Semileptonic B-meson decays to light pseudoscalar mesons with lattice QCD

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for the Fermilab Lattice and MILC Collaborations

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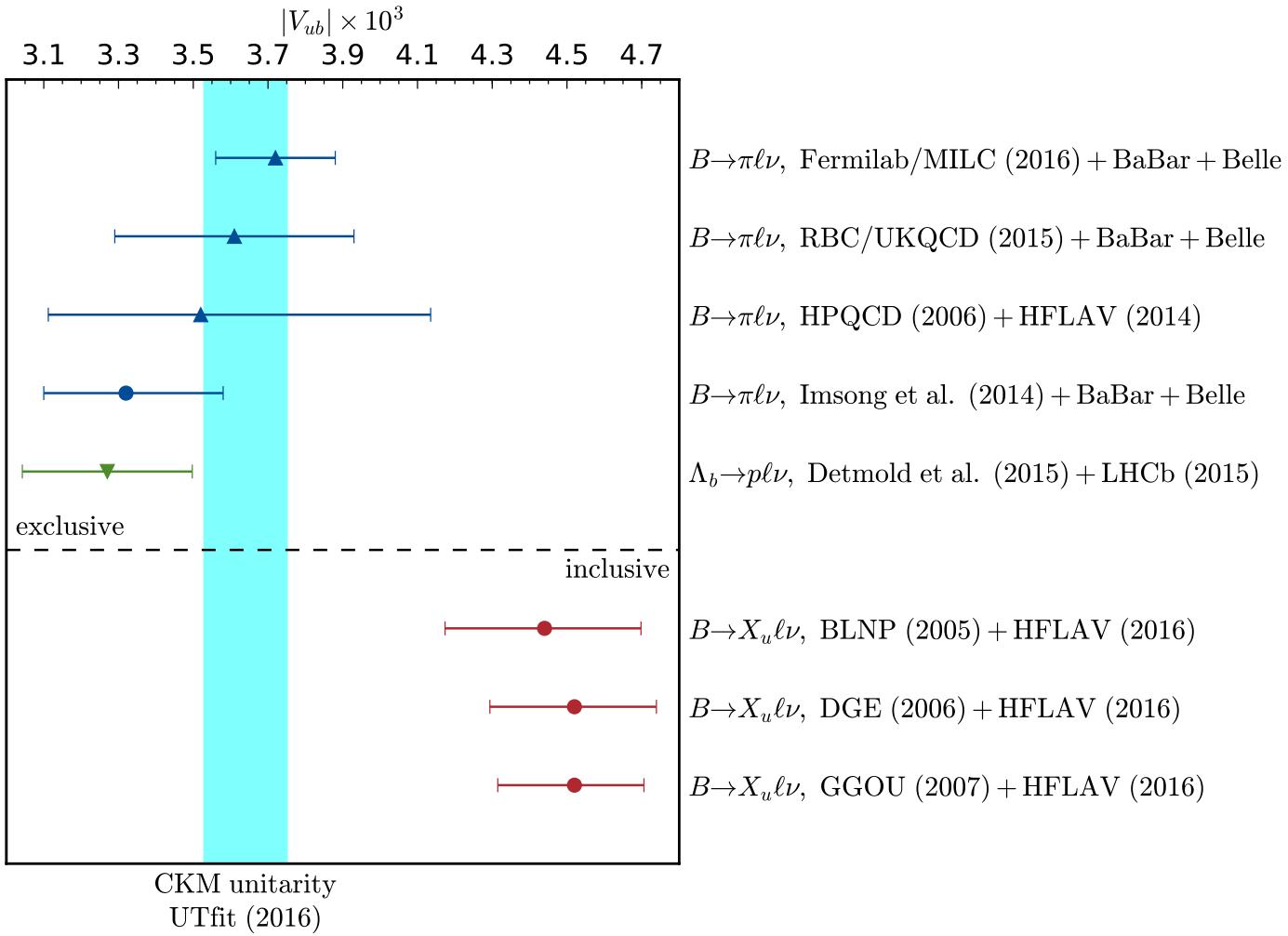
Fermilab
50 Years of Discovery

