SM EFT Contributions to Z Decay at One Loop

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Introduction: EFT

• The canonical example of an EFT is Fermi’s theory of weak decay
  – A real limit of the SM
• We still use this today!
• Captures physics in a particular energy regime
  – Able to be improved in precision systematically
• Ability to systematically improve theory predictions is the key virtue of EFTs
Tree-level Effects

• At tree level SM parameters are modified, e.g.

\[
\delta M_Z^2 \equiv \frac{1}{2 \sqrt{2}} \frac{\hat{m}_Z^2}{\hat{G}_F} C_{HD} + \frac{2^{1/4} \sqrt{\pi} \sqrt{\hat{\alpha}} \hat{m}_Z}{\hat{G}_F^{3/2}} C_{HWB},
\]

\[
\delta M_W^2 = -\hat{m}_W^2 \left( \frac{\delta s^2_\theta}{s^2_\theta} + \frac{c^2_\theta}{s_\theta \sqrt{2} \hat{G}_F} C_{HWB} + \sqrt{2} \delta G_F \right),
\]

\[
\delta G_F = \frac{1}{\sqrt{2} \hat{G}_F} \left( \sqrt{2} C_{HL}^{(3)} - \frac{C_{ul}}{\sqrt{2}} \right),
\]

\[
\delta s^2_\theta = -\frac{s_\theta c^2_\theta}{2 \sqrt{2} \hat{G}_F (1 - 2 s^2_\theta)} \left[ s_\theta c_\theta (C_{HD} + 4 C_{HL}^{(3)} - 2 C_{ul}) + 2 C_{HWB} \right],
\]

• Direct EFT effects also appear in processes at higher energies or with higher multiplicity
Why Loops?

• Electroweak observables have been measured with amazing precision
  – Theory calculations have to match this precision to get full value out of the data

<table>
<thead>
<tr>
<th>Observable</th>
<th>Experimental Value</th>
<th>Ref.</th>
<th>SM Theoretical Value</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{m}_Z$ [GeV]</td>
<td>91.1875 ± 0.0021</td>
<td>[38]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{m}_W$ [GeV]</td>
<td>80.385 ± 0.015</td>
<td>[39]</td>
<td>80.365 ± 0.004</td>
<td>[40]</td>
</tr>
<tr>
<td>$\sigma_h^0$ [nb]</td>
<td>41.540 ± 0.037</td>
<td>[38]</td>
<td>41.488 ± 0.006</td>
<td>[41]</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>2.4952 ± 0.0023</td>
<td>[38]</td>
<td>2.4942 ± 0.0005</td>
<td>[41]</td>
</tr>
<tr>
<td>$R^0_\ell$</td>
<td>20.767 ± 0.025</td>
<td>[38]</td>
<td>20.751 ± 0.005</td>
<td>[41]</td>
</tr>
<tr>
<td>$R^0_b$</td>
<td>0.21629 ± 0.00066</td>
<td>[38]</td>
<td>0.21580 ± 0.00015</td>
<td>[41]</td>
</tr>
<tr>
<td>$R^0_c$</td>
<td>0.1721 ± 0.0030</td>
<td>[38]</td>
<td>0.17223 ± 0.00005</td>
<td>[41]</td>
</tr>
<tr>
<td>$A^\ell_{FB}$</td>
<td>0.0171 ± 0.0010</td>
<td>[38]</td>
<td>0.01616 ± 0.00008</td>
<td>[42]</td>
</tr>
<tr>
<td>$A^c_{FB}$</td>
<td>0.0707 ± 0.0035</td>
<td>[38]</td>
<td>0.0735 ± 0.0002</td>
<td>[42]</td>
</tr>
<tr>
<td>$A^b_{FB}$</td>
<td>0.0992 ± 0.0016</td>
<td>[38]</td>
<td>0.1029 ± 0.0003</td>
<td>[42]</td>
</tr>
</tbody>
</table>
Contributing Operators

- 4-fermion operators:

- Scalar-fermionic current operators:
Contributing Operators

• Gauge-Higgs operators:

• Dipole operators:
Input Parameters

• Any calculation depends on the inputs used to set the theory parameters
• We use a canonical set of inputs for the SM
  \[ \alpha_{EM}, G_F, M_Z, M_t, M_h \]
• EFT gives corrections to the extraction of each
• We treat the Wilson coefficients in $\overline{MS}$ at the NP scale as EFT input parameters to be measured and/or constrained
\( \alpha_{EM} \) Corrections

