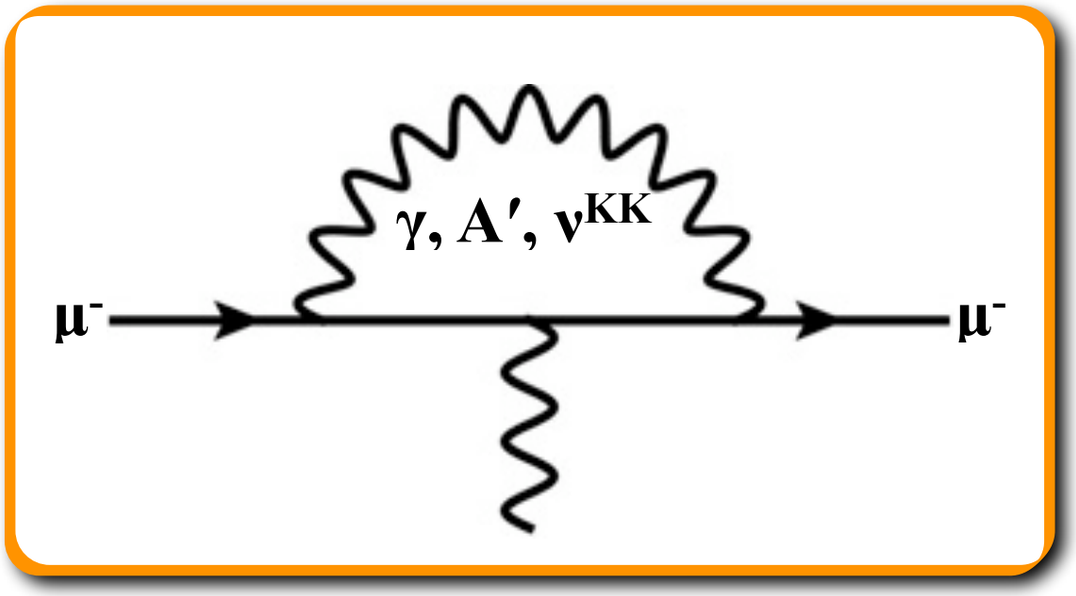
Hadronic contributions to muon g-2 from lattice QCD

Ruth Van de Water
Fermilab

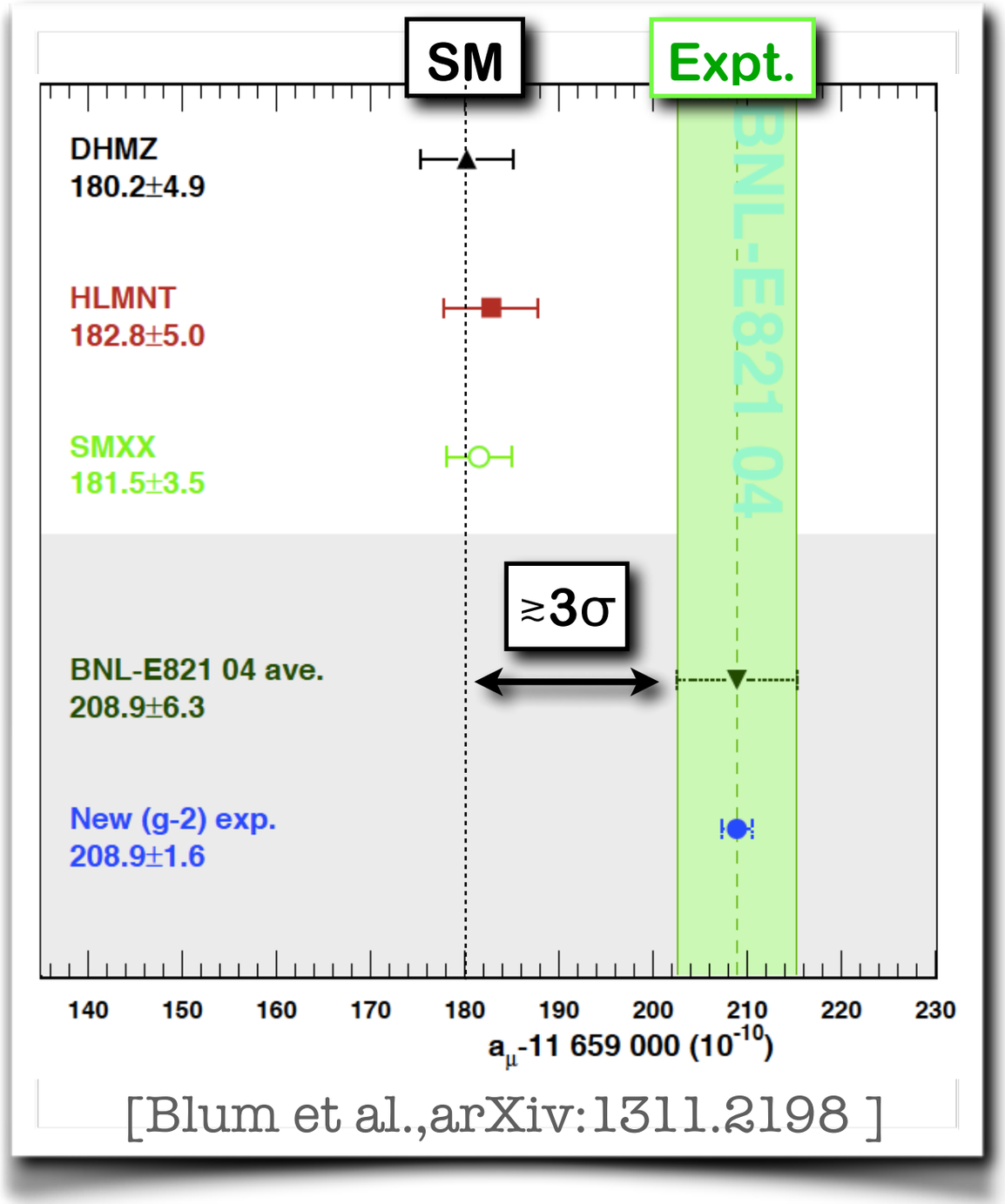
DPF 2017
August 2, 2017

Where is the new physics?

- Muon anomalous magnetic moment (g-2) sensitive to quantum-mechanical effects of new heavy or weakly-coupled particles (e.g SUSY, extra dimensions, dark photons, …)



- Measured to very high precision (0.54 ppm!)
  ➜ **provides some of strongest constraints on extensions of Standard Model**
- BNL measurement disagrees with Standard-Model theory expectations by >3σ

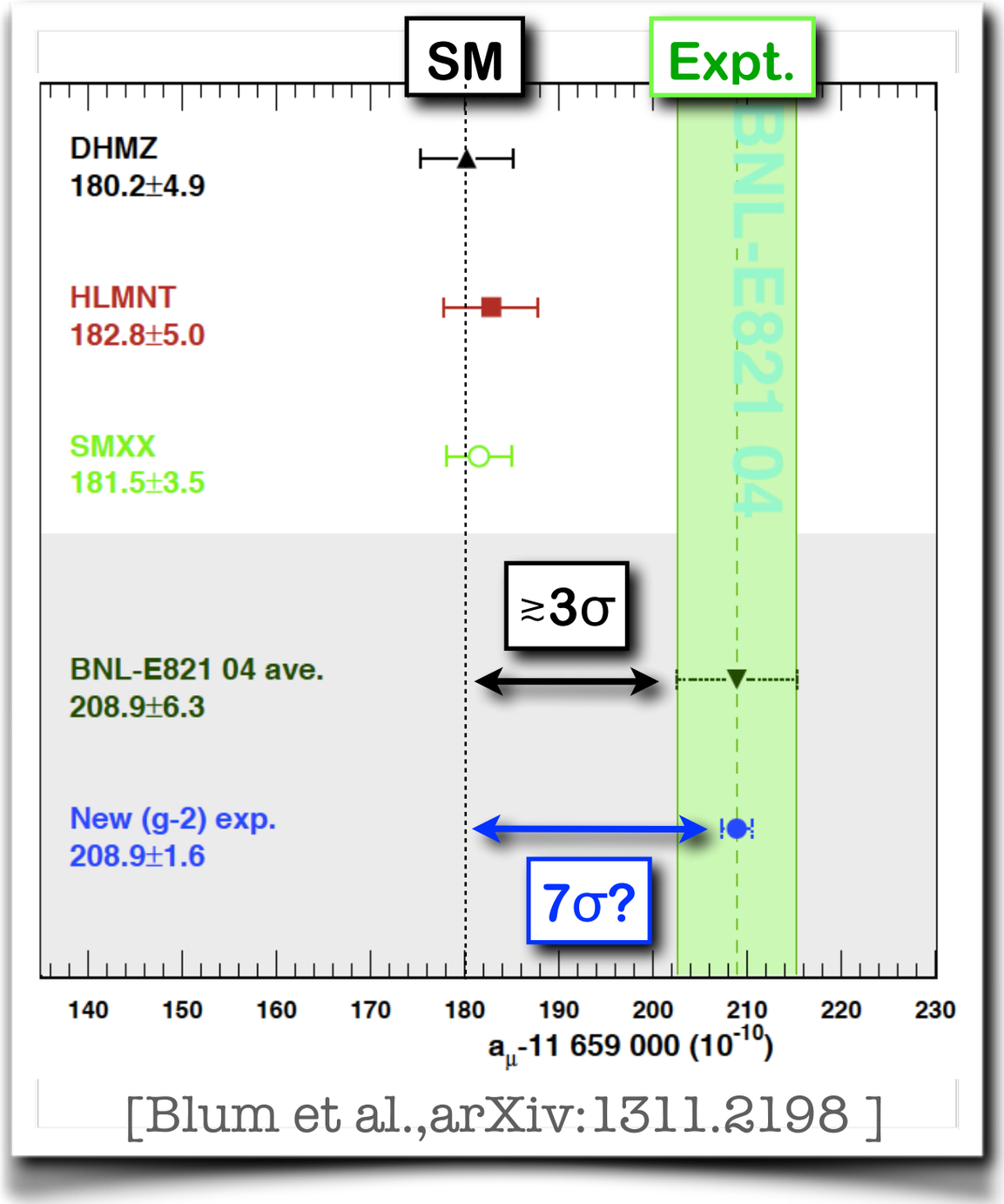


[Blum et al.,arXiv:1311.2198]

Motivation: the Muon g-2 Experiment

- Fermilab Muon g-2 Experiment aims to reduce BNL measurement error by four
  - *Started running in June, and expects first BNL-level results in 2018!*

