Recent Developments in Lattice QCD: Flavor Physics Highlights



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(University of Illinois and Fermilab)

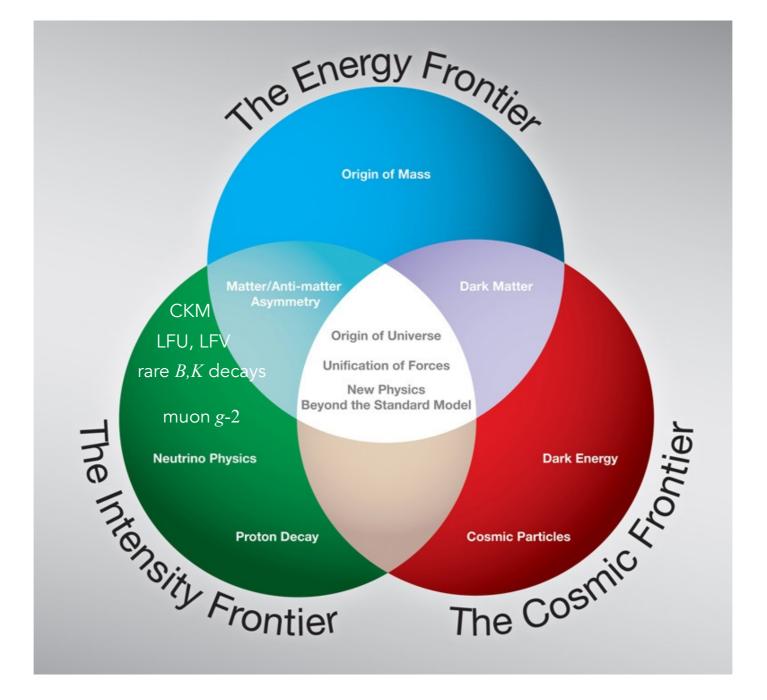




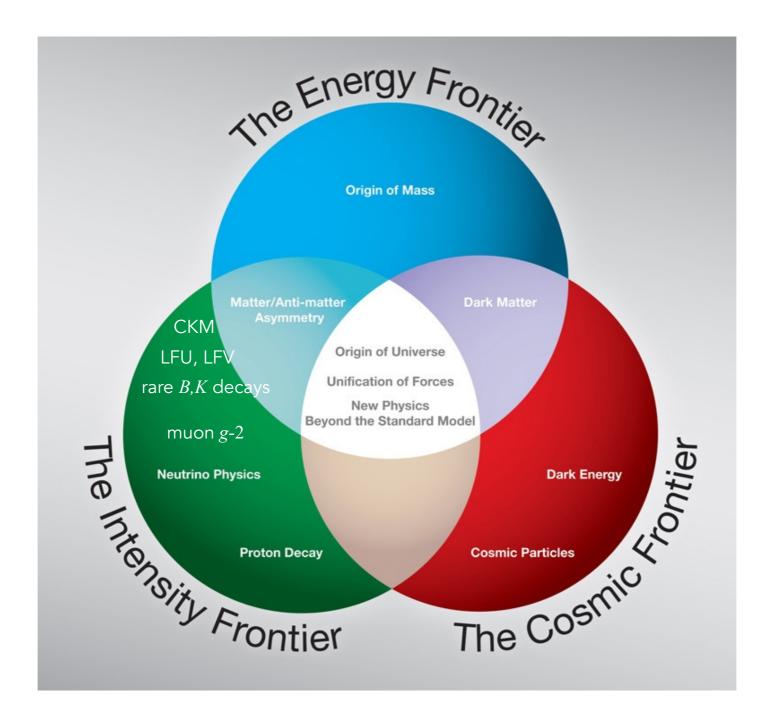
Outline

- Motivation and Introduction
- **Quark Flavor Physics**
- Nucleon matrix elements
- Summary and Outlook

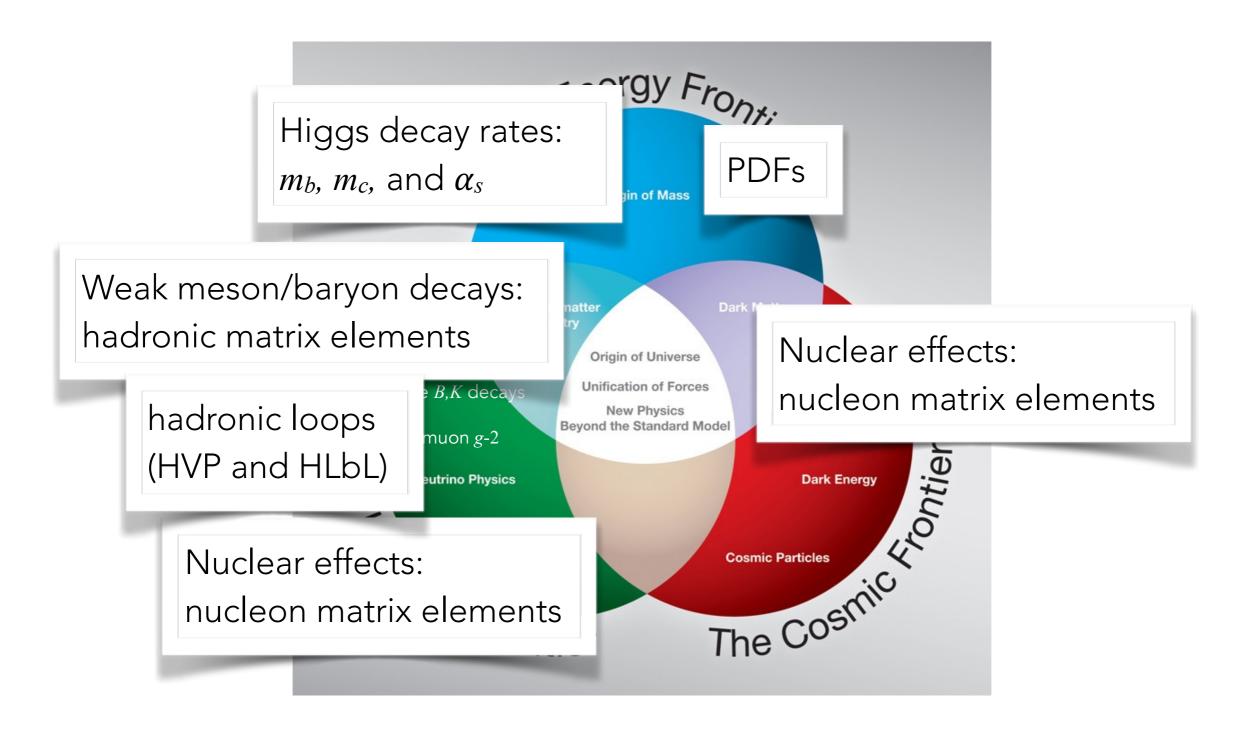
HEP research efforts at the three frontiers are connected by the common goal to discover and understand NP ...

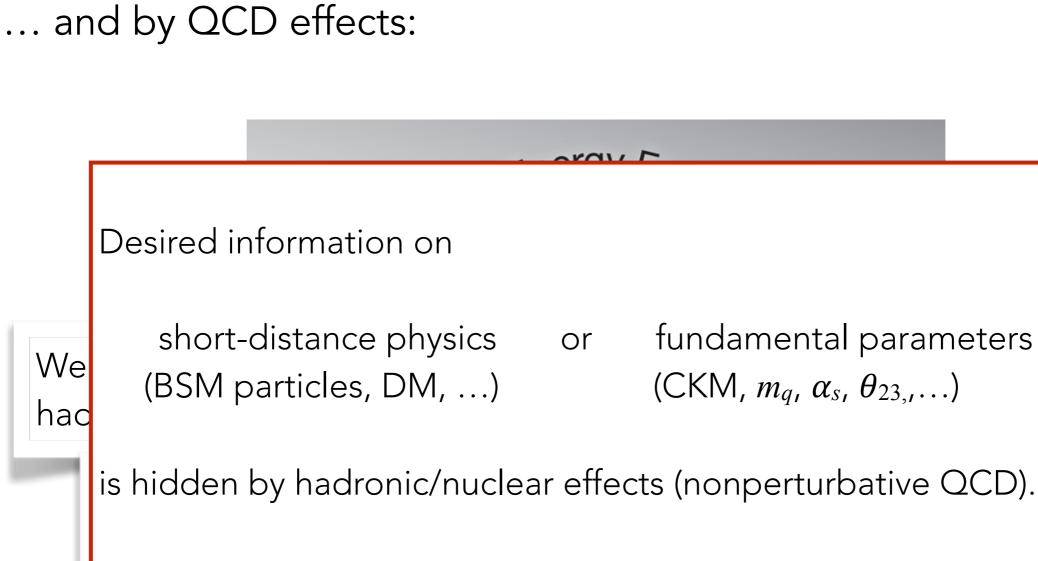


... and by QCD effects:



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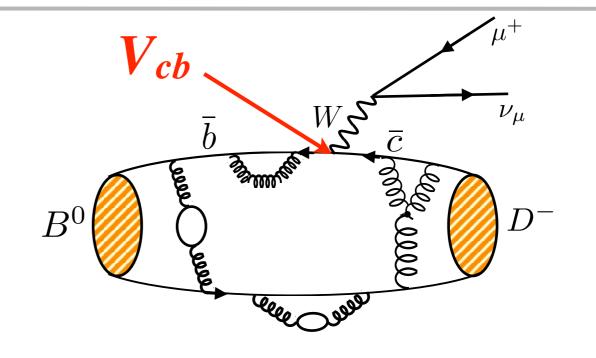


need precise QCD calculations to complement experimental measurements: Lattice QCD

The U

Introduction: quark flavor physics

example:
$$B^0 \to D^- \mu^+ \nu_\mu$$



Experiment vs. SM theory:

(experiment) = (known) x (**CKM factors**) x (had. matrix element)

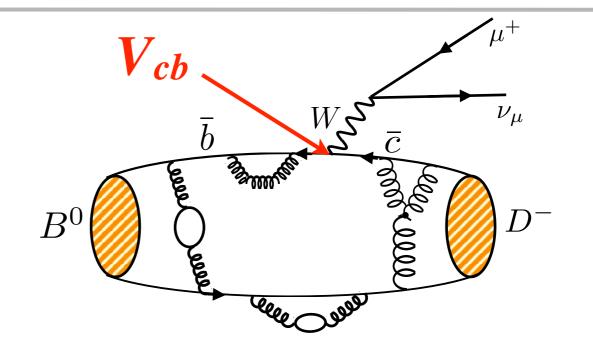
$$\begin{split} & \underbrace{d\Gamma(B \to \pi \ell \nu)}{dq^2}, \underbrace{d\Gamma(B \to K \ell^+ \ell^-)}{dq^2}, \dots \\ & \underbrace{d\Gamma(B \to D \ell \nu)}_{d\omega}, \underbrace{d\Gamma(B \to D \tau \nu)}_{d\omega}, \dots \\ & \Delta m_{d(s)} \end{split}$$



parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

Introduction: quark flavor physics

example:
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Experiment vs. SM theory:

(experiment) = (known) x (**CKM factors**) x (had. matrix element)

Two main purposes:

- combine experimental measurements with LQCD results to determine CKM parameters.
- confront experimental measurements of rare processes or lepton flavor (universality) violating observables with SM theory using LQCD inputs.

Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...



Rich program of ongoing/planned Lattice QCD calculations in support of experiments at all three frontiers...



Quark flavor physics (LHCb, Belle-2, BaBar, BES III, NA 62,...): decay constants, form factors, mixing matrix elements, amplitudes...



Muon g-2 (Fermilab and J-PARC experiments): hadronic corrections (HVP and HLbL)



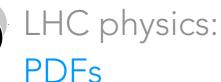
Neutrino, DM experiments (DUNE, NO ν a, μ BooNE, T2K, Mu2e, LZ, SuperCDMS,...):

Nucleon matrix elements



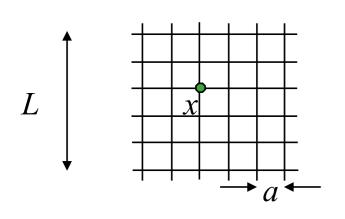
Higgs properties (LHC, ILC, ...):

quark masses and strong coupling



...aligned with the P5 physics drivers

$$\mathcal{L}_{\text{QCD}} = \sum_{f} \bar{\psi}_{f} (\not\!\!\!D + m_{f}) \psi_{f} + \frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu}$$

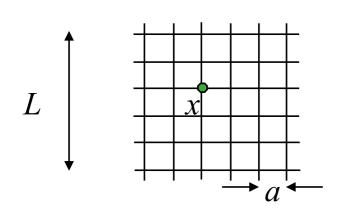


- ◆ discrete Euclidean space-time (spacing *a*)
 derivatives → difference operators, etc...
- finite spatial volume (L)
- finite time extent (T)

adjustable parameters

- ♦ lattice spacing: $a \rightarrow 0$
- ♦ finite volume, time: $L \rightarrow \infty$, T > L
- ♦ quark masses (m_f): $M_{H,lat} = M_{H,exp}$ f → f → f → f,phys M_{ud} m_s m_c m_b
 extrapolations/interpolations

$$\mathcal{L}_{\text{QCD}} = \sum_{f} \bar{\psi}_{f} (\not\!\!\!D + m_{f}) \psi_{f} + \frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu}$$



◆ discrete Euclidean space-time (spacing *a*)
 derivatives → difference operators, etc...

Mud

 m_{s}

 m_c

- finite spatial volume (L)
- ✦ finite time extent (T)

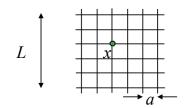
Integrals are evaluated numerically using monte carlo methods.

Mh

adjustable parameters

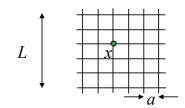
- ♦ lattice spacing: $a \rightarrow 0$
- ♦ finite volume, time: $L \rightarrow \infty$, T > L
- ♦ quark masses (m_f):
 M_{H,lat} = $M_{H,exp}$ tune using hadron masses $m_f
 ightarrow m_{f,phys}$ extrapolations/interpolations

A. El-Khadra



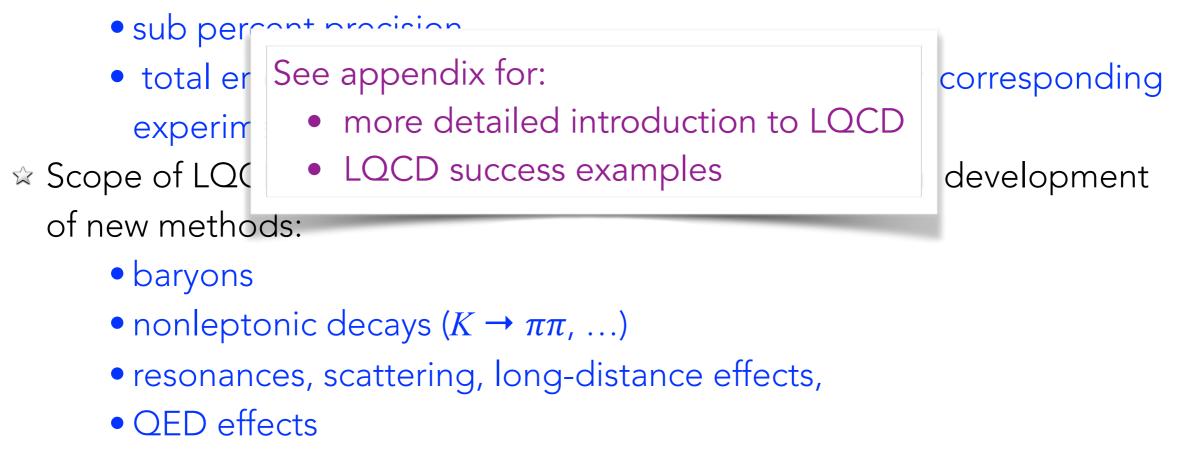
The State of the Art

- Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that quantitatively account for all systematic effects (discretization, finite volume, renormalization,...), in some cases with
 - sub percent precision.
 - total errors that are commensurate (or smaller) than corresponding experimental uncertainties.
- Scope of LQCD calculations is increasing due to continual development of new methods:
 - baryons
 - nonleptonic decays ($K \rightarrow \pi \pi, ...$)
 - resonances, scattering, long-distance effects,
 - QED effects
 - ...



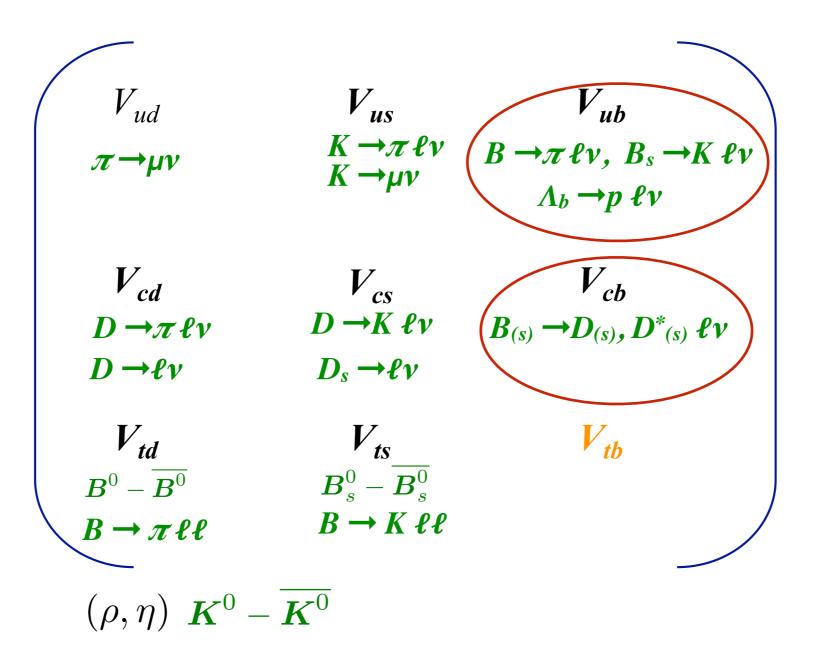
The State of the Art

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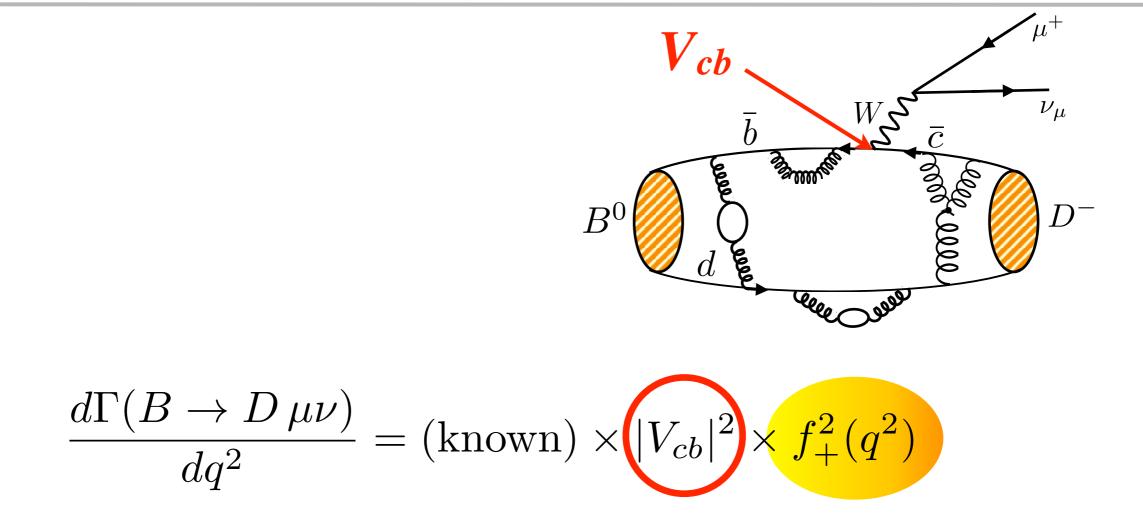
Quark Flavor: CKM determinations



Quark Flavor: CKM determinations

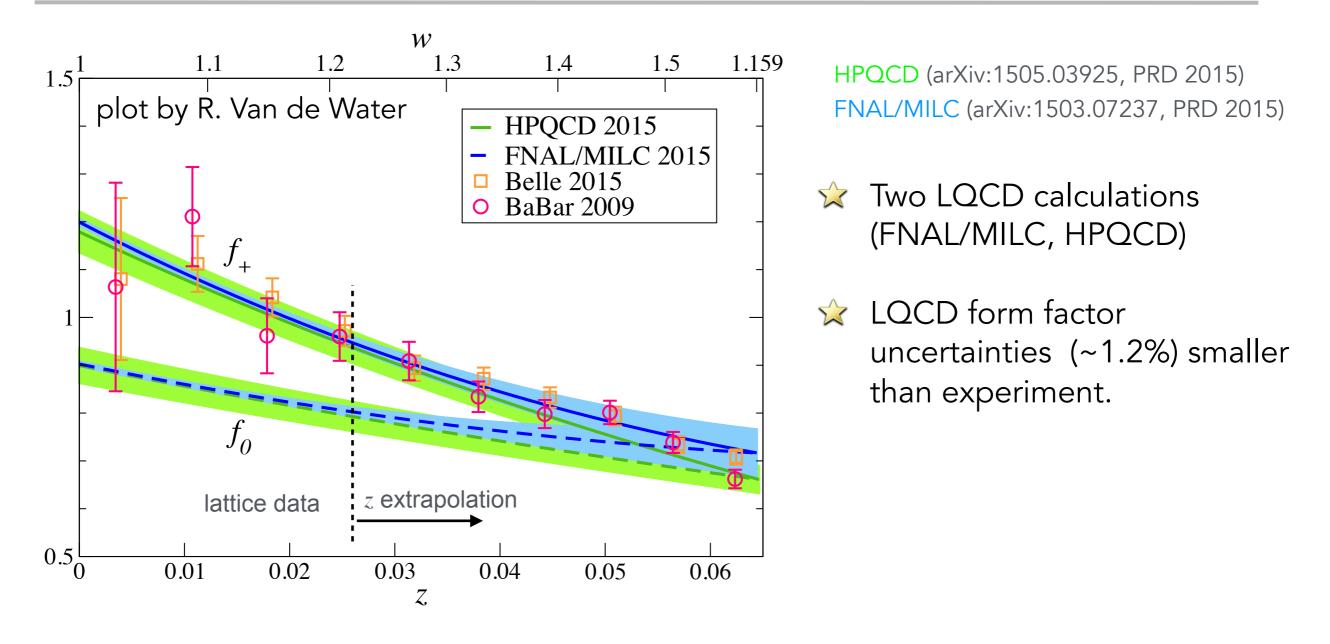
Precise Lattice QCD results with complete systematic error budgets now exist for all these processes improved determinations of the corresponding CKM elements

Form factors for
$$B \to D \,\ell \nu$$
, $(\ell = e, \mu, \tau)$



- \star calculate the form factors in the low recoil energy (high q^2) range.
- * use z-expansion for model-independent parameterization of q^2 dependence.
- ★ calculate both form factors, $f_+(q^2), f_0(q^2)$.
- ★ for $f_+(q^2)$ compare shape between experiment and lattice.
- ★ Tensor form factor(s) are calculable in LQCD using the same methods.

