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INVESTIGATION OF EMITTANCE ANALYSIS METHODS USING THE MUCOOL TEST AREA

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ABSTRACT

Emittance is an important metric in accelerator physics that helps to predict and analyze the beam's behavior and properties. Using the MuCool Test Area beamline, we are able to calculate this parameter with the use of multiwire profile monitors. Emittance is defined as an invariant quantity equal to the area of the envelope of particles in transverse phase space. In a linear transport system this envelope should be elliptical. This analysis aimed to investigate this property and found that emittance was conserved in the horizontal plane. However, an unpredicted behavior of variant emittance as a function of phase advance in the vertical plane leads us to suspect that the beam envelope in transverse phase space is non-elliptical. A more accurate knowledge of the beam's transverse behavior will allow operators to more efficiently inject beam into Booster, which will help to reduce losses.

INTRODUCTION

Fermi National Accelerator Laboratory is home to a multitude of high-energy physics research. A crucial aspect of the laboratory's operations and a major goal of the Accelerator Division is to understand and analyze the proton beam that is sent to various places throughout the complex. This ability to model the beam aids in the minimization of losses. This analysis serves as an investigation into a parameter known as transverse emittance, with the goal of ultimately improving existing models of the beam.

TRANSVERSE BEAM OPTICS

Courant-Snyder Parameters

In the field of accelerator physics, the standard parameters for modeling a beam are known as the Courant-Snyder, or Twiss parameters. These variables (β, α, and γ) describe the behavior of the beam as it travels through a nominal trajectory (we will refer to this as s). For this analysis, we will mainly focus on β, also known as the betatron oscillation. This parameter describes the periodic transverse motion of the beam envelope due to the magnetic fields produced by quadrupoles.

Phase Advance

An important metric to understand for this analysis is the phase advance of a beam. This parameter measures the amount of oscillation period a particle has gone through in a certain arc. Although this measurement will not be used in any calculations, it will be used as a way to parameterize data sets. In order to obtain a different degree of phase advance through a distance in a beamline or circular accelerator, the strengths of the magnet elements can be varied. We will refer to a certain set of magnet settings as the 'tune' of a beam, and characterize the set by the corresponding degree of phase advance through the area of interest. For this analysis, five tunes will be used: 22, 50, 90, 120, and 180 degrees. These values were determined from MAD simulations (explained in further detail in a later section) that were ran prior to the data collection for this study.

Phase Space

The parameter space that this analysis will rely on is known as phase space. This represents the betatron oscillation as a function of distance along the beam path. Each particle in the beam occupies a certain point in transverse phase space (x,x' and y,y') and this distribution can be described with an envelope of all particles. This provides a representation of the particle positions and momenta as the envelope propagates through the beamline.

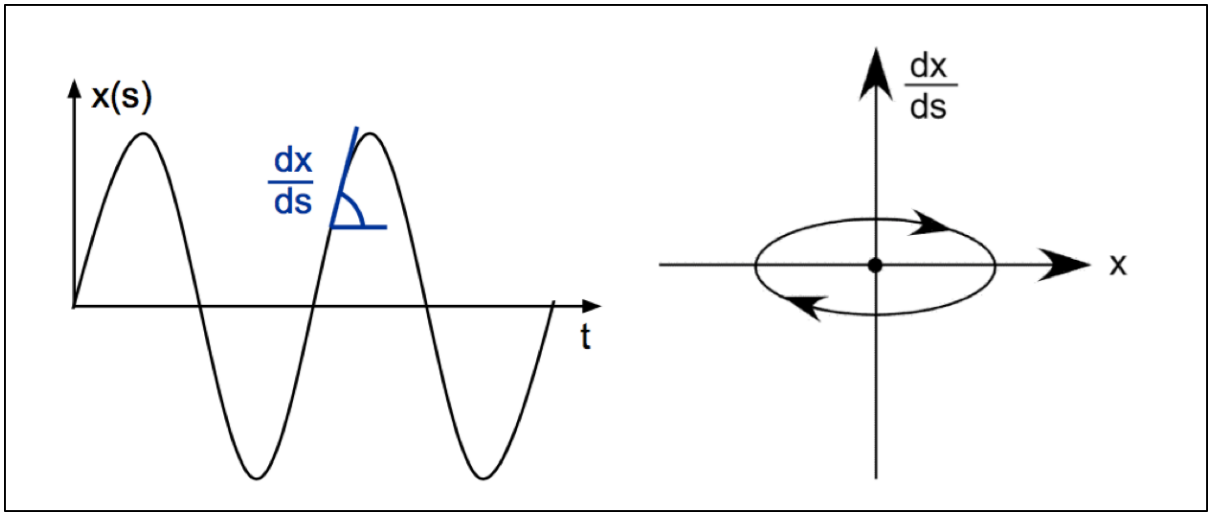


Figure 1: Representation of Phase Space

EMITTANCE

The area of this envelope is known as the transverse emittance, which is predicted to be elliptical when the beam is exposed to linear forces, as is the case with dipole and quadrupole magnets. Although this ellipse may change shape and orientation throughout the beam path (see Figure 2), its area is an invariant quantity in a linear transport system. Emittance should also be independent of phase advance.