The University of Iowa

Overview

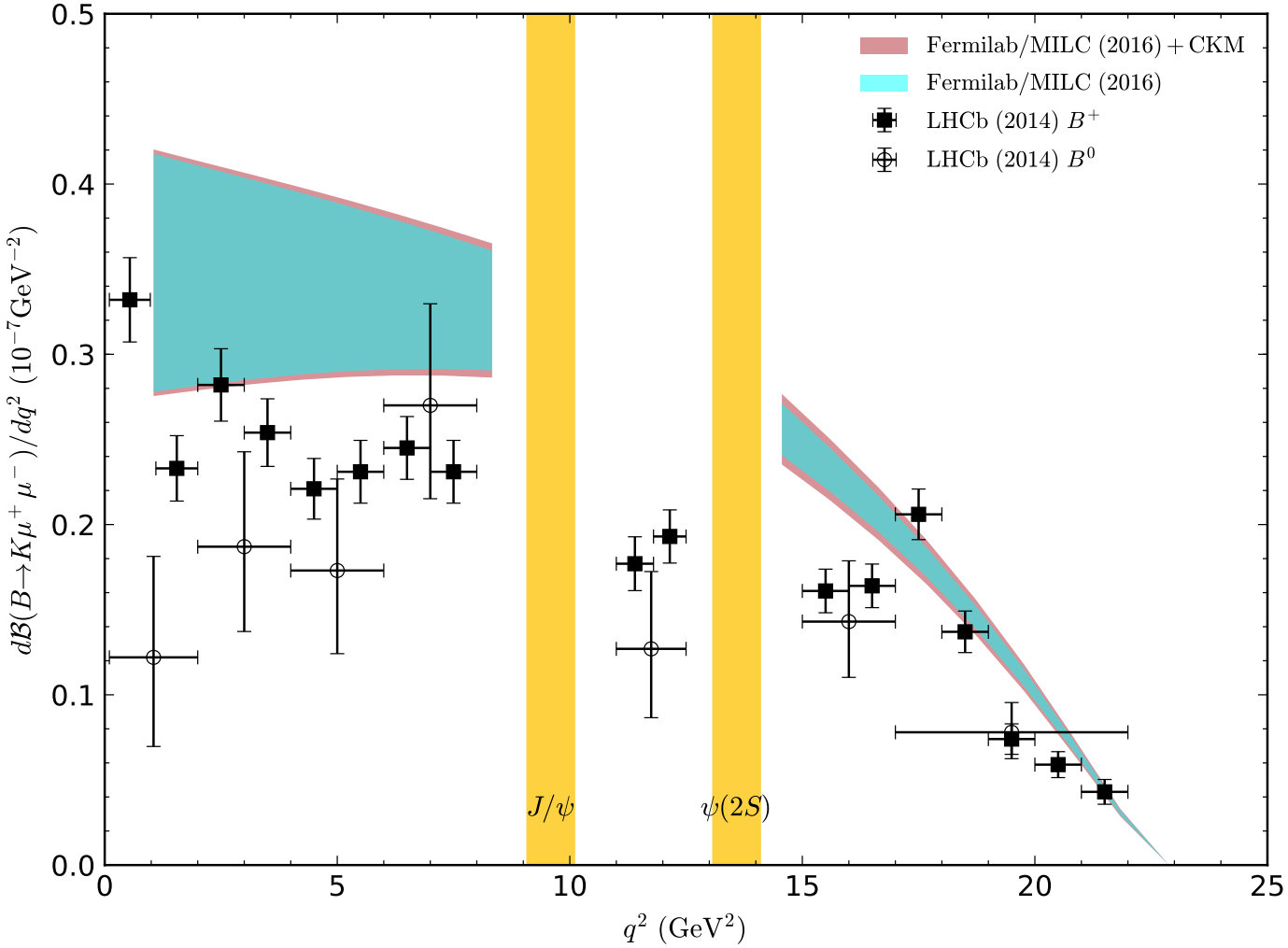
Part I — Introduction

Status of $|V_{ub}|$



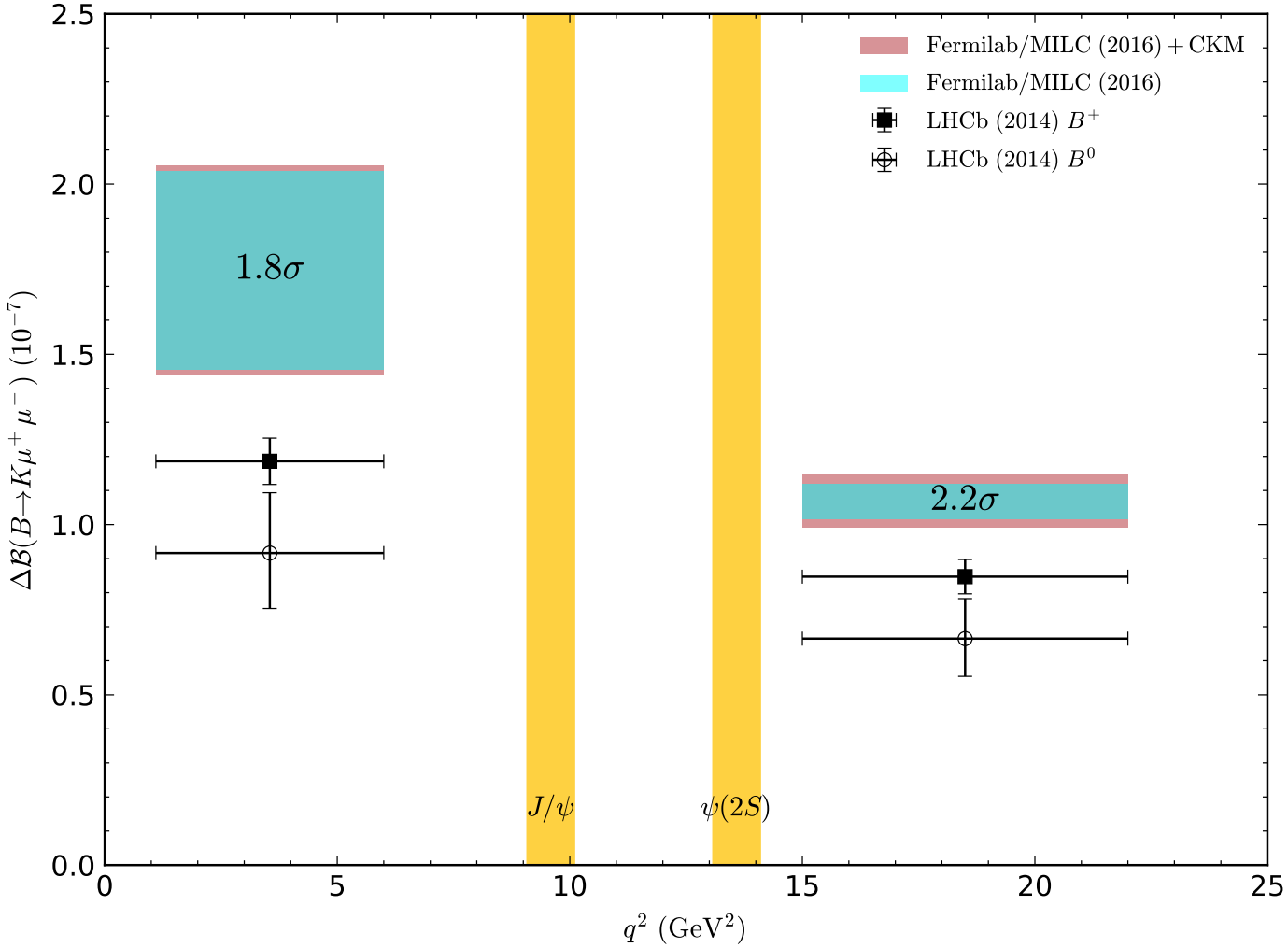
Update of plot in arXiv:1503.07839

Tension in $B \to K\mu^+\mu^-$



Older, less precise experiments omitted; cf. arXiv:1510.02349

Tension in $B \to K\mu^+\mu^-$



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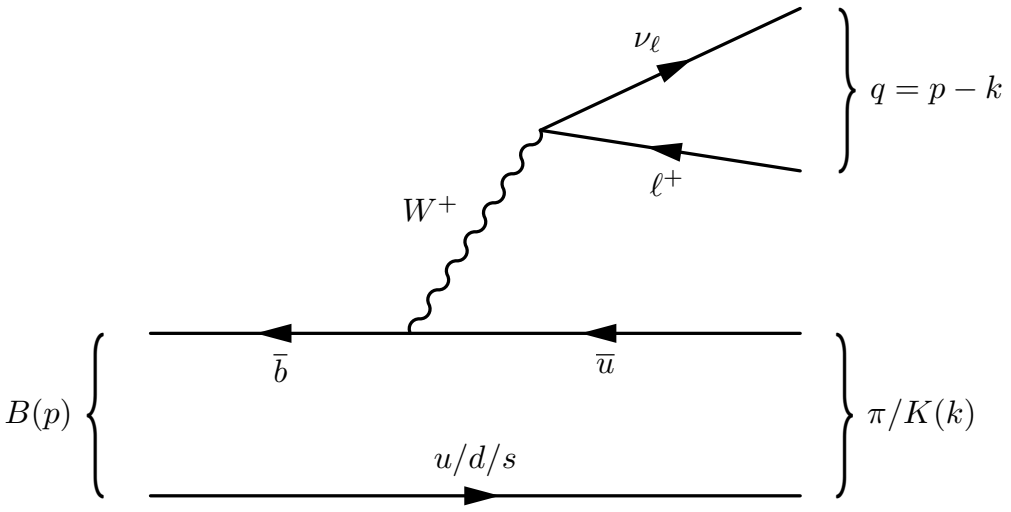
$B_{(s)} \to \pi(K)\ell\nu$: charged currents



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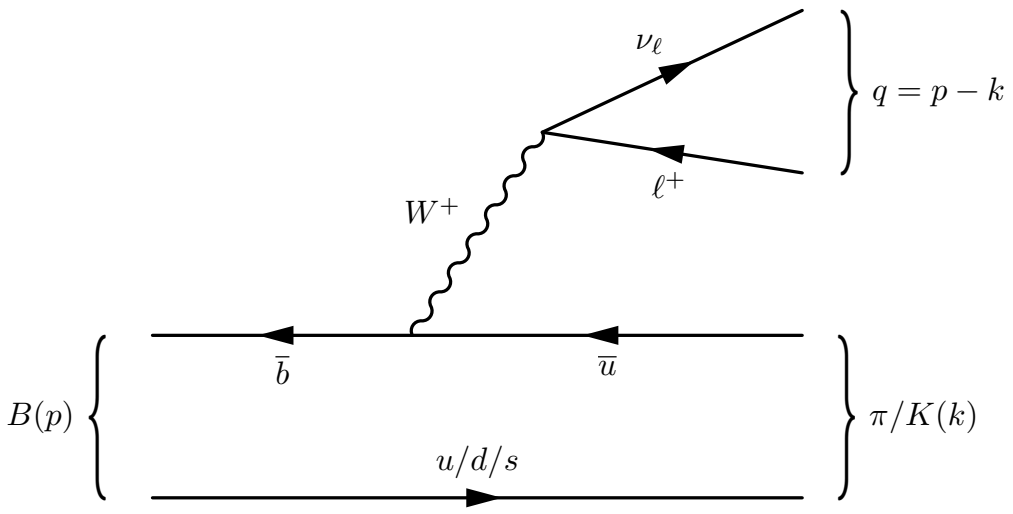
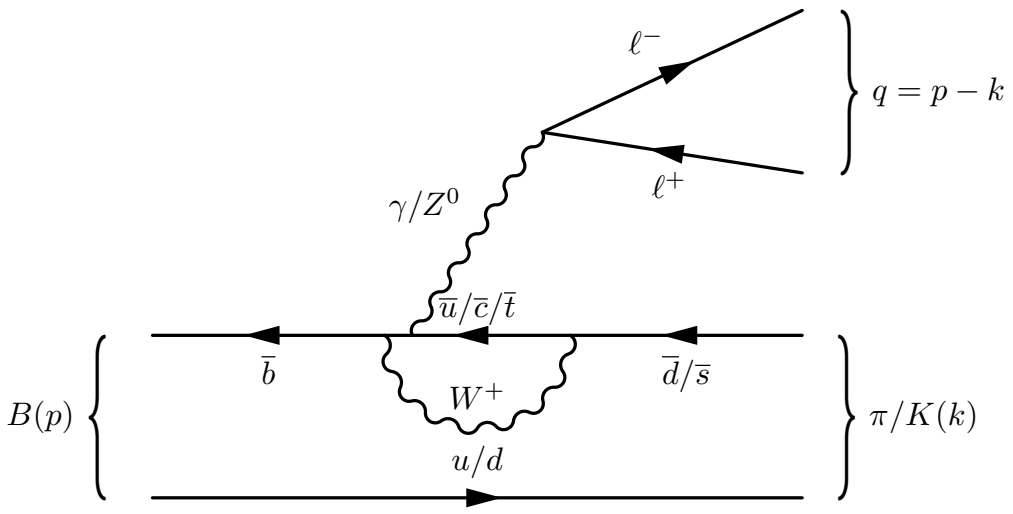


