New developments in the relation between quantum gravity and quantum information

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Black holes

Holography
QFT = gravity

Non trivial phases of condensed matter physics

Quantum information theory, entanglement entropy
Computational Complexity
Quantum error correction
Quantum Chaos
Questions & tools

• How is quantum information stored in the state?
• What is the pattern of entanglement?
• How complex is the state?
• How is quantum information flowing dynamically?

• Computation of entropies of subsystems ("entanglement entropy") tells us about this structure.
• Holography and the Ryu-Takayangi formula
First topic: Entanglement entropy
Entropy as a tool in QFT

• Use subregion entropy to characterize renormalization group flows.

• Entropy of a disk or sphere $\rightarrow$ monotonous under RG flow. (c, f, a theorems).

$$S(A \cup B \cup C) + S(C) \leq S(A \cup C) + S(B \cup C)$$

• Proofs of lower bounds for the null energy.

$$0 \leq \int_{\text{null ray}} dx^- T_-, \quad \frac{\delta^2 S_{\text{ent}}}{\delta \ell^2} \leq T_-$$

Casini, Teste, Torroba, Huerta

Implications for gravity + quantum fields

Wall, Faulkner, Balakirshnan, Khandker, Wang, Leigh, Parrikar...
Entropy as a tool to explore holography

- Main tool: Ryu-Takayanagi formula.

\[ S(A) = \frac{(\text{Area})_{\text{minimal}}}{4G_N} \]

Region \( A_B \) is related to the subregion \( A \).

Ryu, Takayanagi, Hubbeny, Rangamani
Cracking the holographic code

Quantum error correction and holography

Almheiri, Dong, Harlow, Preskill, Yoshida, Pastawski

Protecting a qubit is setting in deeper in the bulk!

Complexity theory

Harlow, Hayden, Brown, Susskind, Zhao, Roberts, Swingle, Stanford.
Now to dynamics
Universal bound on quantum chaos
inspired by black holes
Signature of classical chaos $\rightarrow$ divergence of nearby trajectories
Classical

\[
\frac{\delta q(t)}{\delta q(0)} = \{q(t), p(0)\}, \quad \{q(t), p(0)\}^2
\]

\[
\delta q(t) \propto e^{\lambda t}
\]

Quantum

\[
\begin{align*}
\{Q(t), P(0)\}^2 \\
\{Q_i(t), P_j(0)\}^2
\end{align*}
\]

Quantum General:

\[
\langle [W(t), V(0)]^2 \rangle_\beta \propto \frac{1}{S} e^{\lambda t}
\]
Quantum General:

$$\langle [W(t), V(0)]^2 \rangle_\beta \propto \frac{1}{S} e^{\lambda t}$$

$W, V$ are two ``simple'' (initially commuting) observables.

Imagine we have large entropy system, $S \gg 1$.

This is the definition of the quantum Liapunov exponent.

It is defined by its initial increase.

At very long times it saturates to a quantity of order one.
Period of exponential increase

\[ t_* \sim \frac{1}{\lambda} \log S \]
Use now AdS/CFT and black holes
Black holes in a gravity box

Hot fluid made out of very strongly interacting particles.
Commutator $\rightarrow$ involves the scattering amplitude between these two excitations.

Large $t \rightarrow$ large boost between the two particles.

Leading order $\rightarrow$ graviton exchange

Gravitational interaction has spin 2, Shapiro time delay proportional to energy.

Energy goes as $e^t$

$$\langle [W(t), V(0)]^2 \rangle_\beta \propto \frac{1}{S} e^{\lambda t}$$

$$\lambda = \frac{2\pi}{\beta} = 2\pi T$$
Can it be different?

Graviton phase shift: \( \delta(s) \sim G_N s \rightarrow \lambda = \frac{2\pi}{\beta} \)

Regge limit, \( s >> t, s >> 1 \)

String phase shift

\[ \delta(s) \sim G_N s^{1 + \alpha' t} \rightarrow \lambda = \frac{2\pi}{\beta} \left(1 - \frac{l_s^2}{R^2}\right) \]

\( s, t = \text{Mandelstam invariants} \)

Typical size of string (of graviton in string theory)

Radius of curvature of black hole
It can be less...

More?