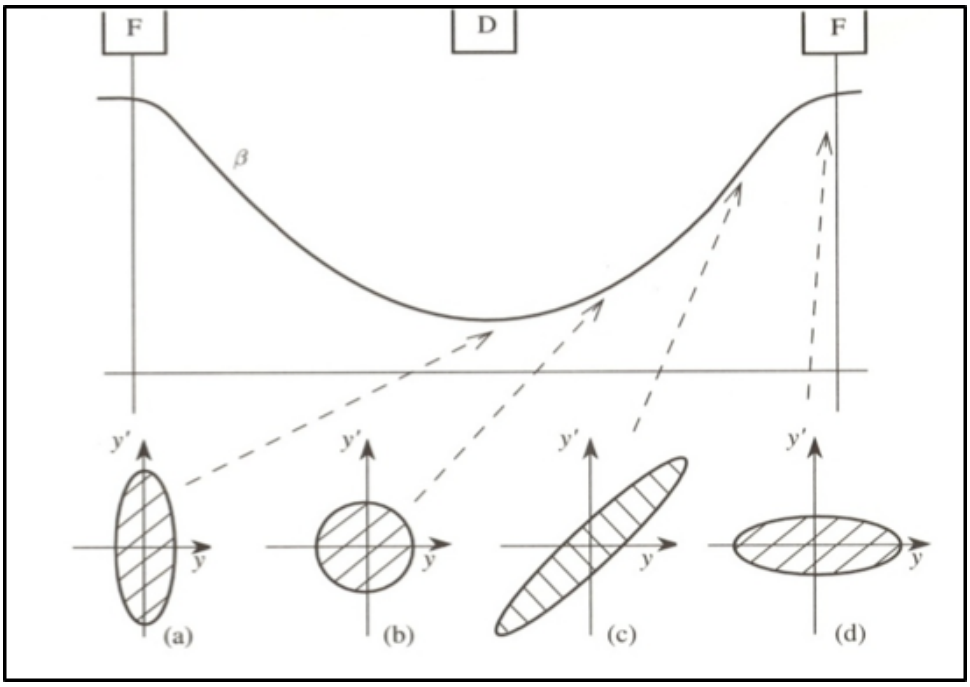


Figure 2: Invariant Ellipse Area

MUON TEST AREA

The MuCool Test Area (MTA) can be used as a diagnostic beamline to explore important parameters and test beam models. MTA is a 400 MeV line off of Fermilab's Linac, and consists of a variety of magnet elements as well as profile monitors. For the purpose of this analysis, the specific focus was on the 10 m straight that begins around 30 m into the beam trajectory. This straight, also known as a drift, contains no quadrupole magnets and therefore creates an environment ideal for emittance investigation. This property greatly simplifies the optical transfer matrices and eliminates the need for confident measurements on magnet parameters such as field uniformity and strength. Data for this analysis was

collected using three Secondary Emission Multiwires, collected in August 2014 and logged by operator Michael Backfish. Five sets of beam profiles at these three wires were collected for five different tunes, characterized by the degree of phase advance through this straight. It is also worth noting that at the exit of Linac, the beam is split between MTA and Booster. Having a better understanding of the transverse emittance in MTA will help optimize matching into Booster and therefore reduce beam losses.

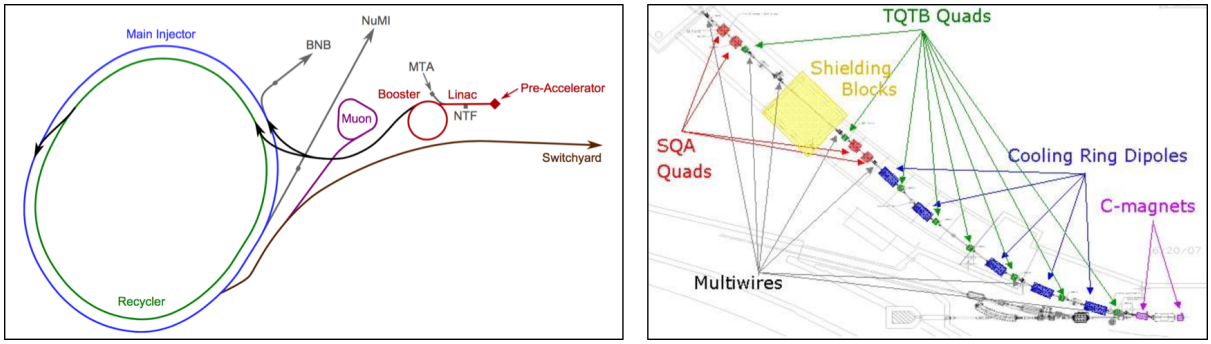
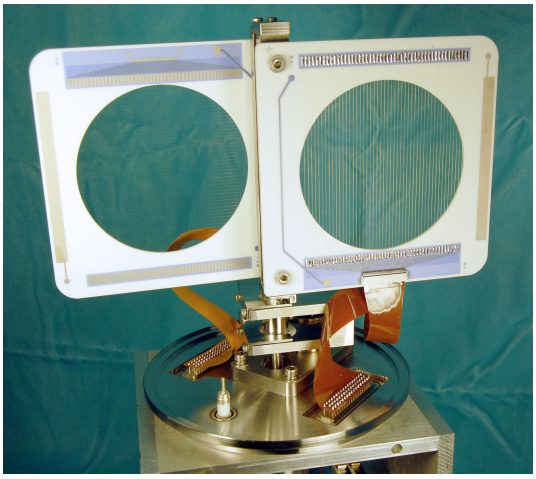


Figure 3: MTA Beamline

SECONDARY EMISSION MULTIWIRES

Knowledge of the transverse size of the beam is key when analyzing and designing a beamline. One of the ways in which we are able to measure this parameter is with the use of Secondary Emission Multiwires (SEMs). These SEMs consist of two paddles (one for each transverse plane) containing 48 evenly spaced wires. The underlying mechanism of these profile monitors relies on the liberation of particles; when protons from the beam strike these wires, electrons are liberated and produce a net charge, which is read out through electronics and recorded. The intensity of the beam is proportional to the amount of charge seen by the wire. The individual wires are spaced at 0.5mm, 1mm, or 2mm in the MTA line. This is known as the pitch and is an important physical property of the monitor when processing data. Because this instrumentation relies on direct interaction with the protons, it can be a source of beam loss. To minimize this, each profile monitor is connected to a motor which can rotate the paddles into the beam path when data collection is needed, but allows the beam to be uninhibited otherwise.