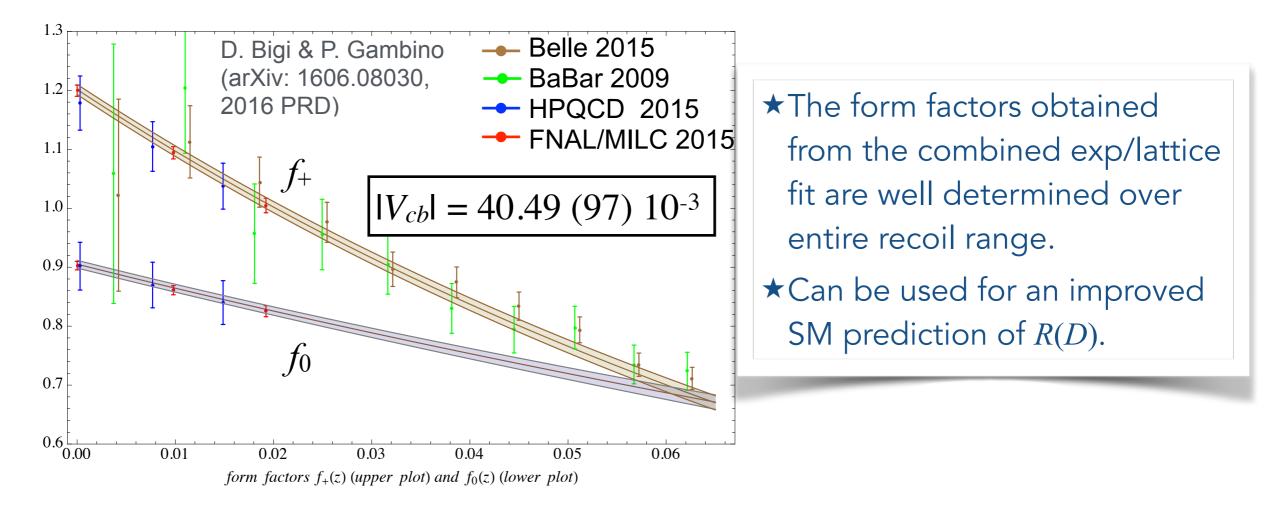
Form factors for $B \to D \,\ell \nu$, $(\ell = e, \mu, \tau)$



 $\cancel{}$ LQCD form factors can be used to calculate the CKM free ratio:

$$R(D) \equiv \frac{\mathcal{B}(B \to D\tau\nu_{\tau})}{\mathcal{B}(B \to D\ell\nu)}$$

combine LQCD form factors with experiment, using the BGL (Boyd, Grinstein, Lebed, hep-ph/9508211, 1996 NPB) parameterization:



☆ FLAG-3 (S. Aoki et al, arXiv:1607.00299, EPJC 2017) performs a similar combined fit using the BCL (Bourrely, Caprini, Lellouch, arXiv:0807.2722, PRD 09) parameterization.

form factor for $B \to D^* \ell \nu$ at zero recoil and V_{cb}

$$\frac{d\Gamma(B \to D^* \ell \nu)}{d\omega} = (\text{known}) \times |V_{cb}|^2 \times (\omega^2 - 1)^{1/2} |\mathcal{F}(\omega)|^2$$

HFLAV 2016: Use CLN^{*} expression to extrapolate exp. data to $\omega=1$:

 $B \to D^* \ell \nu : \eta_{\rm EW} | V_{cb} | \mathcal{F}(1) = (35.61 \pm 0.11 \pm 0.41) \times 10^{-3}$

combine with LQCD calculation of $\mathcal{F}(1)$:

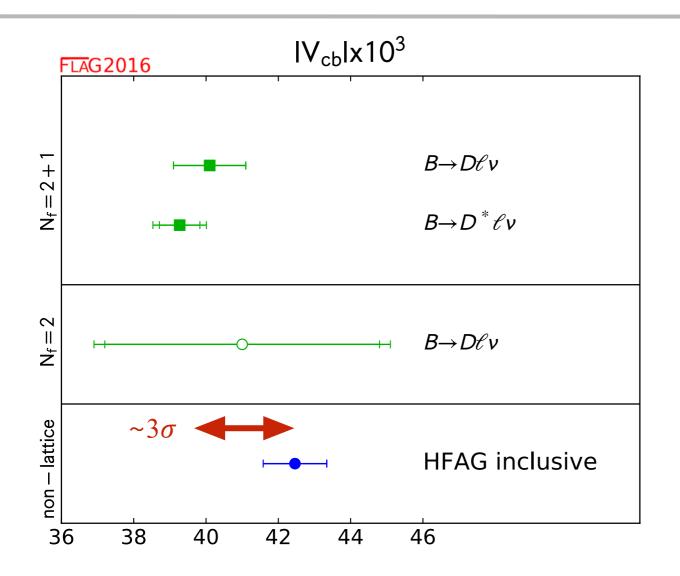
♦ FNAL/MILC 2014 (J. Bailey et al, arXiv:1403.0635, 2014 PRD): $\mathcal{F}(1) = 0.906(4)(12)$

 *CLN (Caprini, Lellouch, Neubert, hep-ph/9712417, NPB 98) is based on the modelindependent z-expansion (just like BGL, BCL), but then add model-dependent assumptions about the parameters

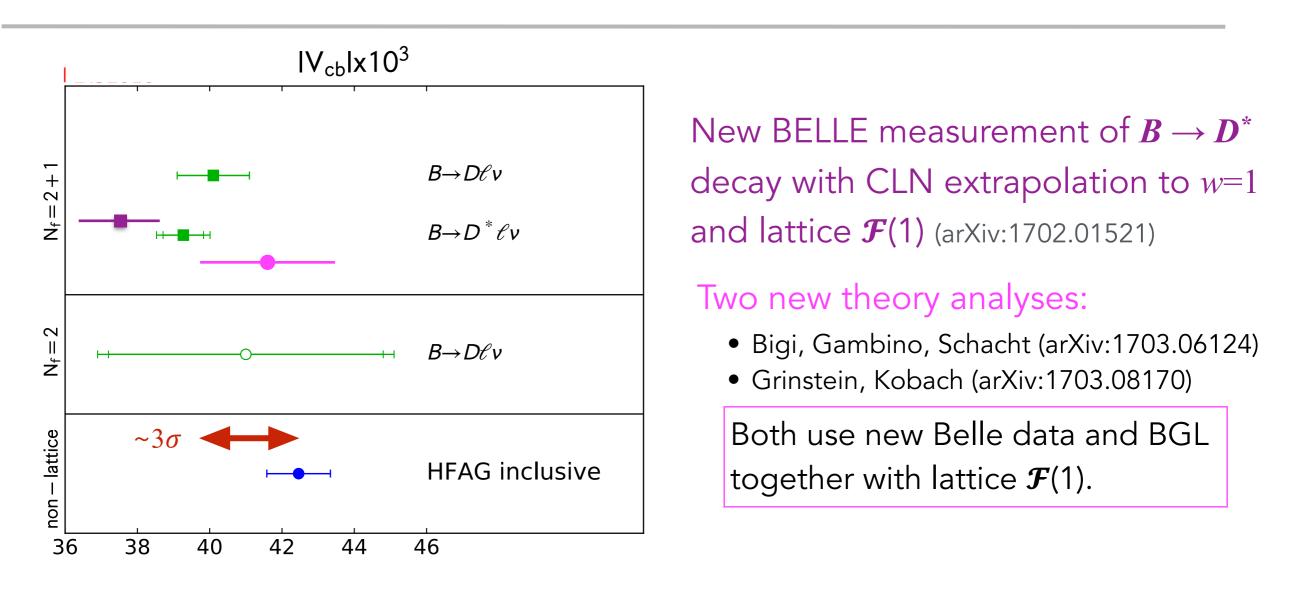
reduces the error from the extrapolation

• LQCD form factor data for $B \rightarrow D^*$ at nonzero recoil are not yet available.

Exclusive $|V_{cb}|$



Exclusive $|V_{cb}|$



- Difference between the new CLN and BGL results hints at possible bias.
- Need lattice form factor data for $B \rightarrow D^*$ at nonzero recoil:

combine with experimental data using BGL (same as for $B \rightarrow D$)

- improve precision and perhaps resolve exclusive/inclusive tension
 - in progress (FNAL/MILC, HPQCD, RBC, LANL/SNU)

BSM phenomenology: LFU τ/ℓ

 $R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_{\tau})}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$

 $\Delta \chi^2 = 1.0$ contours

SM Predictions

R(D)=0.300(8) HPQCD (2015)

0.5

R(D)=0.299(11) FNAL/MILC (2015)

R(D*)=0.252(3) S. Fajfer et al. (2012)

 2σ

HFLAV

FPCP 2017

 $P(\gamma^2) = 71.69$

0.6

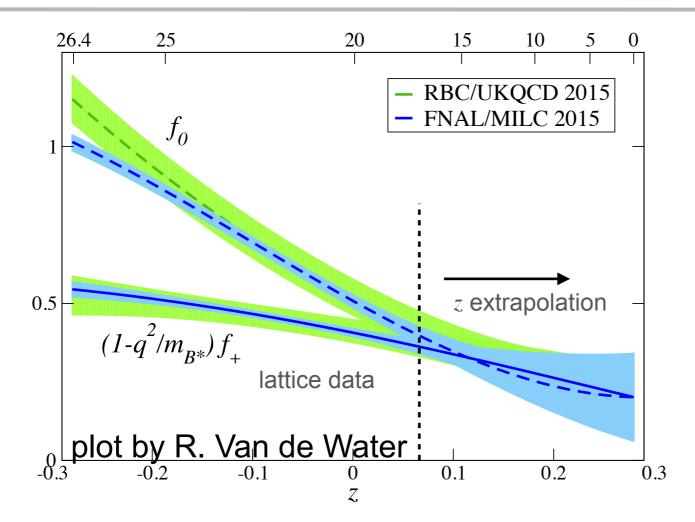
R(D)

$R(D^*)$ BaBar, PRL109,101802(2012) 0.5 Belle, PRD92,072014(2015) **BaBar** LHCb, PRL115,111803(2015) Belle, PRD94,072007(2016) $0.440 \pm 0.058 \pm 0.042$ 0.45 Belle, PRL118,211801(2017) LHCb, FPCP2017 Belle Average 0.4 $0.375 \pm 0.064 \pm 0.026$ 0.35 Average $0.391 \pm 0.041 \pm 0.028$ 0.3 SM prediction 0.25 0.297 ± 0.017 Bigi & Gambino 0.2 FNAL/MILC HPQCD 0.3 0.2 0.4 HFAG Prel. EPS15 HFLAV 2017 average: combined 3.9σ excess 0.2 0.4 0.6

R(D)

HFLAV (EPS 2015)

form factors for $B \to \pi \,\ell \,\nu \,\& \, V_{ub}$



RBC/UKQCD (arXiv:1501.05373, PRD 2015) FNAL/MILC (arXiv:1503.07839, PRD 2015)

FNAL/MILC & RBC/UKQCD form factors are in good agreement.

 \Rightarrow Two independent LQCD **predictions** for $B_s \rightarrow K \ell \nu$ form factors

(HPQCD, arXiv:1406.2279, PRD 2014; RBC, arXiv:1501.05373, PRD 2015)

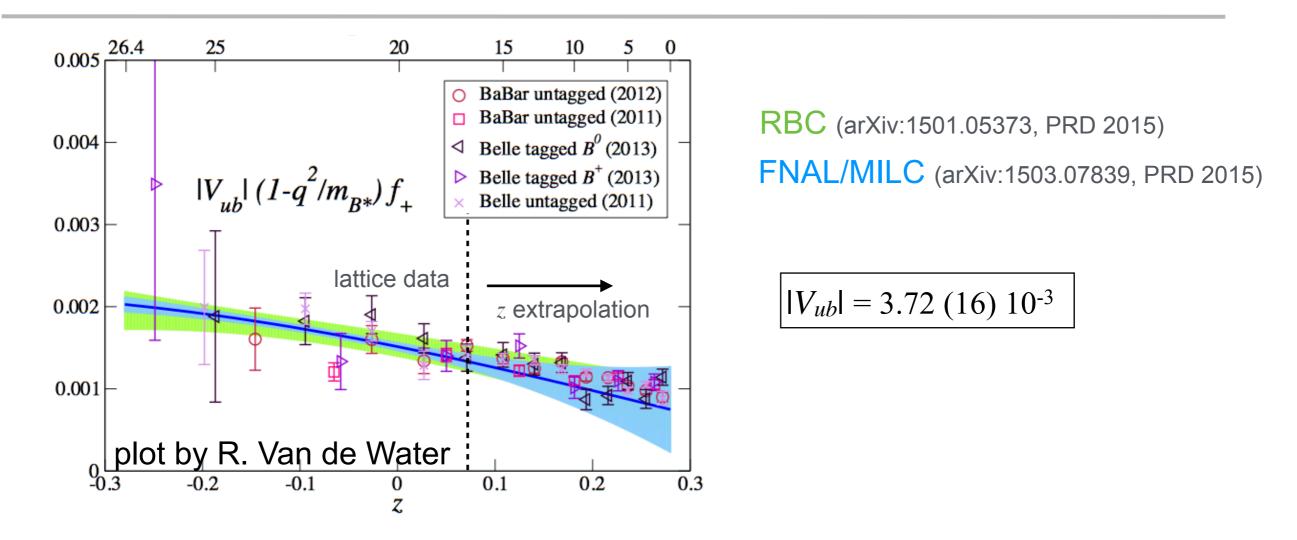
+ ongoing work by FNAL/MILC (see Z. Gelzer talk @11:21 in Curia II) and others (see Lattice 2017 talks by ...)

 \Rightarrow also: LQCD results for form factors for rare ($B \rightarrow K, \pi$) decays:

FNAL/MILC (arXiv:1507.01618, PRL 2015,arXiv:1509.06235, PRD 2016) HPQCD (arXiv:1306.0434, 1306.2384, PRL 2013)

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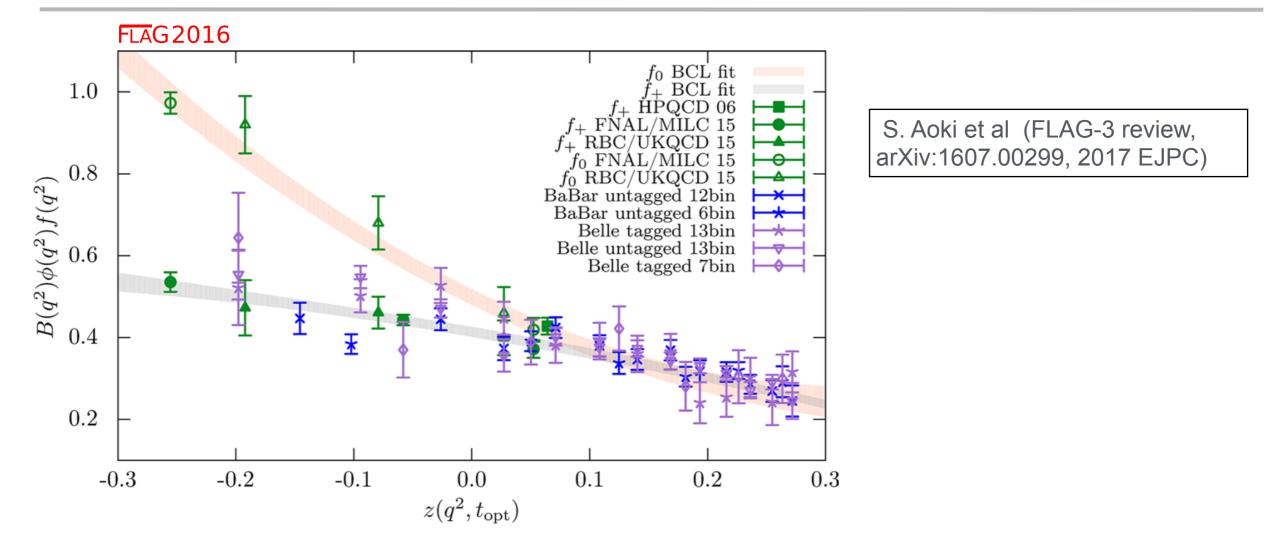
form factors for $B \to \pi \,\ell \,\nu \,\& \, V_{ub}$



 \Rightarrow shape of f_+ agrees with experiment and uncertainties are commensurate

- ★ fit lattice form factors together with experimental data to determine $|V_{ub}|$ and obtain form factors (f_{+}, f_0) with improved precision...
- * similar analysis for $|V_{ub}/V_{cb}|$ from Λ_b decay with LHCb (arXiv:1503.01421, PRD 2015; arXiv:1504.01568, Nature 2015).

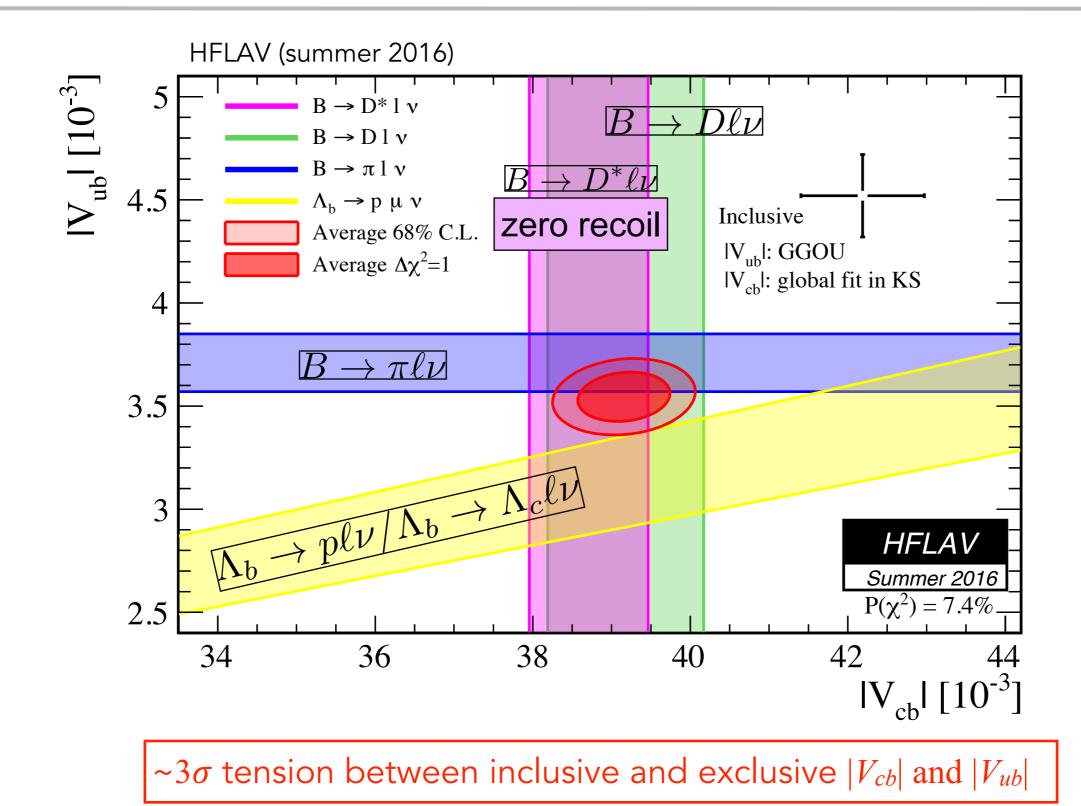
form factors for $B \to \pi \,\ell \,\nu \,\& \, V_{ub}$



