Novel Application of Density Estimation Techniques in MICE

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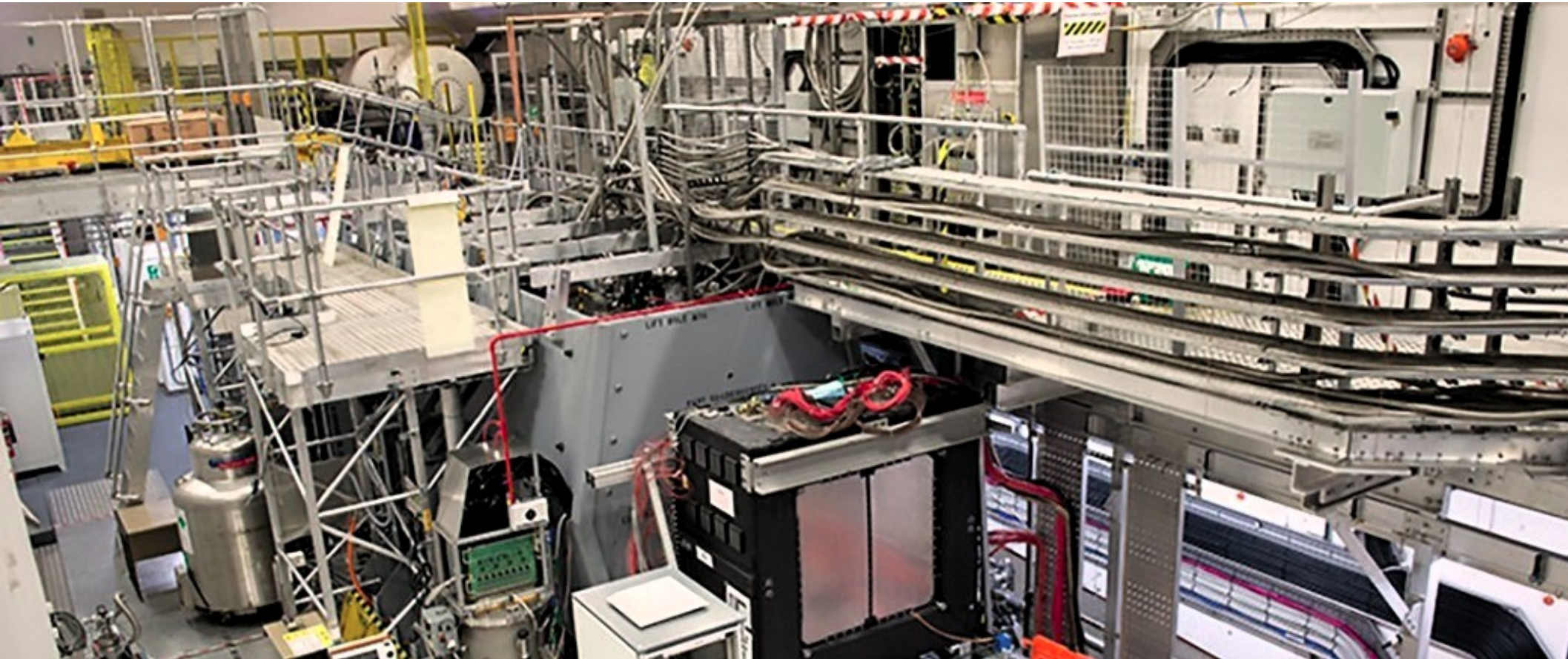
DPF 2017, Fermilab



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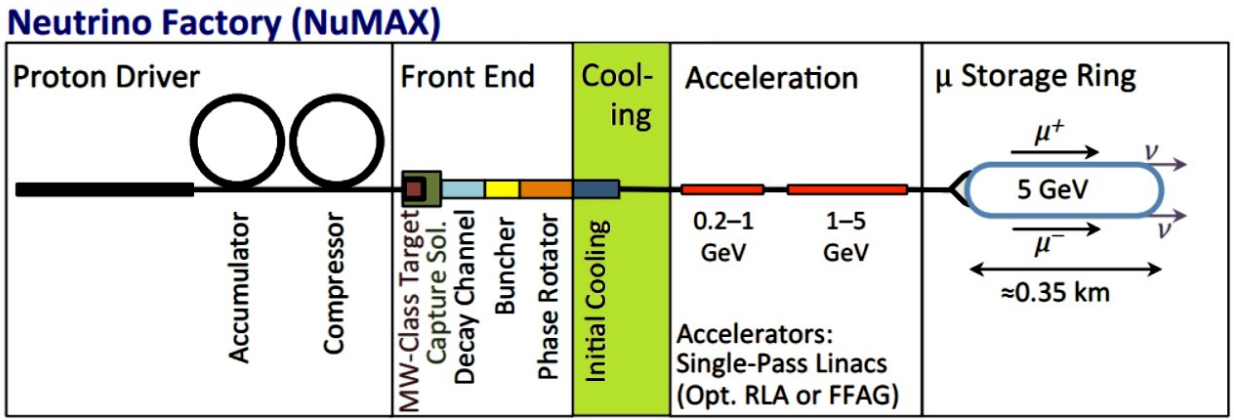
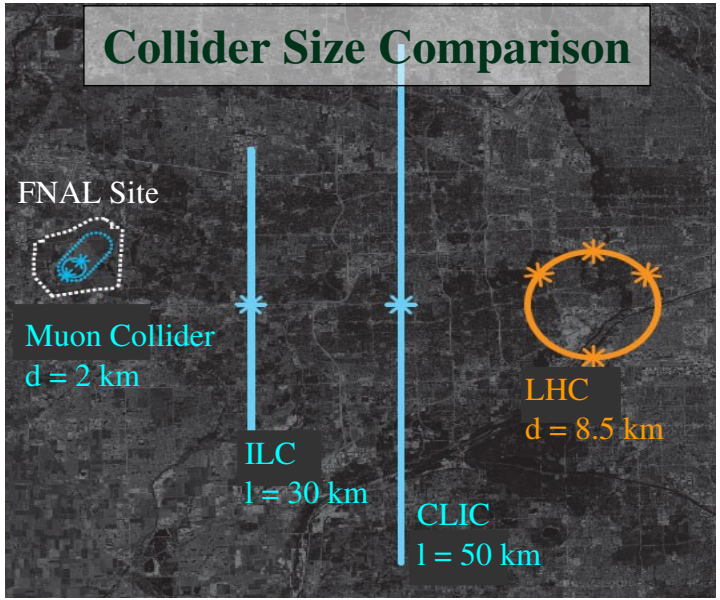
Motivation



- Purpose is higher intensity muon beams for:
  - ★ Neutrino Factory: best neutrino oscillation sensitivity via intense, pure ν_e/ν_μ beams from $\mu^{+/-}$ decay
  - ★ Muon Collider: clean multi-TeV collisions with compact facility
- Challenge:
  - ★ Large phase-space volume of muons and their short lifetime
- Solution:
  - ★ Rapid beam cooling via ionization energy loss
- Test:
  - ★ Muon Ionization Cooling Experiment (MICE)



