Advances in QCD Theory

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August 3, 2017
A handful of Advances in QCD Theory

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August 3, 2017
Fermi and Los Alamos

Valles Caldera, near Los Alamos, May 8, 1945
• Drell-Yan: E772/E789/E866 (NuSea)/E906 (SeaQuest), E1039

Geoff Mills, 1955-2017
My charge

“QCD theory developments over the last ~2 years”
Two years ago

Christopher Lee,
May 27, 1922 — June 7, 2015

750 GeV, 3.6σ
December 2015
— August 2016

CNN/ORC Poll
June 26-28, 2015

Bush 19%
Trump 12%
Huckabee 8%
Carson 7%
Paul 7%
Rubio 6%
Walker 6%
Perry 4%
Christie 3%
Cruz 3%
Santorum 3%
Jindal 2%
Kasich 2%
 Fiorina 1%
Graham 1%

Total 100%
Clinton 59%
Trump 35%
Other 4%
Neither 6%
Historic Day: July 4
Historic Day: July 4

2012

$m_H = 125 \text{ GeV}/c^2$

San Diego
“Big Bay Boom”
Historic Day: July 4

2012

m_H = 125 \text{ GeV}/c^2

San Diego
“Big Bay Boom”

2017

m_{AML} = 1.813 \times 10^{27} \text{ GeV}/c^2

Andreas Maria Lee,
Los Alamos, NM
New observables
Jet algorithms, substructure, grooming

QCD

Phases of QCD
Confinement, QGP

AdS/CFT
AdS/QCD

Unitarity, Amplitudes

pQCD
$N^k$LO, factorization, resummation

EFTs
SCET, NRQCD, HQET, $\chi$PT

Parton showers, Monte Carlo

PDFs
quasi-PDFs, nPDFs

Lattice QCD
Outline

• Soft Collinear Effective Theory
• $N$-Jettiness, SCET$_I$, $N^3$LL resummation
• SCET$_{II}$ and $N^3$LL resummed TMD distributions
• Subtractions and NNLO cross sections
• Non-Global Logarithms, fixed order and resummed
• Jet substructure and SCET$_+$
• Outlook
Formation of Jets in QCD

- Perturbative soft and collinear splittings happen at intermediate time: \( \alpha_s \lesssim 1 \)
- Hadronization at late time at low energy scale: \( \alpha_s \gg 1 \)
- Probability of splitting: \( \sim \frac{1}{E_g (1 - \cos \theta)} \)
- Soft and collinear enhancements
- Production of a new jet suppressed by: \( \alpha_s \ll 1 \)
- Need to resum large perturbative logs
- Separate pert. and non-pert. physics
  - Both are problems of scale separation: a job for EFT
History of Jets in QCD

- Existence of gluons:
- Measurements of strong coupling:
- Boosted heavy particles in SM and BSM
Separation of scales

- Large logs in QCD arise from large ratios of physical scales defining the measurement or degree of exclusivity of a jet cross section.

- For jet cross sections, these are precisely ratios of hard to soft scales and ratios of collinear momentum components.

  - e.g. measurement of jet mass

\[
p^2_J = (p_c + p_s)^2 = m^2_J
\]

\[
p = (\vec{n} \cdot p, n \cdot p, p_{\perp})
\]

Hierarchy of scales

Factorize cross section into pieces depending on only one of these scales at a time.

- Hard
  \[\mu_H = Q\]

- Jet
  \[\mu_J = m_J\]

- Soft
  \[\mu_S = \frac{m^2_J}{Q}\]
Modern tools for high precision resummation, factorization of perturbative and nonperturbative effects


\[ C(Q, \mu) \times \]

Decoupled collinear jet/beam functions

Decoupled soft function

SCET:

QCD:

Hard matching coefficient

Collinear to \( n_1 \)

Collinear to \( n_2 \)

Power expansion

\[ \ln \sigma(\tau) \sim \alpha_s \left( \ln^2 \tau + \ln \tau \right) + \alpha_s^2 \left( \ln^3 \tau + \ln^2 \tau + \ln \tau \right) + \alpha_s^3 \left( \ln^4 \tau + \ln^3 \tau + \ln^2 \tau + \ln \tau \right) + \ldots \]

Leading Log (LL)

Next-to-Leading Log (NLL)

NNLL

\( \ldots \)

\( \ldots \)

\( \ldots \)

\( \ldots \)

\( \ldots \)

Soft Collinear Effective Theory

- **RG Evolution**
  - Hard scale: \( \mu_H = Q \)
  - Jet/beam scale: \( \mu_J, B = Q \sqrt{\tau} \)
  - Soft scale: \( \mu_S = Q \tau \)

