Mu2e Magnetic Field Mapping

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Mu2e Processes

Decay-in-orbit (Background)

Neutrinoless Conversion (Signal)
Uncertainty in field accuracy can shift momentum scale by tens of keV/c. Better field accuracy → better sensitivity!
1. Proton collides with production target.
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2. Pions back-scatter into transport solenoid.
The Mu2e Experiment

1. Proton collides with production target.
2. Pions back-scatter into transport solenoid.
3. Muons and pions transported to detector solenoid.
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2. Pions back-scatter into transport solenoid.
3. Muons and pions transported to detector solenoid.
4. Muons are captured at target.
5. Outgoing electrons pass through detector system.
The Magnetic Field

Detector Solenoid

Bz (T)

Z (mm)

0 4k 6k 8k 10k 12k 14k

0.5 1 1.5 2

Tesla
The Magnetic Field

End of Transport Solenoid and Collimator

Detector Solenoid

2T Field

$B_z$ (T)

$Z$ (mm)
The Magnetic Field

Detector Solenoid

Stopping Target
Muons are captured

Gradient Region
2 to 1 T
The Magnetic Field

Detector Solenoid

Tracker planes

Calorimeter

Uniform Region, 1T

Strictest requirements on field accuracy

Z (mm)

Bz (T)

Tesla

X

Z
The Magnetic Field

Detector Solenoid

~4m < Z < ~13m

Region mapped in upcoming slides

B_z (T)

Z (mm)
Field Mapper will take a sparse set of magnetic field measurements.

- Very demanding hardware requirements!
  (hall probe calibration, laser alignment, etc.)

- A continuous field will be reconstructed.

- Measurement errors must be minimized and quantified.

- Reconstructed field must be accurate to $1 \times 10^{-4}$ w.r.t. true.

Need ~1 G accuracy for 1 T field.

Field mapper in solenoid
Solenoid Field Mapper

Hall Probes

How do we turn discrete measurements into a continuous field?
Maxwell’s Equations

★ Maxwell’s equations for the fiducial region:

\[ \nabla \cdot \vec{B} = 0 \quad \text{and} \quad \nabla \times \vec{B} = 0 \]

★ The B-field can be expressed as gradient of scalar potential:

\[ \vec{B} = -\nabla \Phi \]

★ In cylindrical coordinates, a series solution for \( \Phi \) using modified Bessel’s functions:

\[ \Phi = \sum_{n,m} A_{nm} e^{\pm in\phi} e^{\pm i k_{nm} z} I_n(k_{nm} \rho) \]

★ Will measure field components \( B_\rho \) and \( B_z \) and \( B_\phi \), \textit{not} \( \Phi \).

★ Measurements determine coefficients through a \( \chi^2 \) fit.
Analytical Model

* Derived from solutions to Maxwell’s Equations for a generic solenoid:

\[
B_r = \sum_{n,m} \cos(n\phi + \delta_n)k_{nm}I'_n(k_{nm}r)[A_{nm} \cos(k_{nm}z) + B_{nm} \sin(-k_{nm}z)]
\]

\[
B_z = \sum_{n,m} -\cos(n\phi + \delta_n)k_{nm}I_n(k_{nm}r)[A_{nm} \sin(k_{nm}z) + B_{nm} \cos(-k_{nm}z)]
\]

\[
B_\phi = \sum_{n,m} -\frac{n}{r} \sin(n\phi + \delta_n)I_n(k_{nm}r)[A_{nm} \cos(k_{nm}z) + B_{nm} \sin(-k_{nm}z)]
\]

* All field components fit simultaneously.

* Fit expanded to ~200 terms, ~400 free parameters.
Fit Results

Black dots: Sim data points
Green mesh: Fit
Surface: Residuals (Data-Fit, in units of Gauss)

- Agreement with simulation at $R<800$ mm is excellent.
- Level of disagreement is still on the order of $10^{-5} - 10^{-6}$ (~0.01 Gauss)
- Extrapolation of field is accurate within ~5 Gauss for $800<R<900$ mm

2D Slice Range:
$4 \text{ m} \leq Z \leq 13 \text{ m}$
$R \leq 80 \text{ cm}$
Hall probes will be subject to systematic errors based on positional and measurement accuracy.

- Requirements for Detector Solenoid:
  - **Measurement**: $\sigma |B|/|B| \leq 0.01\%$ *(Shown in next slide)*
  - **Position**: $\sigma$ position $\leq 1\text{ mm}$
  - **Orientation**: $\sigma\phi \leq 0.1\text{ mrad}$

- These effects will translate into slight mis-measurements, which in turn will affect field map.

