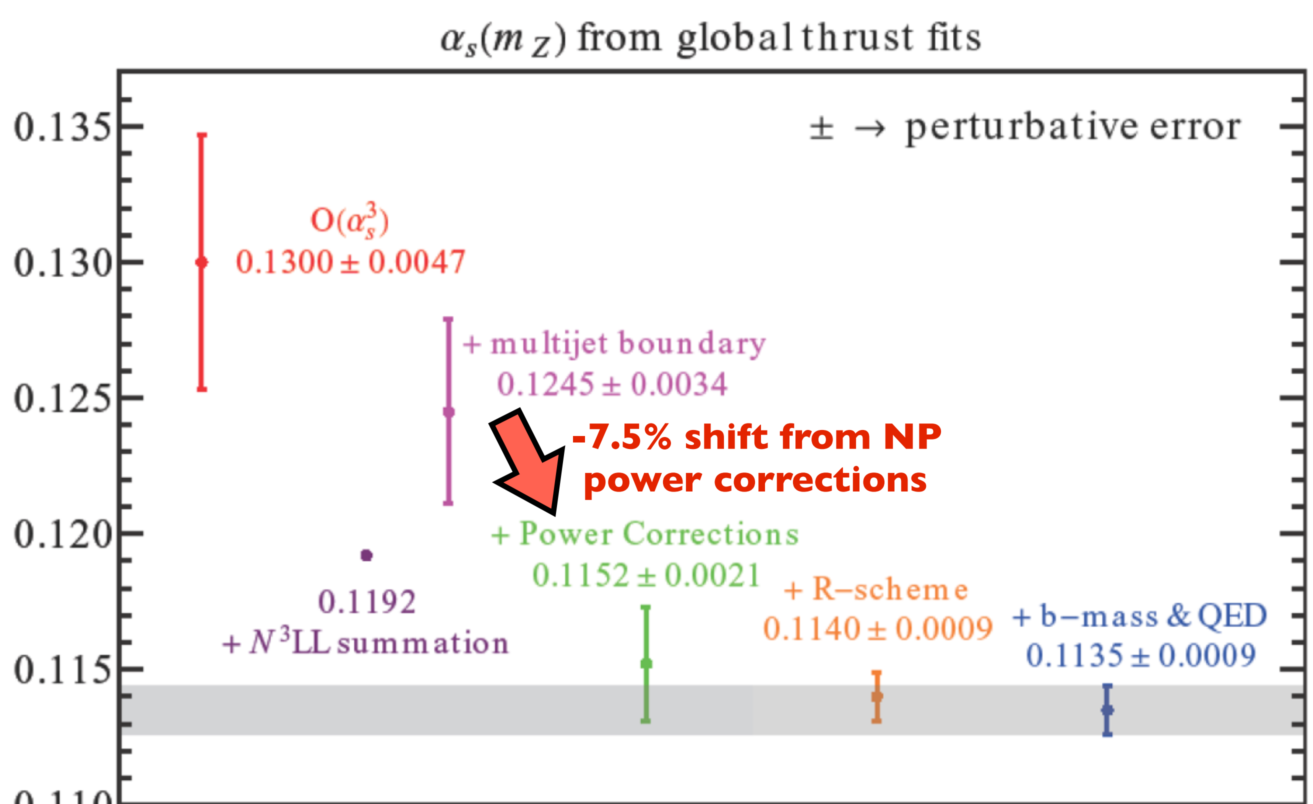
NNNLL resummed perturbative distribution

Becher, Schwartz (2008)



Compare fixed order:





