Nuclear Shell Model and Phase Transitions

Sofia Karampagia

National Superconducting Cyclotron Laboratory,
Michigan State University

Nuclei and Mesoscopic Physics, 03/07/2017
Outline

- Quantum phase transitions in the framework of the shell model
- Collective enhancement of the nuclear level density
Quantum Phase Transitions (QPT) refer to phase transitions, which occur at zero temperature \((T = 0)\), when a non-thermal parameter, \(\lambda\), is changed (pressure, magnetic field). This parameter(s) is called control parameter and the value for which the phase transition takes place is the critical value, \(\lambda = \lambda_c\), for a system described by

\[
H(\lambda) = \epsilon(H_1 + \lambda H_2)
\]

Ising Model in a transverse field (ferromagnetic \(\rightarrow\) paramagnetic)

\[
H = -J \sum_{ij} S^z_i S^z_j - h \sum_i S^x_i
\]
Quantum Phase Transition in atomic nuclei

Algebraic models are used, possible phases of a system obtained by considering possible breakings of the algebra $g$ into its subalgebras, each subalgebra having its own dynamical symmetry, structural shape.

\[
\begin{align*}
g & \supset g_1 \supset \ldots \\
g & \supset g_2 \supset \ldots \\
& \ldots \\
g & \supset g_\nu \supset \ldots
\end{align*}
\]
Quantum Phase Transition in atomic nuclei

Algebraic models are used, possible phases of a system obtained by considering possible breakings of the algebra $g$ into its subalgebras, each subalgebra having its own dynamical symmetry, structural shape.

\[ g \supset g_1 \supset \ldots \]

\[ g \supset g_2 \supset \ldots \]

\[ g \supset g_\nu \supset \ldots \]

\[ H = (1 - \lambda)H_1 + \lambda H_2, \]

\[ \lambda = 0 \rightarrow H_1, \lambda = 1 \rightarrow H_2 \]
Order of Quantum Phase Transition

Ehrenfest criterion:
ground state energy functional and its derivatives, \( n \)th order if its \( n \) derivative with respect to \( \lambda \) is discontinuous

Lincoln D. Carr, *Understanding QPTs* (\( N_B = 10, 100, 200, 300, 400 \))
Order of Quantum Phase Transition

Interacting Boson Model, U(6)

\[ H(\zeta, \chi) = c[(1 - \zeta)\hat{n}_d - \frac{\zeta}{4N_B} \hat{Q}_\chi \cdot \hat{Q}_\chi] \]
\[ \hat{n}_d = d^\dagger \tilde{d} \]
\[ \hat{Q}_\chi = (s^\dagger \tilde{d} + d^\dagger \tilde{s}) + \chi(d^\dagger \tilde{d})^{(2)} \]
\[ \zeta \in [0, 1] \]
\[ \chi \in [0, -1.32] \]

left: first order QPT (U(5) \rightarrow SU(3))
right: second order QPT (U(5) \rightarrow O(6))

Lincoln D. Carr, *Understanding QPTs* \((N_B = 10, 100, 200, 300, 400)\)
Two-Body Matrix Elements:

The interaction scatters 2 nucleons into a different pair of orbits compared to the pair of orbits before the scattering fall into 3 categories

sd shell: 63 matrix elements
pf shell: 195 matrix elements

$V_1$: one particle transfer matrix elements,
$V_2$: the rest matrix elements

$V_1$: deformation

H. Horoi et al., PRC 81, 034396 (2010)
Quantum Phase Transition

\[ H = (1 - \lambda) V_1 + \lambda V_2 \]

✓ Varying the strength of the \( V_1 \) and \( V_2 \) shell model matrix elements in the sd and pf shells using a control parameter \( \lambda \) going from 0 to 1

