Diffusion measurement with transverse beam echoes

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Introduction

- Measuring and managing diffusion is crucial in modern "intensity frontier" machines, where nonlinear phenomena, e.g. intrabeam scattering and space charge effects, can significantly increase emittance over time.
- Traditional methods to measure diffusion, e.g. beam scraping, take up to hours to complete. The transverse echo technique will require minutes or less.
- The echo displays high amplitude sensitivity to small phase space perturbations, making it an ideal tool to probe weak diffusion.
- Simultaneously, we need amplitude-boosting techniques to counter strong diffusion (e.g. space charge effects), so that the echo signal remains measurable.
- In this study, we develop theory and simulation to:
  - Explore the behavior of transverse echoes under diffusion.
  - Investigate pulsed quadrupoles as a method to boost echo amplitude.
  - Provide recommendations for the planned beam echo measurement system in the future IOTA storage ring at Fermilab.

Echo: Theory and Simulation

Theory

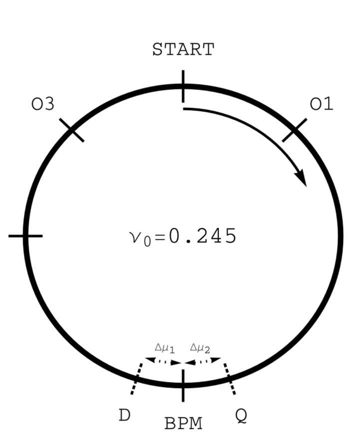
- The transverse echo is a recoherence of the beam distribution, following phase decoherence due to nonlinear ring elements (e.g. octupoles).
- It shows up on the BPM as an oscillation of the beam centroid, some time after an initial disturbance (e.g. dipole kick).
- Typical echo sequence:
  - At t = 0, apply one-turn dipole kick θ.
  - At t = τ, apply one-turn quadrupole kick q.
  - Near time 2τ, the echo signal appears on the BPM.
- The amplitude of the echo is dependent on ring parameters. It is also extremely sensitive to diffusion. (Refer to equations above.)
- Key assumptions:
  - Both dipole and quad kicks are weak (compared to beam spread).
  - The timing of quad kick τ is much greater than decoherence time.

$$a_0 = \theta q \sqrt{\beta \beta_k}\, \omega' J_0 \tau$$

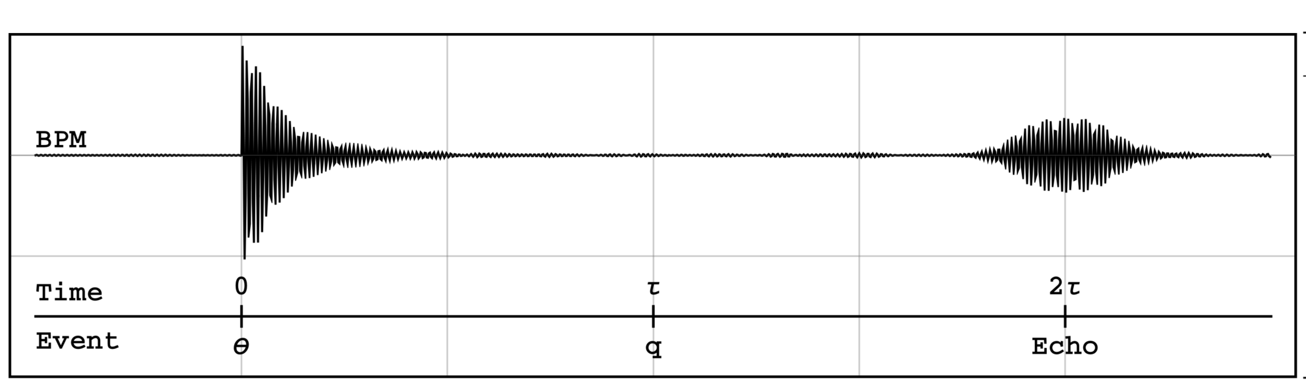
$$a_{\rm difn} = \frac{\theta q \sqrt{\beta\beta_k}\, \omega' J_0 \tau_q}{\alpha_1^3}, \quad \text{where} \quad \alpha_1 = 1 + \frac{2}{3}\left(\frac{D_1 \tau_q}{J_0^2}\right)(\omega' J_0 \tau)^2$$

