Direct CP asymmetry in D→ππ and D→KK in QCD

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Based on arXiv:1706.07780 with A. Khodjamirian

Introduction

- Fundamental problem: CP-violation in the up-quark sector
  - can manifest itself in charm ΔC=1 transitions via

$$\Gamma(D \to f) \neq \Gamma(CP[D] \to CP[f])$$

  - … and observed via CP-violating asymmetry

$$a_{\rm CP}(f) = \frac{\Gamma(D \to f) - \Gamma(\overline{D} \to \overline{f})}{\Gamma(D \to f) + \Gamma(\overline{D} \to \overline{f})}$$

  - this asymmetry has been studied with $\pi^+\pi^-$ and K^+K^- final states

$$a_{CP}(K^-K^+) = (0.04 \pm 0.12\ (\text{stat}) \pm 0.10\ (\text{syst}))\%\,,$$
$$a_{CP}(\pi^-\pi^+) = (0.07 \pm 0.14\ (\text{stat}) \pm 0.11\ (\text{syst}))\%\,.$$

LHCb 2017

- Note that initial state is a D^0 meson: possible CPV in mixing?

Introduction

- Differences of CP-violating asymmetries with D^0 mesons
  - each CPV asymmetry contains three components

$$a_{CP}(f) = \underset{\text{direct}}{a_f^d} + \underset{\text{mixing}}{a_f^m} + \underset{\text{interference}}{a_f^i}$$

  - since for CP-eigenstate final states (such as $\pi^+\pi^-$ and K^+K^-) $a^m_{KK}=a^m_{\pi\pi}$ and $a^i_{KK}=a^i_{\pi\pi}$, mixing asymmetries would (ideally) cancel

$$\Delta a_{\rm CP}^{\rm dir} = a_{\rm CP}^{\rm dir}(f_1) - a_{\rm CP}^{\rm dir}(f_2)$$

SU(3) is badly broken in D-decays e.g. Br(D → KK) ~ 3 Br(D →ππ)

  - recent measurements indicate that

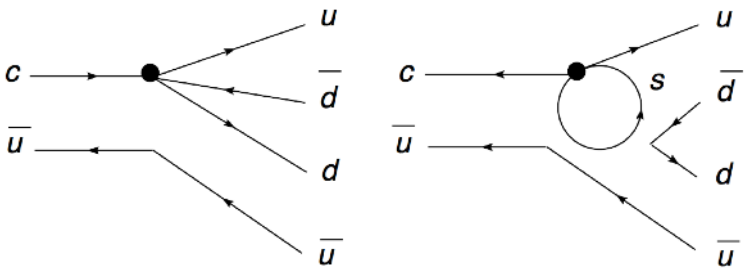
$$\Delta a_{CP}^{dir} = (-0.12 \pm 0.13)\%\,,$$ **HFAG average**

$$\Delta a_{CP}^{dir} = (-0.10 \pm 0.08 \pm 0.03)\%$$ **LHCb 2016**

- Naively, $\Delta a_{CP} = a^d_{KK} - a^d_{\pi\pi} \approx 2a^d_{KK}$. How can it be interpreted?

Introduction: theoretical interpretations

- Naively, there are contributions from trees and penguins
  - … which we need to calculate/fit to predict CPV



$$A_{KK} = \frac{G_F}{\sqrt{2}}\lambda\left[(T+E+P_{sd}) + a\lambda^4 e^{-i\gamma}P_{bd}\right]$$

$$A_{\pi\pi} = \frac{G_F}{\sqrt{2}}\lambda\left[(-(T+E)+P_{sd}) + a\lambda^4 e^{-i\gamma}P_{bd}\right]$$

$$\lambda = \sin\theta_C$$

- Need to estimate size of penguin/penguin contractions vs. tree
  - is there an unknown penguin enhancement (similar to ΔI = 1/2)?
    - SU(3) analysis: how well do we know ME in broken SU(3)?