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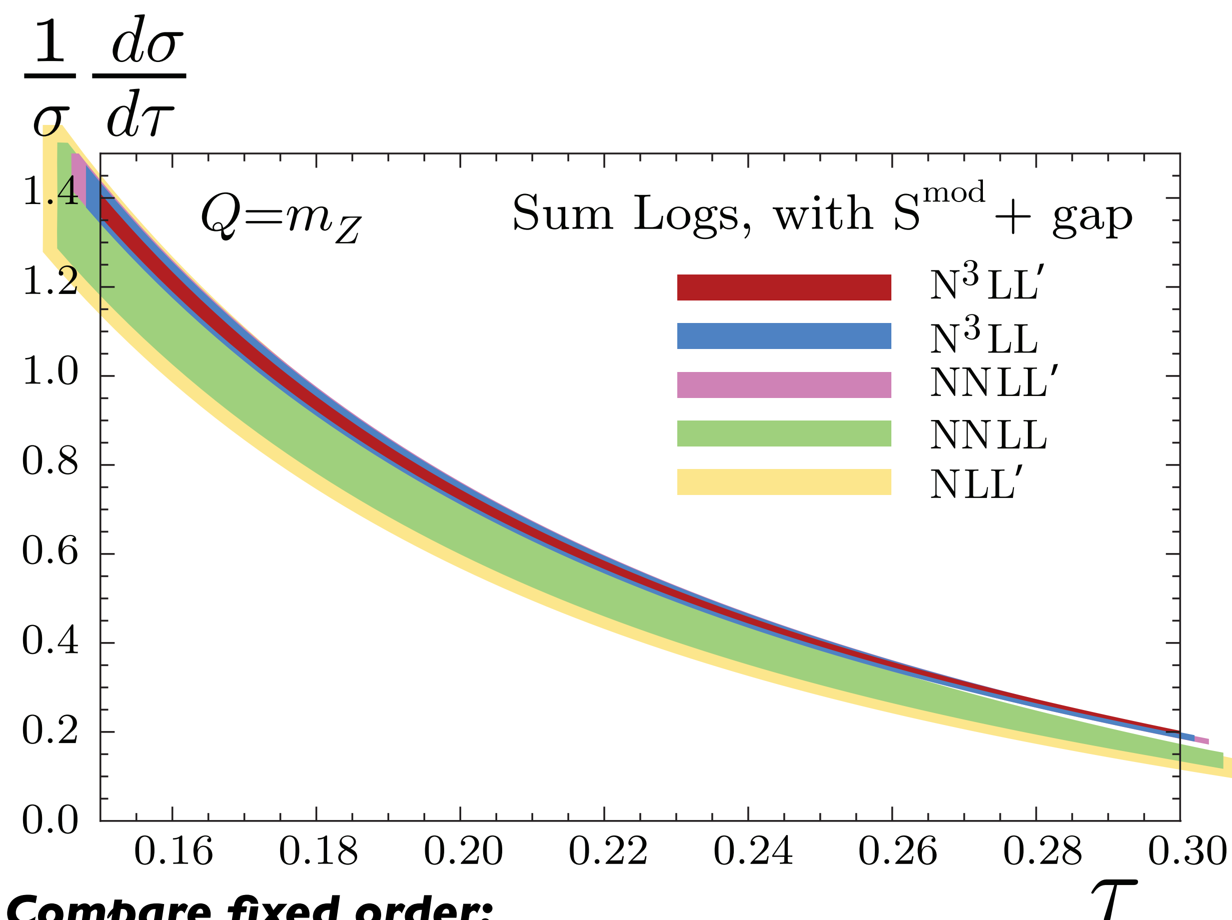
