

# Gauge Model for Minimal Flavor Violation

**K.S. Babu**

Oklahoma State University

DPF Meeting

Fermi National Accelerator Laboratory

July 29 – August 4, 2017

Based on: [K.S. Babu](#), [P. Ko](#), [S. Saad](#) (to appear)

# Outline

- Motivations for gauging flavor symmetries
- $O(3)_L \times O(3)_R$  flavor gauge model
- Explaining  $B$  decay anomalies via  $O(2)_L \times O(2)_R$  gauge bosons
- Conclusions

# Gauging Flavor Symmetries

The Standard Model gauge sector has a  $[U(3)]^5$  global symmetry:

$$U(3)_Q \times U(3)_{u^c} \times U(3)_{d^c} \times U(3)_L \times U(3)_{e^c}$$

Gauge Principle: “All anomaly free symmetries should be gauged”

Maximal Flavor Symmetry that is anomaly free:

- |     |   |   |
|-----|---|---|
| (A) | $O(3)_{\{Q,L\}} \times O(3)_{\{u^c,d^c,e^c\}}$  | $[O(3)_L \times O(3)_R]$                            |
| (B) | $O(3)_{\{Q,u^c,e^c\}} \times O(3)_{\{L,d^c\}}$  | $[O(3)_{10} \times O(3)_{\bar{5}}]$                 |
| (C) | $SU(3)_{\{Q,u^c,d^c\}} \times O(3)_{\{L,e^c\}}$ | $[O(3)_{\text{quark}} \times O(3)_{\text{lepton}}]$ |

Among these,  $O(3)_L \times O(3)_R$  is very promising – from fermion mass generation viewpoint – for gauge model of minimal flavor violation

KB, M. Frank, S. Rai (2011)

# Gauging Flavor Symmetries (cont.)

The 5  $U(1)$  factors of  $[U(3)]^5$  are:

$$Y, (B - L), (B + L), PQ, PQ'$$

Once  $\nu^c$  field is introduced for neutrino mass generation, anomaly-free maximal subgroups of  $[U(3)]^6$  become:

- (A)  $SO(3)_{\{Q,L\}} \times SU(3)_{\{u^c,d^c,e^c,\nu^c\}} \times U(1)_{B-L}$
- (B)  $SU(3)_{\{Q,u^c,d^c\}} \times SU(3)_{\{L,e^c,\nu^c\}} \times U(1)_{B-L}$
- (C)  $SO(3)_{\{Q,u^c,e^c\}} \times SU(3)_{\{L,d^c,\nu^c\}} \times U(1)_{B-L}$
- (D)  $SU(3)_{\{Q,u^c,d^c,L,e^c,\nu^c\}} \times U(1)_{B-L}$
- (E)  $SO(3)_{\{Q,u^c,e^c\}} \times SO(3)_{\{L,d^c\}} \times SO(3)_{\{\nu^c\}}$
- (F)  $SU(3)_{\{Q,u^c,d^c\}} \times SO(3)_{\{L,e^c\}} \times SO(3)_{\{\nu^c\}}$
- (G)  $SO(3)_{\{Q,L\}} \times SO(3)_{\{u^c,d^c,e^c\}} \times SO(3)_{\{\nu^c\}}$

Once  $\nu^c$  acquires large Majorana mass, case (A) will reduce to  $O(3)_L \times O(3)_R$

# $O(3)_L \times O(3)_R$ Flavor Gauge Model

- Left-handed quarks and leptons transform as triplets of  $O(3)_L$ . Right-handed quarks and leptons are triplets of  $O(3)_R$

$$Q : (3, 1), \quad L : (3, 1), \quad u^c : (1, 3), \quad d^c : (1, 3), \quad e^c : (1, 3)$$

- SM Higgs doublet  $H$  is  $(1, 1)$  under  $O(3)_L \times O(3)_R$
- Fermion mass generation requires vector-like isosinglet fermions:

$$U_L(1, 3) + U_R(3, 1); \quad D_L(1, 3) + D_R(3, 1); \quad E_L(1, 3) + E_R(1, 3)$$

- To generate masses for the vector-like fermions, and for  $O(3)_L \times O(3)_R$  symmetry breaking, SM singlet Higgs field  $\Phi(3, 3)$  is necessary
- Fermion masses are generated via a universal seesaw mechanism

