The spin-dependent quark beam function at NNLO

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In collaboration with R. Boughezal, F. Petriello and H. Xing
arXiv:1704.05457

Proton Spin Puzzle

- Proton spin sum rule

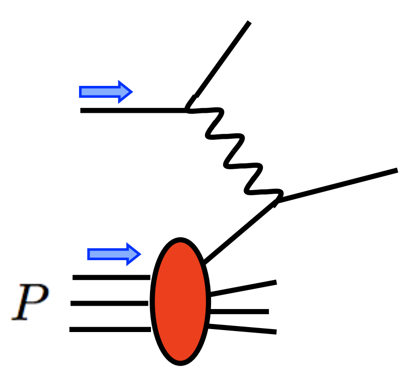
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

$$\Delta\Sigma = \sum_i \int_0^1 dx\, \Delta f_{q_i}(x) \qquad\qquad \Delta G = \int_0^1 dx\, \Delta f_g(x)$$

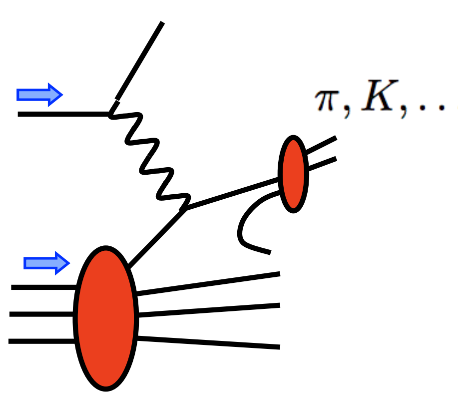
- Contribution from quarks much smaller then expected

$$\Delta\Sigma \approx 0.25$$

- Helicity parton distributions are probed by



DIS



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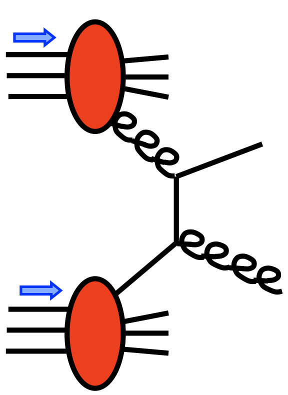
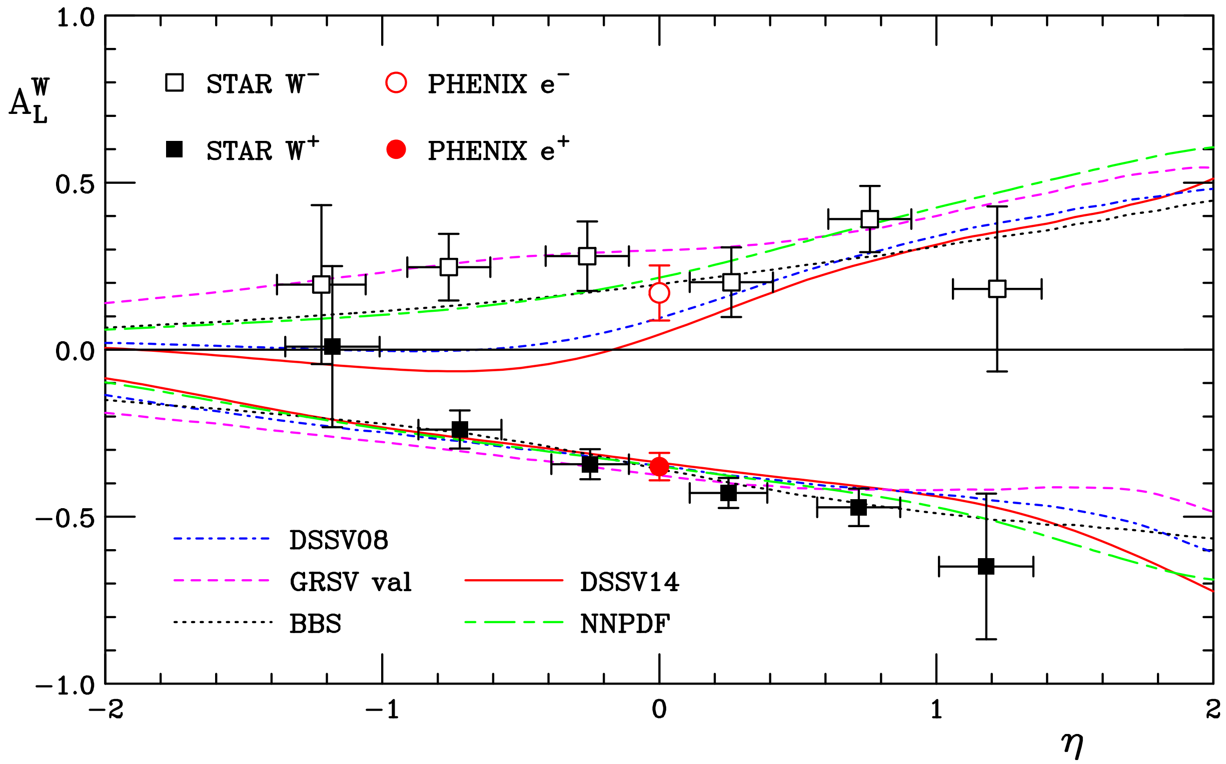