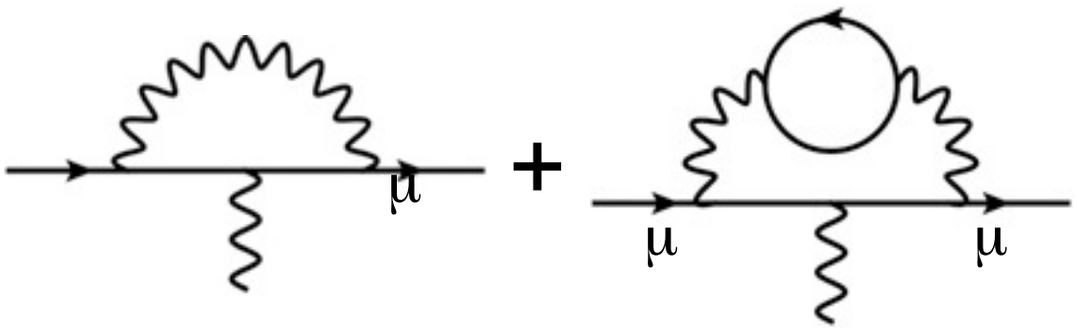
➡ **Must reduce theory error to commensurate level to identify definitively whether any deviation observed is due to new physics**



[Blum et al.,arXiv:1311.2198]

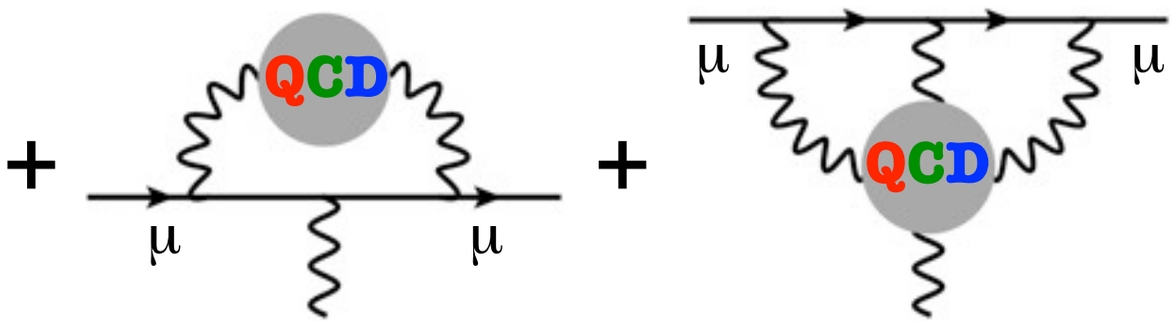
Muon g-2 in the Standard Model

$O(\alpha^5_{EM})$ $O(\alpha_{EM}G_F)$



QED (5 loops!) & EW (2 loops)

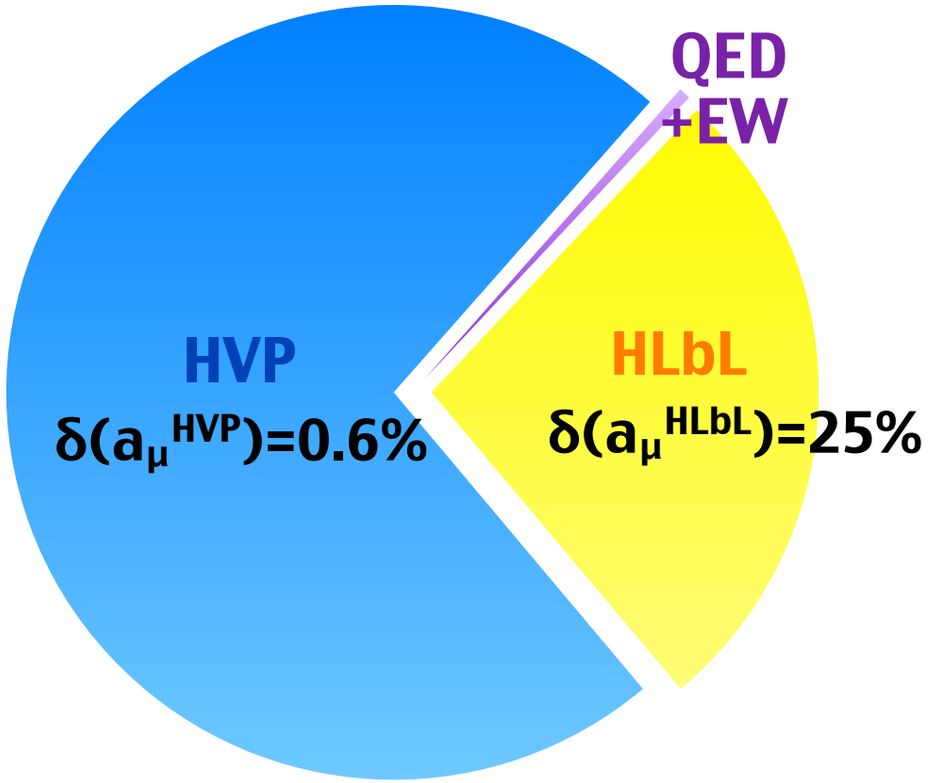
$O(\alpha^2_{EM})$ $O(\alpha^3_{EM})$



Hadronic vacuum polarization (HVP)

Hadronic "light-by-light" (HLbL)

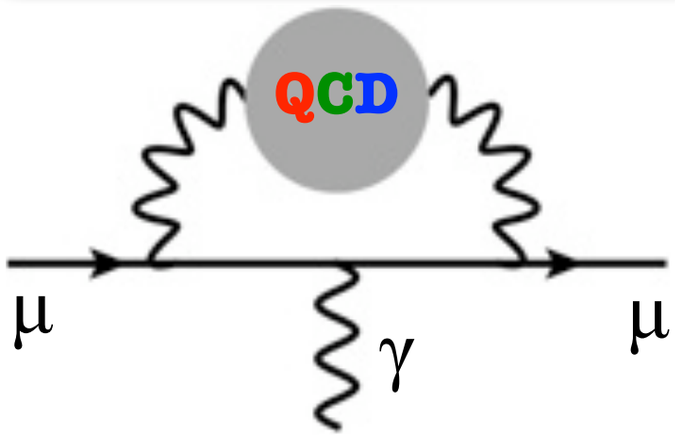
Error budget



- Dominant uncertainty from **nonperturbative hadronic contributions**

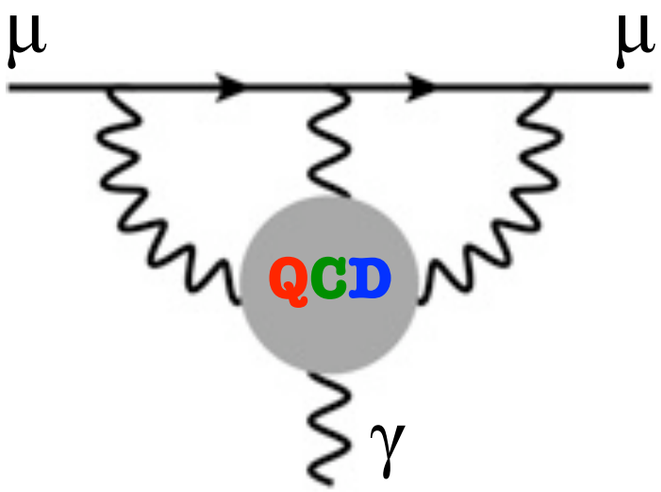
Hadronic contributions to muon g-2

Hadronic Vacuum Polarization (HVP)



- Obtained from experimental measurements for $e^+e^- \to$ hadrons plus dispersion relations
- challenging analysis combining more than 20 exclusive channels with several final-state hadrons [Davier et al., Keshavarzi et al., Jegerlehner]

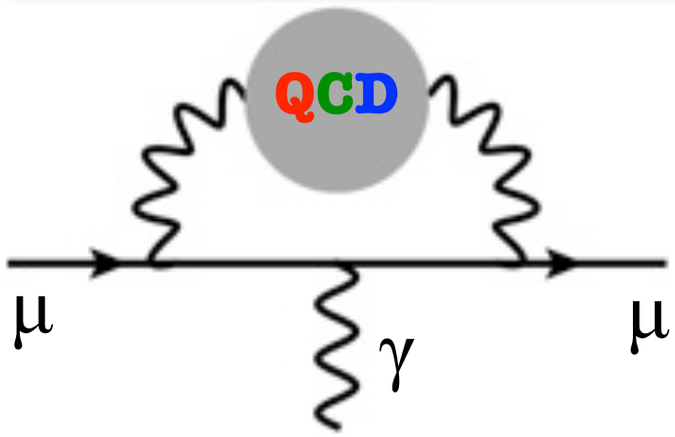
Hadronic Light-by-Light (HLbL)



- Estimated from QCD models such as large N_c, chiral perturbation theory, vector meson dominance, …
- Subjective and not systematically improvable [Glasgow consensus, Prades, de Rafael, Vainshtein, 0901.0306]

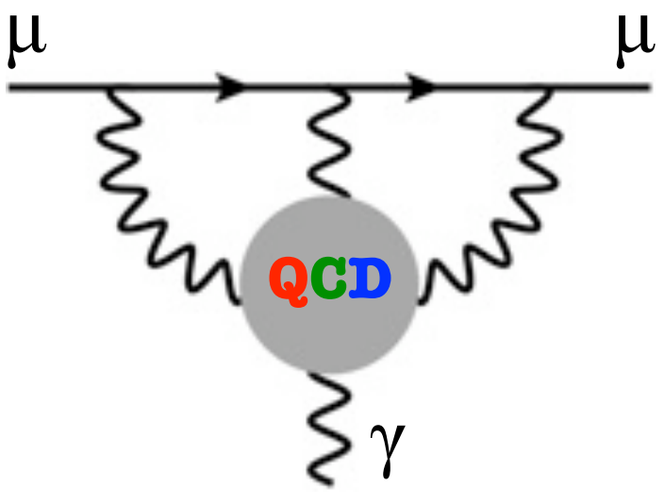
Hadronic contributions to muon g-2

Hadronic Vacuum Polarization (HVP)



- Obtained from experimental

Hadronic Light-by-Light (HLbL)



