Resummation of TMD Distributions

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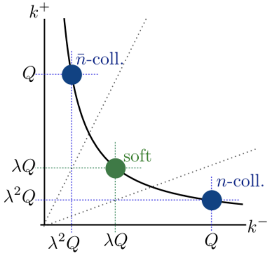
Outline

Factorization

- P+P $\rightarrow$ H+X, P+P $\rightarrow$ $l^+ + l^-$ +X.



$$p_c \sim \mathrm{Q}\,(1, \lambda^2, \lambda)\,,$$
$$p_{\bar{c}} \sim \mathrm{Q}\,(\lambda^2, 1, \lambda)\,,$$
$$p_s \sim \mathrm{Q}\,(\lambda, \lambda, \lambda)\,,$$
$$\lambda = \mathrm{qT} / \mathrm{Q}$$

Figure: IR modes have the same virtuality.

- The gauge boson recoils against soft and collinear radiation
- Need a regulator ν that breaks boost invariance to factorize the Soft from the collinear sector

Factorization

Transverse momentum cross section

$$\frac{d\sigma}{dq_T^2 dy} \propto H(\frac{\mu}{Q}) \times \int d^2 q_{\vec{T}i} S(q_{\vec{T}s}, \mu, \nu) \times$$
$$f_1^{\perp}(x_1, q_{\vec{T}1}, \mu, \nu, Q) f_2^{\perp}(x_2, q_{\vec{T}2}, \mu, \nu, Q) \delta^2(q_{\vec{T}} - q_{\vec{T}s} - q_{\vec{T}1} - q_{\vec{T}2})$$

- Virtuality of hard modes $\sim$ Q
- Virtuality of IR modes spread over a wide range of transverse momentum.
- RG equations in momentum space are convolutions of distributions functions and hard to solve directly [a].

[a] A recent paper 1611.08610 made progress but still challenging numerically

Factorization

b space formulation

$$\frac{d\sigma}{dq_T^2 dy} \propto H(\frac{\mu}{Q}) \int bdbJ_0(bq_T)S(b,\mu,\nu)f_1^{\perp}(x_1,b,\mu,\nu,Q)f_2^{\perp}(x_2,b,\mu,\nu,Q)$$

RG equations in b space are simple

$$\mu\frac{d}{d\mu}F_i(\mu,\nu,b) = \gamma_\mu^i F_i(\mu,\nu,b), \qquad F_i \in (H,S,f_i^{\perp})$$

$$\nu\frac{d}{d\nu}G_i(\mu,\nu,b) = \gamma_\nu^i G_i(\mu,\nu,b), \qquad G_i \in (S,f_i^{\perp})$$

$$\sum_{F_i} \gamma_\mu^i = \sum_{G_i} \gamma_\nu^i = 0$$

Resummation

- Resum large logarithms of the form $log(Q/q_T)$

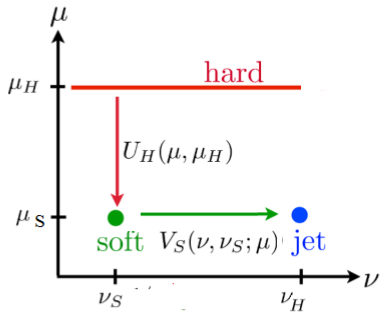


Figure: Choice of resummation path.

Scale choice in b space

Question

Do we make a scale choice in b space or momentum space?

- Scale choice which exactly minimizes Logarithms $\mu_H, \nu_H \sim Q$, $\mu_S, \nu_S \sim 1/b$.
- Equivalent to CSS formalism up to central values $\rightarrow$ resumming log (Qb).
- Landau pole $\alpha_s(1/b) \rightarrow$ modelling non-perturbative effects.
- Independent scale variations in $\mu, \nu \rightarrow$ reliable error estimation.
- Smooth matching to fixed order at high q_T using profiles in μ, ν.

Scale choice in momentum space

Can we choose a particular physical scales in momentum space for μ and ν? No!