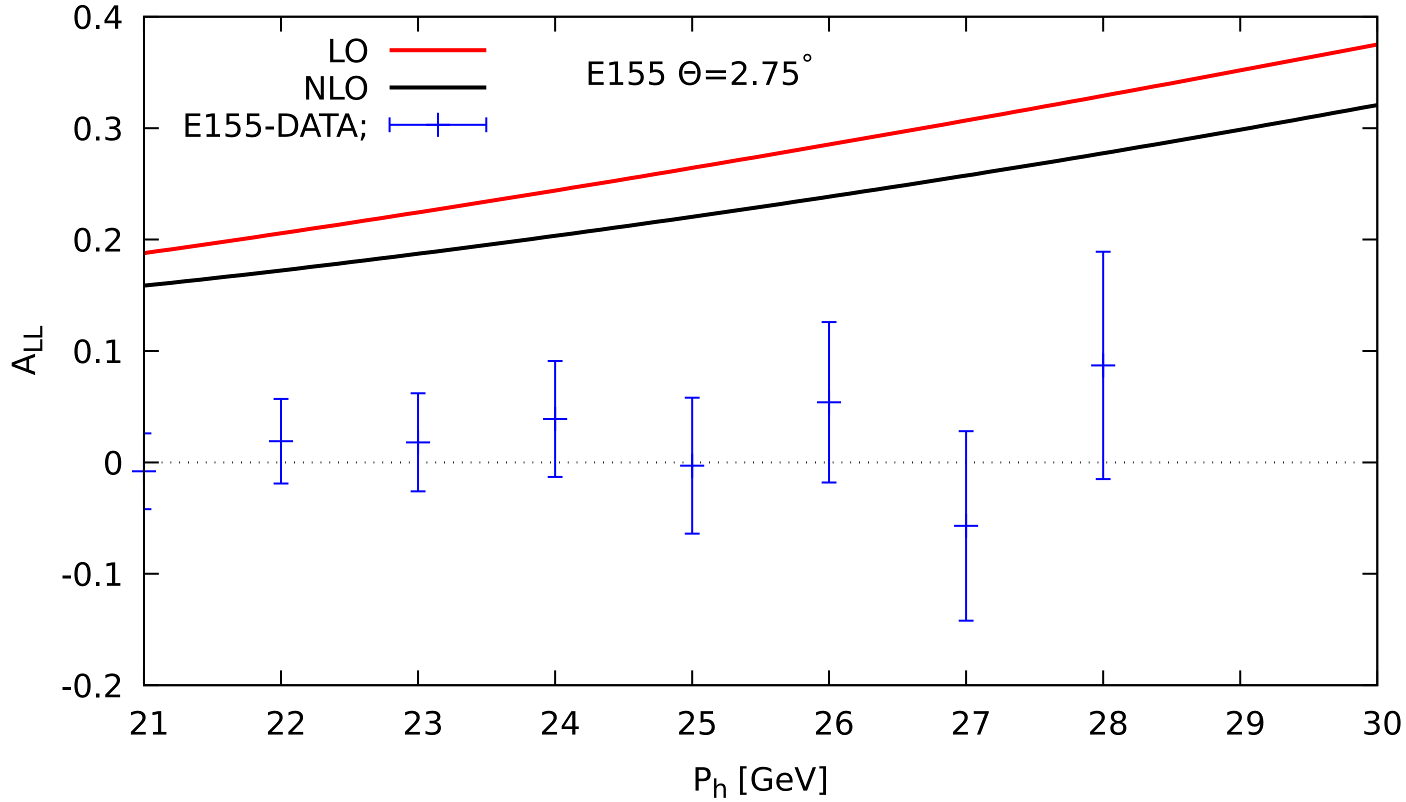
pp (RHIC)

Current Status

- Current data is **not** well described



[Ringer, Vogelsang]