- Estimated from QCD models such as

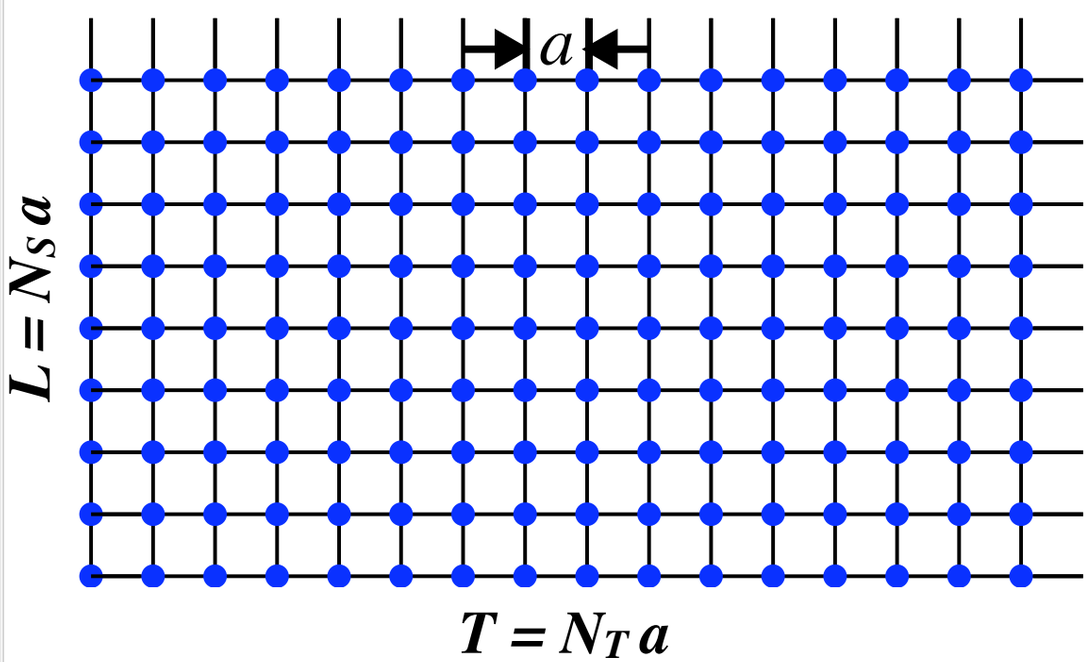
- To match anticipated experimental precision, must reduce **HVP uncertainty to below ~0.2%** and **HLbL uncertainty to below ~10%**
- Both contributions are calculable (in principle) from ab-initio Quantum chromodynamics with numerical lattice QCD!

Keshavarzi et al., Jegerlehner]

0901.0306]

Numerical lattice Quantum ChromoDynamics

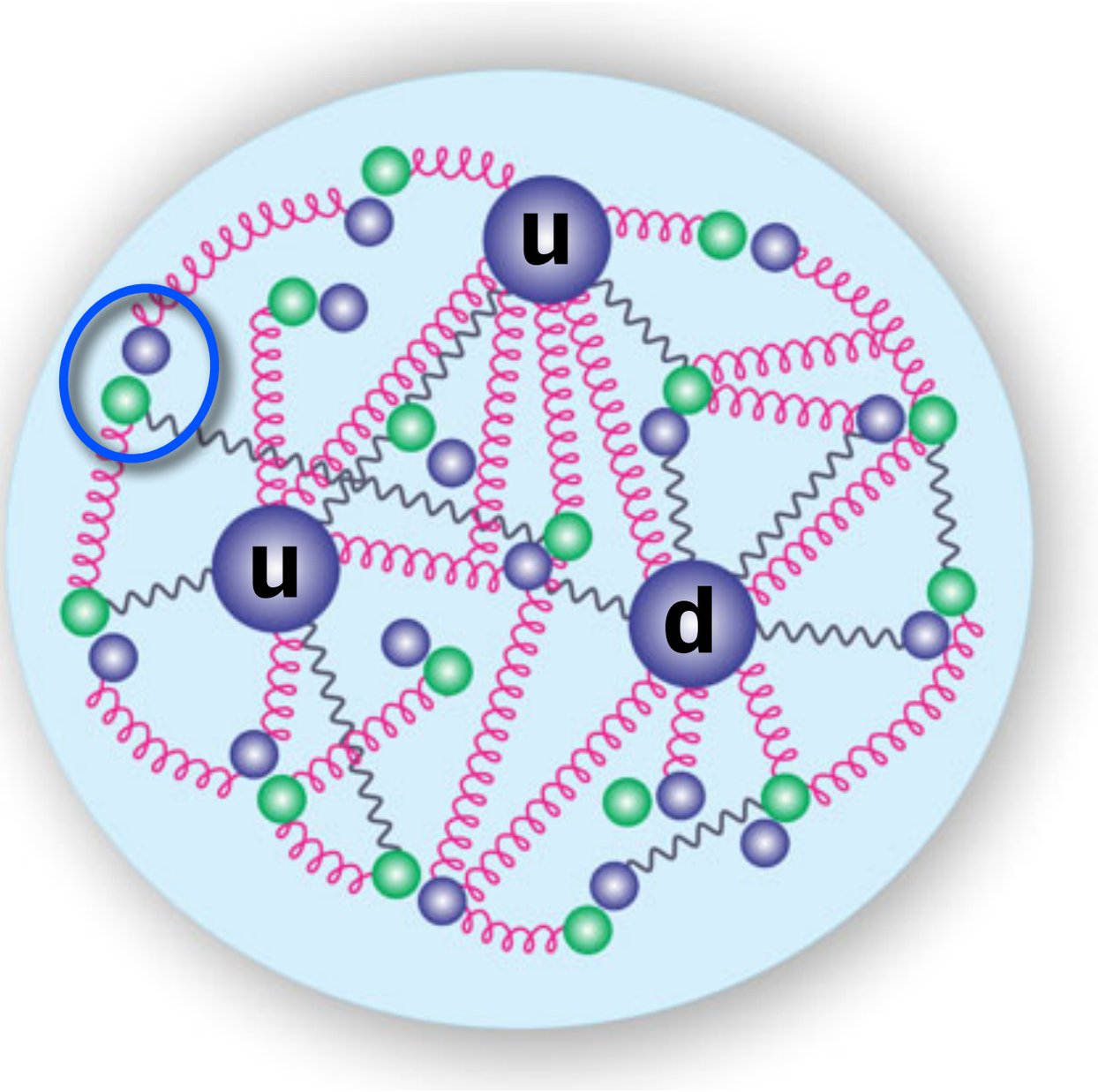
- Systematic method for calculating hadronic parameters from QCD first principles
- Define QCD on (Euclidean) spacetime lattice and solve path integral numerically
  - Recover QCD when lattice spacing $a \to 0$ and box size $L \to \infty$



- Simulate using **Monte-Carlo methods and importance sampling**
  - Sample from all possible field configurations using a distribution given by $\exp(-S_{QCD})$
- Run codes upon **supercomputers** and dedicated clusters

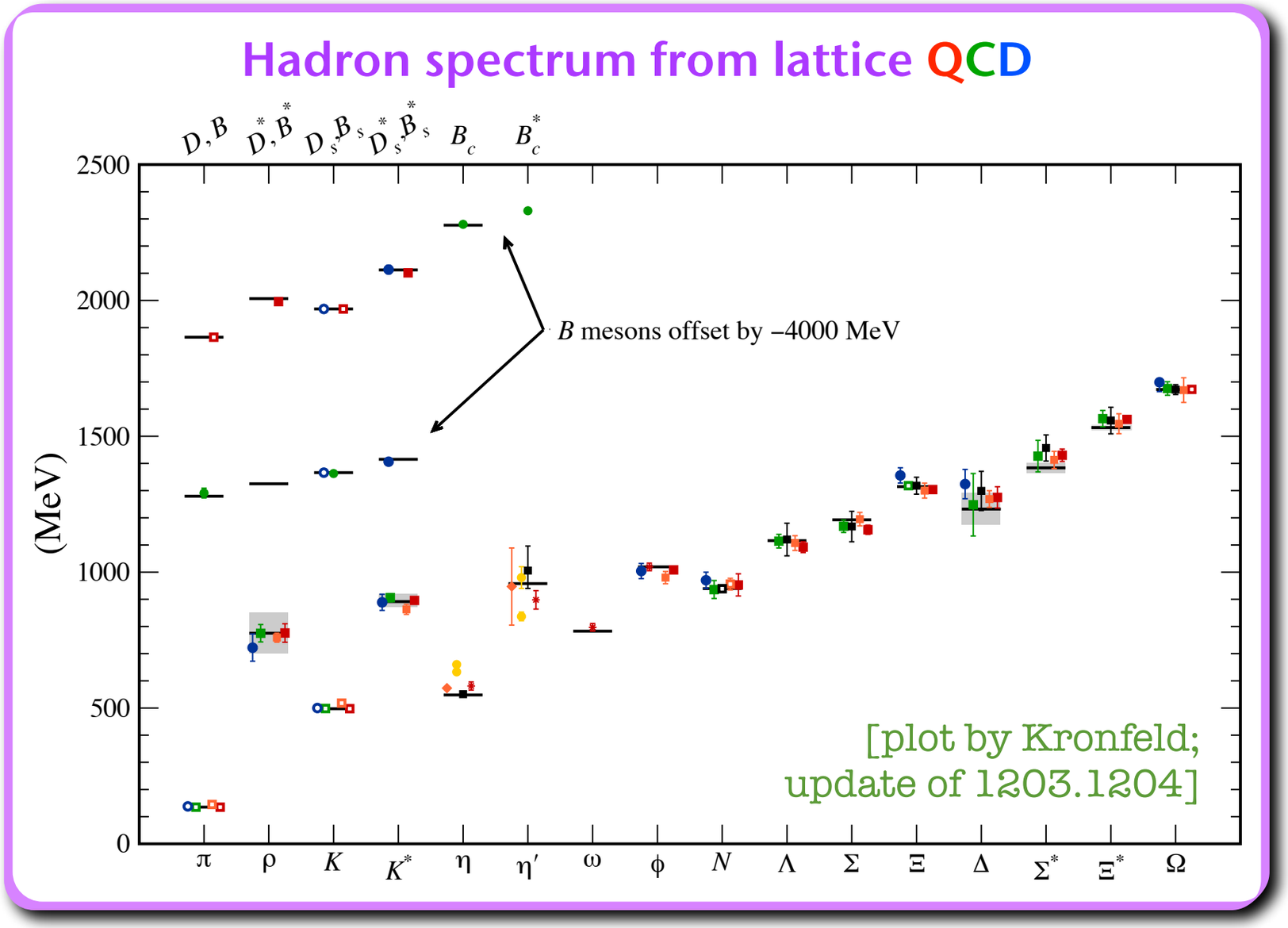
21st-century lattice QCD

- Modern lattice-QCD methods + latest generations of tera/petascale supercomputers have enabled ***calculations of many simple quantities with controlled uncertainties and complete error budgets***
- Standard simulations include **dynamical u, d, & s quarks in the vacuum**
  - Typically isospin symmetric (m_u=m_d)
  - Many now include dynamical charm
- **Control systematic errors using QCD gauge-field ensembles with different parameters** (quark masses, lattice spacings, spatial volumes, …)
  - State-of-the-art calculations employ pions with physical mass ~135 MeV!



