NNLO Jet Phenomenology for Current and Future Colliders

Radja Boughezal

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Santa Fe, New Mexico
QCD yesterday and today

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Is theory keeping up with the impressive experimental progress?
Higgs production

- The dominant component of the systematic error on the signal strength is theory (~10-15%).
- The statistical error from LHC Run I is the largest (~20%), this however will improve during LHC Run II.

LHC Run II prospects:
- x2.5 increase in cross section
- x15 increase in luminosity (300 fb⁻¹)
- ~ 40 times more events

Stat. error in 3-4% range

Theory error becoming a limiting factor in Run II
**W/Z+1jet**

- They provide stringent tests on the SM, as they are measured with small errors over a large energy range. Important for improving PDFs, and detector calibration as well.

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**ATLAS**

$s = 8$ TeV, 20.3 fb$^{-1}$

- Statistical uncertainty
- Total uncertainty

$66$ GeV $\leq m_\perp < 116$ GeV, $|y_\perp| < 2.4$

Total experimental uncertainty up to 200 GeV for the $P_{TZ}$ is $< 1\%$
Jet physics at the EIC

- Numerous physics motivations for studying jet production at a future EIC
  - Measurement of the strong coupling constant
    D. Kang, Lee, Stewart (2013)
  - Determination of parton distribution functions
  - Measure properties of the nuclear medium with event shapes
    Z. Kang, Liu, Mantry, Qiu (2012)

The precision of an EIC plays a critical role in all of these measurements!
Precision theory beyond NLO is important for the interpretation of several experimental measurements.
Ingredients for NNLO calculations

- Need the following ingredients for NNLO cross sections

\[
\begin{align*}
\text{VV} & \quad \begin{array}{c}
\int \left[ \frac{vv_4}{\epsilon^4} + \frac{vv_3}{\epsilon^3} + \frac{vv_2}{\epsilon^2} + \frac{vv_1}{\epsilon} + vv_0 \right] \, d\Phi_2
\end{array} \\
\text{RV} & \quad \begin{array}{c}
\int \left[ \frac{rv_2}{\epsilon^2} + \frac{rv_1}{\epsilon} + rv_0 \right] \, d\Phi_3
\end{array} \\
\text{RR} & \quad \begin{array}{c}
\int [rr_0] \, d\Phi_4
\end{array}
\end{align*}
\]

- IR singularities cancel in the sum of real and virtual corrections and mass factorization counterterms but only after phase space integration for real radiations

- Virtual corrections have explicit IR poles, whereas real corrections have implicit IR poles that need to be extracted.
Ingredients for NNLO calculations

• Need the following ingredients for NNLO cross sections

\[ VV \quad RV \quad RR \]

• IR singularities cancel in the sum of real and virtual corrections and mass factorization counterterms but only after phase space integration for real radiations.

• Virtual corrections have explicit IR poles, whereas real corrections have implicit IR poles that need to be extracted.

• A generic procedure to extract IR singularities from RR and RV was unknown when jets in the final state are involved, until very recently.
A Timeline of NNLO hadron collider cross sections

- Complete NNLO hadron-collider cross sections with control over kinematics:

2005
Higgs  W,Z

2006

2007

2008

2009

2010

2011

2012

2013

2014

2015

WH  YY
ZH  ZZ
\

Wj
Hj
Zj
VBF

NNLO, including for jets, becoming available for 2→2 scattering in time for high-precision Run II data!
RG-assisted methods at NNLO

- **EFT-assisted techniques**: leverage knowledge of effective field theory resummation to remove double-real emission singularities.
- **$q_T$-subtraction**: Catani, Grazzini (2007)

EFT methods are responsible for much of the recent progress and numerous phenomenological results!
N-jettiness

- N-jettiness, $\tau_N$, is an event-shape variable designed to veto final-state jets [Stewart, Tackmann, Waalewijn 0910.0467]

$$\tau_N = \sum_k \min_i \left\{ \frac{2p_i \cdot q_k}{Q_i} \right\}.$$  

- $N=$number of final-state jets
- Momenta of the two beams and the final-state jets
- Measure of the jet hardness (we take $Q_i=2E_i$)
- All final-state partons

Small N-jettiness vetoes events with more than N-jets
N-jettiness

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All final-state partons

Contribution from:
- Two-loop
- Soft and collinear radiation

Contribution from at least 2 hard jets:
- NLO two-jet calculation. Use known results/tools
N-jettiness subtraction

