# Bend Magnet Heat Loads and Out of Orbit Scenarios 

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#### Abstract

This paper presents an analytical calculation of the spatial power spectrum emitted from relativistic electrons passing through a series of bend magnets. Using lattice files from the software Elegant, both the ideal and missteered trajectories taken by the beam are considered in determination of the power profile. Calculations were performed for the Advanced Photon Source Upgrade multi-bend-achromat storage-ring. Results were validated with Synrad, a monte-carlo based program designed at CERN. The power distribution and integrated total power values are in agreement with Synrad's results within one percent error. The analytic solution used in this software gives a both quick and accurate tool for calculating the heat load on a photon absorber. The location and orientation can be optimized in order to reduce the peak intensity and thus the maximum thermal stress. This can be used with any optimization or FEA software and gives rise to $a$ versatile set of uses for the developed program. [7]


## 1 Introduction

### 1.1 Problem Statement

When accelerated through a magnet field, relativistic electrons emit synchrotron radiation. This radiation creates a spatial power distribution on intercepting surfaces which may be used to calculate the resultant heat load. The problem may be broken up into several simpler steps. These involving computing the ideal path that the bunch of electrons take; adding real orbital errors to this trajectory; ray tracing the emitted photons; and determining the intensity of the power where each ray lands.

### 1.2 Solution

A parameterization of the ideal path is used, thus discrete time steps are chosen. Though it can be changed, the program uses 100 equidistant time steps per dipole magnet that is analyzed. This will create 100 photons that impact the input photon absorber per bend magnet. For each of these, a vertical spread of 1000 points is created and the full power spectrum is calculated - this number may also be easily changed. This gives a one-hundred-thousand-point mesh of varying density.
Many initial values such as the magnetic field strength, geometrical constants of the magnets, and beam energies are taken from a lattice file created by the program Elegant. A lattice file from Elegant or of identical formatting must be used for the software to work properly. To simplify the equations that follow, the general rotation matrix is given by Equation 1. Any parameterized function centered at the origin and acted upon by the matrix $\overline{\bar{R}}_{\bar{u}}(\theta)$ is rotated counterclockwise about the vector $\bar{u}$ by an angle $\theta$. Figure 1 displays the global coordinate notation used by the program.


Figure 1: An electron emitting radiation onto a planar surface. Each photon ray creates a vertical distribution. The global ZXY-coordinate system used is analogous to the more common XYZ-cartesian system.

$$
\overline{\bar{R}}_{\bar{u}}(\theta)=\left[\begin{array}{ccc}
\cos \theta+u_{Z}^{2}(1-\cos \theta) & u_{Z} u_{X}(1-\cos \theta)-u_{Y} \sin \theta & u_{Z} u_{Y}(1-\cos \theta)+u_{X} \sin \theta  \tag{1}\\
u_{X} u_{Z}(1-\cos \theta)+u_{Y} \sin \theta & \cos \theta+u_{X}^{2}(1-\cos \theta) & u_{X} u_{Y}(1-\cos \theta)-u_{Z} \sin \theta \\
u_{Y} u_{Z}(1-\cos \theta)-u_{X} \sin \theta & u_{Y} u_{X}(1-\cos \theta)+u_{Z} \sin \theta & \cos \theta+u_{Y}^{2}(1-\cos \theta)
\end{array}\right]
$$

## 2 Theory and Method

### 2.1 Ideal Trajectory

In order to solve for the proper trajectory, values in the lattice files must be used. The bend magnet of interest contains values denoted by subscript 1 while the element preceding it in the lattice file contains values denoted by subscript 0 . In a dipole field, the path taken by the electrons follows an arc of a circle. This can be done using a parametrization with a radius of curvature given by Equation 2 [3].

$$
\begin{equation*}
\rho=\frac{m(\gamma) v}{q_{e} B} \tag{2}
\end{equation*}
$$

The values of which to parametrize the arc are contained in the set $t:\left(0, t_{f}\right)$ where $t_{f}$ is the total time spent in the dipole region given by Equation 3. $\Delta s$ is the arc length of the curve.

$$
\begin{equation*}
t_{f}=\frac{\Delta s}{v} \tag{3}
\end{equation*}
$$

In order to place the trajectory in the proper location in global ZXY-position space, the curve must be oriented and translated properly. If $\theta_{0}$ is the angle that the beam enters a dipole field at with respect to the Z-axis, and if $\bar{r}_{1}$ is the initial position of the entering beam, the parametrization is given by the following equation.

$$
\bar{r}(t)=\overline{\bar{R}}_{\bar{Y}}\left(\theta_{0}\right)\left[\begin{array}{c}
\rho \sin \left(\frac{v}{\rho} t\right)  \tag{4}\\
\rho\left(\cos \left(\frac{v}{\rho} t\right)-1\right) \\
0
\end{array}\right]+\bar{r}_{1}, \quad t=0 . . t_{f}
$$

The initial condition, $\bar{r}_{1}$, is given by Equation 5. $\delta s$ is the effective drift length between the magnet of interest and the preceding element.

$$
\bar{r}_{1}=\left[\begin{array}{c}
Z_{0}  \tag{5}\\
X_{0} \\
0
\end{array}\right]+\delta s\left[\begin{array}{c}
\cos \theta_{0} \\
\sin \theta_{0} \\
0
\end{array}\right]
$$