Attempt at Leading Log $\sim \alpha_s^n log^{n+1}(\mu_H/\mu_L)$

- Assume a power counting $\alpha_s \log(Q/\mu_L), \log(\mu_L b) \sim 1$

$$\frac{d\sigma}{dq_t^2} \propto U_H^{LL}(H, \mu_L)\delta^2(q_t)$$

- Trivial result at non zero q_T
- Necessarily need a b space term

Scale choice in momentum space

Attempt at NLL → running Soft function in ν

$$
\begin{aligned}
\frac{d\sigma}{dq_t^2} &\propto U_H^{NLL}(H,\mu_L)\int dbbJ_0(bq_t)U_S(\nu_H,\nu_L,\mu_L) \\
&= U_H^{NLL}(H,\mu_L)\int dbbJ_0(bq_t)\left(\mu_L^2 b_0^2\right)^{\Gamma_{cusp}^{(0)}\frac{\alpha(\mu_L)}{2\pi}\log(\frac{\nu_H}{\nu_L})} \\
&= 2U_H^{NLL}(H,\mu_L)e^{-2\omega_s\gamma_E}\frac{\Gamma[1-\omega_s]}{\Gamma[\omega_s]}\frac{1}{\mu_L^2}\left(\frac{\mu_L^2}{q_T^2}\right)^{1-\omega_s}, \\
\omega_s &= -\Gamma_{cusp}^{(0)}\frac{\alpha(\mu_L)}{2\pi}\log(\frac{\nu_H}{\nu_L}), \qquad \Gamma_{cusp}^{(0)}<0
\end{aligned}
$$

- Still does not work, singular at $\omega_s \sim 1$

Scale choice in momentum space

- Divergence caused due to single log structure ($log(\mu_L b)$) in the Soft exponent
- Contribution from highly energetic soft mode with $b \sim 1/Q$.
- Need damping at low b to stabilize b space exponent.
- $\mu_H, \nu_H \sim Q$ fixed at the hard scale in momentum space.

Scale choice in momentum space

- Resum all logarithms of the form $\alpha_s \log^2(\mu_L b_0)$

A choice for ν in b space $\rightarrow$ include sub-leading terms

$$\nu_L = \frac{\mu_L^n}{b_0^{1-n}}, \qquad n = \frac{1}{2}\left(1 - \alpha(\mu_L)\frac{\beta_0}{2\pi}\log(\frac{\nu_H}{\mu_L})\right)$$

Soft exponent at NLL $\rightarrow$ Quadratic in $\log(\mu_L b_0)$

$$\log(U_S^{NLL}(\nu_H, \nu_L, \mu_L)) = 2\Gamma_{cusp}^{(0)}\frac{\alpha(\mu_L)}{2\pi} \times$$
$$\left(\log(\frac{\nu_H}{\mu_L})\log(\mu_L b_0) + \frac{1}{2}\log^2(\mu_L b_0) + \alpha(\mu_L)\frac{\beta_0}{4\pi}\log^2(\mu_L b_0)\log(\frac{\nu_H}{\mu_L})\right)$$

Scale choice in momentum space

A choice for μ_L in momentum space

- A choice that justifies the power counting $log(\mu_L b_0) \sim 1$
- A choice that will minimize contributions from residual fixed order logs $log^n(\mu_L b_0)$.
- Scale shifted away from q_T due to complicated b space exponent.

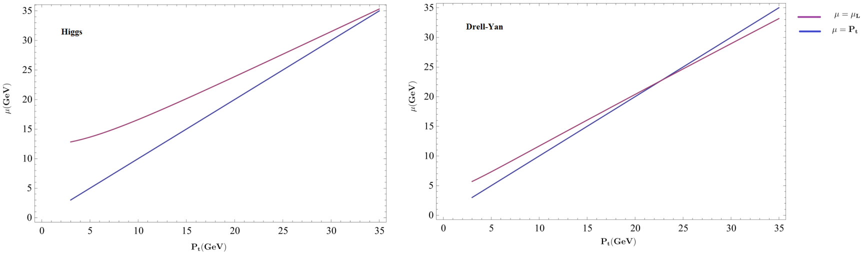


Figure: μ scale choice in momentum space.

