The spin-dependent quark beam function at NNLO

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Proton Spin Puzzle

- Proton spin sum rule
  \[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_q + L_g \]

  \[ \Delta \Sigma = \sum_i \int_0^1 dx \, \Delta f_{q_i}(x) \quad \Delta G = \int_0^1 dx \, \Delta f_g(x) \]

- Contribution from quarks much smaller than expected
  \[ \Delta \Sigma \approx 0.25 \]

- Helicity parton distributions are probed by

  ![DIS](image1)
  ![SIDIS](image2)
  ![pp (RHIC)](image3)
Current Status

- Current data is **not** well described

![Graph showing unpolarized cross sections and longitudinal double-spin asymmetries](image)

- We need more data and more accurate theoretical predictions

  => Extent techniques from unpolarized collision
N-Jettiness  

[Boughezal, Focke, Liu, Petriello; Gaunt Stahlhofen Tackmann, Walsh]

virtual  real virtual  real-real
N-Jettiness

\[ \Theta(\tau_{\text{cut}} - \tau) \quad \tau_{\text{cut}} \quad \Theta(\tau - \tau_{\text{cut}}) \]
N-Jettiness

[The diagram shows three types of functions:

- Hard function (H): virtual corrections, process dependent
- Soft function (S): describes soft radiation
- Jet function (J): describes radiation collinear to final state jets

The text reads:

=> Use factorisation theorem derived from SCET

\[ \frac{d\sigma}{dT_N} = H \otimes B \otimes S \otimes \left[ \prod_{n}^{N} J_n \right] + \text{Power corrections} \]

[Stewart, Tackmann, Waalewijn]

Hard function (H): virtual corrections, process dependent
Soft function (S): describes soft radiation
Jet function (J): describes radiation collinear to final state jets
Beam function (B): describes collinear initial state radiation

\[ \Theta(\tau_{\text{cut}} - \tau) \quad \tau_{\text{cut}} \quad \Theta(\tau - \tau_{\text{cut}}) \]

=> NLO N+1 jet calculation

[Boughezal, Focke, Liu, Petriello; Gaunt Stahlhofen Tackmann, Walsh]
Polarized Collisions

- Above cut piece can simply be polarised
- Similar factorization theorem for the below cut piece

\[
\frac{d\sigma_{LL}}{dT_N} = \Delta H \otimes \Delta B \otimes S \otimes \left[ \prod_{n=1}^{N} J_n \right] + \cdots
\]

Soft function: unchanged from unpolarized version [Boughezal, Liu, Petriello]

Jet function: unchanged from unpolarized version [Becher, Neubert; Becher, Bell]

Hard function: known for DIS and DY

\[
\Delta H = H^+ - H^-
\]

Beam function: previously unknown, discussed here

\[
\Delta B = B^+ - B^-
\]
Beam function

\[
\Delta B_i(t, x, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \Delta I_{ij} \left( t, \frac{x}{\xi} \right) \Delta f_j(\xi, \mu)
\]

- Parton j with momentum distribution determined by PDF emits collinear radiation, which builds up jet described by $I_{ij}$

- These emissions might change the parton i entering the hard scattering (type, momentum fraction)

- $I_{ij}$ can be calculated perturbatively
Outline of Calculation

- Generate squared amplitude

\[ \Delta B_{ij}^{\text{bare}}(t, z) = \ldots \]
Outline of Calculation

• Generate squared amplitude

\[ \Delta B_{ij}^{\text{bare}}(t, z) = \ldots \]

• Reverse Unitarity

[Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello]

• Integration-by-parts (IBP)

[Chetyrkin, Tkachov]

\[ \Delta B_{ij}^{\text{bare}}(t, z) = \sum_{i=1}^{n} c_i(t, z) I_i(t, z) \]
Outline of Calculation

• Generate squared amplitude
  \[ \Delta B_{ij}^{\text{bare}}(t, z) = \sum_{i=1}^{n} c_i(t, z) I_i(t, z) \]