The resulting data is a profile of the beam in each plane. The width of this distribution, which is predicted to be Gaussian, represents the beam radius. The pitch is a crucial mea-



(a) Secondary Emission Multiwire

(b) SEM in MTA Line

surement to have because the measured width needs to be scaled by the resolution of the profile.

Data Analysis

In order to calculate the beam radius, two methods were used: a Gaussian curve fit and a Root Mean Squared (RMS) calculation. The former assumes a predicted distribution based on the elliptical property of the beam in phase space, while the latter treats each data point independently. Because the beam profile is not always well-defined by a Gaussian distribution, it is crucial to have an RMS method with which to compare and verify a correct measurement.

Two main factors need to be accounted for when processing this data. Firstly, these profiles contain a small background contribution due to the physical properties of the SEMs. In addition, the RMS method is highly sensitive to outlying 'tails' in the data, since it does not assume any distribution. Because of this, we need a method for selecting which wires represent data that is relevant to our analysis. This is done in two main steps. First, to address the signal background, the data is fit with a Gaussian + linear contribution. The fit is done using a non-linear least squares method in Python. The intercept of this linear component is then subtracted from all data points as a background. Secondly, in order to eliminate any tails that may introduce inaccuracies in the RMS method, a cut is made 3 standard deviations from the peak (the value of which comes from the Gaussian component to the aforementioned fit). Our initial qualitative analysis of this cut revealed that 3σ may be too narrow, as our RMS calculations were significantly smaller than that of the Gaussian fit. Due to this discrepancy,

the analysis was also run using a cut at 4σ. This discriminator did not have a large effect on the Gaussian method since the fundamental curvature of the data is not affected. However, this had a large effect on the results of the RMS method, as will be discussed in further detail in a later section. After the data has been filtered using the above methodology, the 'clean' data is fit with a normal Gaussian function as well as an RMS analysis.

Gaussian Fit

Python's fit method also conducts a chi-squared goodness of fit test which describes how well the Gaussian is able to describe the data. Since the beam profile does not always follow this distribution well (as seen in Figure 5), this model presents serious limitations. Thus this beam radius calculation may be insufficient for some profiles.

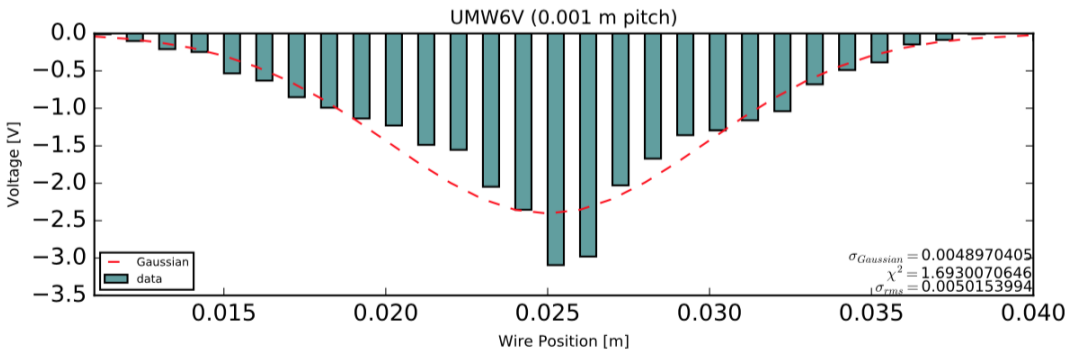


Figure 5: Poor Gaussian Fit

Root Mean Squared Method

As a point of comparison, a root mean squared calculation is also conducted. The RMS method is as follows:

$$\mu = \frac{\Sigma_n n|P(n)|}{\Sigma_n |P(n)|} \qquad \sigma^2_{rms} = \frac{\Sigma_n (x-\mu^2)|P(n)|}{\Sigma_n |P(n)|} \tag{1}$$

where $P(n)$ represents the signal on a particular wire and x represents the wire position. This method is often advantageous over the Gaussian fit because it is independent of a defined distribution.

However, one significant limitation of this method is the need for complete profiles. Certain tunes may cause the beam to scrape the beam pipe, leading to incomplete profiles (Figure 6). This phenomenon is not particularly problematic in the Gaussian analysis since

the overall curve of the data is still present, but because of the RMS reliance on each data point, these types of profiles can severely affect the resulting sigma value using this method.

