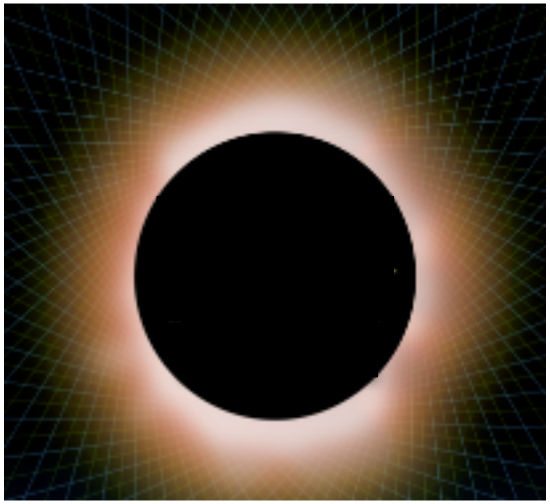
New developments in the relation between quantum gravity and quantum information



Juan Maldacena

Institute for Advanced Study

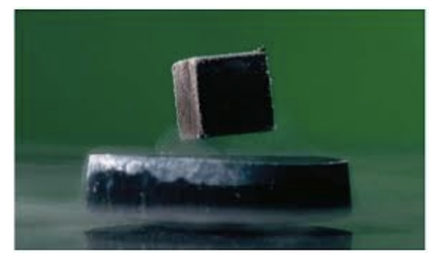
DPF Meeting

Fermilab, 2017

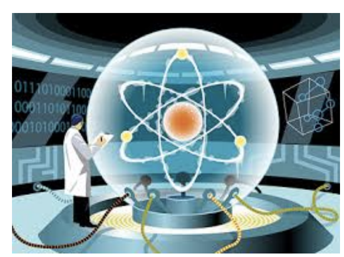
Holography
QFT = gravity

Black holes

Non trivial phases of
condensed matter physics



Quantum information theory,
entanglement entropy
Computational Complexity
Quantum error correction
Quantum Chaos



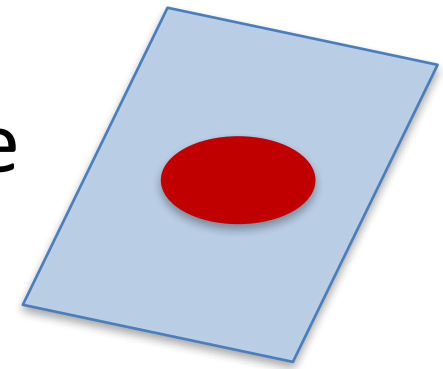
Questions & tools

- How is quantum information stored in the state?
- What is the pattern of entanglement?
- How complex is the state?
- How is quantum information flowing dynamically ?

- Computation of entropies of subsystems ("entanglement entropy")→ tells us about this structure.
- Holography and the Ryu-Takayangi formula

First topic: Entanglement entropy

Entropy as a tool in QFT



- Use subregion entropy to characterize renormalization group flows.

- Entropy of a disk or sphere → monotonous under RG flow. (c, f, a theorems).

Casini, Teste, Torroba, Huerta

$$S(A \cup B \cup C) + S(C) \leq S(A \cup C) + S(B \cup C)$$

- Proofs of lower bounds for the null energy.

$$0 \leq \int_{\text{null ray}} dx^- T_{--} \ , \qquad \frac{\delta^2 S_{\text{ent}}}{\delta \ell^2} \leq T_{--}$$

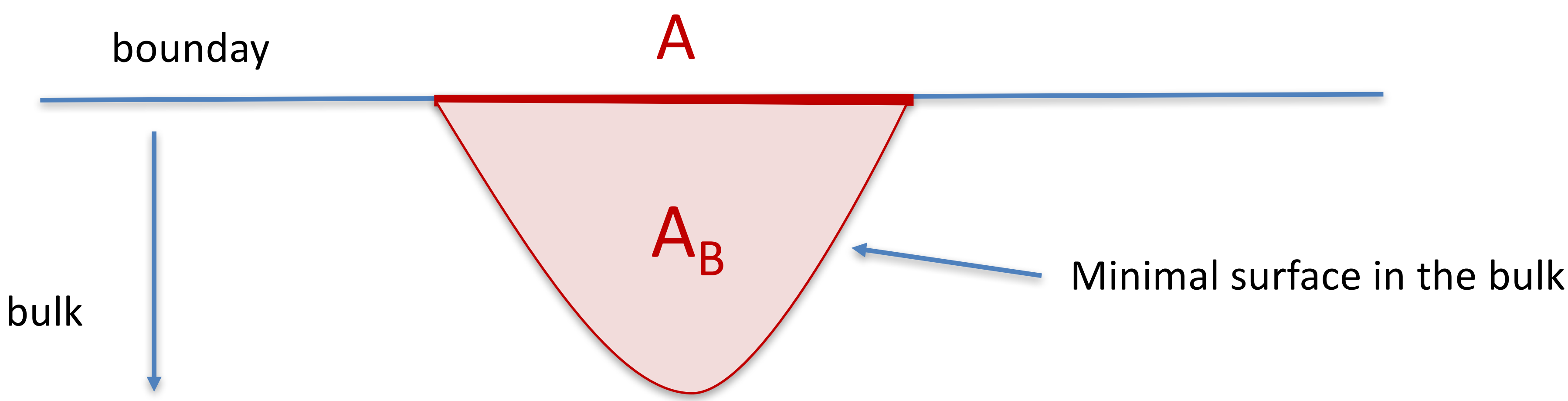
Implications for gravity + quantum fields

Wall, Faulkner, Balakirshnan, Khandker, Wang, Leigh, Parrikar…

Entropy as a tool to explore holography

- Main tool: Ryu-Takayanagi formula.

Ryu, Takayanagi, Hubbeny, Rangamani



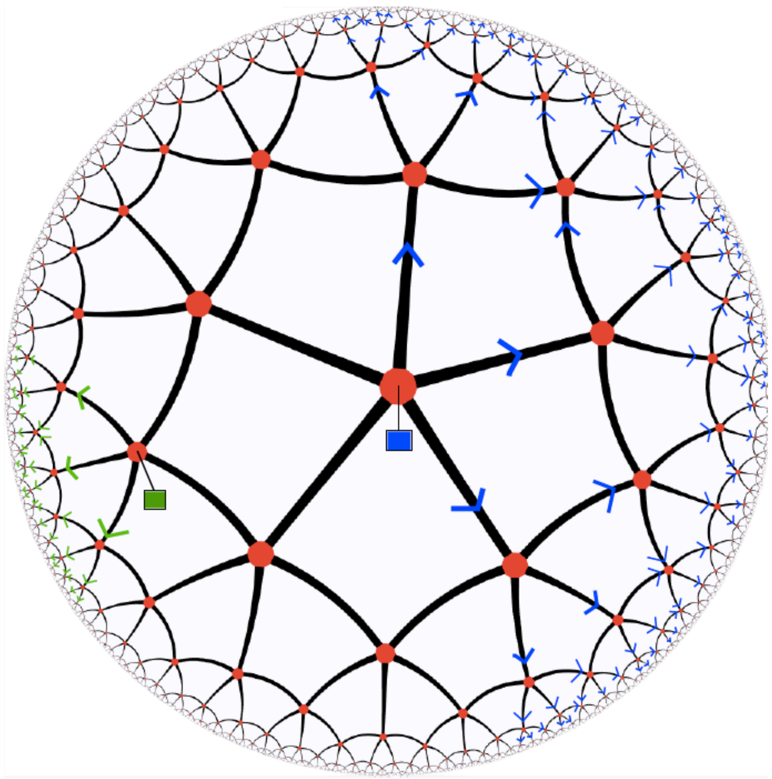
$$S(A) = \frac{(\mathrm{Area})_{\mathrm{minimal}}}{4G_N}$$

Region A_B is related to the subregion A.

