REFINEMENT OF THE PION PDF IMPLEMENTING DRELL-YAN EXPERIMENTAL DATA

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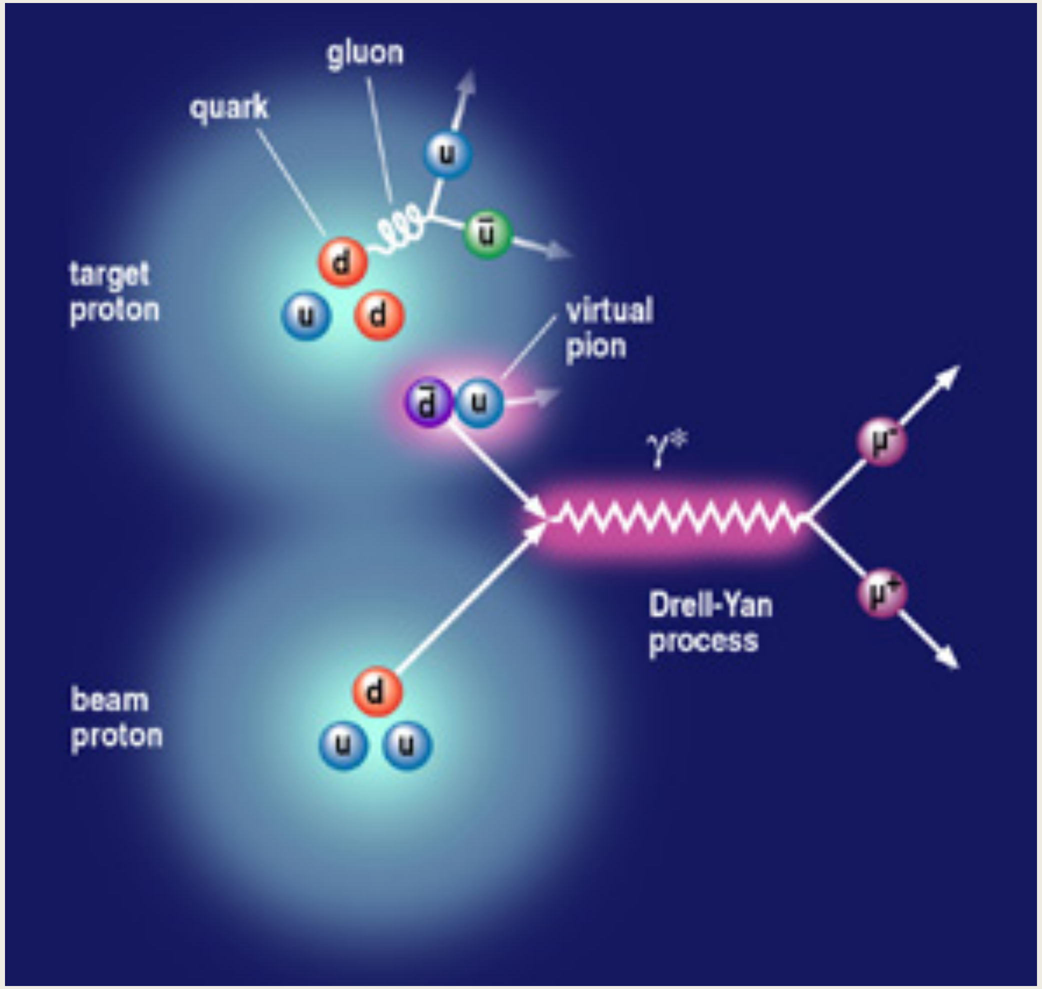
Drell-Yan process



- Two hadrons collide
  - *Do not need to be protons!*
- One donates a quark, other an antiquark
- Quarks annihilate into a virtual photon
- Dilepton production
- Measure differential cross-section of lepton/antilepton pair

Cross section

$$\frac{d^2\sigma}{dQ^2dY} = \frac{4\pi\alpha^2}{3N_cQ^2s}\sum_{i,j}\int dx_1dx_2\tilde{C}_{ij}(x_1,x_2,s,M,\mu_f)f_{i/N_1}(x_1,\mu_f)f_{j/N_2}(x_2,\mu_f)$$

- Cross-section differential in invariant mass of lepton pair, Q^2, and rapidity, Y
- The momentum fractions of the initial hadrons are x_1, x_2
- Parton distribution functions (PDFs) are $f_i(x, Q^2)$
- Sum over all partons

PDFs

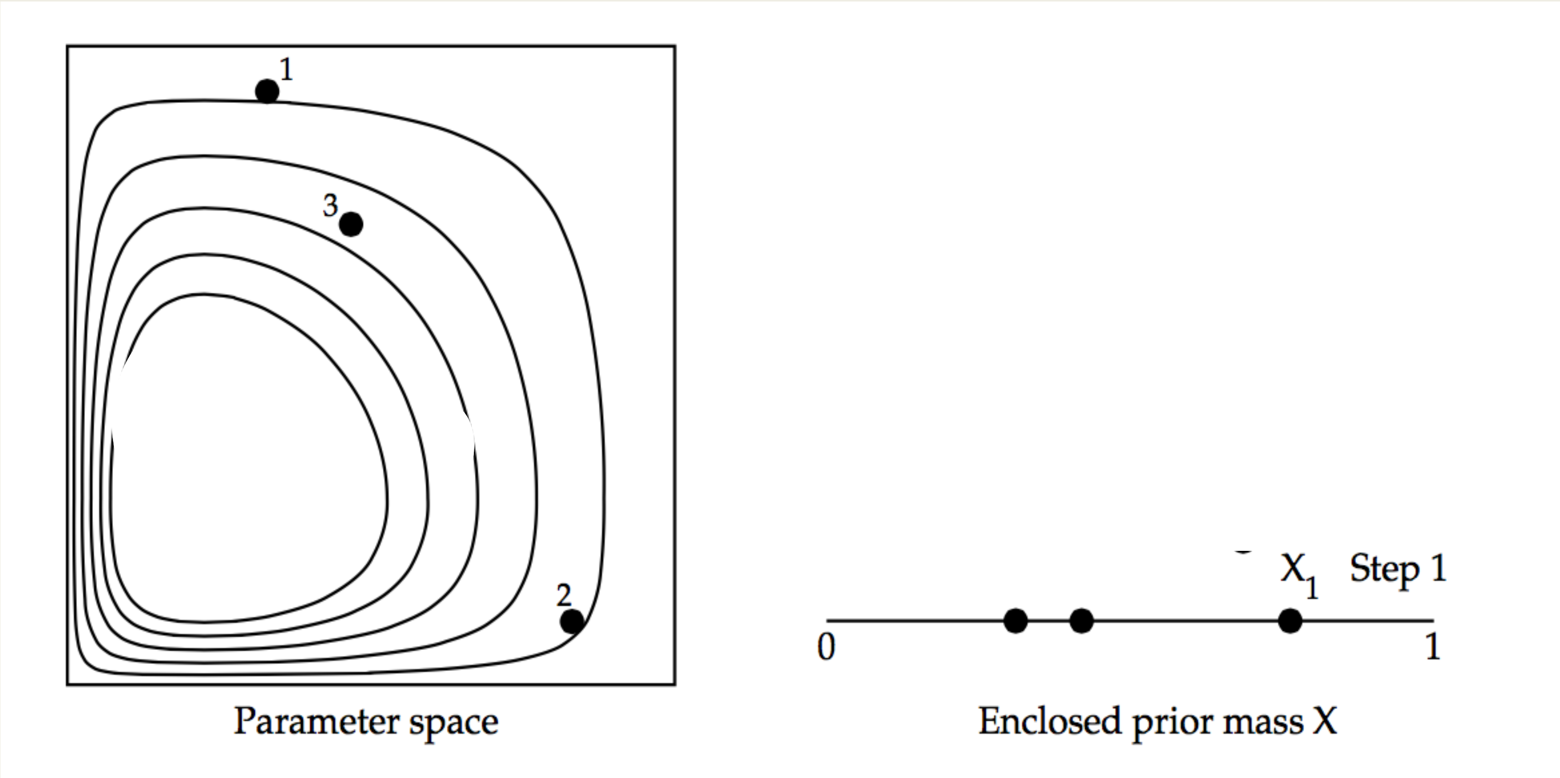
- Parameterize the PDF at $Q_0^2 = 1\text{GeV}^2$ as:
$$f(x, \mu^2) = N\, x^a \left(1 - x^b\right)$$
- Definitions: $q_v = \bar{u}_v = d_v$; $q_s = 2\left(u + \bar{d} + s\right)$; g
- Use sum rules to fix N_{q_v}, N_g
- We fit a, b for the valence, sea, and gluon, and N for the sea
- PDFs are evolved using DGLAP in Mellin space

Nested Sampling

- Monte Carlo fitting method
- Create parameter space to have a uniform prior over a specified range
- Sample points in parameter space closer and closer to the maximum likelihood
- Weights produced with each sample based on proximity to maximum likelihood
- Provides errors without assumption of linear error propagation

$$Var(\mathcal{O}) \propto \sum_k \left(\mathcal{O}_k - E(\mathcal{O})\right)^2$$

Nested Sampling



- Start with random points on line of $0 < X < 1$.
- $X = 0$ is the point of highest likelihood.