Lattice-QCD *validation*

- **Lattice-QCD results agree with experimental measurements for wide variety of hadron properties** including light- and heavy-hadron spectrum & proton-neutron mass difference
- **Lattice-QCD predictions** of B_c mass, $D_{(s)}$ decay constants, & $D \to K l \nu$ form factor appeared **before** measurements with comparable precision
- **Independent lattice-QCD calculations using different actions** & methods provide confirmation for matrix elements inaccessible by experiment
- *Demonstrate that calculations are reliable with controlled errors!*



Results and progress on hadronic vacuum polarization from

Fermilab Lattice, HPQCD, & MILC Collaborations

Participants

- We are using *ab-initio* lattice-QCD to target the hadronic vacuum-polarization contribution, which is the largest source of theory error

Fermilab Lattice Collaboration

- Aida El Khadra (Illinois)
- Andreas Kronfeld (Fermilab)
- Ethan Neil (Colorado)
- Ruth Van de Water (Fermilab)

HPQCD

- Bipasha Chakraborty (JLAB)
- Daniel Hatton (Glasgow)
- Christine Davies (Glasgow)
- Jonna Koponen (INFN, Rome)
- Peter Lepage (Cornell)
- Andrew Lytle (Glasgow)
- Craig McNeile (Plymouth)

MILC Collaboration

- Carleton DeTar (Utah)
- Steve Gottlieb (Indiana)
- Jack Laiho (Syracuse)
- Yuzhi Liu (Indiana)
- Doug Toussaint (Arizona)
- Alejandro Vaquero (Utah)

HVP contribution from lattice QCD

[Blum, PRL91 (2003) 052001]

- Lowest-order hadronic-vacuum polarization contribution obtained from 1-loop QED integral:

$$a_\mu^{\mathrm{HVP(LO)}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2)\left[\Pi(Q^2) - \Pi(0)\right]$$

$$i\Pi_{\mu\nu}(q^2) =$$

  - Kinematic function $f(Q^2)$ accounts for interaction between muon and external photon
  - **Vacuum polarization function $\Pi(Q^2)$ captures nonperturbative QCD effects**
  - $\Pi(Q^2)$ related to inclusive $e^+e^- \to$ hadrons cross section via dispersion relation

☆ **Can also compute Euclidean space $\Pi(Q^2)$ directly in lattice QCD from simple two-point correlation function of electromagnetic quark current**

  - In principle can yield more precise $\Pi(Q^2)$ unlimited by experimental uncertainties

$\Pi(Q^2)$ from moments + Padé approximants

[HPQCD, PRD89, no. 11, 114501 (2014)]

- New method introduced by HPQCD enables precise determination of vacuum polarization function $\Pi(Q^2)$ from zero-momentum vector-current correlation functions:

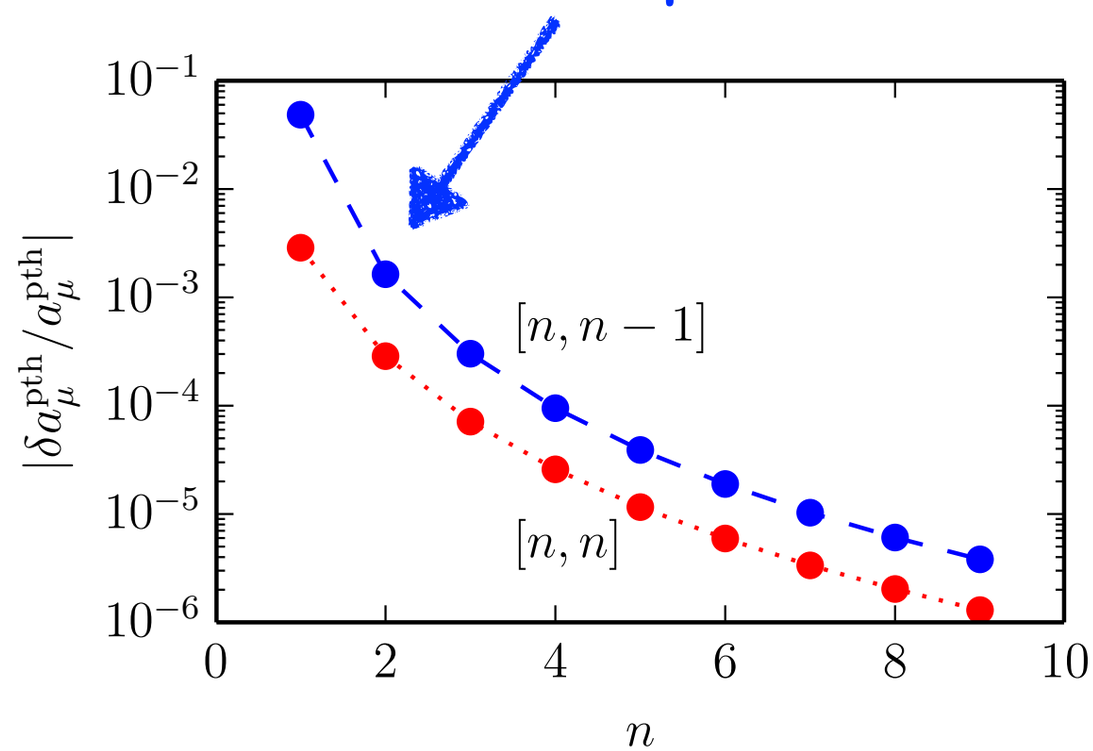
(1) Calculate derivatives of vacuum polarization function $\Pi(Q^2)$ at $Q^2=0$ from time moments of vector-current correlators

$$\hat{\Pi}(Q^2) = \sum_{j=1}^{\infty} Q^{2j}\Pi_j$$

$$\Pi_j = \frac{(-1)^{j+1}}{(2j+2)!}\sum_t t^{2j+2}\langle j^\mu_{\rm EM}(t) j^\mu_{\rm EM}(0)\rangle$$

(2) Use Taylor coefficients to construct [n,n] and [n,n-1] Padé approximants for $\Pi(Q^2)$

[2, 2] approximant sufficient to obtain ~0.5% precision



- Demonstrate method with s-,c-, and b-quark connected contributions [PRD 89, no. 11, 114501 (2014); PRD 91, no. 7, 074514 (2015)]

2016 result for u/d-connected contribution

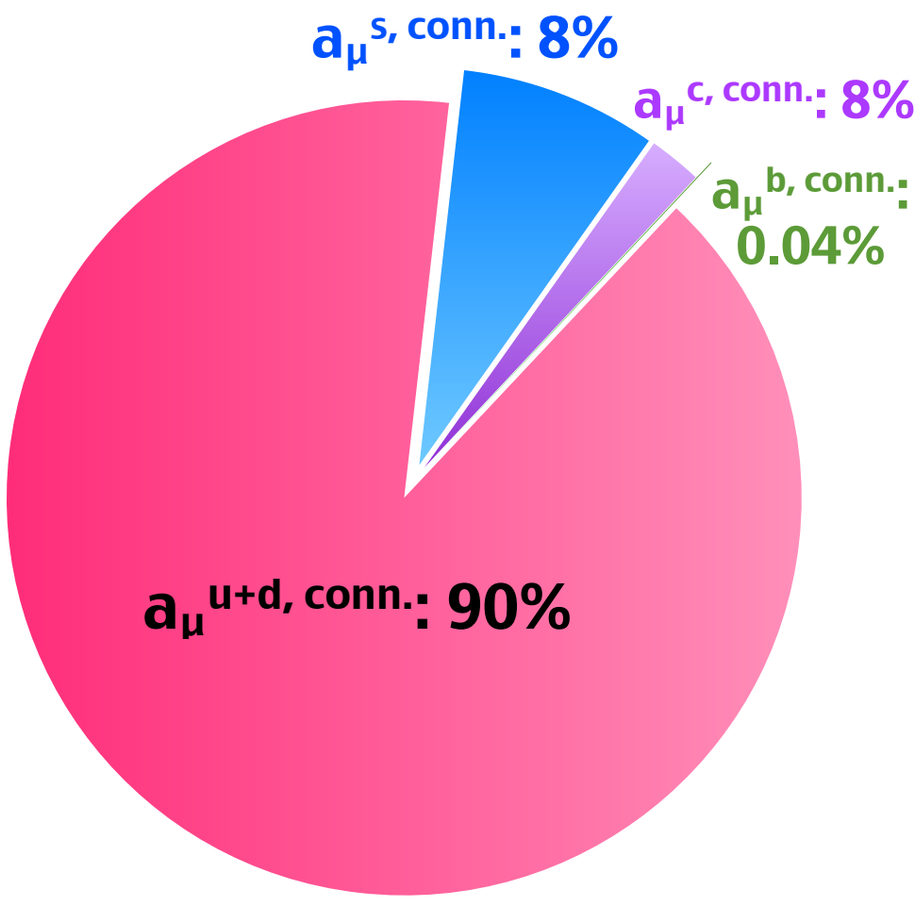
- Employ large set of four-flavor QCD gauge-field ensembles with dynamical u,d,s, & c quarks [MILC Collaboration, PRD82 (2010) 074501]
  - *Three lattice spacings, multiple spatial volumes, physical light-quark masses to control systematic uncertainties*
  - Largest errors from omission of isospin breaking and electromagnetism
  - Errors from the nonzero lattice spacing and finite spatial volume are next

	$a_\mu^{\mathrm{HVP,LO}}(u/d)$
QED corrections:	1.0%
Isospin breaking corrections:	1.0%
Staggered pions, finite volume:	0.7%
Valence m_ℓ extrapolation:	0.4%
Monte Carlo statistics:	0.4%
Padé approximants:	0.4%
$a^2 \to 0$ extrapolation:	0.3%
Z_V uncertainty:	0.4%
Correlator fits:	0.2%
Tuning sea-quark masses:	0.2%
Lattice spacing uncertainty:	$< 0.05\%$
Total:	1.8%

[HPQCD + RV, 1601.03071; to be published in PRD]