Cracking the holographic code

Quantum error correction and holography

Almheiri, Dong, Harlow, Preskill, Yoshida, Pastawski



Protecting a qubit is setting in deeper in the bulk !

Complexity theory

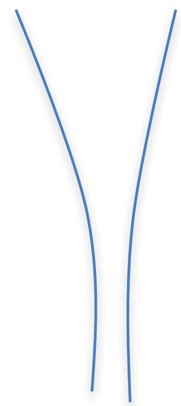
Harlow, Hayden, Brown, Susskind, Zhao, Roberts, Swingle, Stanford.

Now to dynamics

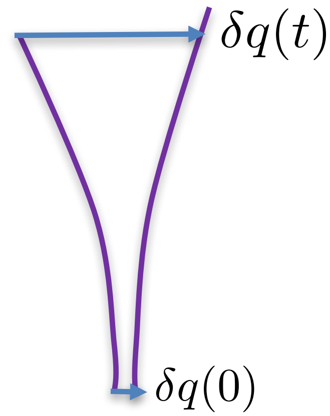
Universal bound on quantum chaos

inspired by black holes

Signature of classical chaos $\rightarrow$ divergence of nearby trajectories



Classical



$$\delta q(t) \propto e^{\lambda t}$$

$$\frac{\delta q(t)}{\delta q(0)} = \{q(t), p(0)\} \ , \qquad \{q(t), p(0)\}^2$$

Quantum

$$[Q(t), P(0)]^2 \qquad [Q_i(t), P_j(0)]^2$$

Quantum General:

$$\langle [W(t), V(0)]^2 \rangle_\beta \propto \frac{1}{S} e^{\lambda t}$$

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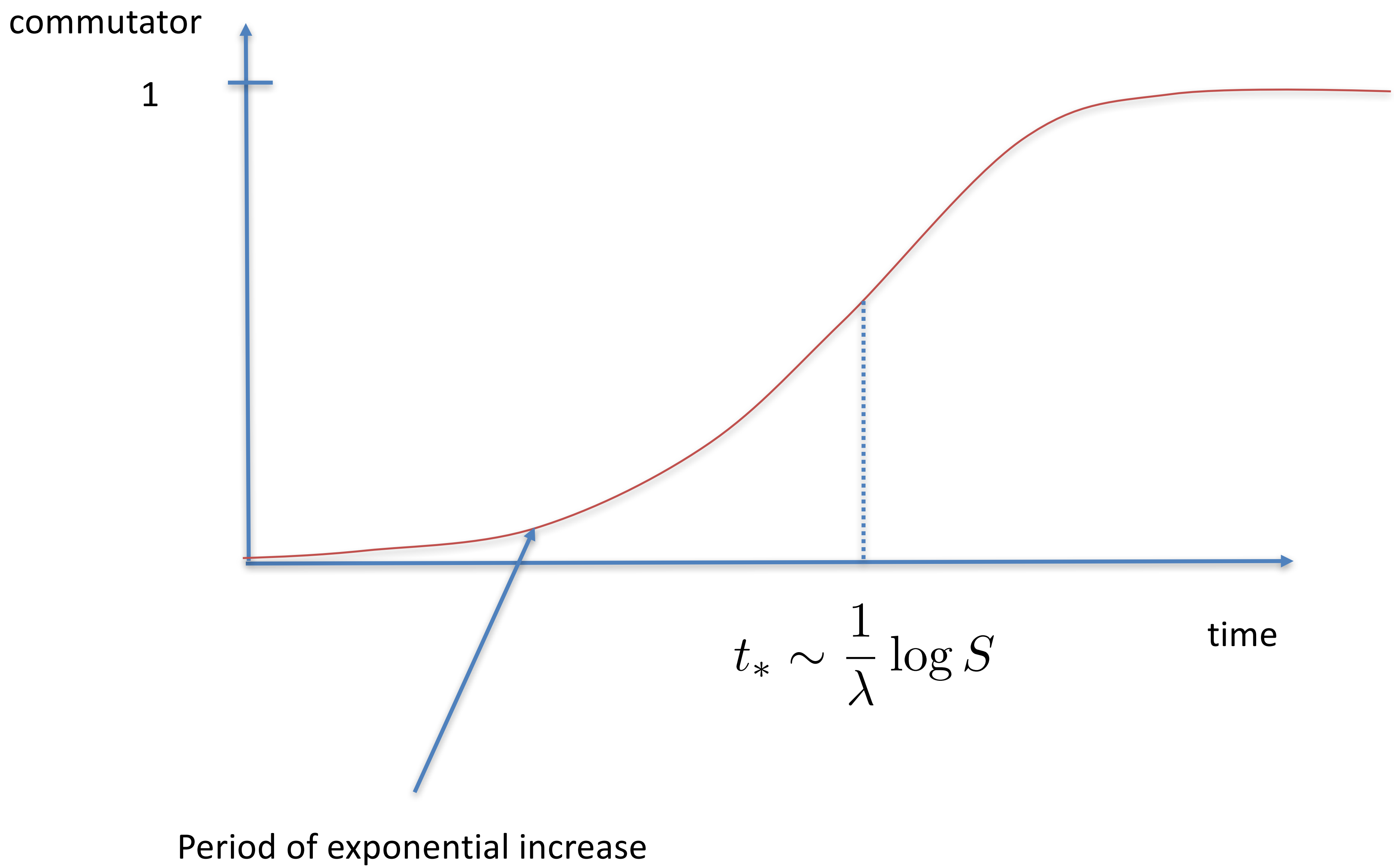
W, V are two ``simple'' (initially commuting) observables.

Imagine we have large entropy system, S>>1.