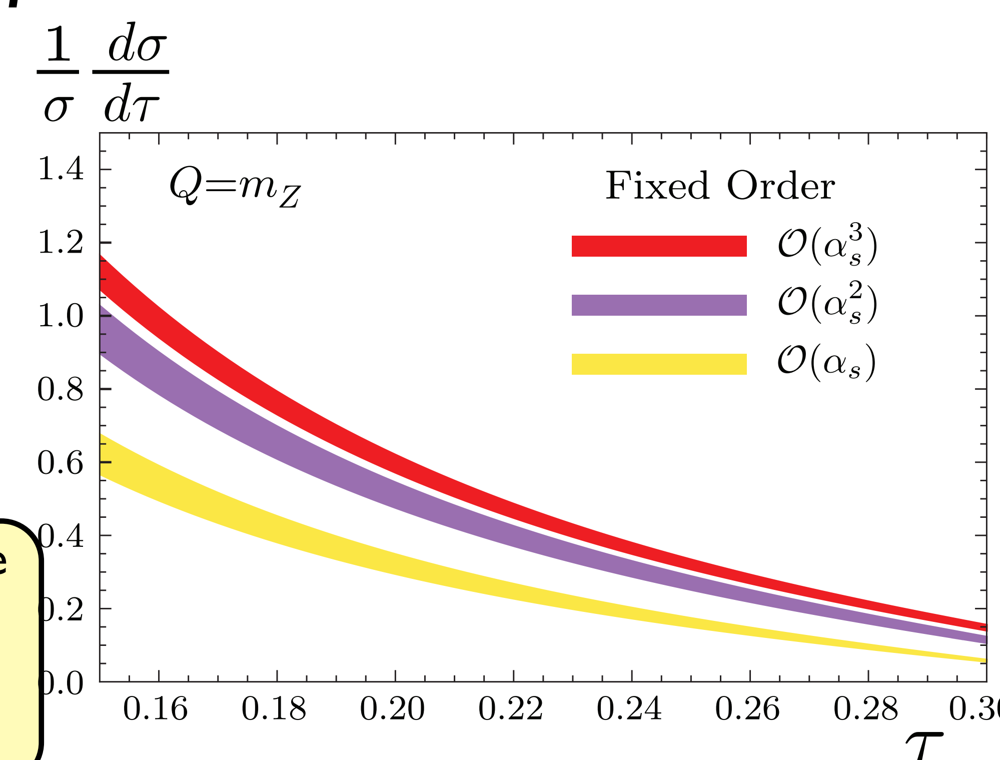
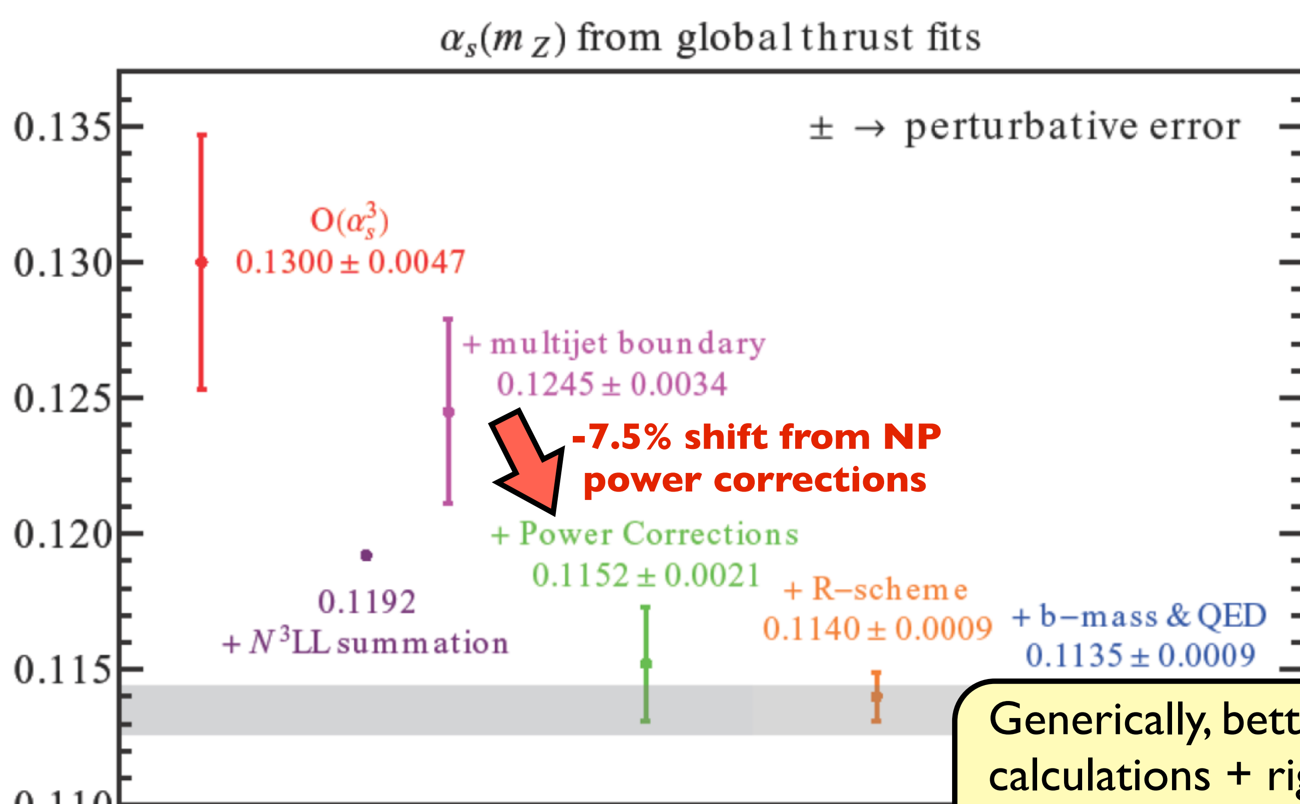