Total LO HVP contribution: *2016*

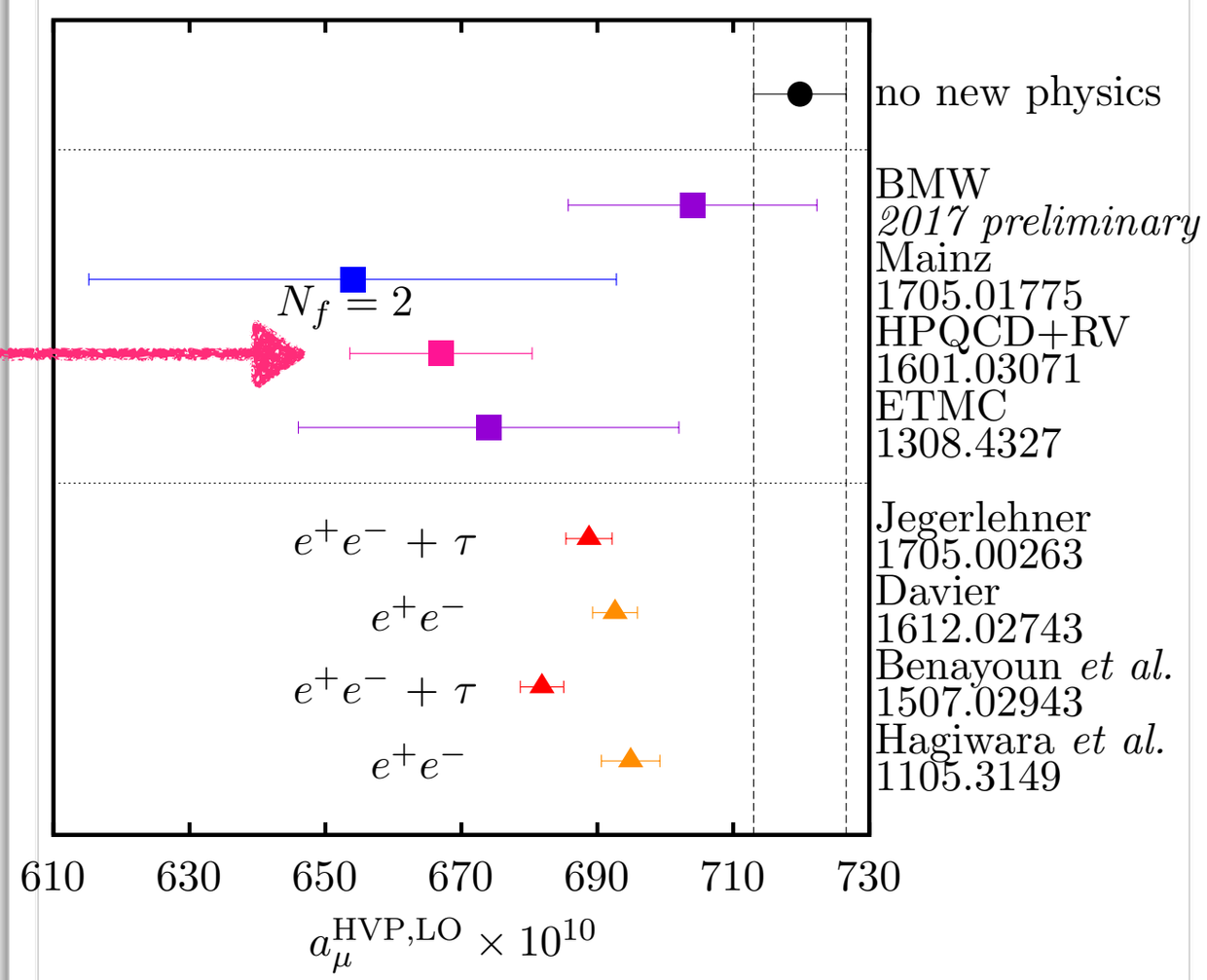
Contributions by quark flavor



$$10^{10} a_\mu^{\mathrm{HVP,LO}} = 667(11)_{u,d}(1)_{s,c,b}(9)_{\mathrm{disc.}}$$

[HPQCD + RV, 1601.03071;
to be published in PRD]

First complete lattice-QCD calculation of leading-order a_μ^{HVP} to reach precision needed to observe significant deviation from experiment



Fermilab/HPQCD/MILC *work in progress*

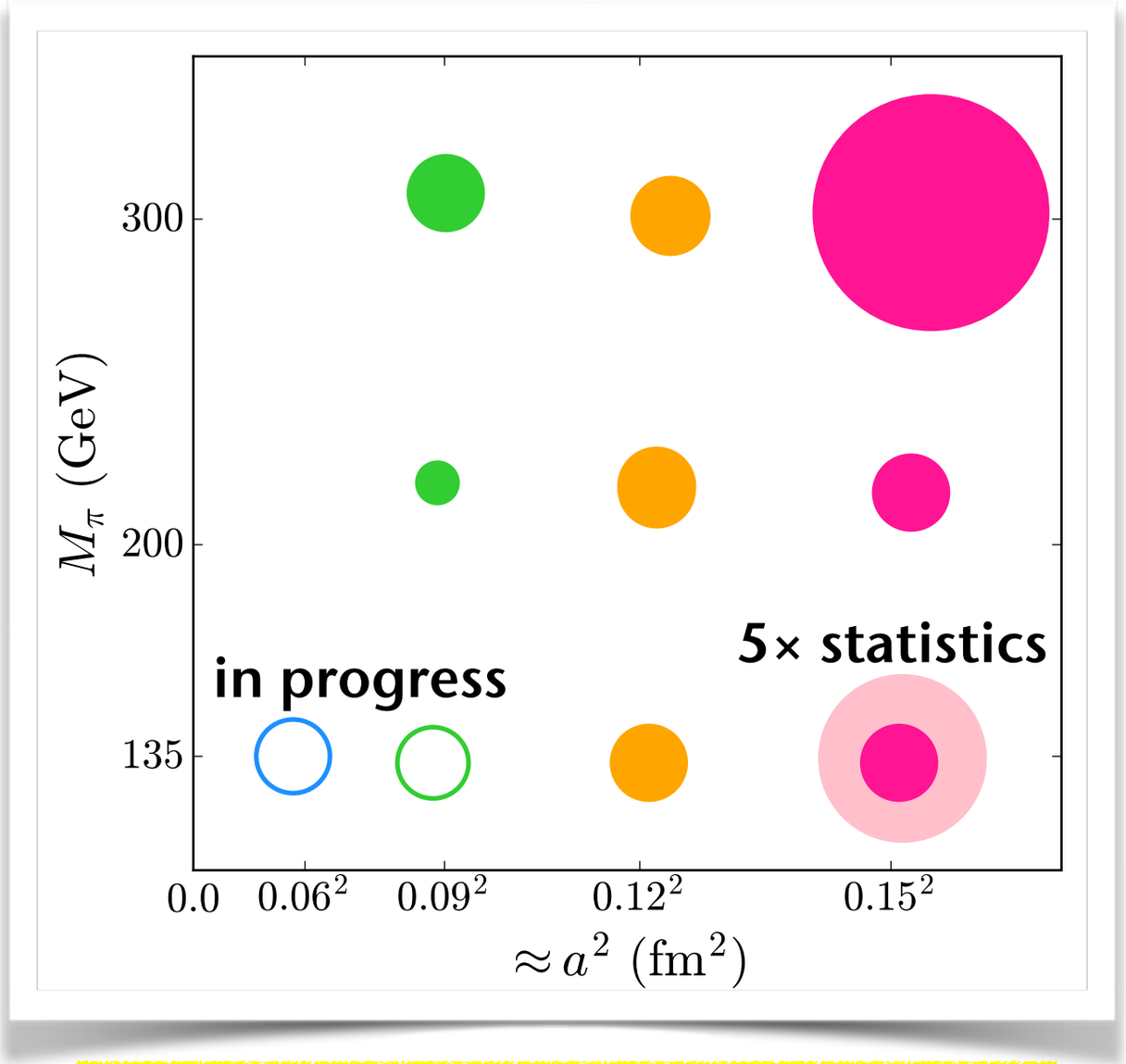
- Fermilab Lattice, HPQCD, & MILC Collaborations joined efforts for follow-up calculation to reduce error and meet target experimental precision
- Addressing leading sources of error from:

  (1) Omission of quarks' EM charges ➜ developing code to generate new ensemble with dynamical photons

  (2) u/d-quark mass difference ➜ generated new 1+1+1+1 ensemble *(see next slide)*

  (3) Quark-disconnected contributions ➜ implemented modern variance-reduction algorithms in MILC code & testing parameters

  (4) Finite spatial volume & nonzero lattice spacing ➜ analyzing ensembles with finer lattice spacings



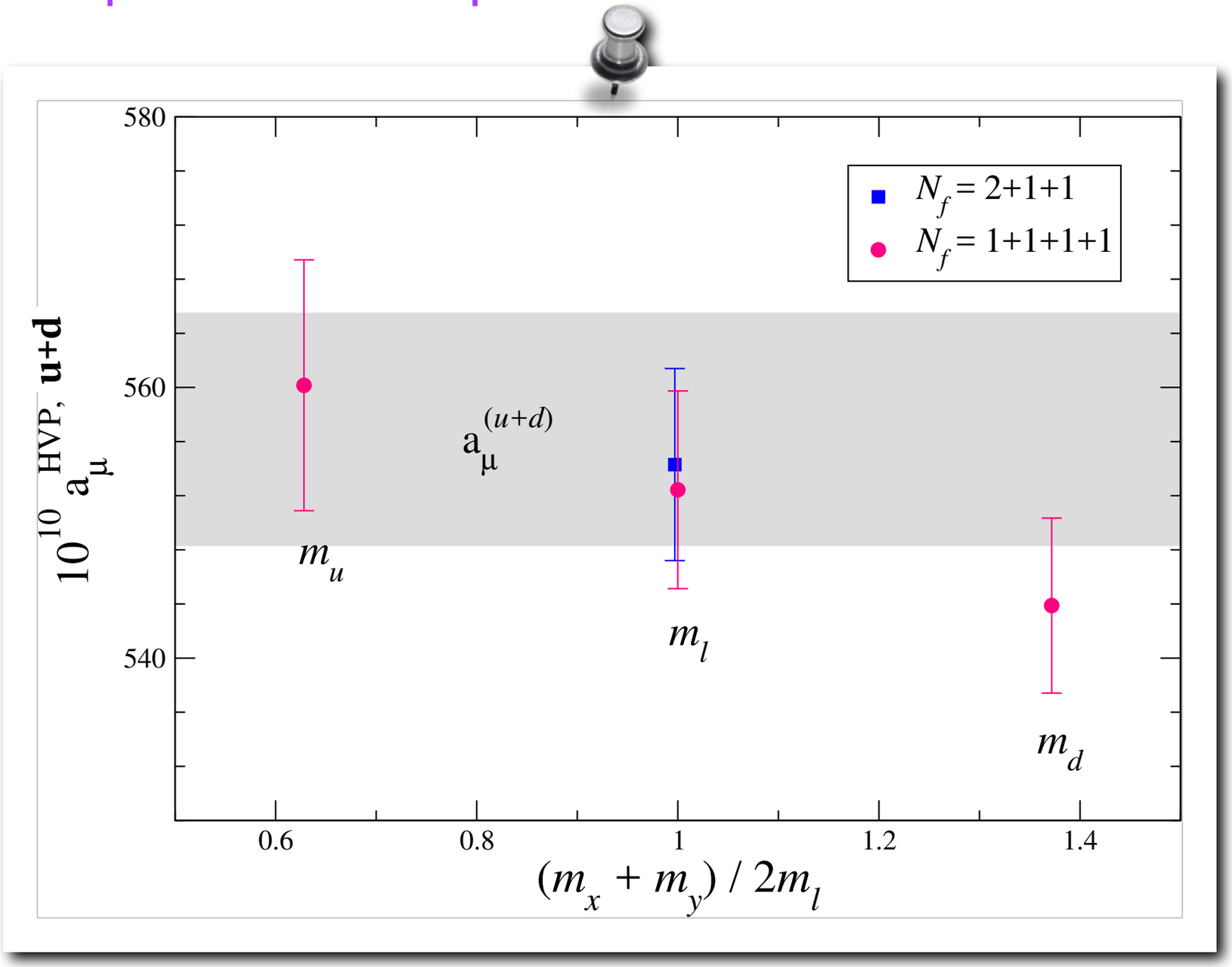
- Aim to reach ~1% uncertainty before first result from Muon g-2 Experiment next year