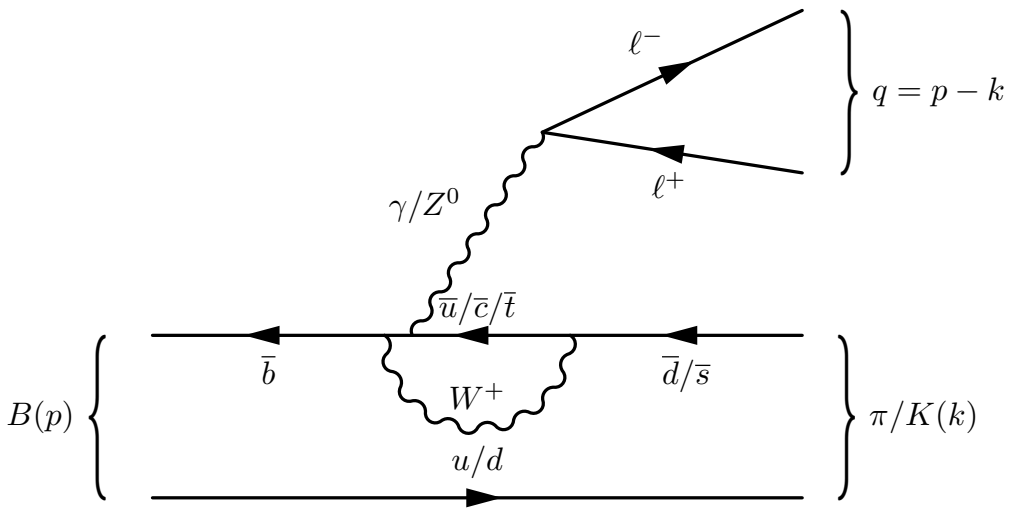
$$\frac{d\Gamma}{dq^2} = \frac{C_P G_F^2 |V_{ub}|^2}{3\,(2\pi)^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 |\boldsymbol{k}| \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) |\boldsymbol{k}|^2 \left| f_+(q^2) \right|^2 + \frac{3m_\ell^2 \left(M_B^2 - M_P^2\right)^2}{8q^2 M_B^2} \left| f_0(q^2) \right|^2 \right],$$

where $B = B,\ B_s$ and $P = \pi,\ K$.

$B \to \pi(K)\ell^+\ell^-$: flavor-changing neutral currents



$B \to \pi(K)\ell^+\ell^-$: flavor-changing neutral currents



$$\frac{d\Gamma}{dq^2} = \frac{C_P G_F^2 \alpha^2 |V_{tb} V_{tq}^*|^2}{4\,(2\pi)^5} \beta_\ell |\boldsymbol{k}| \left[\frac{2}{3}\beta_\ell^2 |\boldsymbol{k}|^2 \left| C_{10}^{\text{eff}} f_+(q^2) \right|^2 + \frac{m_\ell^2 \left(M_B^2 - M_P^2\right)^2}{q^2 M_B^2} \left| C_{10}^{\text{eff}} f_0(q^2) \right|^2 \right.$$
$$\left. + \left(1 - \frac{1}{3}\beta_\ell^2\right) |\boldsymbol{k}|^2 \left| C_9^{\text{eff}} f_+(q^2) + 2C_7^{\text{eff}} \frac{m_b + m_q}{M_B + M_P} f_T(q^2) \right|^2 \right],$$

where $q = d,\ s$; C_i^{eff} are Wilson coefficients; and $\beta_\ell^2 = 1 - 4m_\ell^2/q^2$.

Form factors I

These transitions can be mediated by vector, scalar, and tensor currents.

Taking Lorentz and discrete symmetries into account:

$$\langle P(k)|\mathcal{V}^\mu|B(p)\rangle = f_+(q^2)\left(p^\mu + k^\mu - \frac{M_B^2 - M_P^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_P^2}{q^2}q^\mu$$

$$\langle P(k)|\mathcal{S}|B(p)\rangle = f_0(q^2)\frac{M_B^2 - M_P^2}{m_b - m_q}$$

$$\langle P(k)|\mathcal{T}^{\mu\nu}|B(p)\rangle = f_T(q^2)\frac{2}{M_B + M_P}\left(p^\mu k^\nu - p^\nu k^\mu\right)$$

PCVC ensures that the vector and scalar currents lead to the same f_0.

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$$= \sqrt{2M_B}\left[k_\perp^\mu f_\perp(E_P) + v^\mu f_\parallel(E_P)\right], \quad v = p/M_B$$

$$\langle P(k)|\mathcal{S}|B(p)\rangle = f_0(q^2)\frac{M_B^2 - M_P^2}{m_b - m_q}$$

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Form factors II

It is straightforward to extract the matrix elements

$$f_\perp(E_P) = \frac{\langle P|\mathcal{V}^i|B\rangle}{\sqrt{2M_B}}\frac{1}{k^i}$$
$$f_\parallel(E_P) = \frac{\langle P|\mathcal{V}^0|B\rangle}{\sqrt{2M_B}}$$
$$f_T(E_P) = \frac{M_B + M_P}{\sqrt{2M_B}}\frac{\langle P|\mathcal{T}^{0i}|B\rangle}{\sqrt{2M_B}}\frac{1}{k^i}$$

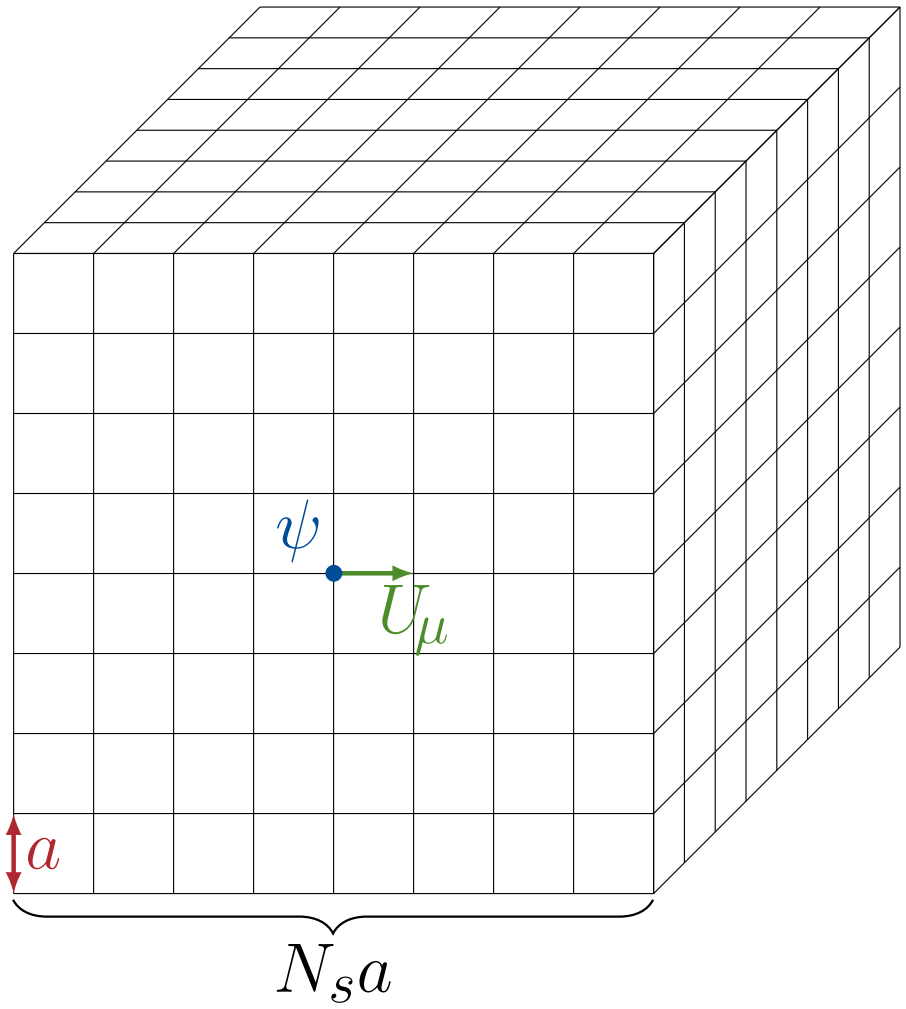
from three-point correlation functions.

Then f_+ and f_0 are linear combinations of $f_\perp$ and $f_\parallel$.