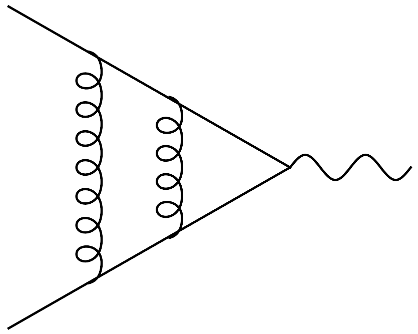
[Hinderer, Schlegel, Vogelsang]

- We need more data and more accurate theoretical predictions

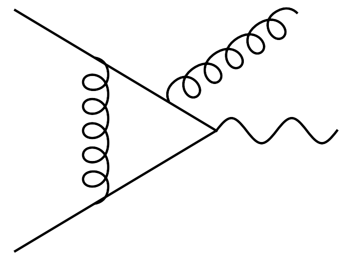
=> Extent techniques from unpolarized collision

N-Jettiness

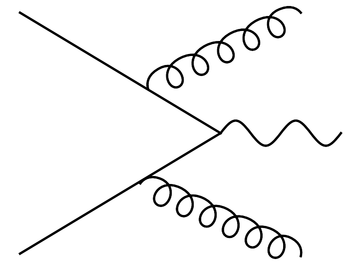
[Boughezal, Focke, Liu, Petriello;
Gaunt Stahlhofen Tackmann, Walsh]



virtual



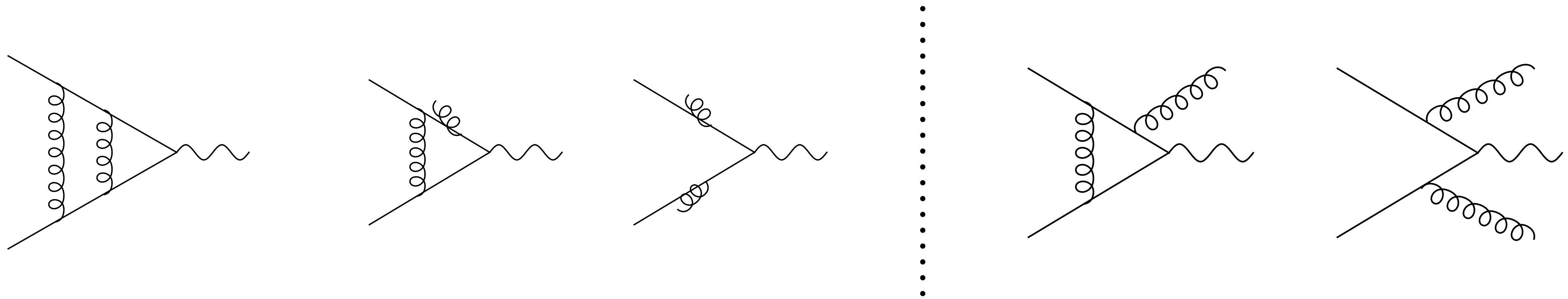
real virtual



real-real

N-Jettiness

[Boughezal, Focke, Liu, Petriello; Gaunt Stahlhofen Tackmann, Walsh]



$\Theta(\tau_{\rm cut} - \tau)$ $\quad$ $\tau_{\rm cut}$ $\quad$ $\Theta(\tau - \tau_{\rm cut})$

N-Jettiness

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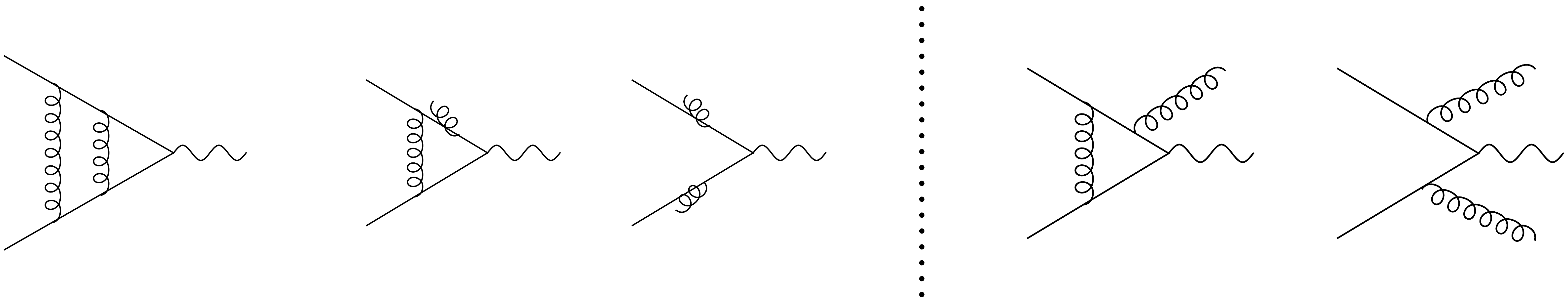