Compare fixed order:





Abbate, Fickinger, Hoang,

Generically, better perturbative calculations + rigorous treatment of nonperturbative corrections gives smaller α_s

Beam Function and PDFs

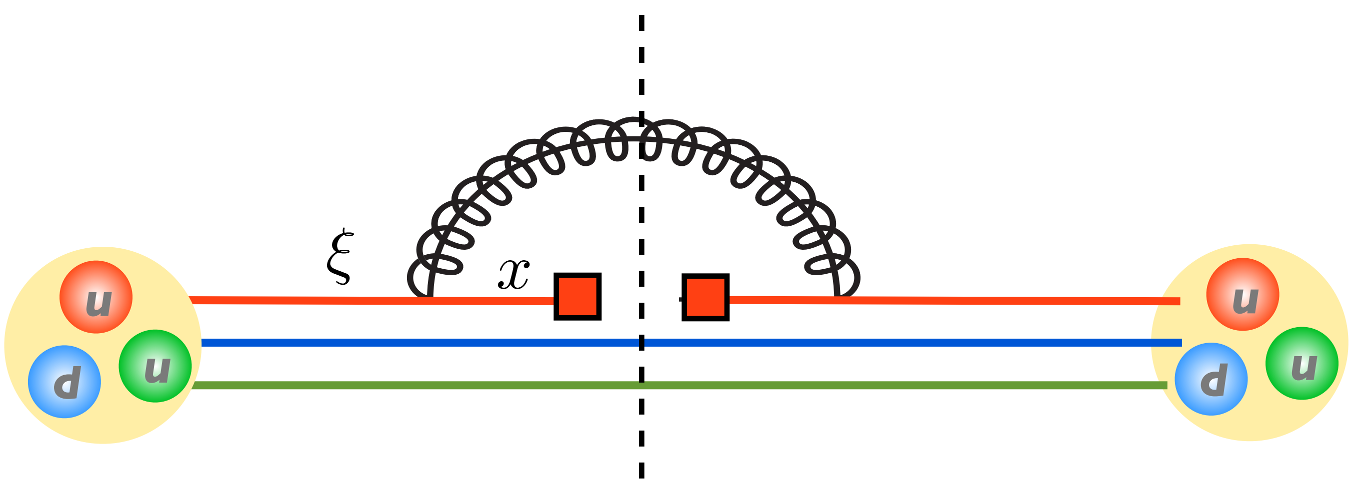
transverse momentum dependent beam function:

$$B(\omega k^+, x, k_\perp^2, \mu) = \frac{\theta(\omega)}{\omega} \int \frac{dy^-}{4\pi} e^{ik^+ y^-/2} \langle P_n(P^-)|\bar{\chi}_n\Big(y^-\frac{n}{2}\Big)\delta(xP^- - \bar{n}\cdot\mathcal{P})\delta(k_\perp^2 - \mathcal{P}_\perp^2)\chi_n(0)|P_n(P^-)\rangle$$

match onto PDF

$$f(x,\mu) = \theta(\omega)\langle P_n(P^-)|\bar{\chi}_n(0)\delta(xP^- - \bar{n}\cdot\mathcal{P})\chi_n(0)|P_n(P^-)\rangle$$

$$\mathcal{B}_q(t, x, \mathbf{k}_\perp^2, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mathbf{k}_\perp^2, \mu\Big) f_j(\xi, \mu)$$



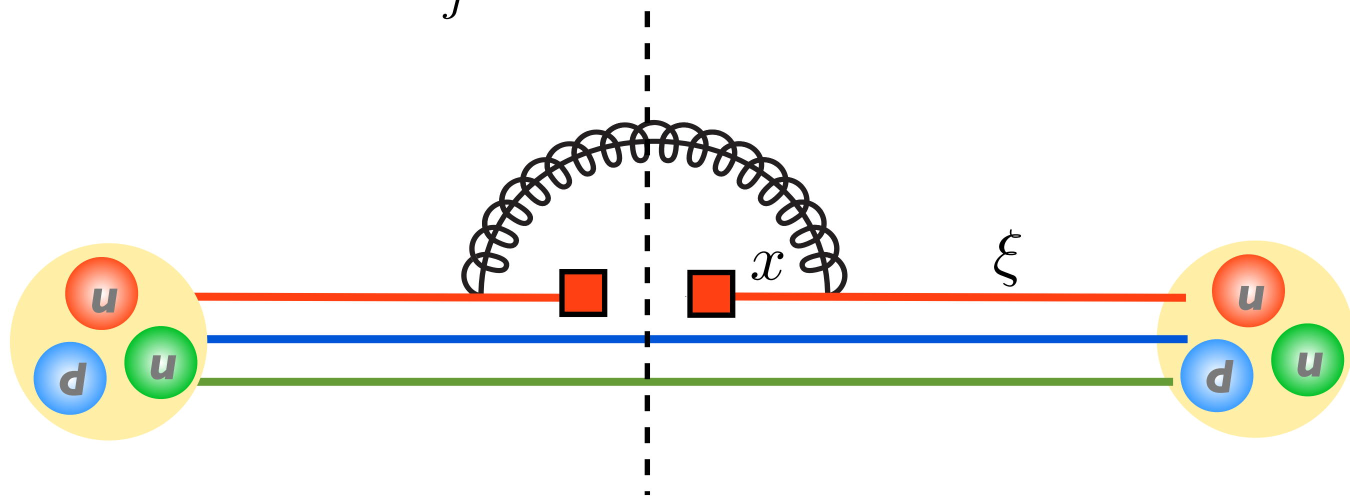
Measure small light-cone momentum $k^+ = t/P^-$
and transverse momentum $\mathbf{k}_\perp$
of initial state radiation