      Golden & Grinstein PLB 222 (1989) 501; Pirtshalava & Uttayarat 1112.5451

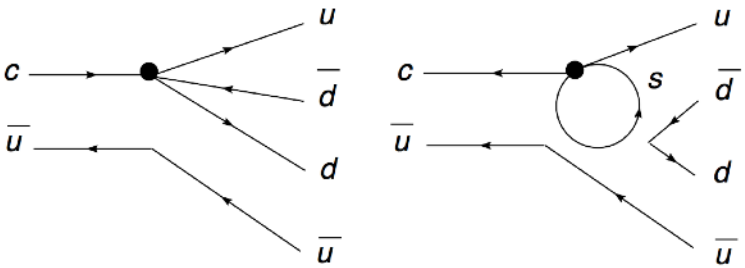
    - QCD factorization-like: how well do we know $1/m_c$ corrections?

      Isidori et al PLB 711 (2012) 46; Brod et al 1111.5000

    - flavor-flow diagram analyses **Broad et al 1203.6659; Bhattacharya et al PRD 85 (2012) 054014; Cheng & Chiang 1205.0580**

Is this a penguin or a tree?

without QCD



with QCD

Calculating CP-asymmetries in QCD

- Effective Hamiltonian for singly Cabibbo-suppressed (SCS) decays
  - drop all "penguin" operators (Q_i for $i \geq 3$) as C_i are small, $\lambda_q = V_{uq} V_{cq}^*$,

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=d,s} \lambda_q \left(C_1 \mathcal{Q}_1^q + C_2 \mathcal{Q}_2^q \right) - \lambda_b \cancel{\sum_{i=3,\ldots,6,8g} C_i \mathcal{Q}_i} \right]$$

  - recall that $\sum_{q=d,s,b} \lambda_q = 0$ or $\lambda_d = -(\lambda_s + \lambda_b)$ and

$$\mathcal{Q}_1^q = (\bar{u}\Gamma_\mu q)\,(\bar{q}\Gamma^\mu c)\,, \qquad \mathcal{Q}_2^q = (\bar{q}\Gamma_\mu q)\,(\bar{u}\Gamma^\mu c)$$

  - define a short-hand notation

$$\mathcal{O}^q \equiv \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i \mathcal{Q}_i^q\,, \qquad \text{with} \quad q = d, s.$$

  - … and matrix elements, e.g.

$$A(D^0 \to \pi^-\pi^+) \;=\; \lambda_d \langle \pi^-\pi^+ | \mathcal{O}^d | D^0 \rangle + \lambda_s \langle \pi^-\pi^+ | \mathcal{O}^s | D^0 \rangle$$

Amplitude decomposition

- Recipe for calculation of CPV asymmetry
  - prepare decay amplitudes

$$A(D^0 \to \pi^-\pi^+) = \lambda_d \langle \pi^-\pi^+|\mathcal{O}^d|D^0\rangle + \lambda_s \langle \pi^-\pi^+|\mathcal{O}^s|D^0\rangle$$
$$A(D^0 \to K^-K^+) = \lambda_s \langle K^-K^+|\mathcal{O}^s|D^0\rangle + \lambda_d \langle K^-K^+|\mathcal{O}^d|D^0\rangle$$

  - add and subtract $\lambda_b \langle \pi^-\pi^+|\mathcal{O}^s|D^0\rangle$, put in a new form

$$A(D^0 \to \pi^-\pi^+) = -\lambda_s \mathcal{A}_{\pi\pi}\left[1 + \frac{\lambda_b}{\lambda_s}\left(1 + r_\pi \exp(i\delta_\pi)\right)\right]$$
$$A(D^0 \to K^-K^+) = \lambda_s \mathcal{A}_{KK}\left[1 - \frac{\lambda_b}{\lambda_s} r_K \exp(i\delta_K)\right]$$

  - define things we cannot compute (extract from branching ratios)

$$\mathcal{A}_{\pi\pi} = \langle \pi^-\pi^+|\mathcal{O}^d|D^0\rangle - \langle \pi^-\pi^+|\mathcal{O}^s|D^0\rangle$$
$$\mathcal{A}_{KK} = \langle K^-K^+|\mathcal{O}^s|D^0\rangle - \langle K^-K^+|\mathcal{O}^d|D^0\rangle$$

