Nuclear Shell Model and Phase Transitions

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Nuclei and Mesoscopic Physics, 03/07/2017

Outline

- Quantum phase transitions in the framework of the shell model
- Collective enhancement of the nuclear level density

Definition of Quantum Phase Transition (QPT)

Quantum Phase Transitions (QPT) refer to phase transitions, which occur at zero temperature ($T = 0$), when a non-thermal parameter, λ, is changed (pressure, magnetic field). This parameter(s) is called control parameter and the value for which the phase transition takes place is the critical value, $\lambda = \lambda_c$, for a system described by

$$H(\lambda) = \epsilon(H_1 + \lambda H_2)$$

Ising Model in a transverse field (ferromagnetic $\rightarrow$ paramagnetic)

$$H = -J \sum_{ij} S_i^z S_j^z - h \sum_i S_i^x$$

Quantum Phase Transition in atomic nuclei

Algebraic models are used, possible phases of a system obtained by considering possible breakings of the algebra g into its subalgebras, each subalgebra having its own dynamical symmetry, structural shape.

$$g \supset g_1 \supset \dots$$

$$g \supset g_2 \supset \dots$$

$$\dots$$

$$g \supset g_\nu \supset \dots$$

Quantum Phase Transition in atomic nuclei

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$$g \supset g_1 \supset \dots$$

$$g \supset g_2 \supset \dots$$

$$\dots$$

$$g \supset g_\nu \supset \dots$$

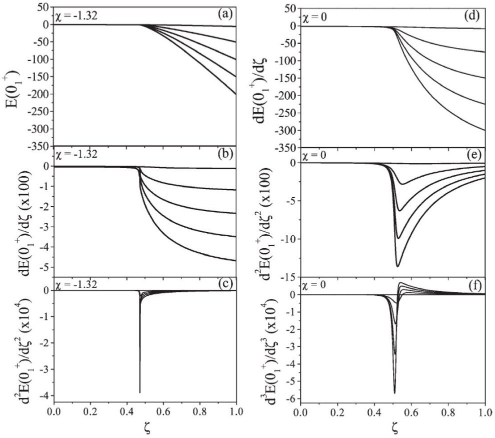
$$H = (1-\lambda)H_1 + \lambda H_2,$$

$$\lambda = 0 \to H_1, \lambda = 1 \to H_2$$

Order of Quantum Phase Transition

Ehrenfest criterion:

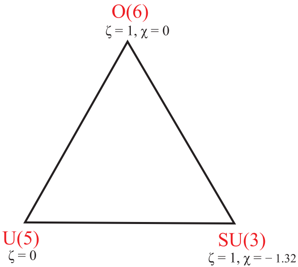
ground state energy functional and its derivatives, *n*th order if its *n* derivative with respect to λ is discontinuous



Lincoln D. Carr, *Understanding QPTs* (N_B = 10, 100, 200, 300, 400)

Order of Quantum Phase Transition

Interacting Boson Model, U(6)



$$H(\zeta,\chi) = c[(1-\zeta)\hat{n}_d - \frac{\zeta}{4N_B}\hat{Q}_\chi \cdot \hat{Q}_\chi]$$

$$\hat{n}_d = d^\dagger \tilde{d}$$

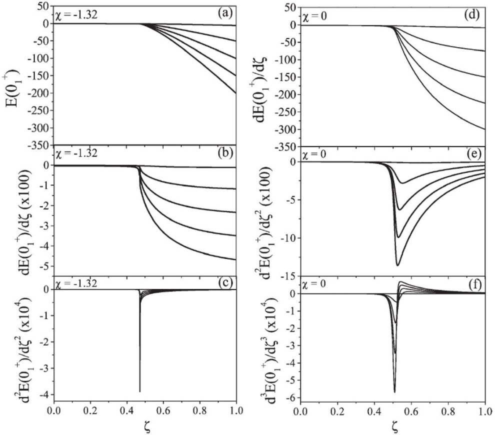
$$\hat{Q}_\chi = (s^\dagger \tilde{d} + d^\dagger \tilde{s}) + \chi (d^\dagger \tilde{d})^{(2)}$$

$$\zeta \in [0,1]$$

$$\chi \in [0,-1.32]$$

left: first order QPT (U(5) → SU(3))

right: second order QPT (U(5) → O(6))