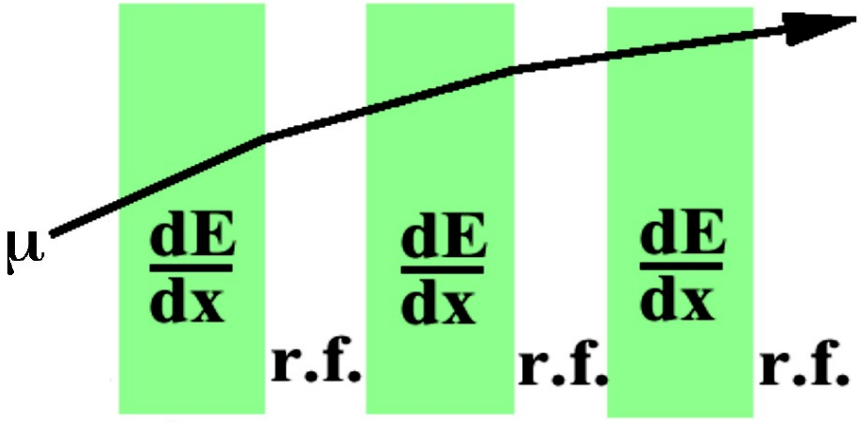
Muon Ionization Cooling

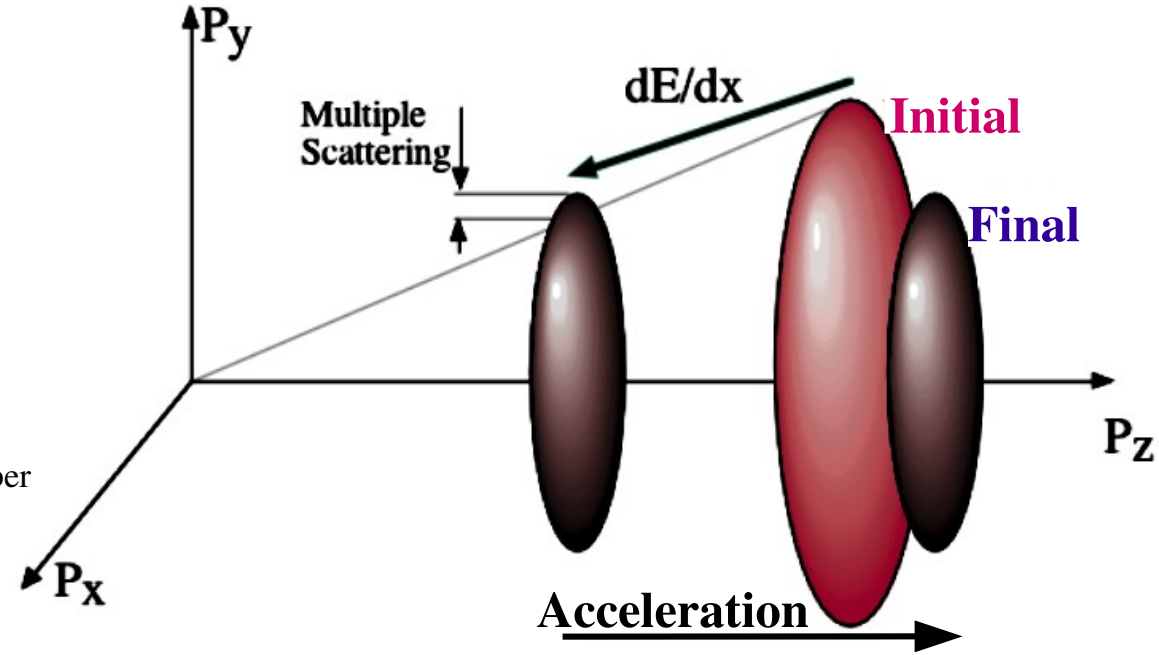
- Cooling by ionization energy loss

$$\frac{d\varepsilon_n}{ds} \simeq -\frac{\varepsilon_n}{\beta^2 E_\mu}\left\langle\frac{dE}{ds}\right\rangle + \frac{\beta_\perp\,(13.6\mathrm{MeV}/c)^2}{2\beta^3 E_\mu m_\mu X_0}$$

- Heating by multiple (Coulomb) scattering

ε_n: normalized emittance, $\beta_\perp$: transverse beta function, X_0: absorber radiation length

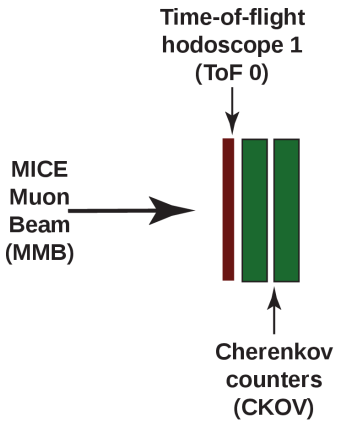


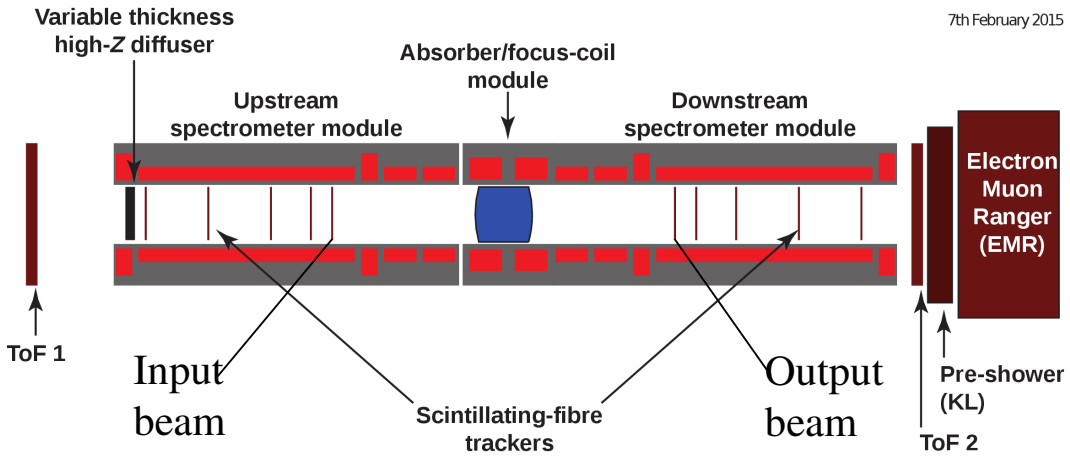


- Measures of muon beam cooling:
  - ★ Reductions: Phase-space volume, emittance
  - ★ Increase: Phase-space density

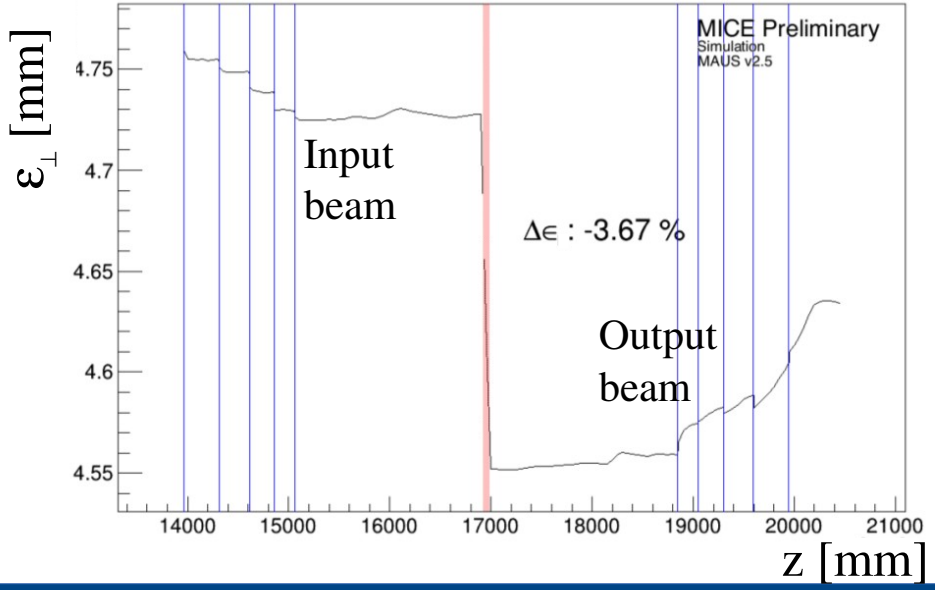
MICE Cooling Channel

- Particle ID with time-of-flight, Cherenkov counters, calorimetry (μ^+ beam slightly contaminated with e^+, π^+)