Simulation

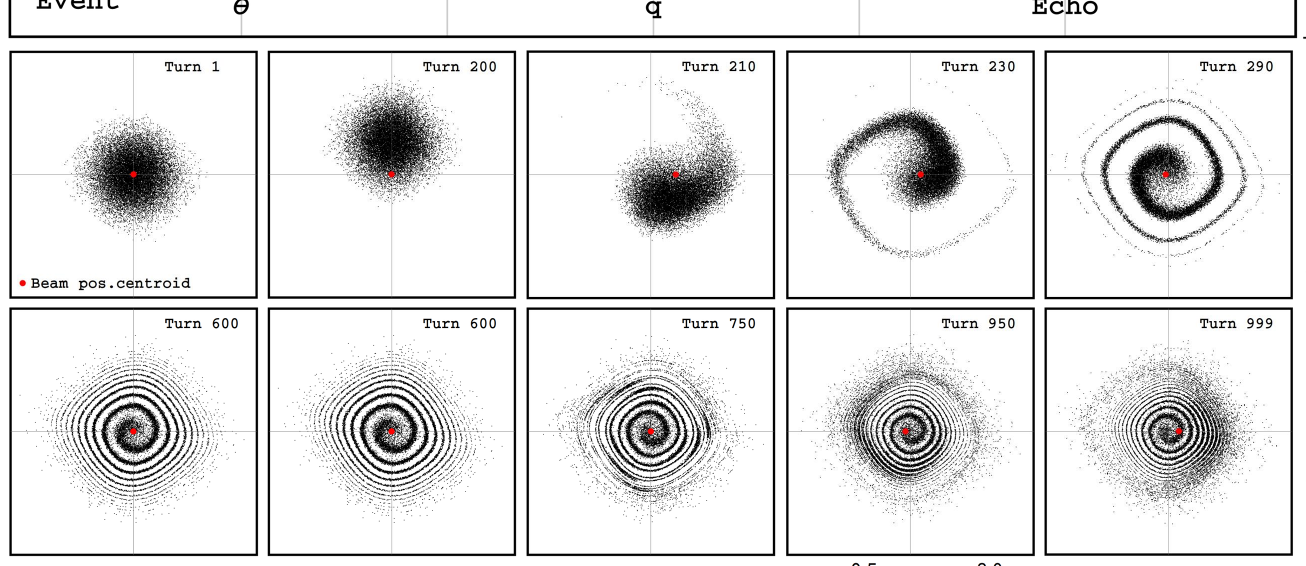
- Simulation written in C, with analysis performed in *Mathematica.*
- Machine parameters based on 2005 RHIC experiment.
- Simulation options include adjustable ring elements, variable starting distribution, variable diffusion model, pulsed quadrupoles and injection oscillation.
- Simulation results agree well with theory.



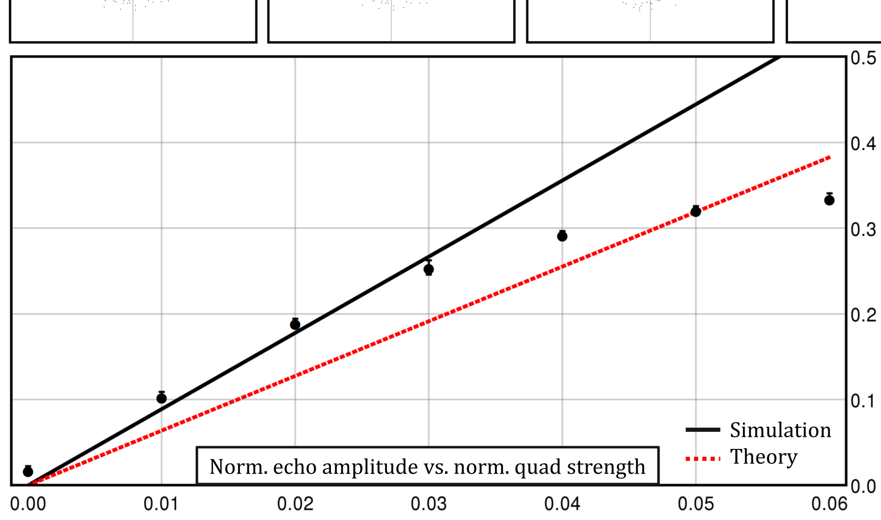
Schematic of ring used in simulation.