Scale choice in momentum space

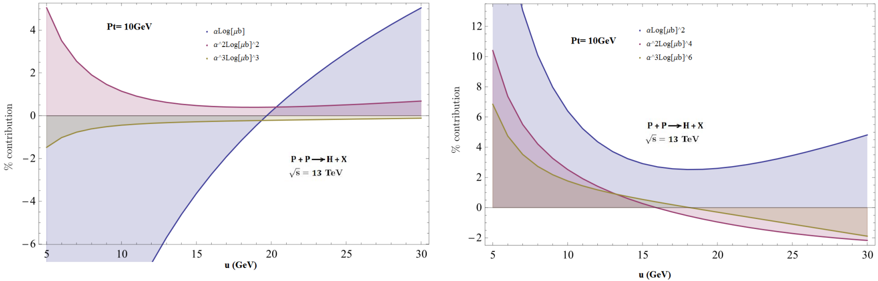


Figure: Percentage contribution of the fixed order logs as a function of the scale choice μ.

- $\mu_L \sim 1/b^*$, b^* is the value at which b space integrand peaks

Analytical expression for cross section

Mellin-Barnes representation of Bessel function

- Polynomial integral representation for Bessel function is needed

$$J_0(z) = \int_{c-i\infty}^{c+i\infty} dt \frac{\Gamma[-t]}{\Gamma[1+t]} \left(\frac{1}{2}z\right)^{2t}$$

b space integral

$$U_S = C_1 Exp[-A \log^2(Ub)]$$

$$I_b = \int_0^\infty bJ_0(bq_T)U_S \qquad \text{No Landau pole}$$

$$= \quad C_1 \int_{-i\infty}^{i\infty} dt \frac{\Gamma[-t]}{\Gamma[1+t]} \int_0^\infty dbb(\frac{bq_T}{2})^{2t} Exp[-A \log^2(Ub)]$$

Analytical expression for cross section

$$I_b = \frac{C_1}{U^2}\sqrt{\frac{\pi}{A}}\int_{c-i\infty}^{c+i\infty} dt \frac{\Gamma[-t]}{\Gamma[1+t]} Exp[(1+t)^2/A - 2t\log(\frac{2U}{q_T})]$$

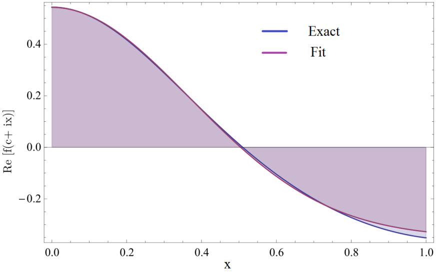
- Integral along contour in t space suppressed by the exponential
- Suppression controlled by 1/A $\sim \frac{4\pi}{\alpha_s} 1/\Gamma^{(0)}_{cusp}$

A gaussian fit for f(t) $= \Gamma[-t]/\Gamma[1+t]$

$$f_R(x) = g_1 + g_2 Exp[-g_3 x^2]$$
$$f_I(x) = f_1 x Exp[-f_2 x^2] + f_3 \sin(f_4 x)$$

- Fit independent of observable or kinematics

Analytical expression for cross section



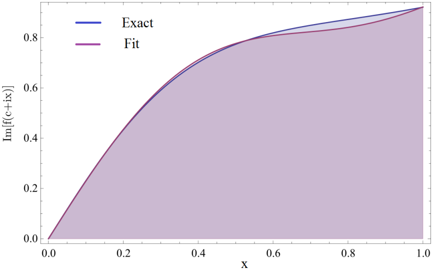


Figure: Fit for real and imaginary parts of f(t), c is chosen to be -0.65

- Contour is parametrized as $t = c + i\,x$
- Region of contribution restricted between $x = \pm 1$ for $A \leq 0.5$