Part II — Analysis

Lattice QCD

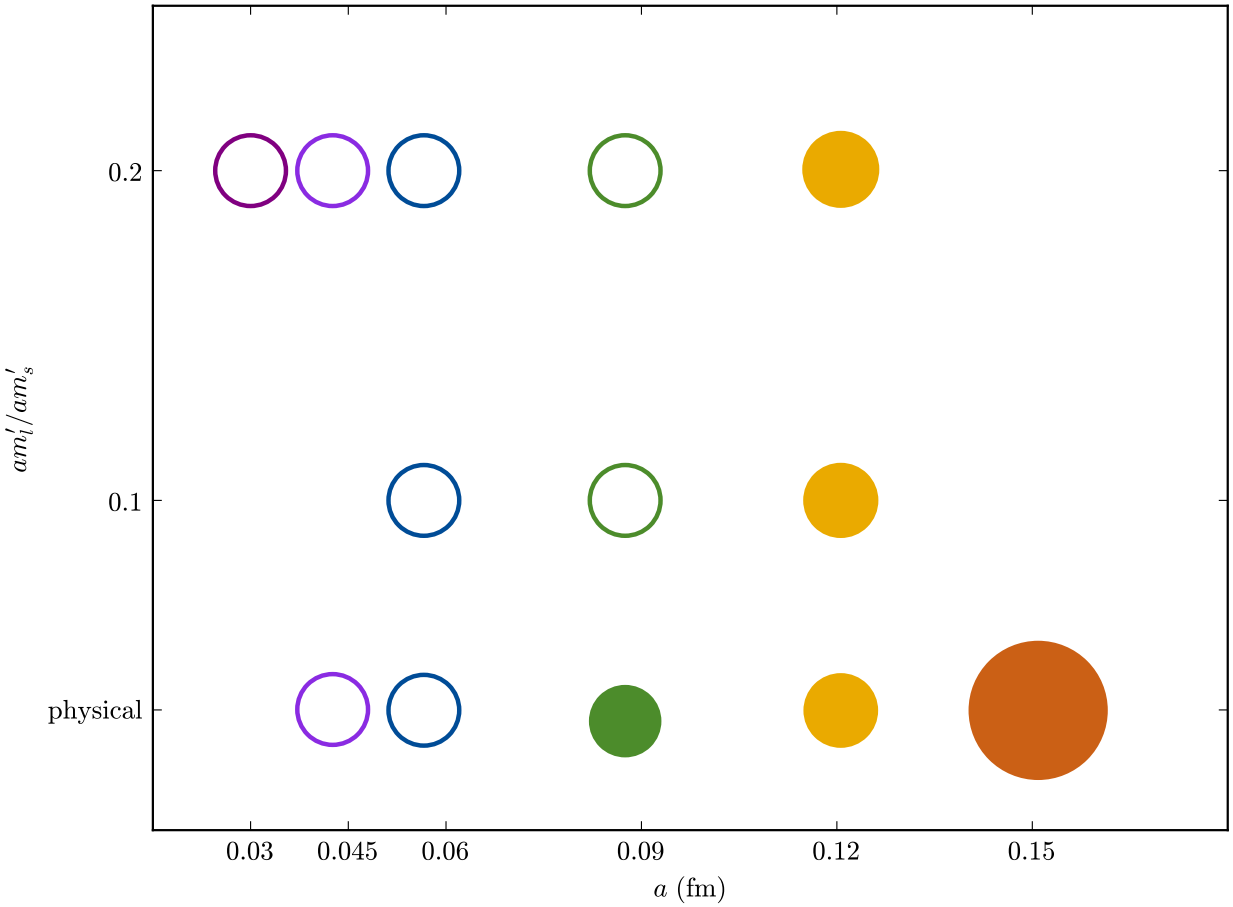
Lattice QCD is not a model; it is a regularization of QCD with quantifiable uncertainties that are systematically improvable.



- Finite theorist lifetime
  $\Rightarrow$ use Euclidean time
- Finite computer memory
  $\Rightarrow$ use finite lattice size

Lattice-QCD datasets

Sample sizes at a glance (area $\propto$ number of samples):



Filled circles: this work; open circles: future work (other HISQ ensembles).

Correlation functions

Energies of pseudoscalar mesons P arise from two-point correlators:

$$C_2(t;\boldsymbol{k}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left\langle \mathcal{O}_P(0,\mathbf{0})\, \mathcal{O}_P^\dagger(t,\boldsymbol{x}) \right\rangle$$

$$= \sum_m (-1)^{m(t+1)} \frac{\left|\left\langle 0|\mathcal{O}_P|P^{(m)}\right\rangle\right|^2}{2E_P^{(m)}} e^{-E_P^{(m)}t}$$

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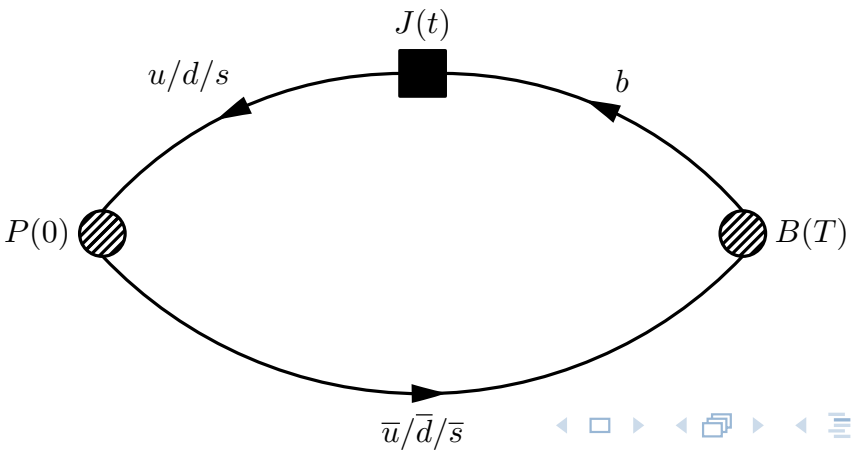
Form factors arise from three-point correlators:

$$C_3^{\mu(\nu)}(t,T;\boldsymbol{k}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{k}\cdot\boldsymbol{y}} \left\langle \mathcal{O}_P(0,\mathbf{0})\, J^{\mu(\nu)}(t,\boldsymbol{y})\, \mathcal{O}_B^\dagger(T,\boldsymbol{x}) \right\rangle$$



Three-point function fits I

Form factors can be obtained directly from fits to $C_3^{\mu(\nu)}$.

Alternatively, and for illustration, form factors arise from fits to ratios:

$$\overline{R}^{\mu(\nu)} \equiv \frac{\overline{C}_3^{\mu(\nu)}(t,T;\boldsymbol{k})}{\sqrt{\overline{C}_{2,P}(t;\boldsymbol{k})\overline{C}_{2,B}(T-t;\boldsymbol{0})}}\sqrt{\frac{2E_P^{(0)}}{e^{-E_P^{(0)}t}e^{-M_B^{(0)}(T-t)}}}$$