Lincoln D. Carr, *Understanding QPTs* (N_B = 10, 100, 200, 300, 400)

Shell model Hamiltonian

Two-Body Matrix Elements:

The interaction scatters 2 nucleons into a different pair of orbits compared to the pair of orbits before the scattering

fall into 3 categories

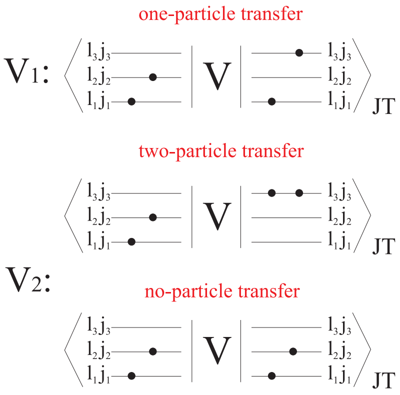
sd shell: 63 matrix elements
pf shell: 195 matrix elements

V_1: one particle transfer matrix elements,

V_2: the rest matrix elements

V_1: deformation

H. Horoi *et al.*, PRC **81**, 034396 (2010)



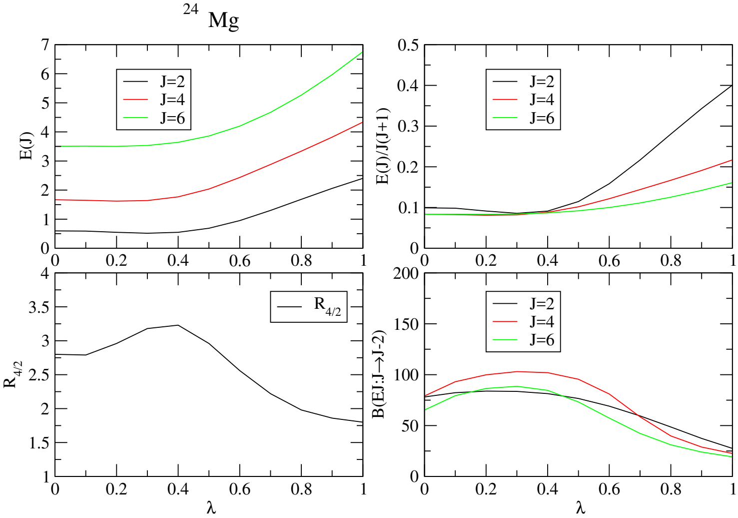
Quantum Phase Transition

$$H = (1-\lambda)V_1 + \lambda V_2$$

✓ Varying the strength of the V_1 and V_2 shell model matrix elements in the sd and pf shells using a control parameter λ going from 0 to 1

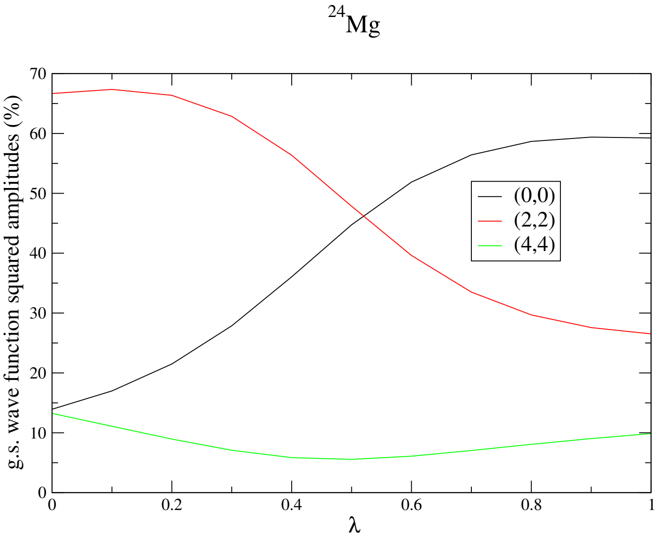
^{24}Mg λ	2_1^+	4_1^+	6_1^+	$R_{4/2}$	$Q(2_1^+)$	B(E2;$2_1^+ \to 0_1^+$)	B(E2;$4_1^+ \to 2_1^+$)	B(E2;$6_1^+ \to 4_1^+$)
0.0	0.596	1.667	3.507	2.80	-16.32	7.81E+01	7.91E+01	6.53E+01
0.1	0.590	1.649	3.512	2.79	-18.02	8.23E+01	9.31E+01	7.94E+01
0.2	0.548	1.620	3.504	2.96	-18.65	8.39E+01	9.99E+01	8.65E+01
0.3	0.515	1.640	3.533	3.18	-18.87	8.36E+01	1.03E+02	8.86E+01
0.4	0.547	1.766	3.642	3.23	-18.68	8.14E+01	1.02E+02	8.46E+01
0.5	0.688	2.036	3.860	2.96	-17.97	7.66E+01	9.55E+01	7.32E+01
...	0.951	2.433	4.201	2.56	-16.65	6.91E+01	8.10E+01	5.74E+01
1.0	2.404	4.337	6.761	1.80	-7.29	2.76E+01	2.26E+01	1.93E+01