  - … and things we can $\mathcal{P}^s_{\pi\pi} = \langle \pi^-\pi^+|\mathcal{O}^s|D^0\rangle\,, \quad \mathcal{P}^d_{KK} = \langle K^-K^+|\mathcal{O}^d|D^0\rangle$

$$r_\pi = \left|\frac{\mathcal{P}^s_{\pi\pi}}{\mathcal{A}_{\pi\pi}}\right|\,, \quad r_K = \left|\frac{\mathcal{P}^d_{KK}}{\mathcal{A}_{KK}}\right|$$

Amplitude decomposition

- Some things to keep in mind
  - "penguin-type amplitudes" $\mathcal{P}^s_{\pi\pi}$ and $\mathcal{P}^d_{KK}$ denote matrix elements of operators that contain quark-antiquark pair that does not match the valence content of the final state mesons; otherwise no relation to penguin topological amplitudes

$$\mathcal{P}^s_{\pi\pi} = \langle \pi^-\pi^+ | \mathcal{O}^s | D^0 \rangle \,, \qquad \mathcal{P}^d_{KK} = \langle K^-K^+ | \mathcal{O}^d | D^0 \rangle$$

  - calculate $\mathcal{P}^s_{\pi\pi}$ and $\mathcal{P}^d_{KK}$ using a modified light-cone QCD sum rules

$$r_\pi = \left| \frac{\mathcal{P}^s_{\pi\pi}}{\mathcal{A}_{\pi\pi}} \right| \,, \qquad r_K = \left| \frac{\mathcal{P}^d_{KK}}{\mathcal{A}_{KK}} \right|$$

$$\delta_{\pi(K)} = \arg\left[\mathcal{P}^{s(d)}_{\pi\pi(KK)} \right] - \arg\left[\mathcal{A}_{\pi\pi(KK)} \right]$$

  - weak phase $r_b e^{-i\gamma} = \dfrac{\lambda_b}{\lambda_s}$

Direct CP-violating asymmetries

- Setting up direct CP asymmetry
  - each amplitude has two parts with own weak and strong phases

$$A(D^0 \to \pi^-\pi^+) = -\lambda_s \mathcal{A}_{\pi\pi}\left[1 + \frac{\lambda_b}{\lambda_s}\left(1 + r_\pi \exp(i\delta_\pi)\right)\right]$$

$$A(D^0 \to K^-K^+) = \lambda_s \mathcal{A}_{KK}\left[1 - \frac{\lambda_b}{\lambda_s} r_K \exp(i\delta_K)\right]$$

  - this implies for the direct CP-violating asymmetries

$$a_{CP}^{dir}(K^-K^+) = -2 r_b r_K \sin\delta_K \sin\gamma$$

$$a_{CP}^{dir}(\pi^-\pi^+) = 2 r_b r_\pi \sin\delta_\pi \sin\gamma$$

  - … and for their difference

$$\Delta a_{CP}^{dir} = -2 r_b \sin\gamma (r_K \sin\delta_K + r_\pi \sin\delta_\pi)$$

- We need to compute $r_{\pi(K)}$ and $\delta_{\pi(K)}$

Calculating matrix elements

Khodjamirian, NPB 605 (2001) 558

- Use modified light-cone QCD Sum Rule (LCSR) method
  - start with the correlation function ($j_5^{(D)} = i m_c \bar{c}\gamma_5 u$ and $j_{\alpha 5}^{(\pi)} = \bar{d}\gamma_\alpha\gamma_5 u$)

$$F_\alpha(p,q,k) = i^2 \int d^4x e^{-i(p-q)x} \int d^4y e^{i(p-k)y} \langle 0| \, T\left\{ j_{\alpha 5}^{(\pi)}(y) \mathcal{Q}_1^s(0) j_5^{(D)}(x) \right\} |\pi^+(q)\rangle$$
$$= (p-k)_\alpha F((p-k)^2, (p-q)^2, P^2) + \ldots,$$

  - use dispersion relation in (p-k) and (p-q), perform Borel transform, extract matrix element:

$$\langle \pi^-(-q)\pi^+(p)|\mathcal{Q}_1^s|D^0(p-q)\rangle = \frac{-i}{\pi^2 f_\pi f_D m_D^2} \int_0^{s_0^\pi} ds e^{-s/M_1^2} \int_{m_c^2}^{s_0^D} ds' e^{(m_D^2 - s')/M_2^2} \mathrm{Im}_{s'} \mathrm{Im}_s F(s, s', m_D^2)$$