### 2.2 Off Orbit Trajectories

In reality, there are orbital errors in the beam trajectory. The lattice files give Courant-Snyder (or Twiss) parameters ( $\beta$ and $\alpha$ ) which can be used to define two phase space ellipses - one for errors along the local x -axis and one for errors along the local y-axis. To achieve the values at the start of the magnet, they must be taken from the previous element in the lattice file and translated across a drift if it is present. Given the values contained within a transfer matrix, $M\left(s_{b}, s_{a}\right)$, between two points on the beam

$$
\overline{\bar{M}}\left(s_{b}, s_{a}\right)=\left[\begin{array}{ll}
m_{11} & m_{12}  \tag{6}\\
m_{21} & m_{22}
\end{array}\right]
$$

one can use the following general translation matrix to solve for the Twiss parameters at the start of a dipole:

$$
\left[\begin{array}{c}
\alpha_{b}  \tag{7}\\
\beta_{b} \\
\gamma_{b}
\end{array}\right]=\left[\begin{array}{ccc}
m_{11} m_{22}+m_{12} m_{21} & -m_{11} m_{21} & -m_{12} m_{22} \\
-2 m_{11} m_{12} & m_{11}^{2} & m_{12}^{2} \\
-2 m_{21} m_{22} & m_{21}^{2} & m_{22}^{2}
\end{array}\right]\left[\begin{array}{c}
\alpha_{a} \\
\beta_{a} \\
\gamma_{a}
\end{array}\right]
$$

where

$$
\begin{equation*}
\gamma=\frac{1+\alpha^{2}}{\beta} \tag{8}
\end{equation*}
$$

The transfer matrix for a drift is given by Equation 9.

$$
\overline{\bar{M}}\left(s_{1}, s_{0}\right)=\left[\begin{array}{cc}
1 & \delta s  \tag{9}\\
0 & 1
\end{array}\right]
$$

which leads to the following equations for the Twiss parameters at the start of the bend magnet [5] .

$$
\begin{align*}
& \alpha_{1}=\alpha_{0}-\gamma_{0} \delta s  \tag{10}\\
& \beta_{1}=\beta_{0}-2 \alpha_{0} \delta s+\gamma_{0} \delta s^{2}  \tag{11}\\
& \gamma_{1}=\gamma_{0} \tag{12}
\end{align*}
$$

The equations defining the phase space ellipses are given below [6].

$$
\begin{align*}
& A_{u}=\gamma_{x, 1} x^{2}+\alpha_{x, 1} x x^{\prime}+\beta_{x, 1} x^{\prime 2}  \tag{13}\\
& A_{u}=\gamma_{y, 1} y^{2}+\alpha_{y, 1} y y^{\prime}+\beta_{y, 1} y^{\prime 2} \tag{14}
\end{align*}
$$



Figure 2: The $\left(x, x^{\prime}\right)$ and $\left(y, y^{\prime}\right)$ phase space ellipses describing orbital errors

To find a properly missteered path, one first selects a point on or within each ellipse corresponding to the orbital errors. The trajectory then needs to be fixed to reflect the error. To do this transformation properly, the local x-axis must be known (denoted by $\bar{x}_{l}$ ). This is orthogonal to the ideal direction of travel, $\bar{s}_{l}$, and the vertical axis.

$$
\begin{equation*}
\bar{x}_{l}=\bar{y}_{l} \times \bar{s}_{l} \tag{15}
\end{equation*}
$$

By definition, $x^{\prime}=\frac{d x}{d s}$ and $y^{\prime}=\frac{d y}{d s}$. In order to apply the correct rotation matrices one should consider the spherical coordinate system shown in Figure 3. Equations 16 through 18 give the off orbit coordinates in terms of the angles $\theta$ and $\phi$. If solved for in terms of $x^{\prime}$ and $y^{\prime}$, rotation matrices may be used.


Figure 3: The angles $\theta$ and $\phi$ may solved for in terms of $x^{\prime}$ and $y^{\prime}$.

$$
\begin{align*}
& d x=\cos \theta \sin \phi \quad d y=\sin \theta \quad d s=\cos \theta \cos \phi  \tag{16}\\
& \frac{d x}{d s}=\tan \phi \quad \frac{d y}{d s}=\tan \theta \sec \phi  \tag{17}\\
& =x^{\prime} \\
& =y^{\prime}  \tag{18}\\
& \phi=\tan ^{-1}\left(x^{\prime}\right) \quad \theta=\tan ^{-1}\left(\frac{y^{\prime}}{\sqrt{1+x^{\prime 2}}}\right)  \tag{19}\\
& \bar{r}(t)_{e r r}=\overline{\bar{R}}_{\bar{x}_{\text {new }}}(-\theta) \overline{\bar{R}}_{\bar{y}_{l}}(\phi)\left(r(t)-\bar{r}_{1}\right)+\left(x \bar{x}_{l}+y \bar{y}_{l}+\bar{r}_{1}\right)  \tag{20}\\
& \bar{x}_{\text {new }}=\overline{\bar{R}}_{\bar{y}_{l}}(\phi) \bar{x}_{l} \tag{21}
\end{align*}
$$

As shown in Equation 20, the ideal trajectory must first be centered on the origin before using the rotation matrices. After orienting the path to contain the $x^{\prime}$ and $y^{\prime}$ errors, the parametrization may be translated back to its initial position, $\bar{r}_{1}$, and then again along the $\bar{x}_{l}$ and $\bar{y}_{l}$ axes by the $x$ and $y$ displacement values obtained by each respective phase space ellipse. This should be done to the first magnet used in the analysis. The orbital errors in the following magnets can be found by using transfer matrices.