$\Theta(\tau_{\rm cut} - \tau)$ $\qquad$ $\tau_{\rm cut}$ $\qquad$ $\Theta(\tau - \tau_{\rm cut})$

=> Use factorisation theorem derived from SCET

=> NLO N+1 jet calculation

$$\frac{d\sigma}{d\mathcal{T}_N} = H \otimes B \otimes S \otimes \left[\prod_n^N J_n\right] + \text{Power corrections}$$

[Stewart, Tackmann, Waalewijn]

Hard function (H): virtual corrections, process dependent

Soft function (S): describes soft radiation

Jet function (J): describes radiation collinear to final state jets

Beam function (B): describes collinear initial state radiation

Polarized Collisions

- Above cut piece can simply be polarised
- Similar factorization theorem for the below cut piece

$$\frac{d\sigma_{LL}}{d\mathcal{T}_N} = \Delta H \otimes \Delta B \otimes S \otimes \left[\prod_n^N J_n\right] + \cdots$$

Soft function: unchanged from unpolarized version
[Boughezal, Liu, Petriello]

Jet function: unchanged from unpolarized version
[Becher, Neubert; Becher, Bell]

Hard function: known for DIS and DY

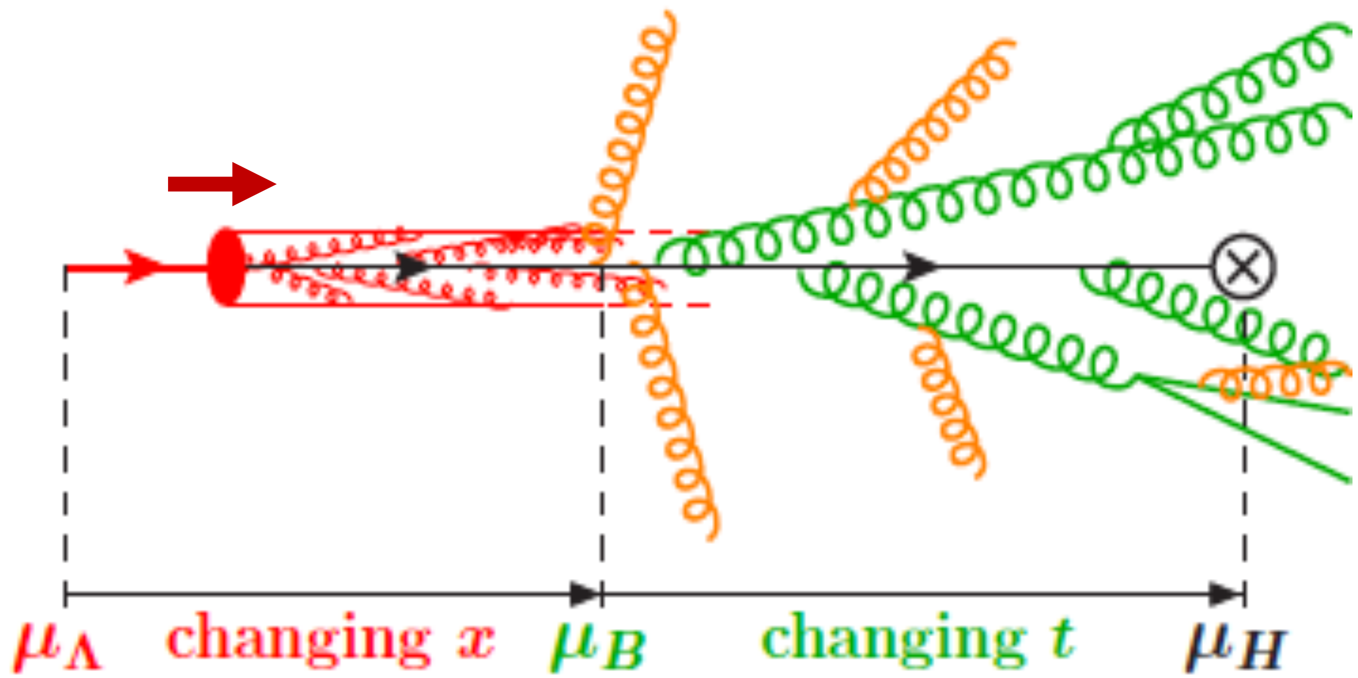
$$\Delta H = H^+ - H^-$$

Beam function: previously unknown, discussed here

$$\Delta B = B^+ - B^-$$

Beam function

[Stewart, Tackmann, Waalewijn]