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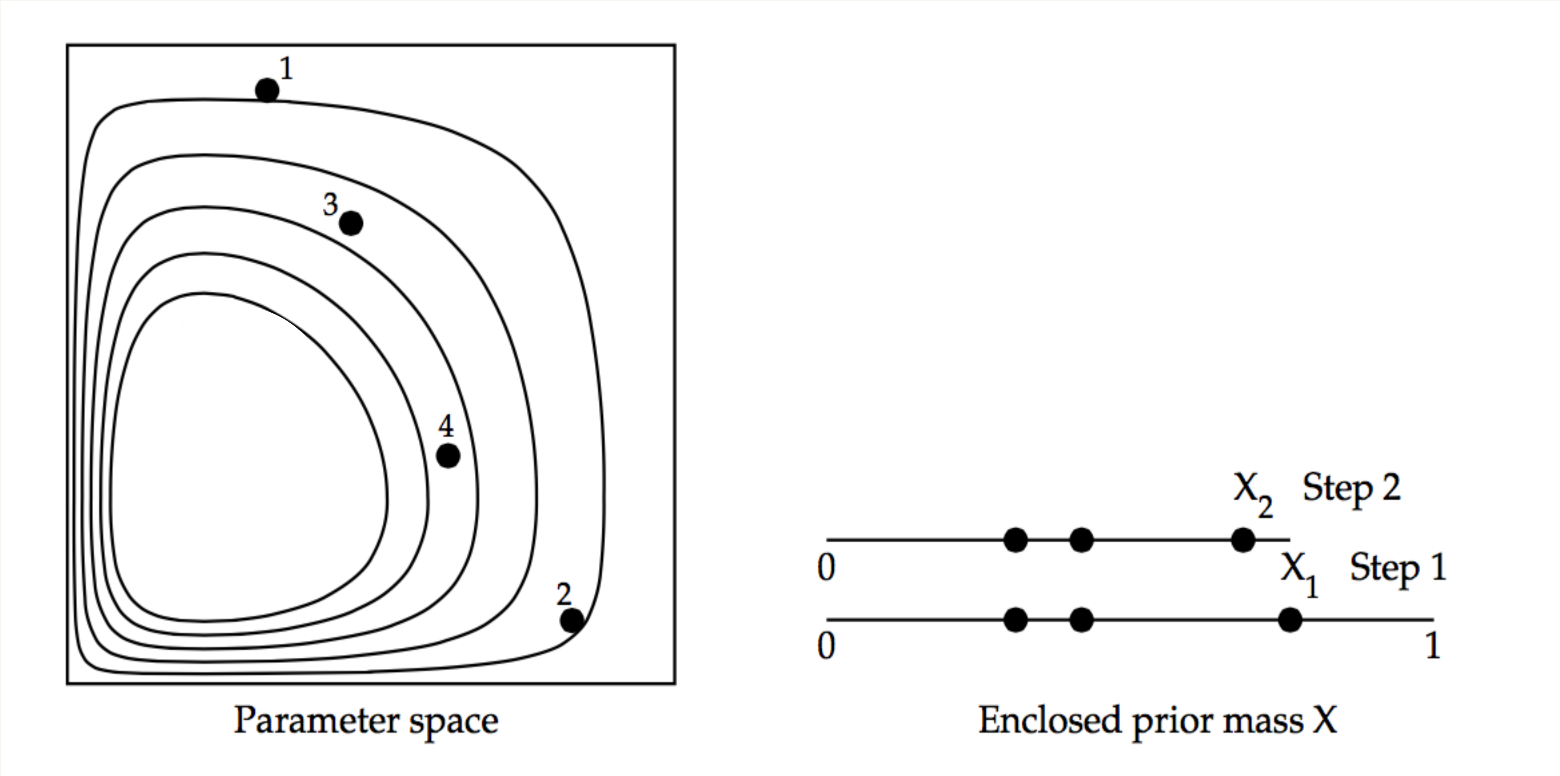

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- Delete point of lowest likelihood and make it the upper-bound on new sampling boundary

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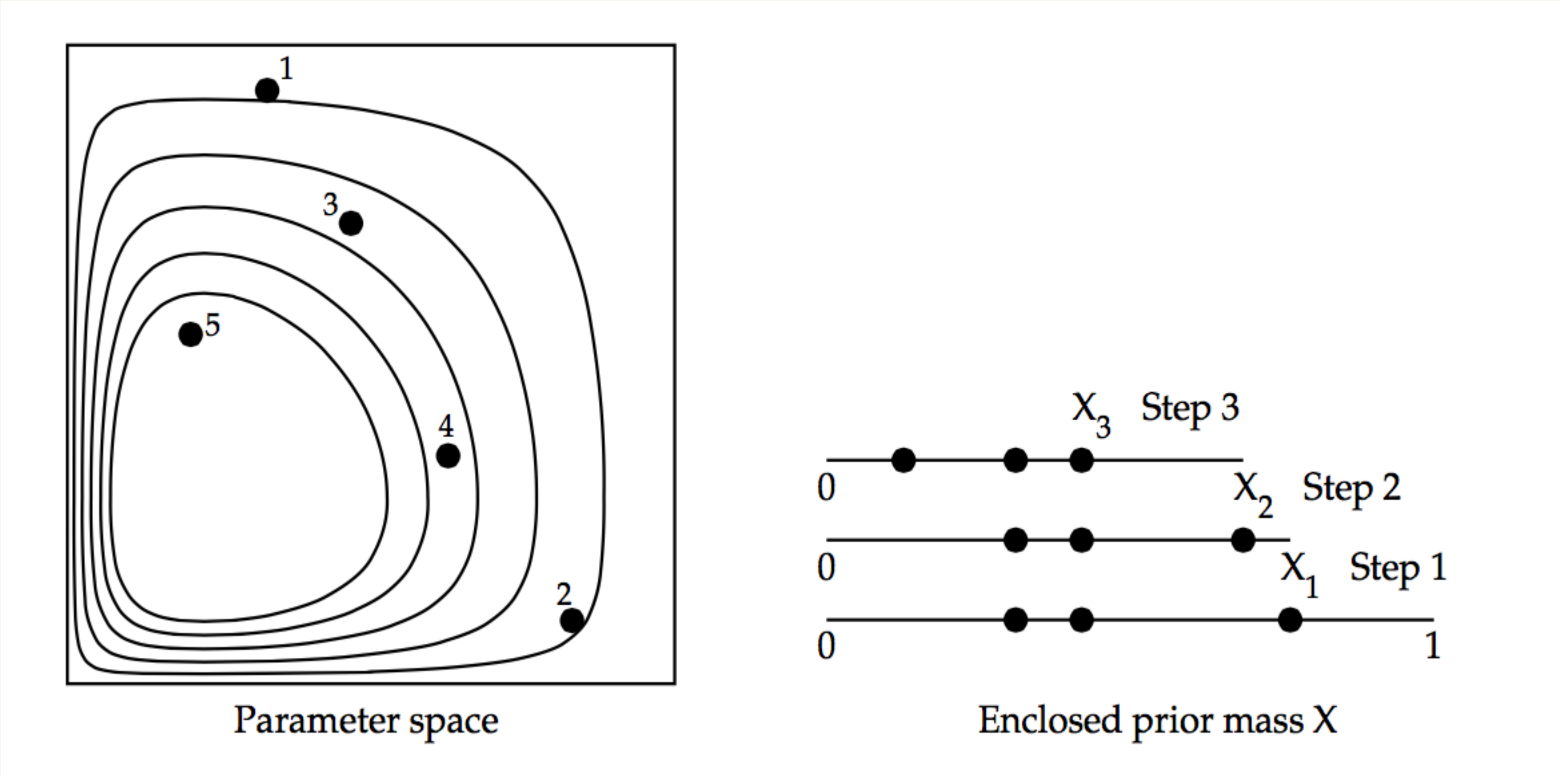

