Novel Application of Density Estimation Techniques in MICE

Tanaz Angelina Mohayai, for the MICE Collaboration
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Motivation

- Purpose is higher intensity muon beams for:
  - Neutrino Factory: best neutrino oscillation sensitivity via intense, pure $\nu_e/\nu_\mu$ beams from $\mu^+/\mu^-$ decay
  - Muon Collider: clean multi-TeV collisions with compact facility
- Challenge:
  - Large phase-space volume of muons and their short lifetime
- Solution:
  - Rapid beam cooling via ionization energy loss
- Test:
  - Muon Ionization Cooling Experiment (MICE)

![Collider Size Comparison Diagram]
Muon Ionization Cooling

Cooling by ionization energy loss

\[ \frac{d\varepsilon_n}{ds} \approx -\frac{\varepsilon_n}{\beta^2 E_\mu} \langle \frac{dE}{ds} \rangle + \frac{\beta_\perp (13.6 \text{MeV/c})^2}{2 \beta^3 E_\mu m_\mu X_0} \]

Heating by multiple (Coulomb) scattering

\( \varepsilon_n \): normalized emittance, \( \beta_\perp \): transverse beta function, \( X_0 \): absorber radiation length

Measures of muon beam cooling:

- Reductions: Phase-space volume, emittance
- Increase: Phase-space density
MICE Cooling Channel

- Particle ID with time-of-flight, Cherenkov counters, calorimetry ($\mu^+$ beam slightly contaminated with $e^+$, $\pi^+$)
- Muons measured **one by one** in the trackers:
  - Changes in density, volume, emittance ($\varepsilon_\perp$) by comparing the beam before (input) and after (output) an absorber
Tracker Reconstruction

- Helical tracks formed in spectrometer solenoids:
  - Phase-space coordinates reconstructed in trackers
Density Estimation (DE) I

- Machine Learning concept: estimates the probability density function or density

- “Data points speak for themselves”
- Powerful tool when density function is not known
- Precisely what is being done in MICE
Density Estimation II

REMARKS ON SOME NONPARAMETRIC ESTIMATES OF A DENSITY FUNCTION

BY MURRAY ROSENBLATT

University of Chicago

1. Summary. This note discusses some aspects of the estimation of the density function of a univariate probability distribution. All estimates of the density function satisfying relatively mild conditions are shown to be biased. The asymptotic mean square error of a particular class of estimates is evaluated.

- Pioneered by M. Rosenblat (1956), P. Whittle, E. Parzen
- Oldest example: histogram
  - ★ Bins of certain widths
- Other examples: kernel density estimation (KDE), $k^{th}$ nearest neighbor (NNDE):
  - ★ Kernels (smooth weight functions) of certain widths

KDE

\[
\hat{f}(x) = \frac{1}{n} \sum_{i=1}^{n} K \left( \frac{x - X_i}{h} \right)
\]

NNDE

\[
\hat{f}(x) = \frac{1}{n} \sum_{i=1}^{n} K \left( \frac{x - X_i}{d_i} \right)
\]

Kernels

\[
\frac{1}{h\sqrt{2\pi}} \exp \left( -\frac{(x - X_i)^2}{h^2} \right)
\]

\[
\frac{1}{d_i\sqrt{2\pi}} \exp \left( -\frac{(x - X_i)^2}{d_i^2} \right)
\]

Histogram

Data Points

h and $d_i$ (distances between points): widths of KDE and NNDE kernels, $n$: sample size
Density Estimation Features

- Mean integrated squared error (MISE), common measure of error: deviation of estimated density from true density

- Optimal kernel width minimizes MISE:
  - True PDF generated from Gaussian
  - $h = 0.005$ (optimal kernel width) reveals a Gaussian
  - $h = 0.05$ ($k^{th} = 9999$ in case of NNDE) over-smooths
  - $h = 0.005$ ($k^{th} = 100$ in case of NNDE) overemphasizes noise
Kernel Density Estimation in MICE