$$\overline{R}^{\mu(\nu)} \overset{\text{fit}}{\propto} F^{\mu(\nu)}$$

$$f_\perp(E_P) = Z_\perp \frac{F^i(\boldsymbol{k})}{k^i}$$

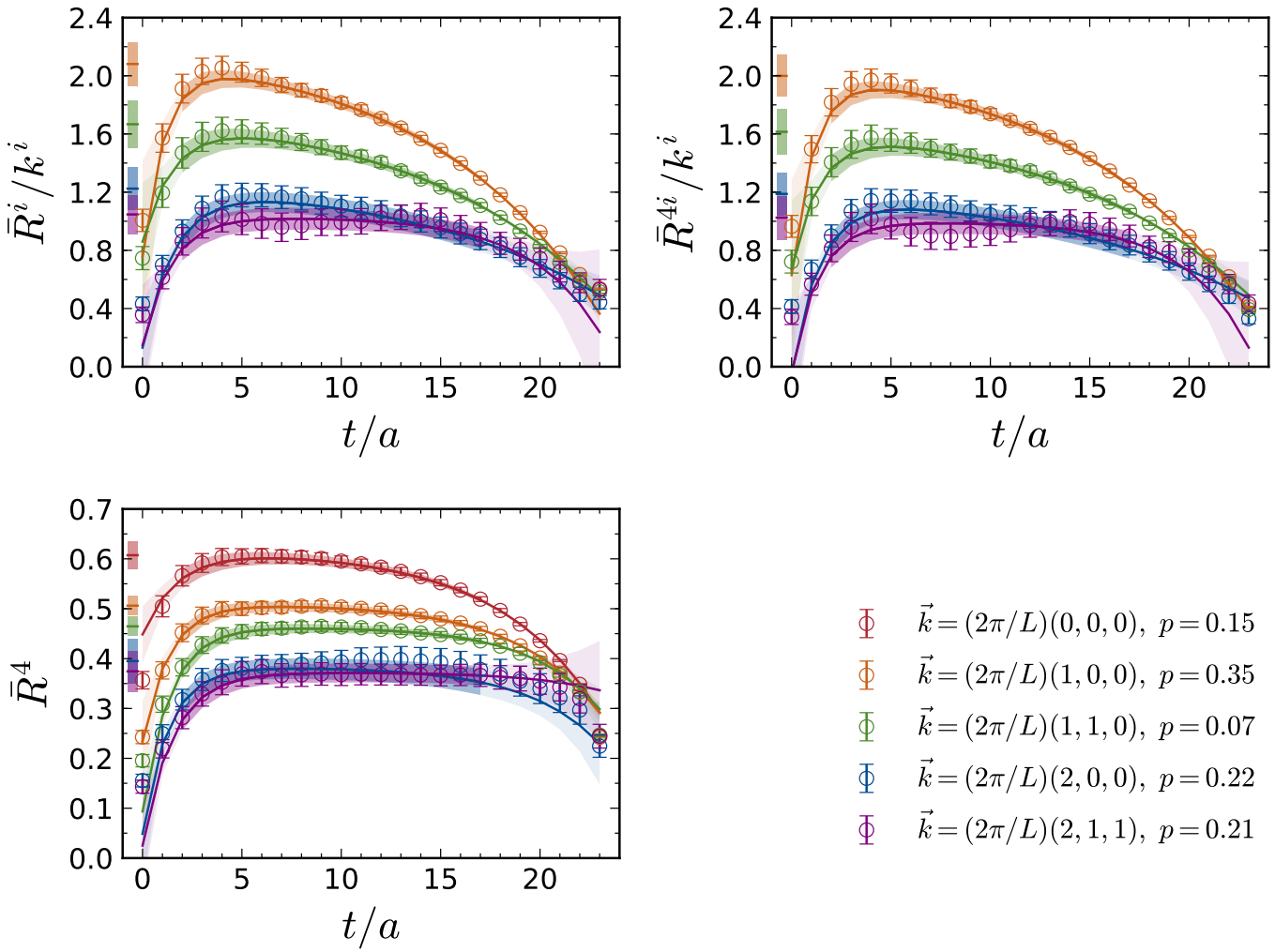
$$f_\parallel(E_P) = Z_\parallel F^4(\boldsymbol{k})$$

$$f_T(E_P) = Z_T \frac{M_B + M_P}{\sqrt{2M_B}} \frac{F^{4i}(\boldsymbol{k})}{k^i}$$

We use mostly nonperturbative matching $Z_J = \rho_J \sqrt{Z_{V^4_{bb}} Z_{V^4_{qq}}}$, multiplying ρ_J by a blinding factor.

Three-point function fits II

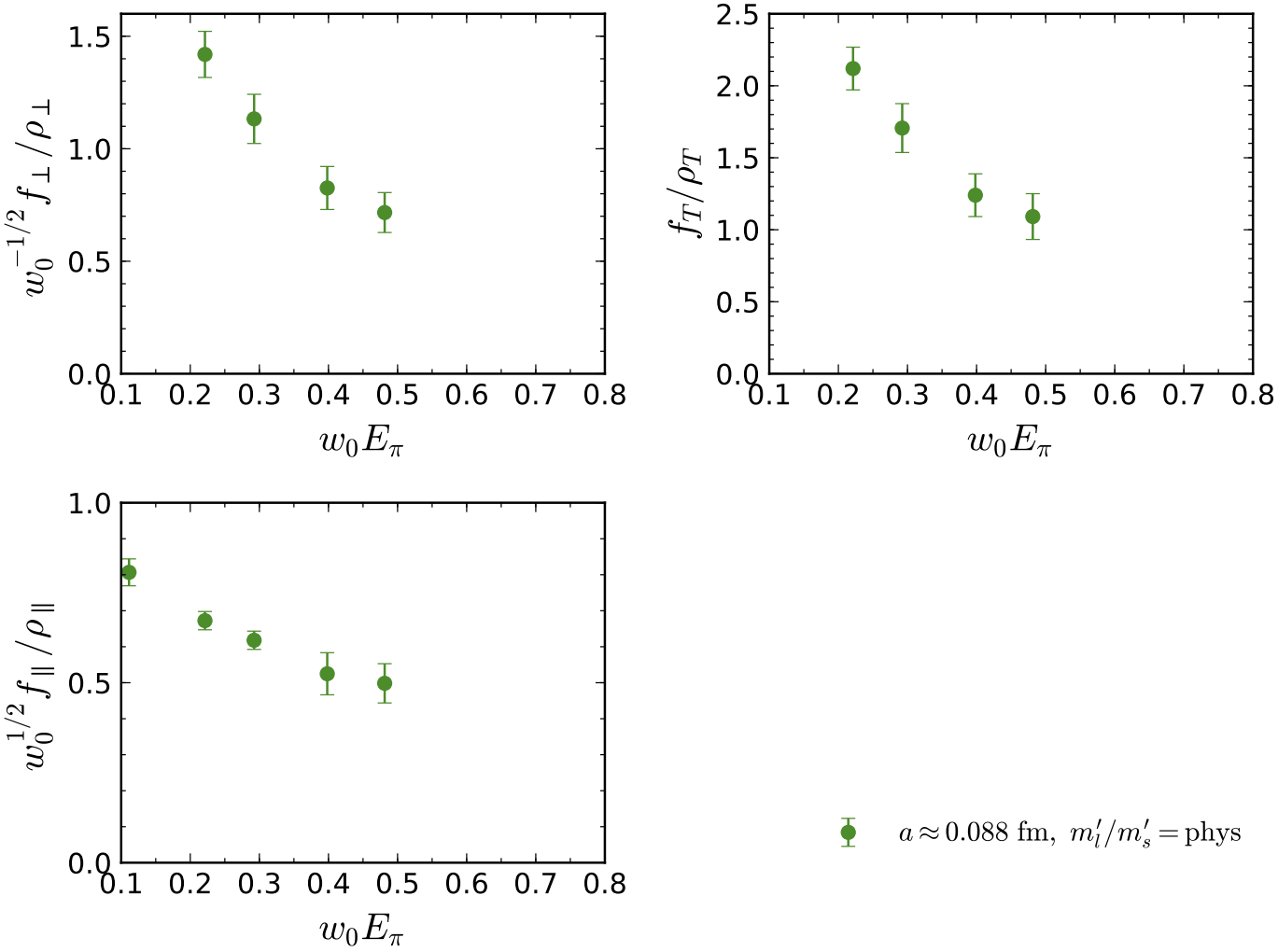
$B \to \pi$, $a \simeq 0.088$ fm, $m_l/m_s =$ physical