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  • Differential Equations(DEQ)
    [Kotikov; Gehrmann, Remiddi]
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\[ + \]

\[ \cdots \]

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- UV renormalization

\[ \Delta B_{ij}^{\text{bare}}(t, z) = \int \! dt' Z_i(t - t', \mu) \Delta B_{ij}(t', z, \mu), \]
Outline of Calculation

- Generate squared amplitude
  \[ \Delta B_{ij}^{\text{bare}}(t, z) = \sum_{\text{diagrams}} \ldots \]

- Reverse Unitarity
  [Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello]
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  \[ \Delta B_{ij}^{\text{bare}}(t, z) = \int dt' Z_i(t - t', \mu) \Delta B_{ij}(t', z, \mu) , \]

- Matching on PDF
  \[ \Delta B_{ij}(t, z, \mu) = \sum_k \Delta \mathcal{I}_{ik}(t, z, \mu) \otimes \Delta f_{kj}(z) \]
Outline of Calculation

- Generate squared amplitude
  \[ \Delta B_{ij}^{\text{bare}}(t, z) = \begin{array}{c}
  
  \end{array} + \begin{array}{c}
  \end{array} \ldots \]

- Reverse Unitarity
  [Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello]
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- Additional renormalization for \(\gamma_5\)
  \[ \Delta B = \left( \Delta \tilde{T} \otimes \tilde{Z}^5 \right) \otimes \left( Z^5 \otimes \Delta \tilde{f} \right) \]
Master Integrals

- Initially $\mathcal{O}(100) - \mathcal{O}(1000)$ integrals

- 9 MIs in real-real channel

- 3 MIs in real-virtual channel

- Generate DEQ

\[ \partial_x \vec{f} = A_x \vec{f}, \quad x = t, z \]
Calculation of Master Integrals

- Bring DEQ in canonical form with Magnus algorithm
  [Henn; Argeri, Di Vita, Mastrolia, Mirabella, Schlenk, Tancredi, U.S.]

\[ \partial_x \vec{g} = \epsilon \hat{A}_x \vec{g} \]

\[ \hat{A}_z = \frac{\hat{A}_1}{z} + \frac{\hat{A}_2}{1 + z} + \frac{\hat{A}_3}{1 - z} \]

- Matrices \( A_i \) have only numeric entries

- Simple alphabet \( \{1 - z, z, 1 + z\} \)

- Solution can be written in terms of Harmonic Polylogarithms

\[ H_{a_1, \ldots, a_n}(z) = \int_0^z dt \frac{H_{a_2, \ldots, a_n}(t)}{t - a_1}, \quad a_i \in 0, -1, 1 \]

\[ H_{0, \ldots, 0}(z) = \frac{1}{n!} \log^n(z) \]
Calculation of Master Integrals

- MI for RR channel behave like \((1 - z)^{-2\epsilon} F(z)\) when \(z \to 1\)  
  [Gaunt, Stahlhofen, Tackmann]

  \[\Rightarrow\] fixes 7 out of 9 boundary constants

- One MI is easily obtained by direct integration

- Last boundary constant obtained by
  - Introduce extra scale
  - Solve DEQ with extra scale
  - Here all boundaries can be fixed easily
  - take scale carefully to zero

- MI for RV behave like \((1 - z)^{-2\epsilon, -\epsilon} F(z)\) when \(z \to 1\)

  \[\Rightarrow\] fixes one boundary constant

- Taking carefully \(z \to 0\) fixes second boundary constant

- Last boundary can be easily obtained by direct integration
UV renormalisation and Matching

- Use standard $\overline{\text{MS}}$ renormalization

\[
\Delta B_{ij}^{\text{bare}(2)}(t, z) = \Delta B_{ij}^{(2)}(t, z, \mu) + Z_i^{(2)}(t, \mu)\delta_{ij}\delta(1 - z) + \int dt'Z_i^{(1)}(t - t', \mu)\Delta B_{ij}^{(1)}(t', z, \mu).
\]