- Matching contributions at low scales where \( \alpha \) is measured are proportional to lepton masses
- Running cannot be neglected for \( \alpha \)
- Two EFT effects here
  - Shift in Weinberg angle from HWB operator
  - Different running from HB, HW operators

\[
\delta \alpha = -\sqrt{2} \frac{4\pi \tilde{\alpha}}{\hat{G}_F} C^{(r)}_{HWB},
\]

\[
\Delta \alpha = -\sqrt{2} \frac{4\pi \tilde{\alpha}}{\hat{G}_F} C^{(r)}_{HWB} \left( \Delta V^2 + \frac{\Delta G_F}{\hat{G}_F} \right) + \frac{\tilde{\alpha}}{\pi} \hat{m}_h^2 \left( C^{(r)}_{HB} + C^{(r)}_{HW} \right) \log \left[ \frac{\hat{m}_Z^2}{p^2} \right],
\]

\[
\simeq -\sqrt{2} \frac{4\pi \tilde{\alpha}}{\hat{G}_F} C^{(r)}_{HWB} \left( \Delta V^2 + \frac{\Delta G_F}{\hat{G}_F} \right) + 0.03 \hat{m}_h^2 \left( C^{(r)}_{HB} + C^{(r)}_{HW} \right). \]
$G_F$ Corrections

• $G_F$ is extracted from the muon lifetime
  – Dominated by $\mu \rightarrow e\nu\nu$

• Corrected by 4-fermion ops and W mass shifts

• Lagrangian parameter extracted as

\[
-\frac{4G_F}{\sqrt{2}} = -\frac{2}{\bar{v}_T^2} \left( 1 - \frac{\Delta V^2}{\bar{v}_T^2} - \frac{\Delta m_W^2}{\bar{m}_W^2} \right) - 4 \hat{G}_F \delta G_F - \Delta \psi^4
\]

\[
\Delta G_F = -\hat{G}_F \Delta V^2 (1 - 2\sqrt{2}\delta G_F) - \frac{\Delta m_W^2}{\sqrt{2}\bar{m}_W^2} + \frac{\Delta \psi^4}{4\hat{G}_F} - \frac{\Delta m_W^2}{\sqrt{2}\bar{m}_W^2} \frac{\delta m_W^2}{\bar{m}_W^2}
\]
$M_Z$ Corrections

- Gauge-Higgs operators correct the Z mass through the graphs

- The Higgs-derivative operator contributes too
Finite Field Normalizations

• Z boson R-factor arises from the graphs:

• B-quark R-factor:

\[ = \frac{\hat{m}_t^2}{16\pi^2} \left( \sqrt{2} \hat{G}_F (1 - 2\delta G_F) + C^{*}_{uH} - 2 C^{(3)}_{Hq} \right) \left[ -\frac{3}{4} - \frac{1}{2} \log \left[ \frac{\Lambda^2}{\hat{m}_t^2} \right] \right] \]
Sample Results

\[
\Delta (g_L^d)_{rr} = \Delta g_Z (g_L^d)_{rr}^{SM} + \frac{N_c \hat{m}_t^2}{16 \pi^2} \log \left[ \frac{\Lambda^2}{\hat{m}_t^2} \right] \left[ C_{qq}^{(1)}_{rr} + C_{qq}^{(1)}_{rr} - C_{qq}^{(3)}_{rr} - C_{qq}^{(3)}_{rr} - C_{qq}^{(1)}_{rr} \right] + \frac{1}{2} \left( \Delta G_F^{1/2} + \Delta V^2 \right) \left( C_{Hq}^{(1)}_{rr} + C_{Hq}^{(3)}_{rr} \right) + \delta_{br} \frac{\hat{m}_t^2}{4 \pi^2} \left[ C_{3333}^{(3)} \left( -1 + \log \left[ \frac{\Lambda^2}{\hat{m}_t^2} \right] \right) \right] - \frac{1}{3} \Delta s_\theta^2,
\]

\[
- \delta_{br} \frac{\hat{m}_t^2}{16 \pi^2} \left[ \left( \frac{1}{4} - \frac{1}{2} \log \left[ \frac{\Lambda^2}{\hat{m}_t^2} \right] \right) C_{Hu} + C_{Hq}^{(1)} \right] - \delta_{br} R_b^L \left( (g_L^d)^{SM}_{rr} + \delta (g_L^d)_{rr} \right),
\]

\[
- \delta_{br} \frac{\hat{m}_t^2}{16 \pi^2} C_{Hq}^{(3)} \left[ \frac{1}{2} - Q_b s_\theta^2 + (3 - 2 Q_b s_\theta^2) \log \left[ \frac{\Lambda^2}{\hat{m}_t^2} \right] \right],
\]