Signs of a QPT in even−even nuclei

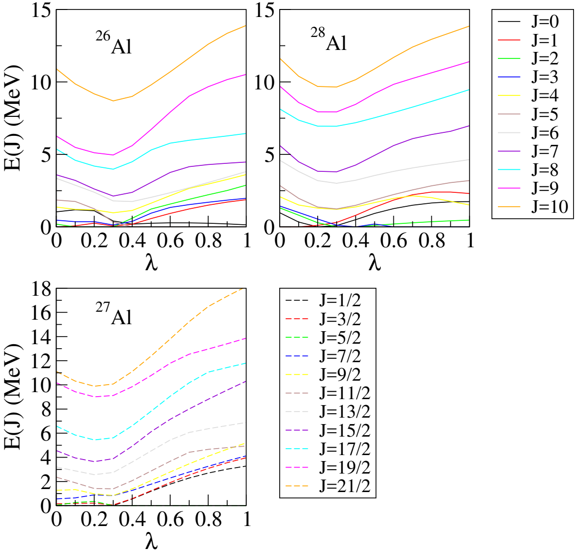


S. Karampagia, V. Zelevinsky, PRC **94** 014321 (2016)

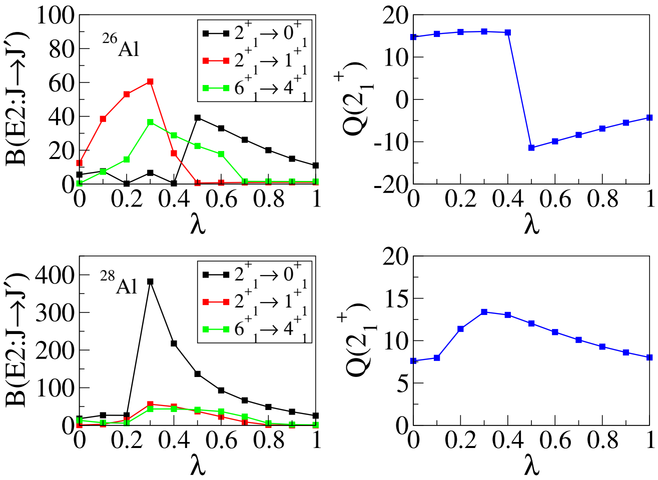
Angular momentum wavefunction decomposition



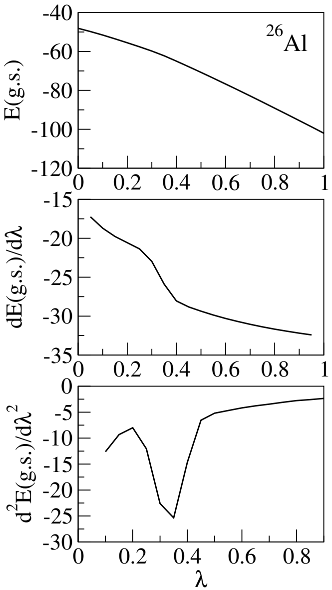
Signs of a QPT in odd−even, odd−odd nuclei