# Fermion Mass Generation

Yukawa couplings of fermions:

$$\mathcal{Y}_{\text{Yukawa}} = y_u \bar{Q}_L \mathbb{I} U_R \tilde{H} + m_u^0 \bar{U}_L \mathbb{I} u_R + Y_U \bar{U}_L \Phi(3, 3) U_R + h.c. + \dots$$

Fermion mass matrices:

$$\mathcal{M}_{u,d,e} = \begin{pmatrix} 0_{3 \times 3} & y_{u,d,e} v \mathbb{I}_{3 \times 3} \\ m_{u,d,e}^0 \mathbb{I}_{3 \times 3} & M_{U,D,E} \end{pmatrix}, M_{U,D,E} = Y_{U,D,E} \begin{pmatrix} V_1 & & \\ & V_2 & \\ & & V_3 \end{pmatrix}$$

Light fermion masses:

$$m_u = \frac{y_u m_u^0 v}{M_1^u}, \quad m_c = \frac{y_u m_u^0 v}{M_2^u}, \quad m_t = \frac{y_u m_u^0 v}{M_3^u}$$

$$m_d = \frac{y_d m_d^0 v}{M_1^d}, \quad m_s = \frac{y_d m_d^0 v}{M_2^d}, \quad m_b = \frac{y_d m_d^0 v}{M_3^d}$$

$$m_e = \frac{y_e m_e^0 v}{M_1^e}, \quad m_\mu = \frac{y_e m_e^0 v}{M_2^e}, \quad m_\tau = \frac{y_e m_e^0 v}{M_3^e}$$

Inverse hierarchy of heavy fermion masses:  $M_1^u \gg M_2^u \gg M_3^u$

# Features of Heavy Fermions/Gauge Bosons

- $m_d : m_s : m_b = M_3^d : M_2^d : M_1^d \Rightarrow$  Inverse mass hierarchy among heavy fermions
- $M_1^d = Y_D V_1, M_2^d = Y_D V_2, M_3^d = Y_D V_3 \Rightarrow V_1 \gg V_2 \gg V_3$
- The 6  $O(3)_L \times O(3)_R$  gauge bosons ( $X_{L,R}, Y_{L,R}, Z_{L,R}$ ) have masses:

$$M_{X_L} = g_L V_2, M_{X_R} = g_R V_2,$$

$$M_{Y_L} = M_{Z_L} = g_L V_1, M_{Y_R} = M_{Z_R} = g_R V_1$$

- Since  $V_1 \gg V_2$ ,  $M_{X_L, X_R} \ll M_{Y_L, Y_R} \simeq M_{Z_L, Z_R}$
- $V_1$  breaks  $O(3)_L \times O(3)_R$  down to  $O(2)_L \times O(2)_R$  leaving  $X_{L,R}$  gauge bosons massless.  $V_2$  breaks  $O(2)_L \times O(2)_R$  to nothing.
- Lighter  $X_{L,R}$  couple to second and third families:
 
$$\begin{aligned} \mathcal{L}^X = & -ig_L \{ (\bar{b}_L \gamma_\mu s_L - \bar{s}_L \gamma_\mu b_L) + (\bar{\tau}_L \gamma_\mu \mu_L - \bar{\mu}_L \gamma_\mu \tau_L) \} X_L^\mu \\ & - ig_R \{ (\bar{b}_R \gamma_\mu s_R - \bar{s}_R \gamma_\mu b_R) + (\bar{\tau}_R \gamma_\mu \mu_R - \bar{\mu}_R \gamma_\mu \tau_R) \} X_R^\mu \end{aligned}$$
- Interesting phenomenology for  $B$  anomalies possible via exchange of  $X_{L,R}$  gauge bosons

# Minimal Flavor Violation?

- With a single bi-fundamental  $\Phi(3, 3)$  coupling to all fermions, there is no CKM mixing. Additional Higgs fields are needed
- The structure of the theory allows Yukawa couplings of these multiplets under  $O(3)_L \times O(3)_R$ :  
 $\chi_3(1, 3); \chi_5(1, 5), \Phi(3, 3)$
- At least two of these fields are needed to generate CKM mixing
- Few possibilities are explored:
  - (i) Two  $\Phi(3, 3)$  fields
  - (ii) One  $\Phi(3, 3)$ , one  $\chi_5(1, 5)$  field and one  $\chi_3(1, 3)$  field
  - (iii) Two  $\Phi(3, 3)$  fields, one  $\chi_5(1, 5)$  field and one  $\chi_3(1, 3)$  field
- Results for cases (i) and (iii) will be presented
- In case (i) there are only 2 Yukawa matrices for  $u, d, e$  – most minimal flavor violation (MMFV)
- Case (iii) is next to minimal FV (NMFV) with 4 Yukawa matrices for  $u, d, e$