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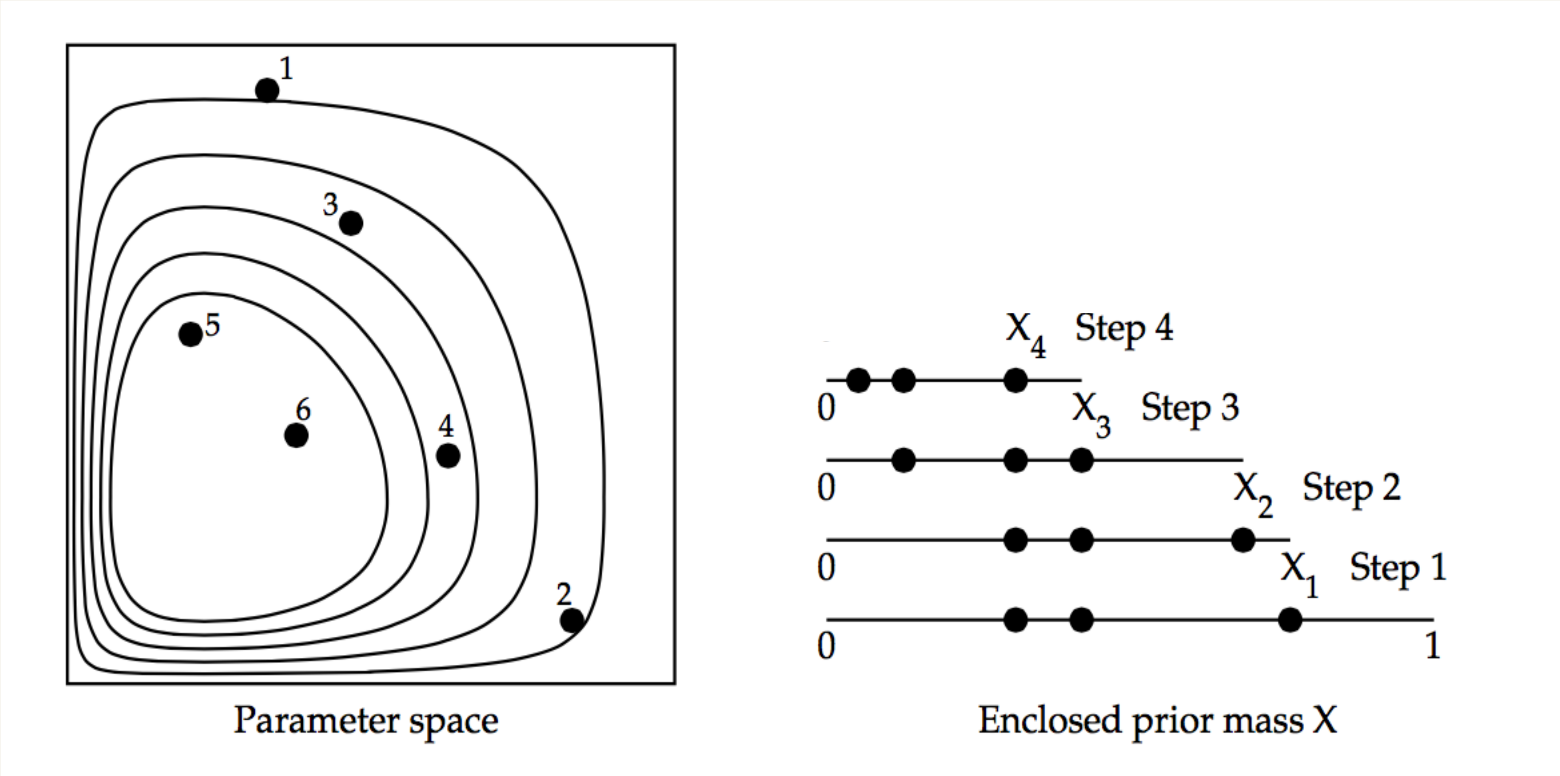

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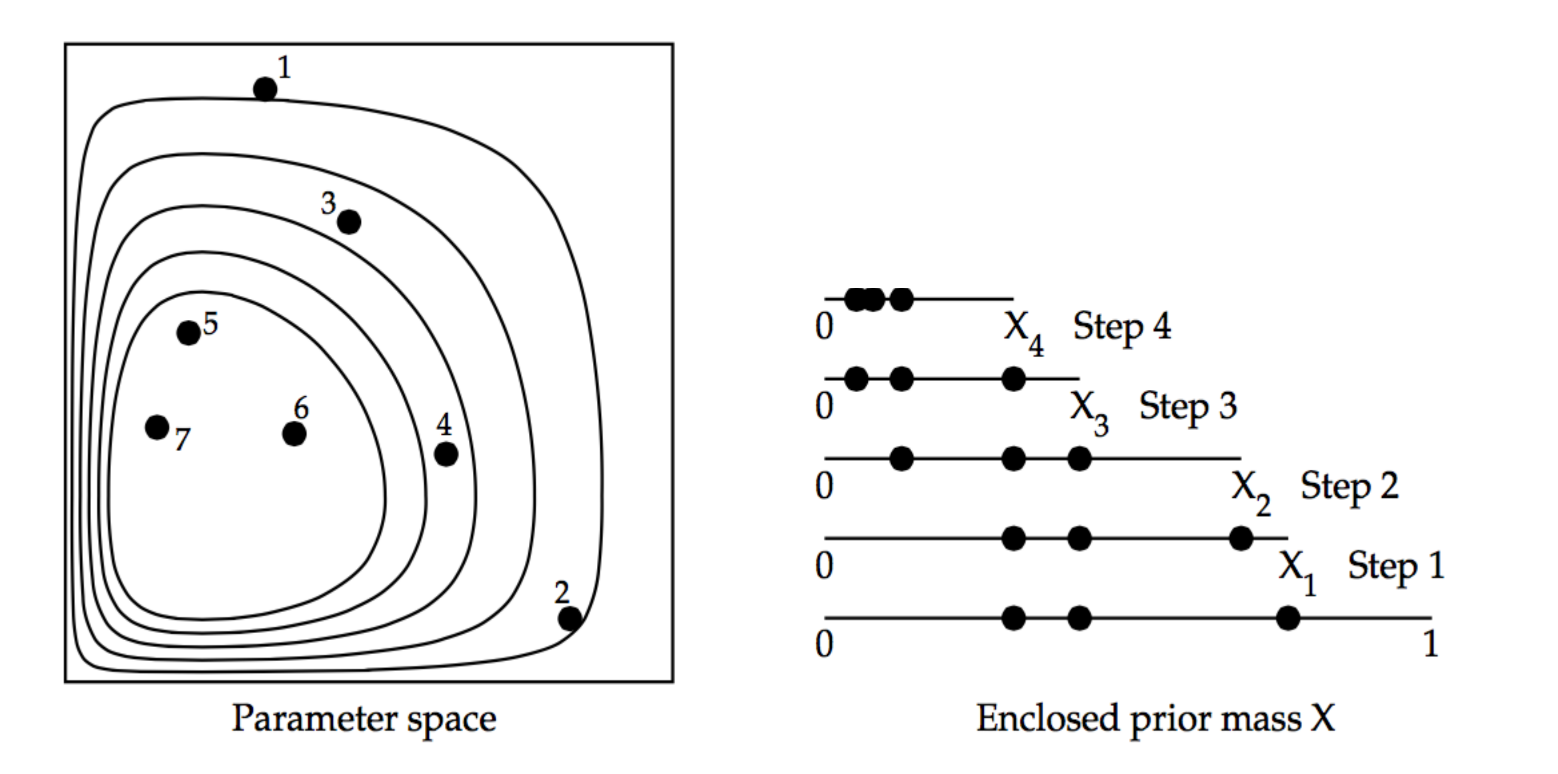