Analytical expression for cross section

$$I_b = 2C_1 e^{\left[-A\log^2\left(\frac{2U}{q_T}\right)\right]} \times \frac{1}{q_T^2}\left(g_1 + \frac{g_2 e^{\left[\frac{A^2C_2^2 g_3}{(1+Ag_3)}\right]}}{\sqrt{1+Ag_3}} - \frac{Af_1 C_2 e^{\left[\frac{A^2C_2^2 f_2}{(1+Af_2)}\right]}}{2(1+Af_2)^{3/2}} - f_3 e^{\left[-\frac{Af_4^2}{4}\right]}\sinh(AC_2 f_4)\right)$$

Parameters at NLL

$$A = -2\Gamma^{(0)}_{cusp}\frac{\alpha(\mu_L)}{4\pi}\left(1 + \frac{\alpha(\mu_L)\beta_0}{2\pi}\log(\frac{\nu_H}{\mu_L})\right)$$

$$C_1 = Exp[A\log^2(\eta)], \quad U = \mu_L\eta$$

$$\eta = Exp\left[\frac{\log(\nu_H/\mu_L)}{1+\frac{\alpha(\mu_L)\beta_0}{2\pi}\log(\frac{\nu_H}{\mu_L})}\right], \quad C_2 = \frac{(1+c)}{A} - \log(\frac{2U}{q_T})$$

- Easily extended to NNLL, b space exponent kept quadratic in $\log(\mu_L b)$

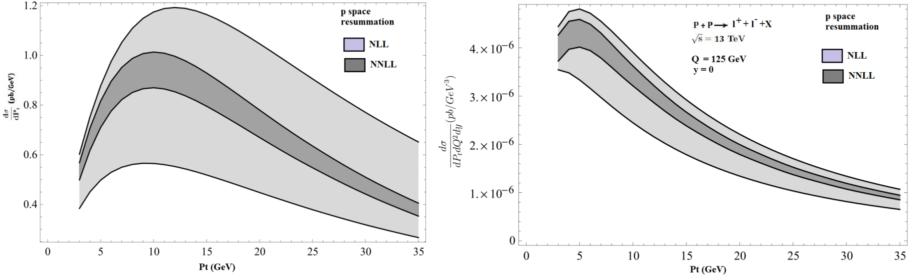


Figure: Resummation in momentum space.

- Excellent convergence for both the Higgs and Drell-Yan spectrum
- No Landau pole and No unphysical soft mode!

Comparison with b space resummation

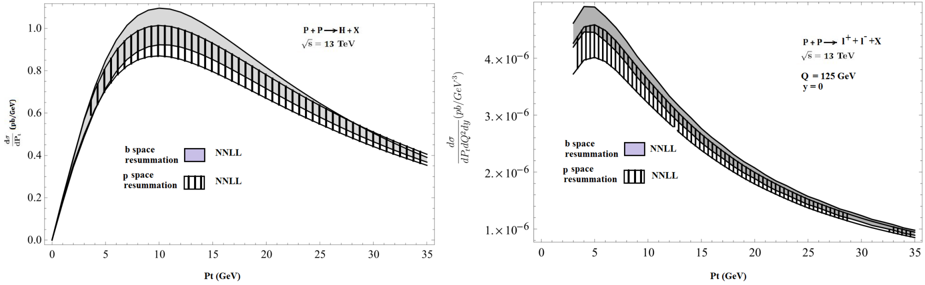


Figure: comparison of NNLL cross section in two schemes

- Difference of the order of sub-leading terms

Summary

- Implementation momentum space resummation for transverse spectra of gauge bosons
- Rapidity choice in impact parameter space
- Virtuality choice in momentum space.
- Analytical expression for cross section obtained for the first time
- Outlook
  - Non-perturbative effects need to be included for low q_T.