# Gauge Model for MMFV

- Two copies of  $\Phi(3, 3)$  Higgs fields are utilized for symmetry breaking and fermion mass generation
- Two Yukawa matrices  $\langle \Phi_1 \rangle$  and  $\langle \Phi_2 \rangle$  must fit up, down and charged lepton masses as well as CKM mixings  $\Rightarrow$  MMFV
- Fermion mass matrices:

$$\mathcal{M}_{u,d,e} = \begin{pmatrix} 0_{3 \times 3} & y_{u,d,e} v \mathbb{I}_{3 \times 3} \\ m_{u,d,e}^0 \mathbb{I}_{3 \times 3} & M_{u,d,e} \end{pmatrix}, M_{u,d,e} = Y_1^{u,d,e} \langle \Phi_1 \rangle + Y_2^{u,d,e} \langle \Phi_2 \rangle$$

- Yukawa coupling are complex, while all VEVs are real
- Only 3 relative phases in Yukawa couplings
- Excellent fit to fermion masses and CKM mixing angles is found

# MMFV Scalar Potential

- Higgs potential with two bifundamental scalar fields  $\Phi_{1,2}(3, 3)$ :

$$\begin{aligned} V = & -\mu_1^2 \text{Tr}[\Phi_1 \Phi_1^T] + \lambda_1 \text{Tr}[\Phi_1 \Phi_1^T]^2 + \lambda_2 \text{Tr}[\Phi_1 \Phi_1^T \Phi_1 \Phi_1^T] + \xi_1 \text{Det}[\Phi_1] \\ & + -\mu_2^2 \text{Tr}[\Phi_2 \Phi_2^T] + \lambda_3 \text{Tr}[\Phi_2 \Phi_2^T]^2 + \lambda_4 \text{Tr}[\Phi_2 \Phi_2^T \Phi_2 \Phi_2^T] + \xi_2 \text{Det}[\Phi_2] \\ & + \lambda_5 \text{Tr}[\Phi_1 \Phi_1^T] \text{Tr}[\Phi_2 \Phi_2^T] + \lambda_6 \text{Tr}[\Phi_1 \Phi_2^T]^2 + \lambda_7 \text{Tr}[\Phi_1 \Phi_1^T \Phi_2 \Phi_2^T] \\ & + \lambda_8 \text{Tr}[\Phi_1 \Phi_2^T \Phi_1 \Phi_2^T] + \xi_3 \Phi_1 \Phi_1 \Phi_2 + \xi_4 \Phi_1 \Phi_2 \Phi_2 \\ & + \lambda_9 \text{Tr}[\Phi_1 \Phi_1^T] \text{Tr}[\Phi_1 \Phi_2^T] + \lambda_{10} \text{Tr}[\Phi_1 \Phi_1^T \Phi_1 \Phi_2^T] \\ & + \lambda_{11} \text{Tr}[\Phi_2 \Phi_2^T] \text{Tr}[\Phi_2 \Phi_1^T] + \lambda_{12} \text{Tr}[\Phi_2 \Phi_2^T \Phi_2 \Phi_1^T] \end{aligned}$$

- Any fit for fermion masses should be generated from this Higgs potential
- This is found to be satisfied for the fermion fit that follows