Under collaboration review

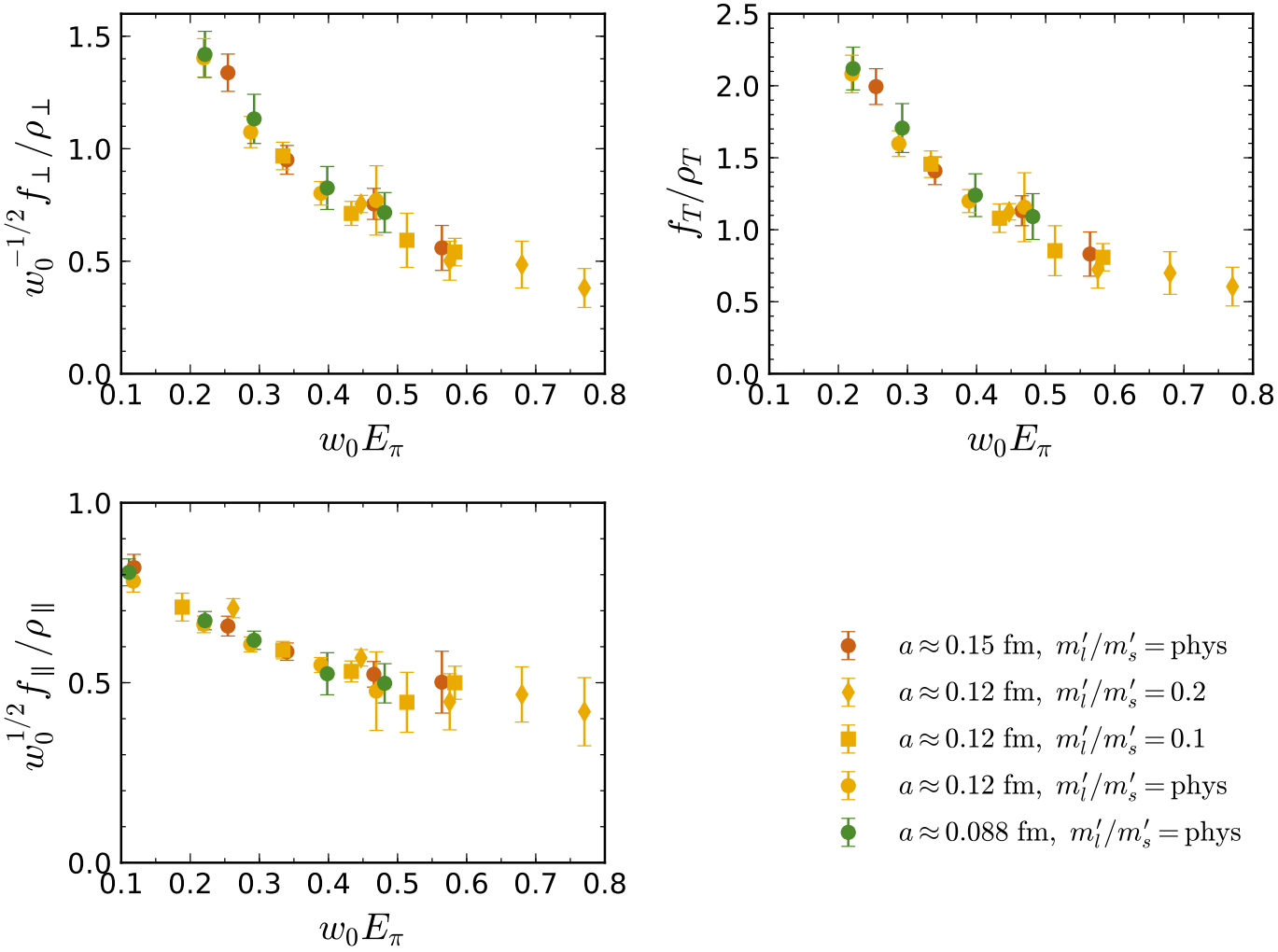
Three-point function fits III

$B \to \pi$, $a \simeq 0.088$ fm, $m_l/m_s =$ physical



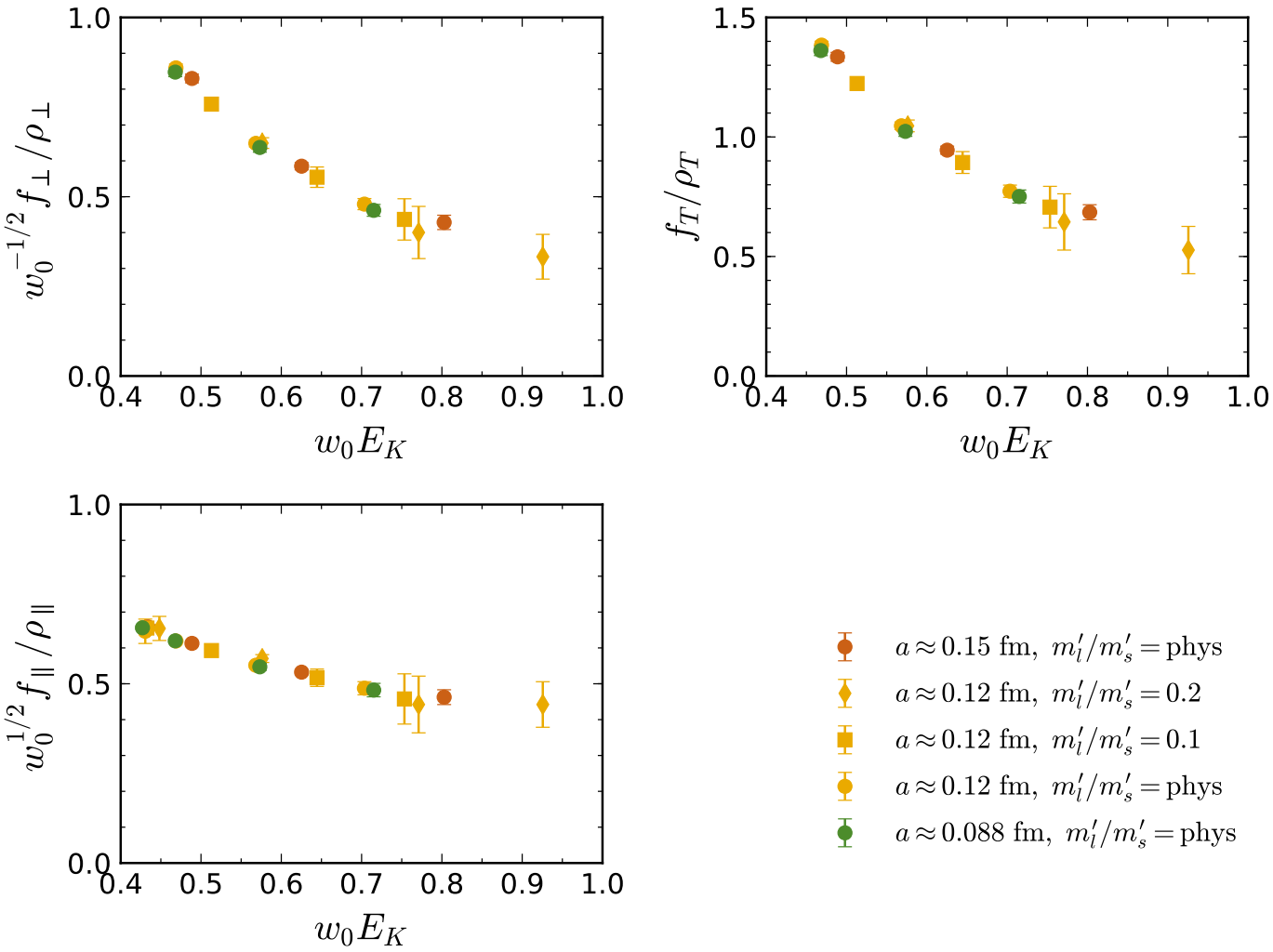
Under collaboration review

$B \to \pi$ form factors



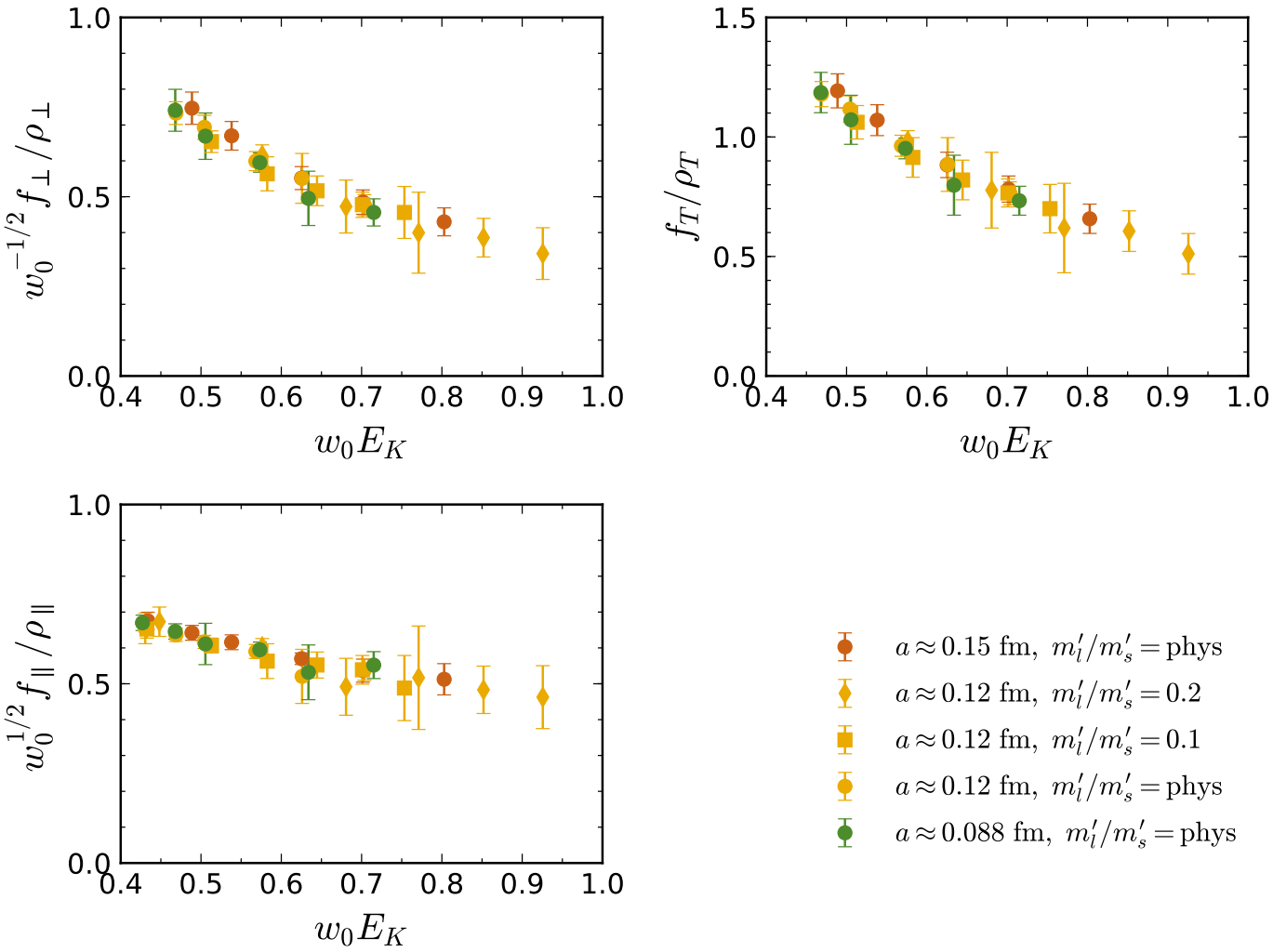
Under collaboration review

$B_s \to K$ form factors



Under collaboration review

$B \to K$ form factors



Under collaboration review

Part III — Conclusion

Outlook

1. Apply blinded current renormalization factors $\rho_\perp$, $\rho_\parallel$, ρ_T
2. Take continuum limit for form factors
3. Extrapolate to full range in q^2 via model-independent z expansion
4. Construct complete error budget
5. Unblind current renormalization factors
6. Confront experiment
   a. Determine $|V_{ub}|$ from charged-current decays
   b. Compare $\mathcal{B}$ observables from neutral-current decays to test for new physics

Thank you!

Actions and parameters

- MILC $N_f = 2 + 1 + 1$ ensembles
- Lüscher-Weisz gauge action $\rightarrow O(\alpha_s^2 a^2)$
- HISQ action for $q_l, s, c \rightarrow O(\alpha_s a^2)$
- Clover action with Fermilab interpretation for $b \rightarrow O(\alpha_s a, a^2)\, f\big((m_b a)^2\big)$
- Setting the scale with w_0
  - $(w_0/a) \times (aM) = w_0 M$
  - $w_0^{a=0} = 0.1714(15)$ fm