Real-life particle beam is non-Gaussian (chromatic and non-linear effects)

Kernel Density Estimation:
- Estimates probability density function or density with few assumptions about the underlying distribution
- Gives detailed diagnostics of the particles in a cooling channel
KDE Density and Volume Measurements – Method

- Assign Gaussian kernel functions at each muon in 4D, then sum each contribution
- Detailed diagnostics of the beam core and halo

Bottom: no change in density (Liouville's theorem)

Top: density increase before and after absorber
KDE Simulation Study I

- Input emittance: $6 \pi \text{ mm-rad}$, momentum: $140 \text{ MeV/c}$, beta function at absorber: $600 \text{ mm}$
- Tracked $9^{\text{th}}$ percentile (beam core in 4D) contour's density and volume across absorber
KDE Simulation Study II

- Input emittance: $10 \pi$ mm-rad, momentum: 140 MeV/c, beta function at absorber: 600 mm
- Tracked 9\textsuperscript{th} percentile (beam core in 4D) contour's density and volume across absorber
Conclusion and Future Prospects

- KDE-based density and volume evolution curves behave as expected
- KDE-based measurements give detailed diagnostics of the muon beam traversing a material
- **First** application of density estimation to beam cooling (as far as I know)
- Other possible applications:
  - Image processing for beam reconstruction
  - Image processing for event reconstruction in time projection chambers
  - Precision studies of particle beams in presence of non-linear effects
- KDE application to **MICE experimental data** and the corresponding **error analysis** *(systematics)* are in progress
- Stay tuned!
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References

Additional Slides
MICE Beam Line

- Protons produced in ISIS proton synchrotron:
  - H⁻ bunches accelerated in the Linac, transported to Al₂O₃ foil for H⁺ production. H⁺ bunches accelerated to 800 MeV in the synchrotron.

- Pions produced via target-ISIS proton beam interactions, \( p + p \rightarrow p + n + \pi^+ \):
  - Quadrupole triplet magnets, Q1-Q2-Q3 for focusing
  - Dipole magnet, D1 for momentum selection

- Muons produced via pions decay in Decay Solenoid, DS, \( \pi^+ \rightarrow \mu^+ + \nu_\mu \):
  - Dipole magnet, D2 for momentum selection
  - Pairs of Quadrupole triplets, Q4-Q5-Q6, Q7-Q8-Q9

DS
Q7-Q8-Q9
Q1-Q2-Q3
D2
Comparisons

Comparison of KDE-based density and volume with emittance:
- Shows consistency of the two methods
- KDE measurements improve upon RMS emittance by accounting for non-linear effects

Channel with 65mm LiH absorber
- 12% density increase
- 9% volume reduction
- 4% emittance reduction

Empty channel absorber

Input $\rightarrow$ Absorber $\rightarrow$ Output

Density

$1/\text{m}^2 \cdot (\text{GeV}/c)^2$

Volume

$\text{m}^3 \cdot (\text{GeV}/c)^2$

Emittance

$\text{π mrad}$

$Z [\text{m}]$
Emittance Measurement in MICE

- Reconstruct position and momentum coordinates using trackers
- Construct covariance matrix
  \[ \Sigma = \begin{pmatrix}
  \sigma_{xx} & \sigma_{px} & \sigma_{yx} & \sigma_{py} \\
  \sigma_{px} & \sigma_{pxx} & \sigma_{pyx} & \sigma_{pyx} \\
  \sigma_{yx} & \sigma_{pyx} & \sigma_{yy} & \sigma_{pyy} \\
  \sigma_{py} & \sigma_{pyx} & \sigma_{pyy} & \sigma_{pyy}
\end{pmatrix}. \]

  \( \sigma_{xp} = \langle xp_x \rangle - \langle x \rangle \langle p_x \rangle. \)
- Compute transverse normalized RMS emittance
  \[ \varepsilon_n = \frac{1}{m} \left| \Sigma \right|^{\frac{1}{4}} \]