### 2.3 Ray Tracing

Ray traces are calculated in two steps. A horizontal distribution of center rays are drawn out by considering each time step in the trajectory parametrization. For each time step, the rays follow in a straight path tangent to the normalized velocity vector, $\bar{t}_{1}(t)$, and start at the position of the particle, $\bar{r}(t)$. The distance traveled is given in Equation 22 by the parameter $d$ and determines the point, $P(d, t)$.

$$
\begin{equation*}
P(d, t)=d * \bar{t}_{1}(t)+\bar{r}(t) \tag{22}
\end{equation*}
$$

The photon absorber lies along a plane specified by the user. This is given by a normal vector, $\hat{n}_{A}$, and an arbitrary point lying on the plane, $P_{0}$. Using the equation of a planar surface, the distance, $d$, can be solved for by substituting in the equation for the ray. This gives the points, $P\left(d^{*}, t\right)$, that the rays impact the absorber at at zero vertical angle.

$$
\begin{align*}
& \hat{n}_{A} \cdot\left(P-P_{0}\right)=0  \tag{23}\\
& d^{*}(t)=\frac{\hat{n}_{A} \cdot\left(P_{0}-\bar{r}(t)\right)}{\hat{n}_{A} \cdot \bar{t}_{1}(t)} \tag{24}
\end{align*}
$$

From each center ray, a set of vertical rays may be drawn. Each one deviates vertically along the direction of the center ray that impacts at $P\left(d^{*}, t\right)$. A secondary plane with a normal vector, $\hat{n}_{R}$, orthogonal to the vertical direction, $\bar{Y}$, and the direction of the center ray, $\bar{t}_{1}(t)$, is used. The line of intersection between this plane and the photon absorber contains the end points of all of the rays in the vertical spread at time $t$. Figure 4 gives a visualization of the vertical spread while Equations 29 and 30 give expressions for the X and Z values on the absorbing surface for a given parameterization of vertical distance, Y . The program currently runs for $Y= \pm 5[\mathrm{~mm}]$ (though this can be easily changed). Each vertical line is centered at $P\left(d^{*}, t\right)$.


Figure 4: A vertical set of rays may be found for each zero-angle ray located at $P\left(d^{*}, t\right)$. This is done using a plane stretched vertically from the zero-angle ray (See the vertical transparent plane above).

$$
\begin{align*}
0= & \hat{n}_{A} \cdot\left(P-P\left(d^{*}\right)\right)  \tag{25}\\
= & n_{A, Z}\left(Z-Z_{d^{*}}\right)+n_{A, X}\left(X-X_{d^{*}}\right)+n_{A, Y}\left(Y-Y_{d^{*}}\right)  \tag{26}\\
0= & \hat{n}_{R} \cdot\left(P-P\left(d^{*}\right)\right)  \tag{27}\\
= & n_{R, Z}\left(Z-Z_{d^{*}}\right)+n_{R, X}\left(X-X_{d^{*}}\right)  \tag{28}\\
& X=\left(\frac{n_{R, Z} n_{A, Y}}{n_{A, Z} n_{R, X}-n_{R, Z} n_{A, X}}\right)\left(Y-Y_{d^{*}}\right)+X_{d^{*}}  \tag{29}\\
& Z=\left(\frac{n_{R, X} n_{A, Y}}{n_{R, Z} n_{A, X}-n_{A, Z} n_{R, X}}\right)\left(Y-Y_{d^{*}}\right)+Z_{d^{*}} \tag{30}
\end{align*}
$$

### 2.4 Power Distributions

For each point $P\left(d^{*}, t\right)$ in the horizontal distribution, the vertical power distribution may be calculated by using the angle, $\theta$, between the zero-angle ray located at $P\left(d^{*}, t\right)$ and the vertical rays previously calculated [4]. The power, $\frac{\partial^{2} P}{\partial \theta \partial \psi}$, may be converted from an angular spread to a spatial spread by dividing it by the square of the distance that a given ray travels, $D^{2}$. The factor, $f$, is the projection of the power onto the photon absorber and is found by dotting the planar normal vector and the unit vector describing each vertical ray's tangential direction together [1].

$$
\begin{align*}
\frac{\partial^{2} P}{\partial \theta \partial \psi} & =f * P_{d, 0} \frac{1}{\left(1+Q^{2}\right)^{\frac{5}{2}}}\left(1+\frac{5}{7} \frac{1}{\left(1+Q^{2}\right)}\right) \quad Q=\gamma \theta  \tag{31}\\
P_{d, 0}\left(W / m r a d^{2}\right) & =5.421 * E(G e V)^{4} I(A) B(T)  \tag{32}\\
\frac{\partial^{2} P}{\partial x_{A} \partial y_{A}}\left(W / m m^{2}\right) & =\frac{1}{D^{2}} \frac{\partial^{2} P}{\partial \theta \partial \psi}  \tag{33}\\
f & =\left|\hat{R}_{v, i} \cdot \hat{n}_{A}\right| \tag{34}
\end{align*}
$$

## 3 Results and Discussion

### 3.1 Accuracy

To verify the accuracy of the program. An analysis was run for the M3.1, M3.2, M2.5, and M2.4 reverse magnets and the planned B-crotch absorber for the APS-U at Argonne National Laboratory. The green fan in Figure 5 in composed of the individual rays from the 400 time steps used in the trajectory parametrization. Figures 6 and 7 show a top down view and a side view of the resultant power spectrum respectively. Numerical data from another distinguished program, Synrad, is overlaid on top of Figure 7 [2]. The total integrated power calculated in Synrad was 2.84 kW while the integrated power given by this program was 2.8428 kW . As shown, the deviation in both the total power and the distribution shape are less than one percent. This verifies the accuracy of the software.


Figure 5: Ray trace results for the M3.1, M3.2, M2.5, and M2.4 magnets radiating onto the B-crotch absorber.


Figure 6: A top down view of the power load.


Figure 7: Side view with Synrad data overlaid. Note: the Synrad data only ranges between 12 and 88 [mm]


Figure 8: A 3D view of the distribution


Figure 9: A 3D view of Synrad's distribution. Note: the Synrad data only ranges between 12 and 88 [mm]. The transverse axis is also mislabeled.