# Fit to Fermion Spectrum with Two $\Phi(3, 3)$

Masses (in GeV) and Mixing parameters	Inputs (at $\mu = 10^8$ GeV)	Fitted values (at $\mu = M_{GUT}$ )	pulls	Heavy Fermions	Heavy Fermion Masses (in GeV)
$m_u/10^{-4}$	$6.5 \pm 2.2$	8.3	0.8	$M_U$	$1 \times 10^8$
$m_c$	$0.35 \pm 0.01$	0.35	-0.07	$M_C$	$2.4 \times 10^5$
$m_t$	$104.9 \pm 0.9$	104.3	-0.7	$M_T$	$8.2 \times 10^2$
$m_d/10^{-3}$	$1.6 \pm 0.1$	1.5	-0.7	$M_D$	$5.6 \times 10^6$
$m_s/10^{-2}$	$3.1 \pm 0.1$	3.1	0.19	$M_S$	$2.7 \times 10^5$
$m_b$	$1.5 \pm 0.01$	1.5	0.03	$M_B$	$5.7 \times 10^3$
$m_e/10^{-4}$	$5.03 \pm 0.05$	5.03	-0.02	$M_E$	$8 \times 10^6$
$m_\mu/10^{-2}$	$10.6 \pm 0.1$	10.6	0.05	$M_\mu$	$3.8 \times 10^4$
$m_\tau$	$1.8 \pm 0.01$	1.79	-0.5	$M_\tau$	$2.2 \times 10^3$
$ V_{us} /10^{-2}$	$22.7 \pm 0.07$	22.7	0.04	Heavy Gauge Bosons	Heavy Gauge Boson Masses (in GeV)
$ V_{cb} /10^{-2}$	$4.5 \pm 0.06$	4.5	0.02	$M_{Z_{L,R}}/g_{L,R}$	$1.71 \times 10^8$
$ V_{ub} /10^{-3}$	$3.9 \pm 0.1$	3.9	-0.08	$M_{Y_{L,R}}/g_{L,R}$	$1.70 \times 10^8$
$\delta_{CKM}$	$1.2 \pm 0.05$	1.19	-0.26	$M_{X_{L,R}}/g_{L,R}$	$1.16 \times 10^6$
$\chi^2$	-	-	2	-	-

Parameter set ( $Y_2^{u,d,e}$  are complex parameters):

$$y_{u,d,e} = \{-0.79, -0.016, -0.01\},$$

$$m_{u,d,e}^0 = \{620, 3110, 2203\} \text{ GeV}$$

$$Y_1^{u,d,e} = \{-0.45, -1.73, -0.13\},$$

$$Y_2^{u,d,e} = \{0.84 - i 0.29, 0.15 - i 0.03, -0.05 - i 3 \times 10^{-6}\}$$

$$\langle \Phi_1 \rangle = \begin{pmatrix} 1.2 \times 10^7 & 0 & 0 \\ 0 & 1.1 \times 10^5 & 0 \\ 0 & 0 & 2.9 \times 10^3 \end{pmatrix} \text{ GeV}, \quad \langle \Phi_2 \rangle = \begin{pmatrix} 1.1 \times 10^8 & 7.3 \times 10^6 & 1.1 \times 10^5 \\ 7.3 \times 10^6 & 7.9 \times 10^5 & 2.3 \times 10^4 \\ 1.1 \times 10^5 & 2.3 \times 10^4 & 2.2 \times 10^3 \end{pmatrix} \text{ GeV}$$

# Flavor Gauge Model and $B$ Decay Anomalies

- The MMFV gauge model is not easily accessible to experiments, owing to large masses ( $> 10^6$  GeV) of flavor gauge bosons
- However, the framework appears to be suitable to explain recently reported  $B$  decay anomalies, especially in the lepton flavor universality violating ratios  $R_K$  and  $R_{K^*}$ . This is because the lightest flavor gauge bosons,  $X_{L,R}$ , couple to second and third family fermions off-diagonally
- We have explored the case of a modified scalar spectrum within the  $O(3)_L \times O(3)_R$  gauge model to explain these anomalies
- Two copies of  $\Phi(3, 3)$  scalars, and one copy of  $\chi_3(1, 3) + \chi_5(1, 3)$  scalar fields are used for fermion masses and symmetry breaking
- This model generates operators  $C_9 = -C_{10}$  that works very well in explaining the  $B$  anomalies

# B Decay Anomalies

- There has been indications for new physics in  $b$  to  $s\mu^+\mu^-$  transition matrix elements for some time
- Hints for new physics come from  $B_s \rightarrow \phi\mu^+\mu^-$  decay (LHCb measurement is  $3\sigma$  lower than SM),  $B^\pm \rightarrow K^\pm\mu^+\mu^-$  (LHCb  $2\sigma$  low in several  $q^2$  bins),  $B \rightarrow K^*\mu^+\mu^-$  angular distribution, ...
- These discrepancies suggest new contributions to  $C_9$  and/or  $C_{10}$  operators:

$$C_9 = (\bar{b}_L\gamma_\mu s_L)(\bar{\mu}\gamma^\mu\mu), \quad C_{10} = (\bar{b}_L\gamma_\mu s_L)(\bar{\mu}\gamma^\mu\gamma_5\mu)$$