- **Resummation of large logs**

  \[ \ln \sigma(\tau) \sim \alpha_s \left( \ln^2 \tau + \ln \tau \right) + \alpha_s^2 \left( \ln^3 \tau + \ln^2 \tau + \ln \tau \right) + \alpha_s^3 \left( \ln^4 \tau + \ln^3 \tau + \ln^2 \tau + \ln \tau \right) + \ldots \]
Soft Collinear Effective Theory

- **SCET\textsubscript{I}**
  
  Theory for jets constrained by mass

- **SCET\textsubscript{II}**
  
  Theory for jets constrained by transverse momentum or for exclusive collinear hadrons

**Equation:**

\[ E + p_z \]

- Hard, collinear, soft all separated by **virtuality**
- Collinear/soft decoupling and factorization
- Dim. Reg. regulates all divergences

**Diagram:**

- Remove hard modes from theory
- \( p^2 \sim Q^2 \)
- \( p^2 \sim Q^2 \lambda^2 \)
- \( p^2 \sim Q^2 \lambda^4 \)

**Bibliography:**

- Bauer, Fleming, Pirjol, Stewart (2001)
Challenges to Precision Jet Cross Sections

- Jet cross sections typically depend on
  - choice of jet algorithm
  - jet sizes
  - jet vetoes (for exclusive jet cross sections)
- These parameters generate a number of logarithms (non-global logs, logs of radii $R$, etc.) in perturbation theory which are challenging to resum
- **N-Jettiness**: a *global* observable picking out $N$-jet final states by measurement of a *single* parameter, logs of which *can* be resummed in perturbation theory by standard RGE
$N$-jettiness

- A global event shape measuring degree to which final state is $N$-jet-like. (small $N$-jettiness vetoes events with more than $N$ jets.)

$$\tau_N = \frac{2}{Q^2} \sum_k \min\{q_A \cdot p_k, q_B \cdot p_k, q_1 \cdot p_k, \ldots, q_N \cdot p_k\}$$

groups particles into regions, according to which vector $q_i$ is closest.

Stewart, Tackmann, Waalewijn (2010)

Factorization and Resummation-friendly
**N^3LL resummation with SCET**

\[ \frac{1}{\sigma} \frac{d\sigma}{d\tau} \quad \text{(2-Jettiness)} \]

**e^+e^- Thrust**

\[ Q=m_Z \]  
Sum Logs, with \( S^{\text{mod}} + \text{gap} \)

- \( N^3LL' \)
- \( N^3LL \)
- \( \text{NLL}' \)
- \( \text{NNLL} \)
- \( \text{NNLL}' \)

**e^+e^- Hemisphere Jet Mass**

\[ \frac{1}{\sigma} \frac{d\sigma}{d\rho} \]

- 1st order
- 2nd order
- 3rd order
- 4th order

**e^+e^- C Parameter**

**DIS ep 1-Jettiness**

- NLL
- NNLL
- \( Q=100 \text{ GeV} \)
- \( x=0.1 \)

**Compare fixed order:**

- \( \mathcal{O}(\alpha_s^3) \)
- \( \mathcal{O}(\alpha_s^2) \)
- \( \mathcal{O}(\alpha_s) \)

Chien, Schwartz (2010)

Hoang, Kolodrubetz, Mateu, Stewart (2014)

Kang, CL, Stewart (preliminary, 2017)
High precision strong coupling

from SCET predictions for $e^+e^-$ event shapes

Hoang, Kolodrubetz, Mateu, Stewart (2015)
NNLL resummation for generic observables

- We do not always have a factorization theorem available to make SCET and its RG evolution to achieve resummation
- Monte Carlo implementation ARES (successor to NLL CAESAR) of emission amplitudes needed for NNLL

Banfi, McAslan, Monni, Zanderighi (2016)
High precision $p_T$ resummation at LHC

\[ \frac{d\sigma}{dq_T^2 dy} = \sigma_0 H(Q^2, \mu) \int d^2 \tilde{q}_{Ts} d^2 \tilde{q}_{T1} d^2 \tilde{q}_{T2} \delta(q_T^2 - |\tilde{q}_{Ts} + \tilde{q}_{T1} + \tilde{q}_{T2}|^2) \times S(\tilde{q}_{Ts}, \mu, \nu) f_1^\perp(x_1 = \frac{Q}{\sqrt{s}} e^y, Q, \tilde{q}_{T1}, \mu, \nu) f_2^\perp(x_2 = \frac{Q}{\sqrt{s}} e^{-y}, Q, \tilde{q}_{T2}, \mu, \nu) \]