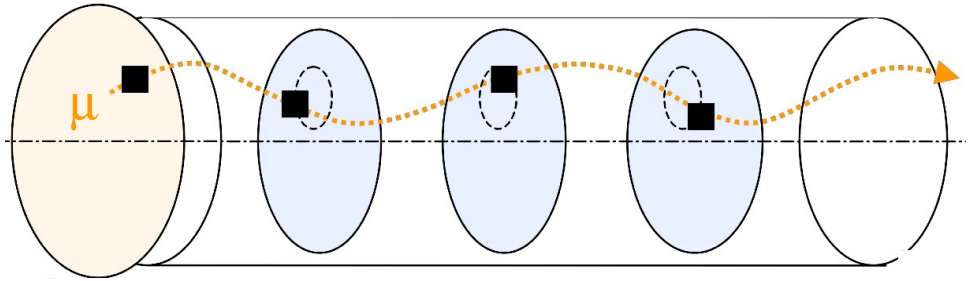


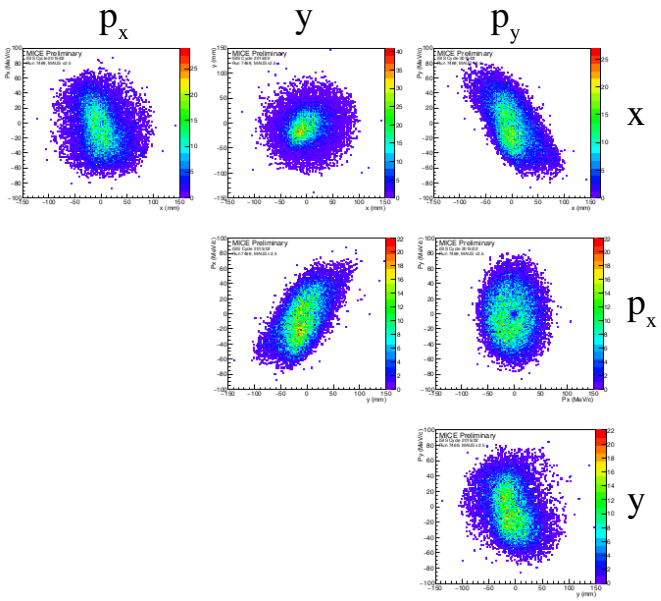
- Muons measured **one by one** in the trackers:
  - ★ Changes in density, volume, emittance ($\varepsilon_\perp$) by comparing the beam before (input) and after (output) an absorber

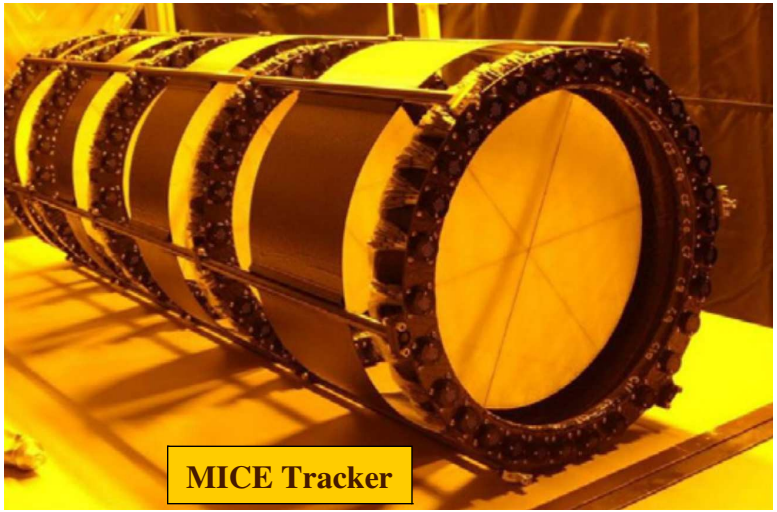


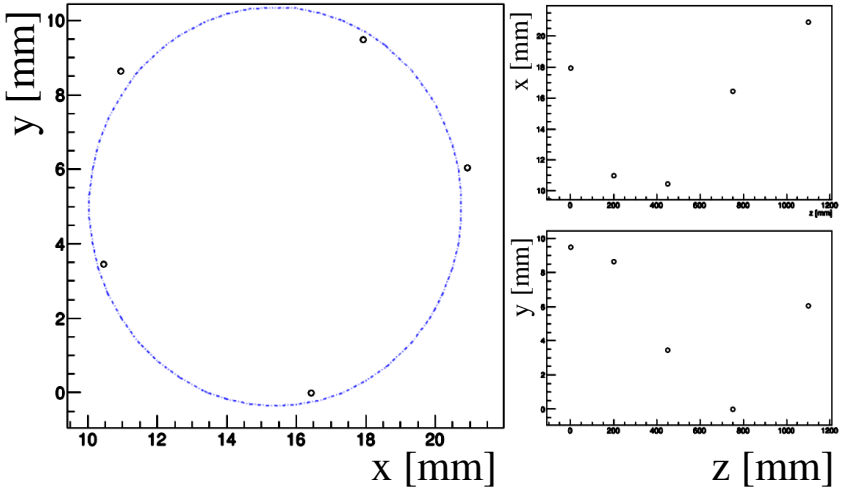
Tracker Reconstruction

- Helical tracks formed in spectrometer solenoids:
  - ★ Phase-space coordinates reconstructed in trackers





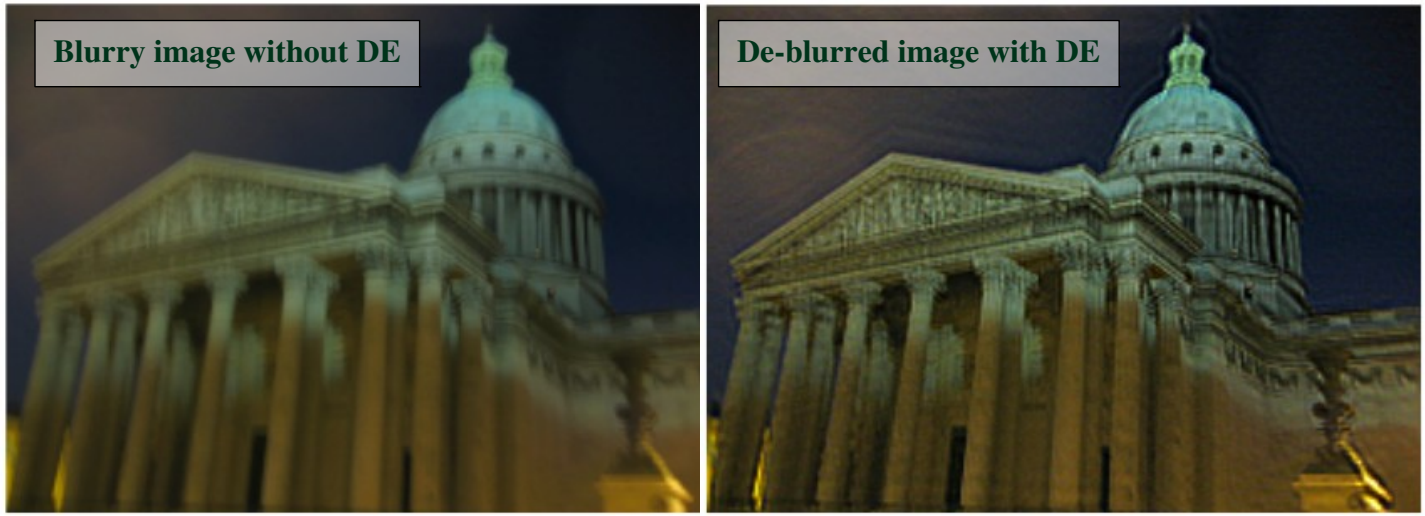




Density Estimation (DE) I

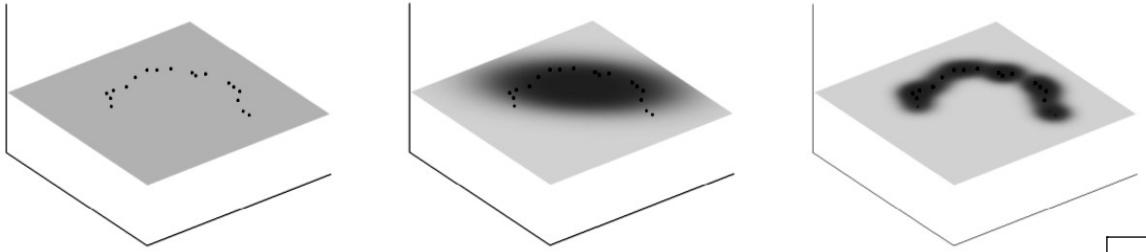
- Machine Learning concept: estimates the probability density function or density