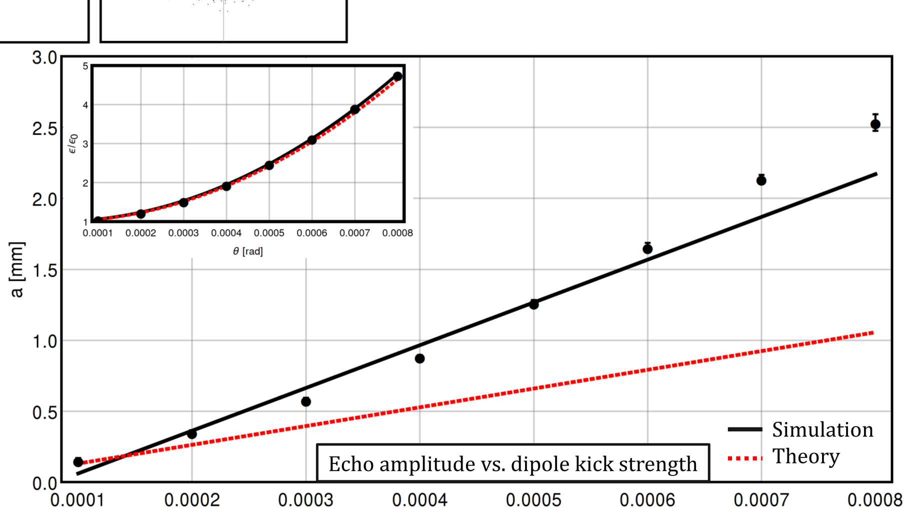


Parameter	Symbol	Value
Number of particles	N_{part}	20000 to 50000
Unnorm. rms emittance [m-rad]	ϵ_0	1.5×10^{-7}
Rms beam size [m]	σ_x	3×10^{-3}
Transverse tune	ν_0	0.245
Betatron at BPM, dipole and quad [m]	β, β_k, β_q	10, 10, 10
Detuning	μ	10^{-4} to 10^{-3}
Dipole kick [rad]	θ	10^{-4} to 10^{-3}
Norm. quad strength	q	0.01 to 0.05
Timing of quad kick [turns]	τ (or τ_q)	400 to 1000



Above: *Typical simulation parameters, based on 2005 RHIC experiment.*
Above left: *Evolution of simulated beam centroid over time, displaying a typical echo sequence.*
Left: *Phase space portraits showing the dipole kick and phase decoherence, followed by the quadrupole kick and formation of the echo.*
Below left: *Plot of norm. echo amplitude against quadrupole kick strength. Simulation results in black, theory prediction in red. Saturation starts to set in at high kick strengths.*
Below: *Plot of norm. echo amplitude against dipole kick strength. Theory (red) and simulation (black) agree well for small kick values.* ***Inset:*** *Plot of fractional emittance increase against dipole kick strength.*