a (fm)	0.1509(14)	0.1206(14)	0.1206(11)	0.1206(11)	0.08750(80)
$N_{\text{cfg}} \times N_{\text{src}}$	3630×8	1053×8	1000×8	986×8	925×8
$N_s^3 \times N_4$	$32^3 \times 48$	$24^3 \times 64$	$32^3 \times 64$	$48^3 \times 64$	$64^3 \times 96$
am_l'	0.00235	0.0102	0.00507	0.00184	0.0012
am_s'	0.0647	0.0509	0.0507	0.0507	0.0363
am_c'	0.831	0.635	0.628	0.628	0.432
κ_b'	0.07732	0.08574	0.08574	0.08574	0.09569
w_0/a	1.1468(3)	1.3835(7)	1.4047(6)	1.4168(10)	1.9473(11)
$\alpha_V(2/a)$	0.45275	0.38138	0.38138	0.38138	0.31391

return to ensembles

Correlator averaging procedure I

Suppress oscillating states characteristic of staggered fermions:

$$\overline{C}_2(t;\boldsymbol{k}) \equiv \frac{e^{-M_P^{(0)}t}}{4}\left[\frac{C_2(t;\boldsymbol{k})}{e^{-M_P^{(0)}t}} + \frac{2C_2(t+1;\boldsymbol{k})}{e^{-M_P^{(0)}(t+1)}} + \frac{C_2(t+2;\boldsymbol{k})}{e^{-M_P^{(0)}(t+2)}}\right]$$

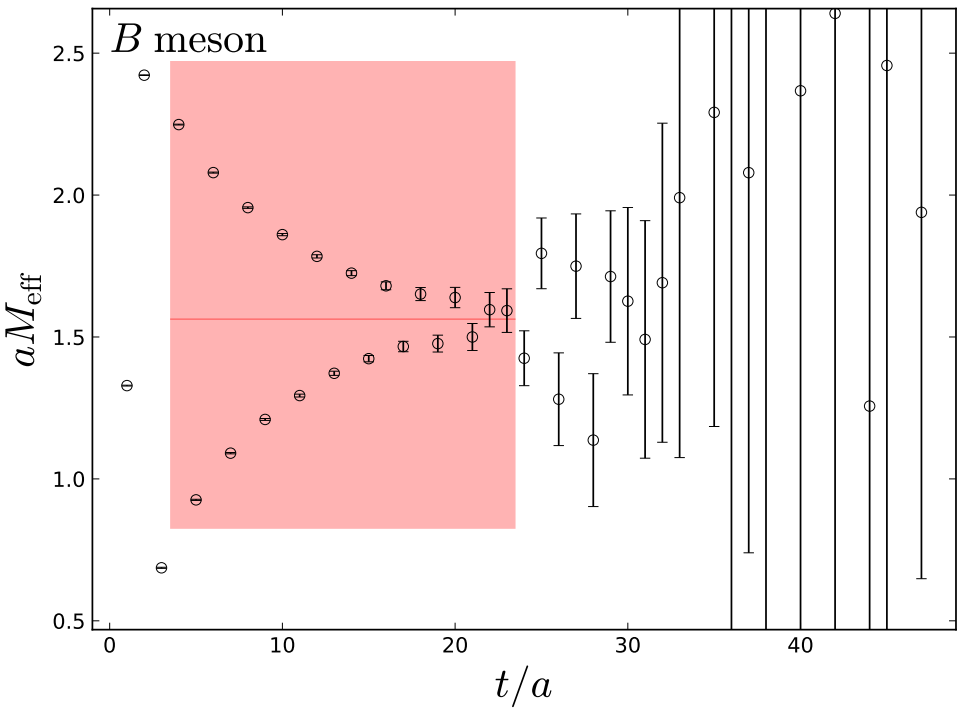
$$\overline{C}_3^{\mu(\nu)}(t,T;\boldsymbol{k}) \equiv \frac{e^{-E_P^{(0)}t}e^{-M_B^{(0)}(T-t)}}{8}\left[\frac{C_3^{\mu(\nu)}(t,T;\boldsymbol{k})}{e^{-E_P^{(0)}t}e^{-M_B^{(0)}(T-t)}} + \frac{2C_3^{\mu(\nu)}(t+1,T;\boldsymbol{k})}{e^{-E_P^{(0)}(t+1)}e^{-M_B^{(0)}(T-t-1)}} + \frac{C_3^{\mu(\nu)}(t+2,T;\boldsymbol{k})}{e^{-E_P^{(0)}(t+2)}e^{-M_B^{(0)}(T-t-2)}} + \{T \to T+1\}\right]$$