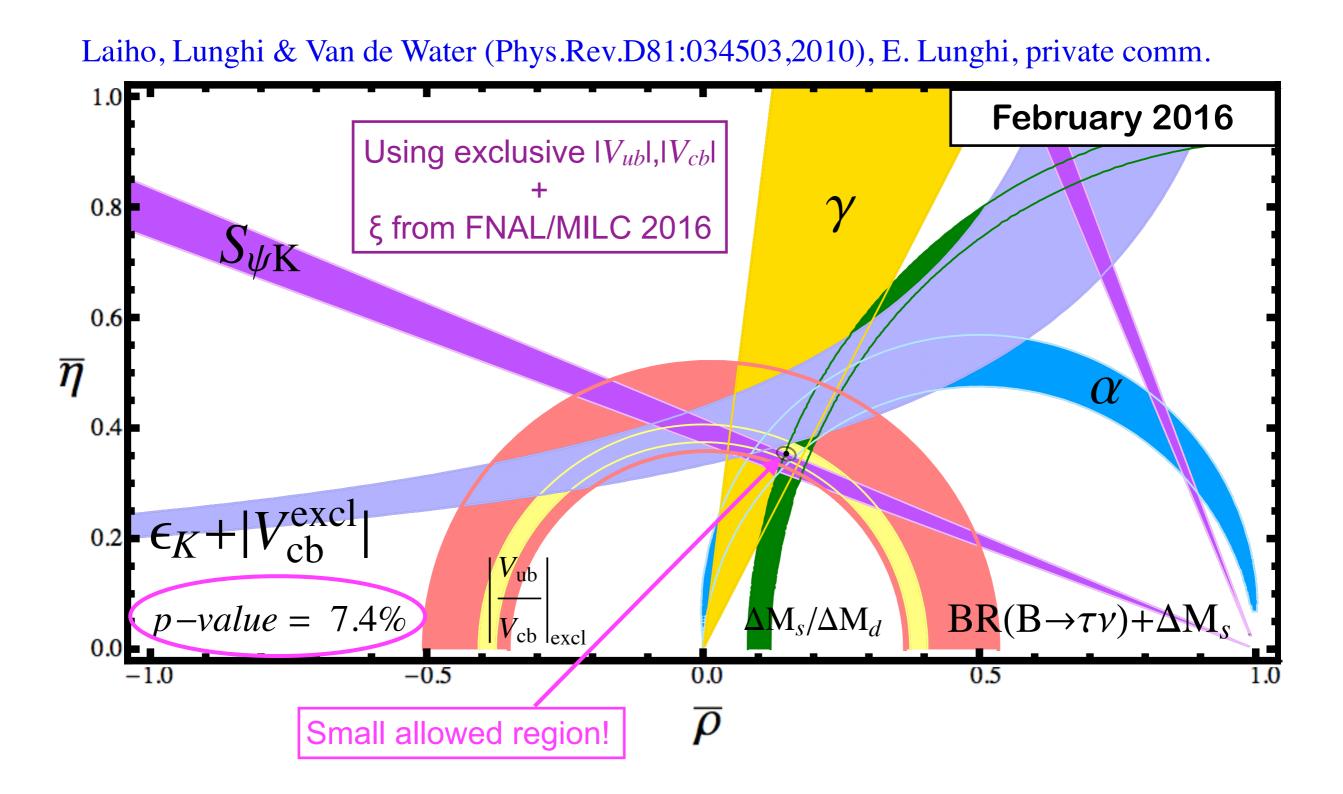
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Exclusive vs. inclusive $|V_{cb}|$ and $|V_{ub}|$



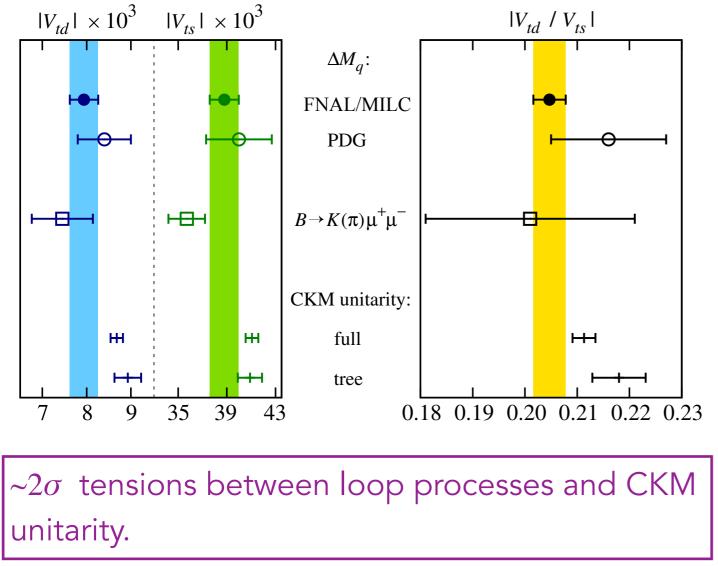
UT analysis



Quark flavor: tensions

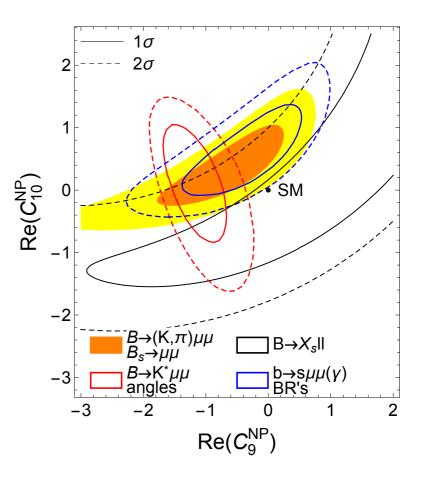
 $B_{(s)}$ -meson mixing, $B \to K, \pi \ell^+ \ell^-$ and $B_s \to \mu^+ \mu^-$



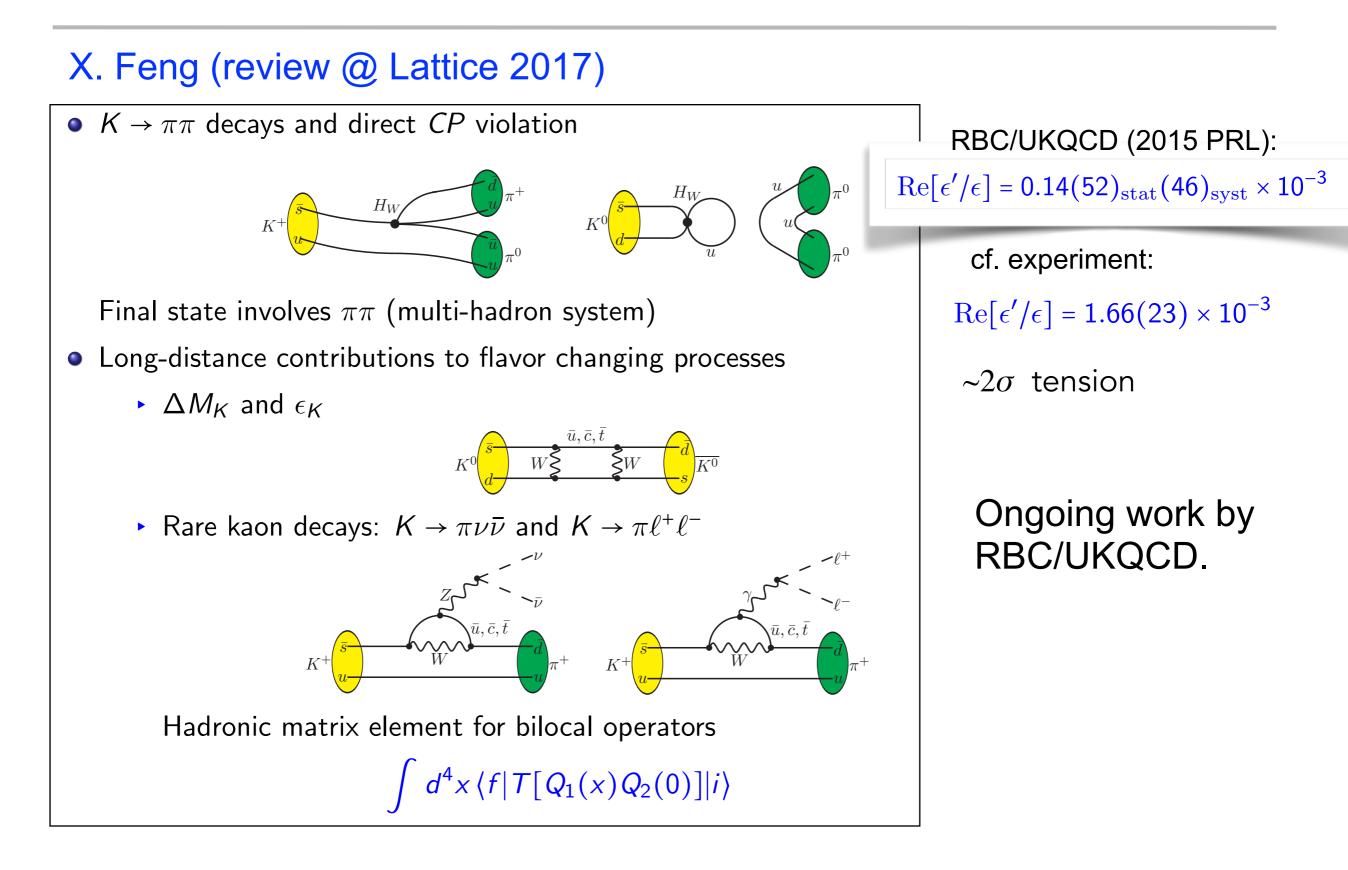


D. Du et al (arXiv:1510.02349, PRD 2016)

Constraints on Wilson coefficients (C_9 , C_{10})



Quark flavor: Kaons - long distance effects



Charged Lepton Flavor: muon g-2

$$= (-ie) \,\overline{u}(p') \left[\gamma^{\mu} F_1(q^2) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2m} F_2(q^2) \right] \, u(p)$$

- ♦ muon anomalous magnetic moment: $a_{\mu} = F_2(0)$
 - is generated by quantum effects (loops).
 - receives contributions from QED, EW, and QCD effects in the SM.
 - is a sensitive probe of new physics.
- ◆ QED + EW correction are known precisely:

3

$$\delta a_{\mu}^{\text{QED}} \times 10^{11} = 0.08 \qquad \delta a_{\mu}^{\text{EW}} \times 10^{11} = 1$$

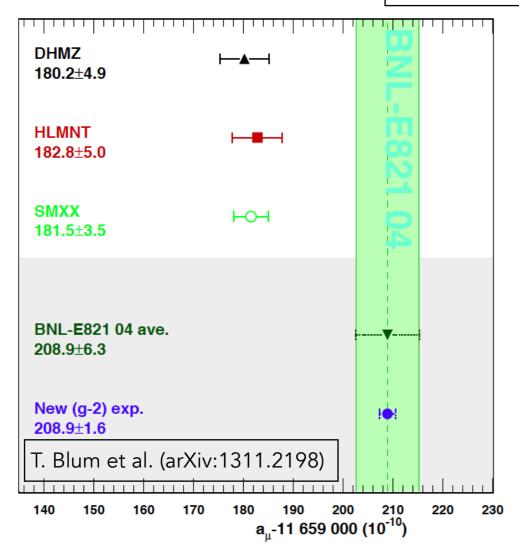
◆ QCD corrections are the dominant source of error in the SM prediction:

$$\delta a_{\mu}^{\rm had} \times 10^{11} \sim 50$$

(Davier et al. 2011, Hagiwara et al 2011, Kurz et al 2014, Prades et al 2009, Colangelo et al 2014, Jegerlehner 2015, Benayoun et al 2015,...)

Charged Lepton Flavor: muon g-2





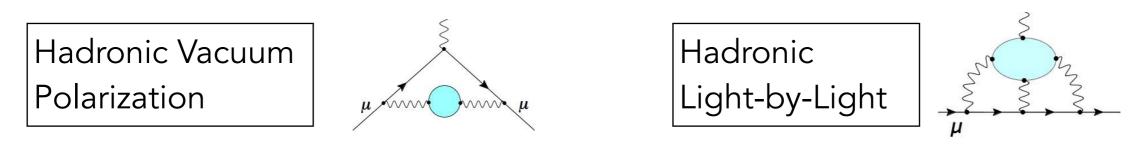
Fermilab g-2 experiment:

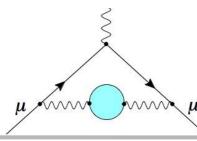
- ✦ reduce exp. error by a factor of 4
- first result with "Brookhaven level" statistics expected in 2018.
- Commissioning of beam, ``wiggle party".

J-PARC experiment:

- ♦ complementary, different exp. method
- expect measurement at 0.3-0.4 ppm level

Need to reduce and better control theory errors of the hadronic corrections:

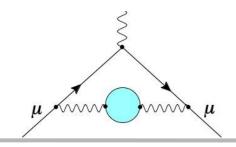




Hadronic vacuum polarization

- ◆ Target: ~0.2% total error
- Dispersion relation + experimental data for $e^+e^- \rightarrow \text{hadrons}$ (and τ data)
 - current uncertainty ~0.4-0.5%
 - can be improved with more precise experimental data
 - new experimental measurements expected/ongoing at BaBar, BES-III, Belle/Belle-II, CMD-3, SND, KLOE,....
- Complete lattice QCD results by several groups. First with ~1.8% total error by HPQCD. LQCD calculations need to
 - be based on physical mass ensembles
 - include disconnected contributions

- see R. Van de Water talk (Wednesday, 2:42pm, Curia II)
- include QED and strong isospin breaking corrections ($m_u \neq m_d$)
- include finite volume corrections
- Compare intermediate quantities (moments, Taylor coefficients) with R-ratio data.
- + Hybrid method: combine LQCD with R-ratio data (Lehner @ Lattice 2017)

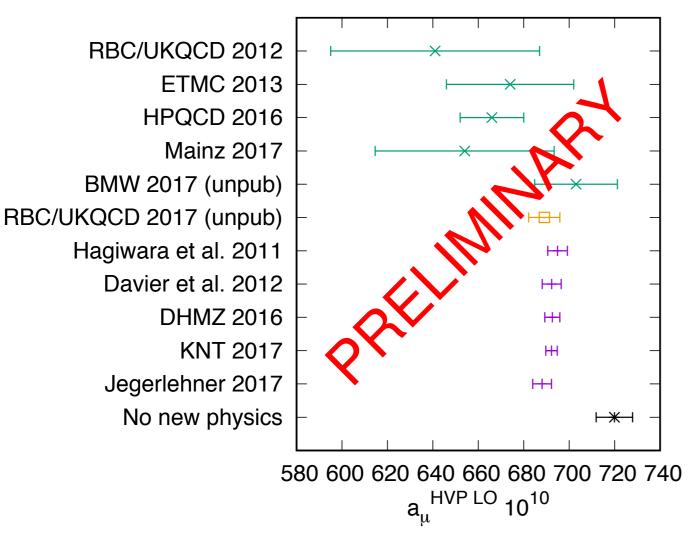


Hadronic vacuum polarization

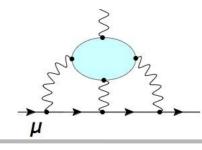
Hybrid method: combine LQCD with R-ratio data (C. Lehner @ Lattice 2017)

LQCD calculations of HVP are still much less precise than dispersive method. But comparisons between R-ratio and lattice data are already useful.

(C. Lehner @ Lattice 2017)



- Convert R-ratio data to Euclidean correlation function (via the dispersive integral).
- Compare lattice/R-ratio data (after adding all the corrections and extrapolating to continuum, infinite volume).
- Use R-ratio data where LQCD errors are large and vice versa.
- Potential for obtaining HVP with reduced error.



Hadronic Light-by-light

Hadronic light-by-light:

- Target: ≤ 10% total error
- ♦ current estimate "Glasgow consensus" based on different QCD models
- theory error not well determined and not improvable

Dispersive approach:

- more complicated than for HVP (Colangelo et al, arXiv:1702.07347; Kubis et al, 2012, 2014; Hoferichter et al, 2012, 2014; Hanhardt et al, 2013; Pascalutsa et al, Pauk et al, Danilkin et al,...)
- combine with exp. data and/or LQCD calculations

Direct lattice QCD calculations:

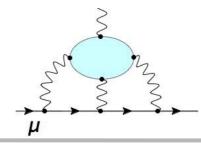
♦ QCD + stochastic QED

(Jin et al, arXiv:1610.04603, 2016 PRL; arXiv:1705.01067)

♦ QCD + exact QED kernel

(Asmussen @Lattice 2017; Green et al, arXiv:PRL 2015)

 dominant contribution from pion pole (transition form factors) (Gerardin et al, arXiv:1607.08174, 2016 PRD; Lattice 2017)



Hadronic Light-by-light

Breakthrough (RBC/UKQCD):

First LQCD calculation of connected and leading disconnected contribution with good statistical significance (T. Blum et al, arXiv:1610.04603, 2017 PRL).

$$a_{\mu}^{\mathrm{HLbL}} = (5.35 \pm 1.35) \times 10^{-10}$$

- \bullet a = 0.11 fm, L = 5.5 fm, physical pion mass, statistical error only.
- ♦ systematic error analysis (finite volume, continuum limit, ...) in progress.
- also: recent work (arXiv:1705.01067) yields exponentially suppressed FV errors.

Mainz group: progress @ Lattice 2017

- ✦ First numerical tests of formulation for direct calculation (Asmussen)
- ♦ New results for the pion transition form factor (Gerardin)

Muon g-2 Theory Initiative

• Steering Committee:

Gilberto Colangelo, Michel Davier, Simon Eidelman, AXK, Christoph Lehner, Tsutomu Mibe (E34), Andreas Nyffeler, Lee Roberts (E989), Thomas Teubner.

- The goal of the initiative is to bring the different communities are engaged in hadronic muon g-2 theory together to facilitate interactions, compare and assess the status of the various efforts and map out a strategy for obtaining the best theoretical predictions ahead of major experimental announcements.
- <u>First Workshop took place at Fermilab (3-6 June 2017)</u>. The second workshop will be in Mainz (18-22 June 2018).
- see the appendix for more information.

First Workshop of the Muon g-2 Theory Initiative

3-6 June 2017 *Q Center* US/Central timezone



66 registered participants, 40 talks, 15 discussion sessions (525 minutes)

A. El-Khadra

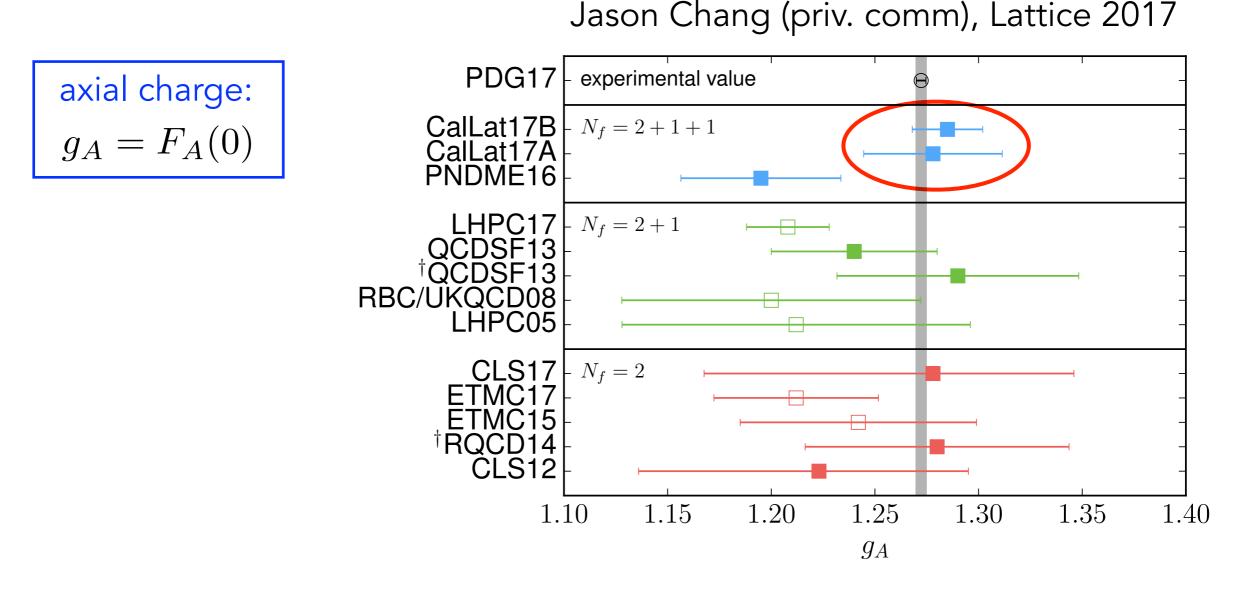
DPF 2017, Fermilab, 31 Jul - 04 Aug 2017

Search

Nucleon matrix elements

- Needed for neutrino and dark matter experiments: Neutrinos and DM particles (in direct detection experiments) scatter off nucleons in nuclear targets.
- Needed matrix elements depend on the underlying quarklevel interactions:
 - axial current (charged and neutral) for v's
 - scalar,... currents for DM

Nucleon matrix elements: g_A



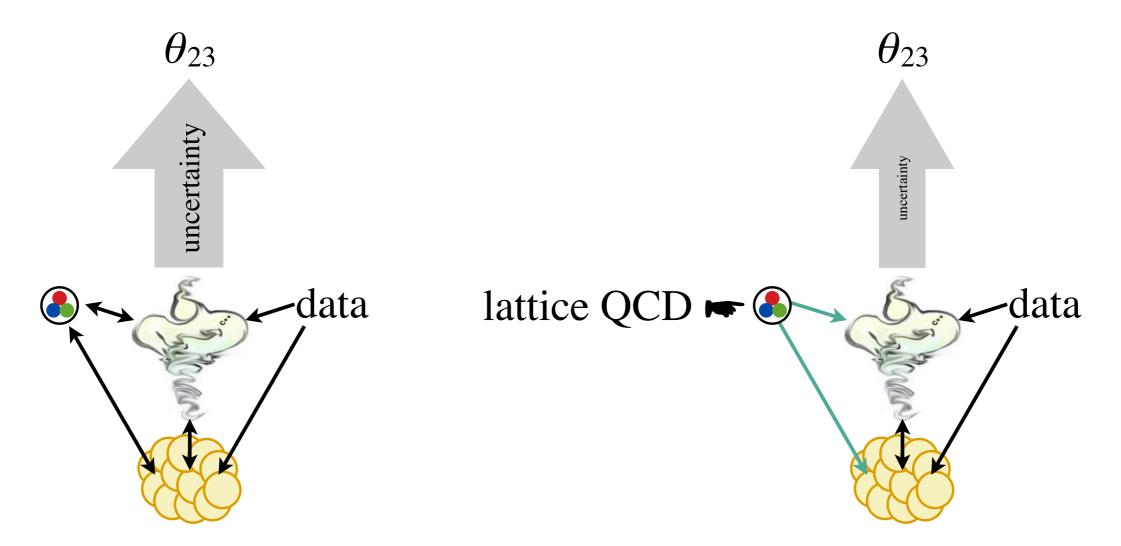
- Several "complete" LQCD calculations.
- Recent CalLat result (arXiv:1704.01114) with small errors in good agreement with exp.