\[
- \delta_{br} \frac{\hat{m}_t^2}{4 \pi} \tilde{\alpha} (c_\theta^2 - s_\theta^2) C_{HWB} (Q_u - 1) \left[ \frac{3}{2} + \log \left[ \frac{\Lambda^2}{\hat{m}_t^2} \right] \right],
\]

\[
\Delta \Gamma_{Z \rightarrow Had} = 2 \Delta \Gamma_{Z \bar{u}u} + 2 \Delta \Gamma_{Z \bar{d}d} + \Delta \Gamma_{Z \bar{b}b},
\]

\[
= \frac{\sqrt{2} \hat{G}_F \hat{m}_Z^3}{6 \pi} \left[ 4 (g_R^u + \delta g_R^u) \Delta g_R^u + 4 (g_L^u + \delta g_L^u) \Delta g_L^u + 4 (g_R^d + \delta g_R^d) \Delta g_R^d + 4 (g_L^d + \delta g_L^d) \Delta g_L^d \right]
\]

\[
+ \frac{\sqrt{2} \hat{G}_F \hat{m}_Z^3}{6 \pi} \left[ 4 (g_R^d + \delta g_R^d) \Delta g_R^d + 2 (g_R^b + \delta g_R^b) \Delta g_R^b + 2 (g_L^b + \delta g_L^b) \Delta g_L^b \right],
\]

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William Shepherd, JGU Mainz
Phenomenology

• Counting is all that’s needed for the most important point

• NLO corrections have introduced dependence on (neglecting flavor indices):
  – 3 Higgs-gauge WCs
  – 2 Dipole WCs
  – 7 Higgs-fermion current WCs
  – 9 four-fermion WCs

• At this level of precision, we can measure only 5 Z pole observables ($A_{FB}$ goes beyond NWA)
Phenomenology

• Recall that at tree level there were flat directions in Z pole observables
  – Lifted by TGC measurements
• With this increase in relevant parameters, all of EWPD not enough to constrain the EFT
• The lesson: loop corrections cannot be constrained by EWPD alone, thus EWPD bounds (at tree level) can never be more precise than a loop factor on WCs
The $\delta$ correction to $\tilde{\Gamma}_{Z \to \tilde{d} \tilde{d}}$ (where $d = \{d, s, b\}$) is given by

$$\frac{\delta \tilde{\Gamma}_{Z \to \tilde{d} \tilde{d}}}{10^{-2}} = -0.939 C_{Hd} - 1.58 C_{HD} - 6.31 C_{H\ell}^{(3)} + 5.10 \left( C_{Hq}^{(1)} + C_{Hq}^{(3)} \right) - 0.510 C_{HWB} + 3.15 C_{\ell\ell}. \quad (7.21)$$

The $\delta \Delta$ correction to $\tilde{\Gamma}_{Z \to \tilde{d} \tilde{d}}$ (where $d = \{d, s\}$) has the contributions

$$\frac{\delta \Delta \tilde{\Gamma}_{Z \to \tilde{d} \tilde{d}}}{10^{-3}} = \left[ (0.071 \Delta \tilde{v}_T + 0.201) C_{Hd} - (0.115 \Delta \tilde{v}_T + 0.144) C_{HD}, -(1.45 \Delta \tilde{v}_T + 1.08) C_{H\ell}^{(3)} + (0.316 \Delta \tilde{v}_T - 0.206) \left( C_{Hq}^{(1)} + C_{Hq}^{(3)} \right) - (0.024 \Delta \tilde{v}_T + 0.064) C_{HWB} + 4.23 \Delta \tilde{v}_T, + (0.727 \Delta \tilde{v}_T + 0.541) C_{\ell\ell} + 0.593 C_{\ell q}^{(3)} + 0.072 \left( C_{HB} + C_{HW} \right) \right], \quad (7.22)$$

and the $\delta \Delta$ corrections to $\tilde{\Gamma}_{Z \to \tilde{d} \tilde{d}}$ (where $d = \{d, s\}$) also has the logarithmic terms