- Requires calculation of $\Delta B_{ij}^{(1)}(t, z, \mu)$ up to $\mathcal{O}(\epsilon^2)$

- Match beam function on PDFs

\[
\Delta \tilde{I}_{ij}^{(2)}(t, z, \mu) = \Delta B_{ij}^{(2)}(t, z, \mu) - 4\delta(t)\Delta \tilde{f}_{ij}^{(2)}(z) - 2 \sum_l \Delta \tilde{I}_{ik}^{(1)}(t, z, \mu) \otimes \Delta \tilde{f}_{kj}^{(1)}(z).
\]

\[
\Delta \tilde{f}_{ij}^{(1)}(z) = - \frac{1}{\epsilon} \Delta \tilde{P}_{ij}^{(0)}(z),
\]

\[
\Delta \tilde{f}_{ij}^{(2)}(z) = \frac{1}{2\epsilon^2} \sum_k \Delta \tilde{P}_{ik}^{(0)}(z) \otimes \Delta \tilde{P}_{kj}^{(0)}(z) + \frac{\beta_0}{4\epsilon^2} \Delta \tilde{P}_{ij}^{(0)}(z) - \frac{1}{2\epsilon} \Delta \tilde{P}_{ij}^{(1)}(z),
\]

- Cancellation of poles provides consistency check
Treatment of Gamma5

- We use HVBM scheme
  \[ \gamma_5 \equiv \frac{i}{4!} \epsilon^{\mu\nu\rho\sigma} \gamma_\mu \gamma_\nu \gamma_\sigma \gamma_\rho \]
  \[ \{\gamma_5, \bar{\gamma}_\mu\} = 0, \quad [\gamma_5, \bar{\gamma}_\mu] = 0. \]

- Result of Dirac traces depends on d- and 4-d-dimensional momenta

- Map 4-d momenta to auxiliary vectors
  \[ I^d[\hat{k}_1 \cdot \hat{k}_2] = -\frac{2\epsilon}{v_\perp^2} I^d[(k_1 \cdot v_\perp)(k_2 \cdot v_\perp)], \]

- But: HVBM breaks helicity conservation
  \[ \Rightarrow \text{Must be restored with additional } Z^5 \text{ renormalization} \]
  \[ \Delta B = \left( \Delta \tilde{T} \otimes \bar{Z}^5 \right) \otimes \left( Z^5 \otimes \tilde{f} \right) \]

- \( Z^5 \) can be obtained by demanding helicity conservation
  \[ \Delta \mathcal{I}^{(2,V)}_{qq} = \mathcal{I}^{(2,V)}_{qq} \quad \Delta \mathcal{I}^{(2,V)}_{q\bar{q}} = -\mathcal{I}^{(2,V)}_{q\bar{q}} \]
Consistency checks

- HVBM scheme implemented in public code Tracer and in-house Form routine
  [Jamin, Lautenbacher]

- MIs calculated by DEQ and direct integration

- Cancellation of poles during renormalization and matching
  - Confirmed polarised LO and NLO splitting functions
    [Vogelsang]
  - Confirmed UV renormalisation constant
    [Stewart, Tackmann, Waalewijn; Ritzmann, Waalewijn]

- Confirmed unpolarised quark beam function calculation at NLO and NNLO
  [Stewart, Tackmann, Waalewijn; Gaunt, Stahlhofen, Tackmann]

- $Z^5$ consistent with Literature
  [Ravindran, Smith, van Neerven]
Conclusions & Outlook

• Calculated spin-dependent quark beam function

• Last missing ingredient to apply N-jettiness subtraction to many polarized processes

• Provided independent check on:
  - unpolarized quark beam function up to NNLO
  - polarised splitting function up to NLO

• Ready for phenomenological studies
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Thank you for your attention