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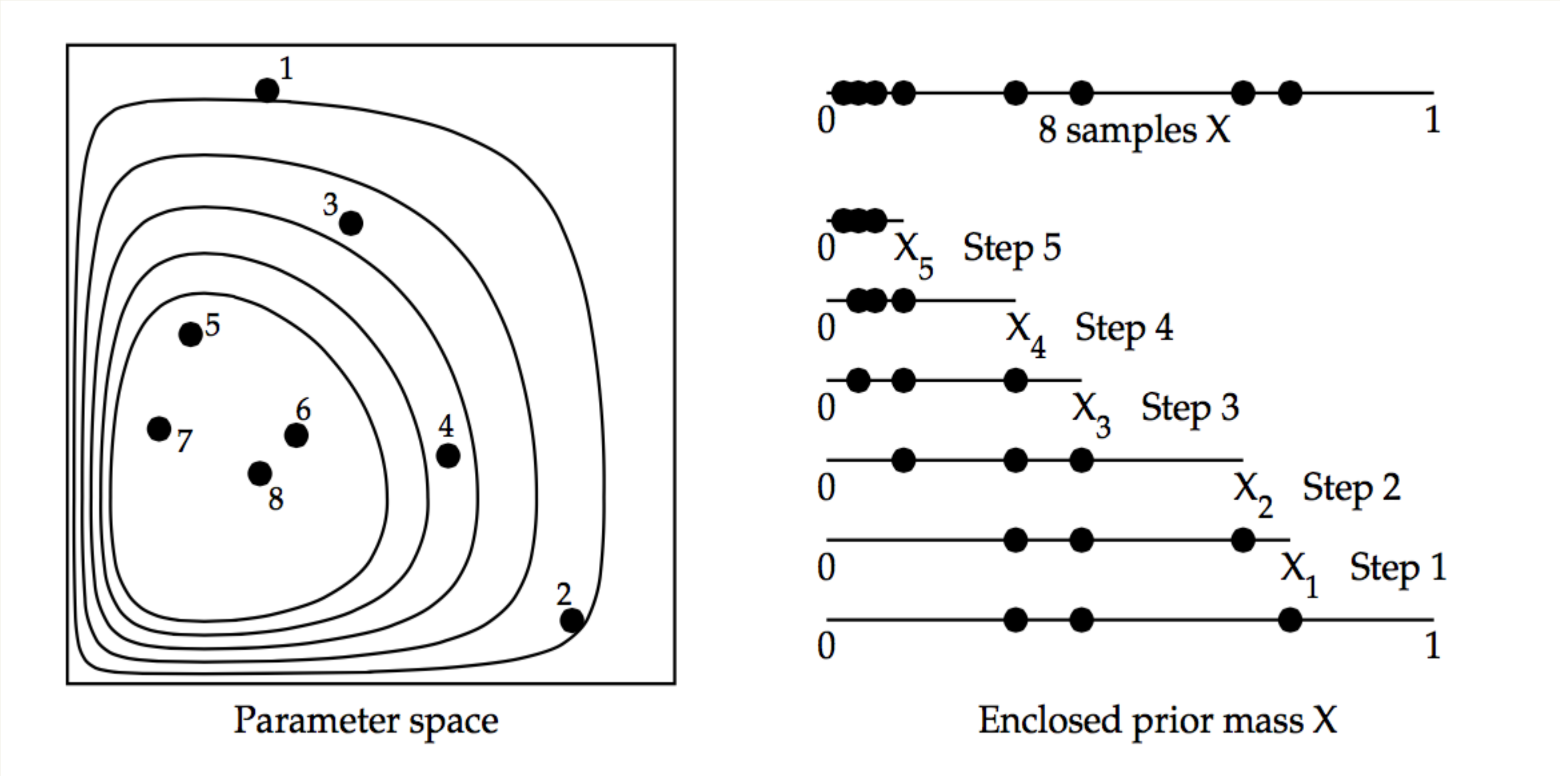

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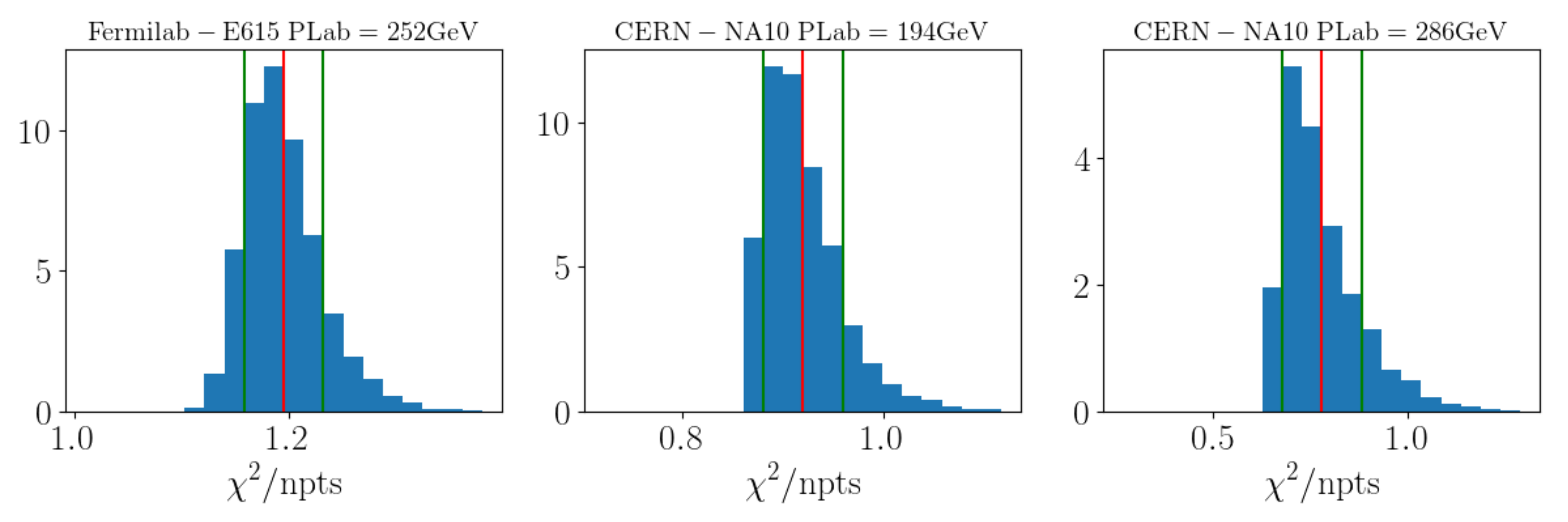
Datasets & Constrictions

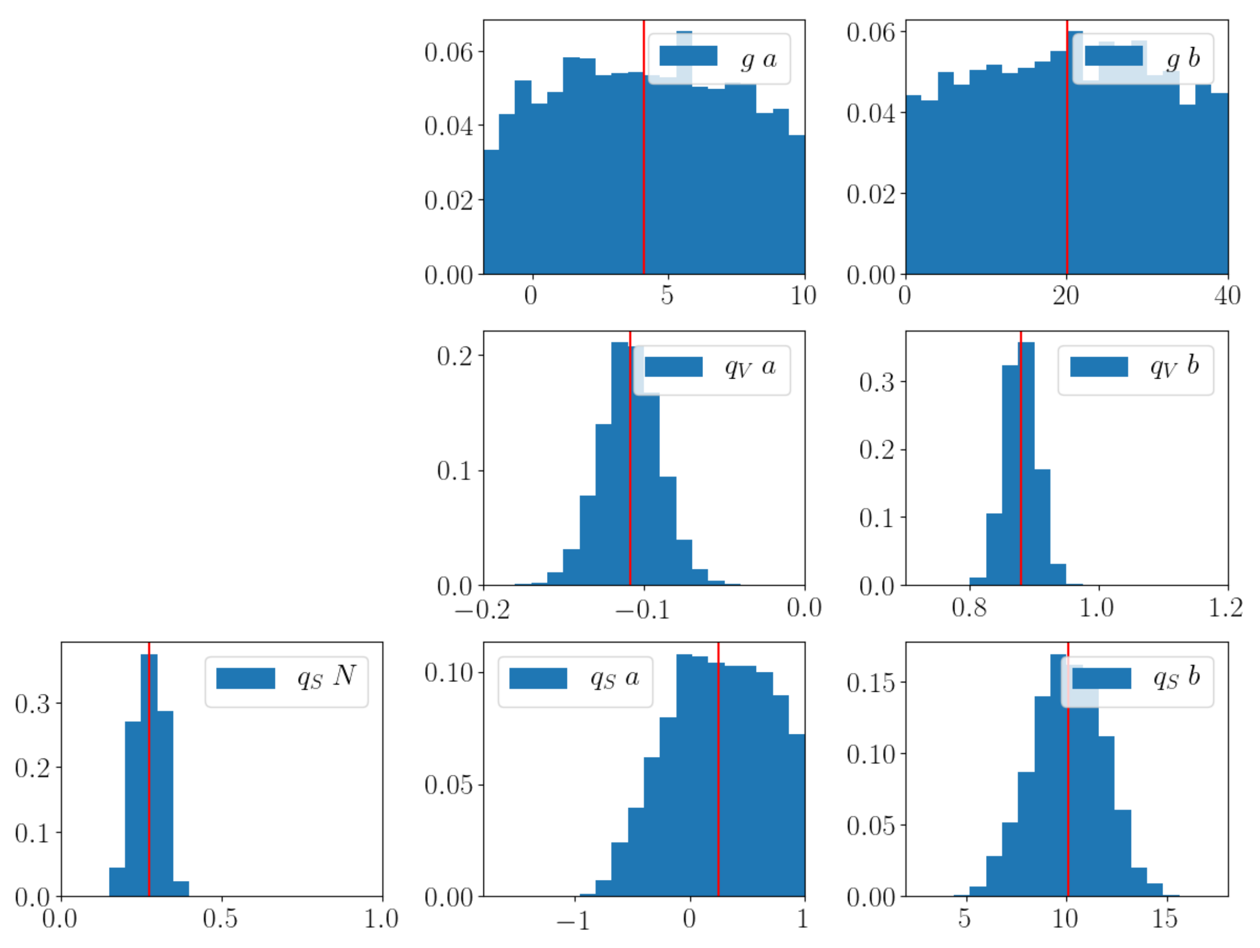
- For Drell-Yan, we use E615 and NA10 datasets
  - π^- beam incident on a Tungsten target
  - Consider only $0 < x_F < 0.9$ and $4.16 < Q < 8.34$ to avoid J/Ψ and Υ production