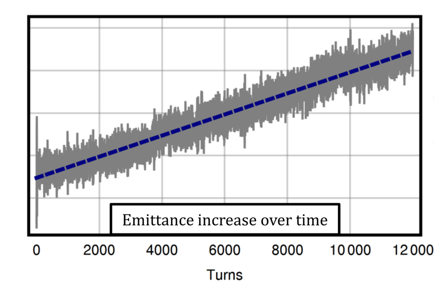


Diffusion

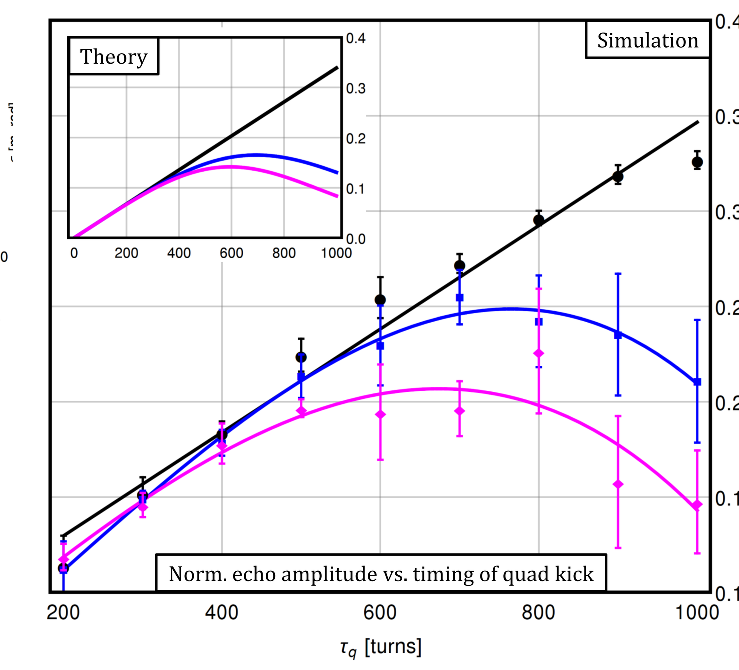
- Linear diffusion model simulated by dipole noise.
- Echo amplitude becomes attenuated with diffusion.
- We directly measure diffusion coefficient by tracking emittance increase over a large number of turns. Results agree excellently with theory.
- Simulation results also demonstrate predicted relationship between echo amplitude and relevant parameters (below).

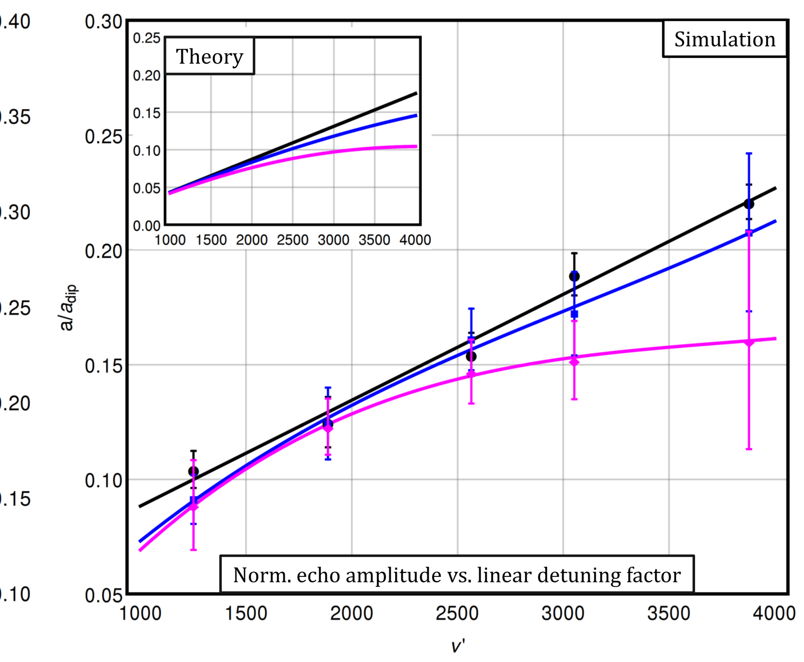
$$\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial J}\left(D(J)\frac{\partial \psi}{\partial J}\right), \quad \text{where} \quad D(J) = D_0 + D_1\left(\frac{J}{J_0}\right)$$

$$D_1 = \pi \epsilon_0 \frac{{\rm d}\epsilon}{{\rm d}t} \qquad \tau_{\max} = \left(\frac{16}{3}\omega'^2 D_1\right)^{-1/3}$$