### 3.2 Limitations

There a few limitations with the current state of the program. Five notable ones include (1) using multiple photon absorbers, (2) having a photon absorber normal vector with a non-zero vertical component, (3) using orbital errors with multiple bend magnets, (4) using downstream magnets that don't emit onto the input absorbing surface, and (5) using absorbing surfaces that are non-planar.

### 3.2.1 Multiple Photon Absorbers

The program applies the heat map to a single photon absorber that is given by the user. This is assumed to be an infinite plane and all regions that intercept photons are plotted. If one wishes to analyze multiple absorbers or an absorber with a gap, running separate simulations is recommended.

### 3.2.2 Normal Vectors of Non-Zero Y-component

When the code runs to completion, the results are accurate. As it is now, however, the code breaks down when the vertical component of the input normal vector is non-zero. In order to see whether some of the synchrotron radiation overlaps and thus superimposes, an interpolation method has to be used. Matlab's built-in function, interpn, requires a very strict monotonically increasing meshgrid to interpolate in space. When the normal vector is titled upwards (or downwards) the mesh may no longer be monotonically increasing and the code throws an error. To fix this, one could either (1) improve the interpolation method or (2) comment it out and instead append the heatmaps together rather than interpolating and then adding. This assumes that the ray traces never overlap however. Commenting out the interpolation code would not show the user whether overlap occurs.

### 3.2.3 Missteering with Multiple Magnets

As it is now, the code that computes the orbital errors will solve for different possible trajectories of a single magnet. The default creates 64 'extreme' paths that incorporate values such as the maximum xdisplacement or the maximum y' value (and all permutations of these values). Unfortunately there is no simple way to match up the missteered paths of one magnet with the missteered paths of the following magnet. In theory, if one starts out with an orbital offset, the orbital errors down stream should be contained on or within the phase space ellipse that corresponds to the location of interest. One may suggest that the current code just start out with an initial position and orientation that reflects an orbital offset and then send it through the rest of the code. The problem with this is that the current program does not handle quadrupoles. If there is a non-zero y' orbital error, the electron will move upwards until a focusing quadrupole is reached. MATLAB's function for reading in excel data (from the Lattice file) does not handle strings and replaces them with 'NaN'. For this, even if the code could handle quadrupoles, there is not a simple method for 'knowing' that the next row of the lattice file to analyze is one. As a result of this issue, the missteering code is currently commented out and set aside for further exploration.

### 3.2.4 Downstream Magnets

If some of the magnets input into the program for analysis are behind the photon absorber, then no rays will be traced out as none of them land on the absorbing surface. Currently as the code is, the code will break down for the same reason previously mentioned in Matlab's interpn function. When the data suddenly cuts off, the mesh grid is no longer monotonically increasing and the interpolation breaks down. As long as the input magnets are realistic in that they precede the photon absorber's position, the code should work well.

### 3.2.5 Non-planar Absorbers

The program only allows for planar surfaces defined by the input normal vector and position. If special curved surfaces are used, one method would be for the user to input a series of nodes and to write a code that determines photon impact location. If the shape can be written using a 2 -variable parameterization (such as a sphere or toroid), that may also be a method for determining ray traces on curved surfaces.

## 4 Conclusion

This project began with the goal of determining the power distributions on arbitrarily placed planar surfaces. As shown, the program completes this with degrees of accuracy within one percent from the software Synrad. The code is versatile in that it can handle any bend magnet that will emit radiation onto the input absorbing surface. The functions are autonomous and set up the possibility of integrating into other programs such as COMSOL or ANSYS. A particle trajectory created in another program may be used in the functions created for this program. Additionally, the analytic method used gives rise to quick runtimes that may be better suited for optimization processes. Though advantageous in some regard, it also has its limitations. As discussed, the code would benefit from utilizing arbitrary surfaces, stable interpolation, and incorporation of missteering effects. The code is easily adaptable however and this paper gives the user the necessary information to improve the program to meet their needs.

## 5 References

## References

[1] Capatina, Dana. private communication (2016).
[2] Carter, Jason. private communication; Synrad data (2016).
[3] Chao, Alexander Wu., and M. Tigner. Handbook of Accelerator Physics and Engineering. River Edge, NJ: World Scientific, 1999. Print.
[4] Dejus, Roger. "Power Distribution from a Dipole Source." Internal APS Memo (2003): 1-8. Print.
[5] Edwards, D. A., and M. J. Syphers. An Introduction to the Physics of High Energy Accelerators. New York: Wiley, 1993. Print.
[6] Harkay, Katherine. "Maximum Beam Orbit in MBA and Ray Tracing Guidelines." 2nd ser. (2014): 1-9. Print.
[7] Suthar, Kamlesh. private communication (2016).