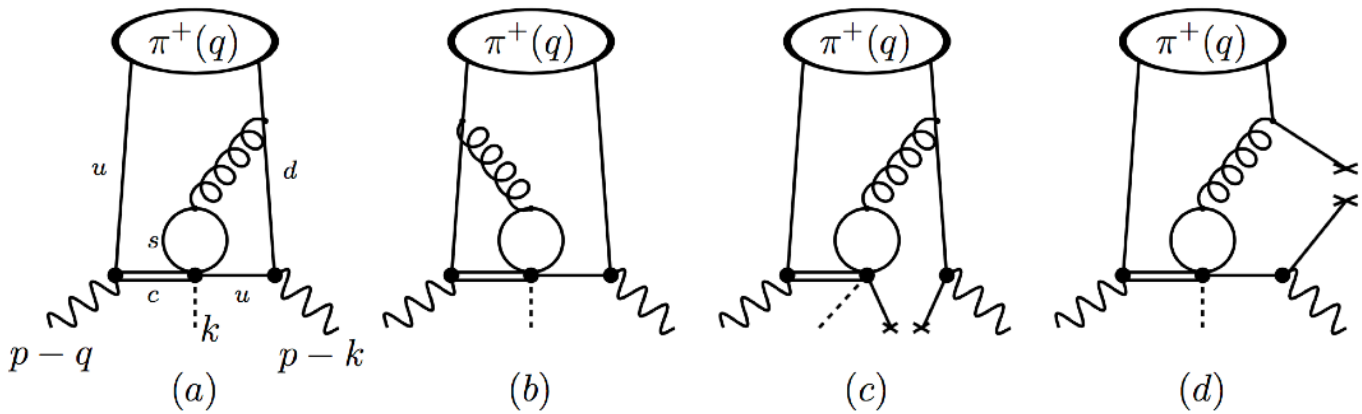
  - perform LC expansion of F(s, s' m_D^2) to get $\mathcal{P}_{\pi\pi}^s$
  - note that $C_1\mathcal{Q}_1^s + C_2\mathcal{Q}_2^s = 2C_1\widetilde{\mathcal{Q}}_2^s + \left(\frac{C_1}{3} + C_2\right)\mathcal{Q}_2^s$ with $\widetilde{\mathcal{Q}}_2^s = \left(\bar{s}\Gamma_\mu \frac{\lambda^a}{2} s\right)\left(\bar{u}\Gamma^\mu \frac{\lambda^a}{2} c\right)$

thus $\mathcal{P}_{\pi\pi}^s = \frac{2G_F}{\sqrt{2}} C_1 \langle \pi^+\pi^-|\widetilde{\mathcal{Q}}_2^s|D^0\rangle$

Calculating matrix elements

- Evaluate (leading) diagrams contributing to the correlation function
  - calculate OPE in terms of known LC DAs

Khodjamirian, NPB 605 (2001) 558;
Khodjamirian, Mannel, Melic, PLB 571 (2003)
Khodjamirian, AAP arXiv: 1706.07780



  - analytically continue from the space-like region of $P^2=(p-k-q)^2$ (with auxiliary 4-momentum $k\neq 0$) to $P^2 = m_D^2$, relying on the local quark-hadron duality
  - extract absolute value and the phase of matrix element $\mathcal{P}^s_{\pi\pi}$
  - vary parameters of the calculation to estimate uncertainties

Results of the calculation

$$\langle \pi^-\pi^+|\widetilde{\mathcal{Q}}_2^s|D^0\rangle = i\frac{\alpha_s C_F m_c^2}{8\pi^3 m_D^2 f_D}\Bigg[\int_0^{s_0^\pi} ds e^{-s/M_1^2}\int_{u_0^D}^1 \frac{du}{u} e^{\left(m_D^2-\frac{m_c^2}{u}\right)/M_2^2}$$

$$\times\Bigg\{P^2\int_0^1 dz I(zuP^2,m_s^2)\Bigg(z(1-z)\varphi_\pi(u)$$

(a)

$$+(1-z)\frac{\mu_\pi}{2m_c}\Bigg[\left(2z+\frac{m_c^2}{uP^2}\right)u\phi_{3\pi}^p(u)+\frac{1}{3}\left(2z-\frac{m_c^2}{uP^2}\right)\left(\phi_{3\pi}^\sigma(u)-\frac{u\phi_{3\pi}^{\sigma\prime}(u)}{2}\right)\Bigg]\Bigg)$$