Generalized Beam Function to 1-loop

$$\mathcal{B}_q(t,x,\mathbf{k}_\perp^2,\mu) = \sum_j \int_x^1 \frac{d\xi}{\xi}\mathcal{I}_{ij}\Big(t,\frac{x}{\xi},\mathbf{k}_\perp^2,\mu\Big) f_j(x,\mu)$$

now known to 2 loops;
anomalous dimension
known to 3 loops

Gaunt, Stahlhofen, Tackmann (2014)



Jain, Procura, Waalewijn (2009)

$$\mathcal{I}_{qq}(t,z,\mathbf{k}_\perp^2,\mu) = \frac{1}{\pi}\delta(t)\delta(1-z)\delta(\mathbf{k}_\perp^2) + \frac{\alpha_s(\mu)C_F}{2\pi^2}\theta(z)\bigg\{\frac{2}{\mu^2}\bigg[\frac{\theta(t)\ln(t/\mu^2)}{t/\mu^2}\bigg]_+ \delta(1-z)\delta(\mathbf{k}_\perp^2)$$
$$+ \frac{1}{\mu^2}\bigg[\frac{\theta(t)}{t/\mu^2}\bigg]_+ \bigg[P_{qq}(z) - \frac{3}{2}\delta(1-z)\bigg]\delta\Big(\mathbf{k}_\perp^2 - \frac{(1-z)t}{z}\Big)$$
$$+ \delta(t)\delta(\mathbf{k}_\perp^2)\bigg[\bigg[\frac{\theta(1-z)\ln(1-z)}{1-z}\bigg]_+ (1+z^2) - \frac{\pi^2}{6}\delta(1-z) + \theta(1-z)\Big(1-z-\frac{1+z^2}{1-z}\ln z\Big)\bigg]\bigg\} \quad (162a)$$

$$\mathcal{I}_{qg}(t,z,\mathbf{k}_\perp^2,\mu) = \frac{\alpha_s(\mu)T_F}{2\pi^2}\theta(z)\bigg\{\frac{1}{\mu^2}\bigg[\frac{\theta(t)}{t/\mu^2}\bigg]_+ P_{qg}(z)\delta\Big(\mathbf{k}_\perp^2 - \frac{(1-z)t}{z}\Big) + \delta(t)\delta(\mathbf{k}_\perp^2)\bigg[P_{qg}(z)\ln\frac{1-z}{z} + 2\theta(1-z)z(1-z)\bigg]\bigg\}, \quad (162b)$$

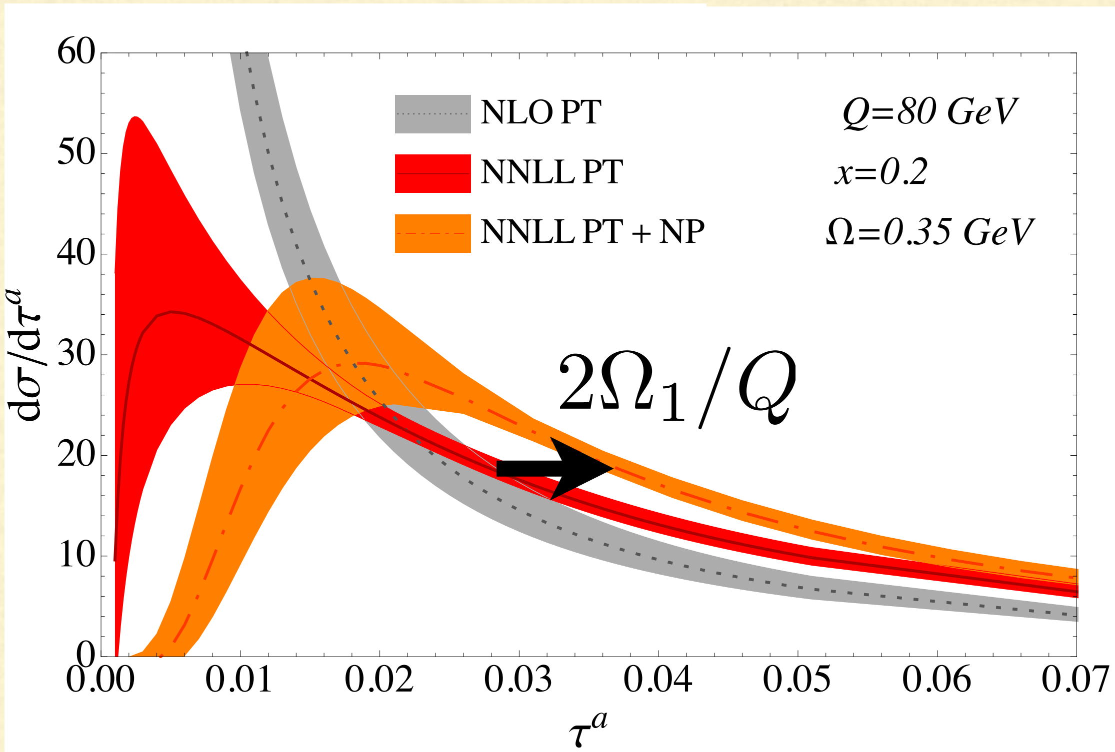
Tells us that PDFs should be evaluated at the beam radiation scale *t*