Drell-Yan fits



Drell-Yan fits

- q_v is well-constrained by Drell-Yan
- q_s has large spread in parameters
- g has almost not constrain



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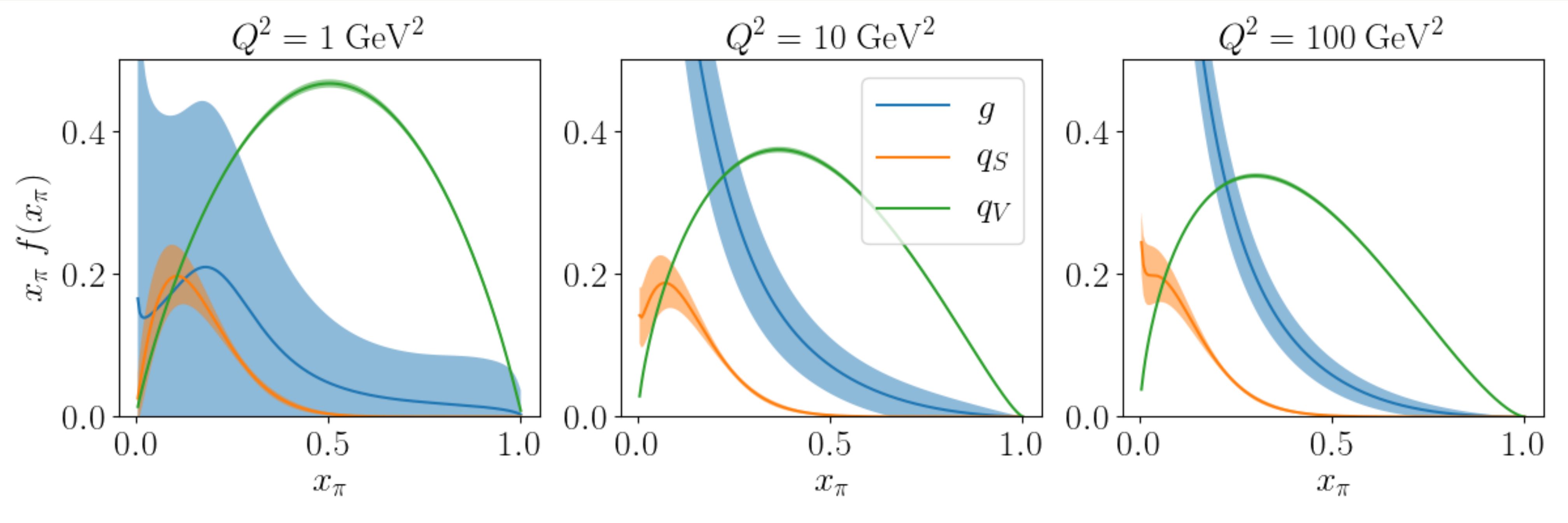
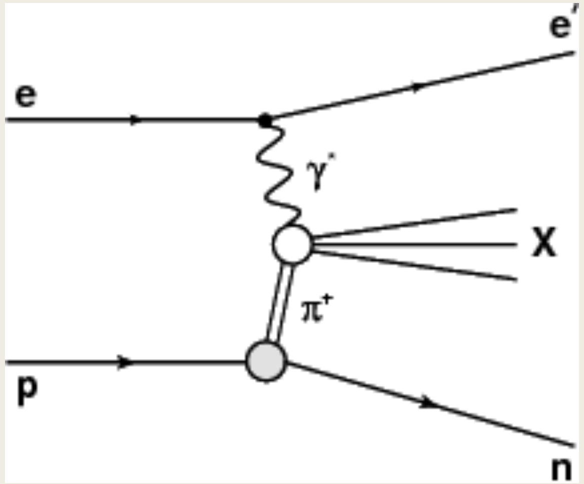


Leading Neutron

- Add in data from HERA (ZEUS & H1) to perform global fit
- Detect neutrons in coincidence with outgoing electrons:
- Neutron has most of the energy of the proton
- Incoming electron barely strikes the surface of the proton, knocking out a pion from the pion cloud
- Focuses on small x_π, whereas Drell-Yan focuses on large x_π



Leading Neutron

- Observable in H1 data is

$$F_2^{LN(3)}(x, Q^2, y) = f_{\pi^+ n}(y)\, F_2^{\pi}(x_\pi, Q^2)$$

– Where $f_{\pi^+ n}(y)$ is the splitting function from the proton, and $F_2^{\pi}(x_\pi, Q^2)$ is the pion structure function (depends on pion PDF)

- Observable in ZEUS is

$$r(x_\pi, Q^2, y) = f_{\pi^+ n}(y) \frac{F_2^{\pi}(x_\pi, Q^2)}{F_2^{p}(x, Q^2)} \Delta y$$