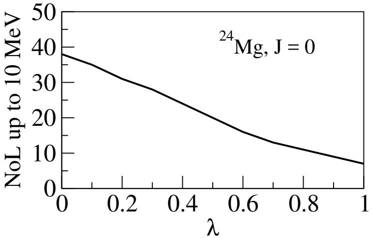
B(E2)s, Q(2_1^+) in odd−even, odd−odd nuclei

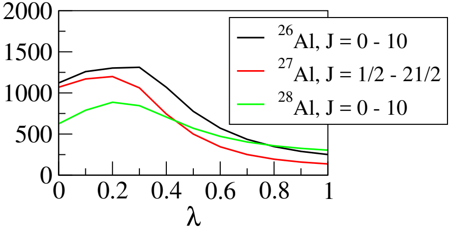


Order of QPT



Level Densities





Signs of Collective Enhancement in even−even nuclei

Collective enhancement: Noticeable change of the low lying nuclear level density due to collective effects

Many cases with signs of collective enhancement, selected just a few...

J=0 (up to 10 MeV)	Deformed $R_{4/2}$	λ	Spherical $R_{4/2}$	λ	Deformed NoL	Spherical NoL
^{28}Si	3.01	0.2	2.15	0.5	45	15
^{24}Mg	3.23	0.4	1.98	0.8	24	11
^{52}Fe	3.10	0.2	2.25	1.0	412	30

Calculation of level density–Moments Method

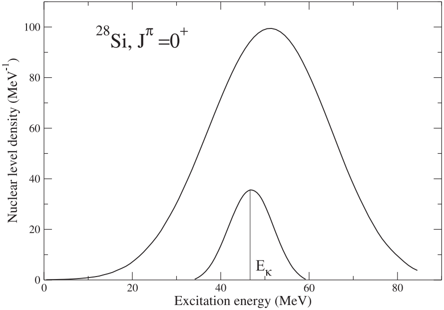
- Many different models (Fermi Gas, Hartree-Fock-Bogolyubov model, ...)
- Shell model, Moments Method (Statistical description)

$\rho(E;\alpha) = \sum_p D_{\alpha p} G(E - E_{\alpha p} + E_{g.s.}; \sigma_{\alpha p})$

$D_{\alpha p}$ – dimension of the class of states

α – quantum numbers of the class of states

p – partition, e.g. 6 particles in sd shell make 15 partitions



p	$d_{5/2}$	$s_{1/2}$	$d_{3/2}$
1	6	0	0
2	5	1	0
3	5	0	1
4	4	2	0
...	...	...	...
15	0	2	4

Moments of H for each partition p:

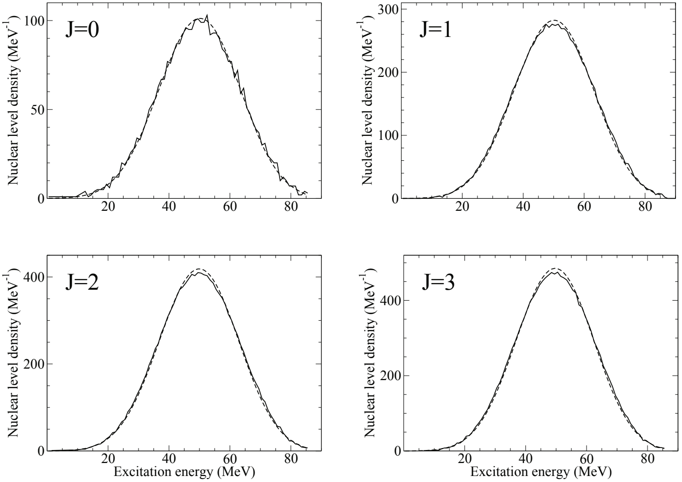
$$E_{\alpha p} = \frac{1}{D_{\alpha p}} Tr^{\alpha p} H$$

$$\sigma^2_{\alpha p} = \frac{1}{D_{\alpha p}} Tr^{\alpha p} H^2 - E^2_{\alpha p}$$