- **SCET\textsubscript{II}**

- **Rapidity Renormalization Group**

**New rapidity regulator and 3-loop anomalous dimension**

\[
\frac{d\sigma}{dq_T^2 dy} = \sigma_0 \pi (2\pi)^2 H(Q^2, \mu) \int db b J_0(bq_T) \tilde{S}(b, \mu, \nu) \tilde{f}_1^+(b, Q, x_1, \mu, \nu) \tilde{f}_2^+(b, Q, x_2, \mu, \nu)
\]

\[
\tilde{f}(\vec{b}) \equiv \int \frac{d^2q_T}{(2\pi)^2} e^{i\vec{b} \cdot \vec{q}_T} f(q_T) \quad \quad \tilde{f}(\vec{b}) \equiv \frac{1}{2\pi} \tilde{f}(\vec{b}) , \quad b \equiv |\vec{b}|
\]

- Computation of beam or soft functions requires regulation of rapidity divergences:

- Regulator: shift separation of soft Wilson lines defining soft function in Euclidean time

\[
\lim_{\nu \to \infty} \int_0^\infty \frac{dk^+}{k^+}
\]

\[
\prod_i \int \frac{d^{d-1}k_i}{2k_i^0(2\pi)^3} \to \left( \prod_i \int \frac{d^{d-1}k_i}{2k_i^0(2\pi)^3} \right) \exp \left( - \sum_j b_j^0 k_j^0 \nu \right)
\]

Li, Neill, Zhu (2016)
$N^3LL$ resummed $p_T$ spectrum

- 3-loop soft function diagrams:
- $N^3LL$ resummed results:
- All three-loop integrals for threshold soft function known

3-loop rapidity anomalous dimension:

- $\gamma_0^R = 0$
- $\gamma_1^R = C_A C_F \left( \frac{28 C_A}{9} - \frac{808}{27} \right) + \frac{112 C_{A,W}}{27}$
- $\gamma_2^R = C_A C_F \left( -\frac{176}{3} C_A + \frac{6302 C_F}{81} + \frac{12328 C_A}{27} + 44 C_4 - 192 C_5 - \frac{207020}{729} \right)$
- $+ C_A C_{A,W} \left( \frac{824 C_A}{81} - \frac{904 C_4}{27} + 8 C_4 + \frac{62626}{729} \right) + \epsilon \beta_0$
- $+ C_A C_{A,W} \left( \frac{32 C_A}{9} - \frac{1856}{729} \right) + C_A C_F N_f \left( \frac{304 C_A}{9} - 16 C_4 + \frac{1711}{27} \right)$

New three loop results!

Li, Neill, Schulze, Stewart, Zhu
(SCET2016, Argonne Advances in QCD 2016)
NNLO Subtractions

- **NNLO:**
  - Real-Real
  - Virtual-Real
  - Virtual-Virtual

- **Local Subtractions:** Colorful NNLO, Sector Decomposition, Antenna Subtraction, …
  - [Del Duca, Duhr, Kardos, Somogyi, Trocsanyi, Tulipant]
  - [Anastasiou, Melnikov, Petriello]
  - [Gehrmann-De Ridder, Gehrmann, Glover et al.]

- **Global Subtractions:** $q_T$, $N$-jettiness, …
  - [Catani, Grazzini]
  - [Boughezal, Focke, Petriello, Liu]
  - [Gaunt, Stahlhofen, Tackmann, Walsh]

\[
\sigma(X) = \int_0 dT \frac{d\sigma(X)}{dT} = \int_0^{T_{\text{cut}}} dT \frac{d\sigma(X)}{dT} + \int_{T_{\text{cut}}}^T dT \frac{d\sigma(X)}{dT}
\]
N-Jettiness Subtractions

• Exploit factorization and 2-loop computations of ingredients for small $\tau_N$

$$\sigma(\tau_{\text{cut}}) = \frac{\tau_{\text{cut}}^{\text{cut}}}{\int_0 d\tau \frac{d\sigma(X)}{d\tau}}$$

Compute using factorization in soft/collinear limits:

$$\frac{d\sigma}{d\tau} = HB_a \otimes B_b \otimes S \otimes J_1 \otimes \cdots \otimes J_{N-1}$$

Additional jet resolved.
Use NLO subtractions.

Boughezal, Liu, Focke, Petriello (2015)
Gaunt, Stahlhofen, Tackmann, Walsh (2015)

• High precision, numerical stability requires power corrections:

$$\sigma(\tau_{\text{cut}}) = \int_0^{\tau_{\text{cut}}} d\tau \frac{d\sigma}{d\tau} = \sum_{n=0}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n \sum_{n=0}^{2n-1} c_{nm}^{(0)} \log^m(\tau_{\text{cut}}) + \tau_{\text{cut}}$$

Leading Power

Power Corrections

$$= \sum_{n=0}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n \sum_{n=0}^{2n-1} c_{nm}^{(2)} \log^m(\tau_{\text{cut}}) + \cdots$$
Subleading Power Corrections

- SCET well formulated to compute power corrections:
  Moult, Rothen, Stewart, Tackmann, Zhu (2016)

