Sommerfeld-Enhanced J-Factors for Dwarf Spheroidal Galaxies

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dwarfs, J-factors, and Sommerfeld

• searches for photons arising from dark matter annihilation in dwarf spheroidal galaxies (dSph)
  – indirect detection strategy with good control over background systematics
• photon flux from dSph, can be factorized into two pieces....
• ... a particle physics factor
  – depends on annihilation cross section, channel, particle mass
• and an astrophysics factor
  – depends on the dark matter density profile of the target
  – encoded in the J-factor
• but if DM annihilation is Sommerfeld-enhanced at low velocity
  – then velocity-distribution also come into play
• goal is to compute the Sommerfeld-enhanced J-factor ($J_s$) ... 
• ... and see impact on dSph searches
main features

• Sommerfeld-enhanced annihilation $\rightarrow \sigma v \propto 1/v$

• relative velocities in dSphs tend to be much smaller than in Milky Way

• can get a large enhancement to annihilation cross section

• considerable variation in velocity distribution between dSphs, and for different choices of density profile

• ordering of dwarf J-factors can change

• important implications for indirect detection searches
what is the J-factor?

- the photon flux depends on ....
  - particle physics of the dark matter model
    - independent of target
  - astrophysics of the target
    - mostly independent of dark matter model
- J-factor is the astrophysics factor
  - larger J = larger flux, regardless of particles physics model
- but factorization based on an assumption
  - $\sigma_A v$ independent of $v$
- what happens for Sommerfeld?

$$\frac{d\Phi}{dE} = \frac{1}{4 \pi} \frac{dN}{dE} \int_{\Delta \Omega} d\Omega \int d\ell$$

$$\int d^3 v_1 \frac{f(\vec{r}(\ell, \Omega), \vec{v}_1)}{m_X} \int d^3 v_2 \frac{f(\vec{r}(\ell, \Omega), \vec{v}_2)}{m_X}$$

$$\times \frac{\langle \sigma_A |\vec{v}_1 - \vec{v}_2| \rangle}{2}$$

$$= \frac{\langle \sigma_A v \rangle dN}{8 \pi m_X^2 dE} \times J$$

$$J \equiv \int_{\Delta \Omega} d\Omega \int d\ell \left[ \rho(\vec{r}(\ell, \Omega)) \right]^2$$

$$\rho(\vec{r}) = \int d^3 v f(\vec{r}, \vec{v})$$

f = dark matter velocity distribution
Sommerfeld-enhancement

- essential setup
  - dark matter annihilation is a contact interaction
  - but dark matter self-interacts through a long range force
    - mediator mass = $m_\Phi$
  - so have to rescale matrix element by wavefunction at the origin
- actual potential is Yukawa
  - can solve numerically
  - but can solve analytically if we approximate it with a Hulthén potential (within 10%)

- $\langle \sigma_A v \rangle \equiv \langle \sigma_A v \rangle_0 \times S(v)$
- $V(r) = -(\alpha_X / r) \exp(-m_\Phi r)$
- four regimes for Hulthén
- $m_\Phi \gg \alpha_X m_X :$ non-enhanced
  - $S = 1$
- $m_\Phi \ll \alpha_X m_X :$ Coulomb limit
  - $S(v) = 2\pi \alpha_X / v$
- $\alpha_X m_X \ll m_\Phi \ll \alpha_X m_X :$ saturation
  - $S(v) = 16 \alpha_X m_X / m_\Phi$
- $m_\Phi = 6\alpha_X m_X / (\pi^2 n^2) \ll \alpha_X m_X :$ resonance
  - $S = 4\alpha_X^2 / v^2 n^2$ (cutoff at small v)
- focus: non-enhanced v. Coulomb
defining $J_S$

- just need to absorb $S(v)$ into definition of astrophysical factor
- new factor, $J_S$, encodes astro. info needed to determine $d\Phi/dE$ for Sommerfeld-enhanced case
- need the DM velocity distribution
  - obtain it from stellar data, using Eddington formula
- what it amounts to:
  - assume $f(r,\nu)$ spherically-sym., isotropic
  - then $f$ depends only on $\epsilon = \nu^2/2 + \Psi(r)$
  - $\rho(r)$ determines $f(r,\nu)$

\[
J_S \equiv \int_{\Delta \Omega} d\Omega \int d\ell \\
\int d^3\nu_1 f(r(\ell,\Omega), \nu_1) \int d^3\nu_2 f(r(\ell,\Omega), \nu_2) \\
\times S(|\nu_1 - \nu_2|)
\]

\[
\frac{d\Phi}{dE} = \frac{\langle \sigma_A \nu \rangle_0}{8\pi m_X^2} \frac{dN}{dE} \times J_S
\]

$\Psi(r)$ = gravitational potential
determining \( f(r,v) \)