R. Sen'kov, V. Zelevinsky, PRC **93** 064304

Moments method vs exact shell model

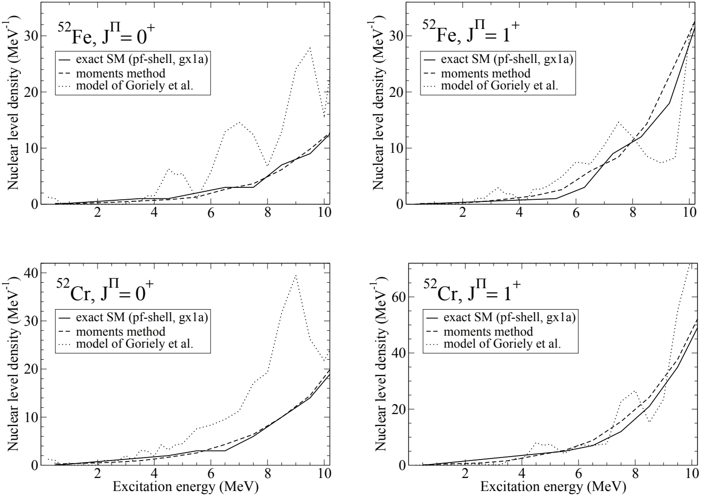
^{28}Si, $J = 0, 1, 2, 3$, SD shell, USDB interaction



R. Sen'kov, V. Zelevinsky, arXiv:1508.03683

Moments method vs exact shell model

^{52}Fe, ^{52}Cr , $J = 0, 1$, PF shell, GX1A interaction



R. Sen'kov, V. Zelevinsky, arXiv:1508.03683

Conclusions

- Found a QPT induced as the strength of the single particle transfer matrix elements changes with respect to the strength of the rest matrix elements
- QPT in even-even (rotational-spherical), odd-even, odd-odd nuclei
- Single particle transfer matrix elements carriers of deformation, provide rotational observables, strong transition probabilities, quadrupole moments and enhanced level densities
- Even in a small system collective motion, phase transitions, thermodynamics are present
- Interesting to further understand how the matrix elements affect the nuclear observables

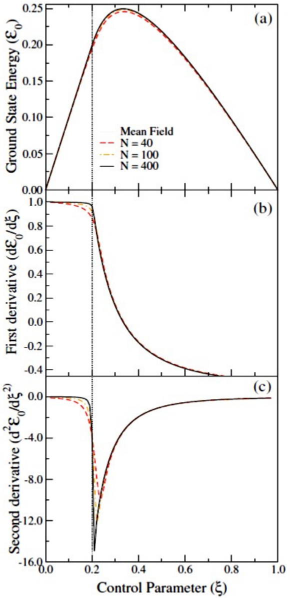
Additional Slides

Order of Quantum Phase Transition

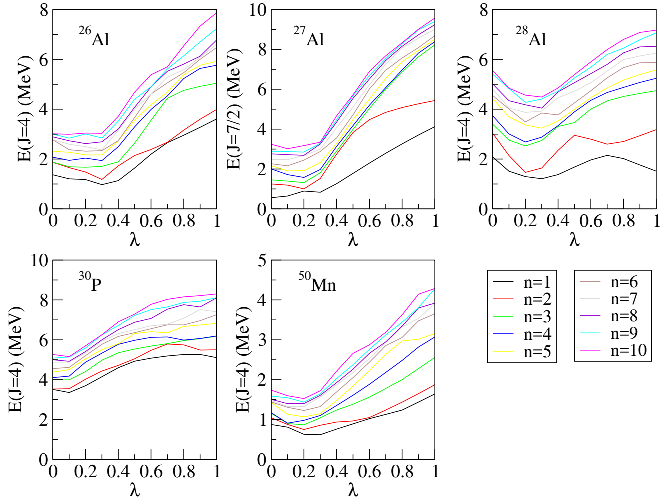
Ehrenfest criterion: ground state energy functional and its derivatives, *n*th order if its *n* derivative with respect to λ is discontinous

image: second order QPT

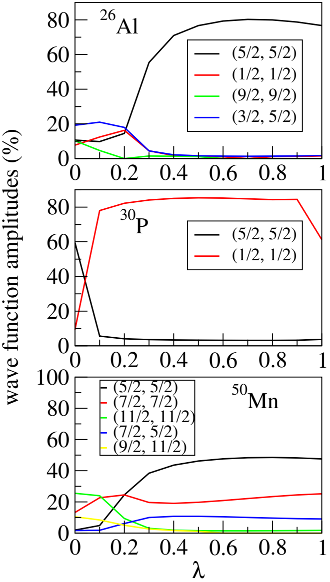
F. Pérex—Bernal *et al.*, PRA **77**, 032115 (2008)