– where $F_2^{p}(x, Q^2)$ is the proton structure function

Datasets & Constrictions

- For Drell-Yan, we use E615 and NA10 datasets
  - π^- beam incident on a Tungsten target
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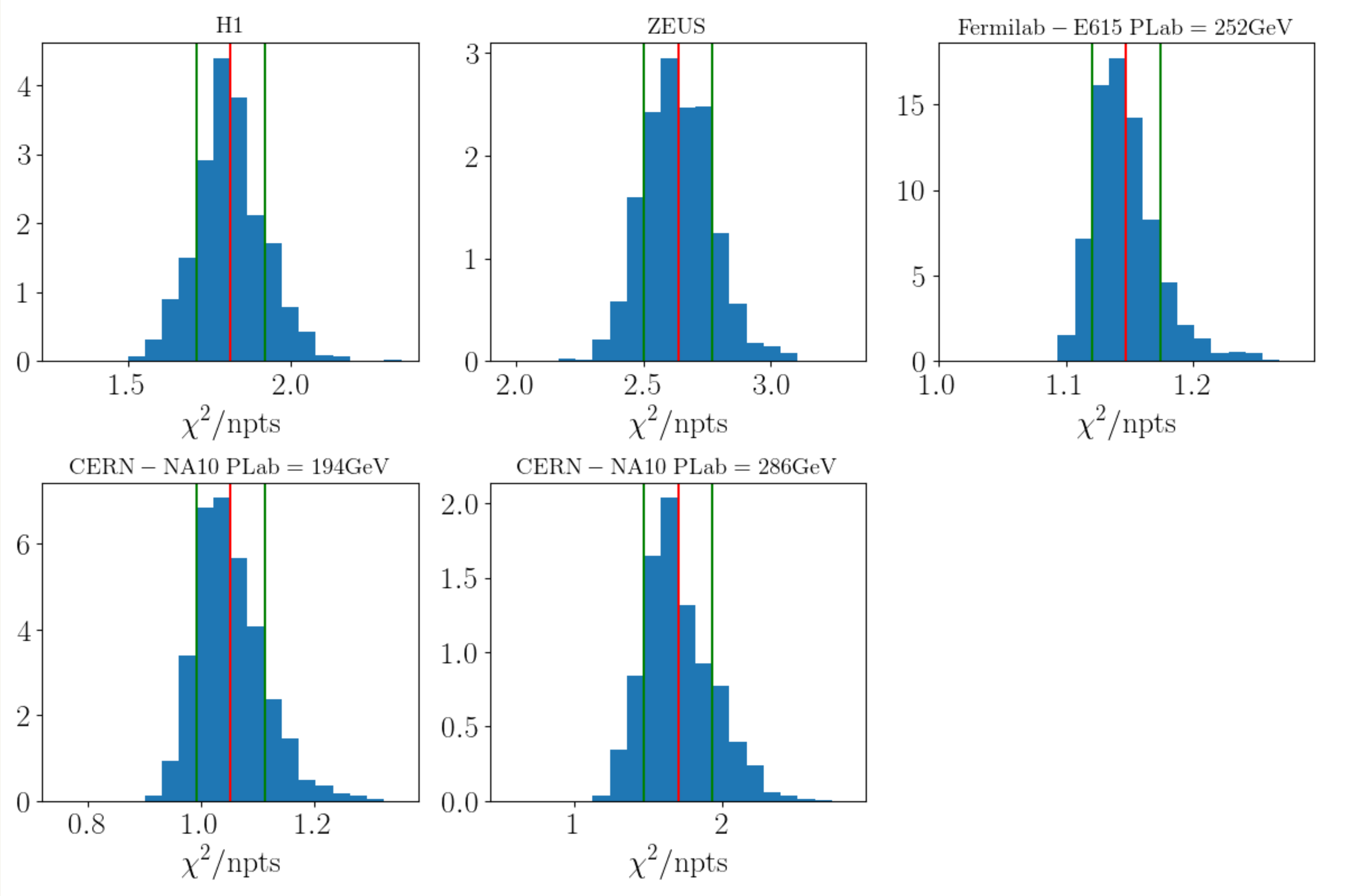
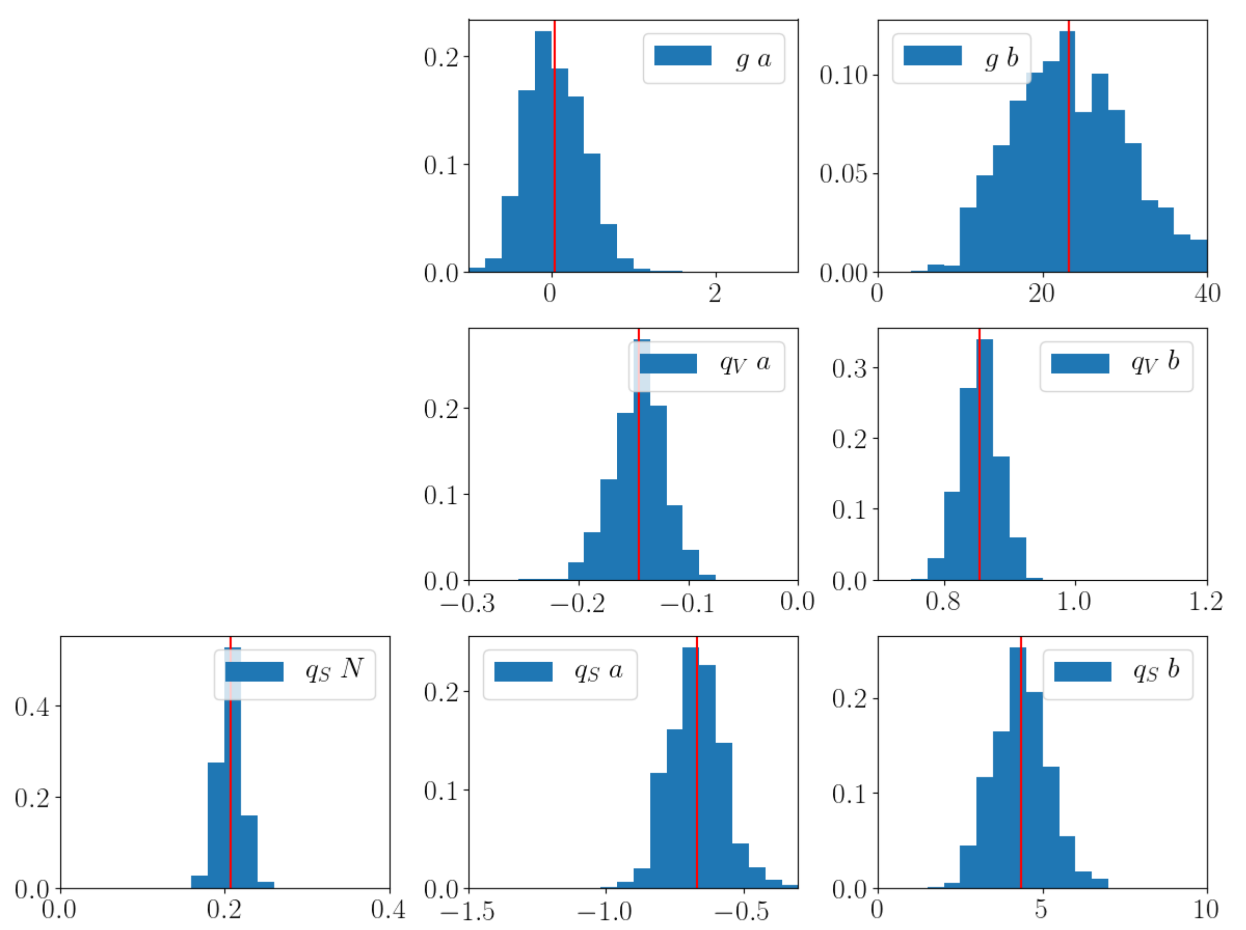
- For Leading Neutron, we use H1 and ZEUS datasets
  - We consider cuts on data based on maximum $y = x_\pi / x$ values

LN results



y-cut of 0.2

LN results



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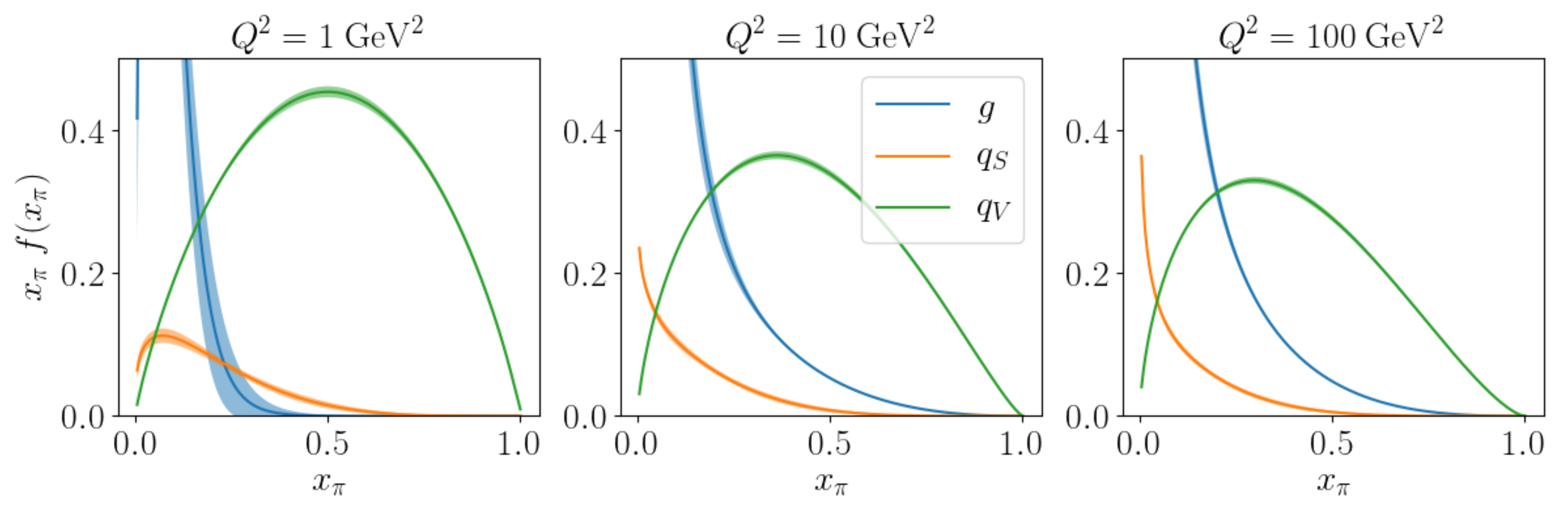


$Q^2 = 1\ \mathrm{GeV}^2$
$Q^2 = 10\ \mathrm{GeV}^2$
$Q^2 = 100\ \mathrm{GeV}^2$
$x_\pi\ f(x_\pi)$
0.0
0.2
0.4
0.0
0.5
1.0
x_π
g
q_S
q_V