This is the definition of the quantum Liapunov exponent

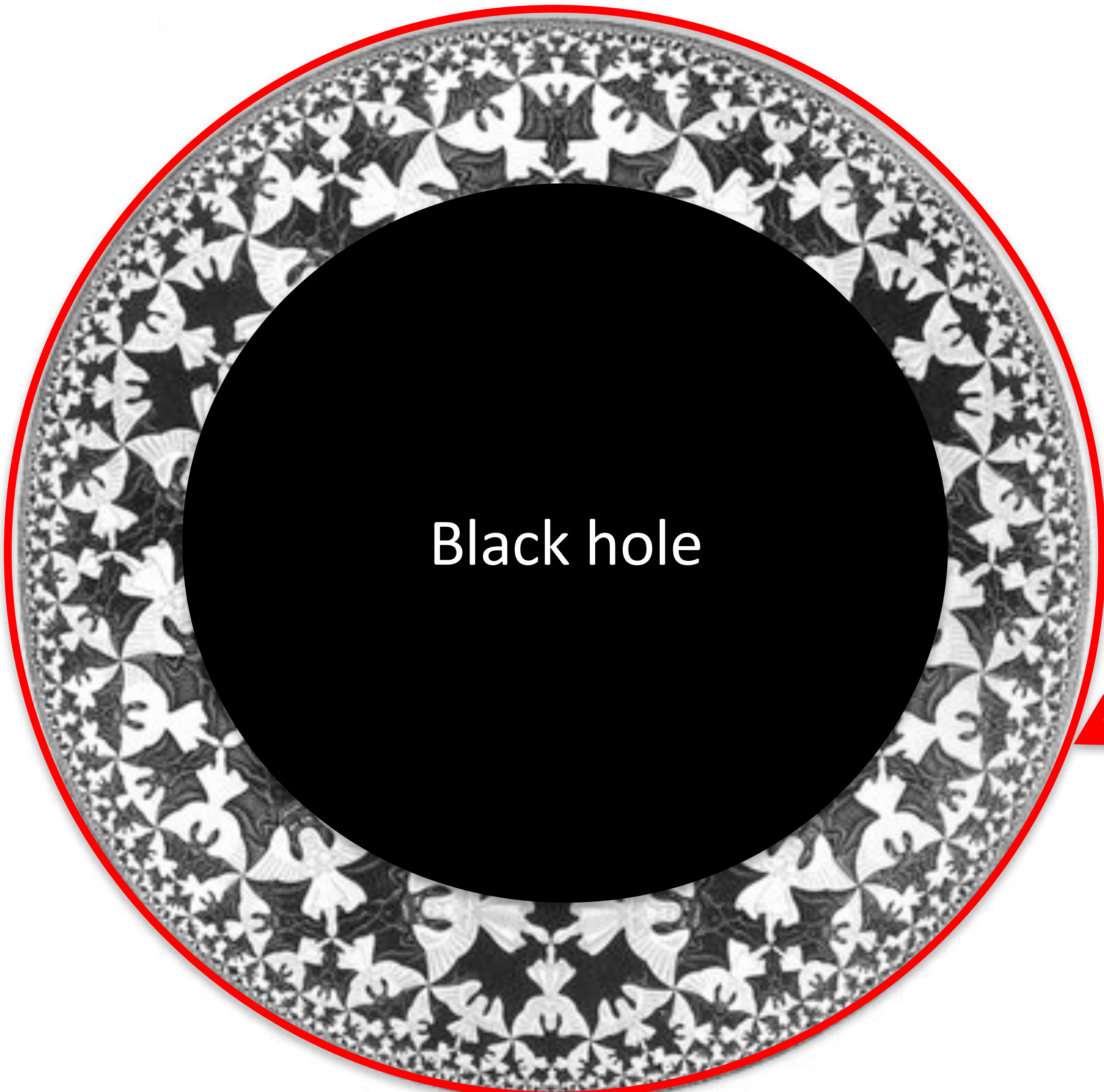
It is defined by its initial increase.

At very long times it saturates to a quantity of order one.


commutator
1
time
$t_* \sim \frac{1}{\lambda} \log S$
Period of exponential increase

Use now AdS/CFT and black holes

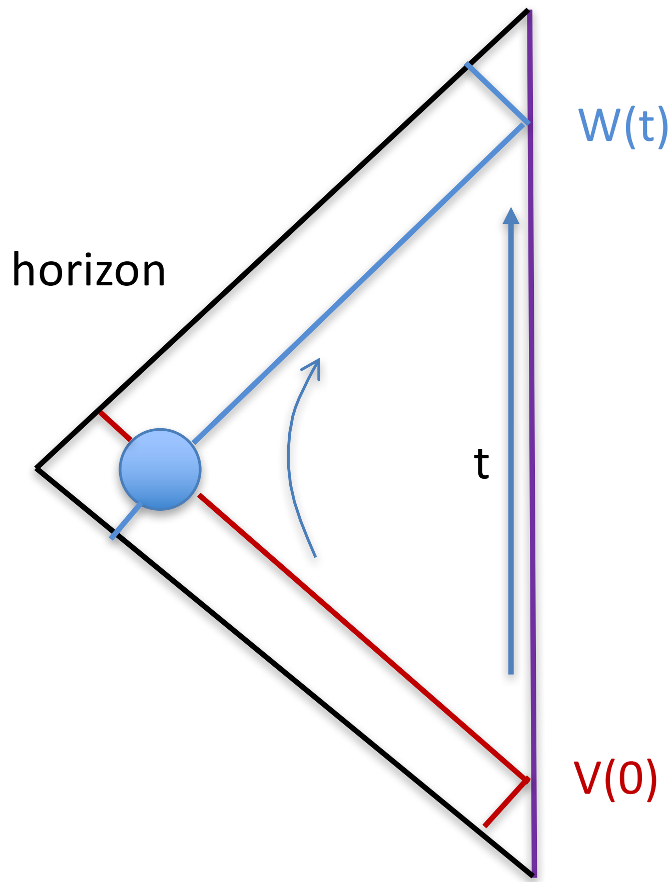
Black holes in a gravity box



Hot fluid made out of
very strongly
Interacting particles.

For quantum systems that have a gravity dual

Shenker Stanford
Kitaev



Commutator → involves the scattering amplitude between these two excitations.

Large t → large boost between the two particles.

Leading order → graviton exchange

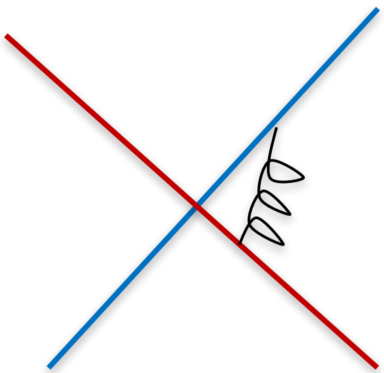
Gravitational interaction has spin 2, Shapiro time delay proportional to energy.

Energy goes as e^t

$$\langle [W(t), V(0)]^2 \rangle_\beta \propto \frac{1}{S} e^{\lambda t}$$

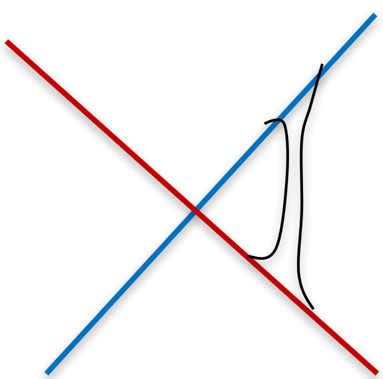
$$\lambda = \frac{2\pi}{\beta} = 2\pi T$$

Can it be different ?