- **strategy**
  - assume NFW profile
    - just need parameters
    - fixes gravitational potential
  - assume Plummer stellar profile
    - find stellar velocity dispersion using Eddington formula and NFW gravitational potential
  - find NFW parameters by matching stellar velocity relation, Aquarius \( V_{\text{max}} - r_{\text{max}} \) relation
  - now Eddington formula determines DM velocity distribution

\[
\rho_{\text{NFW}}(r) = \frac{\rho_s}{\left( \frac{r}{r_s} \right) \left( 1 + \frac{r}{r_s} \right)^2}
\]

\[
\psi_{\text{NFW}}(r) = -4\pi G_N \rho_s \frac{r_s^3}{r} \ln \left( 1 + \frac{r}{r_s} \right)
\]

\[
f(\varepsilon) = \frac{1}{\sqrt{8\pi}^2} \int_{\varepsilon}^{0} \frac{d\psi}{\sqrt{\varepsilon - \psi}} \frac{d^2\rho}{d\psi^2}
\]

\[
\varepsilon \equiv \frac{v^2}{2} + \psi(r) < 0
\]

\[
\rho(r) = 4\pi \int_0^{\sqrt{-2\psi(r)}} dv \ v^2 f(r,v)
\]
velocity profiles

\[ V_{\text{max}} = 0.465(4\pi G \rho_s r_s^2)^{1/2} \text{max. circ velocity, at radius } r_{\text{max}} = 2.16 r_s \]

intersection of stellar fit and fit to Aquarius (Martinez, Bullock, Kaplinghat, Strigari, Trotta 0902.4715)
$J_S$  

$\epsilon_\phi \equiv m_\phi / \alpha_x m_x$

resonances

Reticulum II

Coma Berenices

Segue 1

Coulomb $\propto \alpha_x = 10^{-2}$

non-enhanced

$\Delta \Omega = 2.4 \times 10^{-4}$
upshot

• ordering of $J_S$-factors can change between “ordinary” s-wave limit and Sommerfeld-enhanced Coulomb limit
• affects how we would interpret any gamma-ray excess
• suppose we see an excess in a dwarf
  – ask if an excess is seen in other dwarfs with larger $J$-factors, where you expect a larger flux
  – if not, would call into question the dark matter interpretation
  – but using $J_S$-factor may resolve the tension
• similarly, if multiple excesses are seen, the pattern may point to Sommerfeld-enhancement (or not)
• applications extend to any new dwarfs which are found
  – potential to find excesses in new dwarfs
  – important part of analysis of dark matter interpretation
a general analysis

- say $\rho(r) = \rho_s \tilde{\rho}(r / r_s)$
- say we integrate J-factor over essentially entire dwarf
- $J, J_S$-factors \textit{parametrically determined} by dimen. analysis
  - $V_{\text{max}} \propto (G_N \rho_s)^{1/2} r_s$ (Virial Thm)
  - $J \propto \rho_s^2 r_s^3 / D^2 \propto V_{\text{max}}^4 / r_{\text{max}} D^2$
  - $J_S \propto \rho_s^{3/2} r_s^2 / D^2 \propto V_{\text{max}}^3 / r_{\text{max}} D^2$
    - Coulomb limit
- if one point is to upper left of another, J-$J_S$ ordering changes
- valid in the large angle limit, but instructive even for fixed angle

D = distance to dwarf
conclusion

- J-factors of dwarf spheroidal galaxies change dramatically if dark matter annihilation is Sommerfeld-enhanced
- J-factors enhanced, as expected, but ordering can change
- modifies standard consistency check for dark matter interpretation

- more dSphs being found by DES, etc. (see talk by Alex Drlica-Wagner)
- if excesses are found, need to remember Sommerfeld-enhancement when checking consistency
- Reticulum II?

Mahalo!
Back-up slides
idea behind Eddington formalism

• velocity distribution $f(r,v)$ is essentially the phase space density
• assume particles move only under a collective gravitational central potential (not two-body scattering)
• classical path depends only on integrals of motion, $E$ and $L$
• Jean’s Theorem – phase space distribution depends only on integrals of motion --> why?
  – if two phase space points have the same integrals of motion, any particles at one point will be (or once were) at the other
  – phase space density along path is constant (Liouville’s Theorem)
  – so phase space density has to be a function on only the integrals of motion
• if velocity distribution is spherically symmetric (depends on $r$, not $r$) and isotropic (depends on $v$, not $v$), then velocity distribution depends only on $E$, not $L$
Plummer profile

• fit M and a from stellar data
• \( r_h \sim 1.3 \ a = \) half-light radius
• from Eddington formula, stellar velocity dispersion now depends on NFW parameters, \( \rho_s \) and \( r_s \)
• matching to stellar velocity dispersion to data determines an allowed band for \( r_{\text{max}} \), \( V_{\text{max}} \)

\[
\rho_p(r) = \left( \frac{3M}{4\pi a^3} \right) \left( 1 + \frac{r^2}{a^2} \right)^{-\frac{5}{2}}
\]
Hulthen potential

\[ V_H(r) = -\frac{\alpha_x \left( \frac{\pi^2 m_\phi}{6} \right) e^{-\left(\frac{\pi^2 m_\phi}{6}\right)r}}{1 - e^{-\left(\frac{\pi^2 m_\phi}{6}\right)r}} - \frac{\alpha_x e^{-m_\phi r}}{r} \]

\[ S(v) = \frac{\pi}{\varepsilon_v} \frac{\sinh \left( \frac{2\pi \varepsilon_v}{\pi^2 \varepsilon_\phi / 6} \right)}{\cosh \left( \frac{2\pi \varepsilon_v}{\pi^2 \varepsilon_\phi / 6} \right) - \cos \left( 2\pi \frac{1}{\pi^2 \varepsilon_\phi / 6} - \frac{\varepsilon_v^2}{\left( \pi^2 \varepsilon_\phi / 6 \right)^2} \right)} \]

\[ \varepsilon_v \equiv \frac{v}{2\alpha_x} \]

\[ \varepsilon_\phi \equiv \frac{m_\phi}{\alpha_x m_x} \]

Cassel, 0903.5307