<table>
<thead>
<tr>
<th>( ^{24}\text{Mg} )</th>
<th>( \lambda )</th>
<th>( 2^+_1 )</th>
<th>( 4^+_1 )</th>
<th>( 6^+_1 )</th>
<th>( R_{4/2} )</th>
<th>( Q(2^+_1) )</th>
<th>( B(E2; 2^+_1 \rightarrow 0^+_1) )</th>
<th>( B(E2; 4^+_1 \rightarrow 2^+_1) )</th>
<th>( B(E2; 6^+_1 \rightarrow 4^+_1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.596</td>
<td>1.667</td>
<td>3.507</td>
<td>2.80</td>
<td>-16.32</td>
<td>7.81E+01</td>
<td>7.91E+01</td>
<td>6.53E+01</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.590</td>
<td>1.649</td>
<td>3.512</td>
<td>2.79</td>
<td>-18.02</td>
<td>8.23E+01</td>
<td>9.31E+01</td>
<td>7.94E+01</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.548</td>
<td>1.620</td>
<td>3.504</td>
<td>2.96</td>
<td>-18.65</td>
<td>8.39E+01</td>
<td>9.99E+01</td>
<td>8.65E+01</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>0.515</td>
<td>1.640</td>
<td>3.533</td>
<td>3.18</td>
<td>-18.87</td>
<td>8.36E+01</td>
<td>1.03E+02</td>
<td>8.86E+01</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.547</td>
<td>1.766</td>
<td>3.642</td>
<td>3.23</td>
<td>-18.68</td>
<td>8.14E+01</td>
<td>1.02E+02</td>
<td>8.46E+01</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.688</td>
<td>2.036</td>
<td>3.860</td>
<td>2.96</td>
<td>-17.97</td>
<td>7.66E+01</td>
<td>9.55E+01</td>
<td>7.32E+01</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>0.951</td>
<td>2.433</td>
<td>4.201</td>
<td>2.56</td>
<td>-16.65</td>
<td>6.91E+01</td>
<td>8.10E+01</td>
<td>5.74E+01</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>2.404</td>
<td>4.337</td>
<td>6.761</td>
<td>1.80</td>
<td>-7.29</td>
<td>2.76E+01</td>
<td>2.26E+01</td>
<td>1.93E+01</td>
<td></td>
</tr>
</tbody>
</table>
Signs of a QPT in even–even nuclei

S. Karampagia, V. Zelevinsky, PRC 94 014321 (2016)
Angular momentum wavefunction decomposition

\[ ^{24}\text{Mg} \]

The graph illustrates the g.s. wave function squared amplitudes (\%) for different states as a function of \( \lambda \). Each curve corresponds to a specific angular momentum state:

- **(0,0)**
- **(2,2)**
- **(4,4)**

The data points for each state are marked with different colors and line styles. The x-axis represents \( \lambda \) ranging from 0 to 1, while the y-axis shows the g.s. wave function squared amplitudes in percentage.
Signs of a QPT in odd–even, odd–odd nuclei

\[ E(J) \text{ (MeV)} \]

\[ \lambda \]

\[ 26 \text{Al} \]

\[ 28 \text{Al} \]

\[ 27 \text{Al} \]

Sofia Karampagia

Nuclear Shell Model and Phase Transitions
B(E2)s, $Q(2^+_1)$ in odd–even, odd–odd nuclei

$^{26}$Al

$^{28}$Al

Sofia Karampagia

Nuclear Shell Model and Phase Transitions
Order of QPT

$E(g.s.)$

$\frac{dE(g.s.)}{d\lambda}$

$\frac{d^2E(g.s.)}{d\lambda^2}$
Level Densities

$^{24}\text{Mg}, J = 0$

$^{26}\text{Al}, J = 0 - 10$
$^{27}\text{Al}, J = 1/2 - 21/2$
$^{28}\text{Al}, J = 0 - 10$
Collective enhancement: Noticeable change of the low lying nuclear level density due to collective effects

Many cases with signs of collective enhancement, selected just a few...