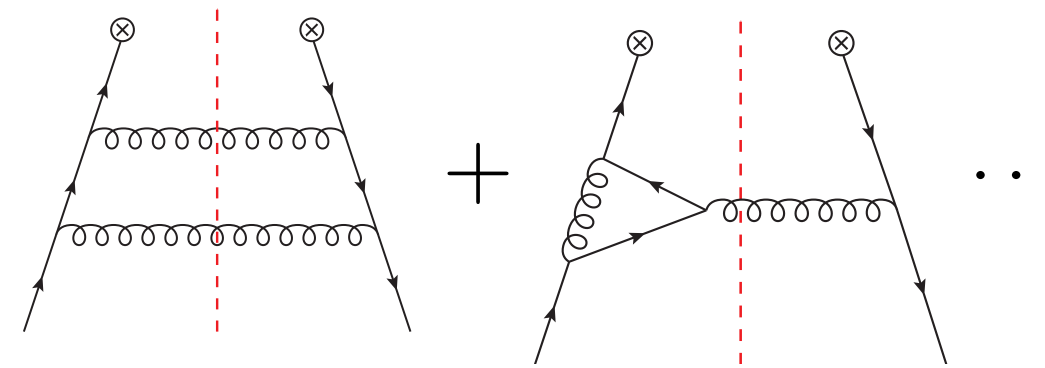
$$\Delta B_i(t,x,\mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \Delta\mathcal{I}_{ij}\left(t, \frac{x}{\xi}\right) \Delta f_j(\xi,\mu)$$

- Parton j with momentum distribution determined by PDF emits collinear radiation, which builds up jet described by $\mathcal{I}_{ij}$
- These emissions might change the parton i entering the hard scattering (type, momentum fraction)
- $\mathcal{I}_{ij}$ can be calculated perturbatively

Outline of Calculation

- Generate squared amplitude

$$\Delta B_{ij}^{\mathrm{bare}}(t,z) =$$



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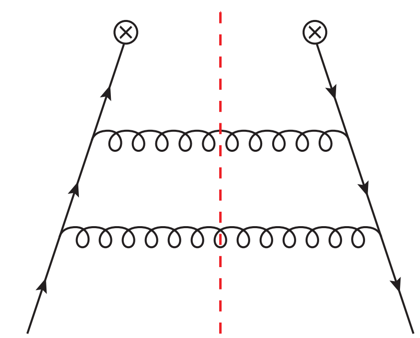
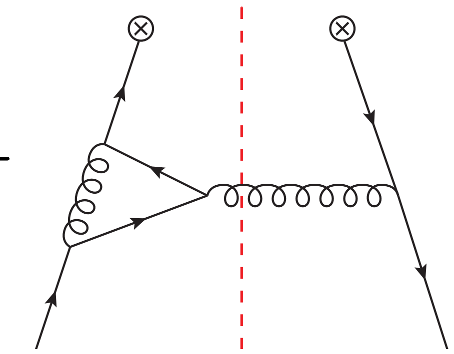


$$+$$



$$\cdots$$

- Reverse Unitarity
  [Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello]
  - Integration-by-parts(IBP)
    [Chetyrkin,Tkachov]

$$\Delta B_{ij}^{\text{bare}}(t,z) = \sum_{i=1}^{n} c_i(t,z) I_i(t,z)$$

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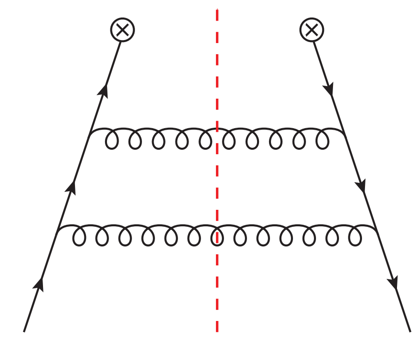

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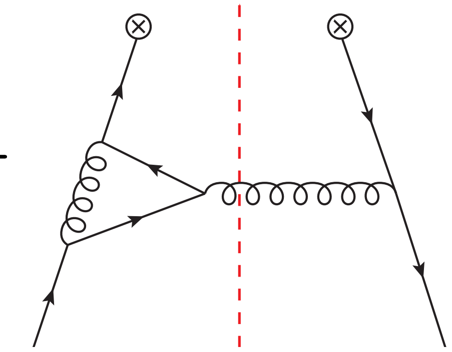