Image Processing



D. Krishnan et al., "Blind Deconvolution Using a Normalized Sparsity Measure", DOI: 10.1109/CVPR.2011.5995521

Actual distribution | Gaussian fitting | Density estimation



M. Rousson et al., "Efficient Kernel Density Estimation of Shape and Intensity Priors for Level Set Segmentation", DOI:10.1007/978-0-387-68343-0_13

- "Data points speak for themselves"
- Powerful tool when density function is not known
- Precisely what is being done in MICE

Power of DE

Density Estimation II

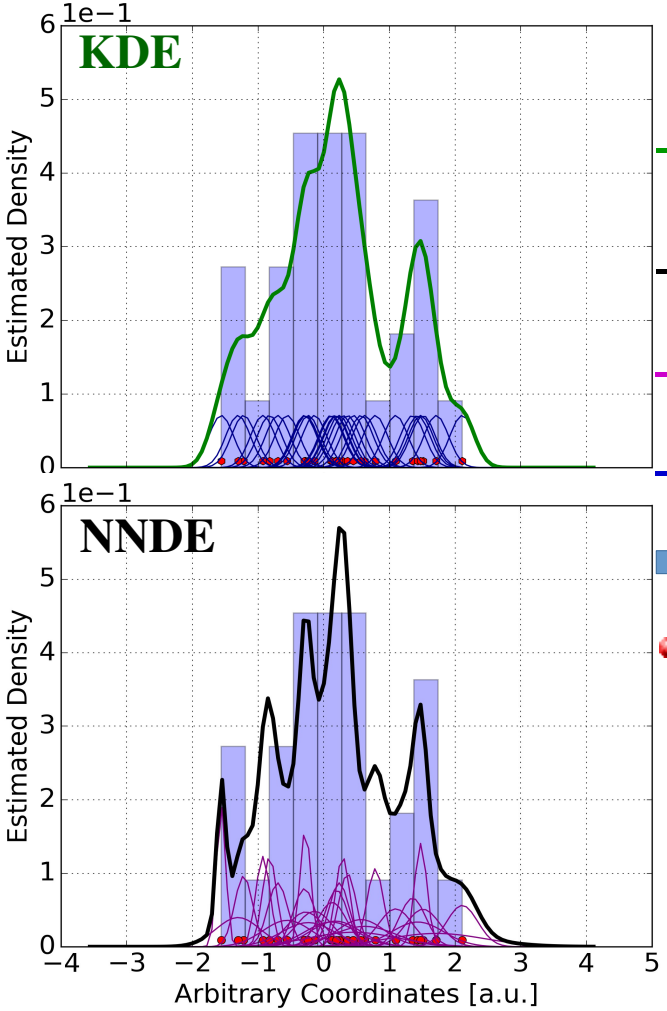
REMARKS ON SOME NONPARAMETRIC ESTIMATES OF A DENSITY FUNCTION[1]

By Murray Rosenblatt[2]

University of Chicago

1. Summary. This note discusses some aspects of the estimation of the density function of a univariate probability distribution. All estimates of the density function satisfying relatively mild conditions are shown to be biased. The asymptotic mean square error of a particular class of estimates is evaluated.

- Pioneered by M. Rosenblat (1956), P. Whittle, E. Parzen
- Oldest example: histogram
  - ★ Bins of certain widths
- Other examples: kernel density estimation (KDE), kth nearest neighbor (NNDE):
  - ★ Kernels (smooth weight functions) of certain widths



KDE $\frac{1}{n}\sum_{i=1}^{n} K\left(\frac{x-X_i}{h}\right)$

NNDE $\frac{1}{n}\sum_{i=1}^{n} K\left(\frac{x-X_i}{d_i}\right)$

Kernels $\frac{1}{h\sqrt{2\pi}}\exp\left(\frac{x-X_i}{h}\right)$

Kernels $\frac{1}{d_i\sqrt{2\pi}}\exp\left(\frac{x-X_i}{d_i}\right)$

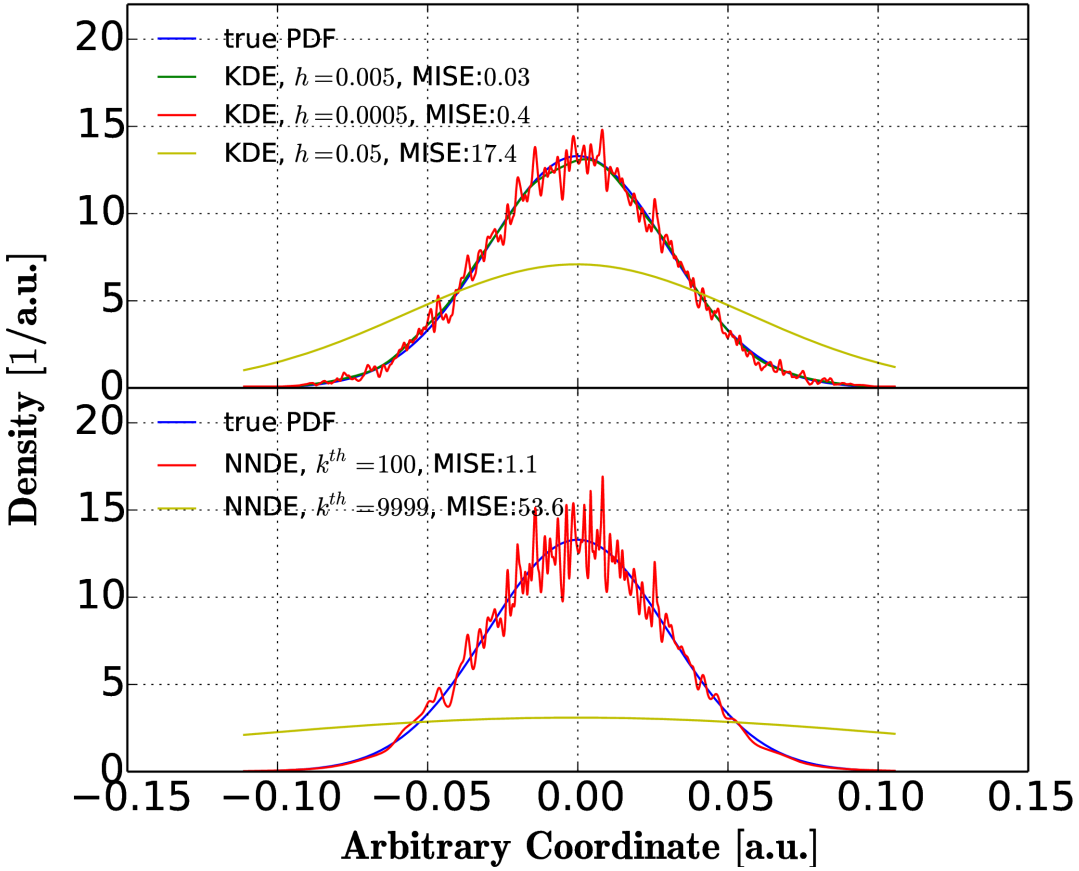
Histogram

Data Points

h and d_i (distances between points): widths of KDE and NNDE kernels, n: sample size