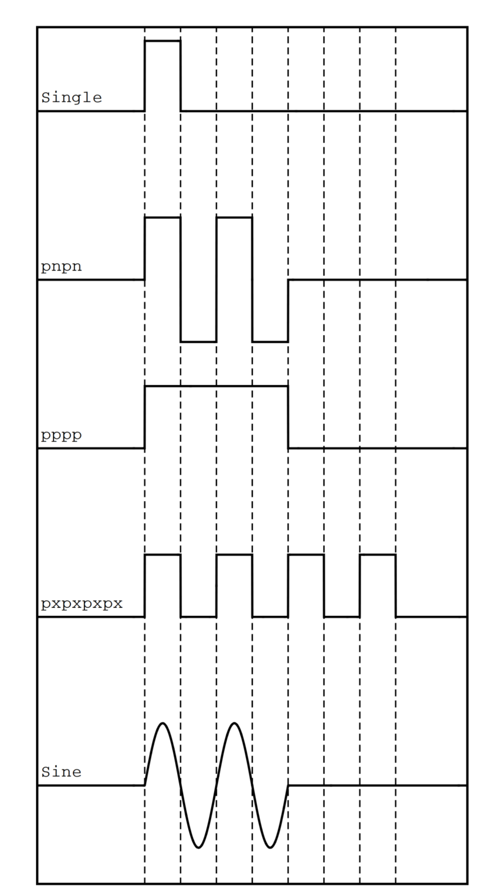
Top: *Emittance increase over time due to dipole noise, from which we deduce diffusion coefficient D_1.*
Right: *Plot of norm. echo amplitude versus timing of quad kick, with different D_1 parameters.* ***Inset:*** *Predicted theoretical relationships.*
Far right: *Plot of norm. echo amplitude versus detuning, again with different D_1 values.* ***Inset:*** *Predicted theoretical relationships.*