- N-jettiness can be applied to obtain exact NNLO cross sections

- Introduce $\tau_{N}^{\text{cut}}$ that separates the $\tau_{N}=0$ doubly-unresolved limit of phase space from the single-unresolved and hard regions

\[
\sigma_{NNLO} = \int d\Phi_{N} |M_{N}|^{2} + \int d\Phi_{N+1} |M_{N+1}|^{2} \theta_{N}^{<} \\
+ \int d\Phi_{N+2} |M_{N+2}|^{2} \theta_{N}^{<} + \int d\Phi_{N+1} |M_{N+1}|^{2} \theta_{N}^{>} \\
+ \int d\Phi_{N+2} |M_{N+2}|^{2} \theta_{N}^{>}
\]

\[\equiv \sigma_{NNLO}(\tau_{N} < \tau_{N}^{\text{cut}}) + \sigma_{NNLO}(\tau_{N} > \tau_{N}^{\text{cut}})\]

\[\theta_{N}^{<} = \theta(\tau_{N}^{\text{cut}} - \tau_{N}) \quad \text{and} \quad \theta_{N}^{>} = \theta(\tau_{N} - \tau_{N}^{\text{cut}})\]
N-jettiness subtraction

• For $\tau_N > \tau_N^{\text{cut}}$, at least one of the two additional radiations that appear at NNLO is resolved; this region of phase space contains the NLO correction to the N+1 jet process. Can be obtained from any NLO program.

• For $\tau_N < \tau_N^{\text{cut}}$, both additional radiations are unresolved. A factorization theorem giving the all-orders result for small N-jettiness was derived Stewart, Tackmann, Waalewijn 0910.0467

\[
\sigma(\tau_N < \tau_N^{\text{cut}}) = \int H \otimes B \otimes B \otimes S \otimes \left[ \prod_n J_n \right] + \cdots .
\]

describes hard radiation; in dim-reg, coincides with the 2-loop virtual corrections
describes radiation collinear to an initial-state beam
describes soft radiation
describes radiation collinear to a final-state jet
N-jettiness subtraction

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$$\sigma(\tau_N < \tau_{N}^{\text{cut}}) = \int H \otimes B \otimes B \otimes S \otimes \left[ \prod_{n}^{N} J_n \right] + \cdots.$$ 

$\tau_{N}^{\text{cut}}$ must be much smaller than any hard scale in the process and any experimental cuts in order to suppress power corrections. Final result must be independent of $\tau_{N}^{\text{cut}}$. 

Radja Boughezal, ANL
Ingredients for the factorization theorem

\[ \sigma(\tau_N < \tau_N^{cut}) = \int H \otimes B \otimes B \otimes S \otimes \left[ \prod_{n=1}^{N} J_n \right] + \cdots . \]

- Expand this formula to \( O(\alpha_s^2) \), and turn off all resummation, to get the NNLO cross section below the cut. Need each of these separate functions to NNLO.
- The beam and jet functions depend only on the flavor of the parton (quark, gluon); the soft function depends only on the parton flavors and the external hard directions; the hard function is process-dependent piece.

- **\( H@NNLO \):** for W/Z/H+j, Gehrmann, Tancredi (2012); Gehrmann, Jaquier, Glover, Koukoutsakis (2012)
- **\( B@NNLO \):** Gaunt, Stahlhofen, Tackmann (2014)
- **\( S@NNLO \):** RB, Liu, Petriello (2015)
- **\( J@NNLO \):** Becher, Neubert (2006); Becher, Bell (2011)

Within the past two years all ingredients have become available to apply this idea to 2→2 scattering processes at colliders!
Results from N-jettiness subtraction

- Obtained new results for $W/H/Z+\text{jet}$ (RB, Focke, Liu, Petriello; RB, Focke, Giele, Liu, Petriello; RB, Campbell, Ellis, Focke, Giele, Liu, Petriello (2015))

- Confirmed existing results for color-singlet processes: $H/W/Z$ (Gaunt, Stahlhofen, Tackmann, Walsh (2015); RB, Campbell, Ellis, Focke, Giele, Petriello, Williams (2016)); $VH$ (Campbell, Ellis, Williams (2016))

A comparison to the NNLO structure function by integrating over the final state jet in the DIS process

MCFM 8.0: RB, Campbell, Ellis, Focke, Giele, Liu, Petriello, Williams (2016)
Vector boson plus jet production