(b)

$$-\frac{\mu_\pi m_c}{4}\int_0^1 dz I(-z\bar{u}m_c^2/u,m_s^2)\frac{\bar{u}^2}{u}\Bigg[\left(1+\frac{3m_c^2}{uP^2}\right)\phi_{3\pi}^p(1)+\left(1-\frac{5m_c^2}{uP^2}\right)\frac{\phi_{3\pi}^{\sigma\prime}(1)}{6}\Bigg]\Bigg\}$$

(c)

$$+\frac{2\pi^2}{3}m_c(-\langle\bar{q}q\rangle)\int_{u_0^D}^1\frac{du}{u^2}e^{\left(m_D^2-\frac{m_c^2}{u}\right)/M_2^2}\Bigg\{I(uP^2,m_s^2)\Bigg(2\varphi_\pi(u)+\frac{\mu_\pi}{m_c}\Bigg[3u\phi_{3\pi}^p(u)$$

$$+\frac{\phi_{3\pi}^\sigma(u)}{3}-\frac{u\phi_{3\pi}^{\sigma\prime}(u)}{6}\Bigg]\Bigg)\Bigg\}\Bigg]_{P^2\to m_D^2}, \qquad I(\ell^2,m_q^2)=\frac{1}{6}+\int_0^1 dx x(1-x)\ln\left[\frac{m_q^2-x(1-x)\ell^2}{\mu^2}\right]$$

Amplitude predictions

- As a result...

$$\langle\, \pi^+\pi^-|\widetilde{\mathcal{Q}}_2^s|D^0\rangle \;=\; (9.50 \pm 1.13) \times 10^{-3} \exp[i(-97.5^o \pm 11.6)]\,\mathrm{GeV}^3$$

$$\langle K^+K^-|\widetilde{\mathcal{Q}}_2^d|D^0\rangle \;=\; (13.9 \pm 2.70) \times 10^{-3} \exp[i(-71.6^o \pm 29.5)]\,\mathrm{GeV}^3$$

- Extract $\mathcal{A}_{\pi\pi}$ and $\mathcal{A}_{KK}$ amplitudes from experimental data

$$\mathcal{B}(D^0 \to \pi^-\pi^+) = (1.407 \pm 0.025) \times 10^{-3}$$

$$\mathcal{B}(D^0 \to K^-K^+) = (3.97 \pm 0.07) \times 10^{-3}$$

$$|\mathcal{A}_{\pi\pi}| \;\simeq\; \lambda_s^{-1}\left|A(D \to \pi^-\pi^+)\right| \;\; = (2.10 \pm 0.02)$$

$$|\mathcal{A}_{KK}| \;\simeq\; \lambda_s^{-1}\left|A(D \to K^-K^+)\right| = (3.80 \pm 0.03)$$

- Thus, $r_\pi = \dfrac{|\mathcal{P}^s_{\pi\pi}|}{|\mathcal{A}_{\pi\pi}|} = 0.093 \pm 0.011\,,\qquad r_K = \dfrac{|\mathcal{P}^d_{KK}|}{|\mathcal{A}_{KK}|} = 0.075 \pm 0.015$

Predictions for the Acp

- Assuming the phases of $r_{\pi\pi(KK)}$ are given by the phases of $\mathcal{P}^{s(d)}_{\pi\pi(KK)}$

$$
\begin{aligned}
a_{CP}^{dir}(\pi^-\pi^+) &= -0.011 \pm 0.001\%, \\
a_{CP}^{dir}(K^-K^+) &= \quad 0.009 \pm 0.002\%. \\
\Delta a_{CP}^{dir} &= \quad 0.020 \pm 0.003\% \,.
\end{aligned}
$$

