Critical behavior of SU(3) lattice gauge theory with 12 light flavors

Yannick Meurice

University of Iowa

Work done in part with Diego Floor and Zech Gelzer

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Talk Content

- Dynamical mass generation
- QCD with N_f light flavors (where is the conformal window?)
- Renormalization Group (RG) flows for $N_f = 12$
- Fisher zeros for $N_f = 12$ flavors of unimproved staggered fermions
- The Tensor Renormalization Group (TRG) method (summary)
- Entanglement entropy and central charge (2D $O(2)$ model)
- Conclusions

Dynamical mass generation

- Very appealing: a lot of structure from simple input
- Very common: in the standard model, more than 98 percent of the mass of the proton comes from quark-gluon interactions (QCD).
- Can be described by an effective theory: a Yukawa coupling $g_{\sigma NN}$ between the σ and the Nucleons:

$$m_N \sim g_{\sigma NN} < \sigma >$$

- In the standard model

$$m_f \sim g_{Hff} < H >$$

  f: quark or lepton, H: Higgs. Is the Higgs also composite?

-

$$\Gamma_\sigma \approx m_\sigma \approx \Lambda_{QCD}$$

  but

$$\Gamma_H < 13MeV \ll m_H(125GeV) \ll \Lambda_{new}??$$

- If $\Lambda_{new} > 2$TeV, is the lightness of the Higgs the remnant of an approximate conformal symmetry?

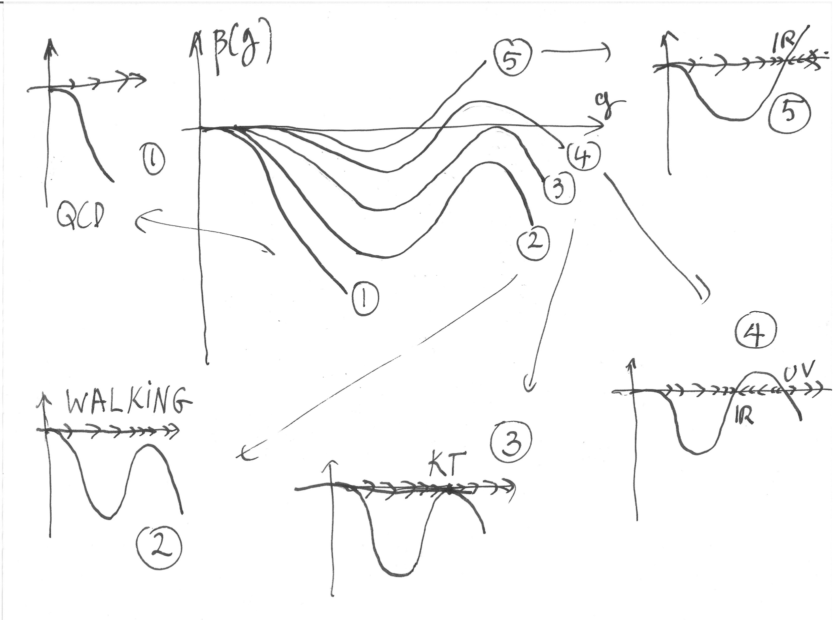
Simple models: $SU(3)$ gauge group with N_f light (or massless) fundamental Dirac quarks

- Familiar setup but the asymptotic scaling can be slower than QCD
- 2-loop perturbative beta function: nontrivial zero for $N_f > 8$

N_f	α_c
9	5.23599
10	2.20761
11	1.2342
12	0.753982
13	0.467897
14	0.278017
15	0.1428
16	0.0416105
QED	$0.0073 \simeq 1/137$

- It is doubtful that we can trust perturbation theory near the perturbative conformal fixed point α_c for $N_f \simeq 12$ massless flavors.
- Could $m_H \ll \Lambda_{new}$ be the remnant of some conformal symmetry?