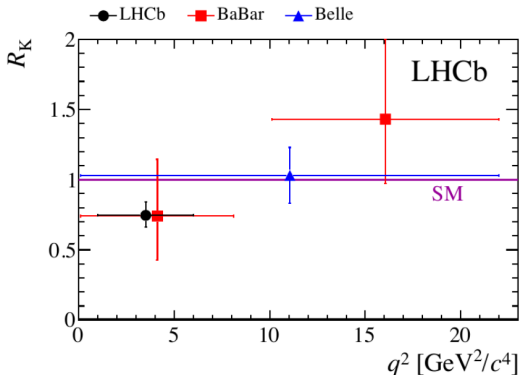
- For  $C_9 = -C_{10}$  (only left-handed fields involved in new physics), best fit is  $C_9 = -0.67$ , which is about  $4.8\sigma$  improvement compared to SM in a global fit
- $R_K$  and  $R_{K^*}$  measurements add to the significance

# Global Fit to $B$ Observables Without $R_K$ and $R_{K^*}$

Coeff.	best fit	$1\sigma$	$2\sigma$	pull
$C_9^{\text{NP}}$	-1.21	[-1.41, -1.00]	[-1.61, -0.77]	$5.2\sigma$
$C'_9$	+0.19	[-0.01, +0.40]	[-0.22, +0.60]	$0.9\sigma$
$C_{10}^{\text{NP}}$	+0.79	[+0.55, +1.05]	[+0.32, +1.31]	$3.4\sigma$
$C'_{10}$	-0.10	[-0.26, +0.07]	[-0.42, +0.24]	$0.6\sigma$
$C_9^{\text{NP}} = C_{10}^{\text{NP}}$	-0.30	[-0.50, -0.08]	[-0.69, +0.18]	$1.3\sigma$
$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$	-0.67	[-0.83, -0.52]	[-0.99, -0.38]	$4.8\sigma$
$C'_9 = C'_{10}$	+0.06	[-0.18, +0.30]	[-0.42, +0.55]	$0.3\sigma$
$C'_9 = -C'_{10}$	+0.08	[-0.02, +0.18]	[-0.12, +0.28]	$0.8\sigma$

Altmannshofer, Niehoff, Stangl, Straub [arXiv: 1703.09189]

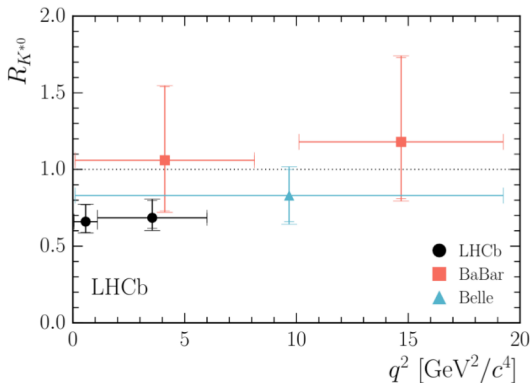
# $R_K$ Anomaly



$$R_K[1, 6] = \frac{Br(B \rightarrow K \mu^+ \mu^-)}{Br(B \rightarrow K e^+ e^-)} = 0.745_{-074}^{+090} \pm 0.036 \text{ [LHCb 1406.6482]}$$

Deviates from SM  $R_K \simeq 1$  by  $\sim 2 \sigma$

# $R_{K^*}$ Anomaly



$$R_{K^*}[0.045, 1.1] = 0.660_{-0.070}^{+0.110} \pm 0.024, \quad R_{K^*}[1.1, 6] = 0.685_{-0.069}^{+0.113} \pm 0.04$$

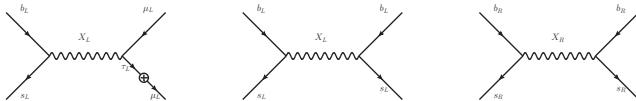
Deviates from SM  $R_{K^*} \simeq 1$  by  $\sim 2.5 \sigma$  in each bin

LHCb, arXiv:1705.05802 [hep-ex]