ordinary beam function: $B(t,x,\mu) = \int d^2k_\perp \mathcal{B}(t,x,\mathbf{k}_\perp^2,\mu)$

Stewart, Tackmann, Waalewijn (2009)

POWER CORRECTIONS IN *PP* AND DIS

- Universal nonperturbative shift in 3 versions of DIS 1-jettiness:

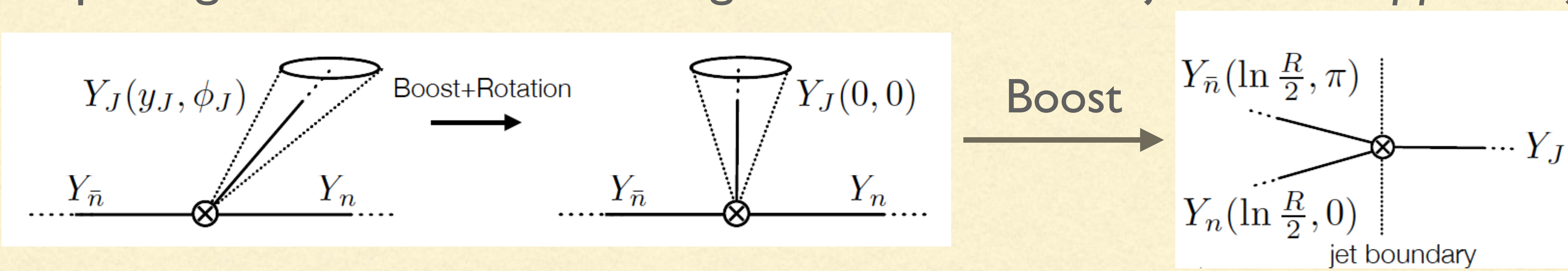


Using factorization theorems and boost invariance properties of soft Wilson lines, can prove that:

$$\boxed{\Omega_1^{\mathrm{a}} = \Omega_1^{\mathrm{b}} = \Omega_1^{\mathrm{c}}}$$