$$\frac{\delta \Delta \tilde{\Gamma}_{Z \to \tilde{d} \tilde{d}}}{10^{-3}} = \left[ 0.342 C_{Hd} - 0.266 C_{HD} - 0.995 C_{H\ell}^{(3)} - 0.225 \left( C_{Hq}^{(1)} + C_{Hq}^{(3)} \right) - 0.110 C_{HWB}, + 1.09 C_{\ell \ell} - 1.19 C_{\ell q}^{(3)} + 0.176 \left( C_{qd}^{(1)} - C_{ud}^{(1)} \right) + 1.92 \left( C_{qq}^{(3)} - C_{qq}^{(1)} \right) + 0.958 C_{qu}^{(1)}, - 0.091 C_{uW} - 0.055 C_{uB} \right] \log \left[ \frac{\Lambda^2}{\tilde{m}_t^2} \right] + \left[ (2.43 \times 10^{-5}) C_{HD} + 0.015 C_{Hd}, + 0.103 C_{Hl}^{(3)} - 0.083 \left( C_{hq}^{(1)} + C_{hq}^{(3)} \right) - 0.005 C_{HWB} - 0.052 C_{\ell \ell} \right] \log \left[ \frac{\Lambda^2}{\tilde{m}_h^2} \right]. \quad (7.23)$$
Numerics

The $\delta$ correction to $\tilde{R}_b^0$ is given by

$$\frac{\delta R_b^0}{10^{-2}} = -0.192 C_{Hd} + 0.039 C_{HD} + 0.158 C_{Hq}^{(3)} + 2.13 C_{Hq}^{(1)} - 0.055 C_{Hq}^{(3)} - 0.494 C_{Hu} + 0.043 C_{HWB} - 0.079 C_{\ell\ell}. \quad (7.35)$$

Similarly, the $\delta \Delta$ correction to $\tilde{R}_b^0$ has the contributions

$$\frac{\delta \Delta R_b^0}{10^{-3}} = \left[ (0.036 \Delta \bar{v}_T + 0.083) C_{Hd} + (0.011 \Delta \bar{v}_T + 0.013) C_{HD} + (0.084 \Delta \bar{v}_T - 0.014) C_{Hq}^{(3)}, \right.$$

$$\left. - (0.085 \Delta \bar{v}_T + 0.152) C_{Hq}^{(1)} - (0.016 \Delta \bar{v}_T + 0.019) C_{Hq}^{(3)} + (0.099 \Delta \bar{v}_T + 0.208) C_{Hu}, \right.$$

$$\left. - (0.042 \Delta \bar{v}_T - 0.007) C_{\ell\ell} + (0.013 \Delta \bar{v}_T + 0.009) C_{HWB} - 0.015 C_{\ell q}^{(3)}, \right.$$

$$\left. + 0.597 C_{qq}^{(3)} + 0.047 C_{uH} - 0.006 (C_{HB} + C_{HW}) - 0.106 \Delta v \right], \quad (7.36)$$

and the $\delta \Delta$ correction to $R_b^0$ also has the logarithmic terms

$$\frac{\delta \Delta R_b^0}{10^{-3}} = \left[ 0.129 C_{Hd} + 0.025 C_{HD} + 0.067 C_{Hq}^{(3)} - 0.559 C_{Hq}^{(1)} + 0.383 C_{Hq}^{(3)} + 0.240 C_{Hu}, \right.$$\n
$$\left. + 0.023 C_{HWB} - 0.049 C_{\ell\ell} + 0.030 C_{\ell q}^{(3)} + 0.036 \left(C_{qd}^{(1)} - C_{ud}^{(1)}\right) - 0.618 C_{qq}^{(3)}, \right.$$\n
$$\left. - 0.803 C_{qq}^{(1)} + 0.494 C_{qq}^{(1)} - 0.002 C_{uB} + 0.032 C_{uH} - 0.004 C_{uW} - 0.186 C_{wu} \right] \log \left[ \frac{\Lambda^2}{\hat{m}_T^2} \right]$$\n
$$+ \left[ -8.94 \times 10^{-7} C_{HD} + \left( 0.313 C_{Hd} - 3.49 C_{Hq}^{(1)} + 0.090 C_{Hq}^{(3)} - 0.258 C_{Hq}^{(3)}, \right. \right.$$\n
$$+ 0.808 C_{Hu} + 0.129 C_{\ell\ell} - 0.020 C_{HWB} \right] 10^{-2} \log \left[ \frac{\Lambda^2}{\hat{m}_h^2} \right]. \quad (7.37)$$
Conclusions

• We have excellent data available, and must have enough respect for that to understand our new physics predictions at comparable precision
• In the case of LEP data, especially at the Z pole, this requires NLO accuracy
• In the most model-independent formulation of heavy new physics, the NLO predictions are under-constrained by low energy data
  – Setting shifts in EW observables to zero for the purposes of further searches does not give model-independent results
• A truly global analysis will be needed to properly constrain the EFT without UV assumptions
• Thank goodness we have the LHC with its forthcoming unprecedented data set to constrain new physics at higher energies!
Thank You!