- **Procedure**:
  - Modify hall probe measurements with systematic errors.
  - **Fit** function to modified probe values.
  - Compare resulting map to **true** field.
A scale factor representing a miscalibration of each probe measurement, satisfying $B_{\text{measured}}$ is within 0.01% of $B_{\text{true}}$.

- e.g., $B \rightarrow B^*(1+\varepsilon)$ where $-0.0001<\varepsilon<0.0001$
- Represents correlated systematic effect, not random error

Fit function resists miscalibration, more accurate than simple interpolation!
The spread of expected residuals is \(~0.25\ G\), which corresponds to a relative error better than \(5 \times 10^{-5}\).
Software Implementation

★ All data manipulation, fitting, and visualization software written in Python with popular open source packages:
  • Numpy, Scipy, pandas, lmfit, matplotlib, plotly…
  • Easy to integrate results into any software framework.

★ Minimization time is good:
  • ~500 parameter fit run over ~20,000 data points takes ~30 min on an i7 laptop.
  • Using numba (with CUDA for GPU acceleration), time reduced by 2x-10x using current-gen GPU.
Mu2e will improve current CLFV sensitivity by over 4 orders of magnitude.

- Great discovery potential!

Demanding performance requires precise and accurate knowledge of magnetic field.

- Novel hardware and software solutions needed.

Leveraging magnetostatics and modern-day computing, semi-analytic fitting technique can produce continuous, accurate maps, even in non-ideal scenarios.
Backup
Lepton Flavor Violation (LFV) is a well known and defining phenomena in the neutrino sector.

But what about Charged Lepton Flavor Violation (CLFV)?

- Has not yet been detected → only limits have been placed.
- Greatly suppressed in SM (BR < $10^{-50}$).

Mu2e is designed to probe CLFV with 10,000 times the sensitivity of previous experiments!

If a single signal event is observed, it will be a clear sign of New Physics.
The Experiment Goal

Key Metric: \( R_{\mu e} = \frac{\mu^- + A(Z,N) \rightarrow e^- + A(Z,N)}{\mu^- + A(Z,N) \rightarrow \nu_{\mu} + A(Z - 1,N)} \) (Rate of neutrinoless conversion) (Rate of ordinary muon capture)

Model Independent Effective Lagrangian:

\[
L_{\text{CLFV}} = \frac{m_{\mu}}{(1 + \kappa)} \Lambda^2 \bar{\mu}_R \sigma_{\mu\nu} e_L F_{\mu\nu} + \frac{\kappa}{(1 + \kappa)} \Lambda^2 \bar{\mu}_L \gamma_{\mu} e_L \left( \sum_{q = u, d} \bar{q}_L \gamma_{\nu} q_L \right)
\]

- Magnetic moment interactions
- Four-fermion interactions

\( \Lambda \): New Physics mass scale
\( \kappa \): Dimensionless relative contribution scale

Mu2e will be sensitive to new physics scales up to \(~10,000\) TeV, and to both types of CLFV operators.

André de Gouvêa, NU
This is an example for a single 2D slice of the magnetic field. All slices and components are fit simultaneously.
Residual compared to probes 
(sparse sample).

- Agreement with simulation at $R<800 \text{ mm}$ is excellent.
- Level of disagreement is still on the order of $10^{-5} - 10^{-6}$ (~0.01 Gauss)
- Extrapolation of field is accurate within ~5 Gauss for $800<R<900 \text{ mm}$
Each probe position is shifted by an offset of $\pm 1$ mm in the radial direction.

As expected, greatest effects are in regions of high magnetic gradient with respect to radial position.

- Minimal effect in tracking region.
Each probe is rotated by an angle of ~0.1 mrads in the R-Z plane

- This mainly impacts the value of Br, as the Bz component is much larger.

This mixing should always reduce the Z-component and increase the R-component.

Fit compared to probe measurements

Fit compared to true field
Field Mapping System (FMS) Team

- **Sandor Feher** — L3 Manager, Fermilab – TD/MSD Measurements and Analysis Group Leader, Mu2e Detector Solenoid (DS) L3 Manager
- **Michael Lamm** — L3 CAM, Mu2e Solenoid System L2 manager
- **Argonne National Laboratory team:**
  - Rich Talaga and Robert G. Wagner — Senior Physicists
  - James Grudzinski and Jeffrey L. White — Senior Mechanical Engineers
  - Allen Zhao — Motion Control Expert, Senior Engineer
- **Fermilab team:**
  - Luciano Elementi and Charles Orozco — System Engineers
  - Horst Friedsam — Geodicist
  - Thomas Strauss — Associate Scientist
  - Jerzy Nogiec — Computer Scientist
- **Northwestern University:**
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