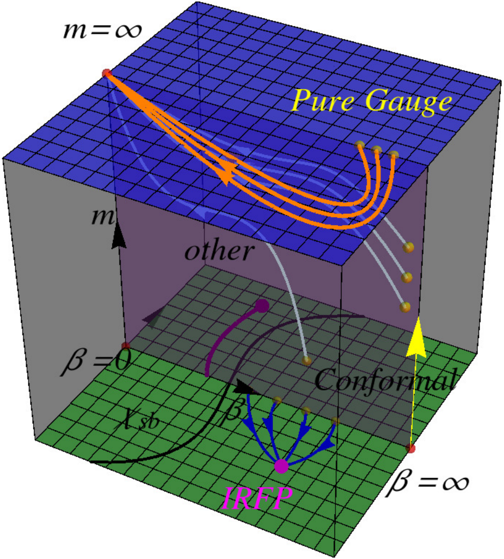
Beta functions beyond perturbation theory (schematic)



N_f=12 and $m = 0$

- The non trivial zero is at $\alpha_s \simeq 0.75 \sim 100\alpha_{QED}$, we need the lattice
- A majority of lattice practitioners think that for N_f=12 and $m = 0$:
  1) There is no confinement
  2) Chiral symmetry is unbroken
  3) The theory is conformal
  (see e. g. Hasenfratz and Schaich 1610.10004 , and Lin, Ogawa, Ramos 1510.05755)
  while a minority believes the exact opposite.
  (see e.g. Fodor et al. 1607.06121)
- A third logical possibility (confinement with unbroken chiral symmetry) seems excluded by 't Hooft anomaly matching + decoupling condition
- Different parts of the RG flows can be connected to hypothetical physical properties, but no realistic attempts are made here
- Recent reviews: DeGrand RMP88, Nogradi and Patella IJMPA 31

Schematic RG flows ($\beta = 6/g^2$)



RG flows from UV to IR (massless case)

1) Setting the scale (initial conditions, far UV)
For a sufficiently high scale we can use the universal perturbative running/dimensional transmutation. The QCD analog is $\alpha_s(M_Z^2) \simeq 0.1$. It is a physical input.

2) The intermediate scale
Using the reference scale in 1), we then reach a physical scale (in TeV) where we are far from both fixed points. From a computational point of view, things look maximally nonlinear/multidimensional in both directions. It is challenging to capture the essential features with small lattices and one-dimensional RG flow approximations.

3) The deep IR scale (assuming $m = 0$ and an attractive IRFP)
As we continue most of the irrelevant features get washed out and if we run all the way down, the intermediate scale does not appear anymore. For model building applications, the standard model interactions will break the conformal symmetry at the EW scale.

Effect of a $SU(12)_V$ symmetric mass term on $\langle\overline{\psi}\psi\rangle$

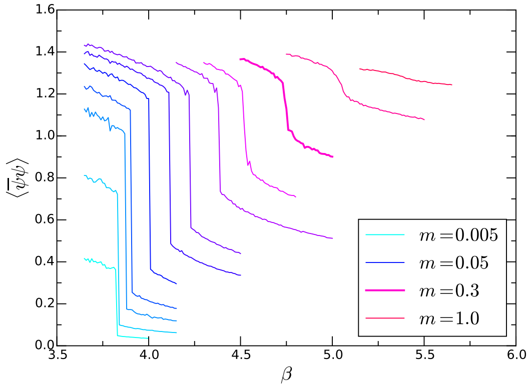


Figure: Unimproved HMC. Chiral condensate vs. β for increasing m, with $N_f = 12$, $V = 4^4$. The masses included (from left to right) are as follows: 0.0050, 0.0105, 0.0200, 0.0300, 0.0500, 0.0755, 0.0995, 0.1505, 0.2002, 0.3000, 0.5000, 0.9999.

The scaling with the size of the lattice suggests a bulk transition. We find that the location of the transitions in the chiral condensate $\langle\overline{\psi}\psi\rangle$ and average plaquette converge rapidly for isotropic lattices $V \equiv L_x^3 \times L_t = L^4$ with $L \geq 12$. This rapid convergence persists even with improved actions.