Nucleon matrix elements

- Needed for neutrino and dark matter experiments: Neutrinos and DM particles (in direct detection experiments) scatter off nucleons in nuclear targets.
- Needed matrix elements depend on the underlying quarklevel interactions:
 - axial current (charged and neutral) for v's
 - scalar,... currents for DM
- Nucleon matrix elements then are used as inputs to
 - event generators (for example, GENIE)
 - Nuclear effective field theory
 - •

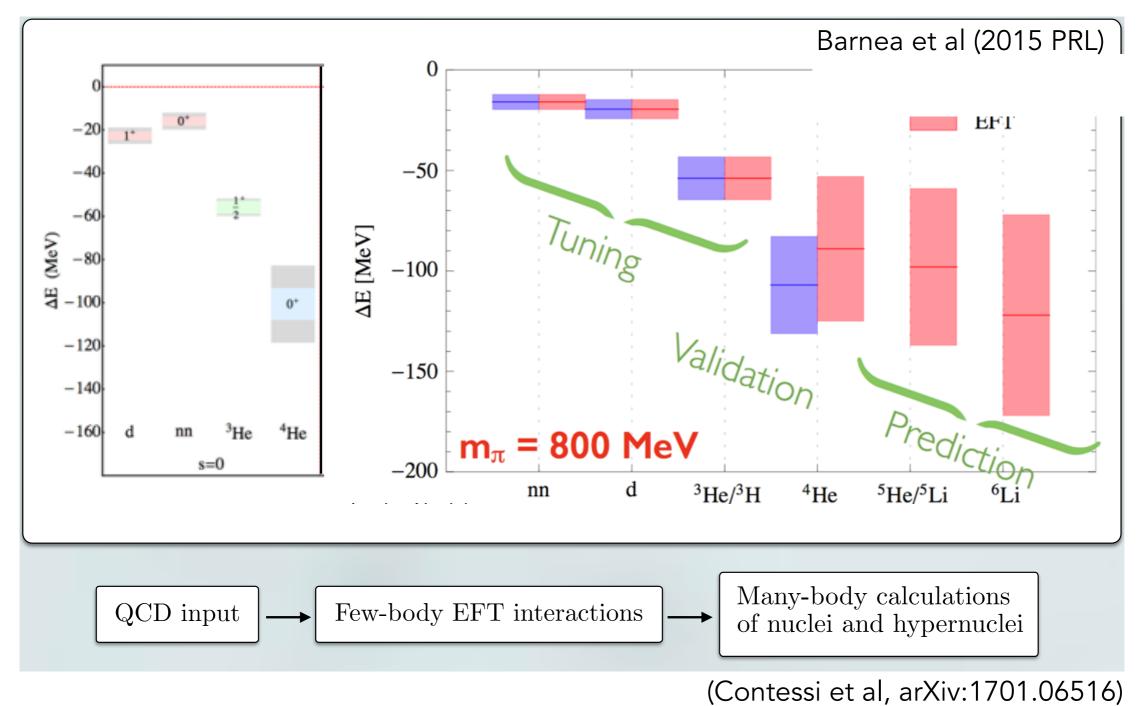
Nucleons to Nuclei





Nucleons to Nuclei

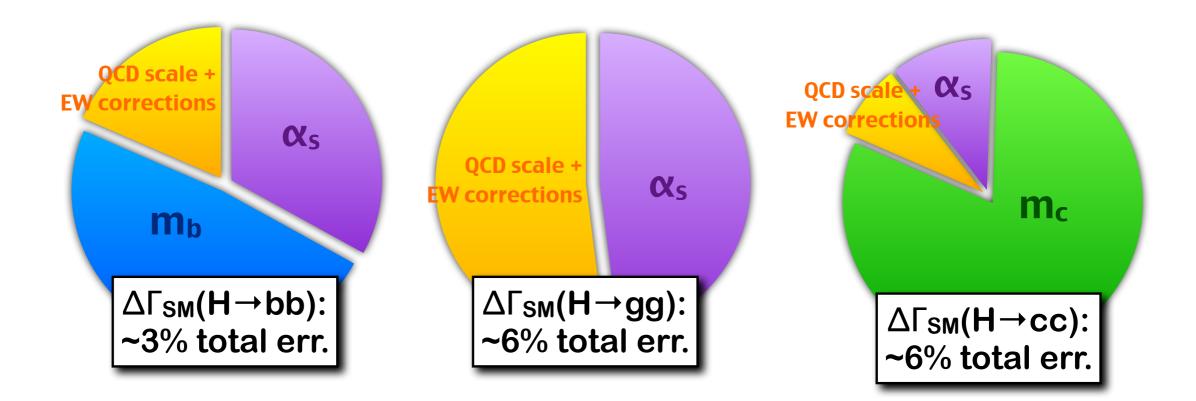
Z. Davoudi @ Lattice 2017



Higgs decays

Aarti Veernala (FNAL/MILC) @ Lattice 2017

Uncertainties in Higgs Branching Ratios and total widths are dominated by parametric uncertainties from α_s , m_c and m_b

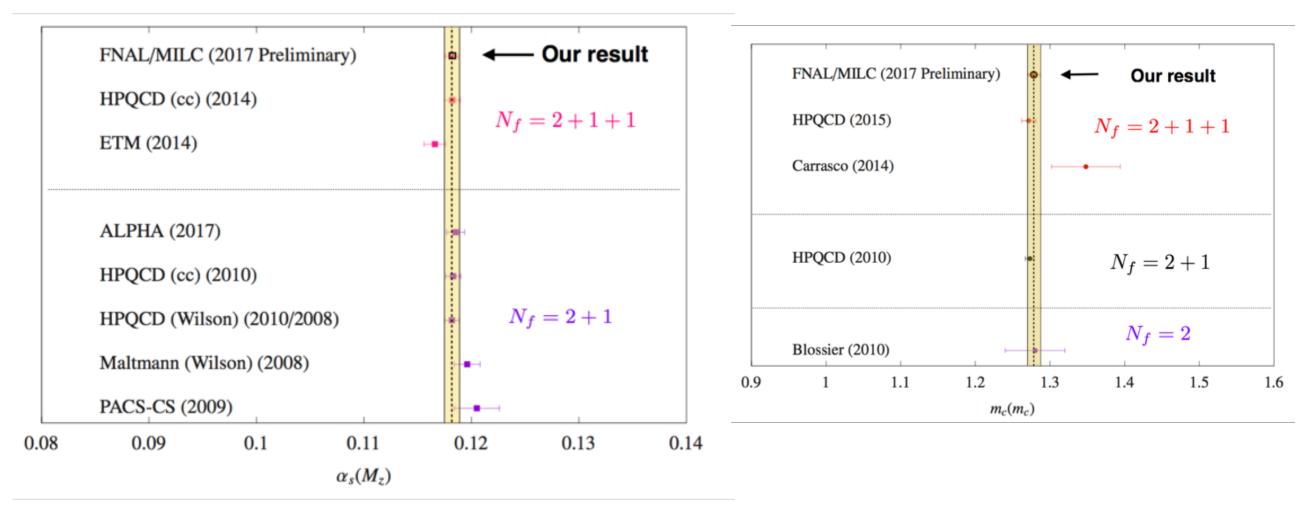


Need to improve parametric uncertainties in order to make use of measuremen Greater precisione in (PAC, MC, MC, MC, MC, S. .)s needed for future high-luminosity colliders. A. El-Khadra

Higgs decays

Aarti Veernala (FNAL/MILC) @ Lattice 2017

 $\alpha_s(M_Z)$



 $\mathcal{M}_{\mathcal{C}}$

New precise results for α_s and m_c , which will improve with inclusion of more ensembles at finer lattice spacings.

A. El-Khadra

$$\langle \bar{B}_{q}^{0} | \mathcal{O}_{i}^{\Delta B=2} | B_{q}^{0} \rangle$$

$$\langle \bar{D}^{0} | \mathcal{O}_{i}^{\Delta C=2} | D^{0} \rangle$$

$$\hat{B}_{K} \cdots$$

$$f_{H,0}^{B \to D^{(*)}}$$

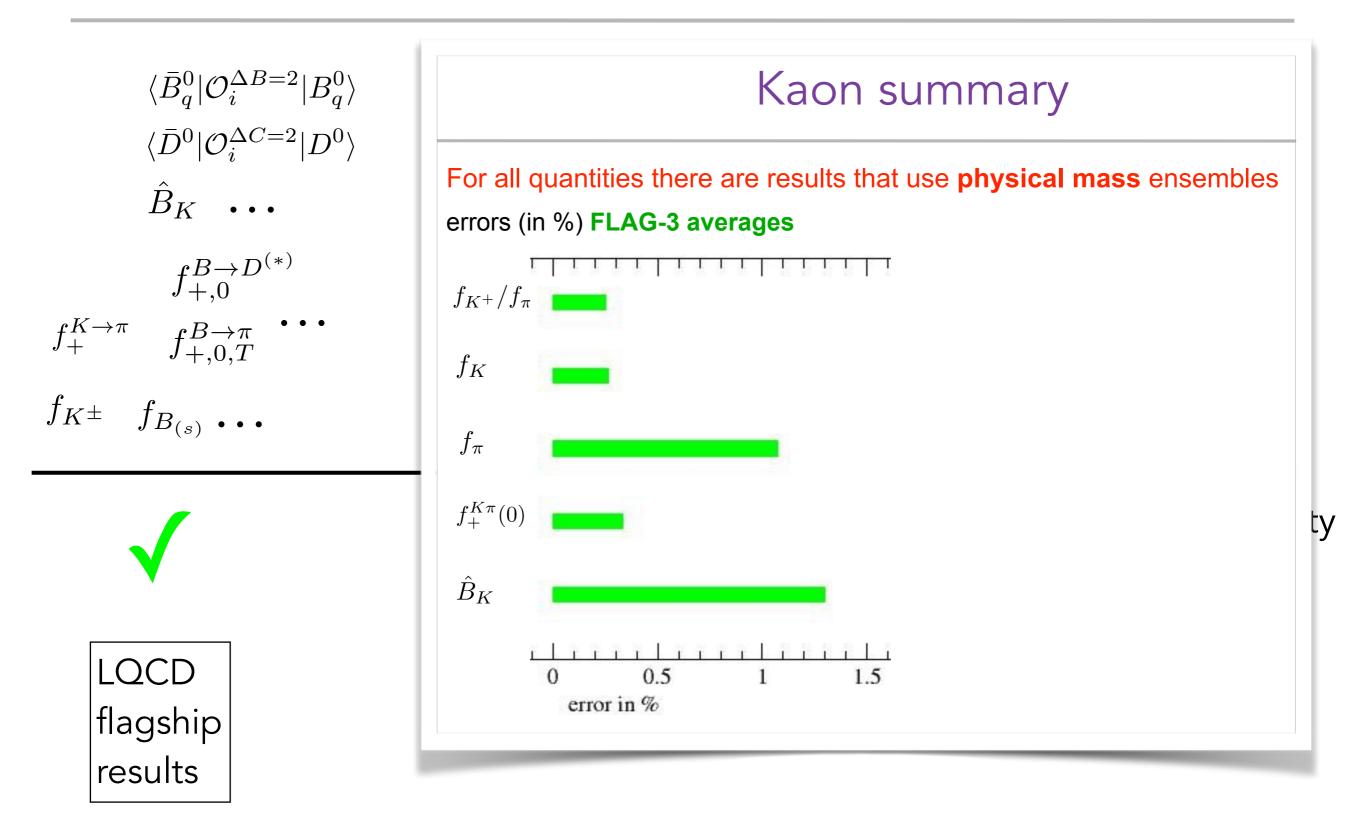
$$f_{H,0}^{K \to \pi} f_{H,0,T}^{B \to \pi} \cdots$$

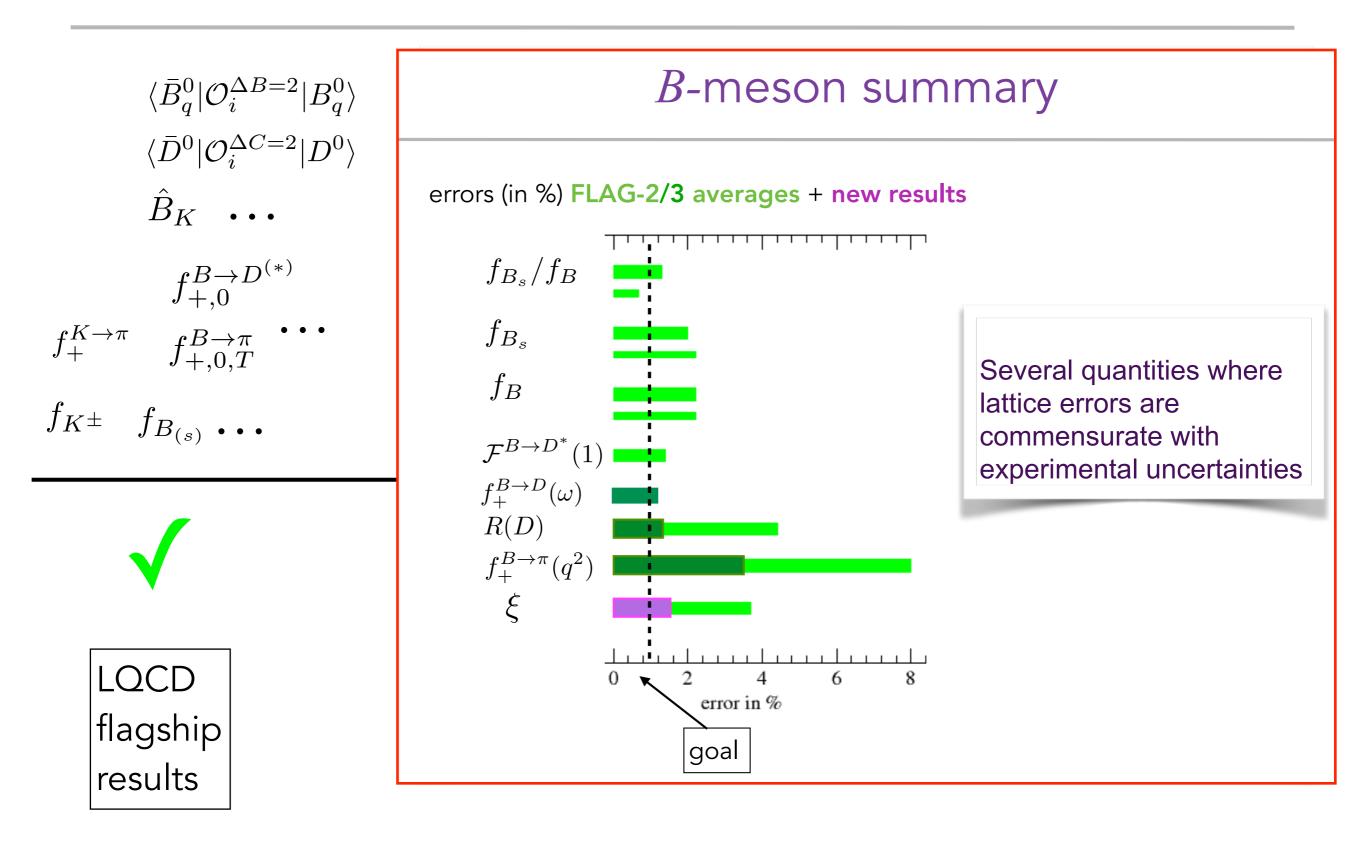
$$f_{K^{\pm}} f_{B_{(s)}} \cdots$$

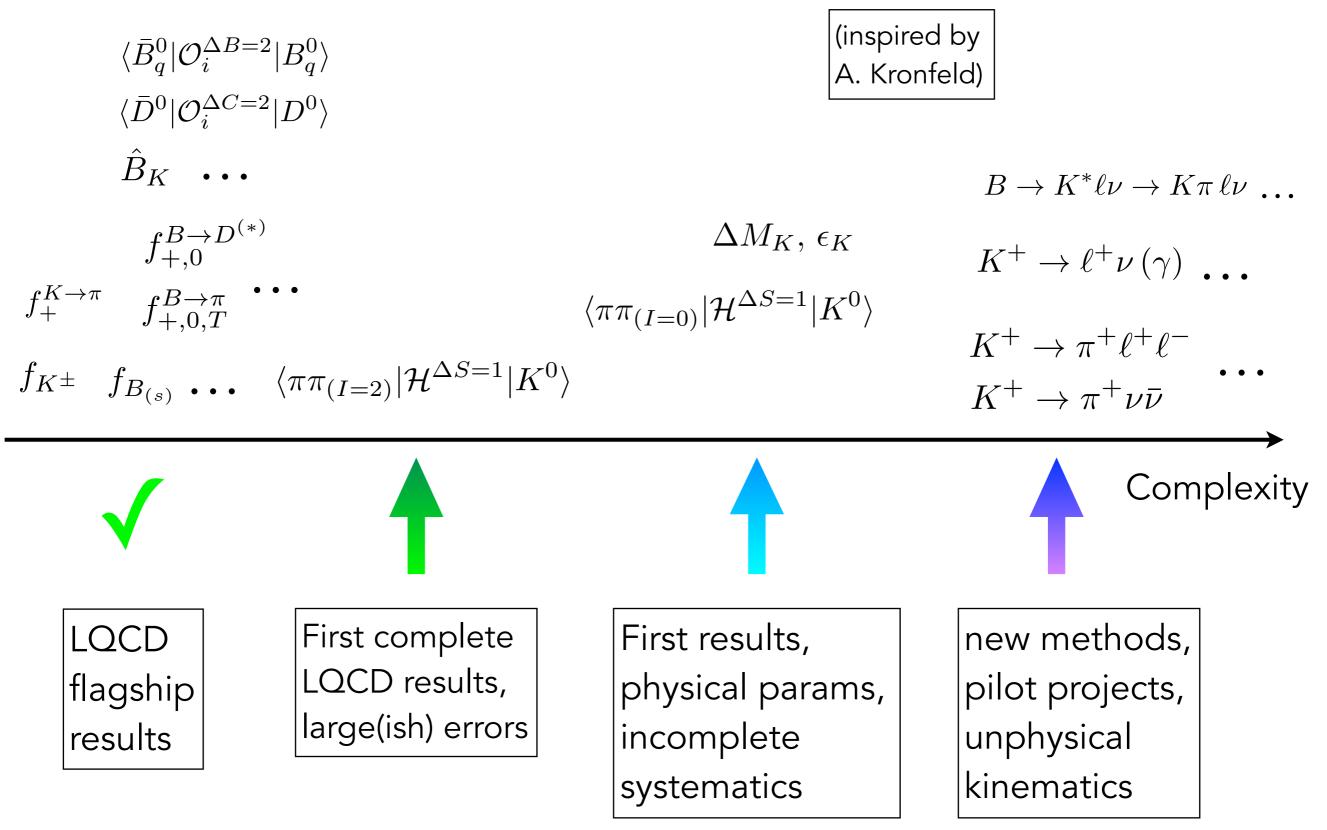
 \checkmark

LQCD flagship results (inspired by A. Kronfeld)

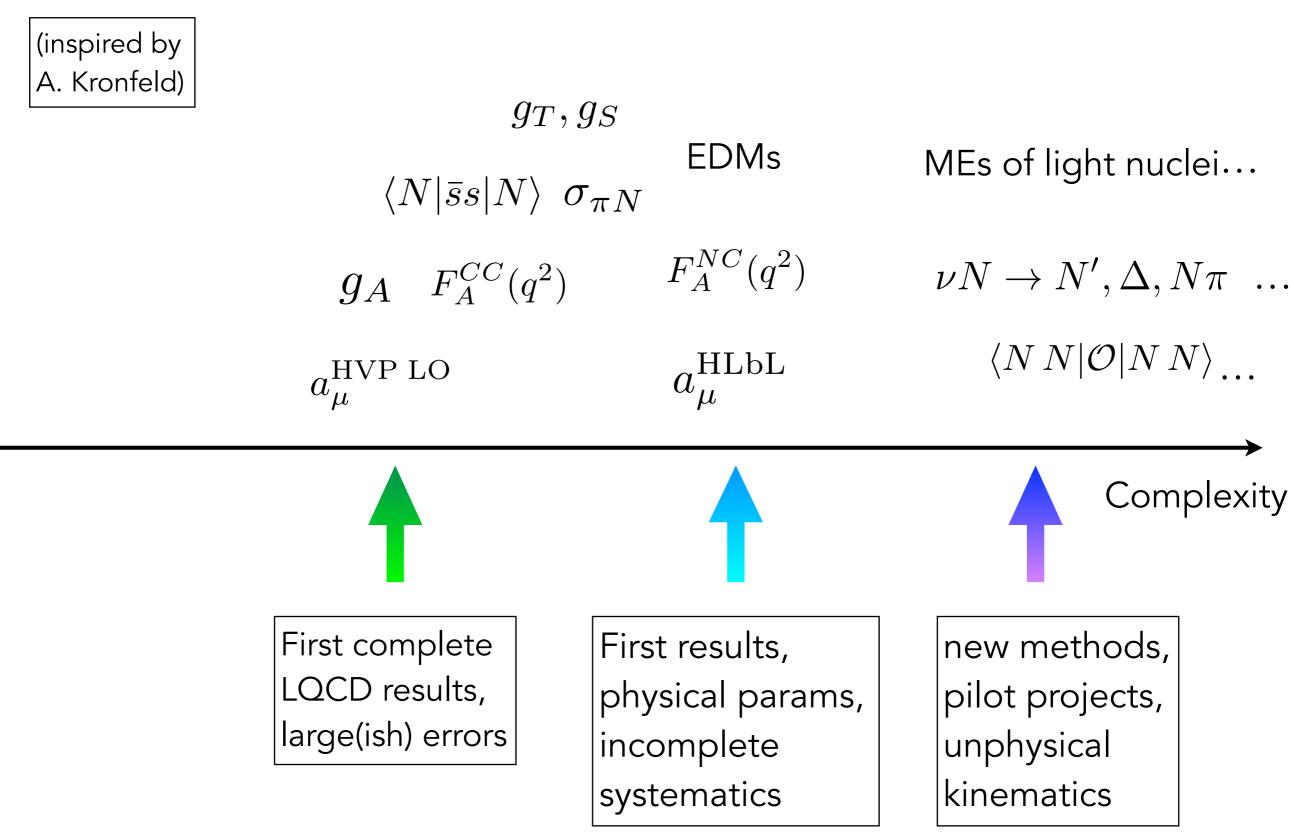
Complexity







Summary: lepton flavor physics



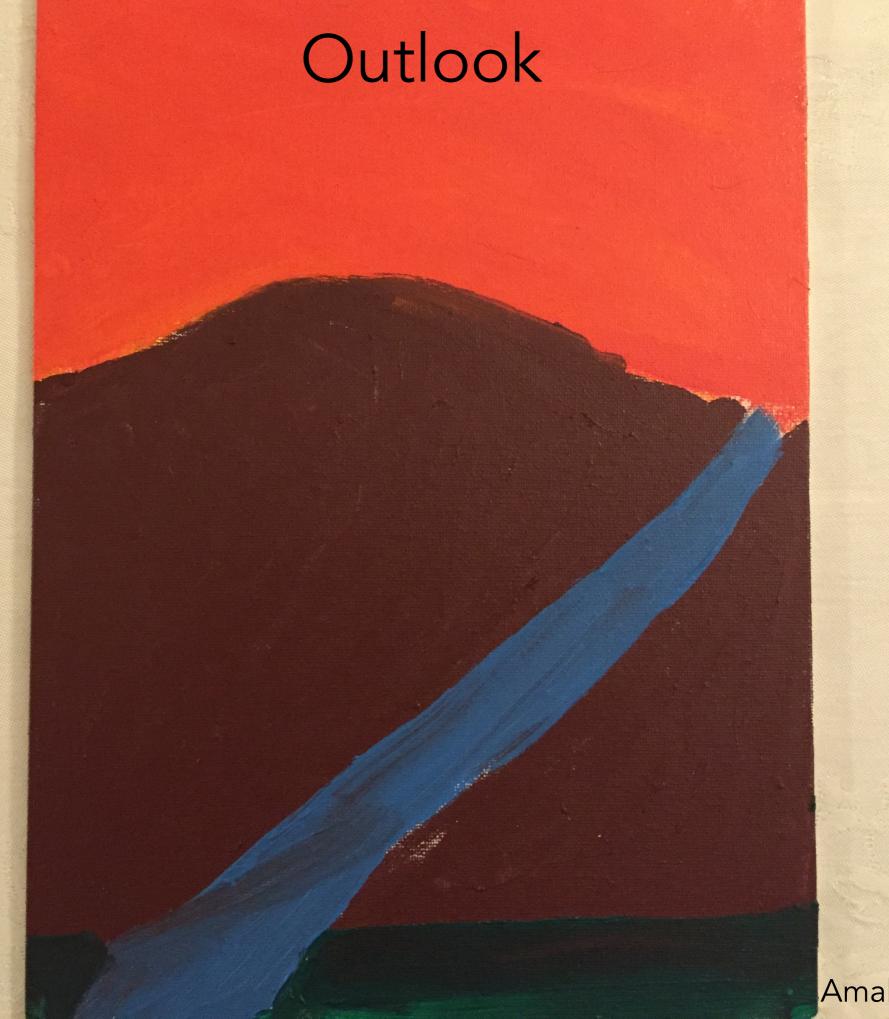
Summary

☆ Lattice QCD calculations are driven by the needs of the (high energy) experiments and provide essential support to them.