- Also computable in fixed-order QCD, dramatic improvement in $\tau_{\text{cut}}$ independence:
  Boughezal, Liu, Petriello (2016)
NNLO Results $V+\text{jet}$

- $N$-jettiness subtraction method vs. antenna subtraction:

- vs. data:

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Boughezal, Campbell, Ellis, Focke, Giele, XL, Petriello Gehrmann-De Ridder, Gehrmann, Glover, Huss, Morgan

Boughezal, Liu, Petriello (2016)
NNLO Revolution

explosion of calculations in past 18 months

X. Liu DPF 2017
Non-global logs

- **Global observable:** 
  
  *(thrust, N-jettiness)*

- **Non-global observable:**
  
  *(double hemisphere mass, jet vetoes)*

D. Neill SCET 2017
Non-global logs

- Start to spoil “global” resummation at 2 loops:

\[
\sigma(m_H/m_L) = \sigma_{gl}(m_H/m_L) \left[ 1 + \frac{\alpha_s^2}{(2\pi)^2} C_F C_A \frac{\pi^2}{3} \ln^2 \frac{m_H}{m_L} + \cdots \right]
\]

Conjecture / fit to Monte Carlo resummation (large \( N_c \)):

\[
S_{ng} = \exp \left[ -C_F C_A \frac{\pi^2}{3} \left( \frac{1 + (at)^2}{1 + (bt)^c} \right) t^2 \right]
\]

\[
t = \frac{1}{4\pi\beta_0} \ln \frac{1}{1 - 2\beta_0 \alpha_s L}
\]

\[
L = \ln \frac{m_H}{m_L}
\]

\[
a = 0.85C_A, \quad b = 0.86C_A, \quad c = 1.33
\]
Fixed-order computations

- Soft functions for non-global observables in SCET, two-loop computations, and subleading (single) NGLs
  
  Kelley, Schabinger, Schwartz, Zhu (2011); Hornig, CL, Stewart, Walsh, Zuberi (2011)

- 5 loops:

\[
g_{\bar{n}\bar{n}}(L) = 1 - \frac{\pi^2}{24} L^2 + \frac{\zeta(3)}{12} L^3 + \frac{\pi^4}{34560} L^4 + \left( -\frac{\pi^2 \zeta(3)}{360} + \frac{17 \zeta(5)}{480} \right) L^5 + \ldots
\]

Schwartz, Zhu (2014)

- 12 loops!

Caron-Huot
Factorization and Resummation of NGLs

Larkoski, Moult, Neill (2015)

\[ e^+ e^- \rightarrow 2j + 1_{sj} : \]

\[ \frac{d\sigma}{de_2^{(\alpha)} de_2^{(\beta)} de_3^{(\beta)} dB} = H_{nn} H_{nn}^{sj} (e_2^{(\alpha)}, e_2^{(\beta)}) J_{n_{sj}} (e_3^{(\beta)}) \otimes S_{n_{sj} n_{sj}} (e_3^{(\beta)}) \]

\[ \otimes S_{n_{n_{sj}}} (e_3^{(\beta)}; B) \otimes J_n (e_3^{(\beta)}) \otimes \bar{J}_n (B) \]

RG evolve, integrate over to obtain original non-global distribution, now resummed!
Resummed computations

Larkoski, Moult, Neill (2015)
Singularities and Buffers

- Boundary Soft RG implies:
  \[ U_{abj} \propto \left( 1 - \frac{\tan^2 \theta_{sj}}{\tan^2 \frac{R}{2}} \right)^L \]

- Cross-section for production of a jet at the boundary vanishes!

- Buffer region and singularities in \( L \) reproduced by resummed calculation with jets, but not fixed-order calculation with partons

- Take fixed order series and apply conformal mapping obeying proper singularities in \( L \) and buffer region:
  \[ g_{ab}(L) \to g_{ab}(u). \]
  \[ g_{ab}(u) = 1 + b_1 u + b_2 u^2 + \ldots. \]

Example \( u \)'s:

\[ u(L) = \begin{cases} 
\ln(1 + L), & \text{log mapping} \\
\frac{\sqrt{1 + L} - 1}{\sqrt{1 + L} + 1}, & \text{disc mapping}
\end{cases} \]

Determine \( b \)'s by matching taylor series at \( L = 0 \).