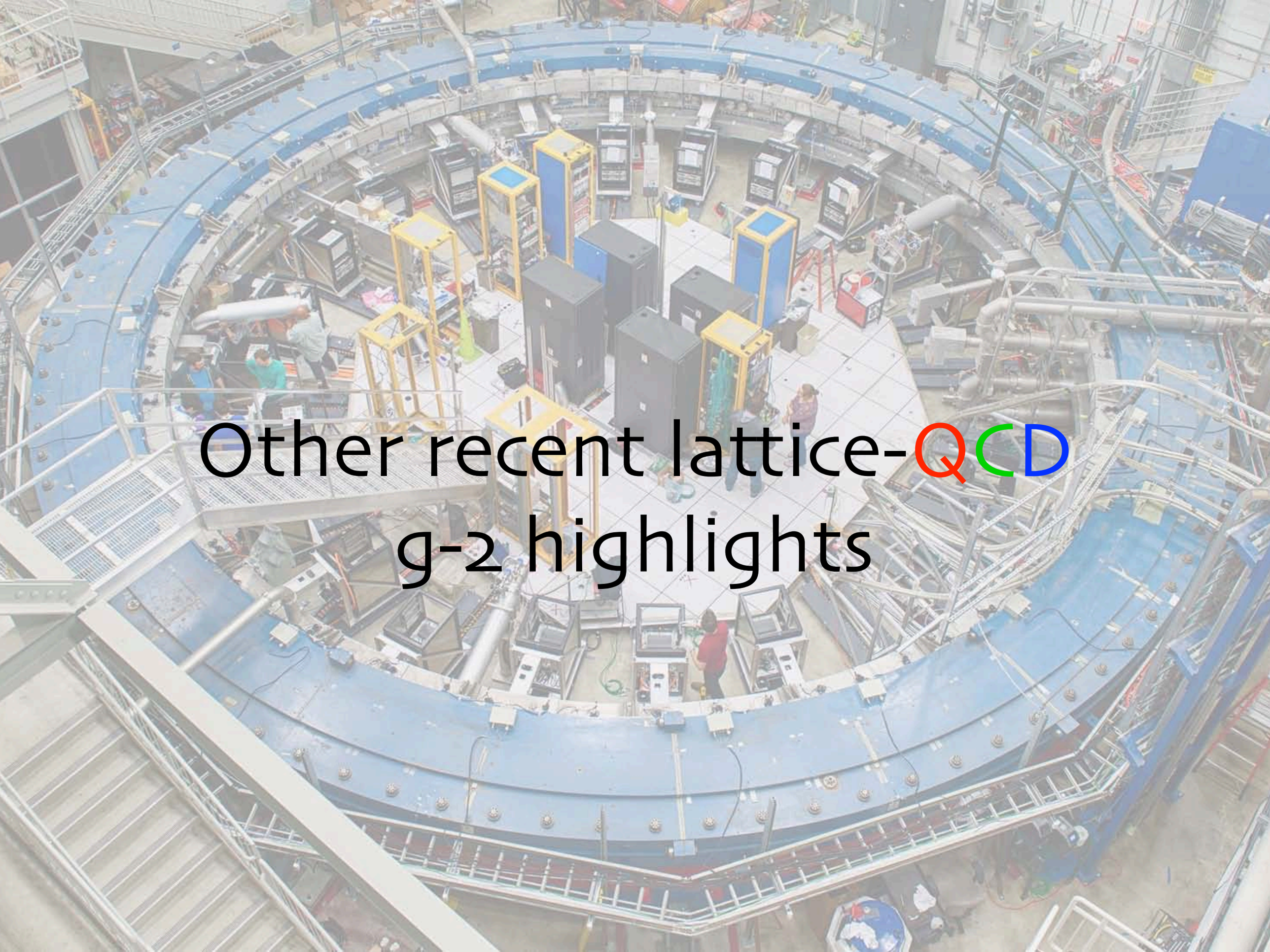
New! Isospin-breaking correction

- First result from joint effort is direct calculation of $m_u \neq m_d$ correction using new ensemble with non-degenerate up and down sea quarks

- Fix average $(m_u+m_d)/2$ = 27.35(11) and ratio m_u/m_d = 0.4582 to reproduce physical M_{π^0} and electromagnetic splitting $(M_{\pi^+} - M_{\pi^0})$ [MILC, PRD90 (2014) no.7, 074509 ;1606.01228]

- First lattice-QCD calculation of the strong-isospin-breaking contribution to $a_\mu^{HVP,LO}$ at the physical pion mass





Other recent lattice-QCD g-2 highlights

Progress on hadronic light-by-light

- Most recent lattice calculations ***combine nonperturbative QCD*** gauge-field configurations ***with perturbative QED*** for the photons to directly obtain desired $O(\alpha_{EM}^3)$ contribution

- New method from **RBC Collaboration** employs stochastically-generated photon propagators to control statistical noise [Blum et al., PRD93, 014503 (2016); PRL118 (2017) no.2, 022005]
  - Obtain **≲10% statistical errors at the physical pion mass** in ballpark of Glasgow consensus value $a_\mu^{HLbL,GC} \times 10^{10} = 10.5(2.6)$

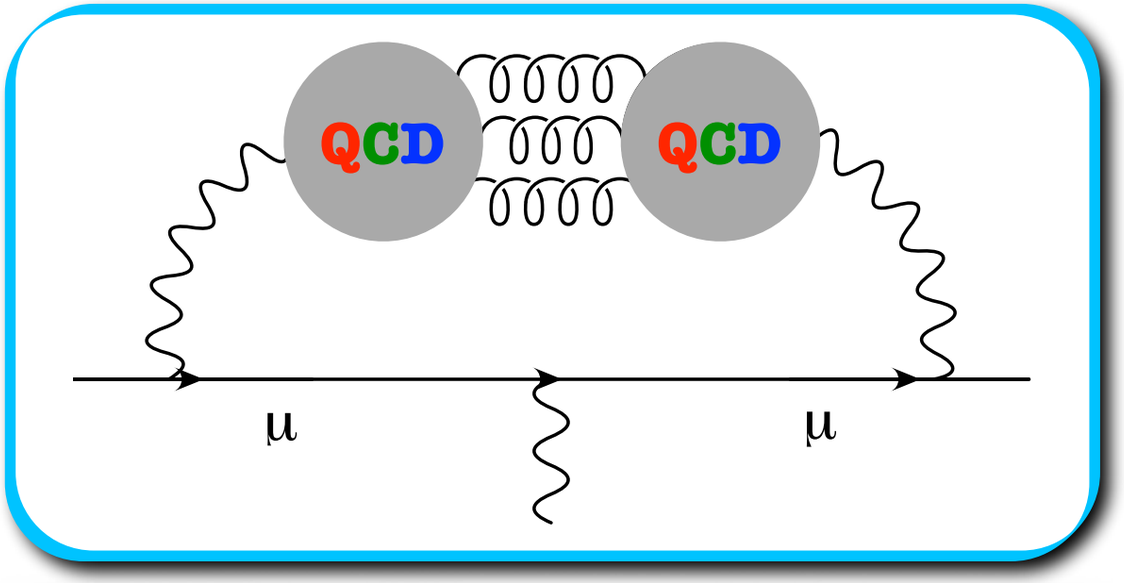
$$a_\mu^{\mathrm{HLbL}} \times 10^{10} = \begin{cases} 11.60(0.96)_{\mathrm{stat.}} & \text{connected} \\ -6.25(0.80)_{\mathrm{stat.}} & \text{disconnected} \end{cases}$$