Access to adequate computational resources is essential to the continued success of the LQCD efforts!

☆ Recent progress is due to access to Petascale computing resources and many creative solutions to the varying challenges of these complex computations.

☆ For the many topics I didn't cover: See the talks at the Lattice 2017 conference: <u>http://wpd.ugr.es/~lattice2017/</u>



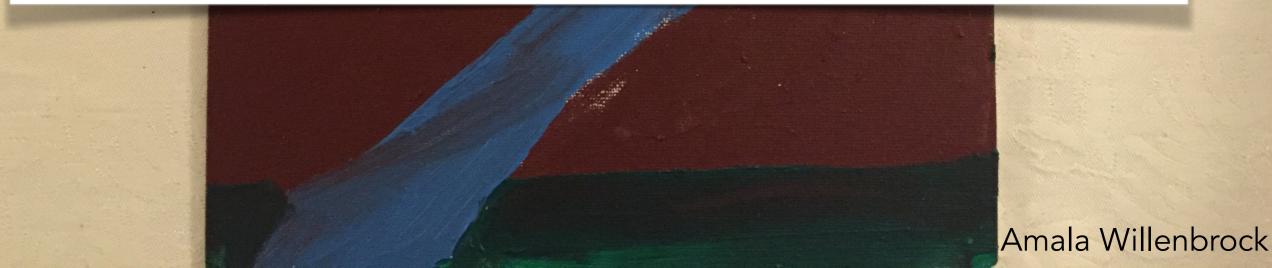
Amala Willenbrock

Outlook

☆ Gauge field ensembles with light sea quarks at their **physical masses** are being used in a growing number of LQCD calculations.

will need to include

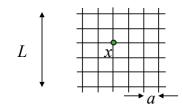
- ◆ structure-dependent QED effects
- program being developed for kaon quantities, muon g-2 extend to B-meson quantities
- ☆ Calculations of an increasing number of quantities (muon g-2, nucleon matrix elements, resonances,...) will become "LQCD flagship" results.
- ☆ While new quantities are added (inclusive decays, multi-hadron states, ...)



Thank you!

Farah Willenbrock

Appendix



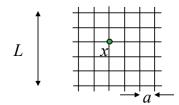
 $\langle \mathcal{O} \rangle \sim \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \,\mathcal{O}(\psi,\bar{\psi},A) \,e^{-S} \qquad \qquad S = \int d^4x \left[\bar{\psi}(\not\!\!\!D+m)\psi + \frac{1}{4} (F^a_{\mu\nu})^2 \right]$

use monte carlo methods (importance sampling) to evaluate the integral.

Note: Integrating over the fermion fields leaves det(D + m) in the integrand. The correlation functions, O, are then written in terms of $(D + m)^{-1}$ and gluon fields.

steps of a lattice QCD calculation:

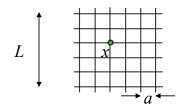
- 1. generate gluon field configurations according to $det(D+m) e^{-S}$
- 2. calculate quark propagators, $(D + m_q)^{-1}$, for each valence quark flavor and source point
- **3.** tie together quark propagators into hadronic correlation functions (usually 2 or 3-pt functions)
- 4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, from correlation functions
- 5. systematic error analysis



systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD → ab initio

- finite *a*: Symanzik EFT
- light quark masses: Chiral Perturbation Theory
- heavy quarks: HQET
- finite *L*: finite volume EFT
- need large enough L and small enough a and simulations with several a, L, ...

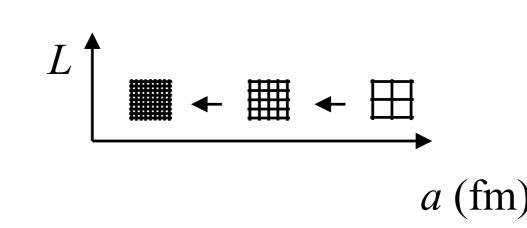


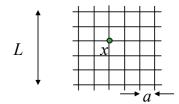
discretization effects — continuum extrapolation

- typical momentum scale of quarks gluons inside hadrons: $\sim \Lambda_{\rm QCD}$
- make *a* small to separate the scales: $\Lambda_{\rm QCD} \ll 1/a$
- Symanzik EFT: $\langle \mathcal{O} \rangle^{\mathrm{lat}} = \langle \mathcal{O} \rangle^{\mathrm{cont}} + O(a\Lambda)^n$, $n \geq 2$

♀ can be used to build improved lattice actions

- Gen be used to anticipate the size of discretization effects
- to control and reliably estimate the error, repeat ...





light quark mass effects

Simulations with $m_{\text{light}} = 1/2 (m_u + m_d)$ at the physical u/d quark masses are now available, but they are computationally expensive and many still have $m_{\text{light}} > 1/2 (m_u + m_d)_{\text{phys}}$

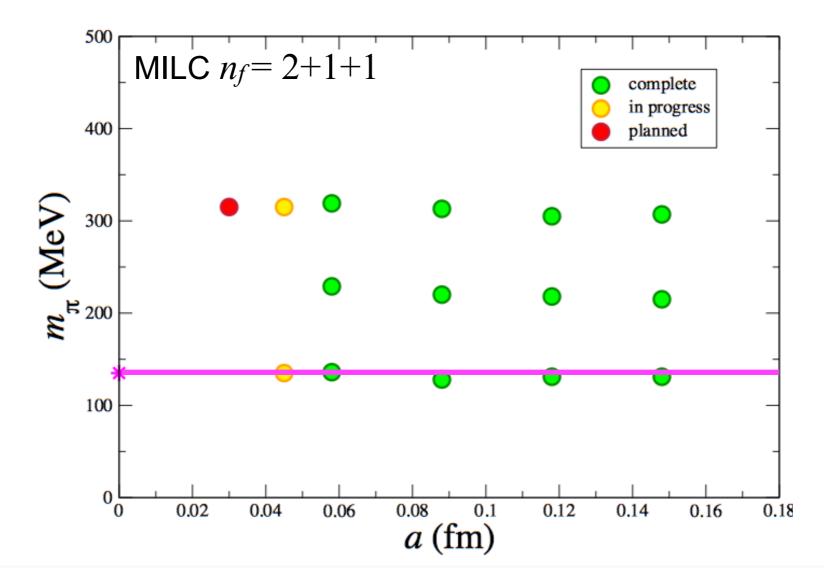
Chiral Perturbation Theory (χ PT) can be used to extrapolate/interpolate to the physical point.

Can include discretization effects

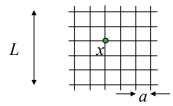
It is now common practice to perform a combined continuum-chiral extrapolation/interpolation

chiral-continuum extrapolation

Example: Set of ensembles by MILC collaboration

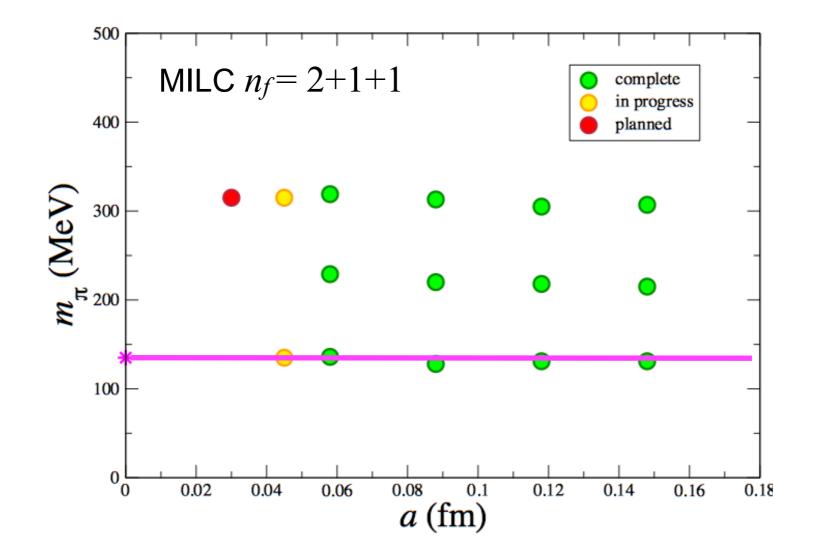


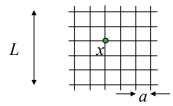
Five collaborations have now generated sets of ensembles that include sea quarks with physical light-quark masses: PACS-CS, BMW, MILC, RBC/UKQCD, ETM



Lattice **QCD** Introduction

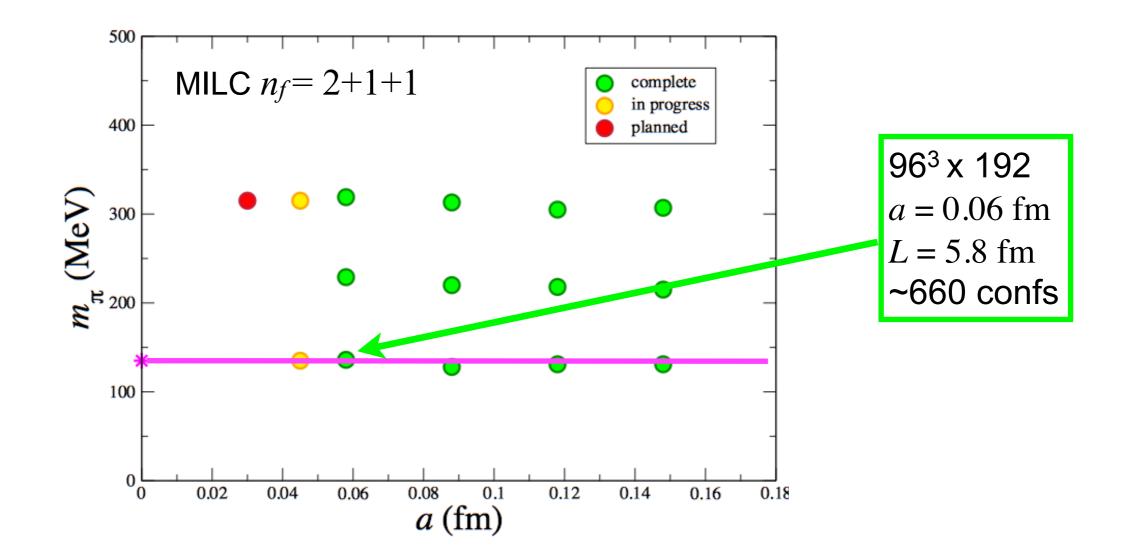
combined chiral-continuum extrapolation

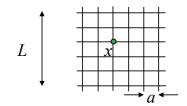




Lattice **QCD** Introduction

combined chiral-continuum extrapolation





Lattice **QCD** Introduction

finite volume effects

One stable hadron (meson) in initial/final state:

If L is large enough, FV error $\sim e^{-m_{\pi} L}$

 \bigcirc keep $m_{\pi} L \gtrsim 4$

To quantify residual error:

 \bigcirc include FV effects in χ PT

Sompare results at several Ls (with other parameters fixed)

The story changes completely with two or more hadrons in initial/final state or if there are two or more intermediate state hadrons.

"simple quantities":

no more than one stable hadron in initial/final state

If QED is included, FV effects also become more complicated...

Heavy Quark Treatment

- For light quarks ($m_\ell < \Lambda_{
 m QCD}$), leading discretization errors ~ $lpha s_s^k (a \Lambda_{
 m QCD})^n$
- For heavy quarks, leading discretization errors ~ $\alpha_s^k (am_h)^n$ with currently available lattice spacings

```
for b quarks am_b > 1
```

for charm $am_c \sim 0.15$ -0.6

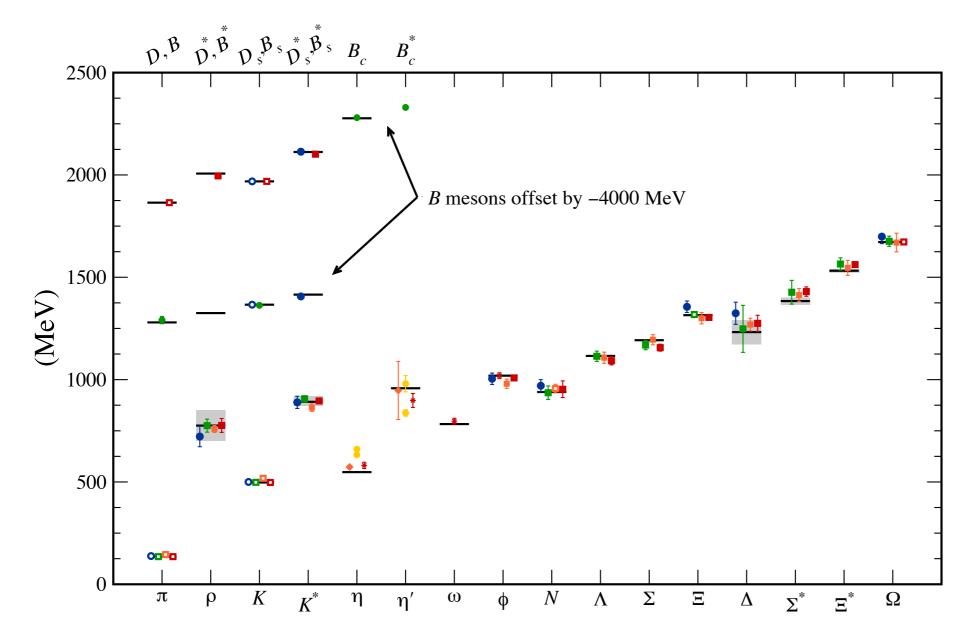
- need effective field theory methods for b quarks for charm can use light quark methods, if action is sufficiently improved
- avoid errors of $(am_b)^n$ in the action by using EFT:
 - relativistic HQ actions (Fermilab, Columbia [aka RHQ], Tsukuba)
 - + HQET
 - + NRQCD

or

- use improved light quark actions for charm (HISQ, tmWilson, NP imp. Wilson,...) and for b:
 - + use same LQ action as for charm but keep $am_h < 1$,
 - use HQET and/or static limit to extrapolate/interpolate to b quark mass

LQCD success: spectrum

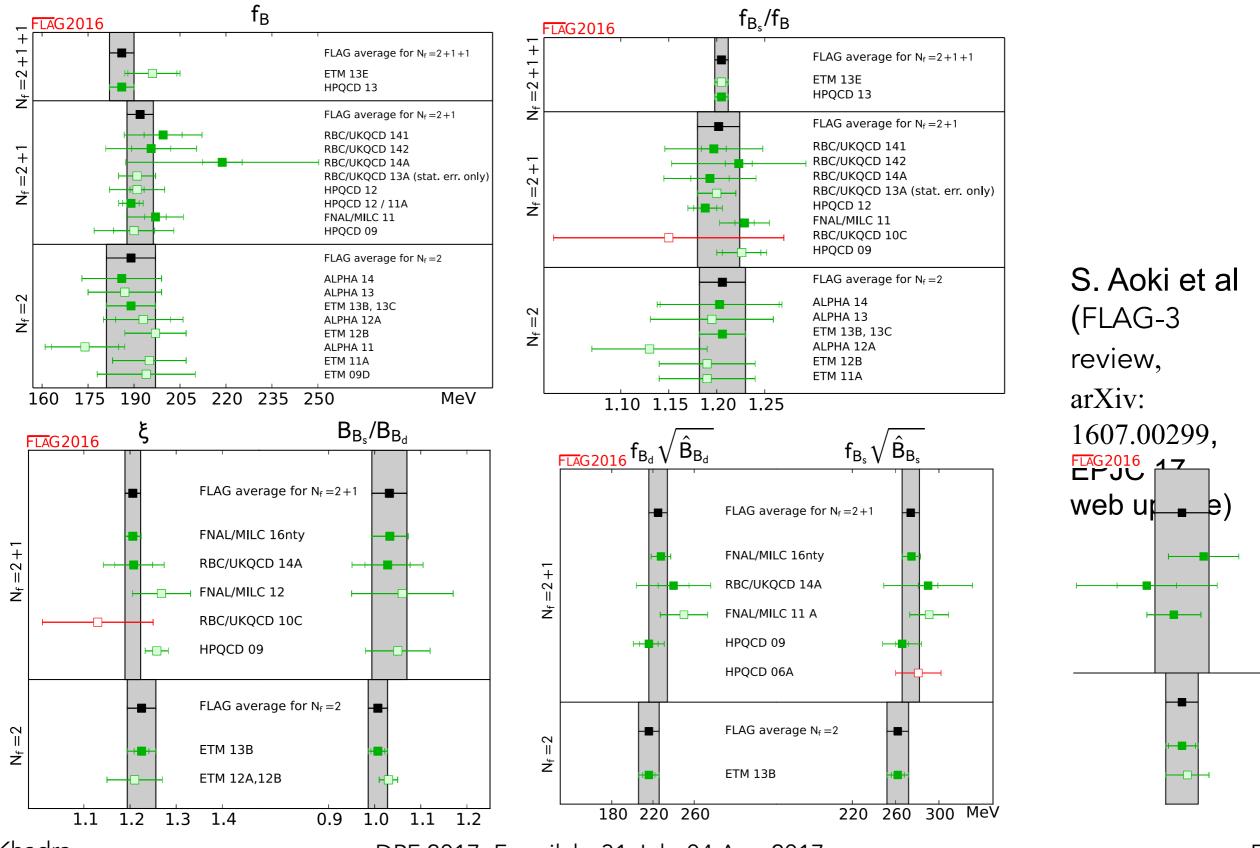
A. Kronfeld (Annu. Rev. Part. & Nucl. Sci, arXiv:1203.1204, updated)



 π ...Ω: BMW, MILC, PACS-CS, QCDSF; η-η': RBC, UKQCD, Hadron Spectrum (ω); *D*, *B*: Fermilab, HPQCD, Mohler-Woloshyn