Larkoski, Moult, Neill (2016)
Conformal improvement of fixed-order NGLs

Caron-Huot

Larkoski, Moult, Neill (2016)
Jet Substructure

1-prong

2-prong

3-prong:
Energy Correlators

\[ v e_n^{(\beta)} = \sum_{1 \leq i_1 < i_2 < \ldots < i_n \leq n_J} z_{i_1} z_{i_2} \ldots z_{i_n} \prod_{m=1}^{v} \min_{s<t \in \{i_1, i_2, \ldots, i_n\}} \{ \theta_{st}^{\beta} \} \]

Moult, Necib, Thaler (2016)

good discriminants:

definite power counting, amenable to factorization and precision calculation

3-prong (top):

\[ N_3^{(\beta)} = \frac{2 e_4^{(\beta)}}{1 e_3^{(\beta)} e_2^{(\beta)}} \]

2-prong:

\[ M_2^{(\beta)} = \frac{1 e_3^{(\beta)}}{1 e_2^{(\beta)}} \]

1-prong (q vs g):

\[ U_i^{(\beta)} = 1 e_{i+1}^{(\beta)} \]

Grooming and Soft Drop

contamination:

out-of-jet perturbative radiation re-emission

Perturbative Radiation

Underlying Event

Pile-up

Larkoski, Marzani, Soyez, Thaler (2014)

Soft Drop: \[ \frac{\min[p_{Ti}, p_{Tj}]}{p_{Ti} + p_{Tj}} > z_{\text{cut}} \left( \frac{R_{ij}}{R} \right)^{\beta} \]

grooming:
Soft Drop

• Simplifies theoretical calculations:

\[ \beta = 0 \]
Groomed substructure

- **Top tagging:**

  - **Groomed $N_3^{(2)}$:**
    - Pythia 8.219, Soft Drop: $\beta=0$, $z_{cut}=0.1$
    - $R=1.0$, $p_T>500$ GeV, $m_{SP}>80$ GeV

  - Plots showing distributions and comparison with various tagging criteria (e.g., $Q_3$, $Q_3^2$).

- **q vs g:**

  - Pythia 8.219, $R=0.6$, $p_T>500$ GeV

  - Plots showing probability density and mis tagged quark/gluon efficiencies.

Moult, Necib, Thaler (2016)
Groomed substructure and SCET+$^+$

- Soft drop groomed energy correlators:

\[
\frac{d^2\sigma}{de_2^{(\alpha)}de_2^{(\alpha)}} = H(Q^2)S_G(z_{cut}) \left[ S_C\left(z_{cut}e_2^{(\alpha)}\right) \otimes J(e_2^{(\alpha)}) \right] \left[ S_C\left(z_{cut}e_2^{(\alpha)}\right) \otimes J(e_2^{(\alpha)}) \right]
\]

free of NGLs; correlated hierarchical emissions groomed away

Frye, Larkoski, Schwartz, Yan (2016)

SCET$^+$: Bauer, Tackmann, Walsh, Zuberi (2011)
NNLL substructure calculations

Frye, Larkoski, Schwartz, Yan (2016)
Many other EFT directions

• Connection of NGLs and small-\(x\) evolution (BFKL)

• SCET with Glauber modes for factorization violating effects, small-\(x\) resummation, forward scattering
  I. Rothstein and I. Stewart (2016)

• \(\text{SCET}_G\) for jets in heavy-ion collisions
  A. Idilbi and A. Majumder (2008)
  G. Ovanesyan and I. Vitev (2011)

• SCET + NRQCD for improved description of quarkonia in jets, discriminate production mechanisms
  Baumgart, Leibovich, Mehen, Rothstein (2014)
  Bain, Dai, Leibovich, Makris, Mehen (2016-17)

• \(\text{SCET}_{\text{EW}}\) for resummation of electroweak logs in colliders, dark matter production and annihilation
  Chiu, Golf, Kelley, Manohar (2007)
  Ovanesyan, Slatyer, Stewart (2014)
  Baumgart, Rothstein, Vaidya (2014)
  etc.
In the last several years, we have gained a collection of EFT and other powerful tools for high precision calculations of observables in multiscale jet-like processes, making possible fixed-order and resummed calculations to orders previously unachievable and the solution of problems previously intractable in QCD.
The future holds great promise
Extra slides
Jet Algorithms and Radii

- Example: $e^+e^-$ to two jet cross section:

- One-loop cross section in QCD:
  - in a cone algorithm:
    \[
    \frac{\sigma_{2\text{-jet}}}{\sigma_0} = 1 + \frac{\alpha_s C_F}{\pi} \left( -4 \ln \frac{2E_0}{Q} \ln R - 3 \ln R - \frac{1}{2} + 3 \ln 2 \right)
    \]
  - in a kT-type recombination (or Sterman-Weinberg) algorithm:
    \[
    \frac{\sigma_{2\text{-jet}}}{\sigma_0} = 1 + \frac{\alpha_s C_F}{\pi} \left( -4 \ln \frac{2E_0}{Q} \ln R - 3 \ln R - \frac{\pi^2}{3} + \frac{5}{2} \right)
    \]
  - Natural to use SCET to factorize and resum, but structure of logs is surprisingly subtle.
Soft and Soft-Collinear phase space