Graviton → phase shift : $\delta(s) \sim G_N s \longrightarrow \lambda = \frac{2\pi}{\beta}$

Regge limit , s>>t, s >> 1



String → phase shift

Typical size of string
(of graviton in string theory)

$$\delta(s) \sim G_N s^{1+\alpha' t} \longrightarrow \lambda = \frac{2\pi}{\beta}(1 - \frac{l_s^2}{R^2})$$

Radius of curvature of black hole

s, t = Mandelstam invariants

It can be less…

More ?

In flat space a phase shift has to scale with a power of s less than one in order to have a causal theory

Maybe there is a bound…

Universal upper bound on chaos

$$\lambda \leq \frac{2\pi}{\beta} = 2\pi T = \frac{2\pi T}{\hbar}$$

Sekino Susskind

JM, Shenker, Stanford

Proof: Uses analyticity in Euclidean time, unitarity, and that simple observables thermalize.

For <u>any</u> large N (small $\hbar$) quantum system.

Black holes are the most chaotic systems

Are there others ?

Turn to a condensed matter inspired model

Sachdev Ye Kitaev model

Sachdev Ye
Kitaev
Georges, Parcollet

N Majorana fermions in 0 + 1 dimensions.
All to all interactions.

$$H = \sum_{i_1, \cdots, i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

Coefficient are random gaussian variables.

It has the virtue of being solvable at large N.

At low temperatures (but not exponentially small), the system is also maximally chaotic.

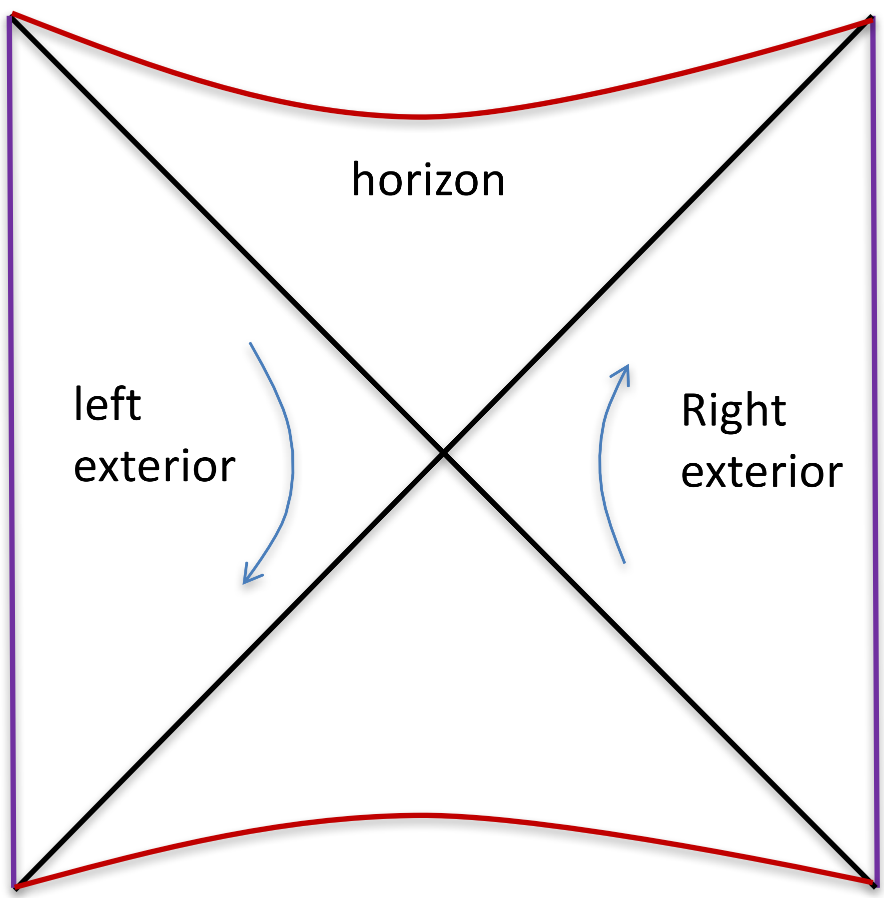
Few comments

- Thinking of black holes as quantum systems we were lead to a particular Liapunov exponent.
- We also used, for inspiration, properties of flat space scattering amplitudes.
- Final argument does not require black holes and is valid for general quantum systems.
- But it was useful to consider the interrelation between different points of view.

Entanglement and geometry:

Exploring wormholes

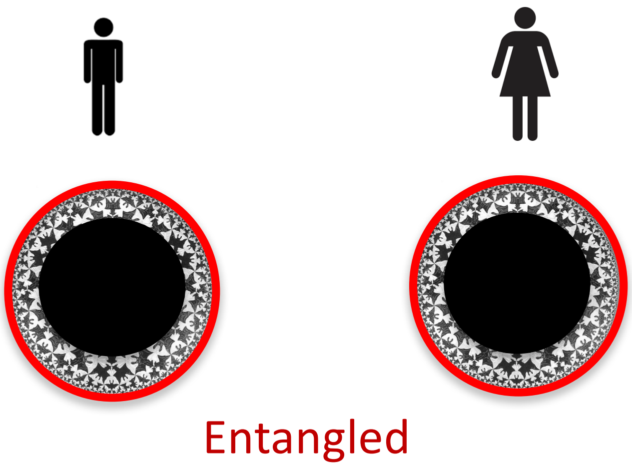
The full Schwarzschild wormhole

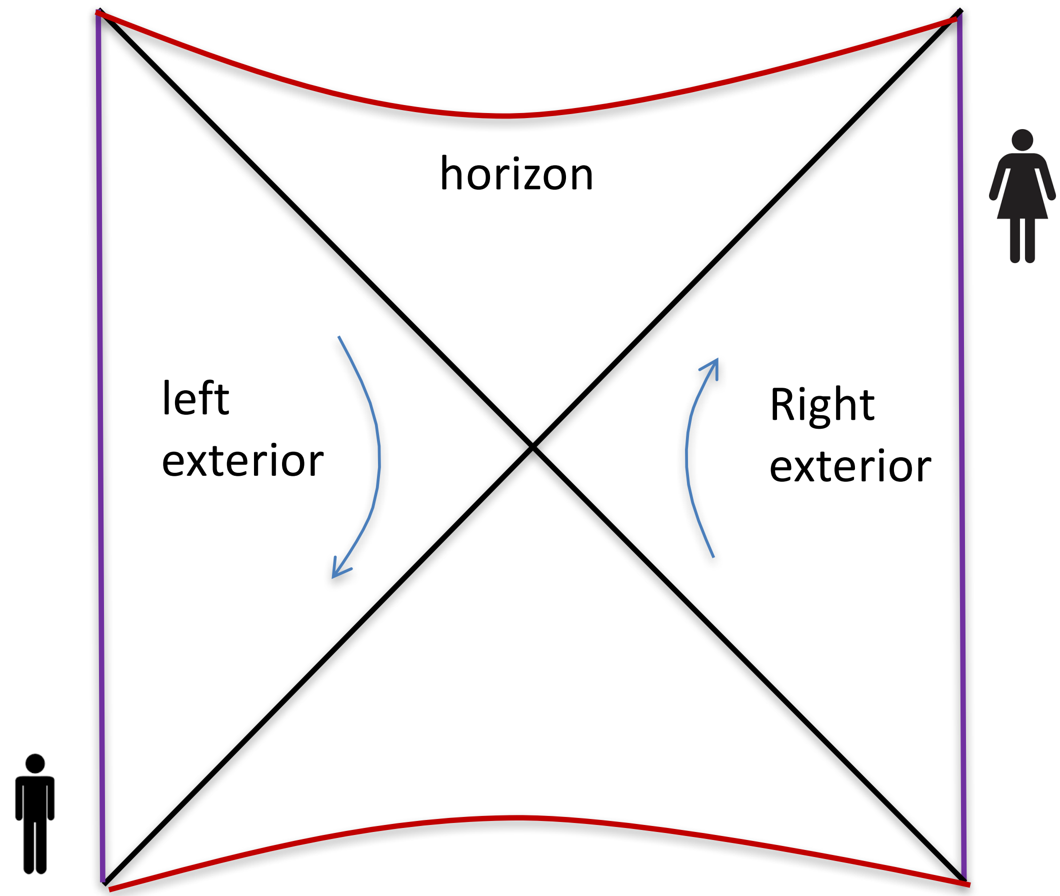


No need to postulate any exotic matter

No matter at all !

View it as an entangled state

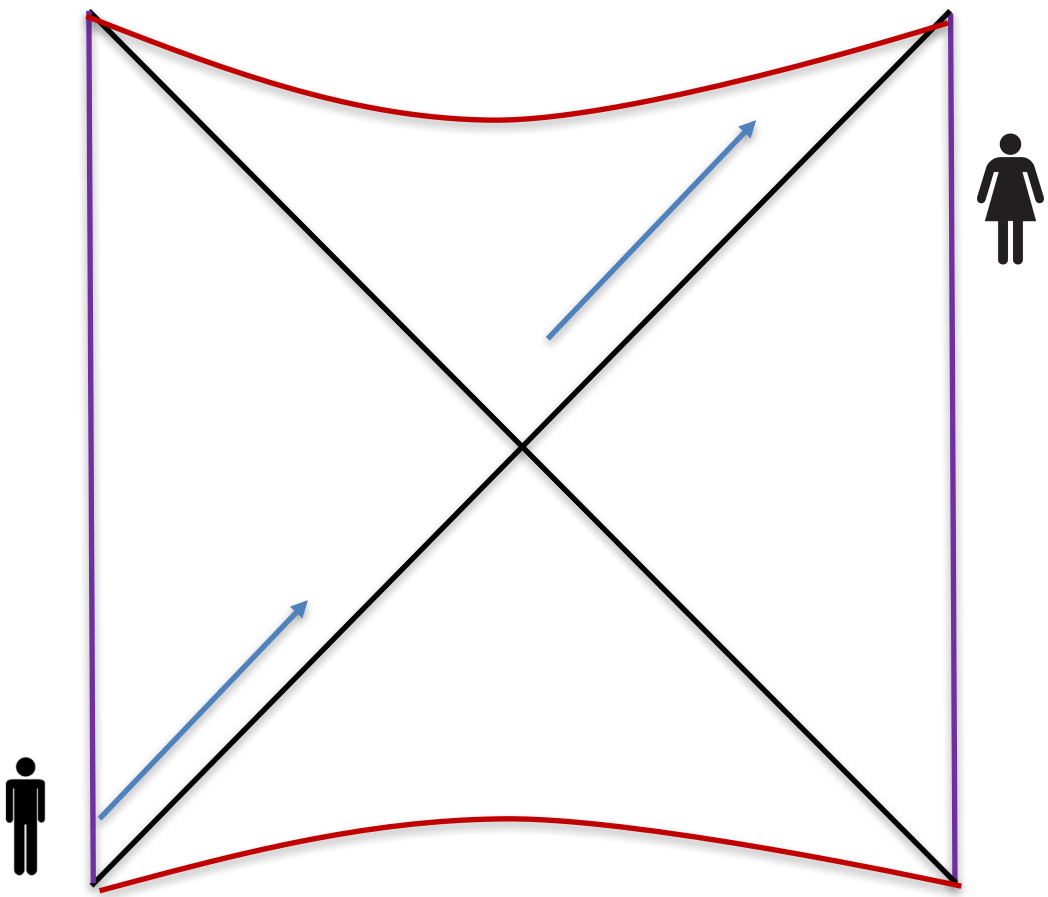




$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Israel 70's
JM 00's
JM Susskind

True causal separation



If Bob sends a signal , then
Alice cannot receive it.