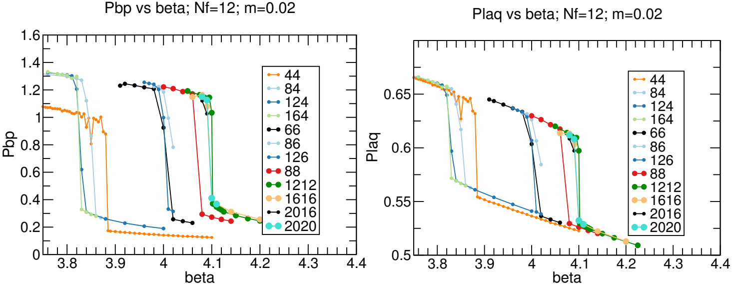


Figure: Unimproved HMC. Left panel: chiral condensate vs. β, with $N_f = 12,\ m = 0.02$. Right panel: plaquette vs. β, with $N_f = 12,\ m = 0.02$. In the legend, “124” signifies a volume of $V = 12^3 \times 4$.

Zeros of the partition function in the complex $\beta = 6/g^2$ plane (Fisher zeros)

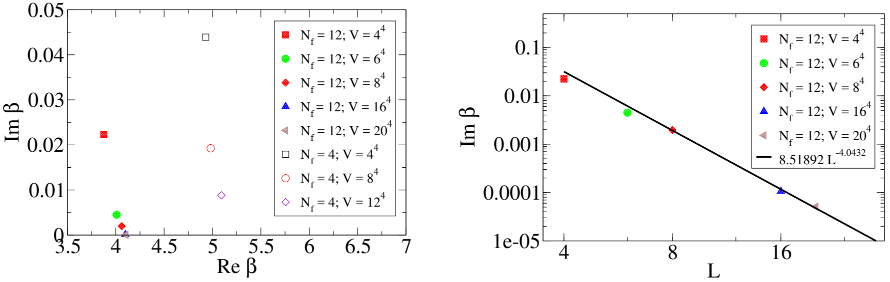


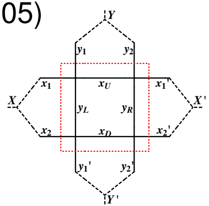
Figure: Left Panel: Fisher zeros for $N_f = 4$ and $N_f = 12$. Right panel: Scaling of the lowest zeros with L for $N_f = 12$. $Im\beta \propto L^{-4}$ is consistent with a first order transition. By increasing the mass we expect to reach an endpoint where a second order transition takes places (4D Ising, with a light weakly interacting 0^+?). At the critical mass we would expect $Im\beta \propto L^{-1/\nu}$ instead of L^{-4} (in progress). The unconventional continuum limit near this endpoint should be studied.

Description of the $N_f = 12$ massive theory with a linear sigma model?

- In lattice QCD, chiral perturbation theory provides useful parametrizations of the dependence on the mass, volume and lattice spacing; with more flavors, it seems that the σ is lighter and cannot be integrated (linear theory?)
- Linear sigma model: $\phi = (S_j + iP_j)\Gamma_j$; $(S_0 : \sigma,\ P_0 : \eta')$
- $Tr(\Gamma_i \Gamma_j) = (1/2)\delta_{ij}$ for 3 flavors: $j = 0, 1, 2, \ldots 8$
- $\mathcal{L} = Tr\partial_\mu \phi \partial^\mu \phi^\dagger - V(M) - \chi(det\phi + det\phi^\dagger) - bS_0$
- $M \equiv \phi^\dagger \phi$
- $V(M) \equiv -\mu^2 TrM + (\lambda_1/2 - \lambda_2/3)(TrM)^2 + \lambda_2 Tr(M^2)$
- For 3 flavors with equal masses, (Y.M. MPLA 2 699)
  $M^2_{\eta'} - M^2_\pi = (3/2)\chi f_\pi$
  $M^2_\sigma - M^2_\pi = (3/2)\lambda_1 f^2_\pi - (1/2)\chi f_\pi$
  $M^2_a - M^2_\pi = \lambda_2 f^2_\pi + \chi f_\pi$
- How does this generalizes for more flavor? (in progress)

The Tensor Renormalization Group (TRG) method

- Ideal tool to study conformal symmetry
- Exact blocking (spin and gauge, PRD 88 056005)



Unique feature: the blocking separates the degrees of freedom inside the block (integrated over), from those kept to communicate with the neighboring blocks. The only approximation is the truncation in the number of "states" kept.