- collinear and soft phase space for cone and kT algorithms:

\[ \begin{align*}
Q & \quad p_g^+ \\
2E_0 & \quad p_g^+ = p_g^- / R \\
Q & \quad (p_g^-) \\
2\Lambda & \quad p_g = (p_g^-, p_g^+, p_g^\perp) \\
Q & \quad p_g^+ \\
2E_0 & \quad p_g^- \\
Q & \quad Q \\
2\Lambda & \quad Q \\
Q & \quad Q
\end{align*} \]
Soft and Soft-Collinear phase space

- Soft phase space splits into two, single-scale-sensitive regions:

\[ S_{\text{veto}}(E_0, R, \mu) \]
\[ S_s(E_0, \mu) \]
\[ -2S_{sc}(E_0 R, \mu) \]

\[
\alpha_s C_F \frac{4\pi}{2E_0} \left( 8 \ln R \ln \frac{\mu^2}{4E_0^2 R} - \frac{2\pi^2}{3} \right)
\]
\[
\alpha_s C_F \frac{4\pi}{2E_0} \left( 8 \ln^2 \frac{\mu}{2E_0} - \pi^2 \right)
\]
\[
\alpha_s C_F \frac{4\pi}{2E_0} \left( -8 \ln^2 \frac{\mu}{2E_0 R} + \frac{\pi^2}{3} \right)
\]
Hard scale \( \mu_H = Q \)

Jet scale \( \mu_J = Q\sqrt{\tau} \)

Csoft scale \( \mu_S = Q\tau/R \)

Global soft (veto) scale \( \mu_\Lambda = 2\Lambda \)

Soft-collinear scale \( \mu_{sc} = 2\Lambda R \)

\[ Q(1, \tau, \sqrt{\tau}) \]

\[ Q\tau\left(\frac{1}{R^2}, 1, \frac{1}{R}\right) \]

\[ (E_0, E_0, E_0) \]

\[ E_0(1, R^2, R) \]

Chien, Hornig, CL (2015)
Resummed jet thrust cross section

- Integrated jet thrust in $e^+e^-$:

$$\sigma_{e^+e^-}(T)$$

With s-c refactorization

$$R = 0.2, \Lambda = 10 \text{ GeV}, Q = 100 \text{ GeV}$$

No s-c refactorization

$$R = 0.2, \Lambda = 10 \text{ GeV}, Q = 100 \text{ GeV}$$

- Improved perturbative convergence thanks to additional logs resummed after soft-collinear refactorization
Resummed jet thrust cross section

- $pp$ jet angularity differential distribution:

A. Hornig, Y. Makris, T. Mehen (2016)

- Larger impact on differential shape

without soft-collinear refactorization

with soft-collinear refactorization
NP Corrections

- Reminder: Dokshitzer-Webber model

\[ \langle e \rangle = \langle e \rangle_{PT} + c_e \frac{\Omega_1}{Q} \]

- SCET: First rigorous proof (and **field theory** definition of \( \Omega_1 \)) from factorization theorem and boost invariance of soft radiation:

\[ \Omega_1 = \frac{1}{N_C} \text{Tr} \langle 0 | \bar{Y}_n Y_n^{\dagger} E_T(\eta) Y_n \bar{Y}_n | 0 \rangle \]

“energy flow” operator

soft radiation sees only direction, not energy, of original collinear partons, invariant to boosts along z

\[ c_e \text{ observable dependent, calculable coefficient} \]

\[ \Omega_1 \text{ universal nonperturbative parameter} \]

(conjecture from single soft gluon emission: Dokshitzer, Webber (1995, 1997))

(proof to all orders in soft gluon emission: CL, Sterman (2006, 2007))
Momentum Flow Operators

generic form of event shapes: \[ e(X) = \frac{1}{Q} \sum_{i \in X} f_e(\eta_i) |P_T^i| \]
e.g. angularities \[ f_{\tau_\alpha}(\eta) = e^{-|\eta|(1-a)} \]

operator action in terms of transverse momentum flow operator:

\[ \hat{e} |X\rangle \equiv e(X) |X\rangle = \frac{1}{Q} \int_{-\infty}^{\infty} d\eta f_e(\eta) \mathcal{E}_T(\eta; \hat{t}) |X\rangle \]

\[ \mathcal{E}_T(\eta) |X\rangle = \sum_{i \in X} |P_T^i| \delta(\eta - \eta_i) |X\rangle \]

construct out of energy-momentum tensor of QCD:

\[ \mathcal{E}_T(\eta) = \frac{1}{\cosh^3 \eta} \int_0^{2\pi} d\phi \lim_{R \to \infty} R^2 \int_0^{\infty} dt \hat{n}_i T_{0i}(t, R\hat{n}) \]

measures total transverse momentum \(|P_T|\)
flowing through slice of sphere at rapidity \(\eta\)
from collision time \(t=0\) to detector at \(t \to \infty\)
\(R \to \infty\)

since Lagrangian of SCET factors into collinear and soft sectors, so does the energy-momentum tensor:

\[ T_{\mu\nu} \rightarrow T^c_{\mu\nu} + T^s_{\mu\nu} + T^s_{\mu\nu} \]
Proof of universality