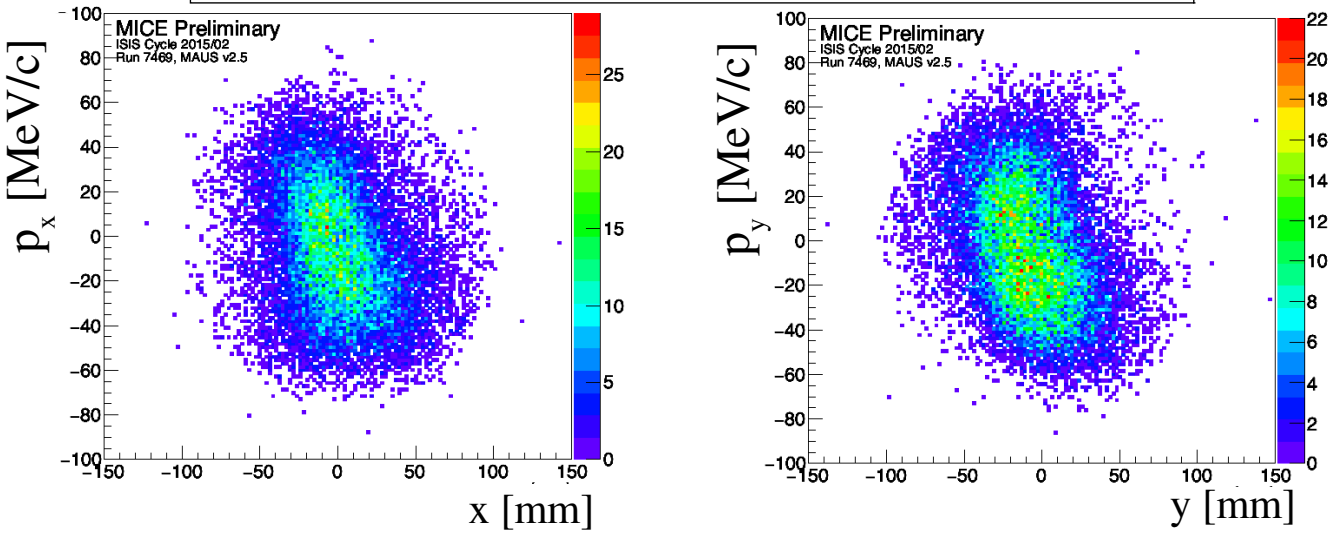
Density Estimation Features

- Mean integrated squared error (MISE), common measure of error: deviation of estimated density from true density
- Optimal kernel width minimizes MISE:
  - ★ True PDF generated from Gaussian
  - ★ h = 0.005 (optimal kernel width) reveals a Gaussian
  - ★ h = 0.05 (k^{th} = 9999 in case of NNDE) over-smooths
  - ★ h = 0.005 (k^{th} = 100 in case of NNDE) overemphasizes noise



Kernel Density Estimation in MICE

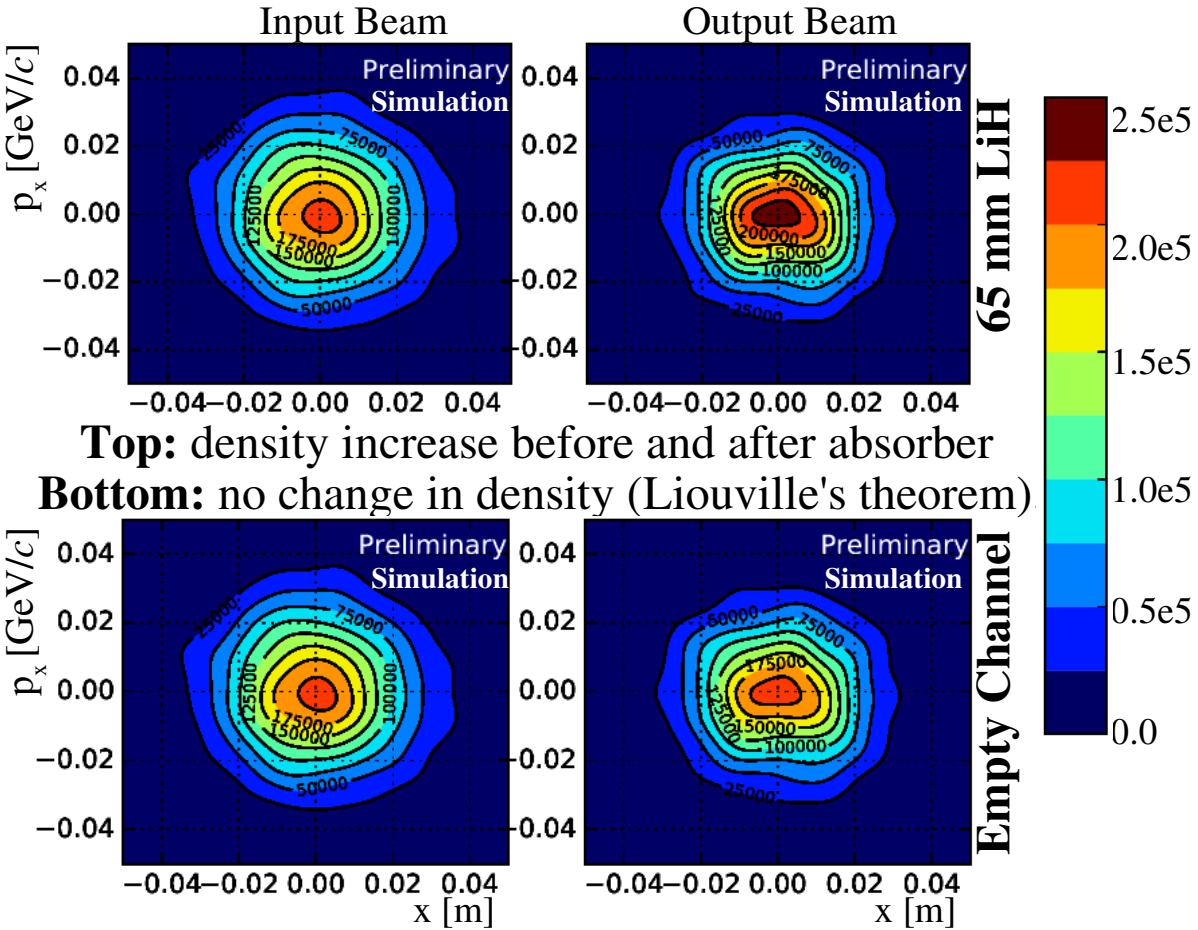
Non-Gaussian Phase Space of Measured Muon Beam



- Real-life particle beam is non-Gaussian (chromatic and non-linear effects)
- Kernel Density Estimation:
  - ★ Estimates probability density function or density with few assumptions about the underlying distribution
  - ★ Gives detailed diagnostics of the particles in a cooling channel