Effective mass defined from two-point correlators C_2 as

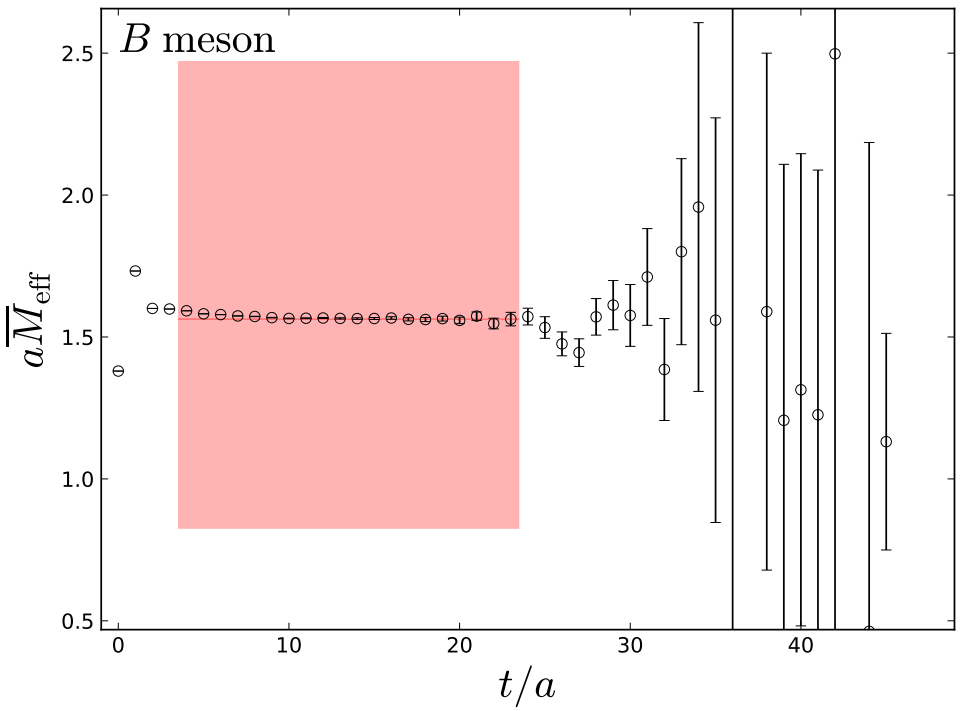
$$aM_{\text{eff}}(t;\boldsymbol{k}) = \cosh^{-1}\left(\frac{C_2(t+1;\boldsymbol{k}) + C_2(t-1;\boldsymbol{k})}{2C_2(t;\boldsymbol{k})}\right)$$

For corresponding effective mass with $C_2 \to \overline{C}_2$, write $a\overline{M}_{\text{eff}}$.

Correlator averaging procedure II



oscillating states evident

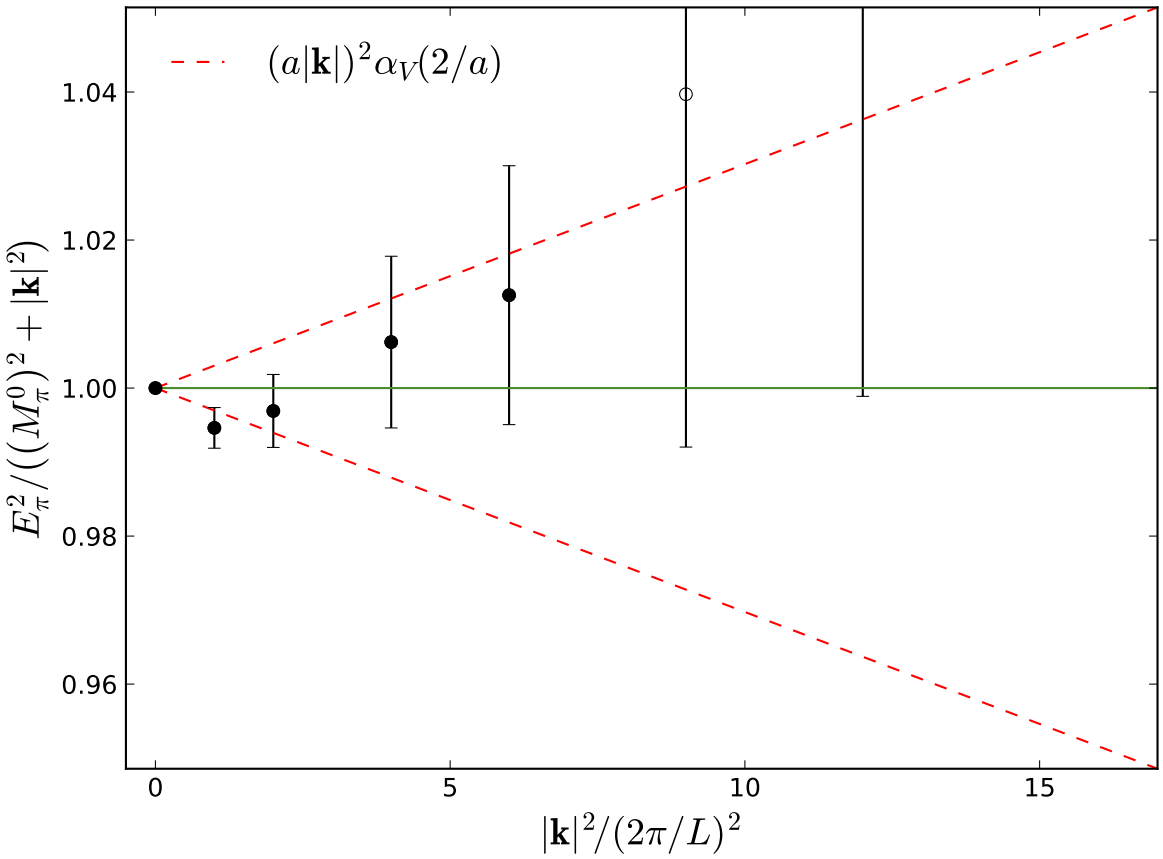


oscillating states suppressed

- Averaging procedure is effective at suppressing oscillations
- $\langle M_{\text{eff}} \rangle$ provides natural prior for the ground-state energy $E^{(0)}$; we take $\tilde{E}^{(0)} = \langle M_{\text{eff}} \rangle \pm \frac{1}{2} \langle M_{\text{eff}} \rangle$.

return to form factor fits

Dispersion relation tests

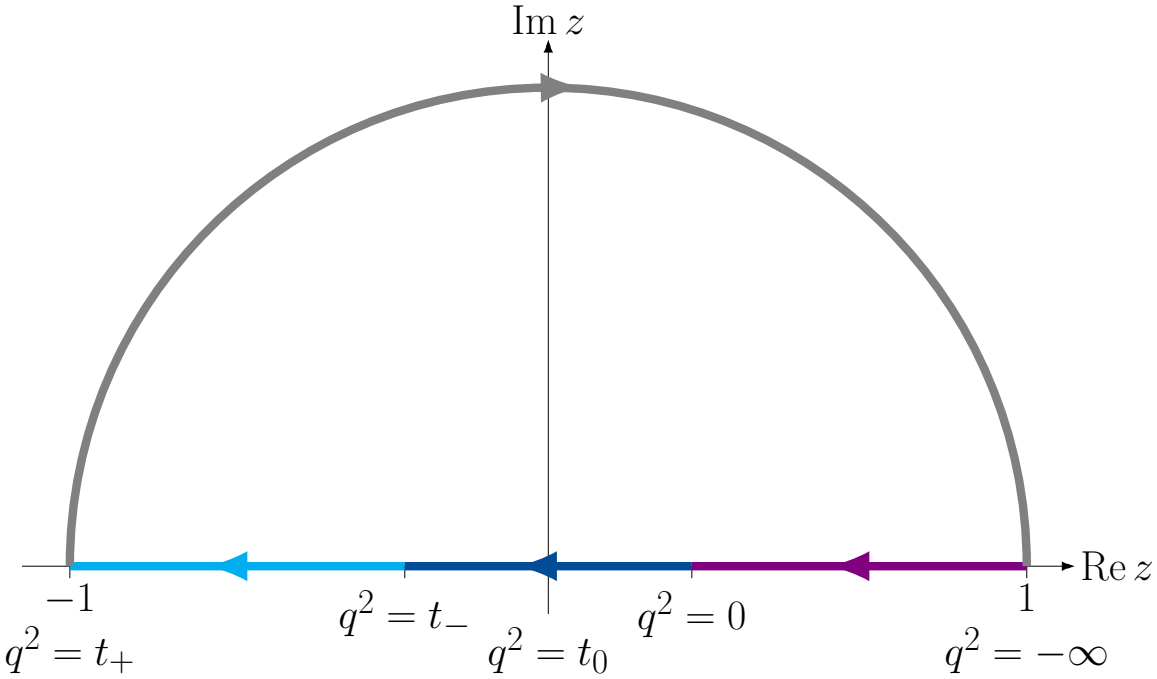


$\Rightarrow$ Replace $E_P^{(0)}$ with $\mathcal{E}_P^{(0)} = \sqrt{\left(M_P^{(0)}\right)^2 + |\boldsymbol{k}|^2}$ in form factor fits

return to form factor fits

z expansion

Conformal mapping $q^2 \mapsto |z| \leq 1$ exploits analytic structure in complex plane to extend chiral-continuum form factors to low q^2.



$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

where $t_\pm = (M_B \pm M_P)^2$

$$\Downarrow$$

$$f(q^2) = \frac{1}{1 - \frac{q^2}{M^2}} \sum_n a_n z^n(q^2)$$

- t_0^{opt} minimizes $|z|$ in physical region $\Rightarrow |z| \leq \{0.30, 0.15\}$ for $P = \{\pi, K\}$
- Smallness of $|z|$ controls truncation
- Unitarity guarantees convergence

return to conclusions