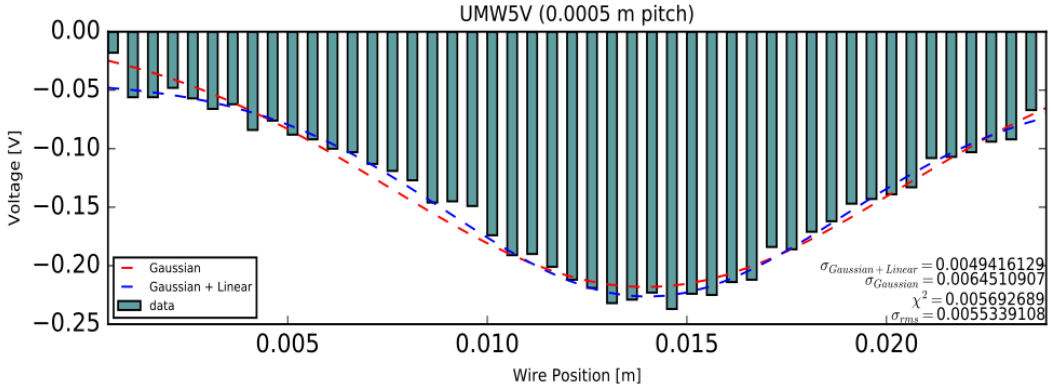


Figure 6: Fractured Beam Profile

These issues can contribute to the overall uncertainty in the radius measurements, which will propagate throughout the analysis. However, to mitigate this to our best ability, we compare our emittance methods using both data analysis approaches in order to check for consistent behavior. After the sigma values have been calculated in each plane for all wires at all phase advance tunes, the emittance can be computed.

Emittance Calculations

MAD Simulation

Developed at CERN, the Methodical Accelerator Design (MAD) program was designed to simulate any beamline given a list of magnet elements and their respective positions and strengths along a nominal trajectory, known as a lattice. Using Courant-Snyder optical transfer matrices, the simulation is able to calculate parameters such as the betatron oscillation, dispersion, and phase advance throughout the line. A limitation that this program has is that it is dependent on initial conditions. For purposes of the MTA line, this requires confident values of the Twiss parameters coming out of Linac, which were not available at the time of this analysis. However, this method of analysis was pursued with the values provided by previous Linac measurements even though they were collected before upgrades in the source for the purpose of testing a MAD approach to emittance calculations.

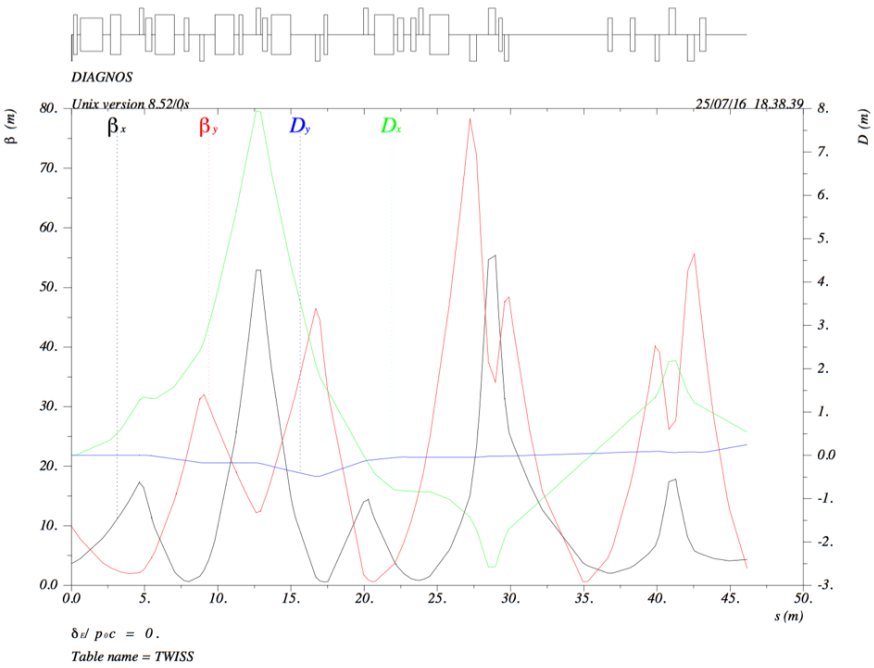
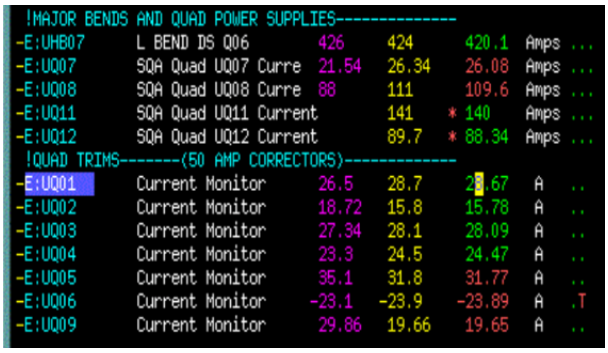


Figure 7: Example of MAD Simulation Output

Magnet Current Conversion

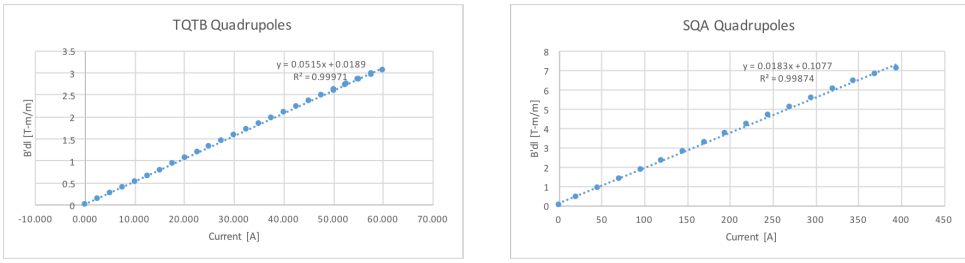
In order to utilize the MAD simulation, a unique lattice file was created for each tune to reproduce beamline conditions at the time of data collection. The current values of each quadrupole in the MTA line were recorded in the External Beamlines elog at the time of each multiwire profile data entry. This line consists of eight TQTB quadrupoles and four SQA quadrupoles, both of which have unique transfer functions for current to k-value conversions.