- Applies to many lattice models: Ising model, $O(2)$ model, $O(3)$ model, $SU(2)$ principal chiral model (in any dimensions), Abelian and $SU(2)$ gauge theories (1+1 and 2+1 dimensions)
- Solution of sign problems: complex temperature (PRD 89, 016008), chemical potential (PRA 90, 063603)
- Checked with worm sampling method of Banerjee et al.
- Critical exponents of Ising (PRB 87, 064422; Kadanoff RMP 86)
- Used to design quantum simulators: $O(2)$ model (PRA 90, 063603), Abelian Higgs model (PRD 92 076003) on optical lattices

TRG blocking: Ising model example

For each link:

$$\exp(\beta\sigma_1\sigma_2) = \cosh(\beta)(1+\sqrt{\tanh(\beta)}\sigma_1\sqrt{\tanh(\beta)}\sigma_2) =$$
$$cosh(\beta)\sum_{n_{12}=0,1}(\sqrt{\tanh(\beta)}\sigma_1\sqrt{\tanh(\beta)}\sigma_2)^{n_{12}}.$$

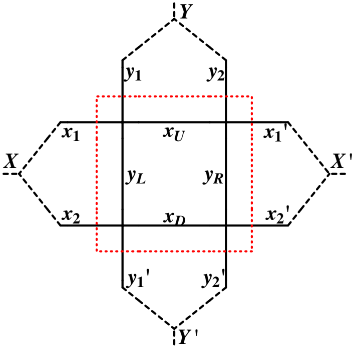
Regroup the four terms involving a given spin σ_i and sum over its two values ± 1. The results can be expressed in terms of a tensor: $T^{(i)}_{xx'yy'}$ which can be visualized as a cross attached to the site i with the four legs covering half of the four links attached to i. The horizontal indices $x,\ x'$ and vertical indices $y,\ y'$ take the values 0 and 1 as the index n_{12}.

$$T^{(i)}_{xx'yy'} = f_x f_{x'} f_y f_{y'} \delta\left(\mathrm{mod}[x+x'+y+y',2]\right) \ ,$$

where $f_0 = 1$ and $f_1 = \sqrt{\tanh(\beta)}$. The delta symbol is 1 if $x+x'+y+y'$ is zero modulo 2 and zero otherwise.

TRG blocking (graphically)

Exact form of the partition function: $Z = (\cosh(\beta))^{2V} \, \mathrm{Tr} \prod_i T^{(i)}_{xx'yy'}$.
Tr mean contractions (sums over 0 and 1) over the links.
Reproduces the closed paths of the HT expansion.
TRG blocking separates the degrees of freedom inside the block which are integrated over, from those kept to communicate with the neighboring blocks. Graphically :



We consider the subdivision of AB into A and B (two halves in our calculation) as a subdivision of the spatial indices.

$$\hat{\rho}_A \equiv \mathrm{Tr}_B \hat{\rho}_{AB}; \quad S_{EvonNeumann} = -\sum_i \rho_{A_i} \ln(\rho_{A_i}).$$

We use blocking methods until A and B are each reduced to a single site.

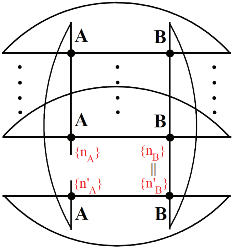


Figure: The horizontal lines represent the traces on the space indices. There are L_t of them, the missing ones being represented by dots. The two vertical lines represent the traces over the blocked time indices in A and B.