• In general NP part of soft function must be modeled and is observable-dependent:

\[ S(e, \mu, \Lambda) = \int_0^\infty de' S_{PT}(e - e', \mu) F_{NP}(e', \Lambda) \]

• The universality of the first moment, however, can be proven exactly:

\[ \Delta \langle e \rangle_s = \frac{1}{Q} \int_{-\infty}^\infty d\eta \, f_e(\eta) \frac{1}{N_C} \text{Tr} \langle 0 | T[Y_n^\dagger Y_n] \mathcal{E}_T(\eta) T[Y_n^\dagger Y_n] | 0 \rangle \]

Theorem by rapidity \( \alpha \) along \( z \):

\[ Y_n = P \exp \left[ i g \int_0^\infty ds \, n \cdot A_s(ns) \right] \quad \Rightarrow \quad Y_n \]

\[ |0\rangle \quad \Rightarrow \quad |0\rangle \]

\[ \mathcal{E}_T(\eta) \quad \Rightarrow \quad \mathcal{E}_T(\eta + \alpha) \]

\[ \Delta \langle e \rangle_s = \frac{1}{Q} \left\{ \int_{-\infty}^\infty d\eta \, f_e(\eta) \right\} \left\{ \frac{1}{N_C} \text{Tr} \langle 0 | T[Y_n^\dagger Y_n] \mathcal{E}_T(0) T[Y_n^\dagger Y_n] | 0 \rangle \right\} \]

\[ c_e \quad \quad \quad \quad \quad \quad \quad \quad \quad \Omega_1 \]

e.g. \[ c_T = 2 \quad c_C = 3\pi \quad c_{\tau\alpha} = \frac{2}{1 - a} \quad \text{for } e^+e^- \text{ scaling is obeyed well by LEP data} \]
**e^+e^- Thrust: Precision extraction of \( \alpha_s \) (2-jettiness)**

NNNLL perturbative prediction + nonperturbative soft power correction led to most precise extraction of strong coupling from event shapes

Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

NNNLL resummed perturbative distribution

Becher, Schwartz (2008)

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**Compare fixed order:**

![Graph showing comparison of fixed order predictions](image)

**Fixed Order**

- \( \mathcal{O}(\alpha_s^3) \)
- \( \mathcal{O}(\alpha_s^2) \)
- \( \mathcal{O}(\alpha_s) \)

![Graph showing fixed order predictions](image)

**Notes**

- NNNLL: Next-to-next-to-next-to-leading order
- NP: Nonperturbative
- \( \alpha_s(m_Z) \): 
  - 0.1300 ± 0.0047
  - 0.1245 ± 0.0034
  - 0.1192
  - \( N^3\text{LL} \) summation
  - + multijet boundary
  - + Power Corrections
  - + \( N^3\text{LL} \) summation

- -7.5% shift from NP power corrections

Abbate, Fickinger, Hoang, Mateu, Stewart (2010)
e⁺e⁻ Thrust: Precision extraction of $\alpha_s$ (2-jettiness)

NNNLL perturbative prediction + nonperturbative soft power correction led to most precise extraction of strong coupling from event shapes

Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

NNNLL resummed perturbative distribution Becher, Schwartz (2008)

$\alpha_s(m_Z)$ from global thrust fits

-7.5% shift from NP power corrections

Generically, better perturbative calculations + rigorous treatment of nonperturbative corrections gives smaller $\alpha_s$
Beam Function and PDFs

transverse momentum dependent beam function:

\[
B(\omega k^+, x, k_\perp^2, \mu) = \frac{\theta(\omega)}{\omega} \int \frac{dy^-}{4\pi} e^{i k^+ y^- / 2} \langle P_n(P^-) | \bar{\chi}_n \left( y^- \frac{n}{2} \right) \delta(x P^- - \bar{n} \cdot \mathcal{P}) \delta(k_\perp^2 - P_\perp^2) \chi_n(0) | P_n(P^-) \rangle
\]

\[
f(x, \mu) = \theta(\omega) \langle P_n(P^-) | \bar{\chi}_n(0) \delta(x P^- - \bar{n} \cdot \mathcal{P}) \chi_n(0) | P_n(P^-) \rangle
\]

\[
B_q(t, x, k_\perp^2, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij} \left( t, \frac{x}{\xi}, k_\perp^2, \mu \right) f_j(\xi, \mu)
\]