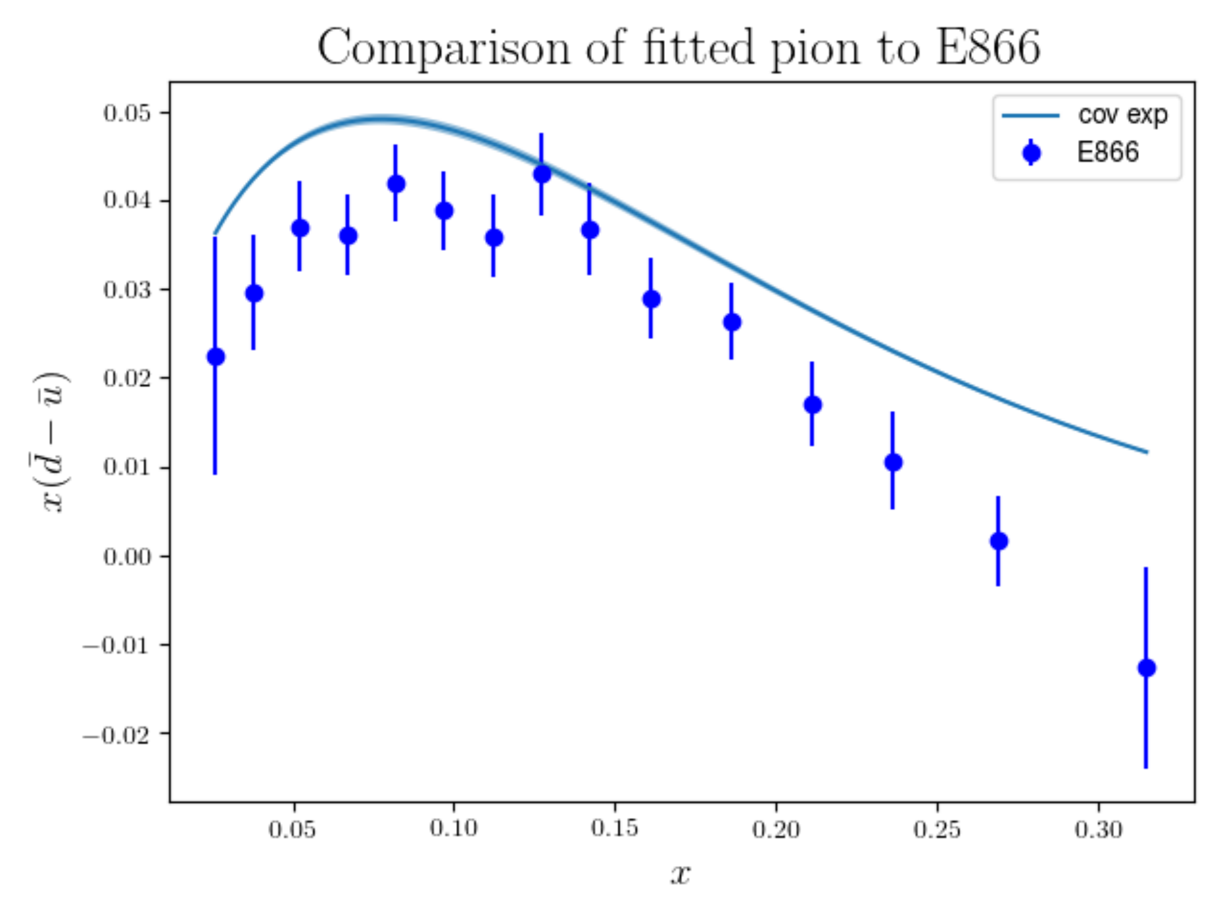
Conclusion

- First attempted fit to both high-x_π and low- x_π regions using Drell-Yan and Leading Neutron data
- Use of nested sampling algorithm to improve errors
- Next steps: to include threshold resummation in our calculation

BACKUP SLIDES

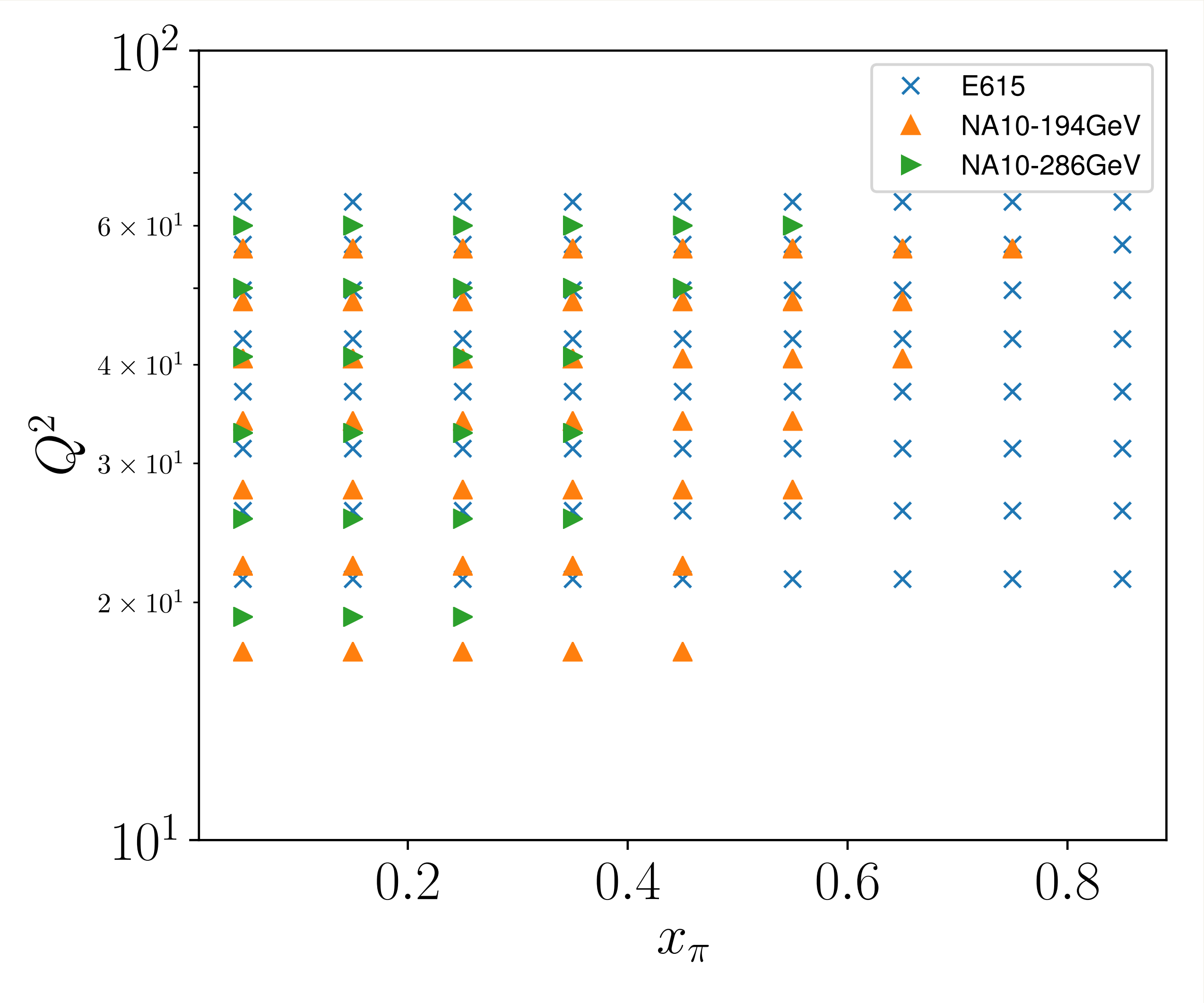
Prediction of E866

- Can make a prediction of E866 data for $\bar{d} - \bar{u} = (f_{\pi^+ n} - \frac{2}{3} f_{\pi^- \Delta^{++}}) \otimes \bar{q}_v^\pi$ using our valence π PDF, where $f_{\pi^+ n}$ and $f_{\pi^- \Delta^{++}}$ are the splitting functions from the proton