D. Kang, CL, I. Stewart (2013)

- Surprising relation also to leading NP correction to jet mass in *pp* to 1 jet



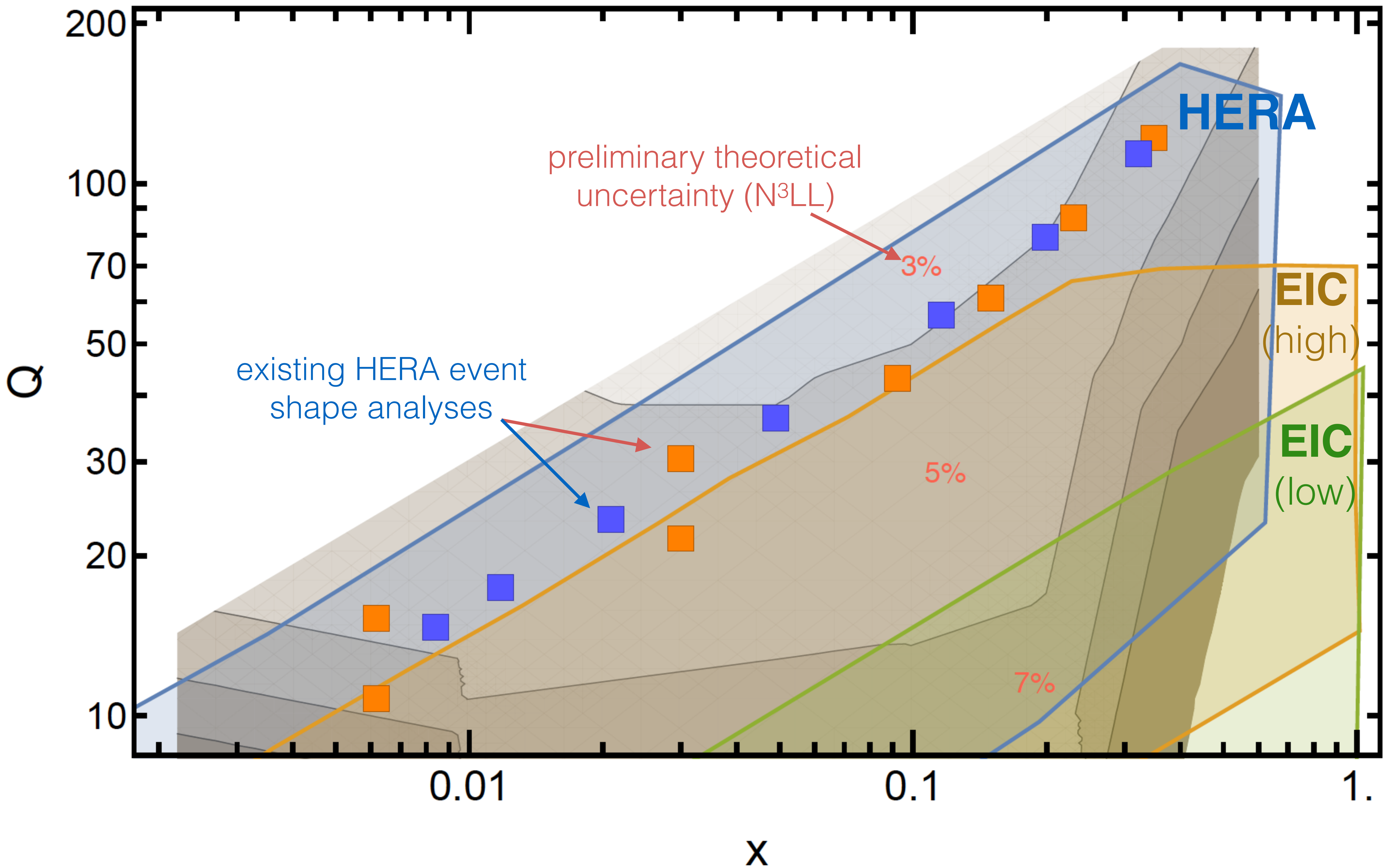
Stewart, Tackmann, Waalewijn (2014)

For $R \ll 1$, the beam Wilson lines fuse and $\Omega = \frac{R}{2}\,\Omega_0 + \ldots$

The universal Ω_0^q can be extracted from DIS event shapes

Experimental Coverage

Preliminary: Kang, CL, Stewart (2016)



New analyses of HERA data for 1-jettiness under way!