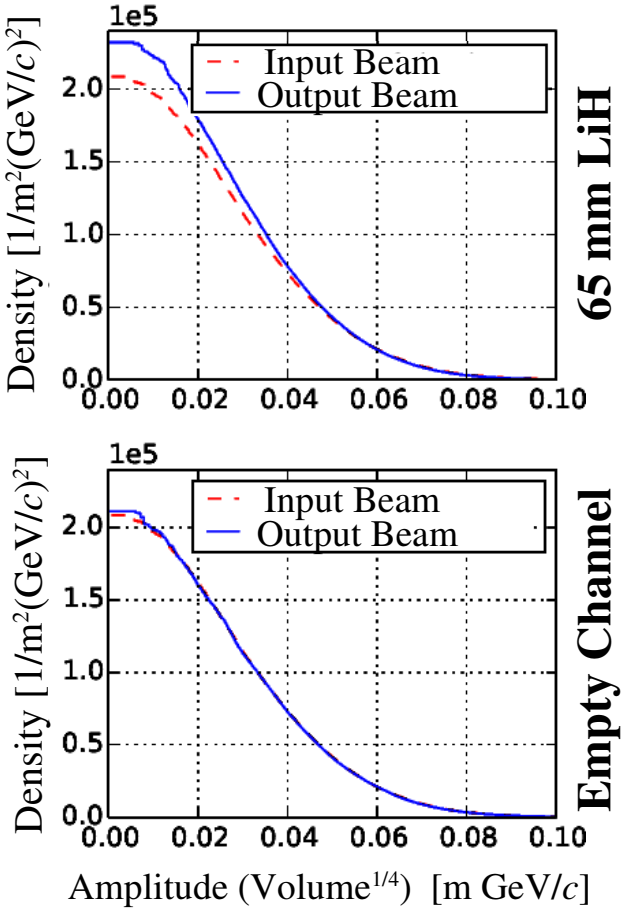
KDE Density and Volume Measurements – Method

- Assign Gaussian kernel functions at each muon in 4D, then sum each contribution
- Detailed diagnostics of the beam core and halo



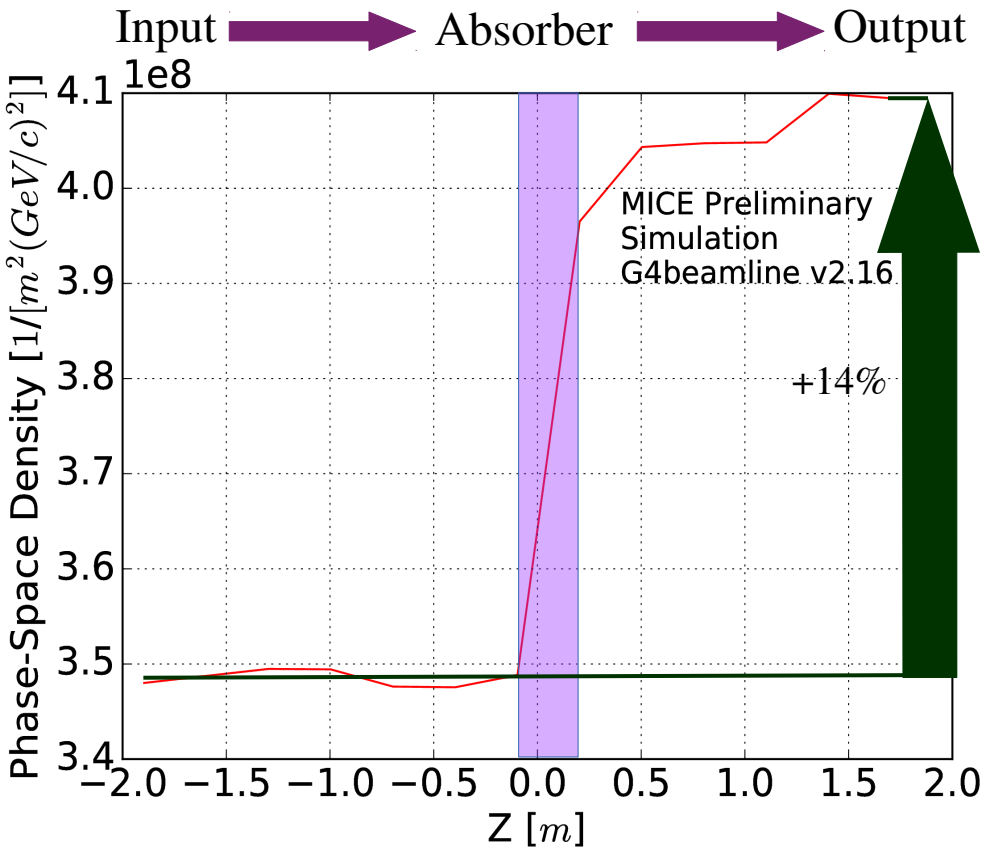
Top: density increase before and after absorber

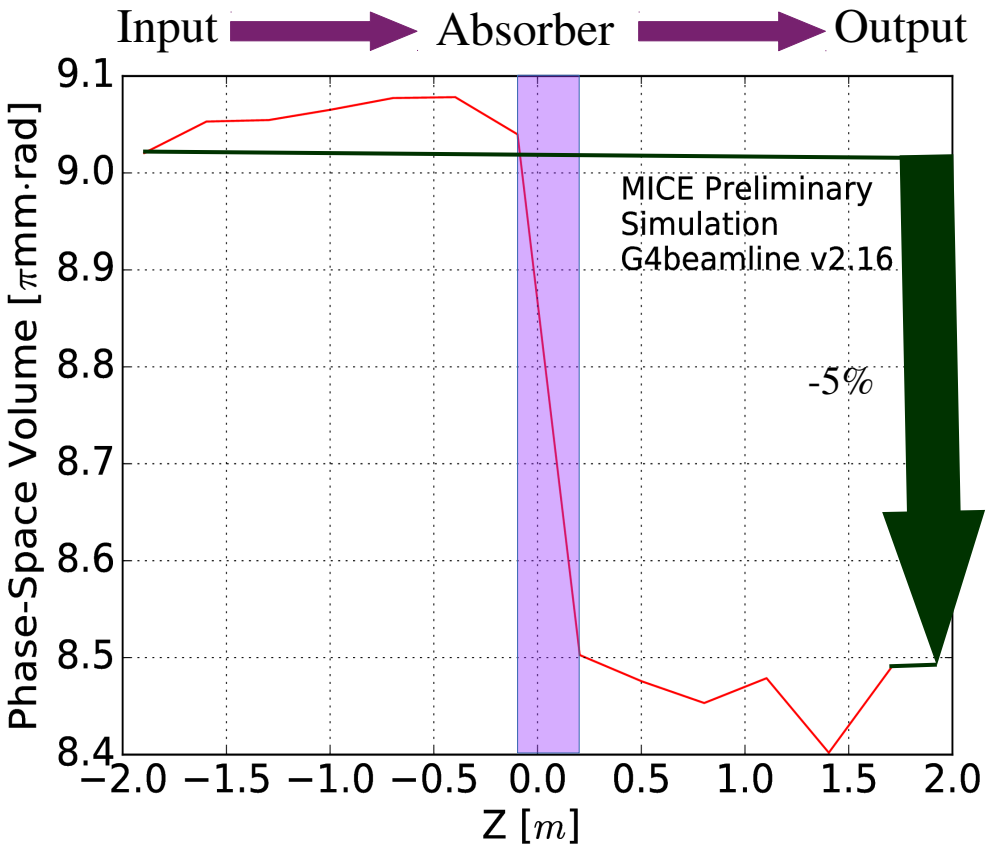
Bottom: no change in density (Liouville's theorem)