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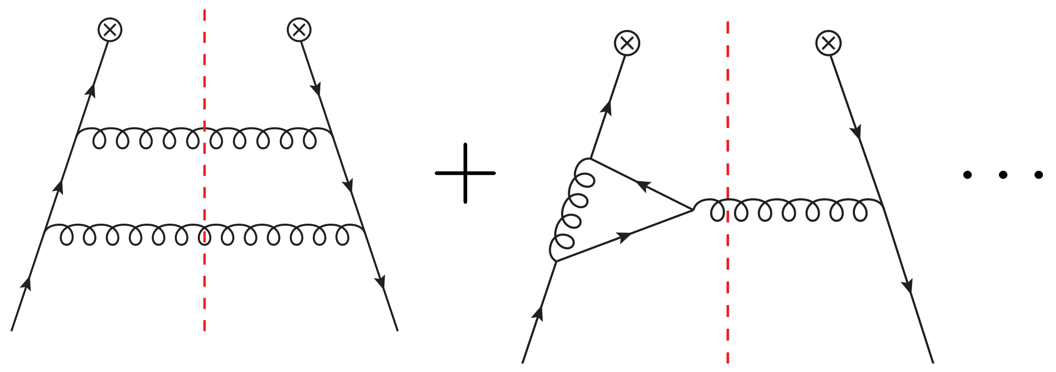
$$\Delta B_{ij}^{\text{bare}}(t,z) =$$ 

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$$\Delta B_{ij}^{bare}(t,z) = \int dt' Z_i(t-t',\mu)\Delta B_{ij}(t',z,\mu)\,,$$

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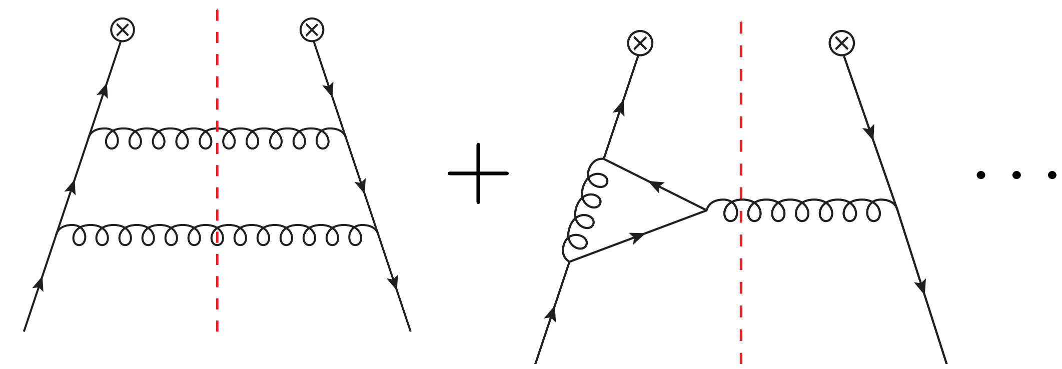

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$$\Delta B_{ij}(t,z,\mu) = \sum_k \Delta\mathcal{I}_{ik}(t,z,\mu) \otimes \Delta f_{kj}\,(z)$$

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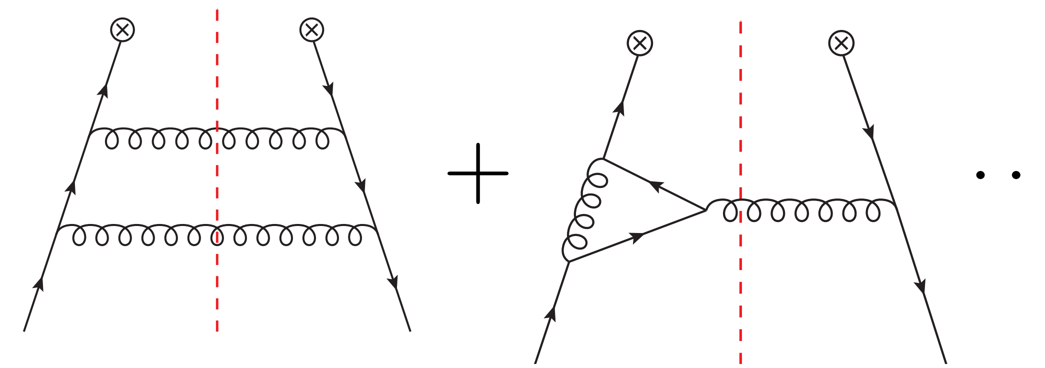