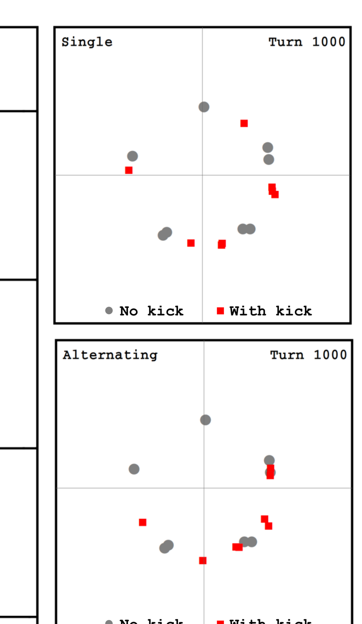




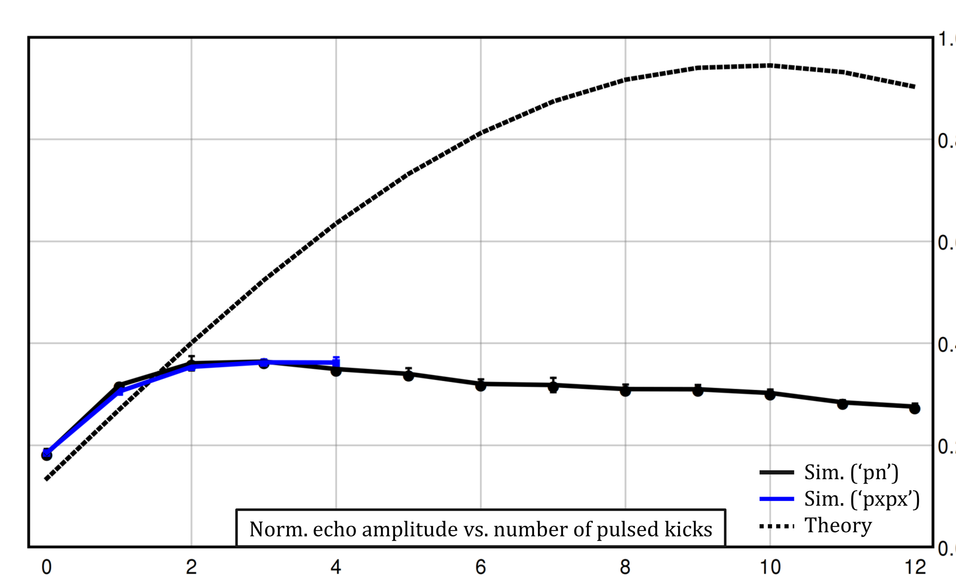
Pulsed quadrupoles

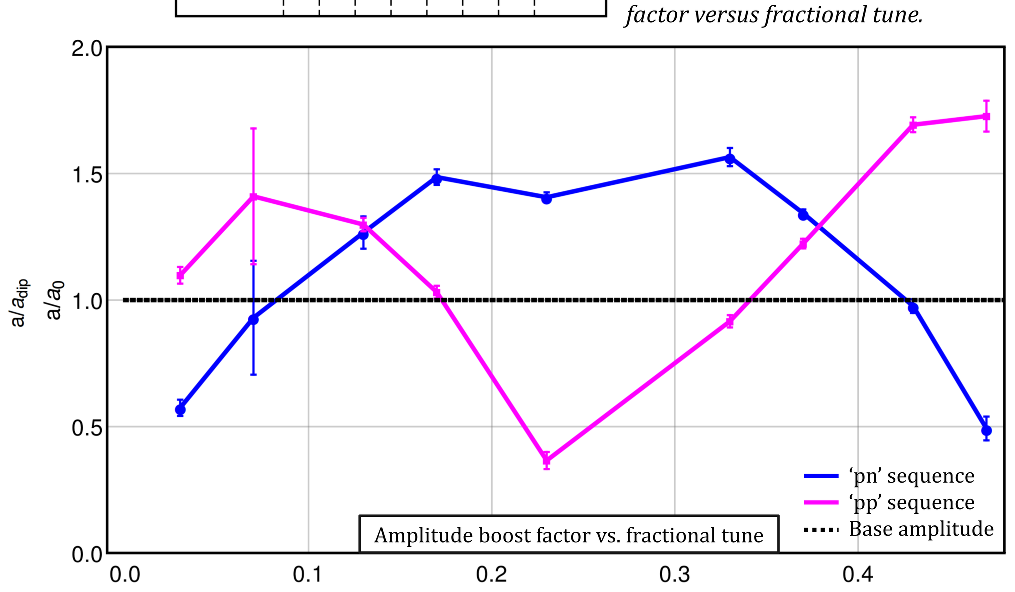
- Based on gradient echoes in NMR.
- A single quad kick introduces a small, position-dependent ΔJ to the particle distribution. With linear detuning, this leads to particles "clumping" together in phase space at time 2τ.
- Pulsed kicks apply a sequence of small ΔJ 's that amplify each other, resulting in a tighter "clump" in phase space.
- Optimal sequence highly dependent on fractional tune. We investigated several possible sequences.
- Maximum echo amplification close to 100% (up to saturation point).





Top: *Phase space portraits at time 2τ with single kick vs. alternating pulsed kicks.*
Left: *Various tested sequences.*
Below left: *Plot of norm. echo amplitude vs. pulse length*
Below: *Plot of echo boost factor versus fractional tune.*





Conclusions and Further Work

- Key findings: Consistent measurement of diffusion coefficient based on $\tau_{\max}$; echo amplitude boost by up to 100% using pulsed quads; optimal sequence depends on fractional tune; pulsed sequence of single polarity can be just as effective.
- Some further questions:
  - What is the optimum pulse sequence for a given fractional tune?
  - Echo amplitude saturation observed empirically at A ≈ 0.4. How do we explain it? Is it possible to surpass this limit?
  - How will echo dynamics change in 2D? Any coupling effects?

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