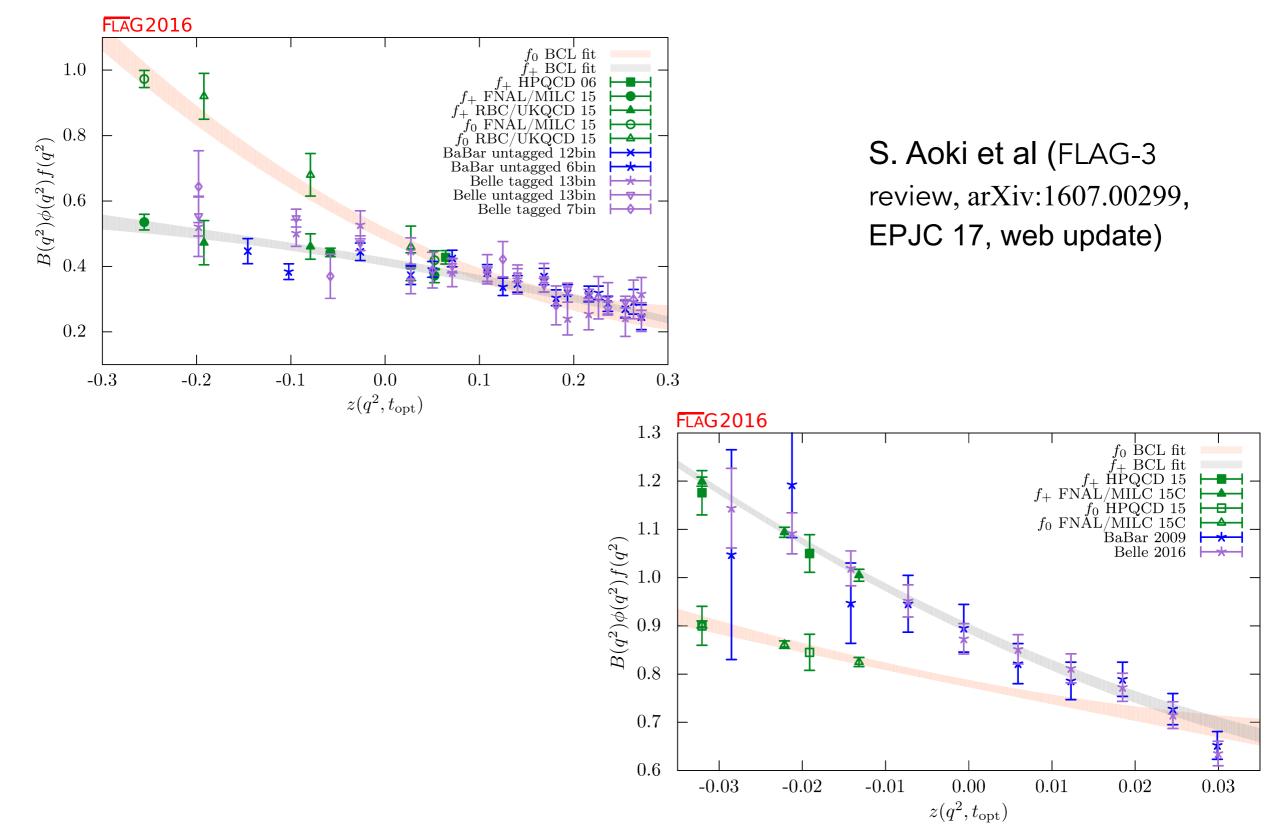
A. El-Khadra

FLAG review of B-meson quantities



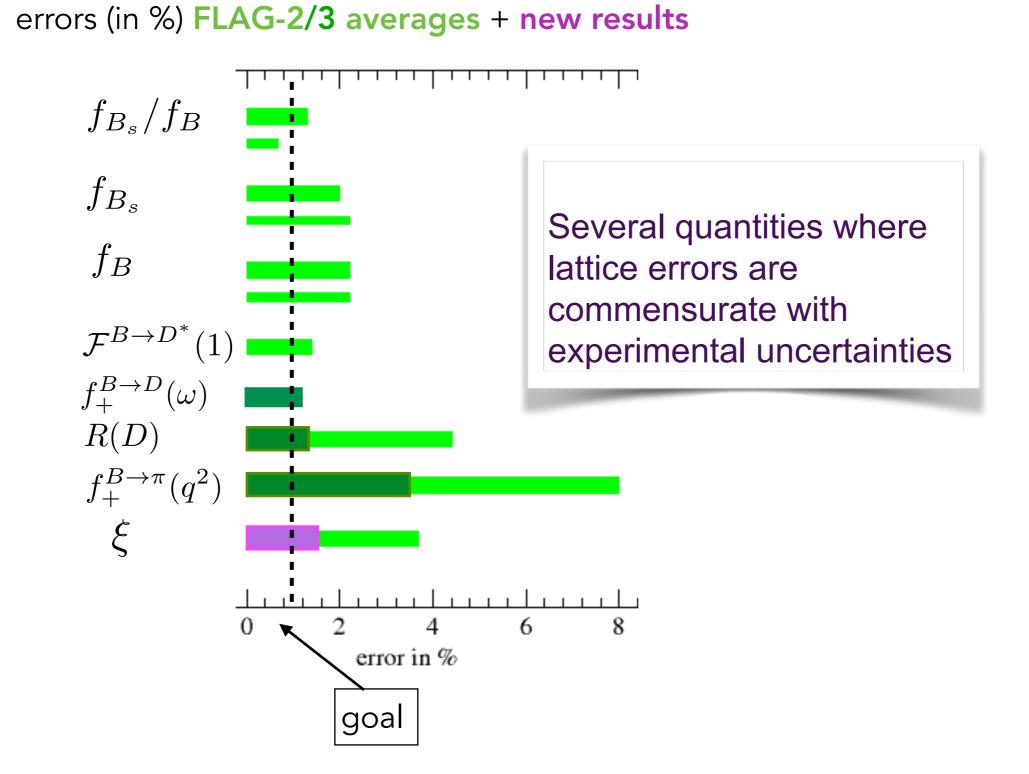
A. El-Khadra

FLAG review of B-meson quantities

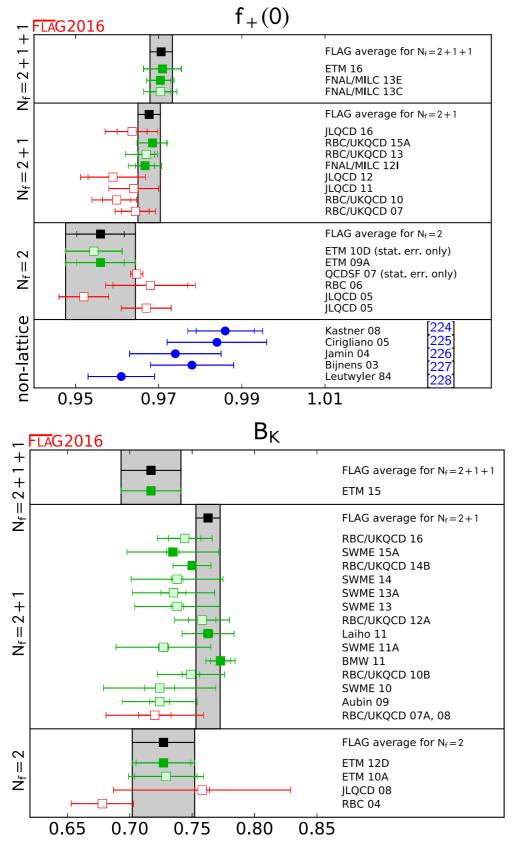


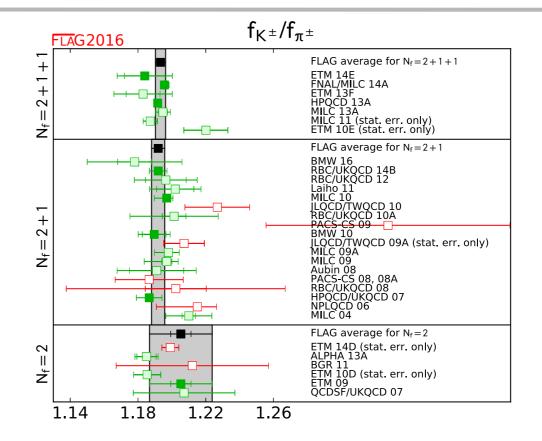
A. El-Khadra

B-meson summary



FLAG review of Kaon quantities





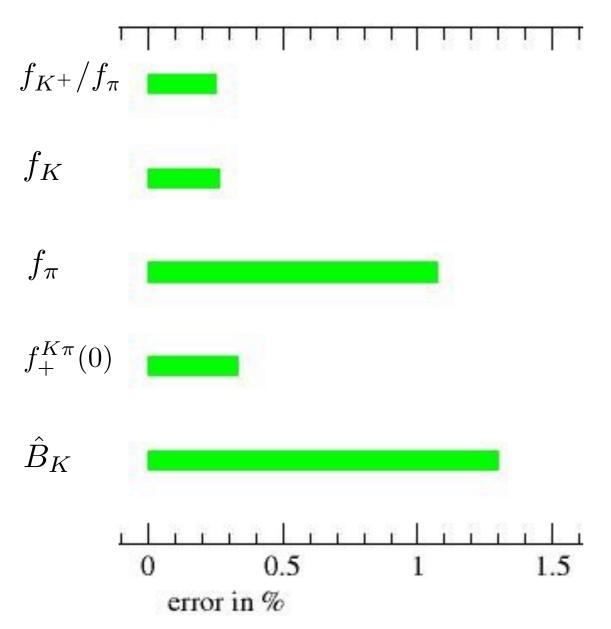
S. Aoki et al (FLAG-3 review, arXiv:1607.00299, EPJC 17, web update)

A. El-Khadra

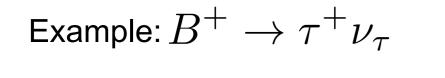
Kaon summary

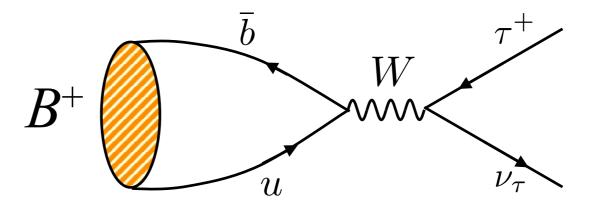
For all quantities there are results that use **physical mass** ensembles

errors (in %) FLAG-3 averages



Leptonic *B*-meson decay





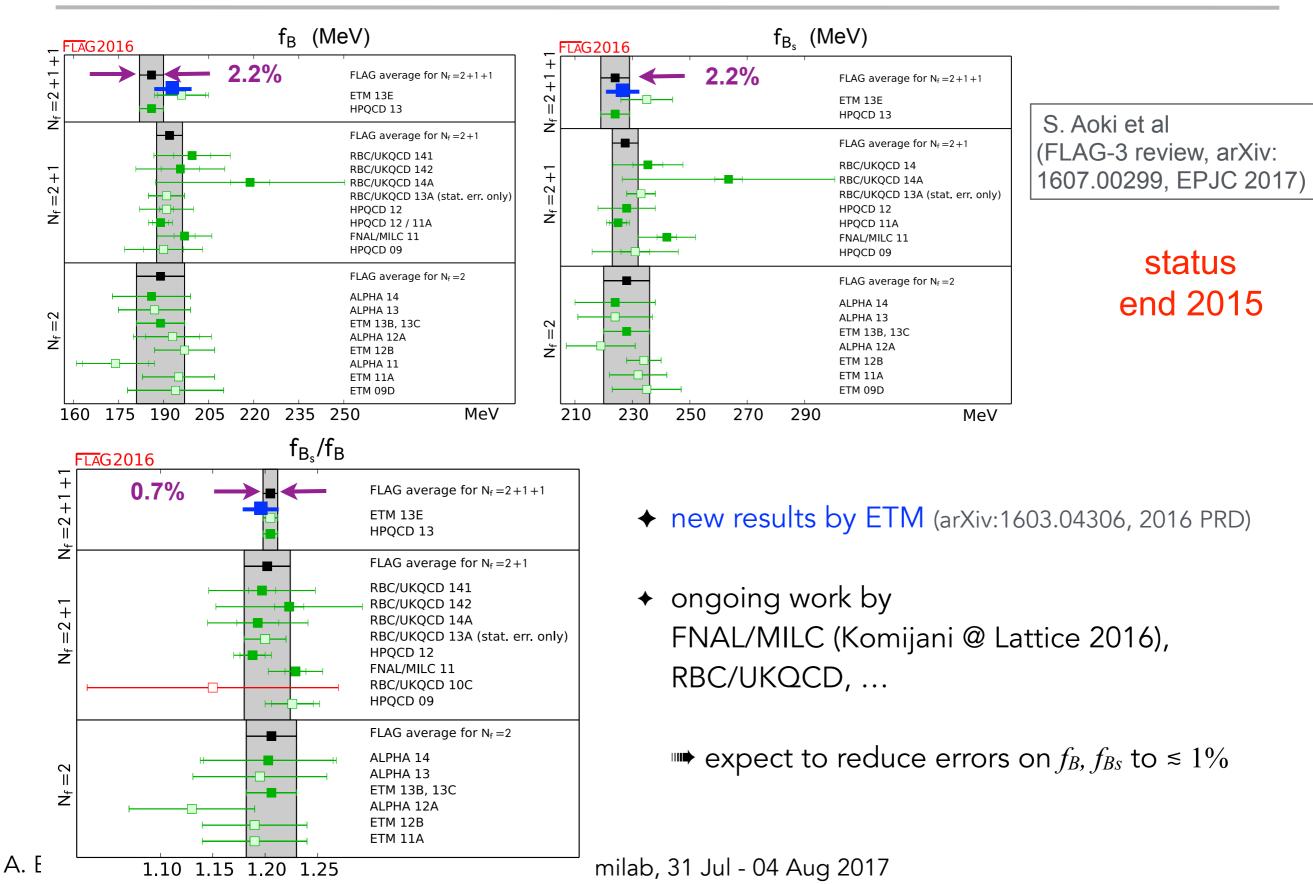
$$\Gamma(B^+ \to \tau^+ \nu_\tau) = (\text{known}) \times |V_{ub}|^2 f_B^2$$

Search for new physics.
Search for new physics.

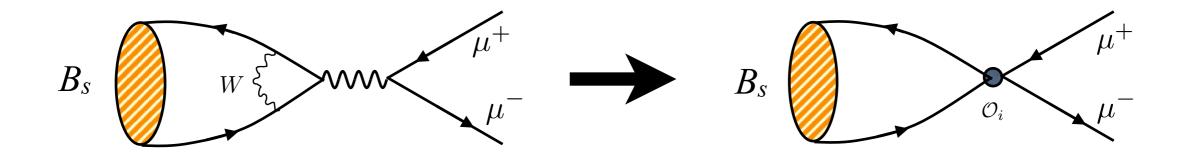
 \bigcirc SU(3) ratio f_{B_s}/f_{B_d} : statistical and systematic errors tend to cancel.

Q Decay constants are also needed for rare leptonic decay, $B_{s(d)}$ →µµ.

B decay constant summary







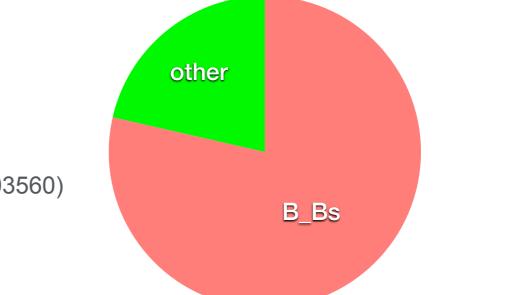
Standard Model prediction: Buras, et al (arXiv:1303.3820, JHEP 2013), Bobeth, et al (arXiv:1311.0903, PRL 2014).

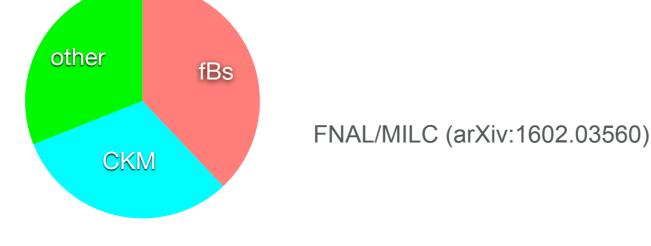
LQCD decay constant

$$\bar{\mathcal{B}}(B_s \to \mu^+ \mu^-) = 3.53(11)(9)(9) \times 10^{-9}$$

B_s mixing measurement + LQCD bag parameter

$$\bar{\mathcal{B}}(B_s \to \mu^+ \mu^-) = 3.22(22)(6) \times 10^{-9}$$

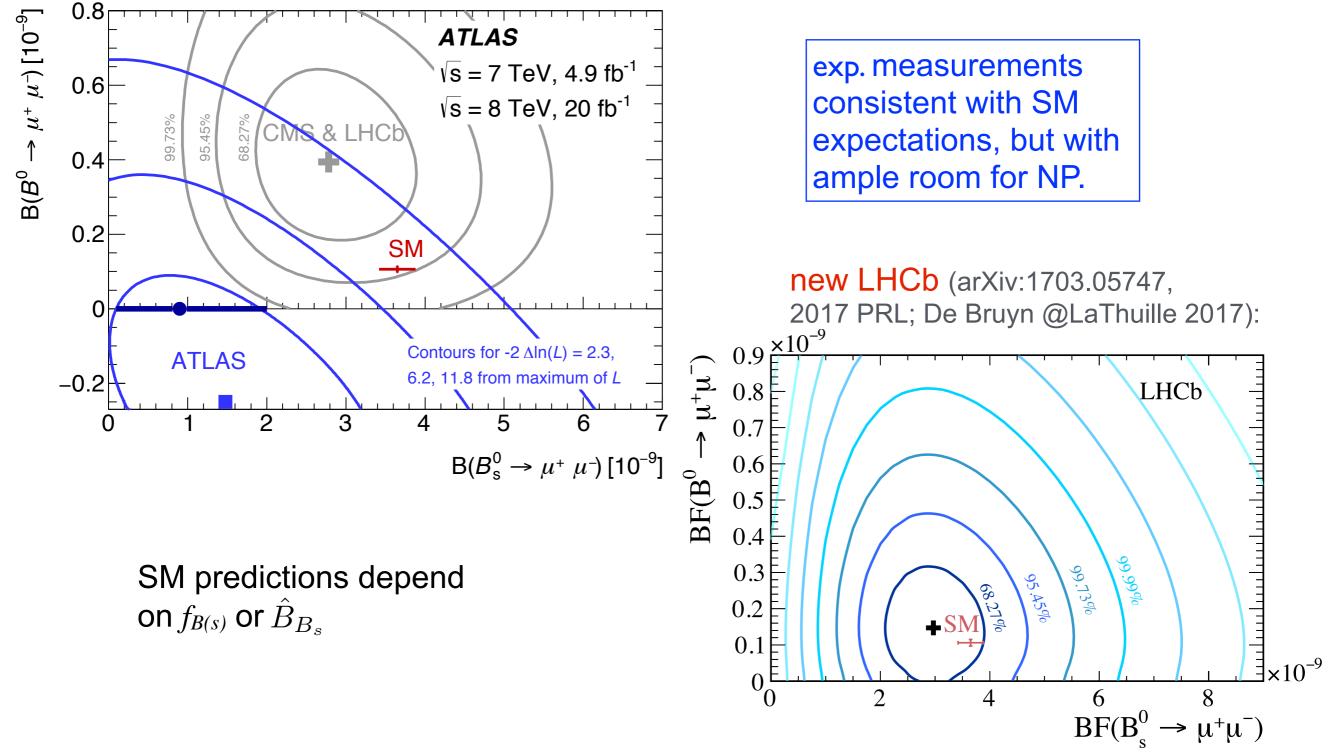




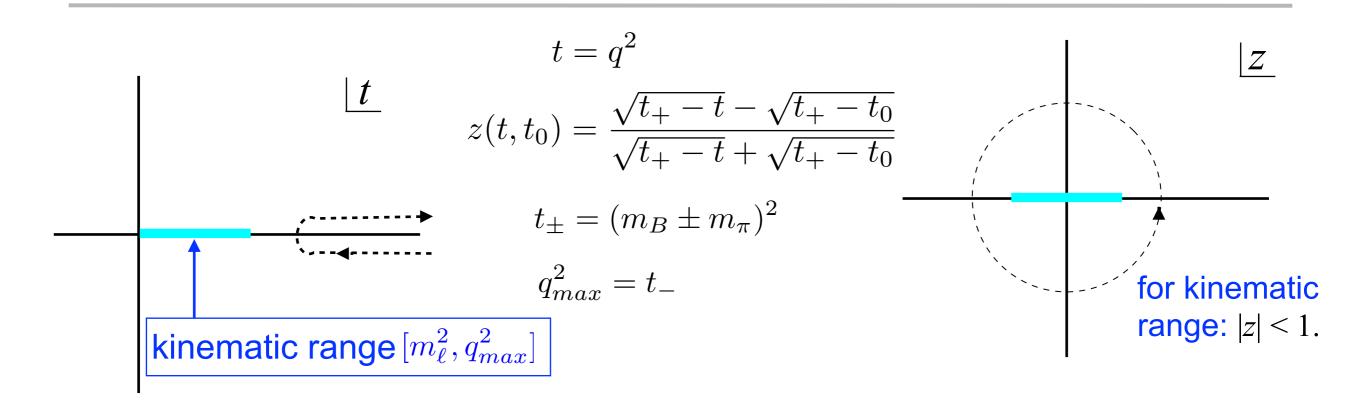
A. El-Khadra



CMS+LHCb combined (arXiv:1411.4413, Nature 2015) and ATLAS (arXiv:1604.04263)



The *z*-expansion



The form factor can be expanded as:

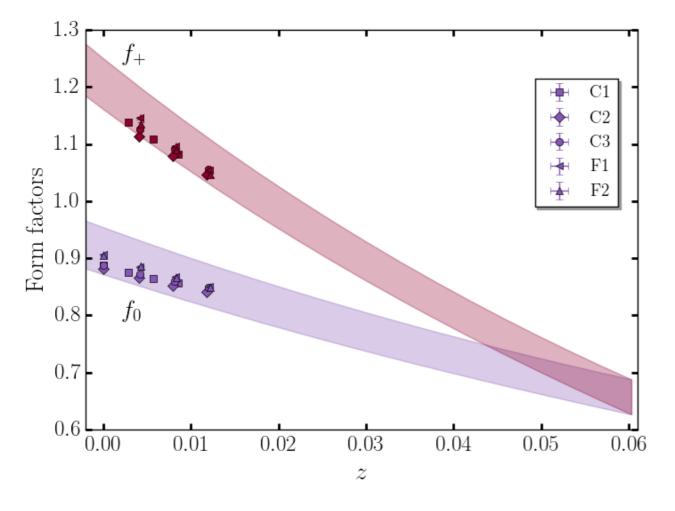
$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0) z(t,t_0)^k$$

Bourrely at al (Nucl.Phys. B189 (1981) 157) Boyd, Grinstein, Lebed (hep-ph/9412324, PRL 95; hep-ph/9504235, PLB 95; hep-ph/ 9508211, NPB 96; hep-ph/9705252, PRD 97) Lellouch (arXiv:hep- ph/9509358, NPB 96) Boyd & Savage (hep-ph/9702300, PRD 97) Bourrely at al (arXiv:0807.2722, PRD 09)

- P(t) removes poles in $[t_{-},t_{+}]$
- The choice of outer function ϕ affects the unitarity bound on the a_k .
- In practice, only first few terms in expansion are needed.