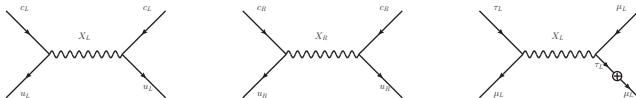


# $O(3)_L \times O(3)_R$ Model for $B$ Anomalies

- $C_9 = -C_{10} \sim -0.63$  can be induced in next to minimal FV model



- Requires  $M_{X_L}/g_L \sim (10 - 22)$  TeV
- $B_s - \bar{B}_s$  mixing provides a strong constraint. However,  $X_R$  exchange almost exactly cancels the  $X_L$  contribution. **Note:**  
 $M_{X_L}/g_L \simeq M_{X_R}/g_R$



- New contributions to  $D^0 - \bar{D}^0$  mixing near experimental limit
- Predicts  $\tau \rightarrow 3\mu$  with a branching ratio of  $\sim 10^{-10}$

# Main Features of NMFV Model

- Uses two copies of  $\Phi(3, 3)$  scalar fields. Their VEVs are taken to be nearly diagonal
- One  $\chi_3(1, 3)$  and one  $\chi_5(1, 3)$  scalars are used.  $\chi_3$  has flavor antisymmetric Yukawa couplings,  $\chi_5$  has flavor-symmetric couplings
- Mass matrices are given by:

$$M^{u,d,e} = \begin{pmatrix} 0 & y_{u,d,e} v \mathbb{I} \\ m_0^{u,d,e} \mathbb{I} + y_3^{u,d,e} \langle \chi_3 \rangle + y_5^{u,d,e} \langle \chi_5 \rangle & Y_1^{u,d,e} \langle \Phi_1 \rangle + Y_2^{u,d,e} \langle \Phi_2 \rangle \end{pmatrix}$$

- $y_{3,5}^d$  couplings are taken to be small, to suppress  $X_{L,R}$  mediated FCNC in  $K, B_d$  systems
- Desired sign of  $X_L$  contribution to  $B$  anomalies fixes sign of  $X_R$  coupling so as to cancel  $X_{L,R}$  contributions to  $B_s - \overline{B}_s$  mixing

## Fit for NMFV Model: $2 \times \Phi(3, 3) + \chi_3(1, 3) + \chi_5(1, 5)$

$$\mathcal{M}_u \sim -y_u v M_u^{-1} X_u$$

$$M_u^{-1} = \begin{pmatrix} M_1^u & 0 & 0 \\ 0 & M_2^u & 0 \\ 0 & 0 & M_3^u \end{pmatrix}^{-1} = \frac{1}{M_3^u} P_u r_u,$$

$$r_u = \begin{pmatrix} r_{31}^u & 0 & 0 \\ 0 & r_{32}^u & 0 \\ 0 & 0 & 1 \end{pmatrix}, P_u = \begin{pmatrix} e^{i\phi_1^u} & 0 & 0 \\ 0 & e^{i\phi_2^u} & 0 \\ 0 & 0 & e^{i\phi_3^u} \end{pmatrix},$$

$$X_u = m_0^u \mathbb{I}_{3 \times 3} + e^{i\chi_3^u} Y_3^u \langle A \rangle + e^{i\chi_5^u} Y_5^u \langle S \rangle,$$

$$\mathcal{M}_u \sim \left( \frac{-y_u v m_0^u}{M_3^u} \right) P_u r_u (\mathbb{I}_{3 \times 3} + e^{i\chi_3^u} \frac{Y_3^u \langle A \rangle}{m_0^u} + e^{i\chi_5^u} \frac{Y_5^u \langle S \rangle}{m_0^u})$$

$$\frac{-y_u v m_0^u}{M_3^u} = 36 \text{ GeV}, r_{31}^u = 0.06, r_{32}^u = 0.12,$$

$$\chi_3^u = 3.11, \chi_5^u = -0.02,$$

$$\frac{Y_3^u A_{12}}{m_0^u} = 0.33, \frac{Y_3^u A_{13}}{m_0^u} = 0.12, \frac{Y_3^u A_{23}}{m_0^u} = 1.59,$$

$$\frac{Y_5^u S_{12}}{m_0^u} = -0.21, \frac{Y_5^u S_{13}}{m_0^u} = -0.12, \frac{Y_5^u S_{22}}{m_0^u} = 0.28, \frac{Y_5^u S_{23}}{m_0^u} = 2.19, \frac{Y_5^u S_{33}}{m_0^u} = 0.76$$