These wormholes are not traversable,
due to the integrated null energy condition

$$0 \leq \int dx^- T_{--}$$

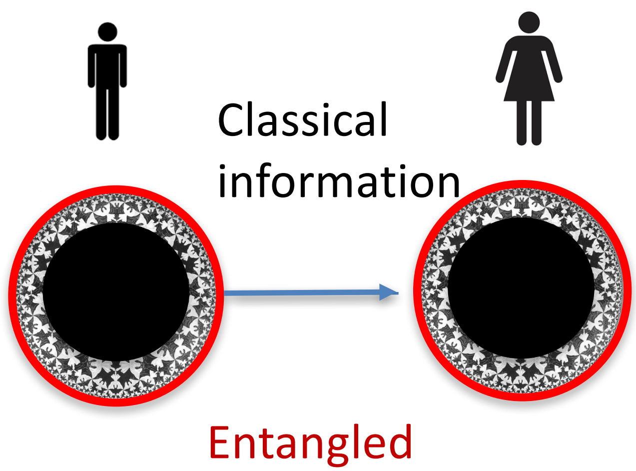
Balakrishnan, Faulkner,
Khandker, Wang

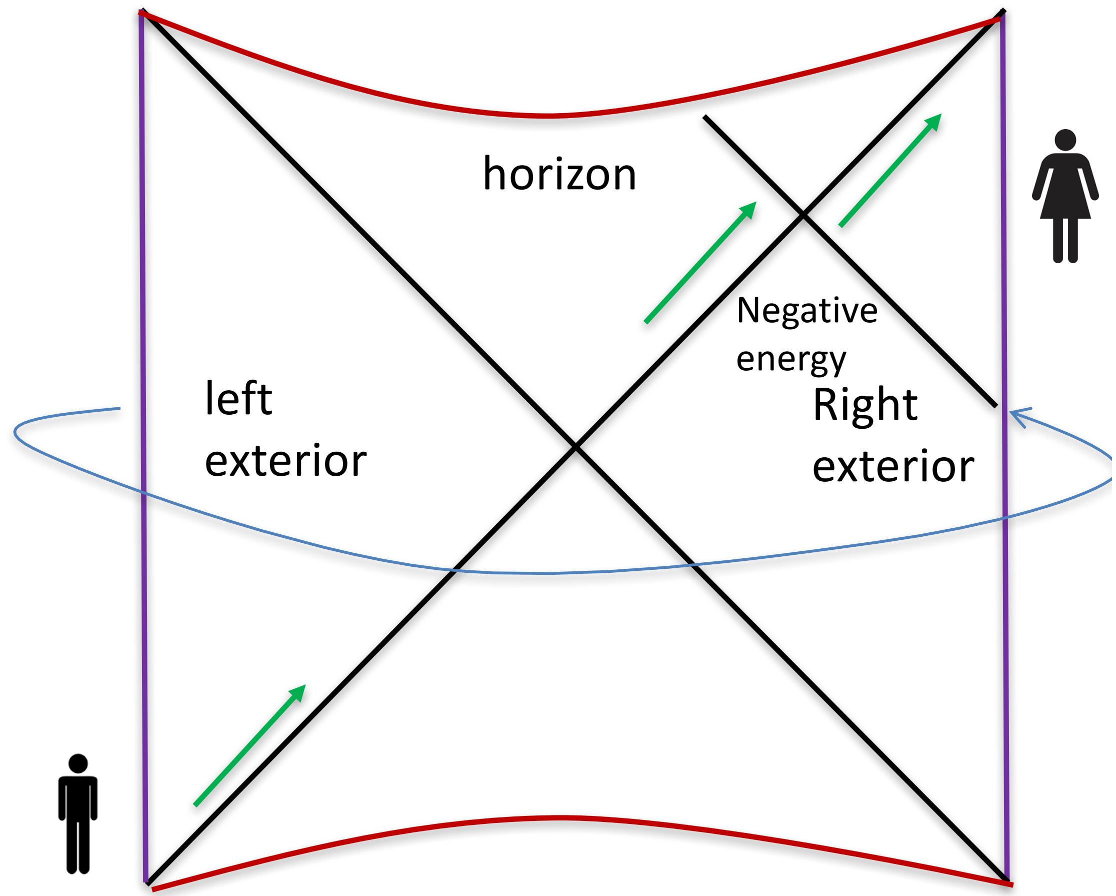
Not good for science fiction.

Good for science!

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Measure and send result → make wormhole traversable





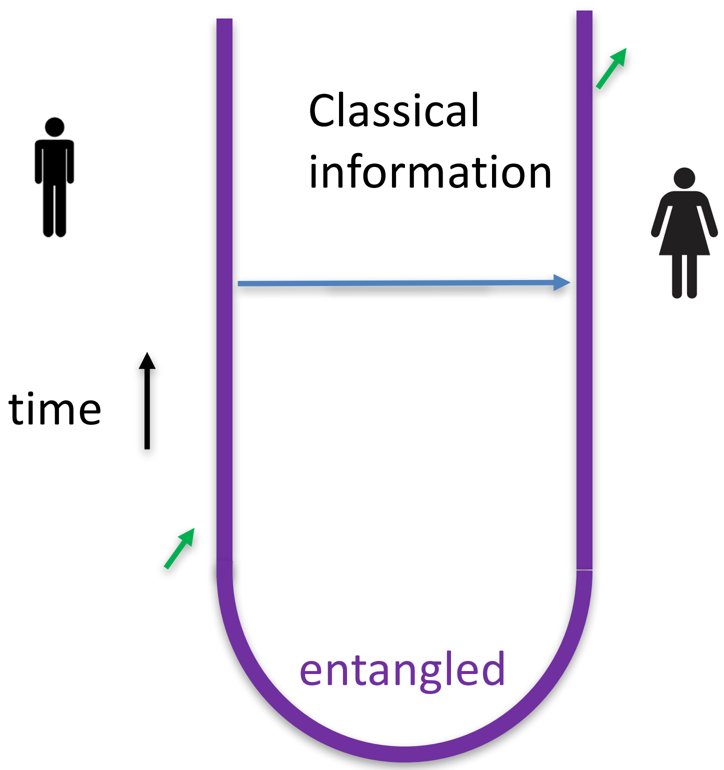
Teleported through the wormhole !

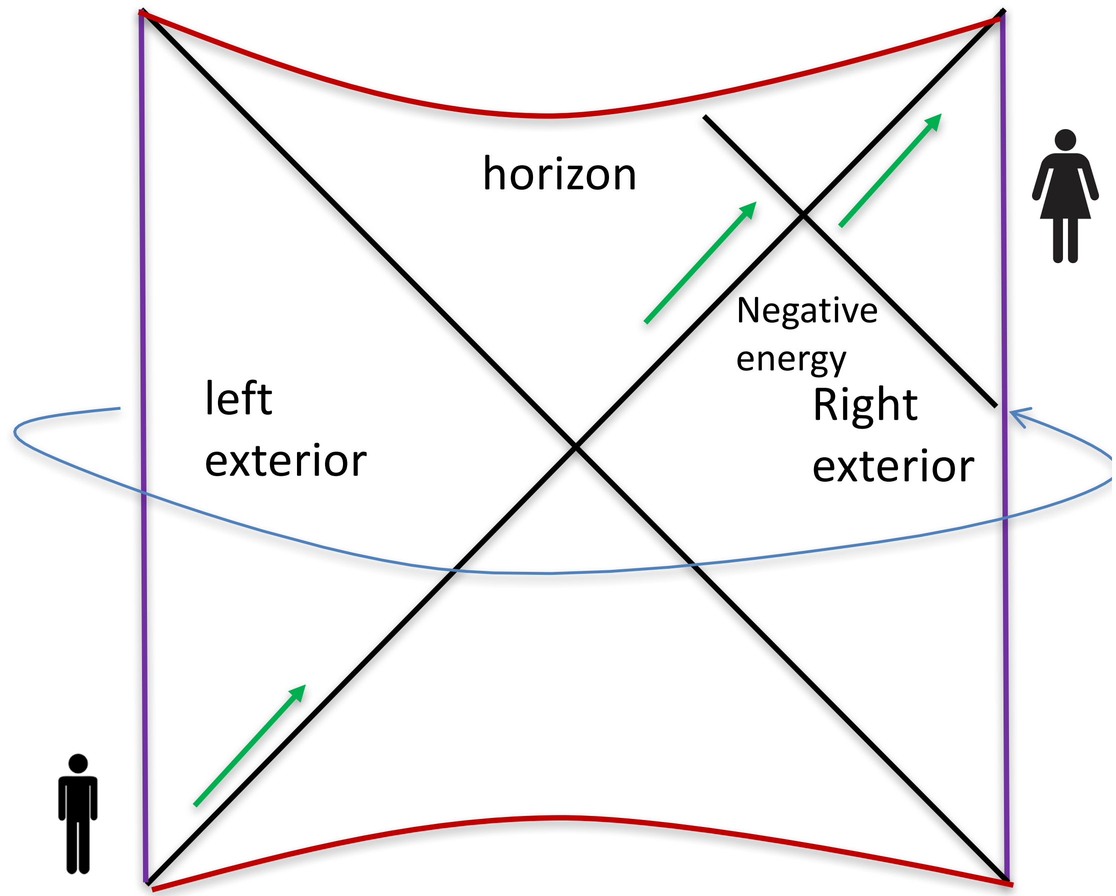
Susskind
Gao, Wall, Jafferis
JM, Stanford, Zhang

<u>Quantum Teleportation</u>: N qubits with 2 N classical bits and N entangled qubits.