$B_s \to D_s$

new LQCD form factors from HPQCD (Monahan et al, arXiv:1703.09728)



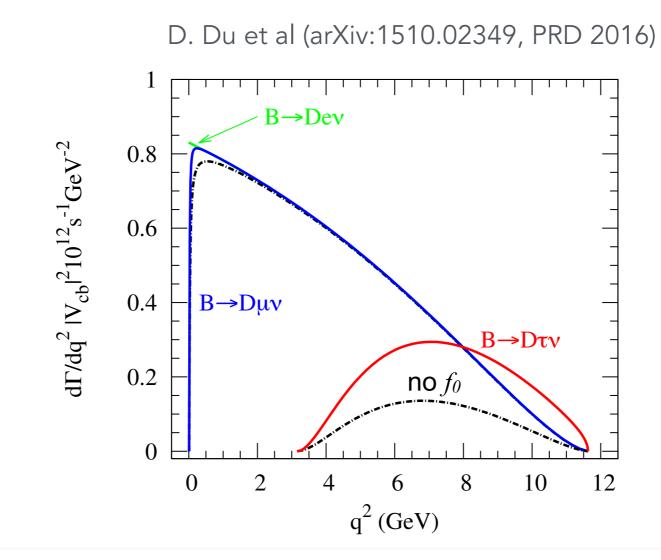
- ★ 5 MILC asqtad (2+1) ensembles two lattice spacings
- ★ NRQCD b and HISQ charm quarks
- ★ O(a) improved current matched through $O(\alpha_s, \Lambda/m_b, \alpha_s a/m_b)$

 $\star R(D_s) = 0.301$ (6)

★ combine results with previous HPQCD calculation of $B \rightarrow D$ form factors to obtain ratios used for f_s/f_d .

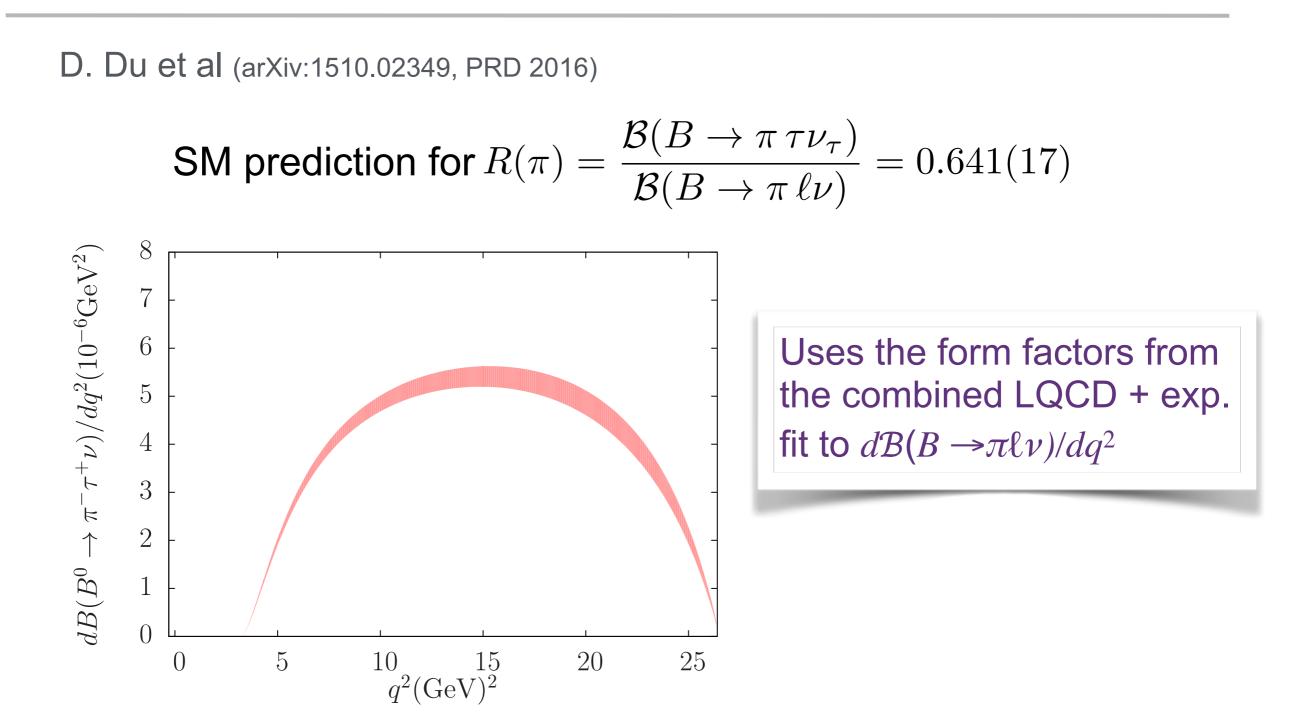
Consistent with previous results by FNAL/MILC (Bailey et al, arXiv:1202.6346, 2012 PRD) and ETMC (M. Atoui et al, arXiv:1310.5238, 2014 EPJC).

BSM phenomenology: LFU τ/ℓ



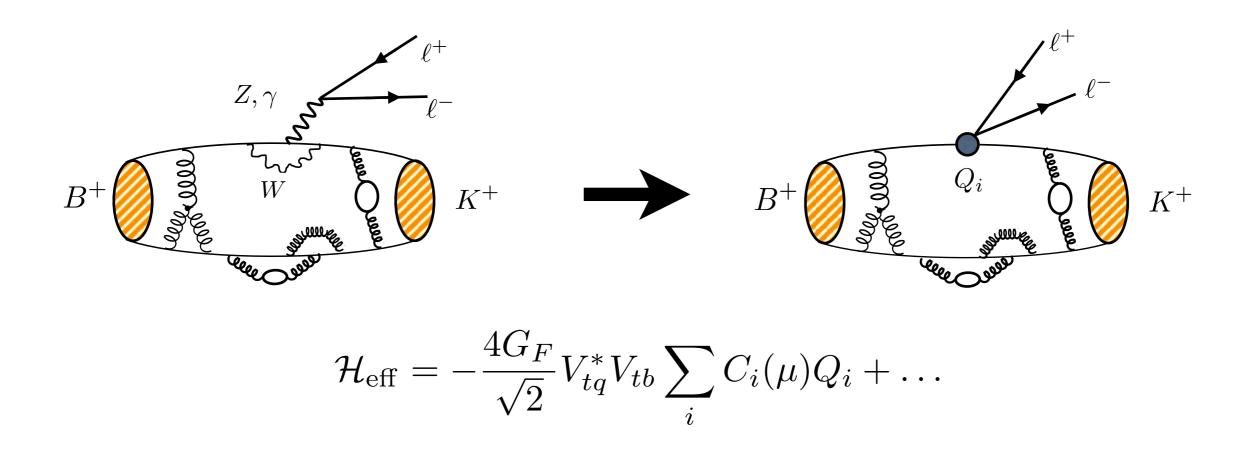
• A shape comparison using (ratios of) differential or binned decay rates to compare theory and experiment would provide useful insights.

BSM phenomenology: LFU τ/ℓ



Hopefully to be measured by Belle II.

Rare semileptonic B decay

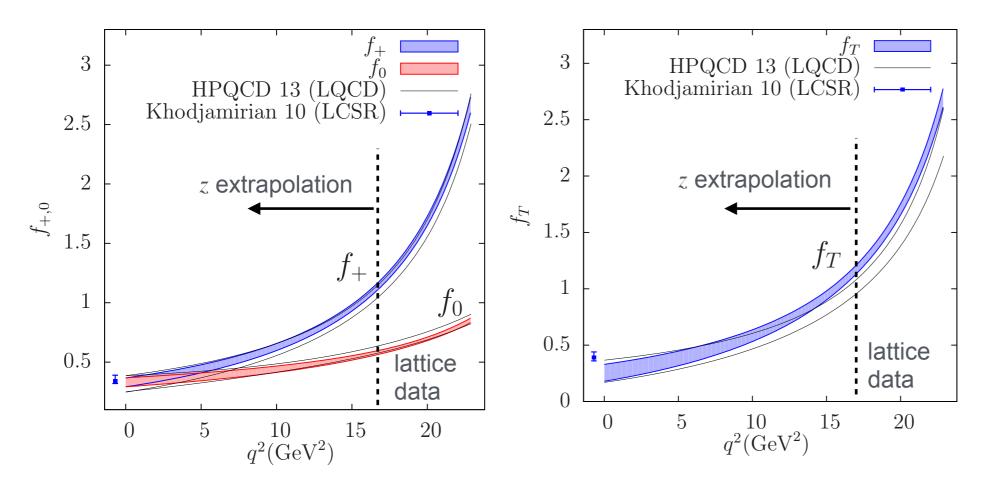


Parameterize the amplitude in terms of the three form factors $f_{+,0,T}(q^2)$:

 $A(B \to P \,\ell \ell) \sim C_7^{\text{eff}} f_T + (C_9^{\text{eff}} + C_{10}) f_+ + \text{nonfactorizable terms}$ see Hurth talk

A. El-Khadra

form factors for $B \to K \, \ell \ell$



HPQCD (arXiv:1306.0434, 1306.2384, PRL 2013)

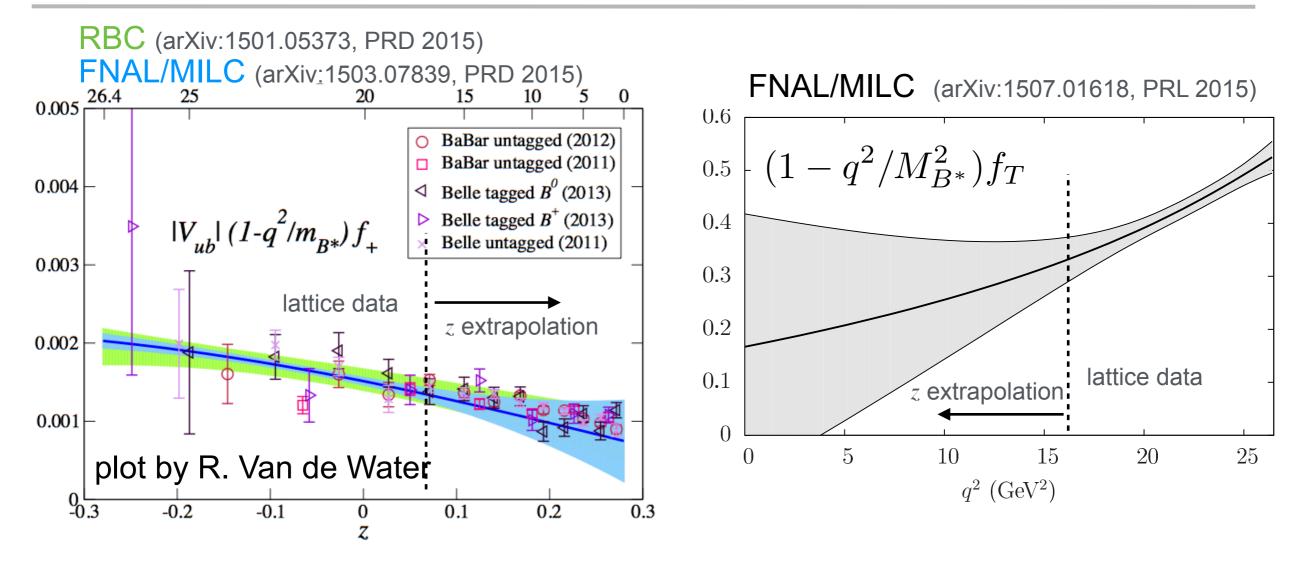
FNAL/MILC (arXiv:1509.06235, PRD 2016)

 \Rightarrow Two LQCD calculations (on overlapping ensemble sets, different valence actions): HPQCD (NRQCD *b* + HISQ), FNAL/MILC (Fermilab *b* + asqtad)

- ☆ consistent results for all three form factors
- * also consistent with LCSR (Khodjamarian et al, arXiv:1006.4945, JHEP 2010)
- ★ Note: First LQCD calculation of $\Lambda_b \to \Lambda \ell^+ \ell^-$ form factors (10 total) (see Meinel talk)



form factors for $B \to \pi \, \ell \ell$

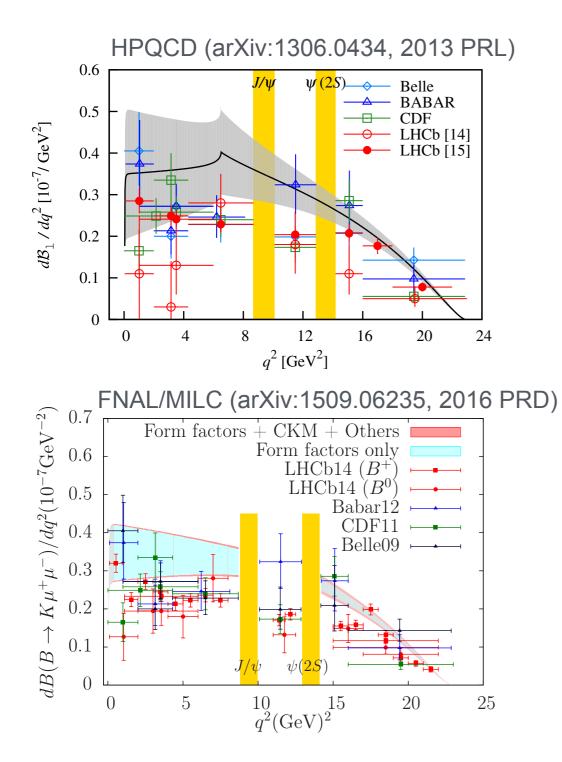


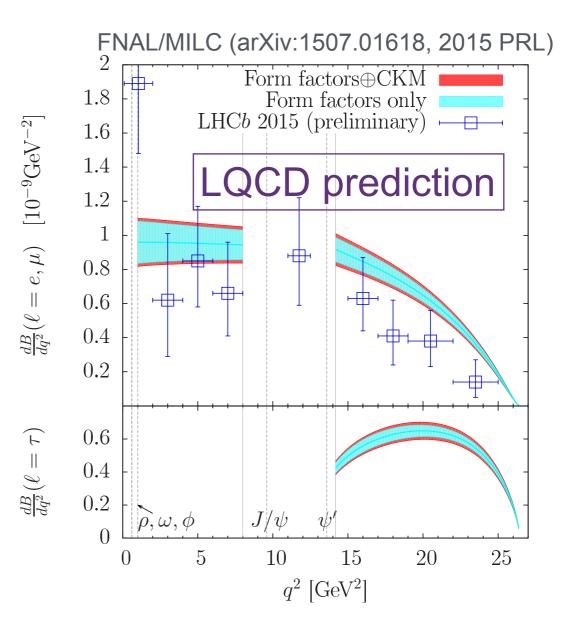
First LQCD calculation of f_T by FNAL/MILC

★ Take f_{+,f_0} from combined fit of lattice form factors + experimental data for $d\mathcal{B}(B \rightarrow \pi \ell \nu)/dq^2$



Experiment vs. Theory



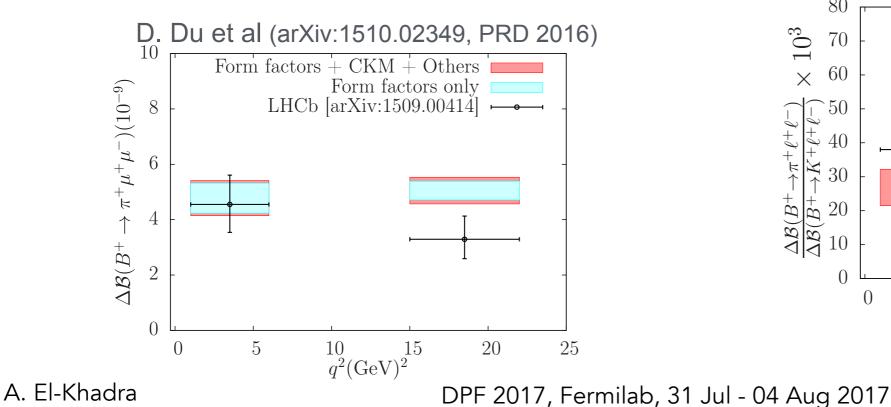




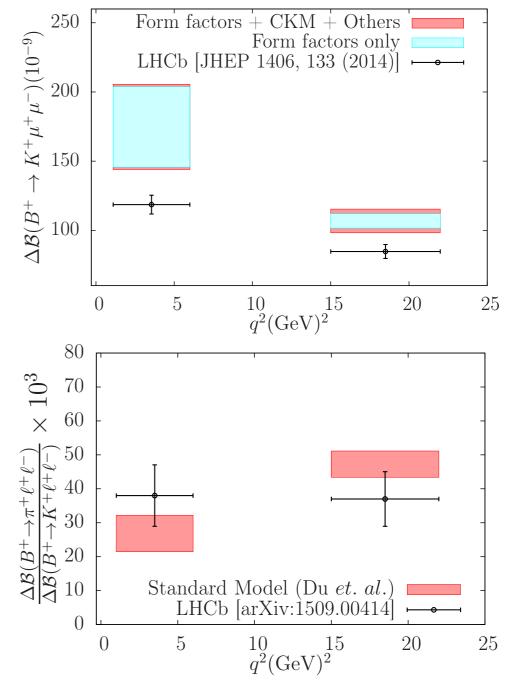
Experiment vs. theory

- LHCb data + FNAL/MILC form factors (arXiv:1509.00414, JHEP 2015;1403.8044, JHEP 2014)
- focus on large bins above and below charmonium resonances
- theory errors commensurate with experiment
- yields $\sim 1-2\sigma$ tensions
- \Rightarrow determine $|V_{td}/V_{ts}, |V_{td}|, |V_{ts}|$