## 6 Appendix

### 6.1 Code




```
\% Missteer
\% If used, this function takes in twis5 parameters and calculates desired
\% missteered paths. The user must input 'extrena' if they want orbital
\% erros that include the extrere points on the phase space ellipses such as
\% \(x\) _max or \(y\) ' max and whatnot. If the user types 'distribution' they must
\% input a number \(N\) to choose \(N\) points on the phase space ellipse. If the
\% user wants to input their own set of errors they must type 'custom.'
function [rs,vs] = Missteer(r, v, betaX, betaY, alphaX, alphaY, DataType, userX's OR N, user's, n)
\(\begin{array}{ll}\text { No Define Global Parameters \& Constants } & \text { \% Global } Y \text {-direction (vertical) }\end{array}\)
\(\begin{array}{ll}Y=[0 ; 0 ; 1] ; & \text { \% Global } Y \text {-direction (vertical). } \\ \mathrm{bx} \_\max =8.177 ; & \text { \% The maximum value of the peta function at the limiting aperature in } \mathrm{X} \text { [m] }\end{array}\)
a \(x=9 * 10^{2}-3\); \(\quad\) \% The chamber halt-aperture in \(x\) [ \(m\) ]
\(A \_x=\left(a x^{*} 2\right) /\left(b x \_\operatorname{tax}\right)\); \(\quad\) \% The motion invariant for all bears in \(\left(x, x^{\prime}\right)\) [m]
by_max \(=5.890 ;\)
\% The maximum value of the peta function at the liniting aperature in \(y\) [ m ]
a \(y=3 * 10^{n}-3\);
\% The charber half-aperture in y [m]
\(A y=\left(a y^{\prime} 2\right) /(\) by max \() ; \quad\) of The motion invariant for all beans in \(\left(y, y^{\prime}\right)\) [m]
SAPS-U Specifications
    gamaX \(=\left(1+\right.\) alphaX \(\left.{ }^{\wedge} 2\right) /\) betaX; \(\quad\) \% Define the gamma twis5 paranter in \(\mathrm{X}\left[\mathrm{m}^{\wedge}-1\right]\)
    gamaY \(=\left(1+a 1\right.\) pha \(\left.^{\prime} 2\right) /\) betaY; \(\quad\) \% Define the gama twis5 paranter in \(y\left[\mathrm{~m}^{\wedge}-1\right]\)
    XII = sqrt (A_xbetaX):
    \% Detine the gamma twis5 paramter in y \(\left[\mathrm{m}^{\prime}-1\right.\) ]
\% The maximum transverse error in ( \(x, x^{\prime}\) ) [ \([\mathrm{m}]\)
    \(x p \times N=-\) alphaX*sqrt \(\left(A, x /\right.\) betaX); \(\quad\) \% The corresponding \(x^{\prime}\) value [rad]
    \(\mathrm{xpH}=\operatorname{sqrt}(\mathrm{A} x * g a m a X)\); \(\quad\) \% The maximum angular deflection error in ( \(\mathrm{x}, \mathrm{x}^{\prime}\) ) [rad]
    x xpH \(=-\) alphaX*sqrt (A_X/gamaX); \(\quad\) \% The corresponding x value [ m ]
    yH = sqrt(A ywbetaY);
    yp_y \(M=-\) alphaY*sqrt(A_y/betaY);
    \% The maximum transverse error in (y,y') [m]
    \% The corresponding \(y^{\prime}\) value [rad]
    ypH = sqrt(A y*gamaY):
    \% The maximum angular deflection error in ( \(y, y^{\prime}\) ) [rad]
    y_ypH = -alphaY*sqrt(A_y/gamaY); \(\quad\) o The corresponding y value [ m ]
    \(\begin{array}{ll}x p_{0} 0=\operatorname{sqrt}(A \quad x / \text { betaX); } & \text { \% The deflection at } x=0[\mathrm{~m}] \text { given in [rad] } \\ y p=\operatorname{sqrt}(A \quad y / \text { betaY); } & \text { \% The deflection at } y=0[\mathrm{~m}] \text { given in [rad] }\end{array}\)
```



```
    \(x{ }^{\prime}=\operatorname{sqrt}\left(\right.\) A_xtgamaX); \(\quad\) \% The displacement error in \(x\) given in [ m ] for \(\mathrm{x}^{\prime}=0\) [rad]
\(y 0=\operatorname{sqrt}\left(A\right.\) y*gamaY); \(\quad\) o The displacement error in \(y\) given in \([\mathrm{m}]\) for \(y^{\prime}=0\) (rad)
So Initial Conditions
if stramp(DataType, 'extrera')
        \% 8 Edge Case Points in \(x\)
```



```
    \% \(\mathrm{x}=\mathrm{X}\) _max
            [ \(0, x\) p_0];... \(\quad\) \% \(x=0\) with \(+x^{\prime}(0)\)
            \(\left[0,-x p \_0\right] ; \ldots . \quad\) of \(x=0\) with \(-x^{\prime}(0)\)
            \(-\left[x N, x p_{2}, x f\right] ; \ldots . \quad\) \% the negated point of \(\left(x_{i}, x^{\prime}\right)\) at \(x=x\) max
            \([\mathrm{X} \times \mathrm{xpH}, \mathrm{xpH}]: \ldots . \quad\) \% x corresponds to \(\mathrm{X}^{\prime}=\mathrm{X}^{\prime}\) _frax
            \([x 0,0]\);... \(\quad\) \& \(x\) corresponds to \(x^{\prime}=0\)
            \(\left[-x \theta^{2}, 0\right]\) i.... \(\% x\) corresponds to \(x^{\prime}=8\) but negated
            \(\left.-\left[x \_x p^{\prime} 1, x p H\right]\right]\); \(\quad\) of the negated point of \(\left(x, x^{\prime}\right)\) at \(x^{\prime}=X^{\prime} \max\)
        \% 8 Edge Case Points in \(y\)
        \(Y_{5}=\left[\left[y \mathrm{H}, \mathrm{yp} \mathrm{yH}_{\mathrm{M}}\right] ; \ldots . \quad\right.\) \% \(\mathrm{y}=\mathrm{Y}_{-} \max\)
            \(\left[0, y p \_0\right] ; \ldots\).
            \(\left[0,-y p \_\theta\right]: \ldots=\quad\) \% \(y=0\) with \(-y^{\prime}(0)\)
            -[yN, yp._M];...
                            \% The negated point of ( \(\mathrm{y}, \mathrm{y}^{\prime}\) ) at \(\mathrm{y}=\mathrm{Y}\) _rax
            \([y, y p H, y p H]: \ldots . \quad\) \% \(y\) corresponds to \(y^{\prime}=Y^{\prime}\) frax
            [yb, 0];... \% y corresponds to \(y^{\prime}=0\) ?
            \([-y 0,0]\) i... \(\quad\) \% \(y\) corresponds to \(y^{\prime}=0\) but negated
```



```
elseif strorp(DataType, 'distribution')
        if (15tloat(userXs_orn) © ...
```



```
        \(\mathrm{N}=\mathrm{user} \mathrm{SS}_{\mathrm{C}}^{\mathrm{R}} \mathrm{RN} \overline{\mathrm{N}}\)
        else
            error( 'The 8th argunent must be an integer');
        end
    \(\mathrm{X}_{5}=\operatorname{zeros}(2 * N, 2) ;\)
    count \(=1\);
    count \(=1 ;\)
for \(1=1\) inspace \(\left(-x_{1} 1, x \neq N\right) \quad\) \% Generate 2 wl points on the \(\left(x, x^{\prime}\right)\) ellipse
            xp_11 = ...
                (-2*alphax*1 + sqrt((2*alphax*1)^2-...
                4*betax*(gamax* \(1^{\wedge} 2-\)-_ X\(\left.\left.)\right) 1\right) /(2 *\) betax \() ;\)
            xp_12 \(=\ldots\)
                (-2*alphax*1 - sqrt((2*alphax*1)^2-...
```



```
            \(\mathrm{Xs}_{\mathrm{s}}\) (count, \(:\) ) \(=\left[1\right.\), xp_11]; \% Add the pairs to the \(\mathrm{X}_{5}\) matrix
```



```
            count \(=\) count \(+1 ;\)
    end
```