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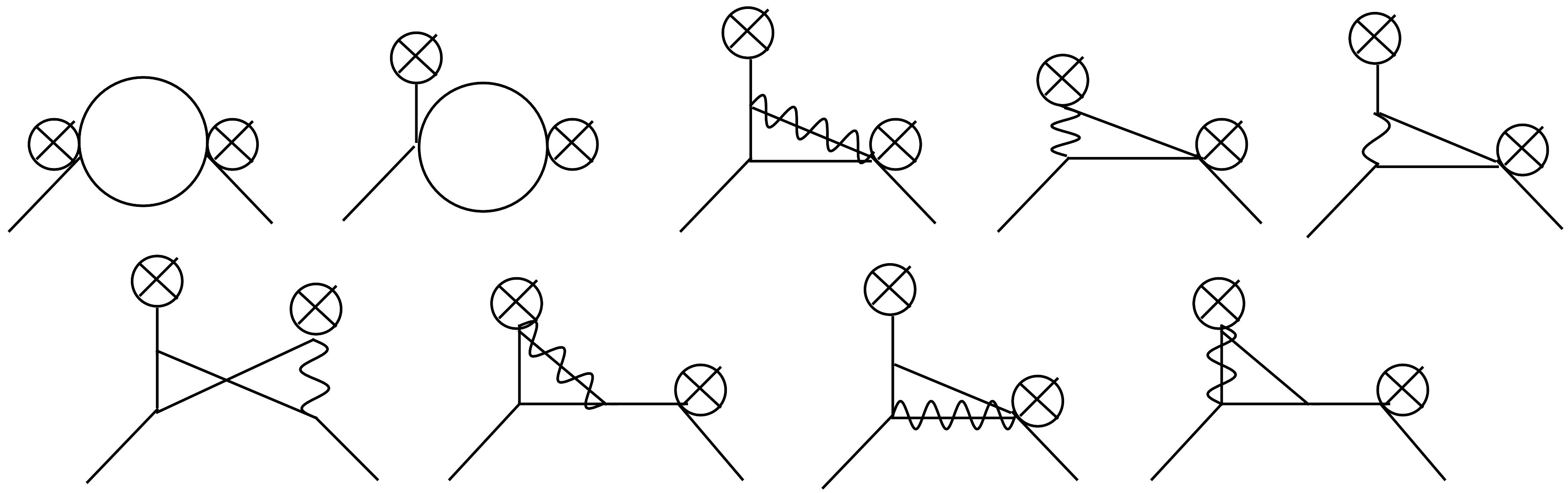
- Additional renormalization for γ_5

$$\Delta B = \left(\Delta\tilde{\mathcal{I}}\otimes\bar{Z}^5\right)\otimes\left(Z^5\otimes\Delta\tilde{f}\right)$$

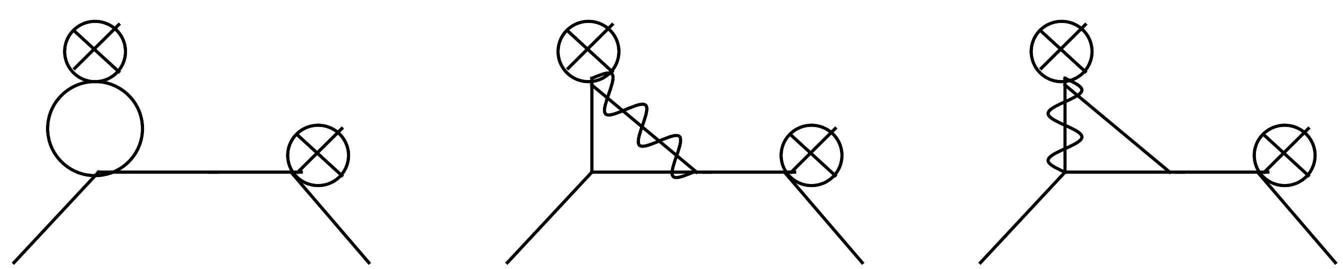
Master Integrals

- Initially $\mathcal{O}(100) - \mathcal{O}(1000)$ integrals

- 9 MIs in real-real channel



- 3 MIs in real-virtual channel



- Generate DEQ

$$\partial_x \vec{f} = A_x \vec{f}, \qquad x = t, z$$

Calculation of Master Integrals

- Bring DEQ in canonical form with Magnus algorithm
  [Henn; Argeri, Di Vita, Mastrolia, Mirabella, Schlenk, Tancredi, U.S.]

$$\partial_x \vec{g} = \epsilon \hat{A}_x \vec{g} \qquad \hat{A}_z = \frac{\hat{A}_1}{z} + \frac{\hat{A}_2}{1+z} + \frac{\hat{A}_3}{1-z}$$

- Matrices A_i have only numeric entries

- Simple alphabet $\{1-z,\, z,\, 1+z\}$

- Solution can be written in terms of Harmonic Polylogarithms

$$H_{a_1,\ldots,a_n}(z) = \int_0^z dt \frac{H_{a_2,\ldots a_n}(t)}{t-a_1}\,, \quad a_i \in 0,-1,1$$

$$H_{0,\ldots,0}(z) = \frac{1}{n!}\log^n(z)$$

Calculation of Master Integrals

- MI for RR channel behave like $(1-z)^{-2\epsilon}F(z)$ when $z \to 1$
  [Gaunt, Stahlhofen, Tackmann]