Measure small light-cone momentum \( k^+ = t / P^- \)

and transverse momentum \( k_\perp \) of initial state radiation
**Generalized Beam Function to 1-loop**

\[ \mathcal{B}_q(t, x, k_{\perp}^2, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}(t, x, k_{\perp}^2, \mu) f_j(x, \mu) \]

now known to 2 loops; anomalous dimension known to 3 loops

Stewart, Tackmann, Waalewijn (2009)

\[ \mathcal{I}_{qq}(t, z, k_{\perp}^2, \mu) = \frac{1}{\pi} \delta(t) \theta(z) \left\{ \frac{\theta(t) \ln(t/\mu^2)}{t/\mu^2} \right\} + \delta(1-z) \delta(k_{\perp}^2) \]

\[ + \frac{1}{\mu^2} \theta(t) \left[ \frac{P_{qq}(z) - 3/2}{t/\mu^2} \right]_{+} + \delta(t) \delta(k_{\perp}^2) \left[ \frac{(1-z)\ln(1-z)}{1-z} \right]_{+} \]

Jain, Procura, Waalewijn (2009)

\[ \mathcal{I}_{qg}(t, z, k_{\perp}^2, \mu) = \frac{\alpha_s(\mu) T_F}{2\pi^2} \theta(z) \left\{ \frac{1}{\mu^2} \left[ \frac{\theta(t)}{t/\mu^2} \right] \right\} P_{qg}(z) \delta\left( k_{\perp}^2 - \frac{(1-z)t}{z} \right) + \delta(t) \delta(k_{\perp}^2) \left[ P_{qg}(z) \ln \frac{1-z}{z} + 2\theta(1-z)z(1-z) \right] \]

\[ \text{(162a)} \]

Tells us that PDFs should be evaluated at the beam radiation scale \( t \)

ordinary beam function: \[ B(t, x, \mu) = \int d^2k_{\perp} B(t, x, k_{\perp}^2, \mu) \]

Stewart, Tackmann, Waalewijn (2009)

Gaunt, Stahlhofen, Tackmann (2014)
POWER CORRECTIONS IN $pp$ AND DIS

- Universal nonperturbative shift in 3 versions of DIS 1-jettiness:

Using factorization theorems and boost invariance properties of soft Wilson lines, can prove that:

$$\Omega^a_1 = \Omega^b_1 = \Omega^c_1$$

D. Kang, CL, I. Stewart (2013)

- Surprising relation also to leading NP correction to jet mass in $pp$ to 1 jet

Stewart, Tackmann, Waalewijn (2014)

For $R \ll 1$, the beam Wilson lines fuse and $\Omega = \frac{R}{2} \Omega_0 + \ldots$

The universal $\Omega^q_0$ can be extracted from DIS event shapes
Experimental Coverage

Preliminary: Kang, CL, Stewart (2016)

HERA EIC (high)
HERA EIC (low)

existing HERA event shape analyses
preliminary theoretical uncertainty (N^3LL)

New analyses of HERA data for 1-jettiness under way!
TMD resummation

\[ \sigma(b, z_1, z_2; \mu_i, \nu_i; \mu, \nu) = U_{\text{tot}}(\mu_i, \nu_i; \mu, \nu) H(Q^2, \mu_H) \tilde{S}(b; \mu_L, \nu_L) \]
\[ \times \tilde{f}_\perp(b, z_1; \mu_L, \nu_H) \tilde{f}_\perp(b, z_2; \mu_L, \nu_H) \]

\[ U_{\text{tot}}(\mu_i, \nu_i, \mu, \nu) = \exp \left\{ 4K_\Gamma(\mu_L, \mu_H) - 4\eta_\Gamma(\mu_L, \mu_H) \ln \frac{Q}{\mu_L} - K_\gamma(\mu_L, \mu_H) \right. \]
\[ + \left. \left[ -4 \eta_\Gamma(1/b_0, \mu_L) + \gamma_{RS} \left[ \alpha_s(1/b_0) \right] \right] \ln \frac{\nu_H}{\nu_L} \right\} \]

Standard scale choices:

\[ \mu_L = \nu_L = \frac{2}{b e \gamma_E} \equiv \frac{1}{b_0} \]
\[ \mu_H = \nu_H = Q \]

Landau pole
TMD resummation in momentum space

Our scale choices:

\[ \nu_L^* = \nu_L(\mu_L b_0)^{-1+n} \]

\[ \nu_L \sim \mu_L \]

chosen in momentum space, after \( b \) integration

Analytic formula:

\[ I_b = \frac{2C}{\pi q_T^2} \sum_{n=0}^{\infty} \text{Im} \left\{ c_{2n} \mathcal{H}_{2n}(\alpha, a_0) + \frac{i \gamma_E}{\beta} d_{2n+1} \mathcal{H}_{2n+1}(\beta, b_0) \right\} \]

Kang, CL, Vaidya (2017)