    \(=\)
    1
11

```
    Y5 = zeros(2*N,2);
    count = 1;
    for 1 = linspace(-yN,yN,N) % Generate 2wN points on the (y,y') ellipse
        yp_11 = ... 
        (-2*alphaY*1 + sqrt((2*alphaY*1)^2-...
        4*betaY*(gammaY*1*2-A_y)))/(2*betaY);
    yp_12 = ...
        (-2*alphaY*1 - sqrt(( 2*alphaY*1)^2-\ldots.
        4*betaY*(gammaY*1*2-A_y)))/(2*betaY);
    Ys(count,: ) = [1, YP_11];
    Y5}(\mathrm{ count+N,: ) = [1, yp_12];
    count = count + 1;
end
elseif stromp(DataType, 'custon')
    if (1smatrix(userX\mp@subsup{S}{_}{\prime}ORNN) }&&\ldots..
            1matrix(userY5))
            XS = userXS_0R_N;
            Ys = user`5;
            [~, CXs] = size(Xs);
            [~,cY5] = size(Ys);
            If (((cXS ~e 2) || (CYS ~e 2)) || ... % Check that each matrix has only 2 columns
                (~15numeric(X5) || ~1snumeric(Y5)))
            error('The 8th and 9th arguments must be 2-column matrices.');
        end
    else
        error('The 8th and 9th arguments must be 2-column matrices.');
    end
else
    error(strcat(...
        sprintf('Please input a DataType for plotting missteered values. \n'),....
        sprintf(' The options are: "extrema" (plots all combinations of points \n'),...
        sprintf(' including at least one extreme value on the phase spase ellipse); \n'),...
        sprintf(' "distribution" (uses a distribution of ZV equally spaced points \n'),...
        sprintf(' in x-xp space and in y-yp space to form 4v'2 combinations of \n'),...
        sprintf(' points); and "custom"' (user must input }2\mathrm{ matrices containing \n'),....
        sprintf(' points); and "custom""(user must input 2 matrices containing \n'),....
        sprintf(' pairs of x-xp and y-yp points respectively).')))
end
    [LX, ~] = size(Xs);
    [Ly, ~] = 51ze(Y5):
Tot = Lx*Ly; }\quad\mathrm{ % Acquire the total number po permutations
    % Acquire the total number of permutations
ICs = zeros(Tot,2,2); }\quad\mathrm{ % There are Lx ( }\textrm{x},\mp@subsup{\textrm{x}}{}{\prime})\mathrm{ point!, and Ly (y,y')
k=1.
k=1;
    for j=1:Ly
        IC5(k,1,:) = X5(1,:);
        IC5(k,2,:) = Y5 (j,:);
        lCs(k,2,!)
        end
    end
N Find the correctly misteered paths
rs = zeros(3,n,Tot+1);
lol
% Prepare a matrix containing all of the paths (including the ideal path).
```



```
r0=r(:,1);
% The current initial displacement.
% The current initial displacem
r_temp = r - T0;
    % Subtract the offset to sirplify rotations about the origin.
va = v(:,1);
x_true = ... 
    cross(Y, va)/norm(cross(Y, va));
        % Use the initial velocity vector to detemmine the true x-transvere
        hold on
hold on
for 1 = 1:Tot
    thetx = ICs(1,1,2); }\begin{array}{ll}{\mathrm{ % The angle about which to rotate the path}}\\{\mathrm{ % around y-axis (z-axis in Cartesian). }}
    thety_true = IC5(1,2,2); }\begin{array}{ll}{\mathrm{ % The angle about which to rotate the path}}\\{\mathrm{ % around }x\mathrm{ -axis (y-axis in Cartesian). }}
    thety_true = IC5(1,2,2); }\quad\begin{array}{ll}{\mathrm{ % The angle about which to rotate the pa}}\\{\mathrm{ % around }x\mathrm{ -ax15 (y-ax15 in Cartesian). }}
    thety = atan(tan(thety_true)*\operatorname{cos(thetx)); % Define the actual angle that must be used}
    % in order to get the rotation correctly.
    % The current displacement error in }x\mathrm{ .
    x_err = ICs(1,1,1); 
    Rz = Rotatehxis(thetx, Y); &% Rotation Natrix about the vertical axis.
```






```
%%getPrev
% This function takes in the lattice file and the current row being
% considered and find the first row above it that isn't a vertex point.
Efunction prev = getPrev(row, Lattice)
    count = 1; % Record how many rows are checked.
    [rws, ~] = size(Lattice); % Acquire the total and hence the number of the last row.
    prev = row - 1; % Set the previous row to be one before the current row.
if (prev <= 0) % If the current row is the first row, the previous loops
        prev = rws; % back and is the last row.
end
currPrev = Lattice(prev, :); % Acquire the information held in the previous row.
〔while (sum(abs(currPrev(9:16))) == 0) % If all parameters are zero one knows this is a VERTEX-POINT.
    if (count >= rws) % If all rows have been considered and failed, there must be a
            error('Bad Lattice Files or Row Input Numbers') % problem with the input or lattice file.
        end
        prev = prev - 1; % Otherwise decrement the previous row by 1.
        if (prev <= 0) % If the current row is the first row, the previous loops
            prev = rws; % back and is the last row.
        end
        currPrev = Lattice(prev, :); % Reset the information of the previous row.
        count = count + 1; % Increment the count.
end
end
```