Bennett, Brassard, Crepeau, Jozsa, Peres, Woetters

Measure and send result → make wormhole traversable





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Conclusions

- Entanglement entropy has emerged as a tool for exploring quantum field theory.
- Quantum error correction and holography
- Dynamical processes spread quantum information. We are exploring speed limits…
- Quantum teleportation can have a geometric picture as information flowing through a wormhole (in some cases).

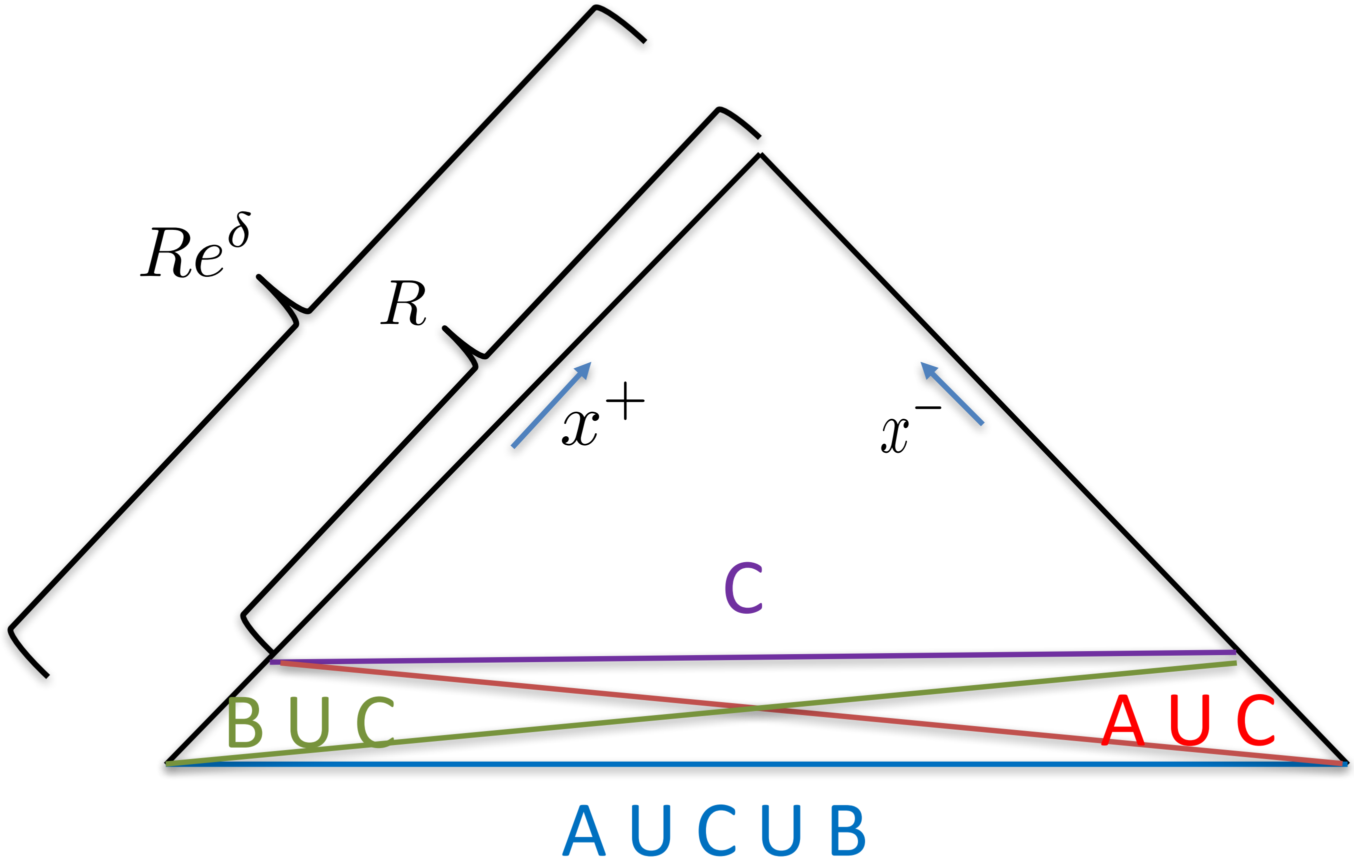
Thank you!

Extra Slides

- The list could go on…
- I will explain in some detail one particular example.

We will consider a 1 +1 dimensional quantum field theory

And derive the c-theorem.



Casini - Huerta

$$S(A \cup B \cup C) + S(C) \leq S(A \cup C) + S(B \cup C)$$

Entropy subaditivity
Lieb Ruskai

$$S(R^2 e^{2\delta}) + S(R^2) \leq S(R^2 e^{\delta}) + S(R^2 e^{\delta})$$

Entropy depends only on the (square of the) proper size of the interval.

$$(R^2 S'(R^2))' \leq 0$$

$$(R^2 S'(R^2))' \leq 0$$

For a CFT: $S(R^2) = \frac{c}{6}\log(R^2/\epsilon^2) \longrightarrow R^2 S'(R^2) = \frac{c}{6}$

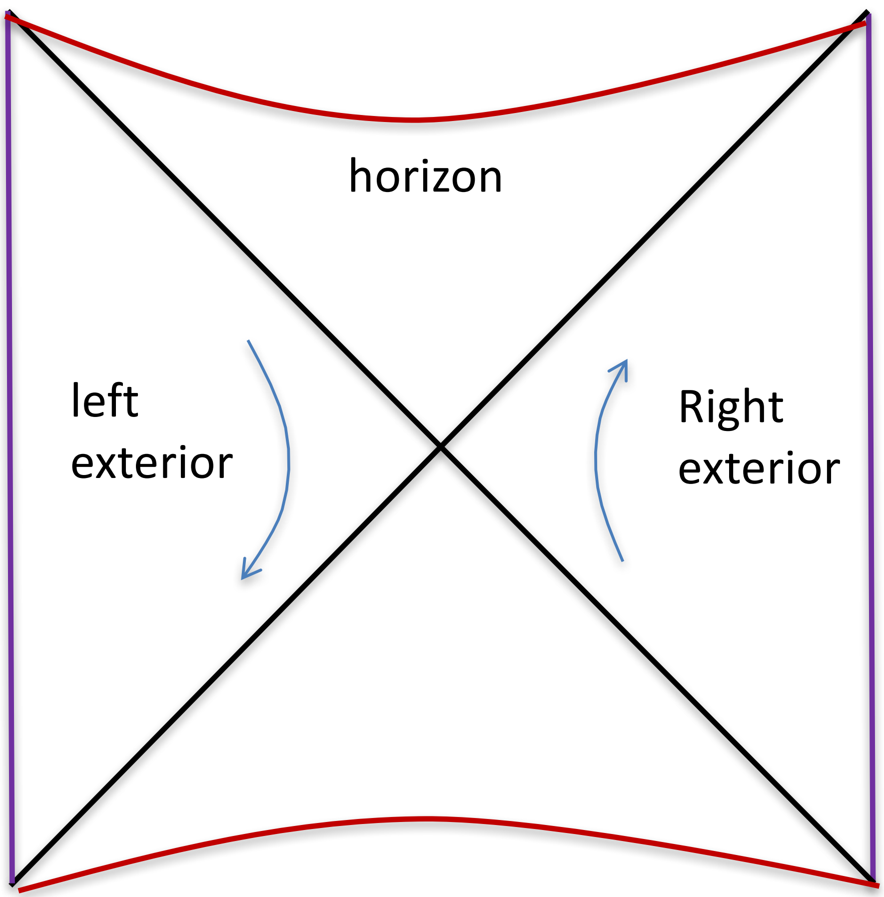
$c_{IR} \leq c_{UV}$ c - theorem Zamolodchikov