Khodjamirian, AAP arXiv: 1706.07780

Things to take home

- The magnitude of hadronic MEs defining CPV are computed

$$r_\pi = \frac{|\mathcal{P}^s_{\pi\pi}|}{|\mathcal{A}_{\pi\pi}|} = 0.093 \pm 0.011\,, \qquad r_K = \frac{|\mathcal{P}^d_{KK}|}{|\mathcal{A}_{KK}|} = 0.075 \pm 0.015$$

- The magnitude of direct CPV asymmetry in $D \to \pi^+\pi^-$ and $D \to K^+K^-$ can be predicted from the calculation of the relevant hadronic matrix elements from LCSRs

$$\Delta a^{dir}_{CP} = \quad 0.020 \pm 0.003\%$$

- No topological amplitude decomposition was used (note that OPE hierarchy sorts out the leading penguin-type diagrams)
- The strong phase difference is not yet reliably accessible: duality violations are not easily identifiable (e.g. broad scalar resonances influencing hadronic matrix elements)

The Evening Herald
EXTRA!
WEATHER
FROM THE WVLRLE

Parameters of the calculation

- Light cone distribution amplitudes

$$\varphi_\pi(u) = 6u\bar{u}\left(1 + a_2^\pi C_2^{3/2}(u-\bar{u}) + a_4^\pi C_4^{3/2}(u-\bar{u})\right)$$

$$\phi_{3\pi}^p(u) = 1 + 30\frac{f_{3\pi}}{\mu_\pi f_\pi}C_2^{1/2}(u-\bar{u}) - 3\frac{f_{3\pi}\omega_{3\pi}}{\mu_\pi f_\pi}C_4^{1/2}(u-\bar{u}),$$

$$\phi_{3\pi}^\sigma(u) = 6u(1-u)\left(1 + 5\frac{f_{3\pi}}{\mu_\pi f_\pi}\left(1 - \frac{\omega_{3\pi}}{10}\right)C_2^{3/2}(u-\bar{u})\right)$$

$$\varphi_K(u) = 6u\bar{u}\left(1 + a_1^K C_1^{3/2}(u-\bar{u}) + a_2^K C_2^{3/2}(u-\bar{u})\right)$$

Parameters of the calculation

Parameter values and references	Parameter rescaled to $\mu = 1.5$ GeV
$\alpha_s(m_Z) = 0.1181 \pm 0.0011$ [6]	0.351
$\bar{m}_c(\bar{m}_c) = 1.27 \pm 0.03$ GeV [6]	1.19 GeV
$\bar{m}_s(2\,\text{GeV}) = 96^{+8}_{-4}$ MeV [6]	105 MeV
$\langle\bar{q}q\rangle(2\,\text{GeV}) = (-276^{+12}_{-10}\,\text{MeV})^3$ [6]	$(-268\,\text{MeV})^3$
$\langle\bar{s}s\rangle = (0.8 \pm 0.3)\langle\bar{q}q\rangle$ [21]	$(-249\ \text{MeV})^3$
$a_2^\pi(1\,\text{GeV}) = 0.17 \pm 0.08$ [22]	0.14
$a_4^\pi(1\,\text{GeV}) = 0.06 \pm 0.10$ [22]	0.045
$\mu_\pi(2\,\text{GeV}) = 2.48 \pm 0.30\,\text{GeV}$ [6]	2.26 GeV
$f_{3\pi}(1\,\text{GeV}) = 0.0045 \pm 0.015\ \text{GeV}^2$ [19]	$0.0036\,\text{GeV}^2$
$\omega_{3\pi}(1\,\text{GeV}) = -1.5 \pm 0.7$ [19]	-1.1
$a_1^K(1\,\text{GeV}) = 0.10 \pm 0.04$ [23]	0.09
$a_2^K(1\,\text{GeV}) = 0.25 \pm 0.15$ [19]	0.21
$\mu_K(2\,\text{GeV}) = 2.47^{+0.19}_{-0.10}$ GeV [6]	2.25
$f_{3K} = f_{3\pi}$	$0.0036\,\text{GeV}^2$
$\omega_{3K}(1\,\text{GeV}) = -1.2 \pm 0.7$ [19]	-0.99
$\lambda_{3K}(1\,\text{GeV}) = 1.6 \pm 0.4$ [19]	1.5