## Fit for NMFV Model: $2 \times \Phi(3, 3) + \chi_3(1, 3) + \chi_5(1, 5)$

$$\mathcal{M}_d \sim \left( \frac{-y_d v m_0^d}{M_3^d} \right) P_d r_d (\mathbb{I}_{3 \times 3} + e^{i\chi_3^d} \frac{Y_3^d \langle A \rangle}{m_0^d} + e^{i\chi_5^d} \frac{Y_5^d \langle S \rangle}{m_0^d})$$

$$\frac{-y_e v m_0^e}{M_3^e} = 2.4 \text{ GeV}, r_{31}^e = 0.002, r_{32}^e = 0.019,$$

$$Y_3^d \sim 0, Y_5^d \sim 0$$

$$\mathcal{M}_e \sim \left( \frac{-y_e v m_0^e}{M_3^e} \right) P_e r_e (\mathbb{I}_{3 \times 3} + e^{i\chi_3^e} f_e \frac{Y_3^u \langle A \rangle}{m_0^u} + e^{i\chi_5^e} g_e \frac{Y_5^u \langle S \rangle}{m_0^u})$$

$$\frac{-y_e v m_0^e}{M_3^e} = 1 \text{ GeV}, r_{31}^e = 0.001, r_{32}^e = 0.38,$$

$$\chi_3^e = 0.91, \chi_5^e = 1.72,$$

$$f_e = \frac{y_3^e}{y_3^u} \frac{m_0^u}{m_0^e} = -0.8, g_e = \frac{y_5^e}{y_5^u} \frac{m_0^u}{m_0^e} = 0.5$$

## Fit for NMFV Model: $2 \times \Phi(3, 3) + \chi_3(1, 3) + \chi_5(1, 5)$

Masses (in GeV) and Mixing parameters	Inputs (at $\mu = 10^8$ GeV)	Fitted values (at $\mu = M_{GUT}$ )	pulls
$m_u/10^{-4}$	$9.8 \pm 3.3$	9.8	0.002
$m_c$	$0.54 \pm 0.01$	0.54	0.001
$m_t$	$151.2 \pm 1.5$	151.2	0.002
$m_d/10^{-3}$	$2.4 \pm 0.2$	2.4	0.0
$m_s/10^{-3}$	$46.9 \pm 2.5$	46.9	0.0
$m_b$	$2.4 \pm 0.02$	2.4	0.0
$m_e/10^{-4}$	$4.95 \pm 0.04$	4.95	0.001
$m_\mu/10^{-3}$	$104.6 \pm 0.1$	104.6	0.001
$m_\tau$	$1.7 \pm 0.01$	1.7	0.002
$\theta_{12}/10^{-2}$	$22.7 \pm 0.07$	22.7	-0.001
$\theta_{23}/10^{-2}$	$4.2 \pm 0.06$	4.2	-0.002
$\theta_{23}/10^{-3}$	$3.7 \pm 0.1$	3.7	-0.005
$\delta_{CKM}$	$1.208 \pm 0.05$	1.208	0.008
$\chi^2$	-	-	0.0001

# Flavor Structure of $X_{L,R}$ Couplings

$$L(X_L, X_R) = -ig_L(\bar{f}_L \gamma_\mu K_L^f f_L) X_L^\mu - ig_R(\bar{f}_R \gamma_\mu K_R^f f_R) X_R^\mu$$

$$K_L^u = \begin{pmatrix} (1 \times 10^{-3} \cos(\phi^u) + 6 \times 10^{-4} \sin(\phi^u))i & (8 \times 10^{-3} - 7 \times 10^{-4}i) \cos(\phi^u) - (7 \times 10^{-3} + 8 \times 10^{-3}i) \sin(\phi^u) & (-0.2 + 0.09i) \cos(\phi^u) + (0.09 + 0.2i) \sin(\phi^u) \\ (8 \times 10^{-3} - 7 \times 10^{-4}i) \cos(\phi^u) - (7 \times 10^{-3} + 8 \times 10^{-3}i) \sin(\phi^u) & -3 \times 10^{-3}i \cos(\phi^u) - 0.08i \sin(\phi^u) & (-0.9 - 1 \times 10^{-4}i) \cos(\phi^u) + (1 \times 10^{-4} + 0.9i) \sin(\phi^u) \\ (0.2 + 0.09i) \cos(\phi^u) + (-0.09 + 0.2i) \sin(\phi^u) & (0.9 - 1 \times 10^{-4}i) \cos(\phi^u) + (-1 \times 10^{-4} + 0.9i) \sin(\phi^u) & 1 \times 10^{-3}i \cos(\phi^u) + 0.08i \sin(\phi^u) \end{pmatrix}$$