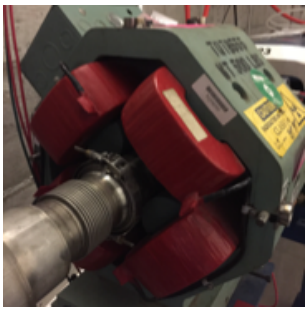
!MAJOR BENDS AND QUAD POWER SUPPLIES----------						
-E:UHB07	L BEND DS Q06	426	424	420.1	Amps	...
-E:UQ07	SQA Quad UQ07 Curre	21.54	26.34	26.08	Amps	...
-E:UQ08	SQA Quad UQ08 Curre	88	111	109.6	Amps	...
-E:UQ11	SQA Quad UQ11 Current		141	* 140	Amps	...
-E:UQ12	SQA Quad UQ12 Current		89.7	* 88.34	Amps	...
!QUAD TRIMS-------(50 AMP CORRECTORS)----------						
-E:UQ01	Current Monitor	26.5	28.7	28.67	A	..
-E:UQ02	Current Monitor	18.72	15.8	15.78	A	..
-E:UQ03	Current Monitor	27.34	28.1	28.09	A	..
-E:UQ04	Current Monitor	23.3	24.5	24.47	A	..
-E:UQ05	Current Monitor	35.1	31.8	31.77	A	..
-E:UQ06	Current Monitor	-23.1	-23.9	-23.89	A	.T
-E:UQ09	Current Monitor	29.86	19.66	19.65	A	..

Figure 8: Quadrupole Current Values

Technical division reports on both types of magnet were conducted upon their manufac-



(a) Technical Specifications for TQTB and SQA Quads



(b) TQTB Quadrupole

(c) SQA Quadrupole

turing that relates $B'dl$ as a function of current. Using this linear relationship, the value of B' was calculated for each quadrupole. The factor of dl was taken to be the integrated length of each magnet. Those calculated values were then entered into the following equation:

$$k = \frac{B'}{Bp} \tag{2}$$

where p is the relativistic momentum of the particle.

Match Command

An especially useful functionality of MAD is known as the 'match' command. The user is able to specify any number of constraints and may choose which beamline elements to vary in order to match those constraints. In addition, the user may specify the amount by which each element is varied (for example, the k value of a certain quadrupole magnet), upper and lower limits for those values, and the acceptable tolerance limit that the algorithm must reach in order for a solution to be considered convergent. Once this command has been executed, MAD will return any relevant parameters upon the user's choosing at the locations of the specified elements. The initial values are listed, followed by the convergent solution along with the number of iterations that the algorithm performed before arriving at a solution, if one was found. Finally, MAD calculates a penalty function at each point of constraint that

defines how closely the parameter was able to be fit and computes the arithmetic sum of each individual penalty for a final cumulative value.

Although powerful, this function does have a number of limitations. If the system is over- or under- constrained, the algorithm will be unable to find a reasonable solution. In addition, the match function often ceases iterations even if the penalty function is not within the defined tolerance. This method was specifically applied to determine the value of emittance in both planes at each phase advance.

Upon completion of the lattice files for each corresponding tune using the above conversion methods, a MAD simulation using the match command was used. This method relies on the parameterization of the ellipse in terms of the Courant-Snyder parameters (below).

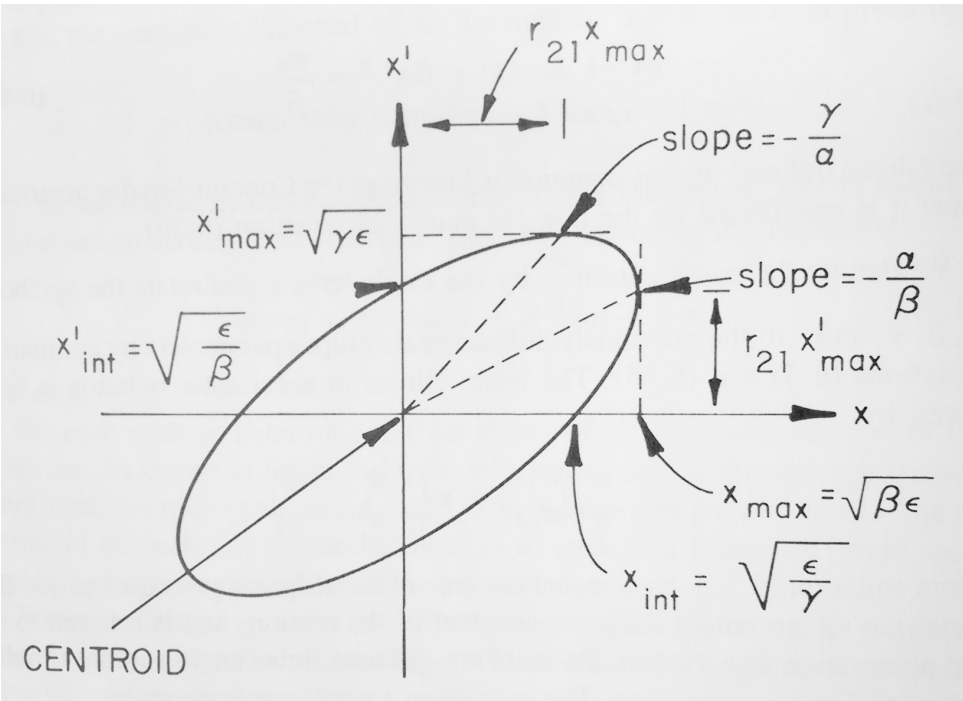


Figure 10: Ellipse in Terms of Courant-Snyder Parameters

This allows us to define the following relationship:

$$\epsilon_{x,y} = \frac{\sigma_{x,y}^2}{\beta_{x,y}} \tag{3}$$

where σ is defined as the measured beam radius from the multiwire profiles. These constraints were defined at multiwires 4, 5, and 6. This simulation was run for each set of σ values from the previously discussed methods, the results of which are below. As a figure of merit, the error bars on the emittance values denote the penalty functions of the match

command, scaled by 1E-9.

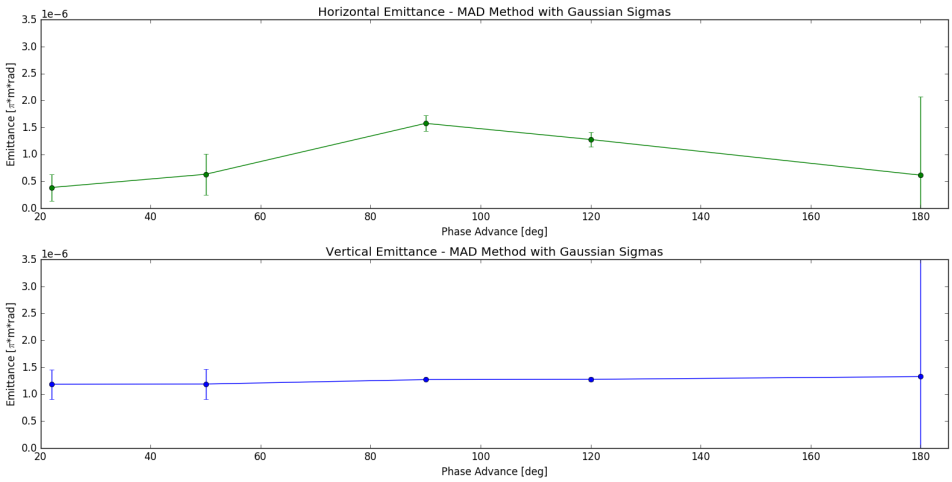


Figure 11: MAD Results Using Gaussian Fit

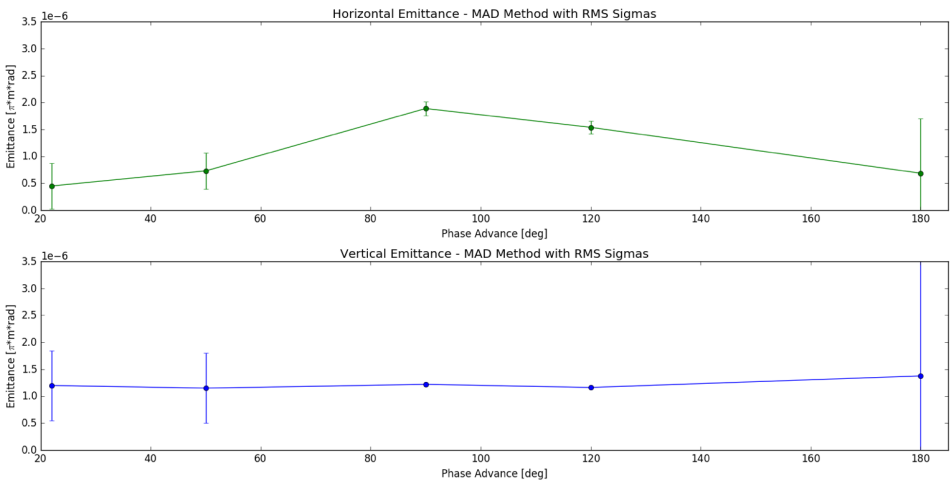


Figure 12: MAD Results Using RMS Method

It was found that this method has serious limitations, the first of which is the reliance on accurate initial conditions for the Twiss parameters. As previously mentioned, the set of parameters that were used in this study may be incorrect and therefore provide a significant source of error. In addition, this method relies on accurate lattice information, including but not limited to beamline element positions, consistency between magnets, and accurate field measurements. This method also inherently uses assumptions about the particle distribution in transverse phase space, namely that it is elliptical, which is precisely the assertion that we are testing. Finally, the penalty function output of this method was found to be rather large

and the algorithm consistently ceased iterating even when the penalty did not fall within a specified tolerance of 1E-12.

3-Wire Method

In contrast, the transverse phase space ellipse can be parameterized without the use of Courant-Snyder parameters as shown:

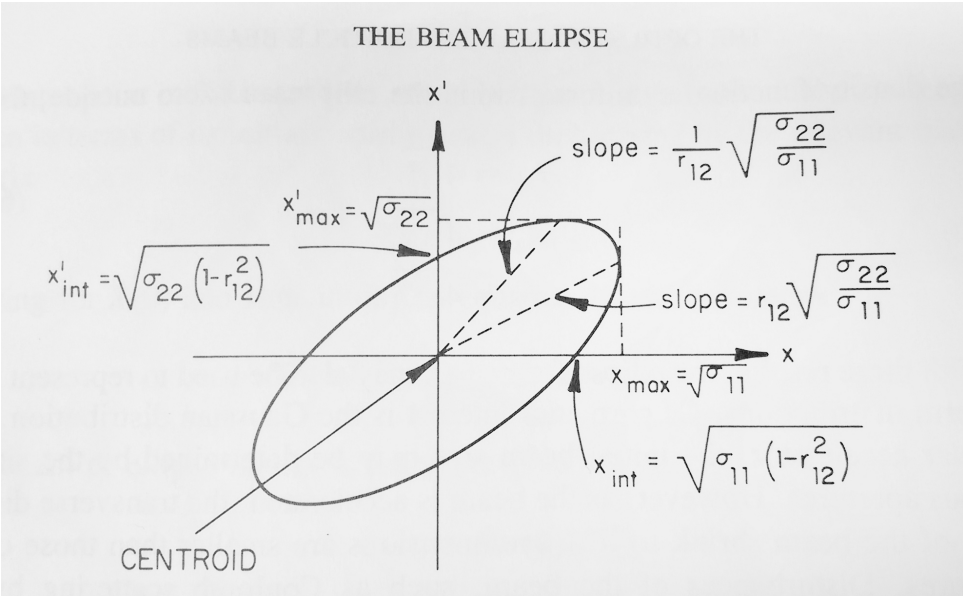


Figure 13: Ellipse in Terms of Sigma Matrix

The 3-Wire Method is a priori and relies only on the transfer matrices which describe how the beam size propagates through a drift, where R values are the measured sigma values at each wire and L_1 and L_2 are the distances between wires as shown in Figure 14 (not to scale).

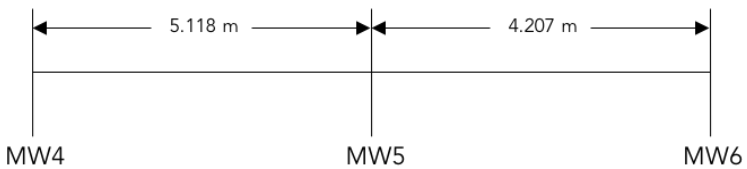


Figure 14: Distances Between Multiwires

σ_{12} and σ_{22} are unable to be directly measured and therefore are calculated by solving the following system:

$$R_2^2 = R_1^2 + 2L_1\sigma_{12} + L_1^2\sigma 22 \tag{4}$$

$$R_3^2 = R_1^2 + 2(L_1 + L_2)\sigma_{12} + (L_1 + L_2)^2\sigma 22 \tag{5}$$

Once those values have been found, the following equation relates the sigma matrix elements to transverse emittance:

$$\epsilon_{rms} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} \tag{6}$$

This analysis was conducted using both the Gaussian (Figure 15) and RMS (Figure 16) sigma values in both planes. Recall the earlier discussion on an appropriate value at which to cut the 'tail' artifacts; the two plots in each plane correspond to a 3σ and 4σ cut. Notice that as expected, this has almost no effect on the Gaussian method and the plots overlap well. The χ^2 goodness of fit parameter is also plotted (scaled by 1E-7) in order to ensure that

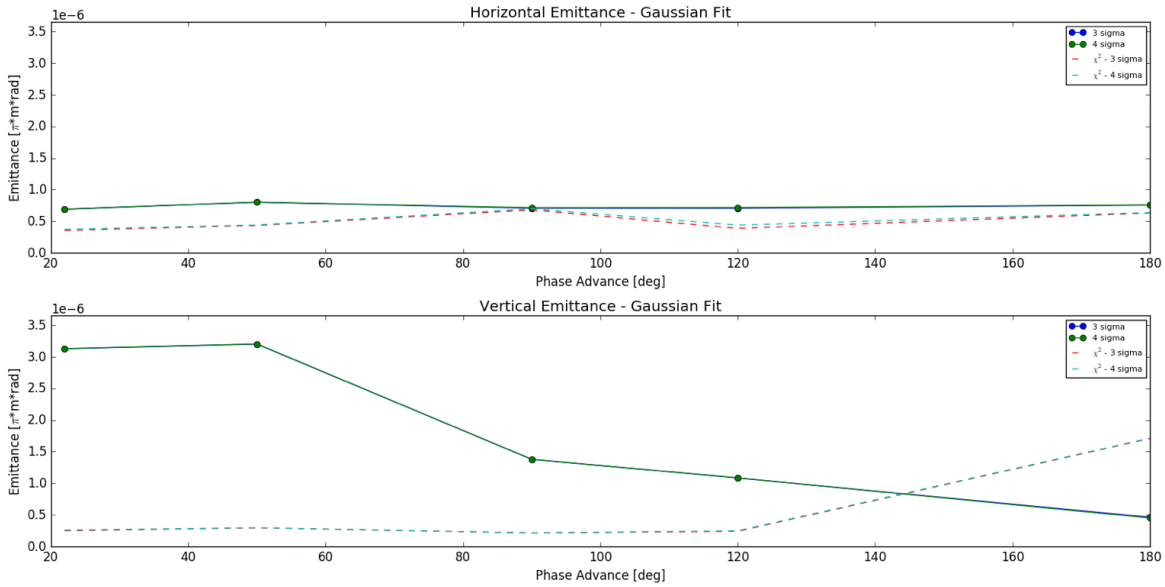


Figure 15: 3-Wire Method Using Gaussian Fit

any observed behavior is not simply due to poor curve fitting. As predicted by the elliptical assumption, the horizontal emittance does not appear to fluctuate drastically. However, the vertical emittance decreases significantly as a function of phase advance. Although the χ^2 is large at the 180-degree tune, giving less confidence in that specific data point, the decreasing behavior of the emittance occurs well before this. This leads us to believe that this trend is physical, and not an artifact of our method.