Rényi entanglement entropy

- The n -th order Rényi entanglement entropy is defined as:

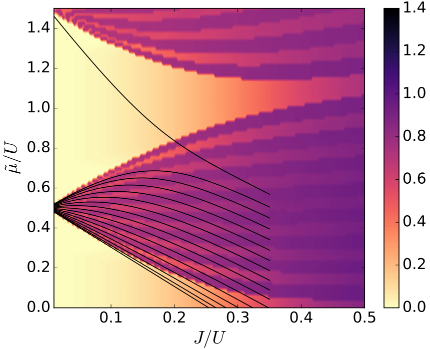
$$S_n(A) \equiv \frac{1}{1-n} \ln(\mathrm{Tr}((\hat{\rho}_A)^n)) \ .$$

- Calabrese-Cardy scaling for a spacial volume N_s with open BC:

$$S_n(N_s) = K + \frac{c(n+1)}{12n} \ln(N_s)$$

- Fits for the central charge for S_2 in the $O(2)$ model with a chemical potential consistent with central charge $c = 1$ arXiv:1703.10577, PRD in press; Quantum simulations with optical lattices: arXiv:1611.05016, PRA in press
- Measuring S_2 for 4D gauge theories with fermions could be an interesting approach to the conformal question

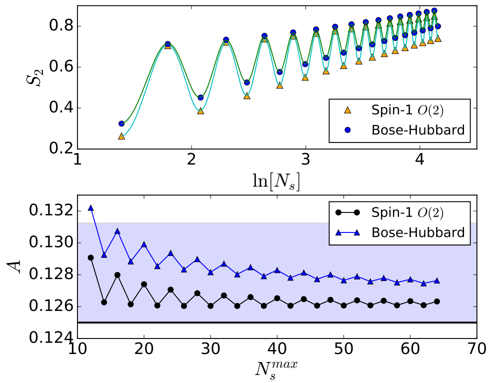
$O(2)$ Phase Diagram arXiv:1611.05016



- $N_s = 16$ lattice
- Color is S_2 for time-continuum $O(2)$.
- The light lobes are Mott insulator regions
- The stripes are jumps in particle number
- In black are the particle number boundaries for a possible quantum simulator on optical lattices (Bose-Hubbard model)

Rényi entropy and fit coefficient for BH and O(2) at $J/U = 0.1$

Fits for the central charge: (coefficient of $Log(N_s)$, $A = c/8 = 1/8?$)
arXiv:1703.10577, PRD in press



Conclusions

- A better understanding of conformal (or near conformal) lattice gauge models (Ie. g. $SU(3)$ with 12 massless, or at least light, quarks) is necessary before attempting model building: it's complicated, there are three scales ...
- Exotic continuum limits (e. g. near the end point of a line of first order transition) should be investigated
- Effective theory involving the 0^+ (the σ) in progress
- The framework is completely natural and can handle $\Lambda_{new} \gg$ 2TeV: experimentalists, keep looking for di-boson resonances!
- Tensor RG methods are promising to study near conformal spin and gauge models in 2D; they need to be extended to 3D and 4D
- Entanglement entropy could be an indicator of conformality in gauge theory with fermions (but it may be difficult to measure)
- Thanks!

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’t Hooft consistency conditions

Assume confinement and unbroken chiral symmetry

Anomaly matching: very cumbersome solutions. Example for $N_f = 8$:

$$-2\,\begin{array}{|c|c|c|}\hline L & L & L \\ \hline\end{array} - 1\,\begin{array}{|c|}\hline L \\ \hline L \\ \hline L \\ \hline\end{array} - 1\left(\begin{array}{|c|}\hline L \\ \hline\end{array} \otimes \begin{array}{|c|c|}\hline R & R \\ \hline\end{array} - \begin{array}{|c|}\hline R \\ \hline\end{array} \otimes \begin{array}{|c|c|}\hline L & L \\ \hline\end{array}\right)$$

$$+2\left(\begin{array}{|c|}\hline L \\ \hline\end{array} \otimes \begin{array}{|c|}\hline R \\ \hline R \\ \hline\end{array} - \begin{array}{|c|}\hline R \\ \hline\end{array} \otimes \begin{array}{|c|}\hline L \\ \hline L \\ \hline\end{array}\right) + 2\,\begin{array}{|c|c|}\hline L & L \\ \hline L & \multicolumn{1}{|c}{} \\ \cline{1-1}\end{array}$$

For $N_f = 12$, no solutions if the 5 indices are less than 12 in absolute value.

If decoupling condition is added: no solutions for $N_f > 2$.

If we relax one of the assumption, there is no conflict: deconfinement means no matching needed (majority point of view), chiral symmetry breaking means no massless baryons (minority point of view).