$$K_R^u = \begin{pmatrix} -1.2 \times 10^{-2}i & 2.9 \times 10^{-5} + 1.02 \times 10^{-4}i & 9.9 \times 10^{-1} - 5.9 \times 10^{-3}i \\ -2.9 \times 10^{-5} + 1.0 \times 10^{-4}i & 5.1 \times 10^{-5}i & 1.1 \times 10^{-2} - 6.3 \times 10^{-2}i \\ -9.9 \times 10^{-1} - 5.9 \times 10^{-3}i & -1.1 \times 10^{-2} - 6.3 \times 10^{-2}i & 1.2 \times 10^{-2}i \end{pmatrix}$$

$$K_L^e = \begin{pmatrix} (-7 \times 10^{-5} \cos(\phi^e) - 5 \times 10^{-5} \sin(\phi^e))i & (-5 \times 10^{-3} - 4 \times 10^{-3}i) \cos(\phi^e) - (4 \times 10^{-3} + 3 \times 10^{-3}i) \sin(\phi^e) & (-6 \times 10^{-3} + 2 \times 10^{-3}i) \cos(\phi^e) + (7 \times 10^{-3} + 2 \times 10^{-3}i) \sin(\phi^e) \\ (5 \times 10^{-3} - 4 \times 10^{-3}i) \cos(\phi^e) - (-4 \times 10^{-3} + 3 \times 10^{-3}i) \sin(\phi^e) & -0.7i \cos(\phi^e) + 0.5i \sin(\phi^e) & (-0.6 - 0.3i) \cos(\phi^e) + (0.3 + 0.7i) \sin(\phi^e) \\ (6 \times 10^{-3} + 2 \times 10^{-3}i) \cos(\phi^e) + (-7 \times 10^{-3} + 2 \times 10^{-3}i) \sin(\phi^e) & (0.6 - 0.3i) \cos(\phi^e) + (-0.3 + 0.7i) \sin(\phi^e) & 0.7i \cos(\phi^e) + 0.5i \sin(\phi^e) \end{pmatrix}$$

$$K_R^e = \begin{pmatrix} -1.2 \times 10^{-2}i & 2.9 \times 10^{-5} + 1.02 \times 10^{-4}i & 9.9 \times 10^{-1} - 5.9 \times 10^{-3}i \\ -2.9 \times 10^{-5} + 1.0 \times 10^{-4}i & 5.1 \times 10^{-5}i & 1.1 \times 10^{-2} - 6.3 \times 10^{-2}i \\ -9.9 \times 10^{-1} - 5.9 \times 10^{-3}i & -1.1 \times 10^{-2} - 6.3 \times 10^{-2}i & 1.2 \times 10^{-2}i \end{pmatrix}$$

- $(K_L^e)_{22}$  is large, while  $(K_R^e)_{22}$  is small. Only  $C_9 = -C_{10}$  will be induced for  $B$  decays
- $(K_L^u)_{12} \sim 8 \times 10^{-3}$  leads to new contributions to  $D^0 - \bar{D}^0$  mixing. Close to experimental limit. CP violation in mixing is also in interesting range
- By construction,  $X_{L,R}$  contributions to  $K^0 - \bar{K}^0$  mixing,  $B_d^0 - \bar{B}_d^0$  mixing are small

# Conclusions

- Gauge model realizations of minimal flavor violation presented based on  $O(3)_L \times O(3)_R$  flavor symmetry
- In the most minimal realization, two flavor matrices can explain up, down and charged lepton flavor structure. However, symmetry breaking scale is  $> 10^6$  GeV
- In a next to minimal flavor violation model,  $B$  decay anomalies can be nicely explained. Lightness of flavor gauge bosons is linked to fermion mass hierarchy
- Future tests can come in  $D^0 - \bar{D}^0$  mixing and CP violation,  $\tau \rightarrow 3\mu$  decay, and precise measurement of  $B_s \rightarrow \mu^+ \mu^-$  decay