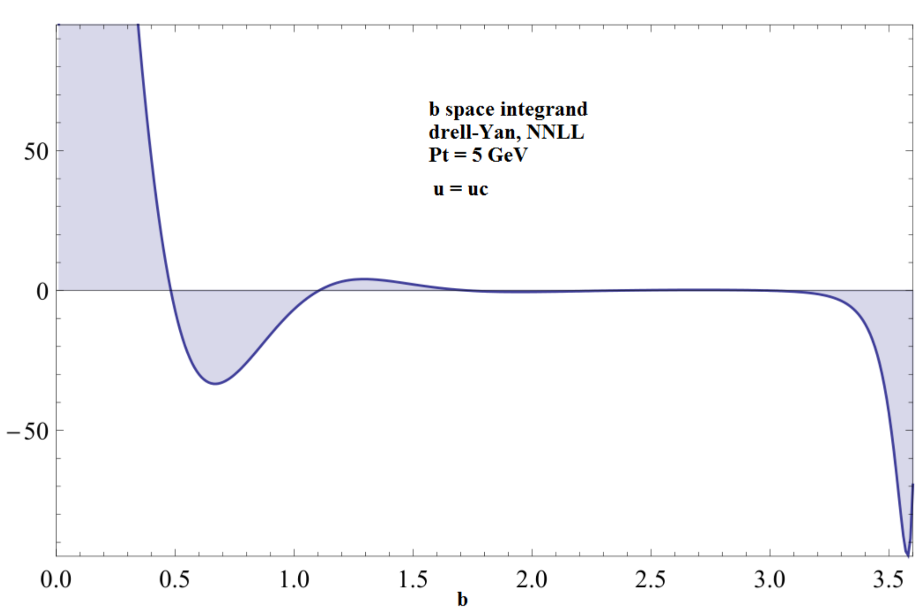
TMD resummation

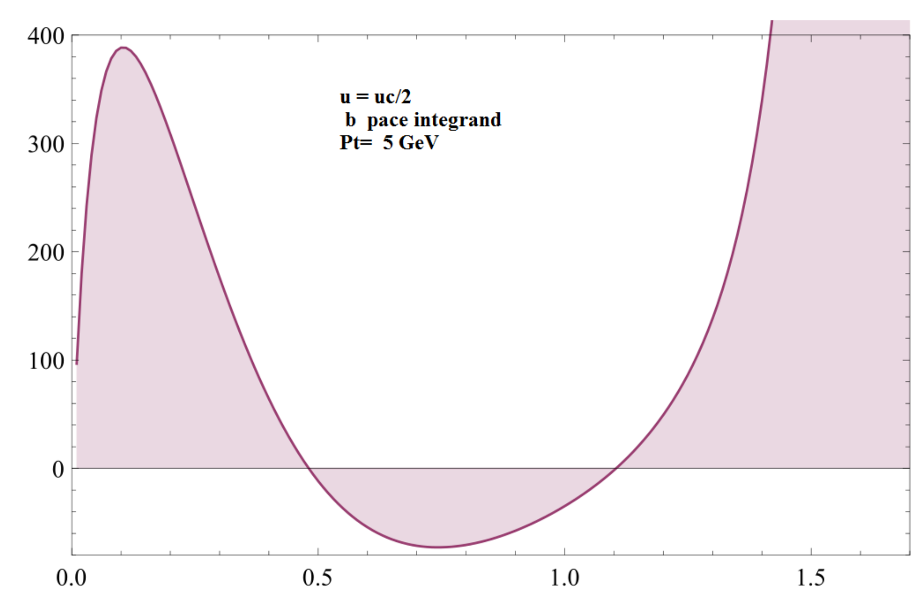
$$\sigma(b, z_1, z_2; \mu_i, \nu_i; \mu, \nu) = U_{\text{tot}}(\mu_i, \nu_i; \mu, \nu)\, H(Q^2, \mu_H) \widetilde{S}(b; \mu_L, \nu_L) \times \widetilde{f}_\perp(b, z_1; \mu_L, \nu_H) \widetilde{f}_\perp(b, z_2; \mu_L, \nu_H)$$

$$U_{\text{tot}}(\mu_i, \nu_i, \mu, \nu) = \exp\Big\{ 4K_\Gamma(\mu_L, \mu_H) - 4\eta_\Gamma(\mu_L, \mu_H) \ln \frac{Q}{\mu_L} - K_{\gamma_H}(\mu_L, \mu_H) + \Big[-4\,\eta_\Gamma(1/b_0, \mu_L) + \gamma_{R\,S}\big[\alpha_s(1/b_0)\big]\Big] \ln \frac{\nu_H}{\nu_L} \Big\}$$

Standard scale choices: $\mu_L = \nu_L = \dfrac{2}{b e^{\gamma_E}} \equiv \dfrac{1}{b_0}$ $\qquad \mu_H = \nu_H = Q$

Landau pole





TMD resummation in momentum space

Kang, CL, Vaidya (2017)

Our scale choices:

$$\nu_L^* = \nu_L(\mu_L b_0)^{-1+n}$$

$$\mu_H \sim \nu_H \sim Q$$

$$\nu_L \sim \mu_L$$

chosen in momentum space, after b integration

$$n = \frac{1}{2}\left[1 - \frac{\alpha_s \beta_0}{2\pi} \log\left(\frac{\nu_H}{\nu_L}\right)\right]$$

automatic damping of b integrand using terms actually in perturbative series

Analytic formula:

$$I_b = \frac{2C}{\pi q_T^2} \sum_{n=0}^{\infty} \mathrm{Im}\left\{ c_{2n} \mathcal{H}_{2n}(\alpha, a_0) + \frac{i\gamma_E}{\beta} d_{2n+1} \mathcal{H}_{2n+1}(\beta, b_0) \right\}$$

$$\mathcal{H}_n(\alpha, a_0) = e^{\frac{-A(L-i\pi/2)^2}{1+a_0 A}} \frac{1}{\sqrt{1+a_0 A}} \frac{(-1)^n n!}{(1+a_0 A)^n} \sum_{m=0}^{\lfloor n/2 \rfloor} \frac{1}{m!} \frac{1}{(n-2m)!} \left\{[A(\alpha^2 - a_0) - 1](1 + a_0 A)\right\}^m (2\alpha z_0)^{n-2m}$$