- Important background process to SUSY, dark matter searches, VH production

NNLO corrections mandatory to properly describe measured $H_T$ distributions in both $W+j$ and $Z+j$ production

RB, Liu, Petriello (2016)
Vector boson plus jet production

Only NNLO correctly describes shape and normalization via a controlled, systematically improvable expansion

RB, Liu, Petriello (2016)

7 TeV CMS Z

$\frac{d\sigma}{dH_T}$ [pb/GeV]

$H_T$ [GeV]
Comparisons with 13 TeV data

- NNLO again offers an excellent description of both shape and normalization
The $Z$ $p_T$ spectrum and the gluon luminosity

NNLO QCD+NLO EW critical to correctly describe data; potentially important impact on gluon distribution!

New data and NNLO predictions have an impact on the gluon PDFs!
Jet physics at electron-nucleon colliders

- N-jettiness subtraction method is not limited to LHC applications, can also help with precision physics at future electron-nucleus colliders

- Large NLO perturbative corrections, O(100%)
- Important, but not dominant, corrections from photon-initiated processes (Weizsacker-Williams)
- Does the perturbative series converge at NNLO?
- Are the NNLO corrections dominated by a single channel?

Hinderer, Schlegel, Vogelsang (2015)
**Definition of the process**

**DIS**: eN → eN

- Lepton tagged
- Cut on $Q^2$
- Hard scale: $Q$

**Inclusive jet production**: eN → jX

- Lepton *not* tagged
- Cut on $p_{Tjet}$
- Hard scale: $p_{Tjet}$

**Quark-lepton recoils against a jet**

$$q(p_1) + l(p_2) \rightarrow q(p_3) + l(p_4)$$
NLO O($\alpha^2\alpha_s$) corrections

• Typical real and virtual corrections to the quark-lepton scattering processes; new contribution from gluon-lepton scattering → calculation amenable to standard techniques

• New configuration: lepton collinear to the beam ($Q^2\sim0$), with two jets balancing in the transverse plane; on-shell photon scattering with quark→differentiates DIS and inclusive jet production

\[
f_{\gamma/l}(\xi) = \frac{\alpha}{2\pi} P_{\gamma/l}(\xi) \left[ \ln \left( \frac{\mu^2}{\xi^2 m_l^2} \right) - 1 \right] + \mathcal{O}(\alpha^2)
\]

\[
P_{\gamma/l}(\xi) = \frac{1 + (1 - \xi)^2}{\xi}
\]
NNLO $O(\alpha^2\alpha_s^2)$ corrections

- **New configuration**: incoming lepton can split into a quark, leading to parton-parton scattering channels. They first appear at this order, and are therefore effectively leading order in our treatment.

- Standard NLO corrections to quark-photon scattering
- Double-virtual, real-virtual, and double-real corrections to quark-lepton scattering
**NNLO jet production in ep collisions**

- **NNLO corrections obtained with N-jettiness subtraction**
  Abelof, RB, Liu, Petriello (2016)

![Graph showing NNLO jet production in ep collisions](image)

- Better perturbative behavior is observed only when NNLO corrections are included!

- **NNLO corrections small for \( \eta_{jet} < 1 \), but increase as \( \eta_{jet} \to 2 \)**

- **Scale dependence increases at NNLO for \( \eta_{jet} < 0 \)**
**η_{jet} distribution: partonic channels**

- **qq** channel drives the large scale uncertainty for η_{jet}<0; it begins at $O(\alpha^2 \alpha_s^2)$, and is effectively leading-order in this result,
- **ql** channel dominates for low η_{jet}; qγ channel dominates at high η_{jet}
- No single channel furnishes a good approximation to the full result

**Single inclusive jet production is a powerful window into proton structure**
Future improvements

- N-jettiness subtraction can be systematically improved by incorporating the power corrections to the factorization formula.
- These corrections are universal and calculable; this allows higher $\tau_N^{\text{cut}}$ and significantly improves the method.
- Leading NNLO power corrections known for 0-jettiness

Moult, Rothen, Stewart, Tackmann, Zhu (2016); RB, Liu, Petriello (2016)
Conclusions

• We have entered the era of NNLO jet phenomenology in time for analysis of LHC Run II data

• New ideas have made NNLO calculations for 2→2 processes far less painful to obtain than was thought possible just a few years ago

• The N-jettiness subtraction scheme allows for precision calculations of jet processes at the LHC and other future colliders; already a significant impact on understanding Run II data

• More results to come!