<table>
<thead>
<tr>
<th>J=0 (up to 10 MeV)</th>
<th>Deformed</th>
<th>Spherical</th>
<th>Deformed</th>
<th>Spherical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_{4/2}$</td>
<td>$\lambda$</td>
<td>$R_{4/2}$</td>
<td>$\lambda$</td>
</tr>
<tr>
<td>$^{28}$Si</td>
<td>3.01</td>
<td>0.2</td>
<td>2.15</td>
<td>0.5</td>
</tr>
<tr>
<td>$^{24}$Mg</td>
<td>3.23</td>
<td>0.4</td>
<td>1.98</td>
<td>0.8</td>
</tr>
<tr>
<td>$^{52}$Fe</td>
<td>3.10</td>
<td>0.2</td>
<td>2.25</td>
<td>1.0</td>
</tr>
</tbody>
</table>

NoL 15 11 30
Calculation of level density–Moments Method

- Many different models (Fermi Gas, Hartree-Fock-Bogolyubov model, ...)
- Shell model, Moments Method (Statistical description)

$$\rho(E; \alpha) = \sum_p D_{\alpha p} G(E - E_{\alpha p} + E_{g.s.}; \sigma_{\alpha p})$$

- $D_{\alpha p}$ – dimension of the class of states
- $\alpha$ – quantum numbers of the class of states
- $p$ – partition, e.g. 6 particles in sd shell make 15 partitions

### Moments of $H$ for each partition $p$:

- $E_{\alpha p} = \frac{1}{D_{\alpha p}} \text{Tr}^{\alpha p} H$
- $\sigma_{\alpha p}^2 = \frac{1}{D_{\alpha p}} \text{Tr}^{\alpha p} H^2 - E_{\alpha p}^2$

---

R. Sen'kov, V. Zelevinsky, PRC 93 064304

Sofia Karampagia  Nuclear Shell Model and Phase Transitions
Moments method vs exact shell model

$^{28}\text{Si}$, $J = 0, 1, 2, 3$, SD shell, USDB interaction

R. Sen'kov, V. Zelevinsky, arXiv:1508.03683
Moments method vs exact shell model

$^{52}$Fe, $^{52}$Cr, $J = 0, 1$, PF shell, GX1A interaction

R. Sen’kov, V. Zelevinsky, arXiv:1508.03683
Conclusions

- Found a QPT induced as the strength of the single particle transfer matrix elements changes with respect to the strength of the rest matrix elements
- QPT in even-even (rotational-spherical), odd-even, odd-odd nuclei
- Single particle transfer matrix elements carriers of deformation, provide rotational observables, strong transition probabilities, quadrupole moments and enhanced level densities
- Even in a small system collective motion, phase transitions, thermodynamics are present
- Interesting to further understand how the matrix elements affect the nuclear observables
Ehrenfest criterion: ground state energy functional and its derivatives, $n$th order if its $n$ derivative with respect to $\lambda$ is discontinuous

image: second order QPT

F. Pérex—Bernal et al., PRA 77, 032115 (2008)
Excited states in odd–even, odd–odd nuclei

\[ E(J=4) \text{ (MeV)} \]

\[ E(J=7/2) \text{ (MeV)} \]

\[ \lambda \]

\[ \lambda \]

\[ \lambda \]

\[ \lambda \]

\[ ^{26}\text{Al} \]

\[ ^{27}\text{Al} \]

\[ ^{28}\text{Al} \]

\[ ^{30}\text{P} \]

\[ ^{50}\text{Mn} \]

\[ n=1 \]

\[ n=2 \]

\[ n=3 \]

\[ n=4 \]

\[ n=5 \]

\[ n=6 \]

\[ n=7 \]

\[ n=8 \]

\[ n=9 \]

\[ n=10 \]
Angular momentum wavefunction decomposition, $1^+_1$

Nuclear Shell Model and Phase Transitions
Expansion of model space

Ways of finding the g.s. energy in larger spaces?

- Exponential method, M. Horoi, A. Volya, V. Zelevinsky, PRL82 8
- Adjusting the level density to the previous space
Moments method vs experimental data

$^{26}\text{Al}, \quad ^{28}\text{Si}$ all $J$, s-p-sd-pf model space, WBT interaction

R. Sen’kov, V. Zelevinsky, arXiv:1508.03683

Sofia Karampagia  Nuclear Shell Model and Phase Transitions
Moments method vs experimental data

$^{26}$Al, $^{28}$Si all $J$, s-p-sd-pf model space, WBT interaction

R. Sen'kov, V. Zelevinsky, arXiv:1508.03683