or constrain Wilson coefficients

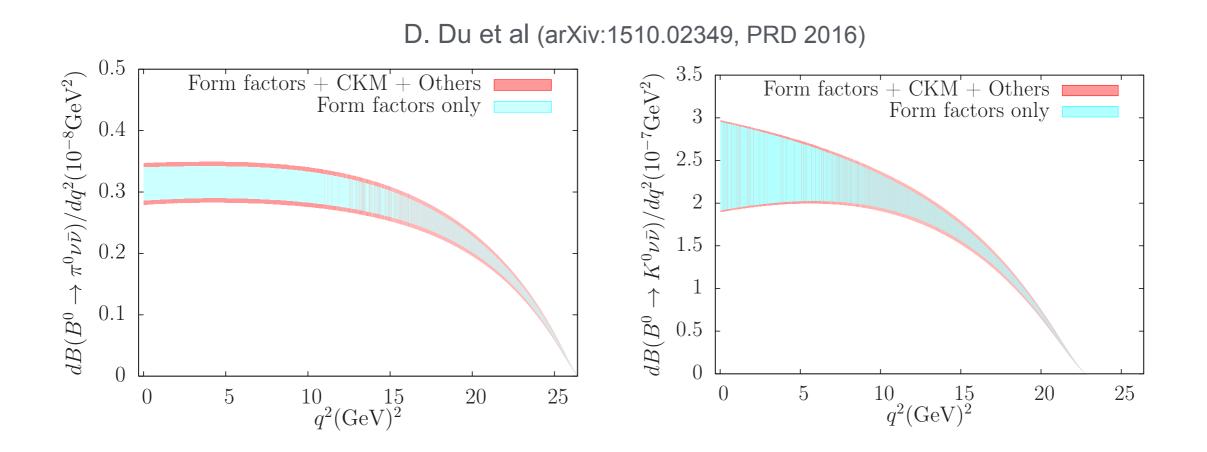


D. Du et al (arXiv:1510.02349, PRD 2016)





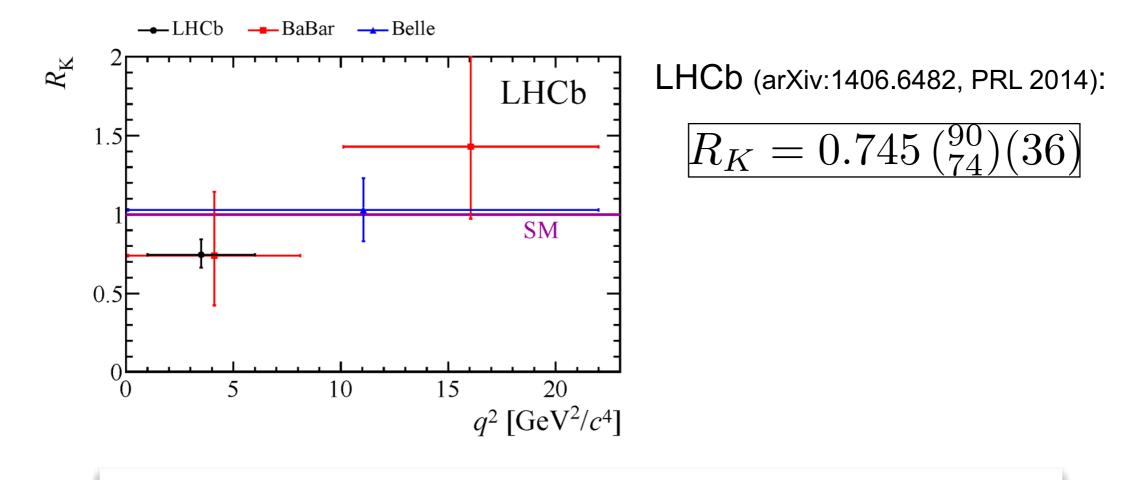
theoretically clean





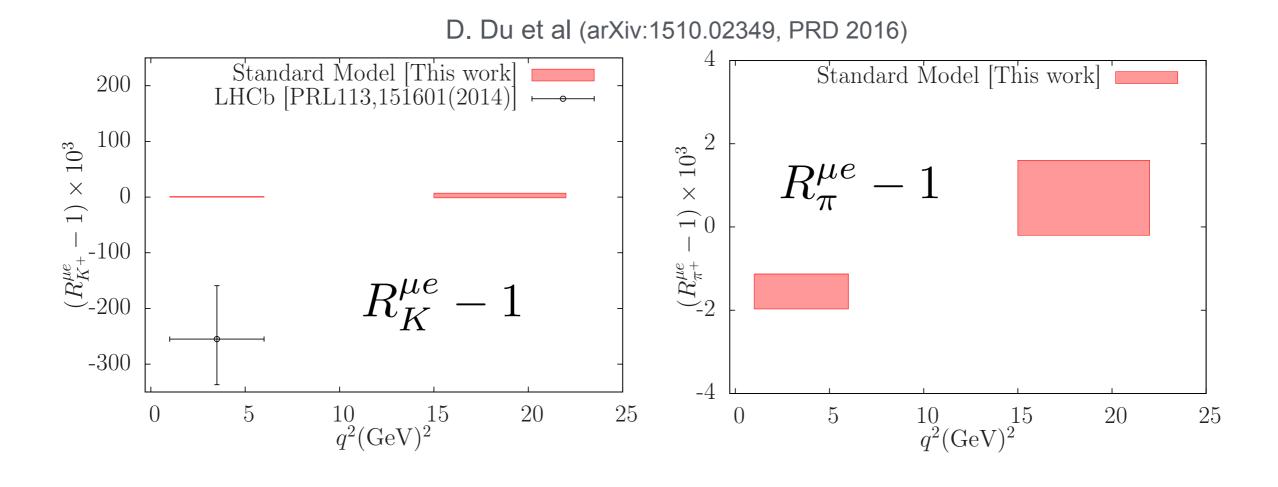
BSM phenomenology: LFU μ/e

Lepton universality test: $B \to K \mu^+ \mu^- / B \to K e^+ e^-$



~2.6 $\sigma\,$ tension between LHCb measurement and SM theory

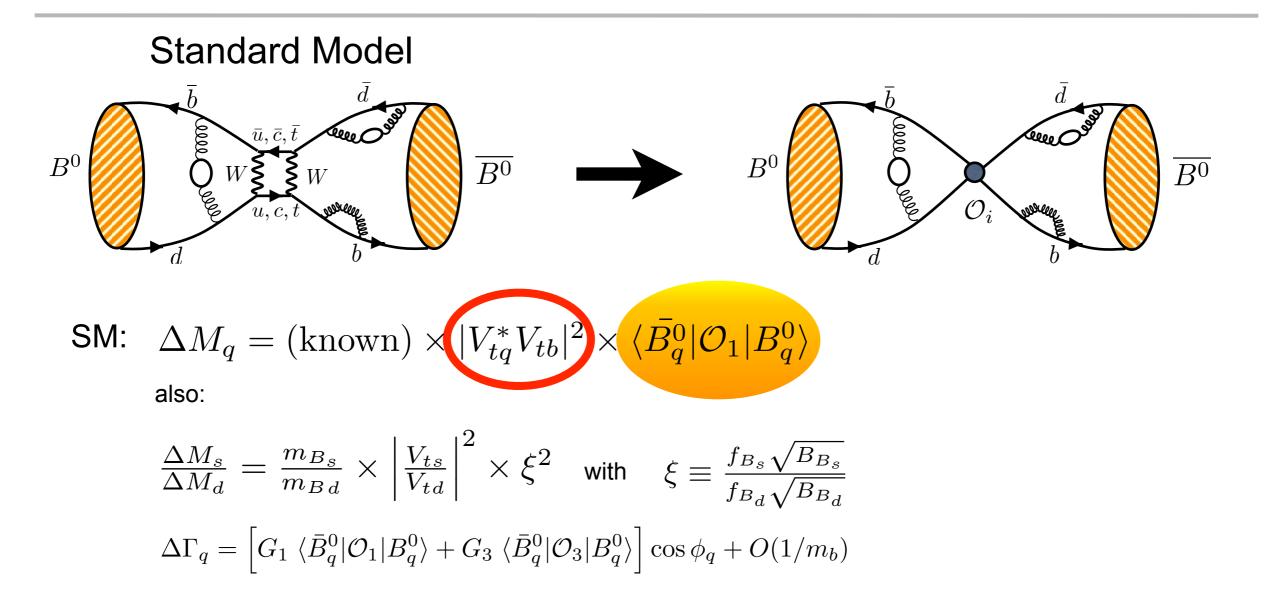




~2.6 σ tension between LHCb measurement and SM theory

In the SM these ratios are insensitive to the form factors (see also C. Bouchard et al, arXiv:1303.0434, PRL 2013)

Neutral B meson mixing

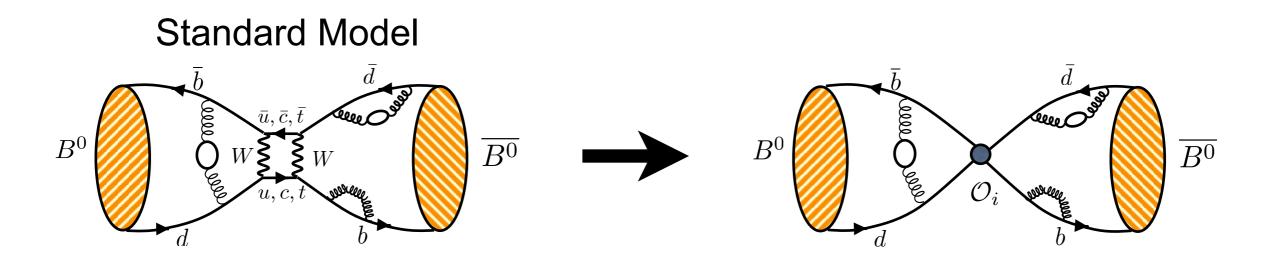


HFAG, PDG 2016 averages:

$$\Delta M_d = (0.5055 \pm 0.0020) \text{ ps}^{-1} (0.4\%) \qquad \Delta \Gamma_d / \Gamma_d = 0.001 \pm 0.010$$

$$\Delta M_s = (17.575 \pm 0.021) \text{ ps}^{-1} (0.1\%) \qquad \Delta \Gamma_s / \Gamma_s = 0.124 \pm 0.009 (7.3\%)$$

Neutral B meson mixing



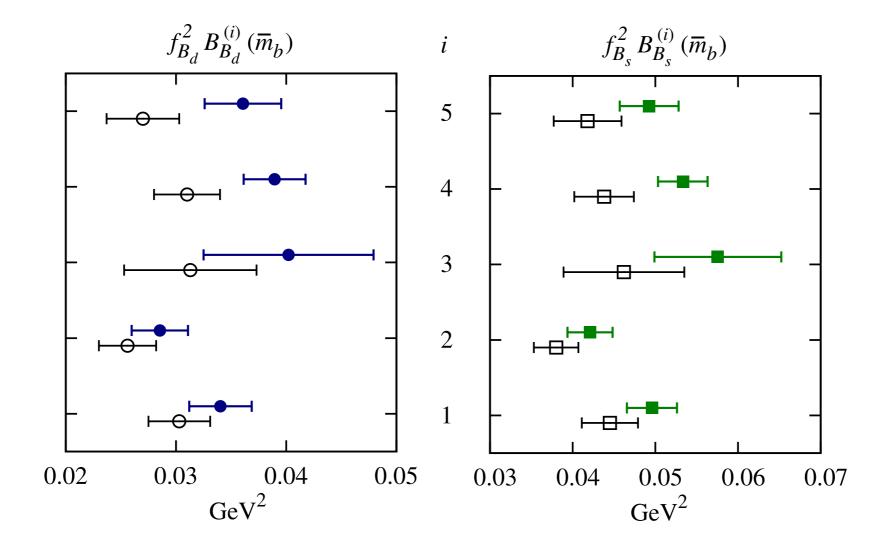
| In general : | SM: | BSM: |
|---|---|---|
| $\mathcal{H}_{\text{eff}} = \sum_{i=1}^{5} c_i(\mu) \mathcal{O}_i(\mu)$ | $\mathcal{O}_{1} = (\bar{b}^{\alpha} \gamma_{\mu} L q^{\alpha}) (\bar{b}^{\beta} \gamma_{\mu} L q^{\beta})$ $\mathcal{O}_{2} = (\bar{b}^{\alpha} L q^{\alpha}) (\bar{b}^{\beta} L q^{\beta})$ | $\mathcal{O}_4 = (\bar{b}^{\alpha} L q^{\alpha}) \ (\bar{b}^{\beta} R q^{\beta})$ $\mathcal{O}_5 = (\bar{b}^{\alpha} L q^{\beta}) \ (\bar{b}^{\beta} R q^{\alpha})$ |
| $i{=}1$ | $\mathcal{O}_3 = (\bar{b}^{\alpha} L q^{\beta}) (\bar{b}^{\beta} L q^{\alpha})$ | |

$$\langle \mathcal{O}_i \rangle \equiv \langle \bar{B_q^0} | \mathcal{O}_i | B_q^0 \rangle(\mu) = e_i \ m_{B_q}^2 \ f_{B_q}^2 \ B_{B_q}^{(i)}(\mu)$$

The matrix elements of all five operators can be calculated in LQCD.

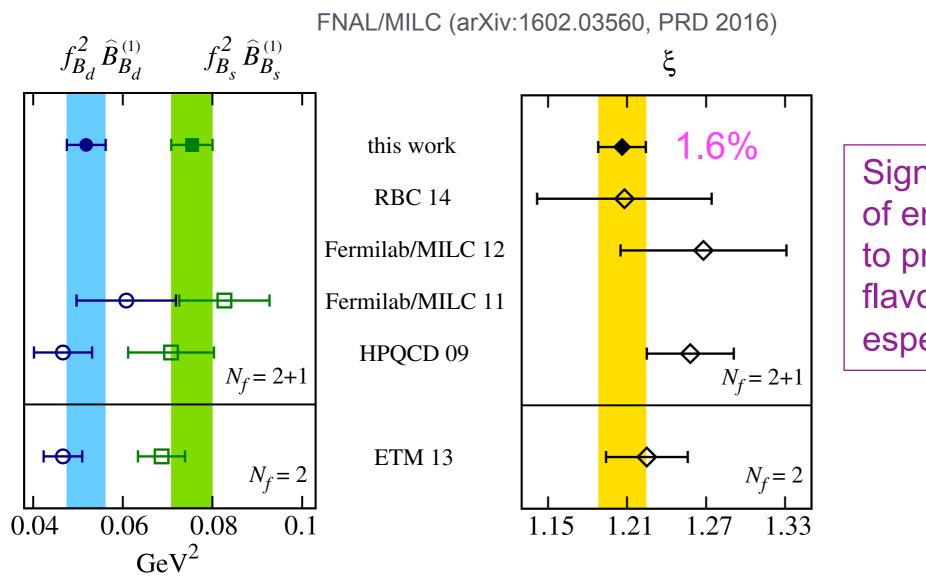
B mixing results in comparison

ETM (*n_f*=2, arXiv:1308.1851, JHEP 2014) vs. FNAL/MILC (*n_f*=3, arXiv:1602.03560, PRD 2016)



First three flavor LQCD results for all five matrix elements including the correlations between all 10 MEs.

B mixing results in comparison

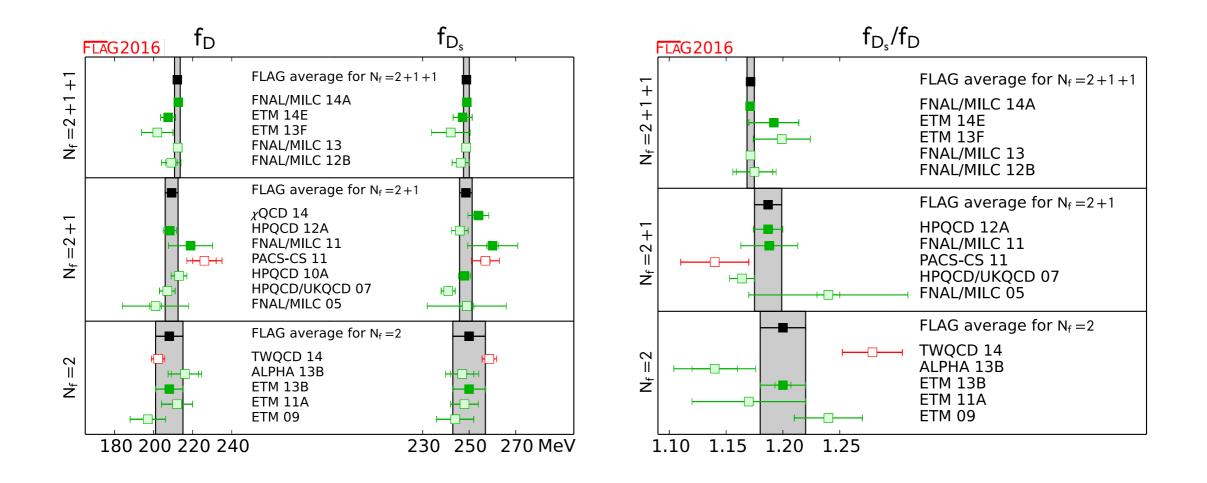


Significant reduction of errors compared to previous three flavor results, especially for ξ

- Note: FLAG-3 is currently updating their averages for B mixing quantities to include the new FNAL/MILC results.
- ongoing LQCD calculations by HPQCD, ETM, RBC/UKQCD, ...

FLAG review of D-meson quantities

S. Aoki et al (FLAG-3 review, arXiv:1607.00299, EPJC 17, web update)





$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

$$\Pi_{\mu\nu} = \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$
Leading order HVP correction:
$$a_\mu^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2 w(q^2) \hat{\Pi}(q^2)$$

 Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic cross section
 talks by Teubner, Jegerlehner, Zhang

Hadronic vacuum polarization

Leading order HVP correction: $a_{\mu}^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dq^2 w(q^2) \hat{\Pi}(q^2)$

- Calculate a_{μ}^{HVP} in Lattice QCD:
 - + Calculate $\hat{\Pi}(q^2)$ and evaluate the integral
 - + Time-momentum representation: reorder the integrations and compute $C(t) = \frac{1}{2} \sum \langle j_i \rangle$

$$= \frac{1}{3} \sum_{i,x} \langle j_i(x,t) j_i(0,0) \rangle$$

(Bernecker & Meyer, EPJ 12)

Time-moments:

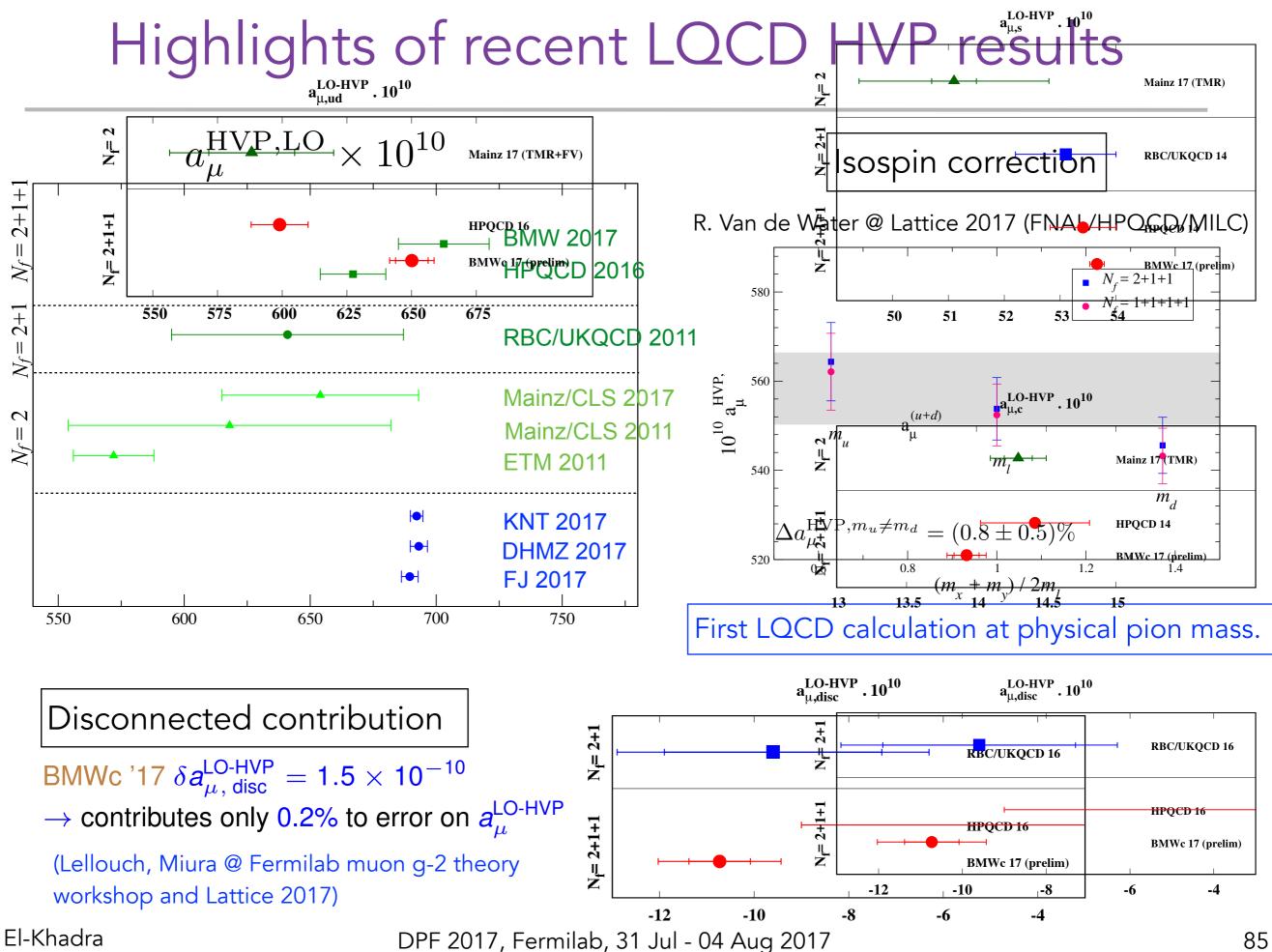
Taylor expand $\hat{\Pi}(q^2) = \sum q^{2k} \Pi_k$

(Chakraborty et al, PRD 14)

and compute Taylor coefficients from time moments:

 $a_{\mu}^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_{0}^{\infty} dt \, \tilde{w}(t) \, C(t)$

$$C_{2n} = a \sum_{t} t^{2n} C(t)$$



A. El-Khadra

85

Muon g-2 Theory Initiative

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 Fermilab E989 experiment
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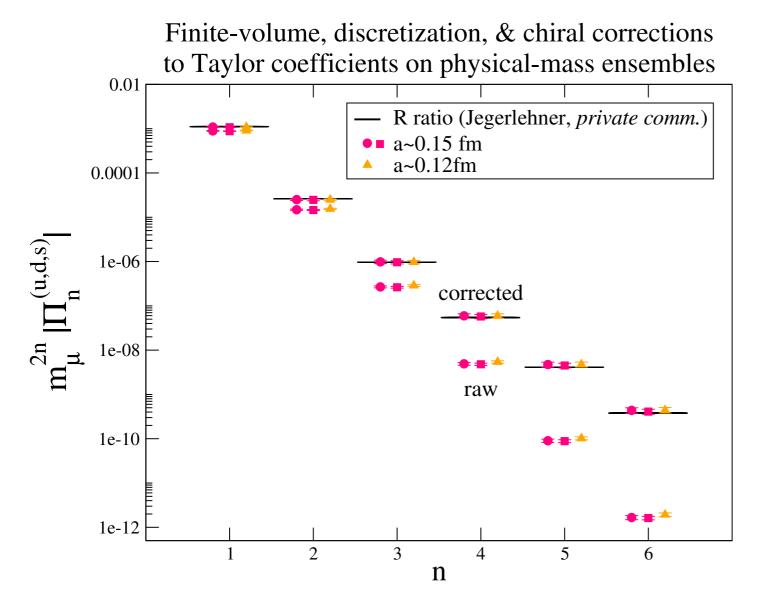
Muon g-2 Theory Initiative: Goals

- It to the Fermilab and J-PARC experiments to maximize their impact
 - need theoretical predictions of the hadronic corrections with reduced and reliably estimated uncertainties
- summarize the theoretical calculations of the hadronic corrections to the muon g-2

 - → assess reliability of uncertainty estimates

Compare HPQCD results to R-ratio data

R. Van de Water @ Lattice 2017



A good test of the corrections, because the comparison was performed after the first version of the HPQCD paper (1601.03071) was posted.

Lowest moments make the largest contributions to a_{μ} .

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- \bigcirc combine to provide theory predictions for $a_{\mu}^{\rm HVP}$ and $a_{\mu}^{\rm HLbL}$ and write a report **before** the Fermilab and J-PARC experiments announce their first results.

Muon g-2 Theory Initiative: Plan

- Organize ``plenary" workshops to bring the different communities together
 - First workshop: held near Fermilab, June 2017: kick-off
 - Second workshop: Mainz, 18-22 June 2018: organize first report
 - 2019 & 2020 workshops: Japan? Seattle?
- Germ two working groups, one for HVP and one for HLbL:
 - invite community participation
 - organize focused workshops to advance the work: winter/spring 2018
- Finalize the first report before the Fermilab experiment announces its first result with "Brookhaven level" statistics target date for first report: September 2018

First Workshop of the Muon g-2 Theory Initiative

3-6 June 2017 *Q Center* US/Central timezone

Search

Sponsors

Committees

Timetable

Registration

Registration Form

List of registrants

List of confirmed speakers

workshop photos

Accommodations

Wilson Hall

Visa Information

In the coming years, experiments at Fermilab and at J-PARC plan to reduce the uncertainties on the already very precisely measured anomalous magnetic moment of the muon by a factor of four. The goal is to resolve the current tantalizing tension between theory and experiment of three to four standard deviations. On the theory side the hadronic corrections to the anomalous magnetic moment are the dominant sources of uncertainty. They must be determined with better precision in order to unambiguously discover whether or not new physics effects contribute to this quantity.

There are a number of complementary theoretical efforts underway to better understand and quantify the hadronic corrections, including dispersive methods, lattice QCD, effective field theories, and QCD models. We have formed a new theory initiative to facilitate interactions between the different groups through organizing a series of workshops. The goal of this first workshop is to bring together theorists from the different communities to discuss, assess, and compare the status of the various efforts, and to map out strategies for obtaining the best theoretical predictions for these hadronic corrections in advance of the experimental results.

All sessions in this workshop will be plenary, featuring a mix of talks and discussions.

Muon g-2 Theory Initiative: WGs

Ito sign-up for the HVP or HLbL WG send email to one of the WG coordinators

⊌ HVP WG coordinators:

- Michel Davier <u>davier@lal.in2p3.fr</u>
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 - Gilberto Colangelo gilberto@itp.unibe.ch
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