In addition, we have confidence in this result due to the following comparison with the RMS σ analysis. Since this calculation has no distribution dependence, the values are not subject to any poor fitting. However, there is an observed effect from the different values at which the tails are cut from the original data. This subjectivity should be kept in mind

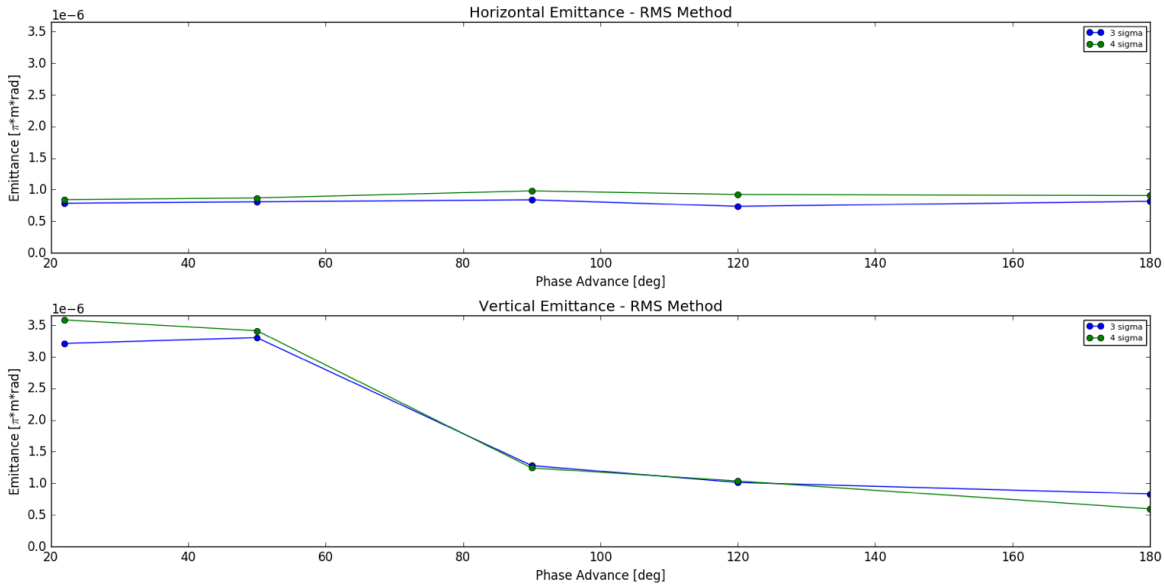


Figure 16: 3-Wire Method Using RMS Method

with further analyses. Although the values vary slightly, the overall behavior in the vertical is identical to that of the Gaussian, giving us agreement between methods and therefore greater confidence in our results.

Dispersion

An additional factor that was explored is known as dispersion. Analogous to a prism effect in optics, this refers to the principle that particles of different momenta will travel through a different trajectory in the magnetic field of a dipole. This is a measurable quantity and is easily calculated with a MAD simulation.

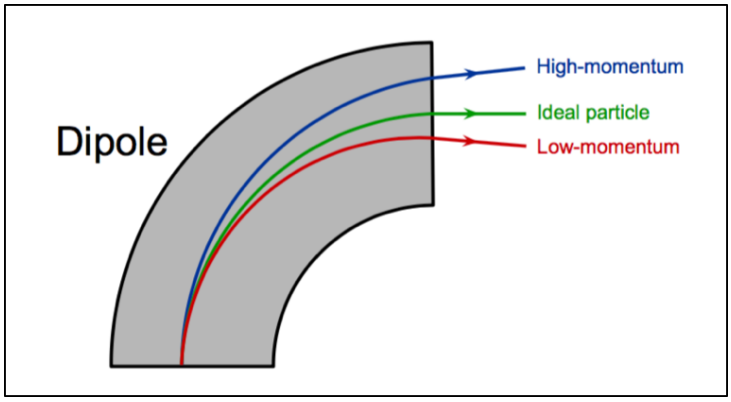


Figure 17: Dispersion Effect

Because dispersion leads to a spread of particles, the measured σ from the SEMs may have been too large. We can account for this overestimation with the following equation:

$$\beta\epsilon = \sigma^2_{measured} - \left(D\frac{\sigma p}{p}\right)^2 \tag{7}$$

where $\beta\epsilon = \sigma^2$, D is the dispersion measurement, and $\frac{\sigma p}{p}$ is the momentum spread of the particles. We recognize that this may have an effect on our final beam radii, however the current momentum spread value that exists was measured at Booster injection and therefore not entirely accurate for this study. In addition, the dispersion through this straight reaches a maximum of around 1 m. Using the provided momentum spread value for the purpose of exploration, this effect was small enough to be considered negligible for this analysis. Future projects may take this into consideration but this effect did not fall under the scope of this particular study.

CONCLUSIONS

There appears to be violation of emittance conservation in the vertical plane but not in the horizontal. We believe the most likely source of this to be a non-elliptical beam in the vertical transverse phase space. Because this is a single-pass beamline, it is not unrealistic for this phenomenon to occur since the beam is not subjected to linear forces for a significant period of time. In order to test this assertion, a tomographic reconstruction algorithm is being developed by Adam Watts for these types of profiles. This type of analysis, like the 3-Wire Method, does not rely on assumptions about the initial shape of the beam. We hope that this analysis will serve as a point of comparison and aid in the development of improved beam models for future operations.

ACKNOWLEDGMENTS

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