  => fixes 7 out of 9 boundary constants

- One MI is easily obtained by direct integration
- Last boundary constant obtained by
  - Introduce extra scale
  - Solve DEQ with extra scale
  - Here all boundaries can be fixed easily
  - take scale carefully to zero

- MI for RV behave like $(1-z)^{-2\epsilon,-\epsilon}F(z)$ when $z \to 1$

  => fixes one boundary constant

- Taking carefully $z \to 0$ fixes second boundary constant
- Last boundary can be easily obtained by direct integration

UV renormalisation and Matching

- Use standard $\overline{\mathrm{MS}}$ renormalization

$$\Delta B_{ij}^{bare(2)}(t,z) = \Delta B_{ij}^{(2)}(t,z,\mu) + Z_i^{(2)}(t,\mu)\delta_{ij}\delta(1-z) + \int dt' Z_i^{(1)}(t-t',\mu)\Delta B_{ij}^{(1)}(t',z,\mu).$$

- Requires calculation of $\Delta B_{ij}^{(1)}(t,z,\mu)$ up to $\mathcal{O}(\epsilon^2)$

- Match beam function on PDFs

$$\Delta \tilde{\mathcal{I}}_{ij}^{(2)}(t,z,\mu) = \Delta B_{ij}^{(2)}(t,z,\mu) - 4\delta(t)\Delta \tilde{f}_{ij}^{(2)}(z) - 2\sum_k \Delta \tilde{\mathcal{I}}_{ik}^{(1)}(t,z,\mu) \otimes \Delta \tilde{f}_{kj}^{(1)}(z) \; .$$

$$\Delta \tilde{f}_{ij}^{(1)}(z) = -\frac{1}{\epsilon}\Delta \tilde{P}_{ij}^{(0)}(z),$$

$$\Delta \tilde{f}_{ij}^{(2)}(z) = \frac{1}{2\epsilon^2}\sum_k \Delta \tilde{P}_{ik}^{(0)}(z) \otimes \Delta \tilde{P}_{kj}^{(0)}(z) + \frac{\beta_0}{4\epsilon^2}\Delta \tilde{P}_{ij}^{(0)}(z) - \frac{1}{2\epsilon}\Delta \tilde{P}_{ij}^{(1)}(z),$$

- Cancellation of poles provides consistency check

Treatment of Gamma5

- We use HVBM scheme

$$\gamma_5 \equiv \frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\rho \qquad\qquad \{\gamma_5, \tilde{\gamma}_\mu\} = 0, \quad [\gamma_5, \hat{\gamma}_\mu] = 0.$$

- Result of Dirac traces depends on d- and 4-d-dimensional momenta
- Map 4-d momenta to auxiliary vectors

$$I^d[\hat{k}_1 \cdot \hat{k}_2] = -\frac{2\epsilon}{v_\perp^2} I^d[(k_1 \cdot v_\perp)(k_2 \cdot v_\perp))],$$

- But: HVBM breaks helicity conservation

  => Must be restored with additional Z^5 renormalization

$$\Delta B = \left(\Delta\tilde{\mathcal{I}} \otimes \bar{Z}^5\right) \otimes \left(Z^5 \otimes \tilde{f}\right)$$

- Z^5 can be obtained by demanding helicity conservation

$$\Delta\mathcal{I}_{qq}^{(2,V)} = \mathcal{I}_{qq}^{(2,V)} \qquad\qquad \Delta\mathcal{I}_{q\bar{q}}^{(2,V)} = -\mathcal{I}_{q\bar{q}}^{(2,V)}$$

Consistency checks

- HVBM scheme implemented in public code Tracer and in-house Form routine
  [Jamin,Lautenbacher]
- MIs calculated by DEQ and direct integration
- Cancellation of poles during renormalization and matching
  - Confirmed polarised LO and NLO splitting functions
    [Vogelsang]
  - Confirmed UV renormalisation constant
    [Stewart, Tackmann, Waalewijn; Ritzmann, Waalewijn]
- Confirmed unpolarised quark beam function calculation at NLO and NNLO
  [Stewart, Tackmann, Waalewijn; Gaunt, Stahlhofen, Tackmann]
- Z^5 consistent with Literature
  [Ravindran, Smith, van Neerven]

Conclusions & Outlook

- Calculated spin-dependent quark beam function
- Last missing ingredient to apply N-jettiness subtraction to many polarized processes
- Provided independent check on:

  - unpolarized quark beam function up to NNLO

  - polarised splitting function up to NLO

- Ready for phenomenological studies

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Thank you for your attention