## \%\% makeGrid

```
\% This function takes in a vector, \(u\), and a number, \(N\), and creates a
    % matrix containing N columns each of vector u.
[function G = makeGrid(u,N)
    rws = length(u); % Obtain the number of rows in the vecotr, u.
    O = ones(rws,N); % Create a 'rws by N' matrix of ones
    T = zeros(rws); % create a 'rws by rws' matrix of zeros
[for i = 1:rws
    T(i,i) = u(i); % For every element in u, place it along the diagnol of T.
    end
    G = T*0; % Multiply T and 0 to create a matrix of u's.
    end
```

```
    %% RotateAxis
```

```
    % This code takes in an angle, theta, and a unit vector, u, about
```

    % This code takes in an angle, theta, and a unit vector, u, about
    % which to create a rotation matrix about.
    % which to create a rotation matrix about.
    function Ru = RotateAxis(theta, u)
    function Ru = RotateAxis(theta, u)
    % Normalize u if needed.
    % Normalize u if needed.
    u = u/norm(u);
    u = u/norm(u);
    % Define the components of the rotation matrix, Ru.
    % Define the components of the rotation matrix, Ru.
    R11 = ...
    R11 = ...
    cos(theta)+u(1)^2*(1-cos(theta));
    cos(theta)+u(1)^2*(1-cos(theta));
    R12 = ...
    R12 = ...
    u(1)*u(2)*(1-cos(theta))-u(3)*sin(theta);
    u(1)*u(2)*(1-cos(theta))-u(3)*sin(theta);
    R13 = ...
    R13 = ...
    u(1)*u(3)*(1-cos(theta))+u(2)*sin(theta);
    u(1)*u(3)*(1-cos(theta))+u(2)*sin(theta);
    R21 = ...
    R21 = ...
    u(2)*u(1)*(1-cos(theta))+u(3)*sin(theta);
    u(2)*u(1)*(1-cos(theta))+u(3)*sin(theta);
    R22 = ...
R22 = ...
cos(theta)+u(2)^2*(1-cos(theta));
cos(theta)+u(2)^2*(1-cos(theta));
R23 = ...
R23 = ...
u(2)*u(3)*(1-cos(theta))-u(1)*sin(theta);
u(2)*u(3)*(1-cos(theta))-u(1)*sin(theta);
R31 = ...
R31 = ...
u(3)*u(1)*(1-cos(theta))-u(2)*sin(theta);
u(3)*u(1)*(1-cos(theta))-u(2)*sin(theta);
R32 = ...
R32 = ...
u(3)*u(2)*(1-cos(theta))+u(1)*sin(theta);
u(3)*u(2)*(1-cos(theta))+u(1)*sin(theta);
R33 = ...
R33 = ...
cos(theta)+u(3)^2*(1-cos(theta));
cos(theta)+u(3)^2*(1-cos(theta));
% Rotation Matrix about axis u in Cartesian.
% Rotation Matrix about axis u in Cartesian.
Ru = ...
Ru = ...
[[R11, R12, R13];...
[[R11, R12, R13];...
[R21, R22, R23];...
[R21, R22, R23];...
[R31, R32, R33]];
[R31, R32, R33]];
end

```
    end
```

```
%% InterpMaps
% This function takes in multiple heatmaps and linearly interpolates them
% so that they can be superimposed.
Efunction HeatMap = InterpMaps(HeatMaps)
[rws, cls, ~, segs] = size(HeatMaps); % Find the dimesnions of the intensity distributions
qZ = zeros(rws, segs*cls);
% Create a new Z matrix to contain the new superimposed heatmap data
qX = zeros(rws, segs*cls); % Create a new X matrix to contain the new superimposed heatmap data
qY = zeros(rws, segs*cls); % Create a new Y matrix to contain the new superimposed heatmap data
for i = 1:segs % Iterate over each map and fill the matrices with the coordinates
    qZ(:,((i-1)*cls+1):i*cls) = ...
        HeatMaps(:,:,1,i);
    qX(:,((i-1)*cls+1):i*cls) = ...
        HeatMaps(:,:,2,i);
    qY(:,((i-1)*cls+1):i*cls) = ...
        HeatMaps(:,:,3,i);
end
HeatMap = zeros(rws, segs*cls, 4);
HeatMap(:,:,1) = qZ;
HeatMap(:,:,2) = qX;
HeatMap(:,:,3) = qY;
for i = 1:segs % Iterate over every heatmap
    Zs = HeatMaps(:,:,1,i);
    Ys = HeatMaps(:,:,3,i);
    In = HeatMaps(:,:,4,i);
    IntIn = interpn(Zs',Y\mp@subsup{s}{}{\prime},In',... %Given the data from the current heatmap, linearly interpolate
        qZ, qY, 'linear', 0); % the power level at the rest of the queried points.
    HeatMap(:,:,4)=\ldots. %Add the values to the final cumulative heatmap so that they superimpose
        HeatMap(:,:,4) + IntIn;
end
end
```