KDE Simulation Study I

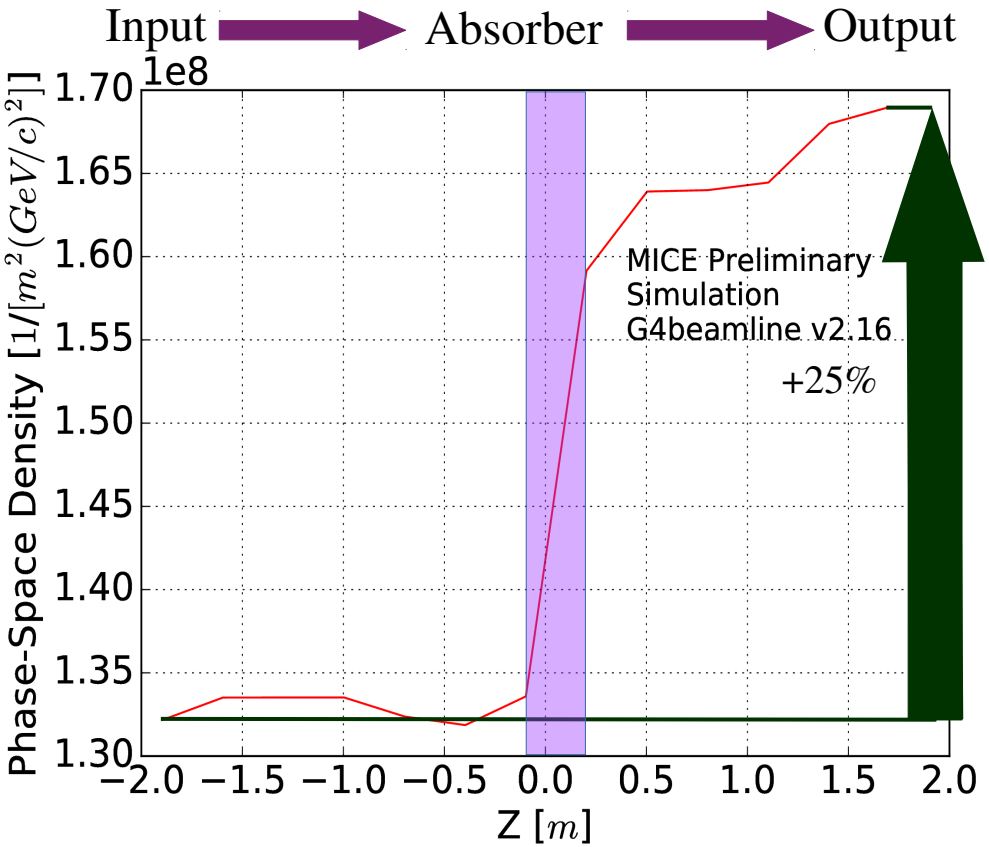
- Input emittance: 6 π mm·rad, momentum: 140 MeV/c, beta function at absorber: 600 mm
- Tracked 9th percentile (beam core in 4D) contour's density and volume across absorber

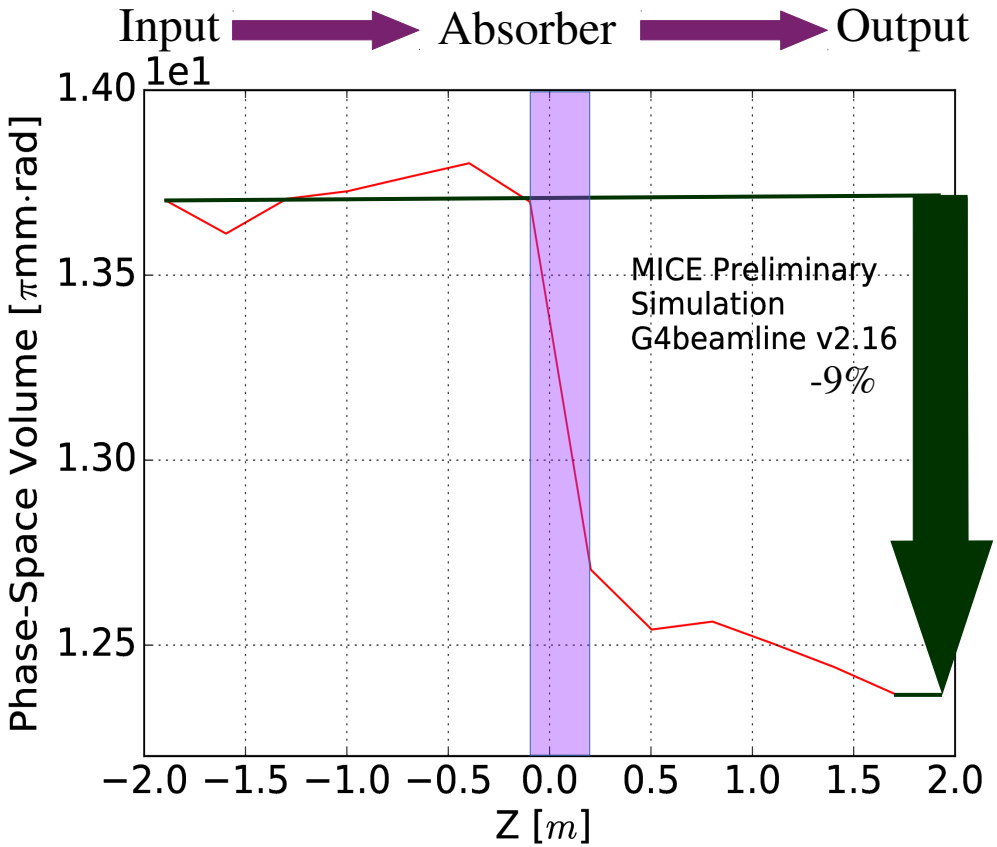




KDE Simulation Study II

- Input emittance: 10 π mm·rad, momentum: 140 MeV/c, beta function at absorber: 600 mm
- Tracked 9th percentile (beam core in 4D) contour's density and volume across absorber





Conclusion and Future Prospects

- KDE-based density and volume evolution curves behave as expected
- KDE-based measurements give detailed diagnostics of the muon beam traversing a material
- **First** application of density estimation to beam cooling (as far as I know)
- Other possible applications:
  - ★ Image processing for beam reconstruction
  - ★ Image processing for event reconstruction in time projection chambers
  - ★ Precision studies of particle beams in presence of non-linear effects
- KDE application to **MICE experimental data** and the corresponding **error analysis (systematics)** are in progress
- Stay tuned!

Acknowledgements

Thank You!

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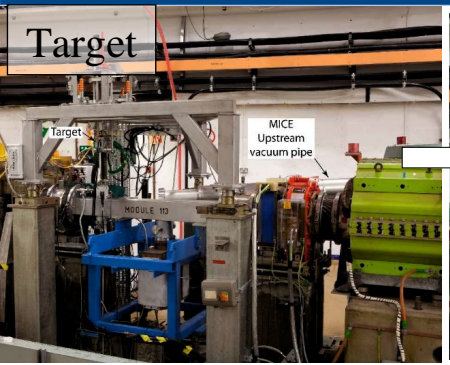
- Many thanks to P. Snopok, D. Neuffer, and C. Rogers.

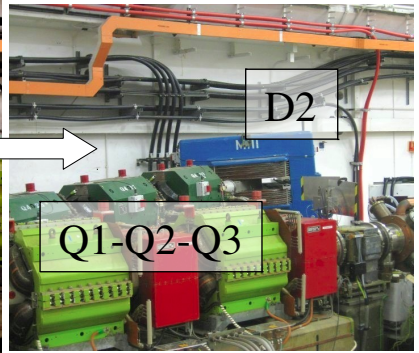
References

1. T. A. Mohayai, et al., “Novel Implementation Of Non-Parametric Density Estimation in MICE”, IPAC’17, IPAC-2017-WEPAB135 (2017).
2. T. A. Mohayai, “Measurements Of Beam Cooling In Muon Ionization Cooling Experiment”, University of Mississippi invited colloquium talk (2017).
3. T. A. Mohayai, “Novel Application of Kernel Density Estimation in MICE”, MICE-Note-506 (2017).
4. T. A. Mohayai, et al., “Measurements of Beam Cooling in the Muon Ionization Cooling Experiment”, APS April Meeting'17 (2017).
5. T. A. Mohayai, et al., “Simulated Measurements of Beam Cooling in Muon Ionization Cooling Experiment”, Proc. NA-PAC’16, NA-PAC-2016-WEPOA36 (2016).
6. T. A. Mohayai, et al., “Simulated Measurements of Cooling in Muon Ionization Cooling Experiment”, Proc. IPAC’16, IPAC-2016-TUPMY011 (2016).