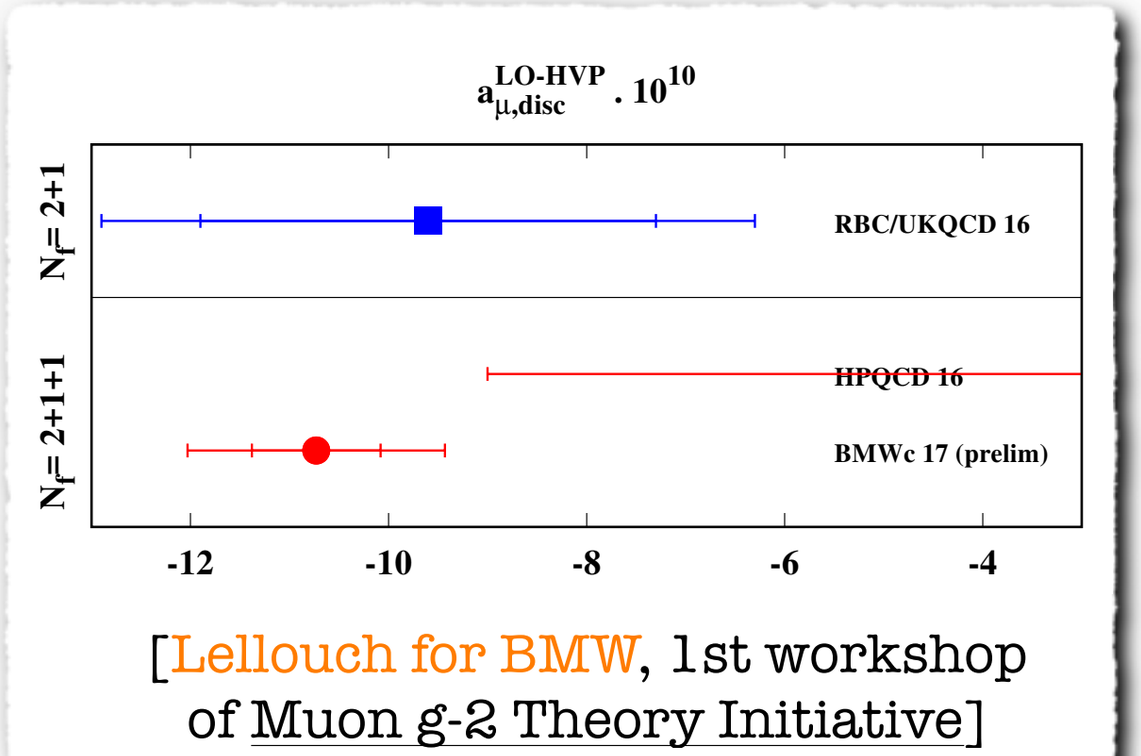
  - *Full study of systematic errors (especially finite-volume effects) needed, but initial results encouraging!*

- Other complementary approaches being studied by **Mainz group**:
  - Combination of lattice QCD for nonperturbative 4-point function + continuum, infinite-volume perturbation theory for QED kernel [Asmussen et al., 1609.08454]
  - Calculation of dominant pion-pole contribution using lattice QCD for $\pi^0 \to \gamma^* \gamma^*$ form factor + dispersion relation [Géradin et al., PRD94 (2016) no.7, 074507]

Quark-disconnected contributions to HVP

- Contributions in which valence quarks are connected by a pair of gluons only few percent of total HVP contribution due to $SU(3)_{flavor}$ and Zweig suppression
- Computationally challenging due to large statistical noise, but **now possible to obtain statistically-significant signal using modern variance-reduction strategies such as all-mode averaging** [Blum et al, PRD 88 (2013) 094503]
- First calculation of quark-disconnected contribution with physical-mass pions last year by RBC/UKQCD [PRL116, 232002 (2016)]
- Preliminary BMW Collaboration result with higher statistics has uncertainty $\delta a_\mu^{HVP,disc.} \sim 1.5\times10^{10}$ ➡ **contributes only 0.2% to error on total $a_\mu^{HVP,LO}$**

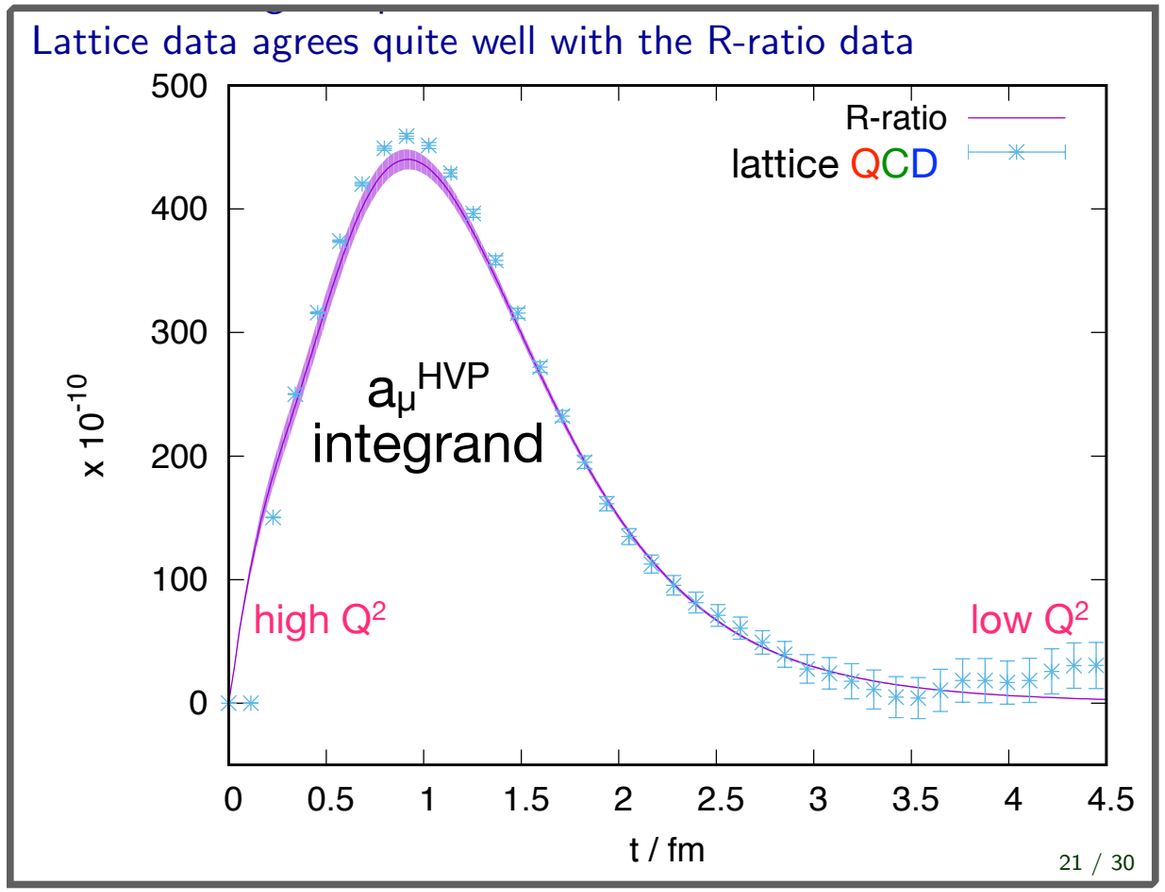


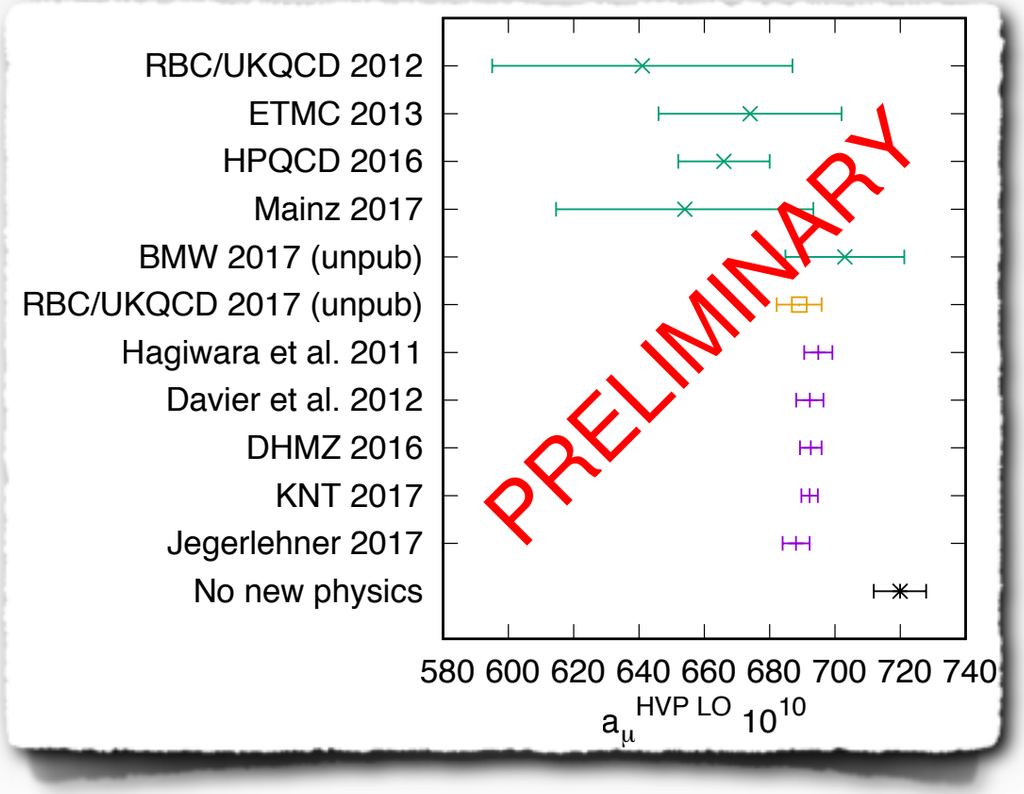


[Lellouch for BMW, 1st workshop of Muon g-2 Theory Initiative]

Hybrid lattice+experiment approach for HVP

- Spectral representation enables comparison of vacuum polarization function from lattice QCD with determination from experimental $e^+e^- \to$ hadrons ("R-ratio") data [Bernecker & Meyer, EPJA47 (2011) 148 , arXiv:1107.4388]
  - **Most precise in complementary Q^2 regions** ➡ combination yields smaller error than lattice QCD or experiment alone
  - ⭐ May provide more immediate path to target experimental precision until first-principles lattice-QCD-calculations reach this goal





[C. Lehner, Lattice 2017]

Conclusions

"Progress in science is based on the interplay between theory and experiment, between having an idea about nature and testing that idea in the laboratory. *Neither can move forward without the other.*" – **Snowmass 2013 Executive Summary**

Summary & outlook

- Lattice-QCD community is working hard to compute hadronic contributions to muon g-2 to the precision needed by the Muon g-2 (and planned J-PARC) Experiment
- **Methods in place for hadronic vacuum polarization contribution**
  - Several independent calculations ongoing using different approaches
  - First results reported for previously omitted effects of isospin breaking & electromagnetism and contributions from quark-disconnected diagrams
  - Improvements to most precise calculation by HPQCD+RV in progress by Fermilab, HPQCD, and MILC Collaborations
  - Hope to reach sub-per cent precision from lattice QCD in 1 or 2 years!
- **Methods being developed and tested for hadronic light-by-light contribution**
  - Study of systematic errors in promising RBC-Collaboration approach in progress
  - Still room for new theoretical developments to make substantial impact

Meeting experimental precision goals contingent on continued support for lattice-QCD hardware & software!