In flat space a phase shift has to scale with a power of s less than one in order to have a causal theory

Maybe there is a bound...
Universal upper bound on chaos

\[ \lambda \leq \frac{2\pi}{\beta} = 2\pi T = \frac{2\pi T}{\hbar} \]

Proof: Uses analyticity in Euclidean time, unitarity, and that simple observables thermalize.

For any large N (small \( \hbar \)) quantum system.
Black holes are the most chaotic systems

Are there others?

Turn to a condensed matter inspired model
Sachdev Ye Kitaev model

$H = \sum_{i_1, \ldots, i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$

N Majorana fermions in $0 + 1$ dimensions.
All to all interactions.

Coefficient are random gaussian variables.

It has the virtue of being solvable at large N.

At low temperatures (but not exponentially small), the system is also maximally chaotic.
Few comments

• Thinking of black holes as quantum systems we were lead to a particular Liapunov exponent.

• We also used, for inspiration, properties of flat space scattering amplitudes.

• Final argument does not require black holes and is valid for general quantum systems.

• But it was useful to consider the interrelation between different points of view.
Entanglement and geometry:
Exploring wormholes
The full Schwarzschild wormhole

No need to postulate any exotic matter

No matter at all!
View it as an entangled state

\[ |TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R \]

Israel 70's
JM 00's
JM Susskind
True causal separation

If Bob sends a signal, then Alice cannot receive it.

These wormholes are not traversable, due to the integrated null energy condition

\[ 0 \leq \int dx^- T_{--} \quad \text{Balakrishnan, Faulkner, Khandker, Wang} \]

Not good for science fiction.

Good for science!

\[ |TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R \]
Measure and send result \(\rightarrow\) make wormhole traversable

Teleported through the wormhole!

Quantum Teleportation: N qubits with 2N classical bits and N entangled qubits.
Bennett, Brassard, Crepeau, Jozsa, Peres, Woetters

Susskind Gao, Wall, Jafferis JM, Stanford, Zhang
Teleported through the wormhole!

Quantum Teleportation: N qubits with 2N classical bits and N entangled qubits.
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Conclusions

• Entanglement entropy has emerged as a tool for exploring quantum field theory.

• Quantum error correction and holography.

• Dynamical processes spread quantum information. We are exploring speed limits...

• Quantum teleportation can have a geometric picture as information flowing through a wormhole (in some cases).
Thank you!
Extra Slides
• The list could go on...

• I will explain in some detail one particular example.

  We will consider a 1 +1 dimensional quantum field theory

  And derive the c-theorem.
Entropy subadditivity

\[ S(A \cup B \cup C) + S(C) \leq S(A \cup C) + S(B \cup C) \]

\[ S(R^2 e^{2\delta}) + S(R^2) \leq S(R^2 e^{\delta}) + S(R^2 e^{\delta}) \]

Entropy depends only on the (square of the) proper size of the interval.

\[ (R^2 S'(R^2))' \leq 0 \]
For a CFT: \( S(R^2) = \frac{c}{6} \log\left(\frac{R^2}{\epsilon^2}\right) \) \( \Rightarrow \) \( R^2 S'(R^2) = \frac{c}{6} \)

\[ (R^2 S'(R^2))' \leq 0 \]

\( C_{IR} \leq C_{UV} \) \( \text{c - theorem} \) \( \text{Zamolodchikov} \)

The argument in 2 + 1 is similar but it involves an large number of boosted circular regions.
(has not been proven in any other way) \( \text{Casini, Huerta} \)

In 3 + 1 one needs to compare the entropy to the one of the UV fixed point. And use that the entropy subaditivity is an equality for the case of regions lying on the lightcone \( \text{Casini, Teste, Torroba} \)
Now to another application
What is this funny “time translation symmetry”? 

$$U = e^{it(H_R - H_L)}$$

Exact symmetry

Exact boost symmetry!

$$|T F D\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$
If they jump in, they can meet in the interior!

But they cannot tell anyone.

\[
|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R
\]
Spacetime connectivity from entanglement

\[ \text{ER} = \text{EPR} \]

Van Raamsdonk
Verlinde
Papadodimas Raju
JM Susskind
Entanglement and geometry
Local boundary quantum bits are highly interacting and very entangled.

\[ S(R) = \frac{A_{\text{min}}}{4G_N} \]

Generalization of the Black hole entropy formula.

Ryu, Takayanagi, Hubeny, Rangamani