Excited states in odd−even, odd−odd nuclei



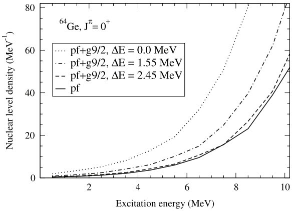
Angular momentum wavefunction decomposition, 1_1^+

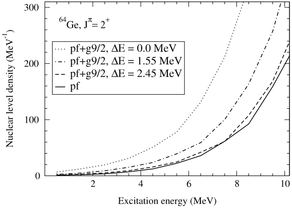


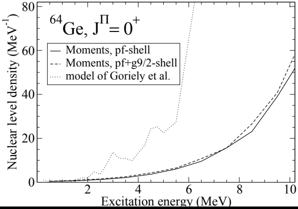
Expansion of model space

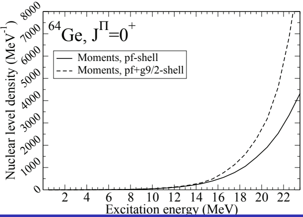
Ways of finding the g.s. energy in larger spaces?

- Exponential method, M. Horoi, A. Volya, V. Zelevinsky, PRL82 8
- Adjusting the level density to the previous space



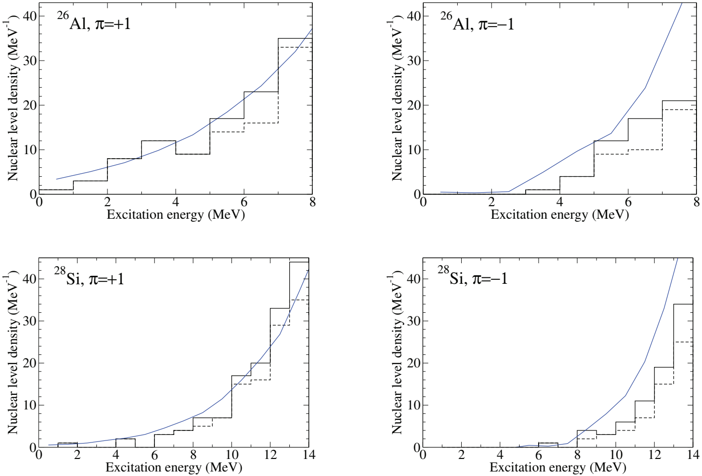






Moments method vs experimental data

^{26}Al, ^{28}Si all *J*, s-p-sd-pf model space, WBT interaction



R. Sen'kov, V. Zelevinsky, arXiv:1508.03683

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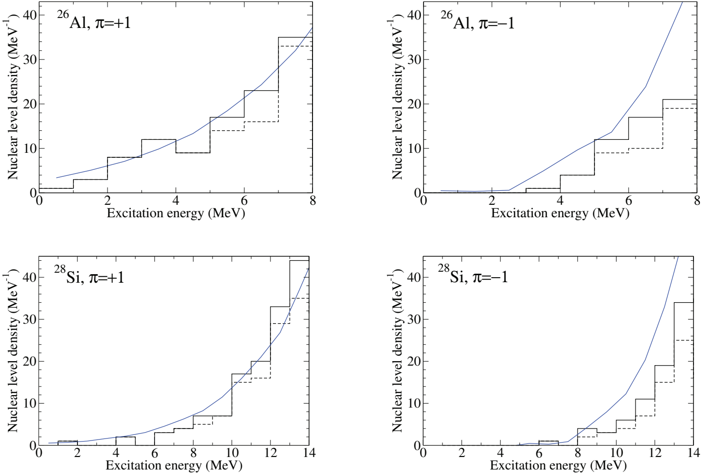

R. Sen'kov, V. Zelevinsky, arXiv:1508.03683