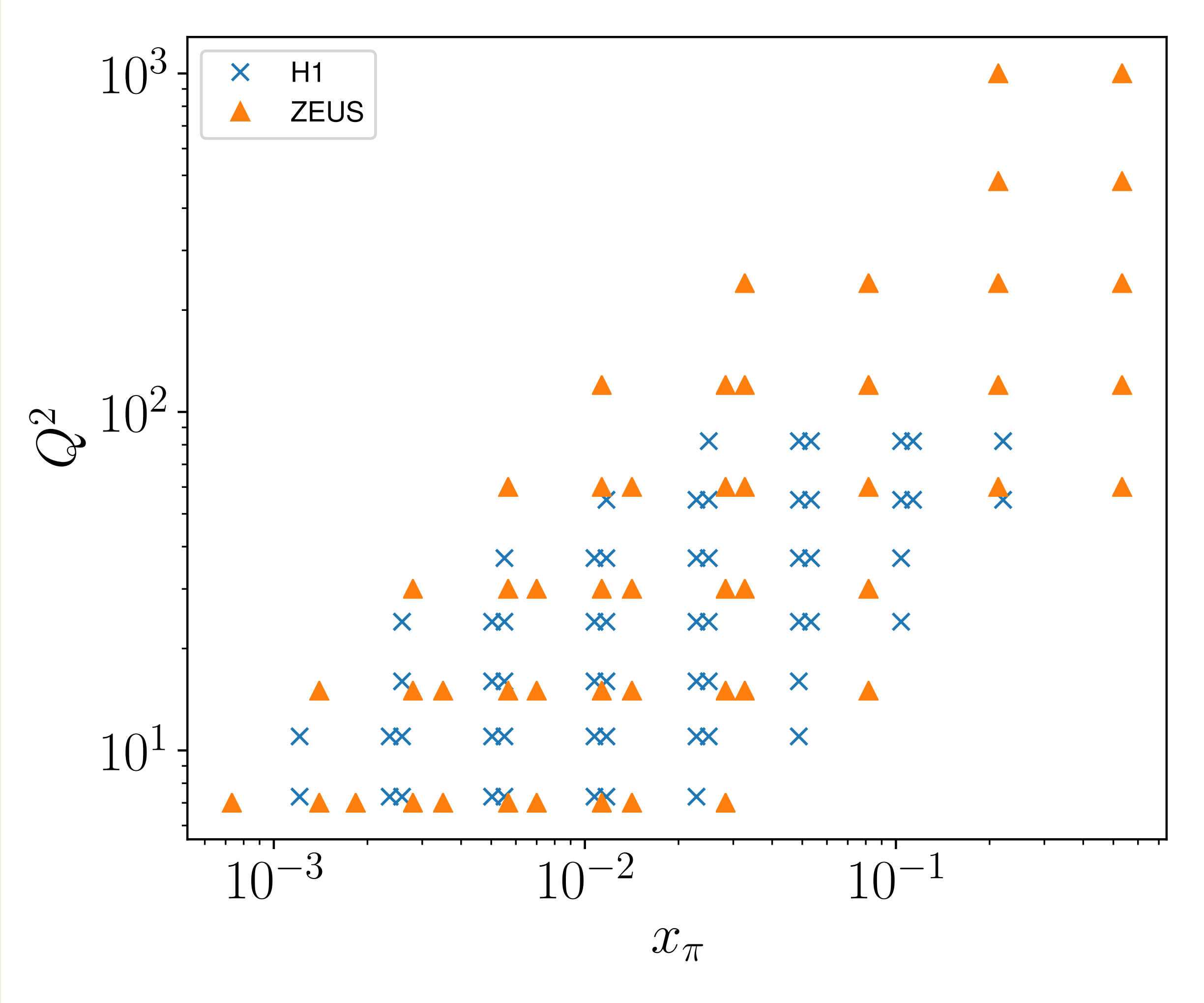
Kinematics - DY

- Hard cut-offs for $4.16^2 < Q^2 < 8.34^2$
- More available data for large-x_π



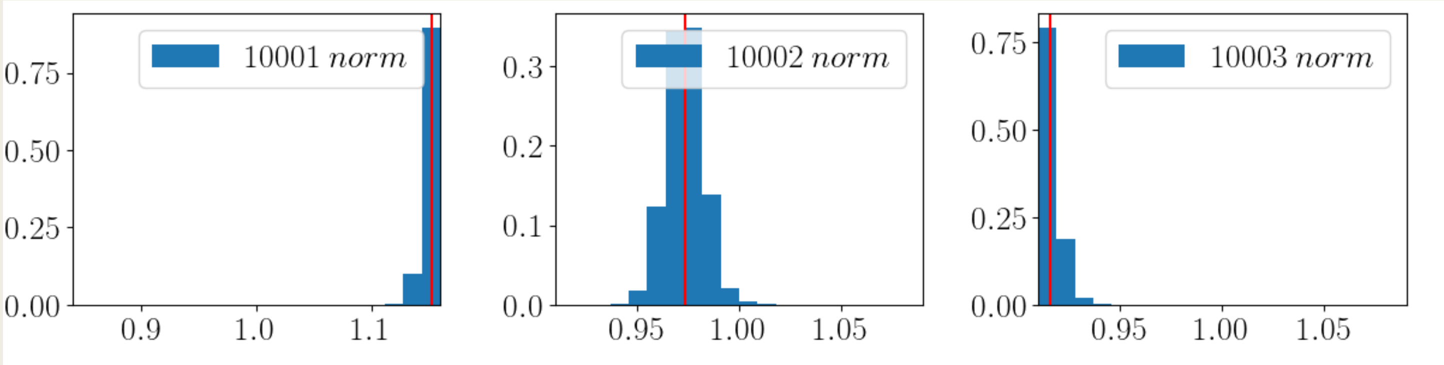
Kinematics - LN

- y-cut of 0.2

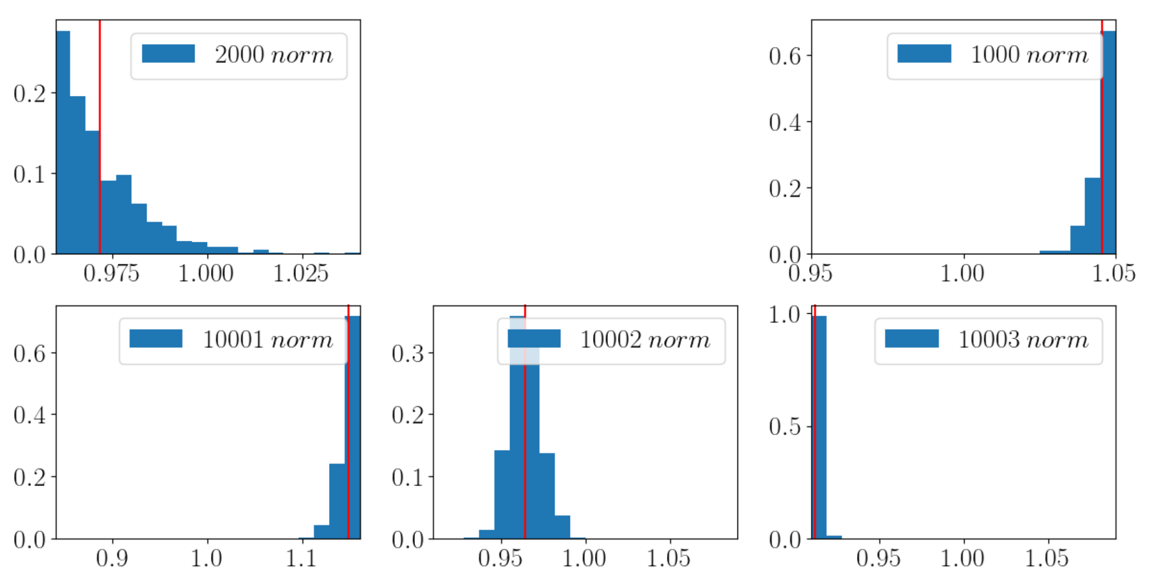


Normalization Parameterization

- For all datasets with overall normalization uncertainty, we fit to within the reported percentage around 1.



DY



LN

Mellin Transformation

$$f_i^n(\mu) \equiv \int_0^1 dx x^{n-1} f_i(x,\mu)$$

- Analogous to the Fourier transform
- Transform from *x*-space to Mellin space (exponents of *x*)

WHY??

- We know how PDFs evolve in scale based on DGLAP:

$$\frac{\partial f_i(\mu_f^2)}{\partial ln(\mu_f^2)} = DGLAP$$

Mellin Inversion

- After evolution, invert back into *x*-space

$$f_i(x,\mu) = \frac{1}{2\pi\mathbf{i}} \int_{C_n} dn x^{-n} f_i^n(\mu)$$

  - For each value on the contour, we do the DGLAP evolution
  - At large enough contour radius, integrand converges