The argument in 2 + 1 is similar but it involves an large number of boosted circular regions.
(has not been proven in any other way) Casini, Huerta

In 3 +1 one needs to compare the entropy to the one of the UV fixed point.
And use that the entropy subaditivity is an equality for the case of regions lying on the lightcone
Casini, Teste, Torroba

Now to another application

Symmetry



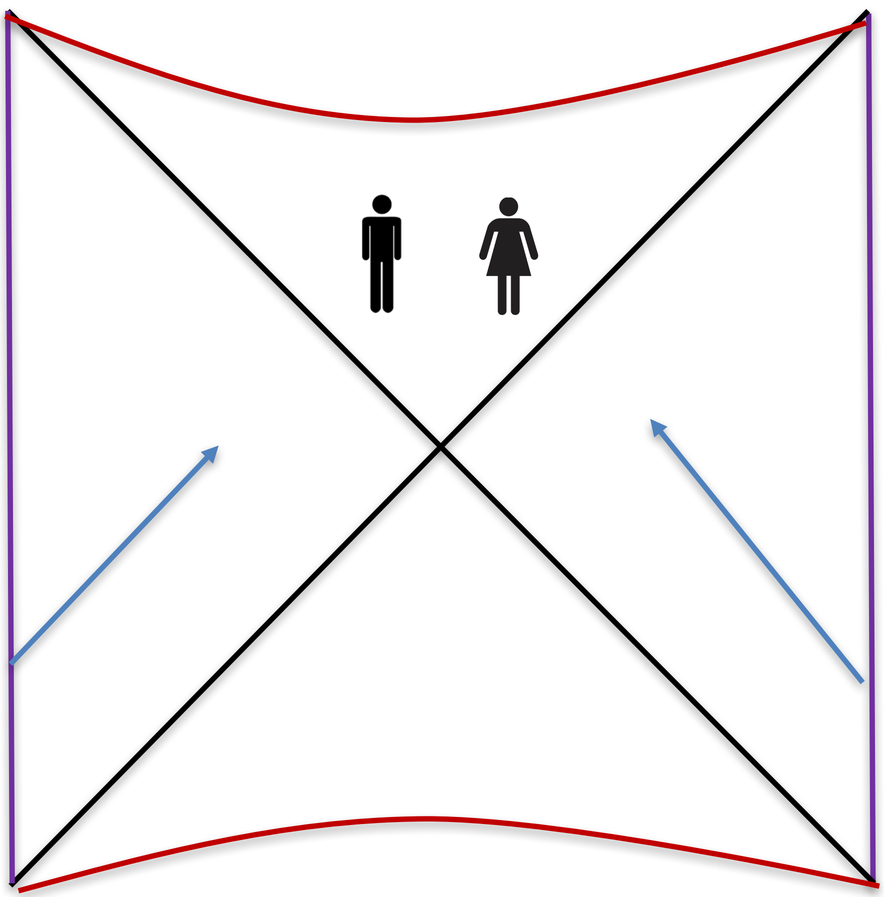
What is this funny
"time translation symmetry" ?

$$U = e^{it(H_R - H_L)}$$

Exact symmetry
Exact <u>boost</u> symmetry!

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Interior is common

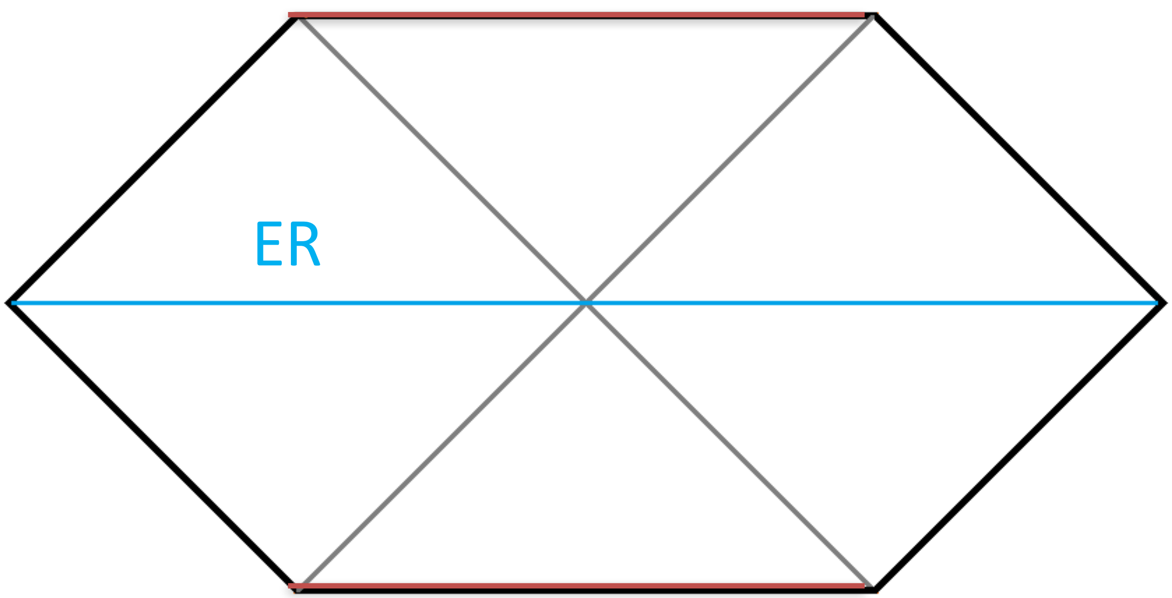


If they jump in,
they can meet in the interior !

But they cannot tell anyone.

$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$$

Spacetime connectivity from entanglement



ER = EPR

Van Raamsdonk
Verlinde^2
Papadodimas Raju

JM Susskind

Entanglement and geometry

Local boundary quantum bits are highly interacting and very entangled

Ryu, Takayanagi, Hubeny, Rangamani

$$S(R) = \frac{A_{\mathrm{min}}}{4G_N}$$

Generalization of the Black hole entropy formula.