## $\%$ Vectorize

\% This code takes in a heatmap composed of 4 meshgrids. 1 for $Z$ values, 1
\% for $X$ values, 1 for $Y$ values, and one for intensity values, and turns it
\% into 4 column vectors of the ZXY-Intenisty data
[function TotIntVecs = Vectorize(TotInt)
[rws, cls, ~] = size(TotInt);
TotIntVecs $=$ zeros(rws*cls,4);
[for $i=1: c l s$

```
        TotIntVecs((rws*(i-1)+1):(rws*i),:) = TotInt(:,i,:);
```

end
end


Figure 10: Lattice file rows containing Vertex-Points


Figure 11: Lattice file rows with removed Vertex-Points

### 6.2 Help

- Don't know how to run the program? Simply specify a point on the absorption plane, a normal unit vector, and a vector containing which rows in the lattice file the analysis should be done on.

```
>>
% Note: inputs are column vectors (spaced with ; not ',')
p0 = [18.7079; -0.8732; 0]; % [global Z value; global X value; global Y value] all in meters
np = [0.9912; -0.1323; 0]; % [global Z value; global X value; global Y value] all in meters
latRows = [46;48;51;52]; % row numbers in the lattice file corresponding to each magnet
[TotIntVecs, TotalPower, Peak] = CalcPower(p0, np, latRows);
```

- Always input SI units.
- Are there gaps in the ray trace fans? This means the path is not continuous. Either there is an error in the lattice file values; the number of each row that is input is incorrect (double check the row numbers!), the wrong lattice file is being used (See the 'CalcPower' script above - lines 10 and 11. Whatever is uncommented, it better be the lattice file being used), or finally perhaps the offset (see the script for 'CalcPower' line 17) is wrong. As shown in Figures 10 and 11, the first 3 rows are nonnumeric so they are not included in the matrix named 'Lattice' in the code. 3 is thus subtracted from the input values to correct for this. If there is a numerical value in one or more of the first few lines then the number currently being subtracted (3) must be changed.
- Want a different mesh size in the transverse direction? Change the value 'n' in line 13 of the script for 'CalcPower'. Note that this mesh is constant size for the particle's trajectory but not for the heatmap. On the heatmap it is more dense where the gradient is higher (often desirable as it is).
- Want a different mesh size in the vertical direction? Change the value 'yN' in line 14 of the script for 'CalcPower'. This is a constant sized mesh on the absorbing surface's vertical axis.
- Want to cover a greater vertical distance on the absorber? Change the value 'm' in line 15 of the script for 'CalcPower'. Currently set at $2[\mathrm{~mm}]$, this is the vertical range that the heatmap covers. 'yN' is the number of points examined within this range.
- If the full trajectory is desired, simply change the output to include the 'totR' matrix defined on line 19 and updated on line 62 of 'CalcPower'.
- Lines 64 through 67 of 'CalcPower' calculate the missteered paths. If one examines the specific script 'Missteer' she or he will see that it is possible for the user to specify specific missteered values rather than just use the extreme errors. The script 'CalcPower' will have to be edited to reflect this input and to output the desired missteered paths.
- If one creates a more stable working interpolation function it would replace line 86 of 'CalcPower'. The format of the input, 'HeatMaps' currently in the code is this: it's an 'yN by n by 4 by seg' matrix (4-dimensional). If 'seg' is the number of rows/bend magnets being analyzed then imagine 'seg' 3 -dimensional matrices all contained within the 4 -dimensional matrix, 'HeatMaps'. As it appears, each of these 3D matrices are a separate heatmap corresponding to each segment (element, bend magnet, etc...). The structure of the individual heatmap is that they have a depth of 4 (or visualized as 4 layers). The first is an [yN by n] matrix of all of the Z values on the absorbing plane, then another of all the X values, then all of the Y values, and finally all of the intensity values. The third layer containing the Y values will have 'n' identical columns each with yN rows that span from '-m' to ' +m '. Each column represents the vertical spread found from each zero-vertical-angle photon that was initially traced out onto the absorber.
- Notice that in addition to the 'Missteer' script, the 'RayTrace' script also has default values in the case that a normal vector and point were not specified. This doesn't work with the code as it is but just goes to show that these scripts are autonomous and can be used independently in other contexts.
- Lines 47 through 64 of the 'RayTrace' script are used to determine if the ray impacts the planar surface and if so to plot it. If a code that handled arbitrary shapes were designed this is where it would go.
- Figure 10 shows a lattice file that contains Vertex-Points. As evident in Row 30, there are sometimes mistakes in the lattice files. The length of the M2.3 magnet here is certainly not 0.02 meters. This messes up the B-field calculation and thus the heat load is incorrect. Figure 11 shows a lattice file with the Vertex-Points removed. Line 28 is the same M2.3 magnet but with a more realistic length of 0.557 meters. It's good to check which lattice file is being used and if the values being input into the code through the lattice files make physical sense.
- The best way to debug or understand Matlab code is to just play around with it. I suggest using the command window and just playing with the individual functions, checking the contents and sizes of different variables by unsuppressing them in the code itself. This would be especially beneficial to someone who plans to improve the project to include things like arbitrary absorbers, quadrupoles, and more.