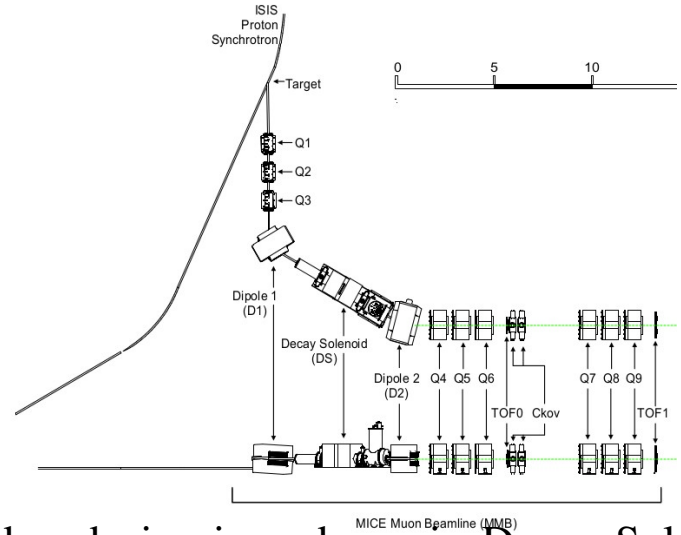
Additional Slides

MICE Beam Line

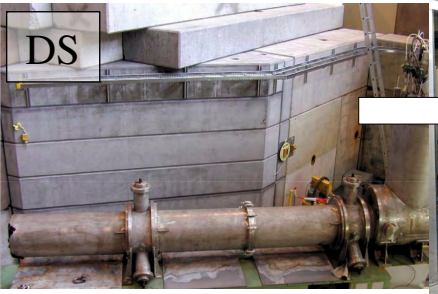




- Protons produced in ISIS proton synchrotron:
  - ★ H^- bunches accelerated in the Linac, transported to Al_2O_3 foil for H^+ production. H^+ bunches accelerated to *800* MeV in the synchrotron.

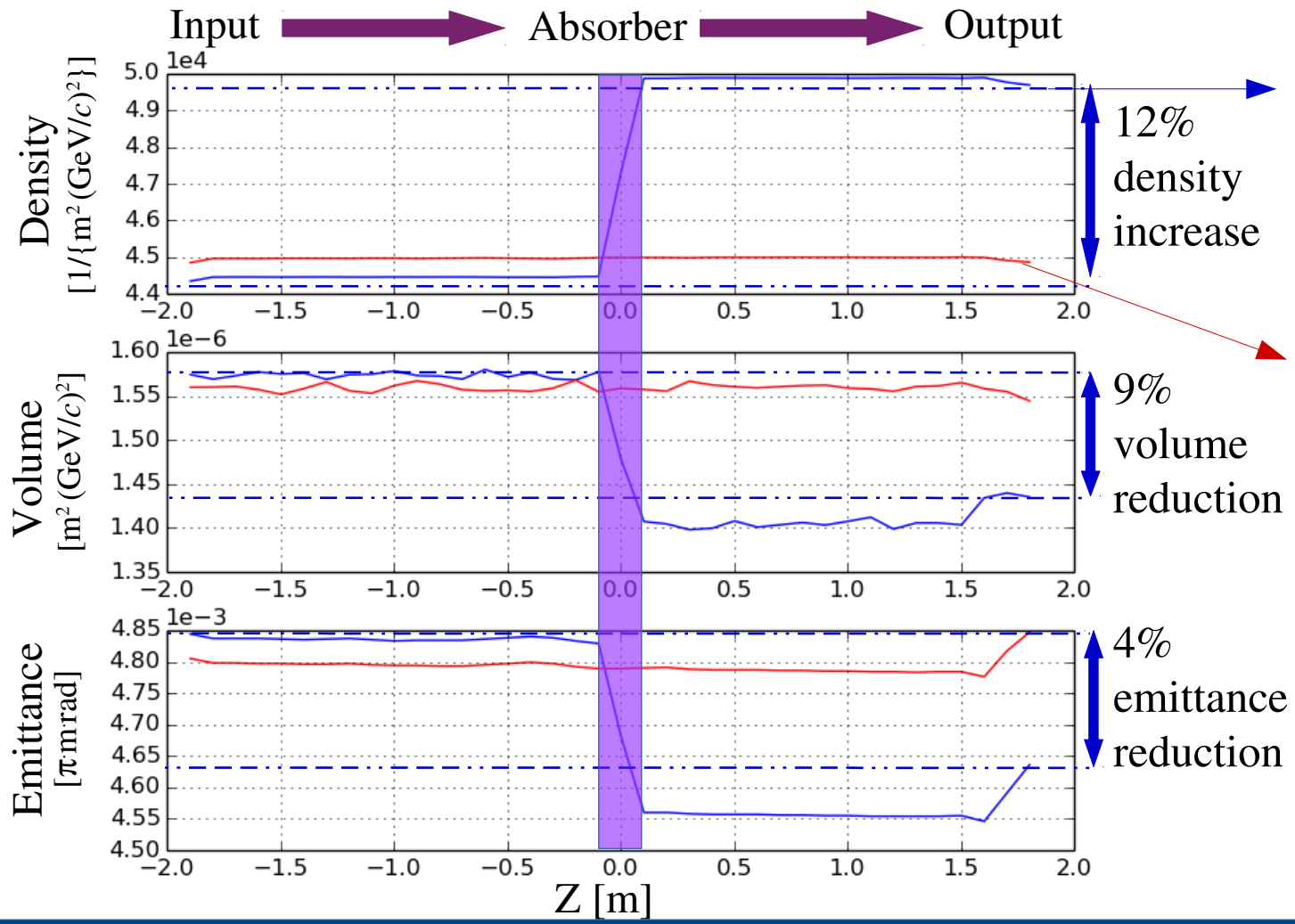


- Pions produced via target-ISIS proton beam interactions, $p+p \rightarrow p+n+\pi^+$:
  - ★ Quadrupole triplet magnets, Q1-Q2-Q3 for focusing
  - ★ Dipole magnet, D1for momentum selection



- Muons produced via pions decay in Decay Solenoid, DS, $\pi^+ \rightarrow \mu^+ + \nu_\mu$:
  - ★ Dipole magnet, D2 for momentum selection
  - ★ Pairs of Quadrupole triplets, Q4-Q5-Q6, Q7-Q8-Q9

Comparisons



Channel with 65mm LiH absorber

Empty channel absorber

- Comparison of KDE-based density and volume with emittance:
  - shows consistency of the two methods
  - KDE measurements improve upon RMS emittance by accounting for non-linear effects

Emittance Measurement in MICE

- Reconstruct position and momentum coordinates using trackers
- Construct covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{p_x x} & \sigma_{yx} & \sigma_{p_y x} \\ \sigma_{x p_x} & \sigma_{p_x p_x} & \sigma_{y p_x} & \sigma_{p_y p_x} \\ \sigma_{xy} & \sigma_{p_x y} & \sigma_{yy} & \sigma_{p_y y} \\ \sigma_{x p_y} & \sigma_{p_x p_y} & \sigma_{y p_y} & \sigma_{p_y p_y} \end{pmatrix} .$$

$$\sigma_{x p_x} = \langle x p_x \rangle - \langle x \rangle \langle p_x \rangle .$$

- Compute transverse normalized RMS emittance

$$\varepsilon_n = 1/m \, |\Sigma|^{\frac{1}{4}}$$

Measurement of Emittance in MICE Upstream Tracker