Muon g-2 Theory Initiative

- **Goal:** provide theoretical predictions for hadronic contributions to g-2 with reliable & precise uncertainties to maximize impact of Fermilab & J-PARC experiments
- **Plan:** organize series of workshops bringing together theorists from lattice & non-lattice communities to:
  - Discuss, assess, & compare efforts and approaches
  - Map out strategies for improving theoretical calculations
  - Write reports summarizing status of relevant theoretical work
- **Timeline:** finalize first report in September 2018 before Muon g-2 Experiment announces first BNL-level measurement

- Kickoff workshop hosted by Fermilab in June with 66 registered participants, 40 talks in 15 discussion sessions over 4 days

Extras...

HVP from $e^+e^- \to$ hadrons

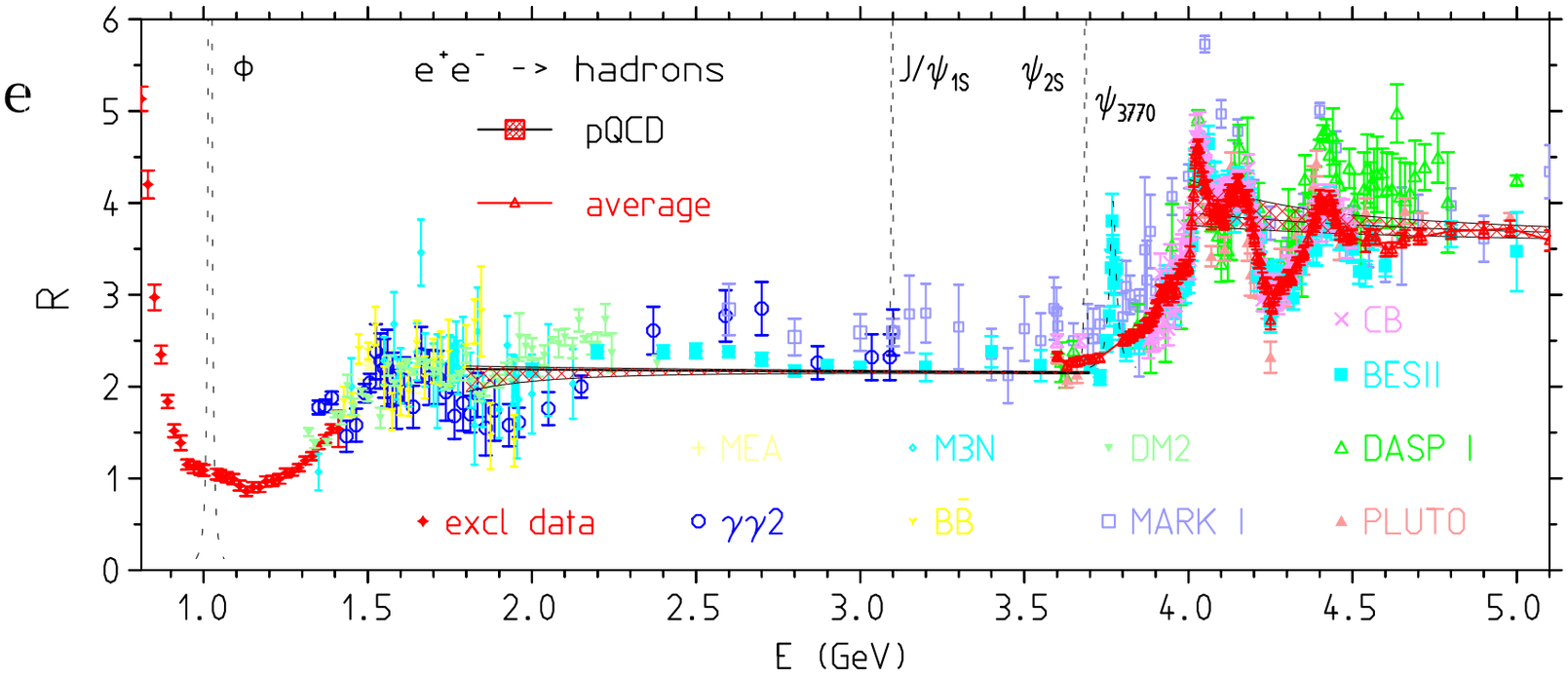
- Standard-Model value for a_μ^{HVP} obtained from experimental measurement of $\sigma_{total}(e^+e^- \to \text{hadrons})$ via optical theorem:

$$a_\mu^{\rm HVP} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} {\rm d}s \frac{R(s)K(s)}{s^2} \qquad R \equiv \frac{\sigma_{\rm total}(e^+e^- \to {\rm hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

  - (Away from quark thresholds, use four-loop pQCD)

- Includes >20 multi-particle channels with up to six final-state hadrons

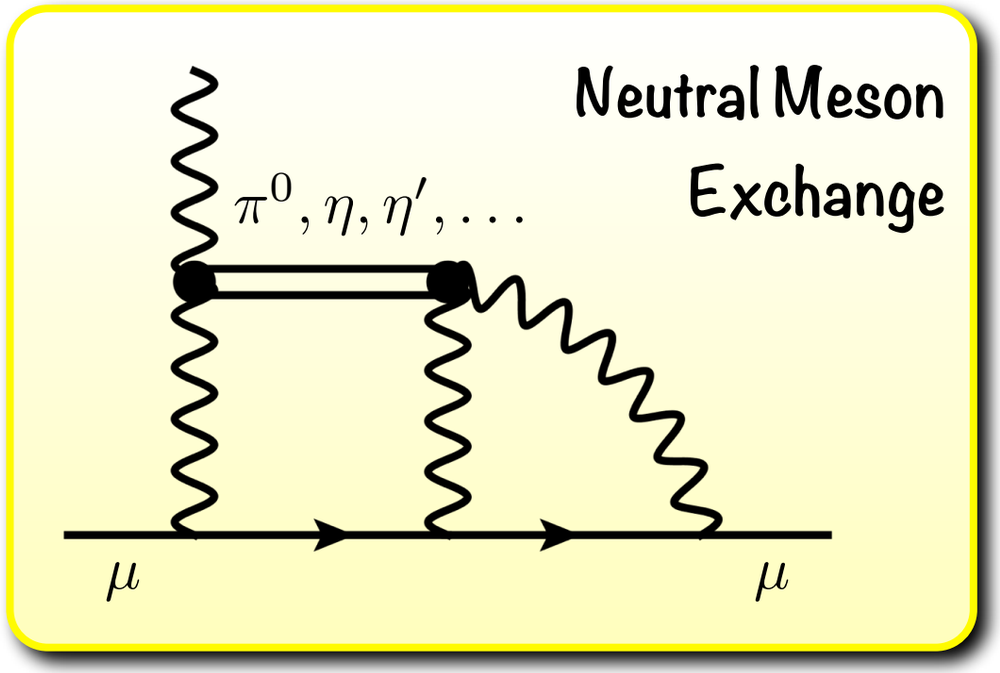
- Multi-hadron channels represent a small absolute contribution to a_μ^{HVP}, but contribute a significant fraction of the total uncertainty

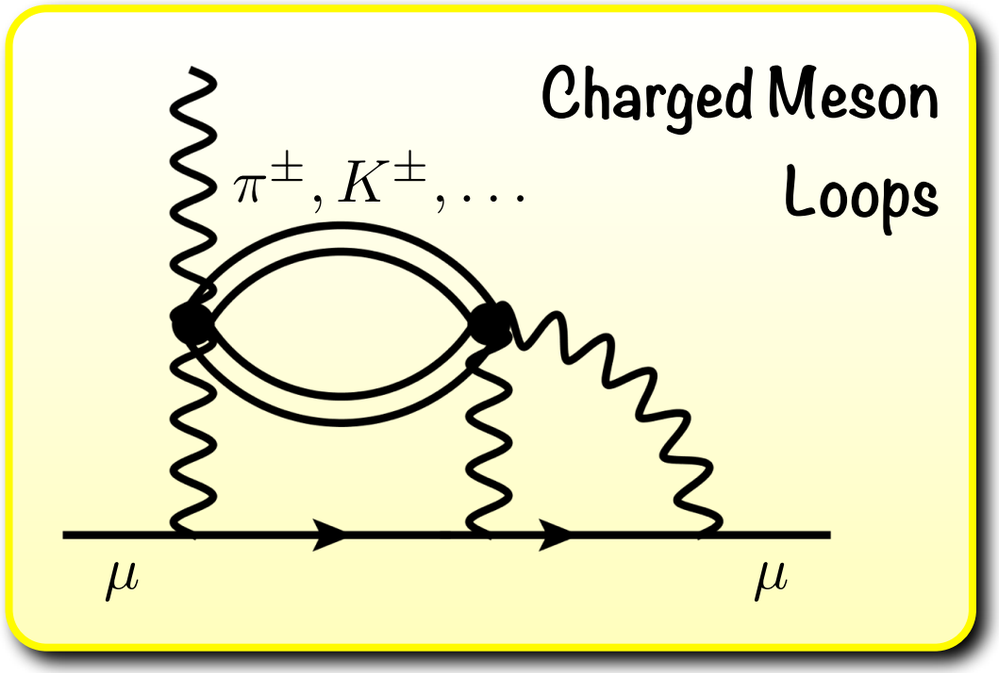


[Jegerlehner and Nyffeler, Phys.Rept. 477 (2009) 1-110]

HLbL from QCD models

- Hadronic light-by-light contribution cannot be expressed in terms of experimental quantities and must be obtained from theory
  [cf. Jegerlehner and Nyffeler, Phys.Rept. 477 (2009) 1-110 and Refs. therein]
  - All recent calculations compatible with constraints from large-N_c and chiral limits
  - All normalize dominant π^0-exchange contribution to measured $\pi^0 \to \gamma\gamma$ decay width
  - Differ for form factor shape due to different QCD-model assumptions such as vector-meson dominance, chiral perturbation theory, and the large N_c limit





The Glasgow consensus for HLbL

- Quoted error on for a_μ^{HLbL} from Prades, de Rafael, Vainshtein, 0901.0306 based range on model estimates
  - π^0-exchange contribution estimated to be ~10 times larger than others
  - Largest contribution to uncertainty ($\pm 1.9\times 10^{-10}$) attributed to charged pion and kaon loop contributions

Table 1: *Contribution to a^{HLbL} from π^0, η and η' exchanges*

Result	Reference
$(8.5 \pm 1.3) \times 10^{-10}$	[7, 8]
$(8.3 \pm 0.6) \times 10^{-10}$	[4, 5, 6]
$(8.3 \pm 1.2) \times 10^{-10}$	[1]
$(11.4 \pm 1.0) \times 10^{-10}$	[9]

$$a^{\mathrm{HLbL}}(\pi\,,\eta\,,\eta') = (11.4 \pm 1.3) \times 10^{-10}$$

- Subjective error estimate does not cover spread of model values, and and is not systematically improvable